Recent developments in optimization of flexible components of multibody systems

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OUTLINE

- Introduction
- Finite Element approach of Multibody system dynamics
- Sensitivity Analysis
- Formulation of optimization problem
- Numerical application: 2-dof robot arm
- Conclusion & Perspectives

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INTRODUCTION

EVOLUTION OF VIRTUAL PROTOTYPING

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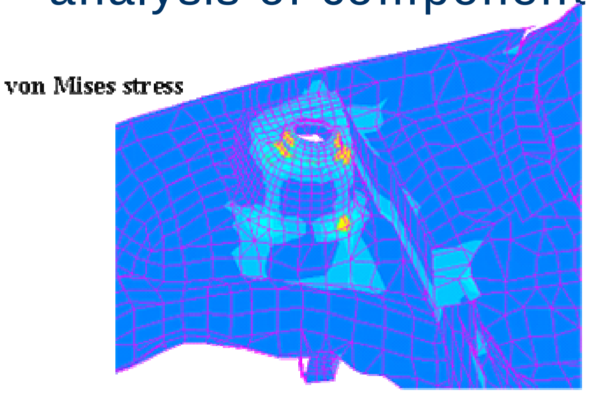
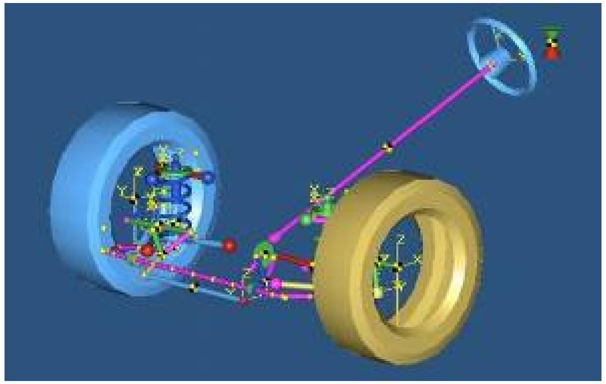
- Finite Element: structural analysis of components

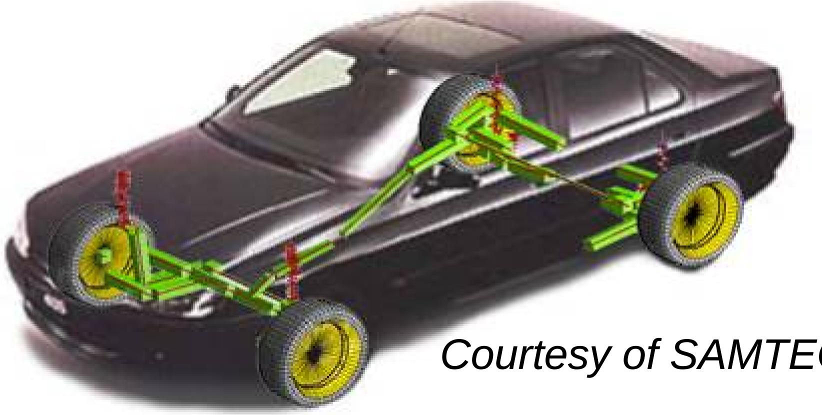


- Multibody system: mechanism of rigid bodies



- Flexible Multibody systems:

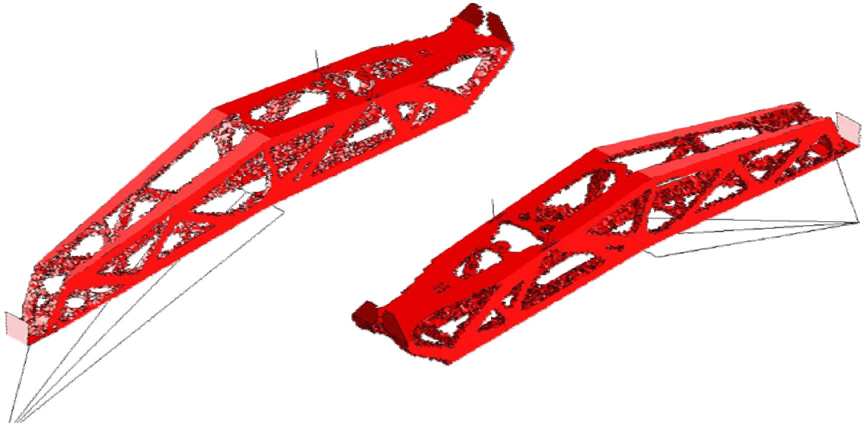
System approach (MBS)
& structural dynamics (FEM)



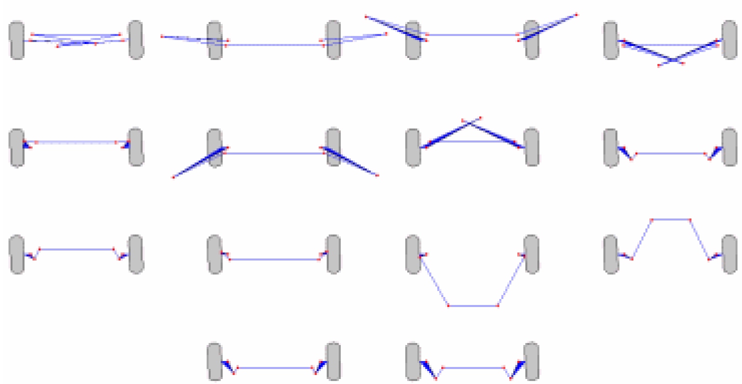
Courtesy of SAMTECH

EVOLUTION OF OPTIMIZATION

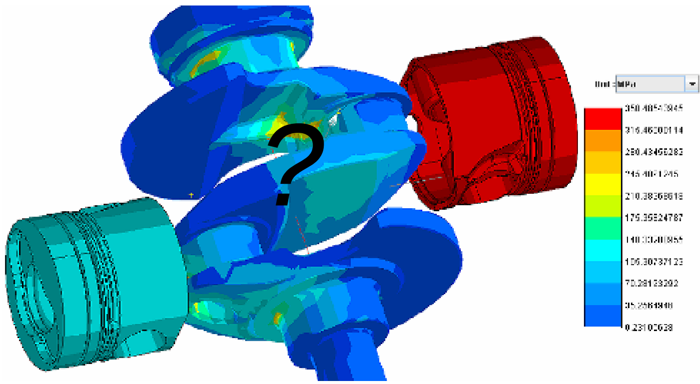
- Structural optimization



- Mechanism synthesis



- Optimization of flexible components in Multibody systems:



AN INDUSTRIAL PERSPECTIVE…

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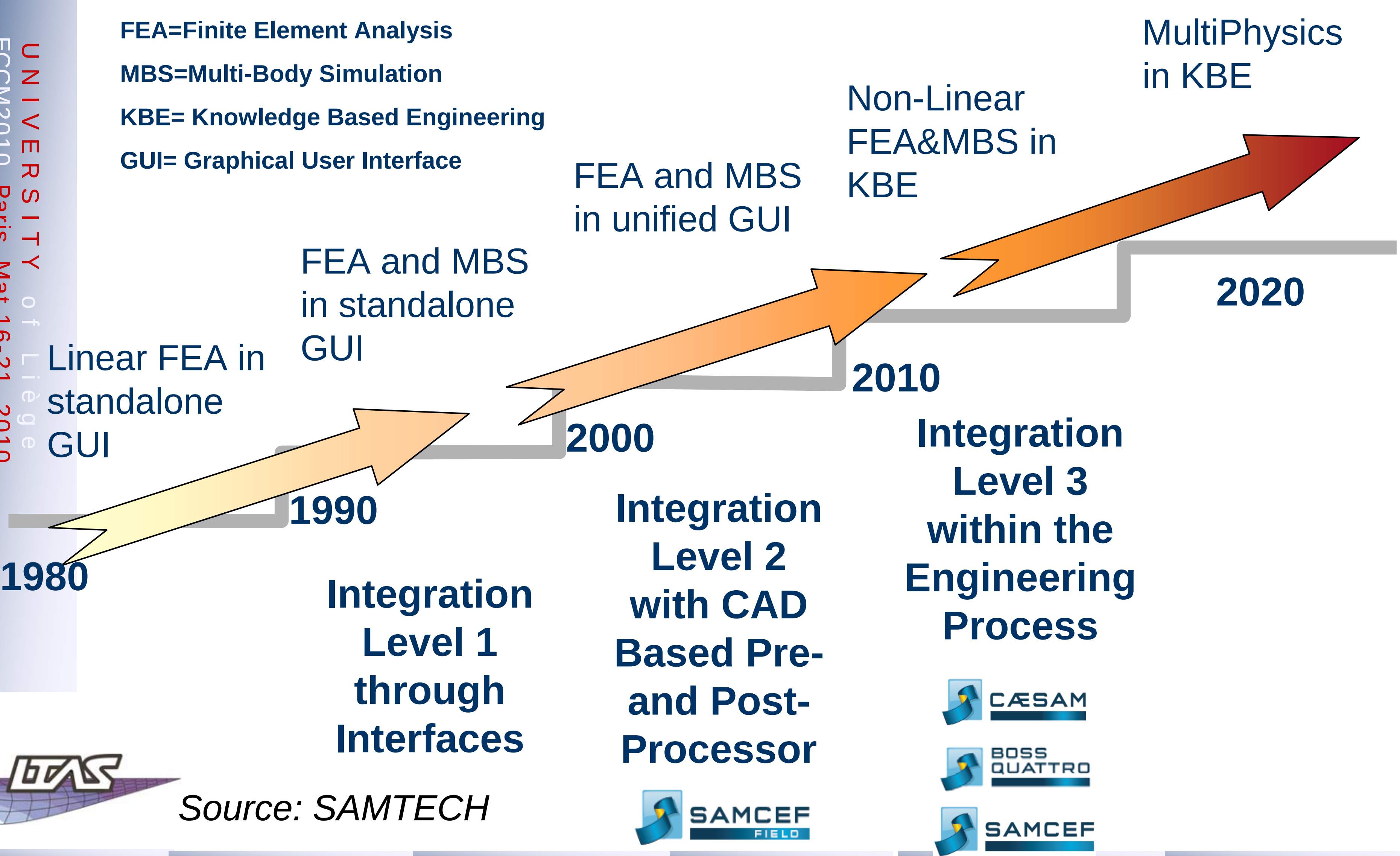
FEA=Finite Element Analysis

