Tijdschrift van de Belg. Ver. Aardr. Studies - BEVAS, 1991-2
Bulletin de la Soc. belge d'Etudes Géog. - SOBEG, 1991-2

SPATIALLY CONTINUOUS PROCESSING WITHIN A RASTER BASED GIS : SOME EXAMPLES OF GEOGRAPHICAL MODELS

Jean-Paul DONNAY
Marc BINARD

RÉSUMÉ

La représentation d'un territoire en mode raster introduit une discrétisation qui peut être considérée, en première approximation, comme le résultat d'un échantillonage à deux dimensions de cette surface. Sous cette forme, un attribut spatialement continu constitue une couche de données directement accessible aux systèmes d'information géographique (S.I.G.) hybrides (raster/vecteur). Les avantages de cette représentation sont déjà exploités pour visualiser et traiter des phénomènes physiques surfaciques, tels que les modèles numériques de terrain. Une procédure identique peut être appliquée aux surfaces théoriques issues de l'application de modèles géographiques. En outre, les propriétés du mode raster sont susceptibles d'offrir des solutions graphiques soit plus simples, soit en l'absence de solutions analytiques. Cet article illustre ces avantages sur quelques modèles bien connus de l'analyse spatiale.

Mots-clés
Système d'Information Géographique en mode maillé, Analyse spatiale, Polygones de Voronoï - Modèle de localisation de Weber - Modèle de potentiel.

1. HYBRID GIS

For a while, GIS softwares tend to offer some raster facilities on top of classical vectorial functions. If a few systems are initially raster based (e.g. GRASS or SPANS), some of the well-known representative vectorial GIS turn towards hybrid systems able to carry out raster processing (e.g. new version of ARC/INFO, ESRI 1990).
Resorting to raster mode is essentially justified by two concerns. On the one hand, it allows an easy handling of digital elevation models (D.E.M.) and

makes their processing to get any derived map easier (gradient and aspect of slopes, watersheds, etc.). On the other hand, hybrid GIS are able to take into account remotely sensed data, which are naturally in raster format and which constitute a more and more appreciated source of information for land and country planning. In this respect, it is worth noting that several remote sensing softwares offer today GIS functionalities in addition to classical image processing operations. So they make up integrated raster based systems, often presented as a collection of modules (e.g. ERDAS).
DEM and remotely sensed images are both data which can put up with the relatively coarse geometrical resolution featuring the raster mode. Generally speaking, it is also the case for any application drawn at a regional scale. With regard to the GIS functionalities, the raster mode is particularly efficient ta run basic operations. As an example, an overday of raster layers can be seen as a simple Boolean operation between the homologous pixels over the binary masks resulting form the selections, whereas the computing of intersection polygons is required in vectorial mode.
Besides the raster data, an hybrid GIS must be able to handle vectorial data. They are required either because of the need of a higher accuracy or, more generally, because the vectorial mode allows more sophisticated data structures storing the topological relations between the geographical objects. Moreover, many hybrid systems tend to put the only vectorial mode in charge of the spatial analysis issues. The solutions afforded in this field by the current systems are rather weak (Burrough 1990) but they could be enhanced by an adequate use of the raster potentialities available through the hybrid GIS. Precisely, this paper deals with such raster facilities applied to some of the common spatially continuous models of geography.

2. SPATIAL DISCRETIZATION

The elementary information of the vectorial mode is the point, or vertex, which has no dimension. Owing to the vertices and according to an adequate data structure, the vectorial mode is able to handle every types of discrete geographical objects : punctual, linear (including network) and polygonal. Recause the vertex bas no dimension, the vectorial mode cannot take into account the notion of surface. As an example, a polygon is only handled in an indirect way, through a sequence of vertices which is supposed to define the boundary of the polygon at a given scale and generalization level.
This incapacity to handle the surfaces makes the vectorial mode unsuitable for a spatially continuous geographical phenomenon. Indeed, in this case the attribute can present a different value at each point over the study area and because there is an infinity of points, they can be neither processed nor straight displayed in vectorial mode.

