Multifractality in the kicked rotator

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Multifractality of wave functions

Multifractal exponents D_q ($q \in \mathbb{R}$) can be defined from the scaling with N of the moments P_q of wave functions in a Hilbert space of dimension N [1],

$$P_q = \sum_{i=1}^{N} |\psi_i|^{2q} \propto N^{-D_q(q-1)}$$

The wave functions in Hilbert space of increasing dimensions are considered as the same distribution at smaller and smaller scales. Alternatively, the multifractal exponents can be obtained through the quantum wavelet transform [2]. The singularity spectrum $f(\alpha)$ is the Legendre transform of $\tau_q \equiv D_q(q-1)$

$$f(\alpha) = \min_q (q\alpha - \tau_q)$$

For systems with disorder ($\Rightarrow$ ensemble of wave functions), two types of exponents should be distinguished according to the kind of average :

- scaling of $\langle P_q \rangle \rightarrow D_q$ (average exponent)
- scaling of $\langle \ln(P_q) \rangle \rightarrow D_q^{\mathrm{typ}}$ (typical exponent)

These two kinds of exponents coincide for q values such that the distribution of moments $\mathcal{P}(P_q)$ falls off faster than $1/P_q^2$ [1]. At the 3D-Anderson transition, the anomalous exponents $\Delta_q \equiv (D_q - 1)(q-1)$ should obey the symmetry $\Delta_q = \Delta_{1-q}$ [1].

[1] F. Evers & A. Mirlin, RMP **80**, 1355 (2008).
[2] I. García-Mata *et al.*, PRA **79**, 052321 (2009).

Quantum maps

In this work, we consider the "intermediate map" defined on the torus by

$$\bar{p} = p + \gamma \pmod 1 \qquad (\gamma \in \mathbb{R})$$
$$\bar{q} = q + 2\bar{p} \pmod 1$$

where p is the momentum and q the angle variable. The corresponding quantum evolution is given by

$$\bar{\psi} = \hat{U}\psi = e^{-2i\pi\hat{p}^2/N} e^{2i\pi\gamma\hat{q}}\psi$$

A whole ensemble of $\hat{U}$ matrices can be constructed by taking instead of $2i\pi\hat{p}^2/N$ independent uniformly distributed random phases. The eigenvectors of $\hat{U}$ show *intermediate spectral statistics* [3] and are *multifractal* [4].

[3] E. Bogomolny & C. Schmit, PRL **93**, 254102 (2004).
[4] J. Martin *et al.*, PRE **77**, 035201(R) (2008).

We also consider a 1D system with incommensurate frequencies, which has been shown to display an Anderson-like transition [5]. The system, which is a generalization of the quantum kicked rotator model, is defined by

$$\bar{\psi} = U\psi = e^{-iV(q,t)} e^{i\phi_p}\psi$$

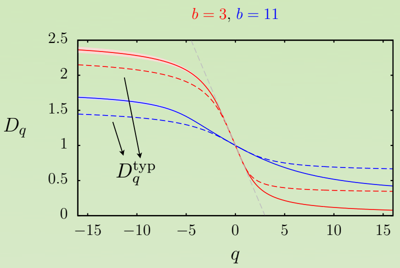
where $V(q,t) = k(1 + 0.75\cos\omega_1 t\, \cos\omega_2 t)\cos q$ with $\omega_1 = 2\pi\lambda^{-1}$, $\omega_2 = 2\pi\lambda^{-2}$, $\lambda \approx 1.3247$, and where ϕ_p are random phases. This model shows the *Anderson transition* at $k \approx 1.8$.

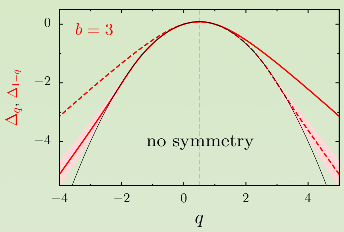
[5] G. Casti *et al.*, PRL **62**, 345 (1989).
[6] J. Chabé *et al.*, PRL **101**, 255702 (2008).

Multifractal exponents : results

Intermediate map : For $\gamma = a/b$ ($a, b \in \mathbb{N}_0$), we find for $|q/b| \lesssim 0.1$

$$D_q \approx 1 - \frac{q}{b} \quad \Leftrightarrow \quad f(\alpha) \approx 1 - \frac{b}{4}\left(\alpha - 1 - \frac{1}{b}\right)^2$$

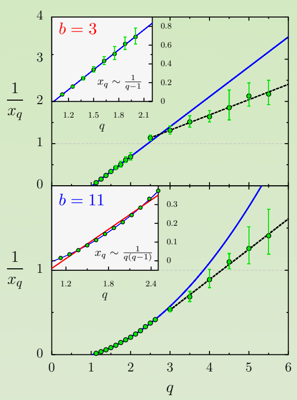




Confirmation of the link between D_2 and the level compressibility χ [7]

$$\chi = \frac{1}{2}\left(1 - \frac{D_2}{D_0}\right) \approx \frac{1}{b}$$

Fall-off exponent x_q : $\mathcal{P}(P_q) \propto 1/P_q^{1+x_q}$

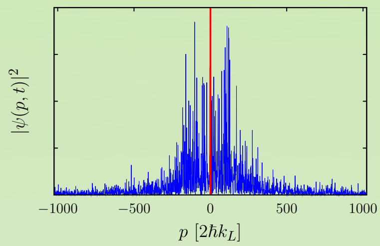


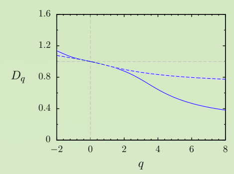
[7] J. T. Chalker *et al.*, JETP Lett. **64**, 386 (1996).

1D Anderson map :

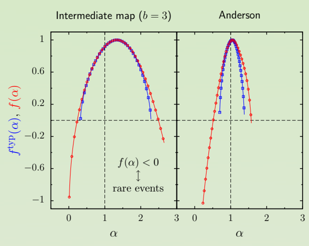
$k < 1.8$: localized
$k \approx 1.8$: multifractal
$k > 1.8$: ergodic

Parameters : $k = 1.81$,
$t = 10^8$ kicks
N=2048





multifractality !



Fall-off exponent x_q : $\mathcal{P}(P_q) \propto 1/P_q^{1+x_q}$