MBS=Multi-Body Simulation

KBE= Knowledge Based Engineering

GUI= Graphical User Interface



Source: SAMTECH

INTRODUCTION

- Classical FEM approach: static load cases (empirical)

- Weak coupling between FEM and MBS:
  - Coupling with pre / post processing
  - Define equivalent quasi-static load cases
  - Optimization of isolated components

- Optimization of flexible components in multibody system dynamics
  - Define realistic dynamic loading
  - Take care of the coupling between large overall rigid-body motions and deformations

    ➔ FEM based approach of MBS

    ➔ Sensitivity of MBS

    ➔ Coupling with optimization

FINITE ELEMENT APPROACH OF MULTIBODY SYSTEM DYNAMICS

EQUATION OF MBS DYNAMICS

- Motion of the flexible body is represented by absolute nodal coordinates **q**

- Dynamic equation of multibody system

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{\mathrm{ext}} - \mathbf{g}^{\mathrm{int}}$$

  - **M** masse matrix of the system
  - **g** external and internal forces

- The motion of the mechanical system is subject to some kinematic constraints (joints, admissible or prescribed motion)

$$\mathbf{\Phi}(\mathbf{q}, t) = 0$$

EQUATION OF MBS DYNAMICS

- Solution based on an augmented Lagrangian approach of total energy and two additional terms related to the constraints including a penalty and the Lagrangian multipliers λ.
- Stationary conditions

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}^T (k\boldsymbol{\lambda} + p\boldsymbol{\Phi}) = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) \\ k\boldsymbol{\Phi}(\mathbf{q}, t) = 0 \end{cases}$$

If

$$\mathbf{B} = \frac{\partial \boldsymbol{\Phi}}{\partial \mathbf{q}}$$

- With initial conditions

$$\mathbf{q}'(0) = \mathbf{q}'_0 \text{ and } \dot{\mathbf{q}}'(0) = \dot{\mathbf{q}}_0$$

TIME INTEGRATION

- The set of nonlinear differential and algebraic equations can be solved using the generalized-α method by Chung and Hulbert (1993)
- Define pseudo acceleration **a**:

$$(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f \ddot{\mathbf{q}}_n$$

- Newmark integration formulae

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1}$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_{n+1} + h^2(1/2-\beta)\mathbf{a}_n + h\beta\mathbf{a}_{n+1}$$

- Solve the dynamic equation system (Newton-Raphson)

$$\left[\begin{array}{l} \mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{C}_t\Delta\dot{\mathbf{q}} + \mathbf{K}_t\Delta\mathbf{q} + \mathbf{B}^T\Delta\boldsymbol{\lambda} = \Delta\mathbf{r} \qquad \mathbf{r} = \mathbf{M}\ddot{\mathbf{q}} - \mathbf{g} + \mathbf{B}^T\boldsymbol{\lambda} \\ \qquad\qquad \mathbf{B} = \mathbf{0} \end{array}\right.$$

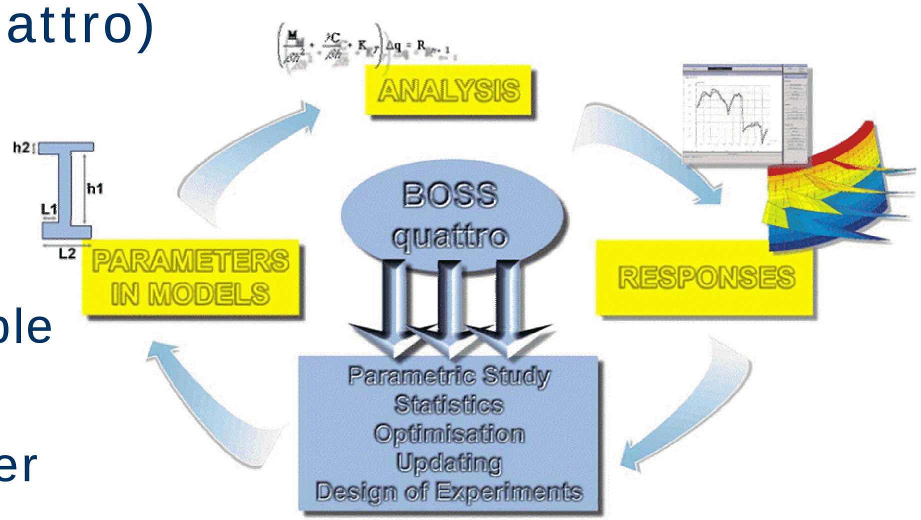
SENSITIVITY ANALYSIS

- Gradient-based optimization methods requires the first order derivatives of the responses

- Finite differences (Boss Quattro)

$$\frac{\partial \mathrm{f}}{\partial x} \approx \frac{\mathrm{f}(x+\delta x)-\mathrm{f}(x)}{\delta x}$$

Perturbation of design variable
Additional call to MBS code
➔ Boss Quattro task manager



- Semi-analytical approach

$$\frac{\partial \mathbf{r}}{\partial x} \approx \frac{\mathbf{r}(x+\delta x)-\mathbf{r}(x)}{\delta x} \qquad \frac{\partial \mathbf{\Phi}}{\partial x} \approx \frac{\mathbf{\Phi}(x+\delta x)-\mathbf{\Phi}(x)}{\delta x}$$

SENSITIVITY ANALYSIS *Bruls and Eberhard (2008)*

- Derivative of response function f

$$f' = f_{,\mathbf{z}}\,\mathbf{z}'(\mathbf{x},t) + f_{,x}$$

- → Evaluate the derivatives of the state variables

- Derivation of the state equations

$$\begin{cases} \mathbf{M}\,\ddot{\mathbf{q}}' + \mathbf{C}_t\,\dot{\mathbf{q}}' + \mathbf{K}_t\,\mathbf{q}' + \mathbf{B}^T\lambda' - \mathbf{r}_{,x} = \mathbf{0} \\ \mathbf{B}\,\mathbf{q}' + \mathbf{\Phi}_{,x} = \mathbf{0} \end{cases}$$

Pseudo loads

- With the initial conditions

$$\mathbf{q}'(0) = \mathbf{q}'_0 \text{ and } \dot{\mathbf{q}}'(0) = \dot{\mathbf{q}}_0$$

SENSITIVITY ANALYSIS *Bruls and Eberhard (2008)*

- One observes that the sensitivity equations are linear with respect to derivatives of q and λ.
- At time step n+1, the sensitivities can be computed using the same integration algorithm as for the dynamic response except for the residuals
- The iteration (tangent) matrix is the same as for the original problem. Hence this matrix should be computed and factorized only once for the sensitivity analysis at time step n+1.
- See Bruls and Eberhard (2008) for details

FORMULATION AND SOLUTION OF OPTIMIZATION PROBLEM

GENERAL FORM OF THE OPTIMIZATION PROBLEM

- Design problem is cast into a mathematical programming problem

$$\begin{array}{lll} \min_{\mathbf{x}} & g_0(\mathbf{x}) & \\ \text{s.t.:} & g_j(\mathbf{x}) \leq \bar{g}_j & j = 1 \ldots m \\ & \underline{x}_i \leq x_i \leq \overline{x}_i & i = 1 \ldots n \end{array}$$

- Provides a general and robust framework to the solution procedure
- Take benefit of the available efficient solvers :
  - CONLIN (Fleury, 1989); MMA (Svanberg, 1987)
  - GA, PSO, etc.