The elementary informations featuring the raster mode are ail identical and they have an area. They are the tells of the raster grid or pixels. Consequently, the raster mode seems to be relevant to the processing and the display of any gegraphical object or phenomeno n covering some area. However the rasterisation introduces alteration. The punctual (without dimension) or linear (one dimension) elements are theoretically altered because they are assigned to isolated pixels or arrays of pixels, respectively, which have not a null area. In the case of a discrete polygon, the collection of an integer number of contiguous pixels, which are supposed to cover the area of the polygon, alter its boundary. In the case of a continuons phenomenon, the finite number of pixels substituting for the infinity of points, alters the variation function of the phenomenon, because it is supposed homogeneous within each pixel. This last alteration can be seen as a discretization of the surface holding the phenomenon.

The discretization of the surface is not necessarily a reducing procedure. On the contrary, with an adequate grid resolution, it constitutes a significant advantage of the raster mode. Owing to the discretization, a continuous phenomenon can be evaluated in a finite number of places, as if a dense systematic sampling was drawn over the surface. Besides, because all the pixels have an area, they can hold their own graphical information (grey tone, colour, pattern, etc.) and their juxtaposition leads to a visualization of the phenomenon. The facilities afforded by the discretization in the field of the graphical representation are used by every raster technology devise (e.g. printers). The advantages of the discretization for the processing of continuous coverages are applied particularly to natural phenomena (e.g. orographical surface). As it will be shown, the discretization can also facilitate dramatically the solution of theoretical models of spatial analysis.

3. VORONOI POLYGONS

The first application deals with the well-known analysis of an irregular distribution of objects over a plane. The objects, whatever their nature is, can be seen as punctual sites on a map. The method aims to get a division of the plane into as many areas as sites, in such a way that each area gathers ail the points on the plane nearer to one site than to any other. The shape of each area is a polygon, called "Voronoi" or "Thiessen polygon". The tessellation of polygons settles a neighbourhood network between the sites, a couple of sites being said neighbour when their polygons share a common boundary. It should be noted that the notion of neighbourhood used here makes reference to the topological neighbourhood and not only to a simple criterion of distance between sites.

Geographical literature gives many algorithms and applications for Voronoi tessellation (Boots 1986). Among the common uses, we shall note for instance : the production of areally wheighted averages, a spatial assignment rule for allocating points to areas, the delineation of hinterland around economic centres, etc. From a more theoretical point of view, the model can be used to analyse statistically the pattern of discrete homogeneous events (Vincent *et al.* 1983).

The classical way to resolve the Voronoi problem, that we shall call the vectorial solution, is to construct in the first tinte the simplicial graph, or Delaunay graph, made of all the straight-lines lining the couples of neighbour sites (neighbour according to the meaning given above). Then the tessellation of Voronoi polygons can be derived easily because the Delaunay graph is the dual of the Voronoi graph. Many of the algorithms of construction of the Delaunay graph are based on the following geometric property :

> 3 sites are considered as neighbour in the Delaunay sense if the cincle passing through them does not include any other site. The straight-line links between the 3 neighbours draw a triangle. The Delaunay graph is constructed in a stepwise way, by adding contiguous triangles.

The Voronoi graph is then achieved by drawing the lines bissecting the sides of the Delaunay triangles. According to the application, Delaunay or Voronoi graph is the end-product required.

The raster solution of the model uses the initial definition of the Voronoi polygons : any point within a polygon is nearer the centre (site) of that polygon than any other polygon. The discretization introduced by the raster mode turns the infinity of points over the study area into a large but finite number of pixels. Then the problem cornes down to compute the distance between each pixel and every site and to assign to the current pixel the identificator of the nearest site. As often in image processing, the statements to be run are few and simple, but they must be repeated many times. In tins application, the statements can be drastically simplified :

- it is not necessary to extract the square roots because the comparisons can be made with the squares of the distances;
- if the sites can be assigned to pixels (with integer coordinates) any required distance cames down to the sum of squares of differences between lines/ columns indices. The whole possible set of squares of differences can be computed only one tinte and stored in an array at the beginning of the procedure.

The Voronoi polygons are drawn quickly and in straight way, without the intermediate step of the Delaunay graph. The tessellation makes up a raster layer of information which can be handled by a hybrid GIS (figure 1).