SOLUTION ALGORITHMS

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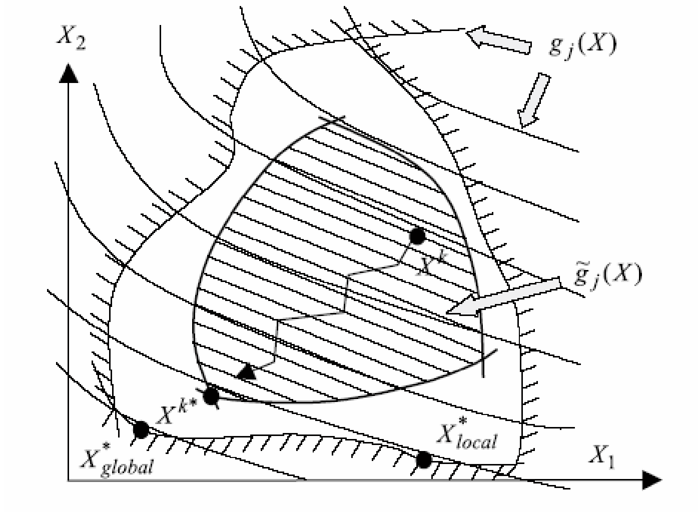
- GRADIENT-BASED ALGORITHMS (SCP)
  - Structural approximation + Efficient solvers
  - CONLIN (Fleury, 1989), MMA (Svanberg, 1987), GCM (Bruyneel, Duysinx, and Fleury, 2002), SQP



- METAHEURISTIC ALGORITHMS: GA
  - Require only the computation of the function values
  - Global convergence
  - Large number of function evaluations

- SURROGATE BASED OPTIMZATION
  - Replace the direct evaluation of the simulation model by a global approximation of the responses (Artificial Neural Network, etc.)

Design variables

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- Optimization of flexible components

➔ Sizing variables: Plate thickness, Bar and beams cross sections, lumped properties (stiffness, mass, etc.)

➔ Shape variables: geometrical parameters of flexible body shape: e.g. control node positions

➔ Topology: Pseudo density variables, e.g. SIMP $E=\mu^3 E°$

- BUT NOT
  - Synthesis variables of mechanisms (Hansen, 2002)
  - Links and joints connectivity (Kawamoto et al. 2004)

Objective function and restrictions

- Mass of the mechanism
  - Source of inertia loads
  - Cost of material

- Stiffness → Compliance
  - Compliance of component i at time step t

$$C_{(i)}(t) = \int_{V_E} \boldsymbol{\varepsilon}^T \mathbf{D} \boldsymbol{\varepsilon} \, dV$$

  - Averaged compliance along the motion (Bruls et al. 2007)

$$\bar{C} = \frac{1}{T} \int_0^T \sum_i C_{(i)}(\tau) \, d\tau$$

Objective function and restrictions

- For robot motion: stiffness → trajectory tracking

  - Perfect trajectory = rigid body trajectory $\mathbf{r}_{rigid}(t)$
  - Flexible trajectory $\mathbf{r}(t)$

  - Average deviation of the flexible trajectory: mean tip deflection (Bruls et al. 2007)

$$\bar{d} = \frac{1}{T}\int_0^T \left\| \mathbf{r}(\tau) - \mathbf{r}_{rigid}(\tau) \right\|^2 d\tau$$

    - Norm 2 of deviation
    - Loose control of instantaneous deviation?
    - Relation between mean square deviation and max deviation?

Objective function and restrictions

- For robot motion: stiffness → trajectory tracking

  - Instantaneous deviation of the flexible trajectory

    $$-\left|d_{\min}\right| \leq dist\left(\mathbf{r}_{rigid}(t) - \mathbf{r}(t)\right) \leq d_{\max}$$

    - Two constraints per time step → large scale optimization problem

  - Global maximum/minimum deviation constraint

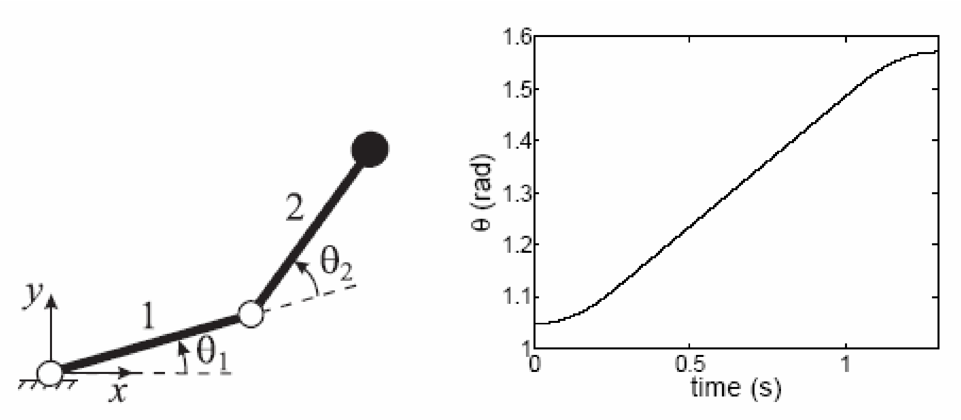
    $$[\max_k dist(\mathbf{r}_{rigid}(t_k) - \mathbf{r}(t_k))] \leq d_{\max}$$

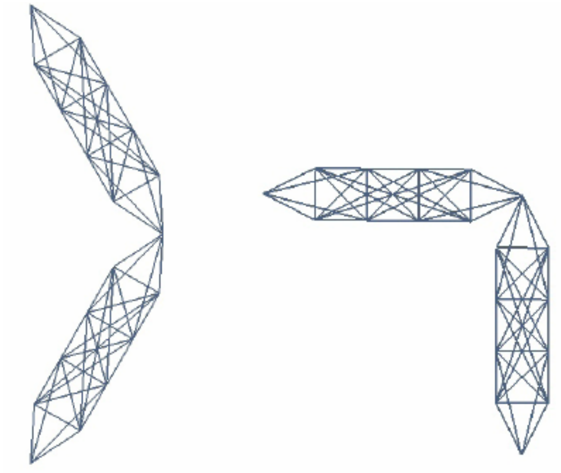
    - One single constraint
    - Non smooth and strongly non linear

NUMERICAL APPLICATIONS

TOPOLOGY OPTIMIZATION OF ROBOT COMPONENTS *(Bruls et al. 2007, 2010)*

- Two-dof robot-arm





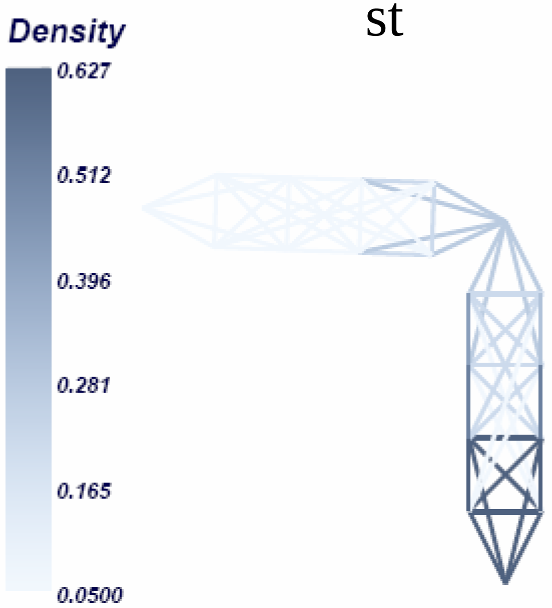
- Topology optimization of truss-like components
  - SIMP model $E = \mu^3 E^0$
  - CONLIN
  - Semi-analytical sensitivity

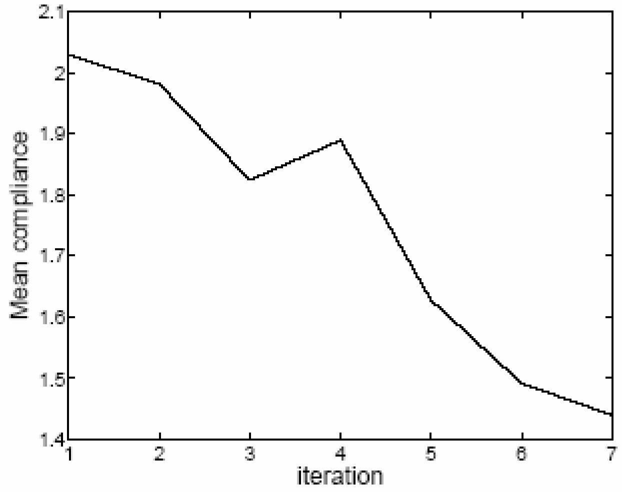
TOPOLOGY OPTIMIZATION OF ROBOT COMPONENTS (Bruls et al. 2007, 2010)

- Minimum compliance

$$\min_x \quad \bar{C} = \frac{1}{T}\int_0^T \sum_i C_{(i)}(\tau)\, d\tau$$

$$\text{st} \qquad V \leq 0.4\bar{V}$$





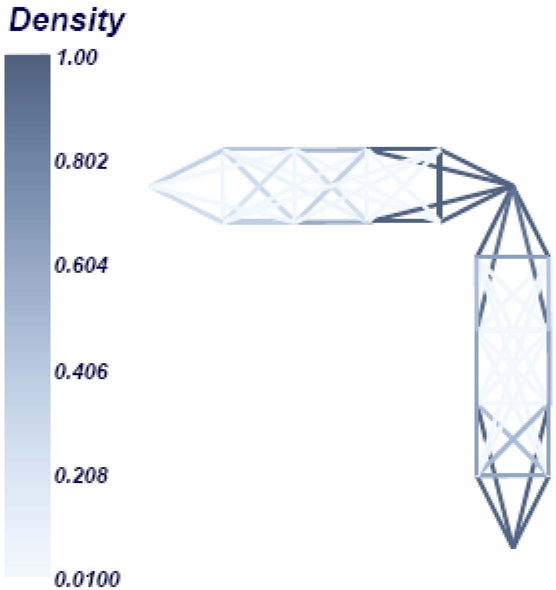
Non convergence at iteration 7

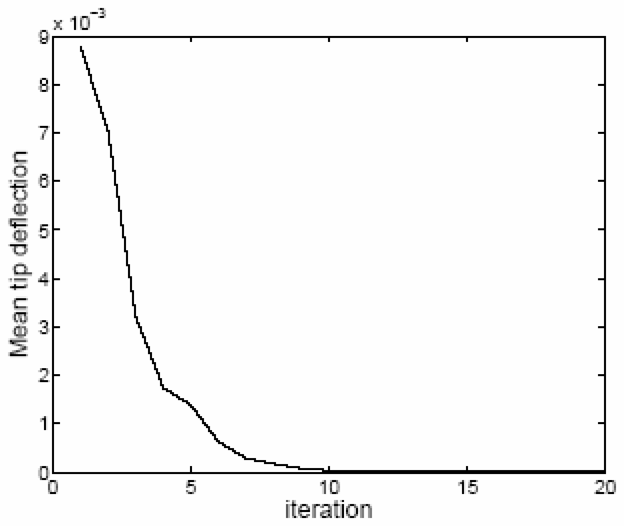
TOPOLOGY OPTIMIZATION OF ROBOT COMPONENTS *(Bruls et al. 2007, 2010)*