For some applications, the Delaunay graph is required (for a variety of applications dealing with the Voronoï spatial adjacency, the interested reader may refer to Gold, 1991). This graph can be constructed owing to raster

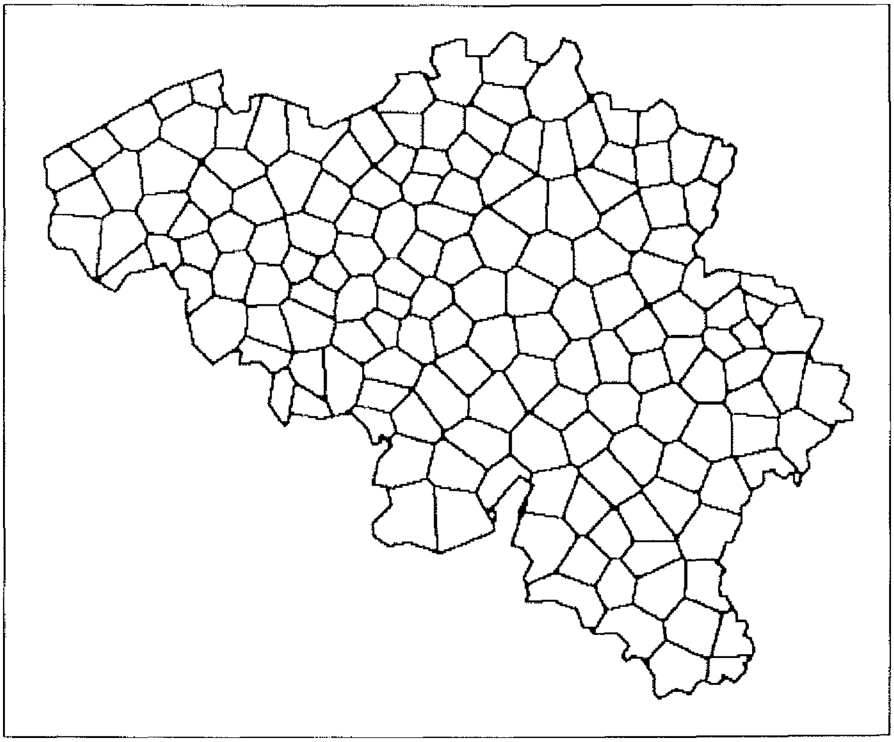

Figure 1. Voronoi tessellation achieved by raster processing and applied to a sample of 13elgian tocans (original data : Grimmeau 1980).

procedures too, starting from the image of the Voronoi tessellation as achieved previously. To be able to construct the Delaunay network we have to know the neighbours of each site. This task is carried out by an "edge filtering"-like procedure (e.g. Roberts filter) slightly modified to update a neighbourhood matrix each time a boundary between two polygons is found inside the image. Because the pixels bold the identificator of the site located at the centre of their polygon, the couple of neighbour sites is directly identified. At last, the drawing of straight-lines between the neighbour sites is conditionned by the state of the cells in the neighbourhood matrix (figure 2). An alternative approach of the Voronoi tessellation is called "the weighted polygon model" (Boots 1987) and its use in GIS has been particularly highlighted by Vincent and Daly (1990). Until now, ail the sites taken into account to settle the tessellation had the same unitary weight. In some situations, especially in marketing applications, it is relevant to use a differentiated weight, that is to say to assign an attribute to the sites. In this case, the shape and the geometric properties of the polygons are different from the standard model. This alternative model can add some complexity to the vectorial solution, but it does not modify the raster resolving way noticeably.

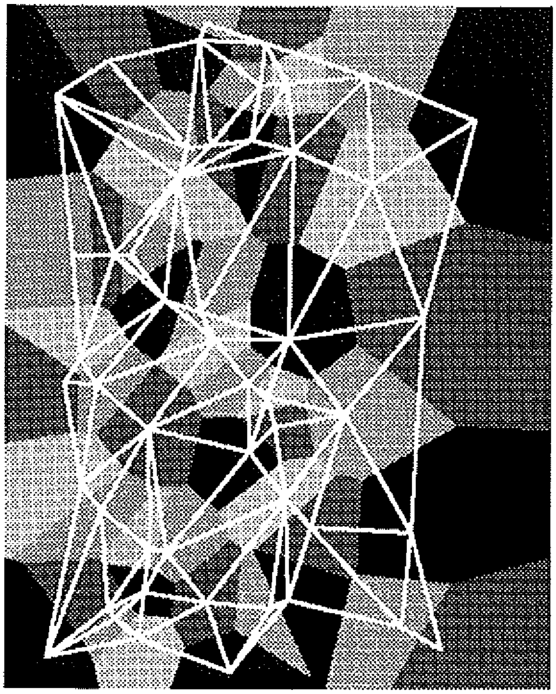

Figure 2. Delaunay triangulation drawn from a Voronoi tessellation with the help of a raster procedure (original data : McCullagh and Ross 1980).

As previously, the polygons are constructed pixel by pixel, but the comparisons now concern the weighted distances between pixels and sites. We may notice that the raster approach avoids the geometric solution of the Voronoi problem, in favour of a graphical algorithm simpler and generally faster than many classical methods. Besicles, it is worth noting that the whole set of procedures is a matter of image processing, including the Delaunay graph construction. Incidentally, the construction order of the two dual graphs is reverted in comparison with the standard approach.

4. WEBER MODEL OF INDUSTRIAL LOCATION

The second application deals with the continuous case of the Weber location model. This model aims to find the location of a factory, for instance, in such a way that the total transportation costs are minimized. The transport concerns simultaneously input and output products from the supplier and to the customer of the factory respectively. For any candidate i, the cost function is given by :

$$F_i = \sum_{j=1}^{n} W_j \,.\, R_j \,.\, d_{ij}$$

where :

- i = punctual candidate for the factory location
- j = punctual supplier or customer of the factory
- W_j = weight of raw material or manufactured goods to be moved from the supplier to the candidate or from the candidate to the customer respectively
- R_j = transportation cost by weight unit and by distance unit
- d_{ij} = distance between the candidate and the supplier/customer

The model must find the candidate i affording the minimal F value. In the discrete case of the Weber model, distances are measured along the actual transportation network and the candidates are a subset of points belonging to the network and known a priori. In the continuous case, discussed here, every point of the plane is candidate for the best location and the distances are computed according to the simple Euclidean criterion. The solution for the discrete case concerns the graph theory and consequently it can be resolved in a vectorial way. On the contrary, the continuous case of the Weber model does not have an analytical solution. The best location is approximated by an iterative tnethod until a user selected accuracy threshold is reached (Scharlïg 1973; Brans 1980).

However the continuous case has a graphical solution. A relatively thin grid is surimposed over the study area. Each node of the grid is seen as a candidate and the cost fonction F is computed at each node. Then, contour-lines are drawn according to an interpolation procedure between the nodes. The lines, so called isodapanes, link the points of saine F values over the model surface. This surface appears as a comb revealing a relatively flat bottom, su the area surrounded by the minimal isodapane shows the best location region with a generally sufficient accuracy.

The raster resolving way of the continuous case of the Weber model is very near from the graphical solution discussed above. The discretization introduced by the raster mode corresponds to the thin superimposed grid and all the pixels of the raster image can be seen as candidates for the best location. So, the cost fonction F is computed at each pixel. The display of the remit, pixel by pixel, leads to a visualization of the Weber surface without having to interpolate and to draw the isodapanes. Statements required by the raster solution are simpler than the iterative analytical approach. However they must be repeated many times. If the sites of the suppliers and the customers can be assigned to pixels (with integer coordinates), the distances computing can be enhanced by preliminar processing and storing of ail the possible squares of differences between the lines/columns indices.

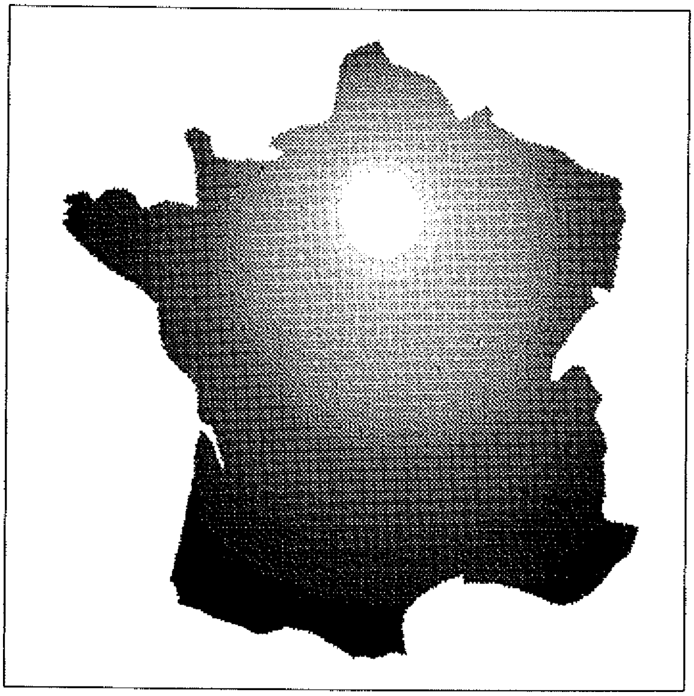

Figure 3. Continuons surface of the Weber location model achieved by raster processing and applied to a didactical example (original data Scharlïg 1973).