- Minimum average deviation

$$\min_x \quad \bar{d} = \frac{1}{T}\int_0^T \left\| \mathbf{r}(\tau) - \mathbf{r}_{rigid}(\tau) \right\|^2 d\tau$$

$$\text{st} \qquad V \leq 0.4\bar{V}$$

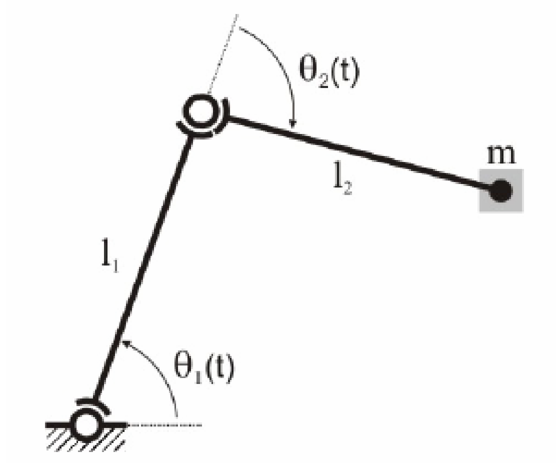




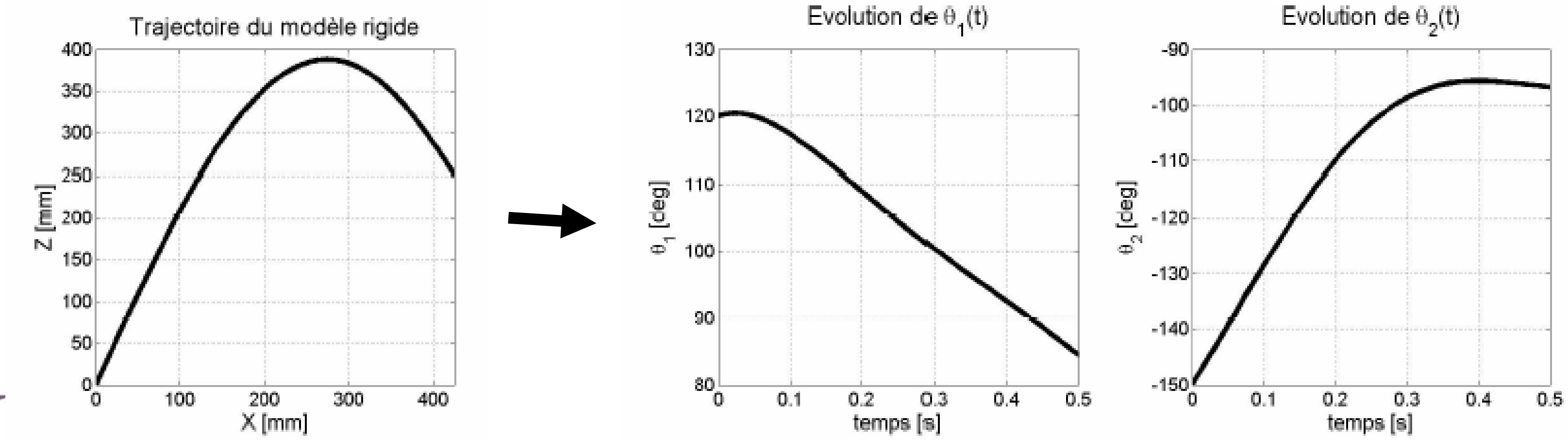
Convergence OK!

COMPONENT OPTIMIZATION OF A 2-DOF ROBOT ARM

- Two-dof robot arm (Ata, 2007)
  trajectory time: 0,5s
  $l_1=l_2=600$ mm
  M= 1kg
  Aluminum: 72 GPa, $\nu=0.3$
  $\rho=2700 kg/m^3$

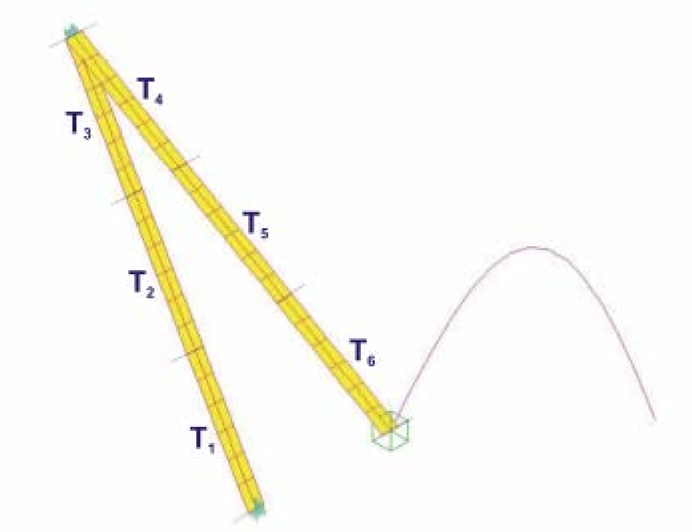


- Trajectory followed by rigid robot → prescribed joint law



SIZING OPTIMIZATION OF ROBOT COMPONENTS

- Modeling of the flexible components as plates of variable thicknesses: T1…T6.



- Prescribe the joint motion from rigid body simulation

- Optimization problem

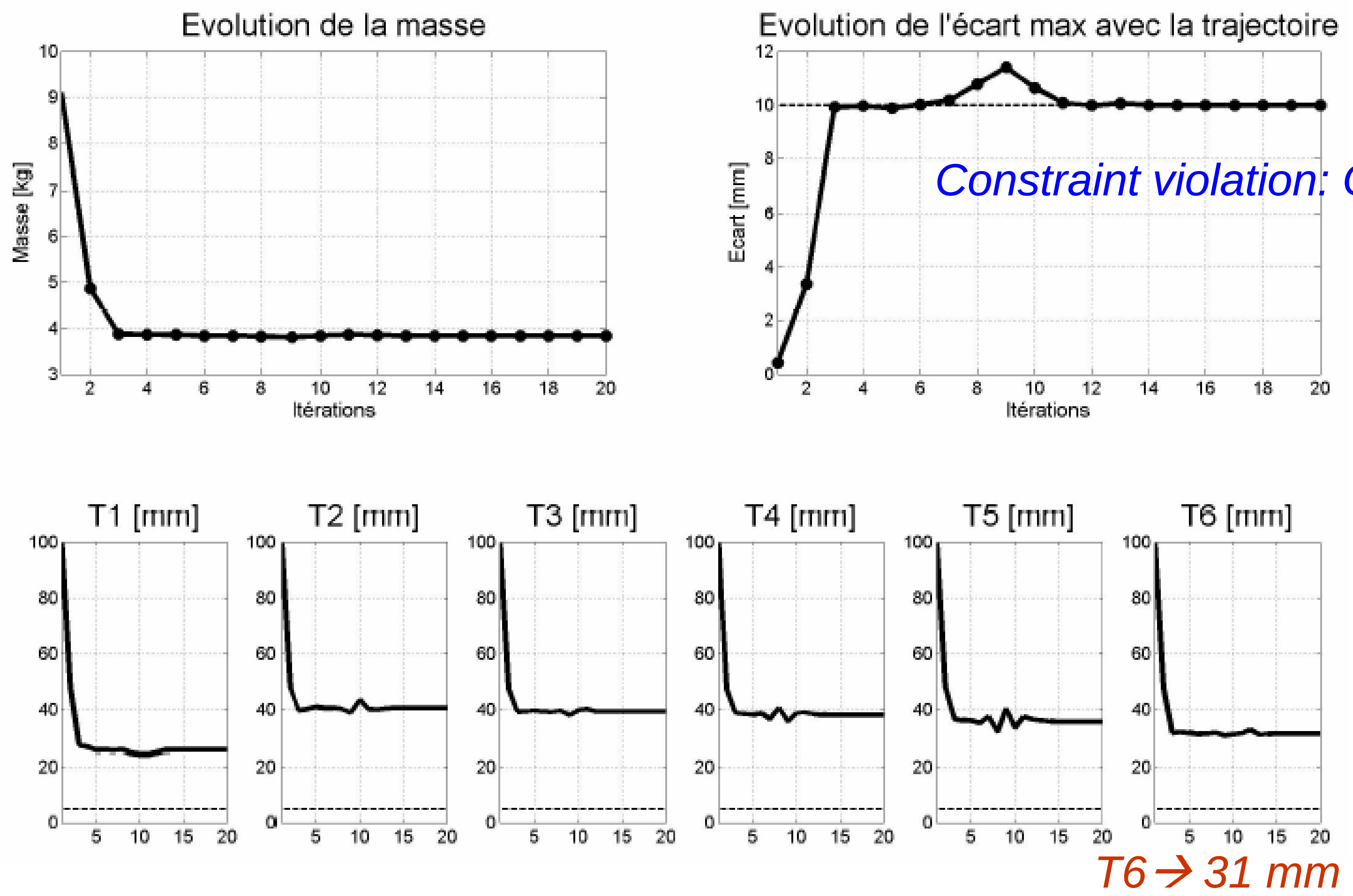
$$\begin{aligned} \min \quad & \text{mass} \\ \text{s.t.} \quad & \text{deviation} \leq 10 \text{ mm} \\ & 5 \leq T_i \leq 200 \text{ mm} \end{aligned}$$