The Weber surface completed according to the raster solution is an information layer, generally coded in real numbers, compatible with the hybrid GIS (figure 3). The pixel affording the minimal F value is identified on the fly, during the computing procedure, but the value of any other pixel can be known interactively during the visualization process. If the isodapanes are required, they can be extracted from the raster image of the Weber surface according to classical image processing procedures. A density slicing of the image, using an equidistance chosen by the user, turns the Weber surface into a stepped surface. An edge-filter applied to this last image gives the contour-lines which, as this step, can be vectorized if required (figure **4).**

5. POTENTIAL MODEL

The third and last application showing the raster facilities in spatial analysis concerns the potential model. This model, appearing frequently in human geography literature, has been interpreted variously, for example as a measure of remote influence, as a generalised measure of concentration or as an indicator of accessibility (Rich 1980; Pooler 1987).

Figure 4. Isodapanes drawn from the previous Weber surface, owing to a density slicing and an edge filtering procedures (original data : Scharlïg 1973).

The interaction between two places is a function of their weight and an inverse function of the distance between them. If there are *n* places in the system, the total potential at a point is the sum of all interactions between this point and all the other places. It is given by :

$$Pot_i = \sum_{j=1}^{n} \frac{W_j}{d_{ij}}$$

where *W* is the weight of a place and *d* is the distance between two places. By computing potentials for all the places, it is possible to generate a potential field or potential surface. The model is particularly interesting because it is a basic way of turning a point coverage into a continuous surface. However this transformation rises two pitfalls. The first difficulty deals with the definition of the self-potential term. Indeed, the computing of the potential requires a sum over all the places available in the system, including the place for which the potential is computed. So it is necessary to remove the indetermination of a null denominator in the expression above. This can be reformulated as :

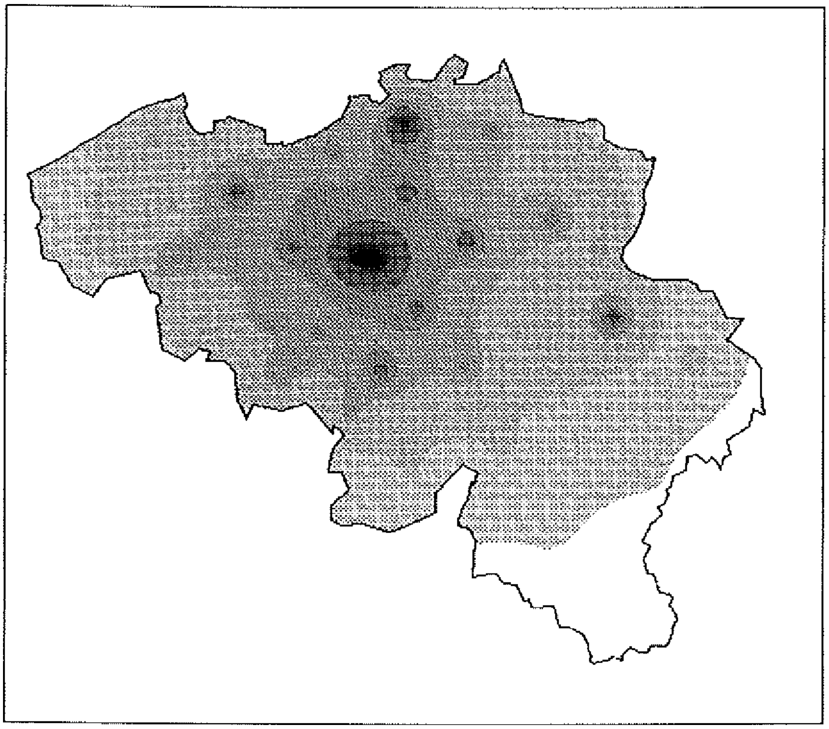

Figure 5. Population (1981) potential field achieved by raster processing over the 43 Belgian districts.