- Deviation each time step

$$-\left|d_{\min}\right| \leq dist(\mathbf{r}_{rigid}(t) - \mathbf{r}(t)) \leq d_{\max}$$

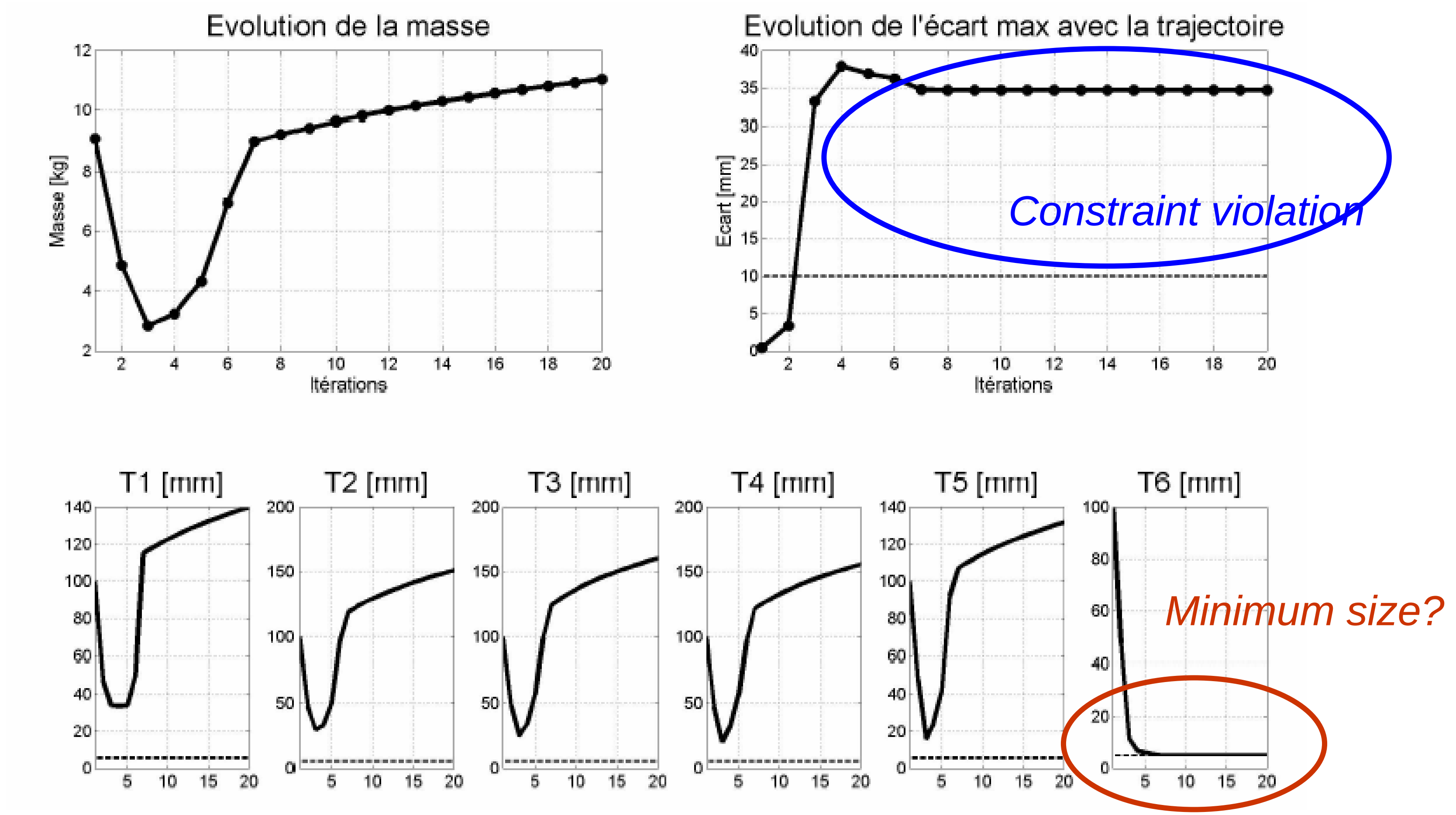
- Solution GCM (Bruyneel et al. 2002)

SIZING OPTIMIZATION OF ROBOT COMPONENTS



By chance, *one feasible solution using* *finite difference* *sensitivity analysis*

SIZING OPTIMIZATION OF ROBOT COMPONENTS



Generally, optimization procedure fails (here semi-analytical sensitivity analysis)

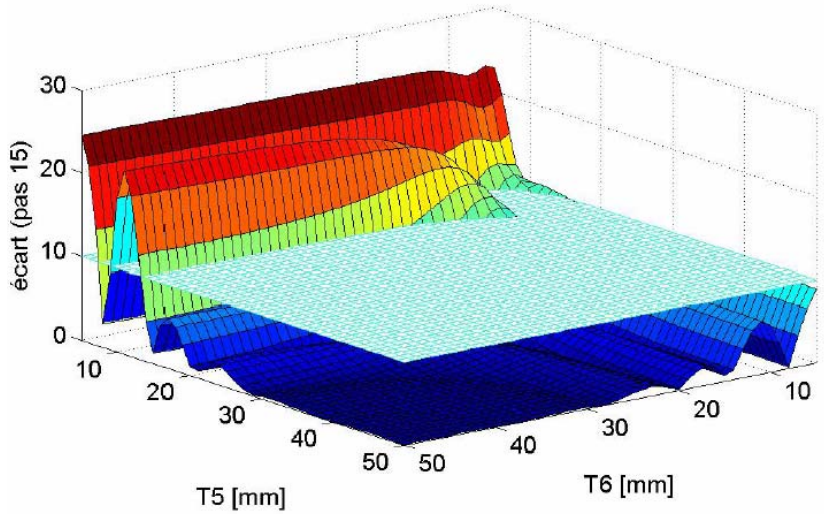
SIZING OPTIMIZATION OF ROBOT COMPONENTS

- Open questions
  - Origin of the problem?
  - Suitable formulation?
    - Deviation at each time step
    - Average deviation
    - Global deviation constraint (max)
  - Choice of optimization algorithm?
    - Gradient-based algorithm
    - GA, surrogate (ANN)

- Answer:
  - Plot design space (slices for selected design variables)
  - Iteration points trajectory in the design space

SIZING OPTIMIZATION OF ROBOT COMPONENTS

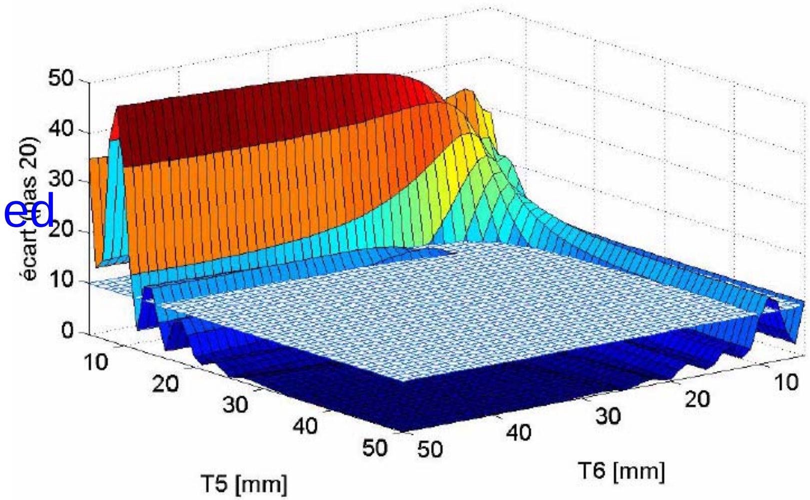
- One deviation constraint per time step
  - Complex feasible design domain



$$-\left|d_{\min}\right| \le dist(\mathbf{r}_{rigid}(t) - \mathbf{r}(t)) \le d_{\max}$$

Is structural approximation procedure adapted to such problems?