$$Pot_i = \frac{W_i}{d_{ii}} + \sum_{\substack{j=1 \\ j<>i}}^{n} \frac{W_j}{d_{ij}}$$

There is a number of ways to define d_{ii}. The common methods use either the half distance to die nearest neighbour of *i,* or the half radius of a circle of die same ares as the place *i.* The second difficulty concerns the shape of the potential surface. If the potential is only estimated at the *n* places having a weight W, then the shape of the surface is not independent of the location of the *n* places. If their distribution is not roughly homogeneous over the study area, the shape of the surface can be altered. To raise this difficulty, it is advisable to compute the potential at a set of complementary control points, regularly distributed but having no own weight, so no self-potential.
The solution of the potential model according to the raster way follows implicitly this last rule because the potential is computed at each pixel over the image. A self-potential terni is only computed for the pixels matching the location of the weighted places. In this case, one of the classical solutions is used to raise the indetermination d_{ii}.
The potential surface so achieved constitutes one layer of an hybrid GIS

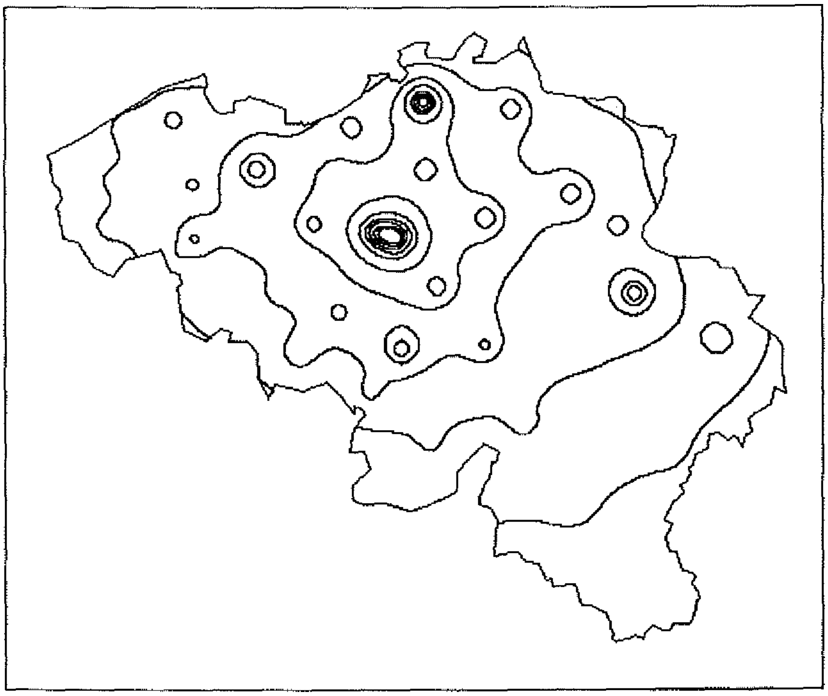

Figure 6. Equipotential contour-limes drawn from the previous potential surface according to a density slicing and an edge filtering procédures.

(figure 5) and contour-lines of equipotential can be drawn owing to a density slicing procedure (figure 6), as it was shown with the isodapanes in the previous example.

If the number of weighted places is large, the potential computing can be a time-consuming process. It is possible to have a good estimation of the potential at one pixel by restricting the interaction contributions to the only weighted places contained in an adequate sized window surrounding the current pixel. The weighted places falling ouside the window are considered as having no significant contribution according to the frictional effect of the distance. In this case, the potential surface is constructed by convoluting the window, as done in many image processing operations.

6. CONCLUSIONS

This paper has illustrated several facilities afforded by the raster mode for resolving continuons models of spatial analysis. Through three examples (Voronoi, Weber and Potential) and owing to the discretization of the study area introduced by the raster mode, we have shown that

- the complex geometric solutions are avoided in favour of simpler graphical solutions;
- the model surface completed according to the raster way constitutes an information layer which can be held by any hybrid GIS;
- various applications derived from continuous surfaces can be drawn using typical raster procedures;
- incidentally, the computer programs are easier to write, including fewer and simpler statements.

Other models or statistical surfaces can be submitted to and take advantage of the raster solving approach, Moreover, some models may be adapted to process strict!)' raster input data such as remote sensing derived data (Nadasdi et al. 1991). When the GIS community becomes aware of the raster domain, we must take advantage of its potentialities to enhance the GIS capabilities in spatial analysis.

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