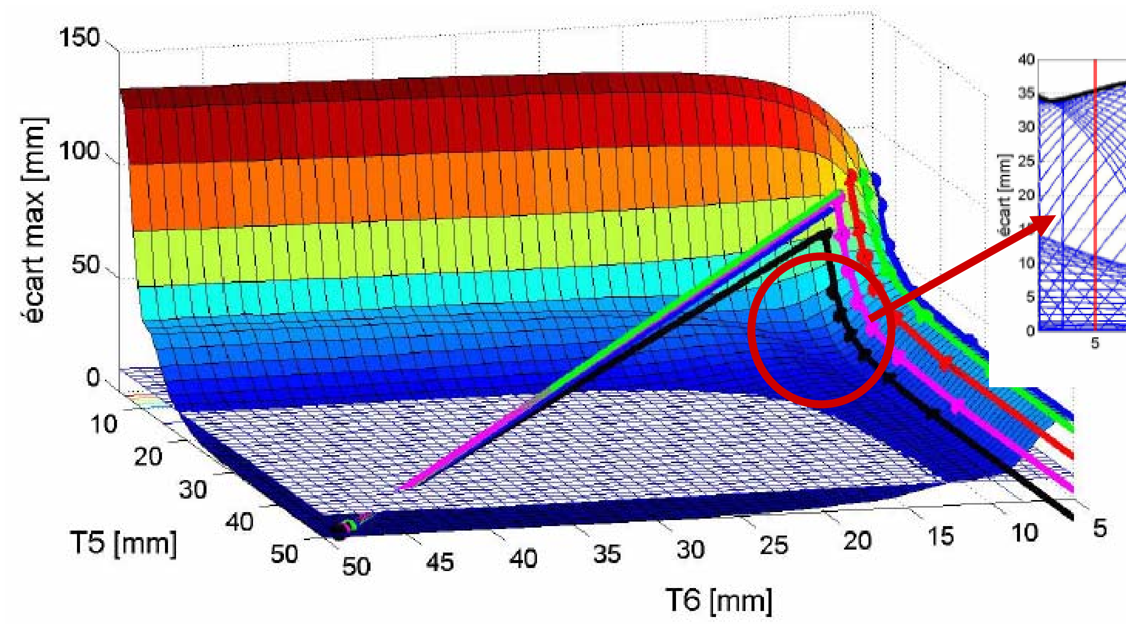
CONLIN, GCM, MMA, SQP failed
Surrogate (ANN) failed
GA: very heavy solution

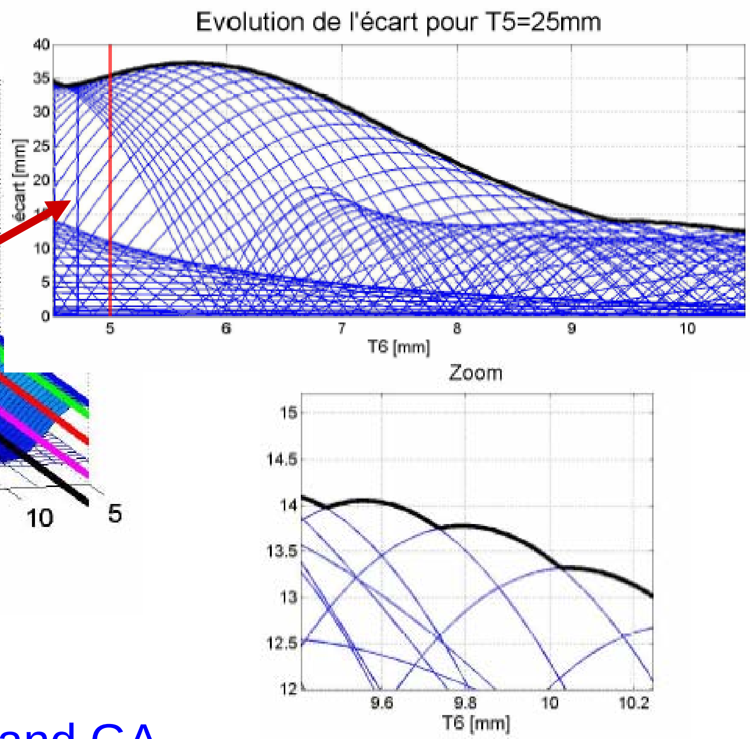


SIZING OPTIMIZATION OF ROBOT COMPONENTS

- Global maximum deviation constraint
- Non smooth but less complex topology of design space

$$[\max_k dist(\mathbf{r}_{rigid}(t_k) - \mathbf{r}(t_k))] \le d_{\max}$$





Failure for gradient-based methods
Success for ANN surrogate models and GA

SIZING OPTIMIZATION OF ROBOT COMPONENTS

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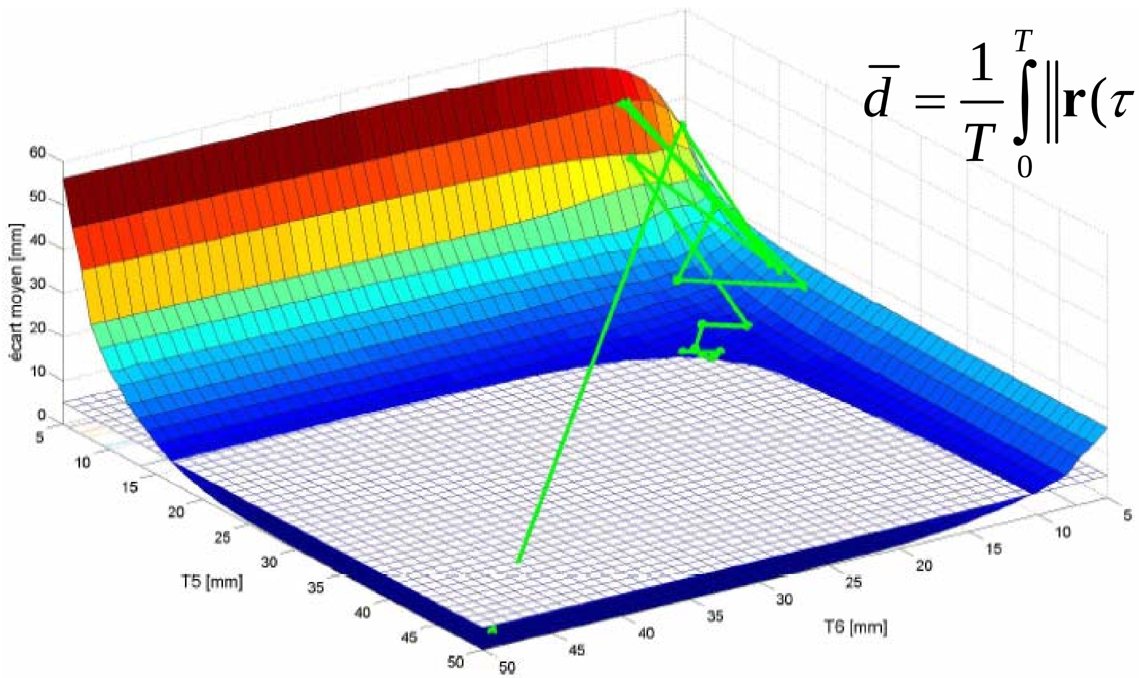
- Average deviation constraint (d<5 mm)
  - Smooth and less complex topology of design space
  - Loose control of the instantaneous deviation

$$\bar{d} = \frac{1}{T}\int_0^T \left\| \mathbf{r}(\tau) - \mathbf{r}_{rigid}(\tau) \right\|^2 d\tau$$



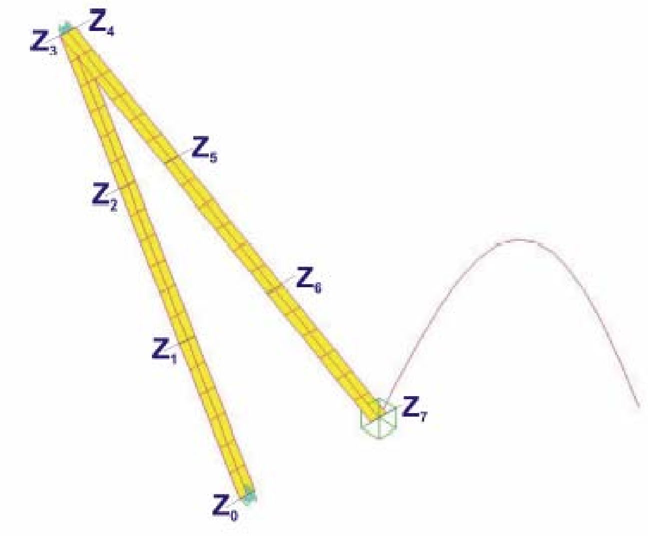
Success for GCM, SQP
Success for ANN surrogate models and GA

SHAPE OPTIMIZATION OF ROBOT COMPONENTS

- Modeling of the flexible components as plates of variable width: Z1…Z8.
- Prescribe joint motion from rigid body simulation
- Optimization problem

$$\min \quad \text{mass}$$
$$\text{s.t.} \quad \text{deviation} \leq 10 \text{ mm}$$
$$5 \leq T_i \leq 200 \text{ mm}$$

- Solution GCM (Bruyneel et al. 2002)

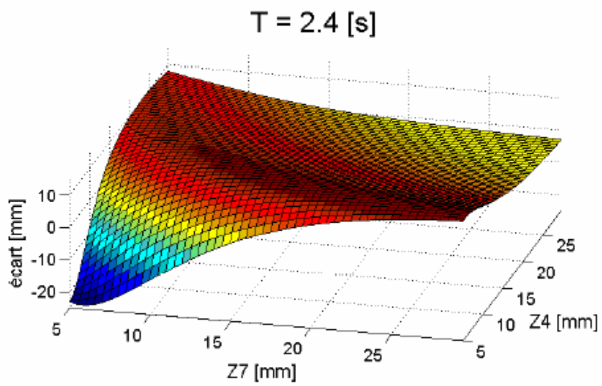


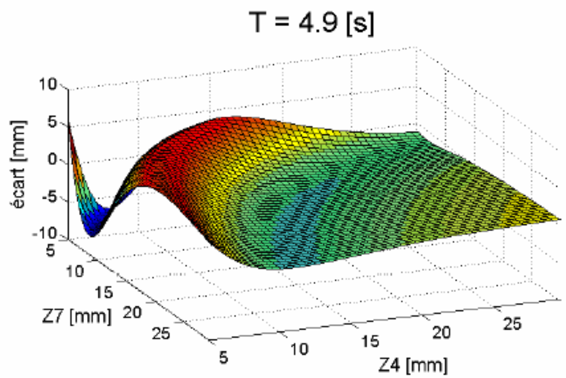
- Semi-analytical sensitivities
- Deviation each time step

$$-|d_{\min}| \leq dist(\mathbf{r}_{rigid}(t) - \mathbf{r}(t)) \leq d_{\max}$$

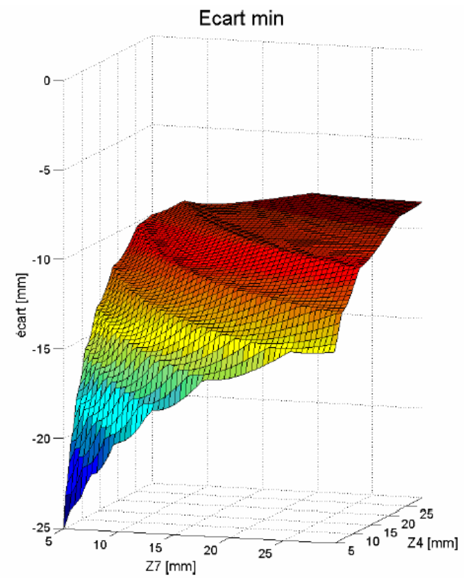
SHAPE OPTIMIZATION OF ROBOT COMPONENTS

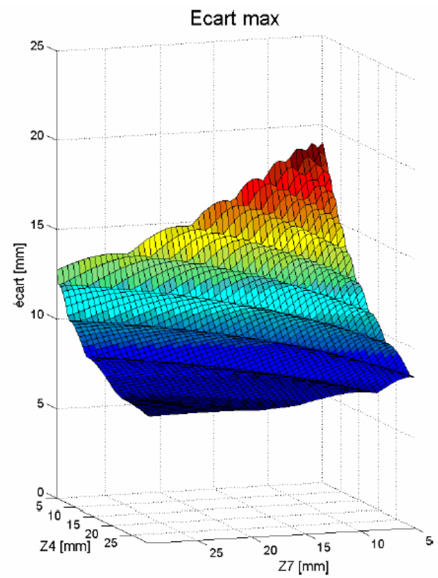
- Nature of deviation constraints at t=15 and t=20





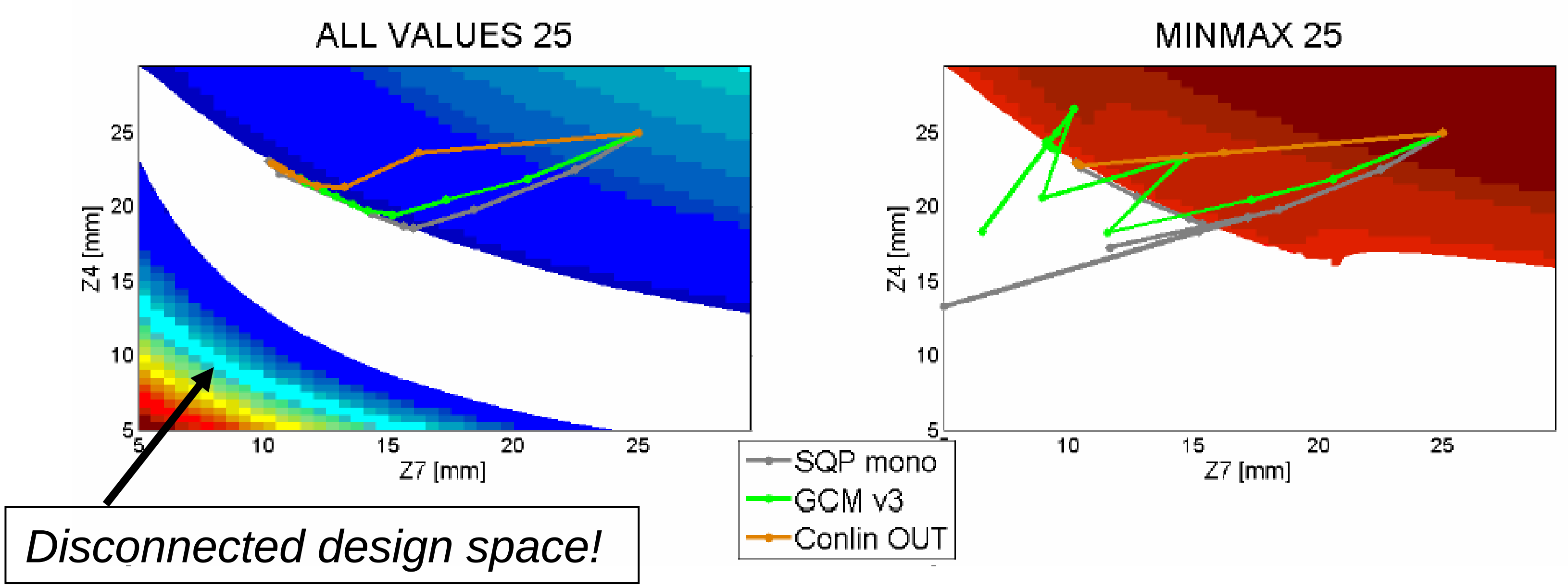
- Global max /min deviation





SHAPE OPTIMIZATION OF ROBOT COMPONENTS

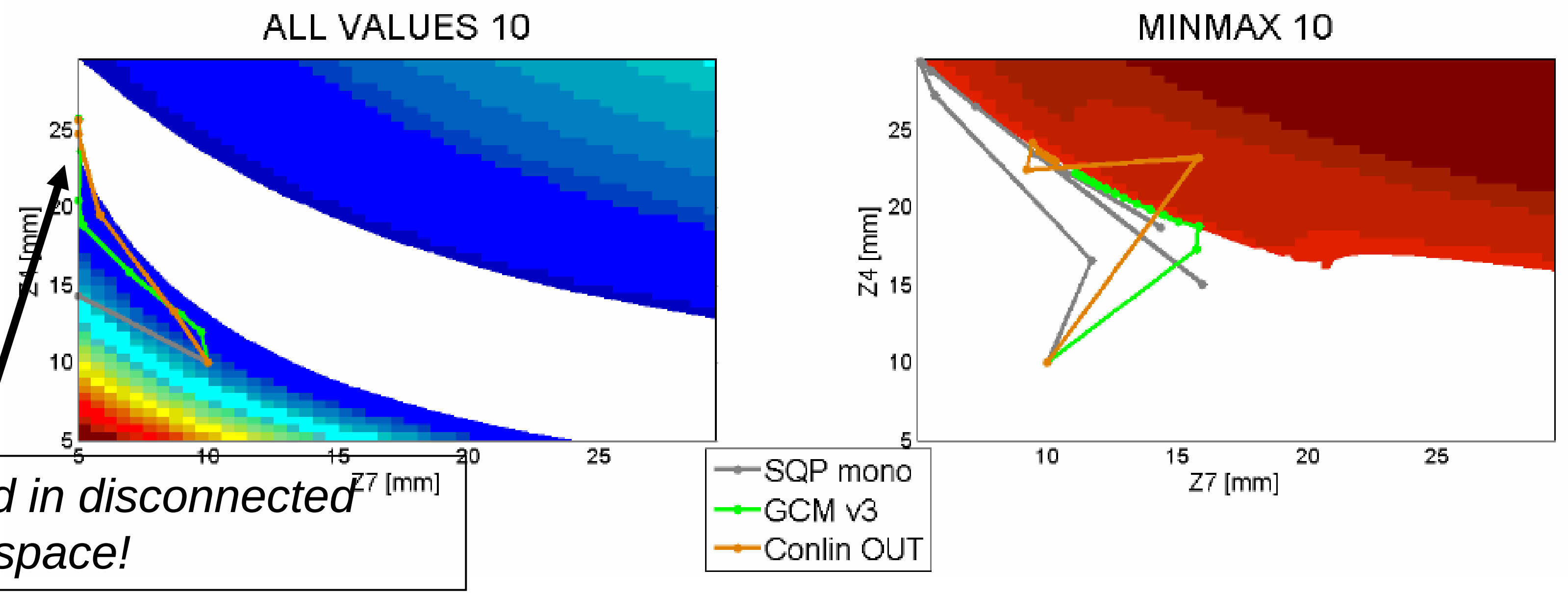
- Comparison between: CONLIN, GCM, SQP
  - Deviation at each time step v.s. max deviation



- All gradient-based algorithms converge when starting from a feasible design point

SHAPE OPTIMIZATION OF ROBOT COMPONENTS

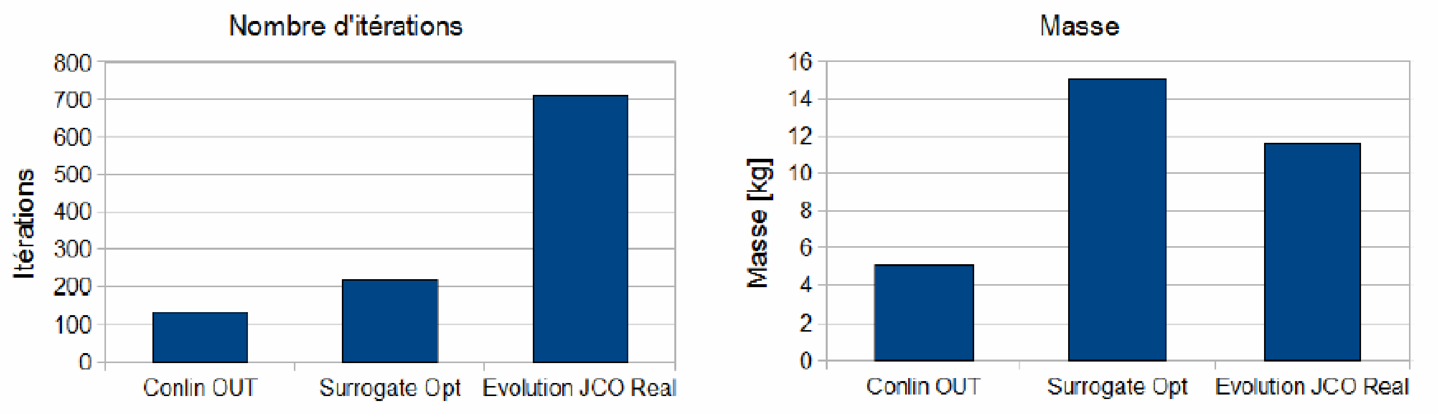
- Comparison between: CONLIN, GCM, SQP
  - Deviation at each time step v.s. max deviation



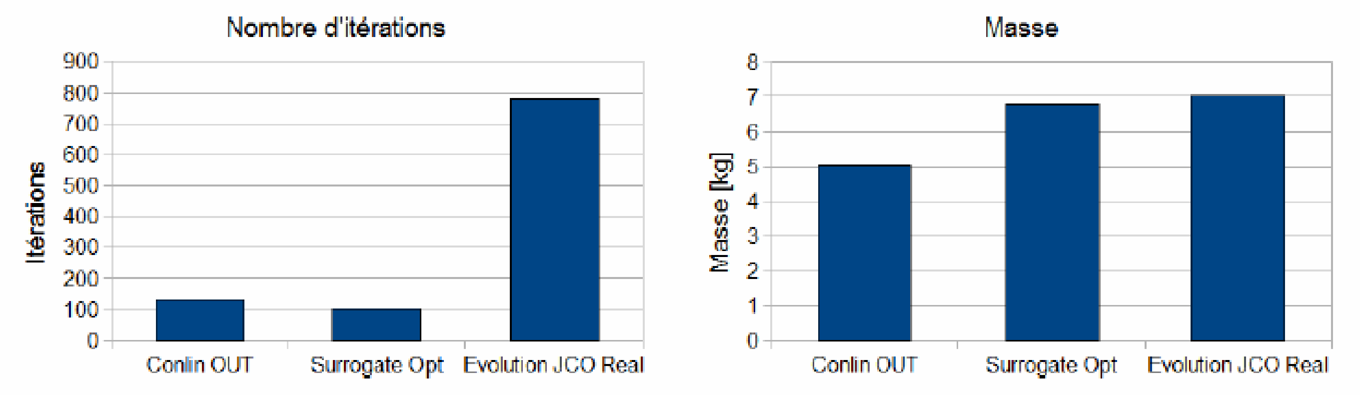
- Gradient-based algorithms + local minima and unfeasible starting point
- Global criteria improves the situation and helps to find global optima

SHAPE OPTIMIZATION OF ROBOT COMPONENTS

- Comparison between: CONLIN, SURROGATE+GA, GA



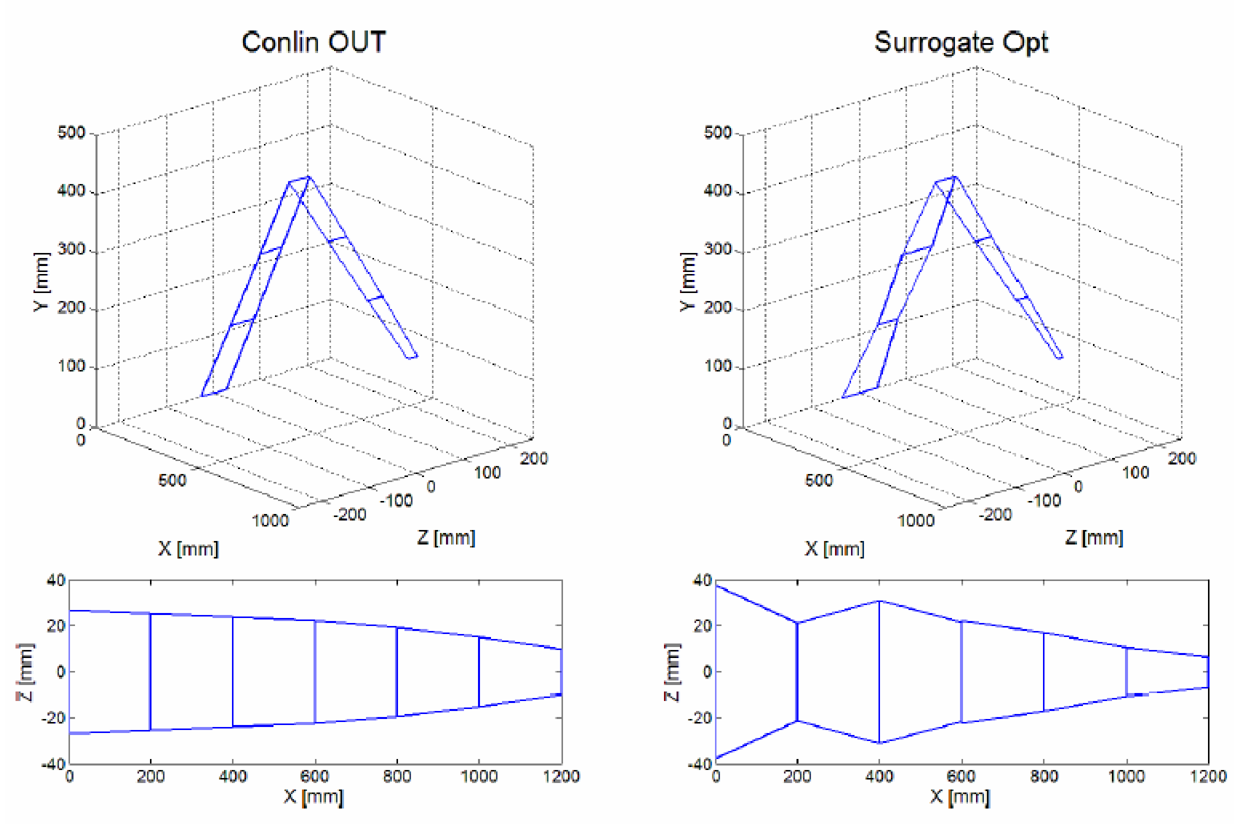
Constraint at each time step – feasible starting point



Constraint max deviation – unfeasible starting point

SHAPE OPTIMIZATION OF ROBOT COMPONENTS

- Comparison between: CONLIN, SURROGATE+GA



Constraint at each time step – feasible starting point

CONCLUSIONS

CONCLUSIONS

- Optimization of structural components can be carried out in the framework of flexible multibody simulations
  - Dynamic coupling between large overall rigid-body motions and deformations
  - Single dynamic analysis instead of a patchwork of (empirical) static analyses
  - Design constraints defined with respect to the actual dynamic problem
- Lack of past experiences in
  - Formulation of optimization problem
  - Solution strategies

CONCLUSIONS

- Design problem is complex: naïve implementations do generally not work!

- Trajectory following
  - Time step deviation constraint are very difficult but may work when starting from feasible starting point
  - Global max constraints are non smooth → GA + surrogate
  - Average deviation are the easiest ones but what is the link to local deviations?

- Solvers
  - Gradient-based solvers can work efficiently when starting from feasible design
  - GA give poor results
  - Surrogate models + GA are reliable and robust

PERSPECTIVES

- Mixed formulations including global (average) constraints and time step constraints should be the best compromise

- Revisiting solution algorithms for dynamic problems!
  - Structural approximations: local / global: trust regions?
  - Reliability and robustness when starting from unfeasible design points

- Other design criteria for time domain analysis of dynamic systems:
  - ISE, ISA
  - Rise time
  - Controller and actuator activity…

THANK YOU VERY MUCH FOR YOUR ATTENTION

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