

## Microbial bioprocesses : current states and future prospects

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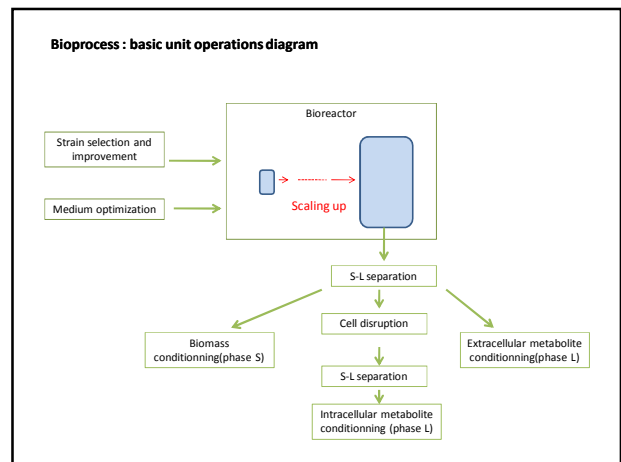
### Content

(fundamental aspects and applications, estimated duration : 7 hrs)

- Section 1 : introduction
- Section 2 : Microbial physiology in process conditions
- Section 3 : Basic bioreactor design
- Section 4 : new trends in bioreactor design
- Section 5 : Bioreactor scale-up : chemical engineering or biological issues ?

- Case study 1 : modelling approaches for bioreactor hydrodynamics
- Case study 2 : whole cell biosensors for the detection of mixing imperfections
- Case study 3 : modelling in chemical engineering and life science, two applications with MatLab

## Section I : introduction



Category of Product	Product	Typical organism	Market value
Whole cells	Baker's yeast	<i>Saccharomyces cerevisiae</i>	
	Lactic acid bacteria	Lactic acid bacteria	
	Single cell protein	Methylotrophic bacteria	
Primary metabolites	Beer, wine	<i>Saccharomyces cerevisiae</i>	
	Ethanol	<i>carlsbergensis</i>	12 billion US\$
	Lactic acid	<i>Saccharomyces cerevisiae</i>	
		<i>Zygomonas mobilis</i>	200 million US\$
		Lactic acid bacteria	
		<i>Rhizopus oryzae</i>	1.5 billion US\$
Secondary metabolites	Citric acid	<i>Aspergillus niger</i>	1 billion US\$
	Glutamate	<i>Corynebacterium glutamicum</i>	500 million US\$
	Lysine	<i>Corynebacterium glutamicum</i>	200 million US\$
	Phenylalanine	<i>Escherichia coli</i>	
	Penicillins	<i>Penicillium chrysogenum</i>	4 billion US\$
	Cephalosporins	<i>Acremonium chrysogenum</i>	11 billion US\$
Recombinant proteins	Streptomycines	<i>Streptomyces clavuligerus</i>	
	Statins	<i>Aspergillus terreus</i>	9 billion US\$
	Taxol	Plant cells	1 billion US\$
	Insulin	<i>Saccharomyces cerevisiae</i>	3 billion US\$
	tPA	<i>Escherichia coli</i>	
	Erythropoietin	Chinese Hamster Ovary cells	1 billion US\$
Enzymes	Human growth hormone	Chinese Hamster Ovary cells	3.6 billion US\$
	Interferons	<i>Escherichia coli</i>	1 billion US\$
	Vaccines	<i>Escherichia coli</i>	2 billion US\$
	Monoclonal antibodies	Bacteria and yeast	
	Detergent enzymes	<i>Bacilli</i> , <i>Aspergilli</i>	700 million US\$
	Starch industry	<i>Bacilli</i> , <i>Aspergilli</i>	600 million US\$
Polymers	Clymosin	<i>Bacilli</i> , <i>Aspergilli</i>	200 million US\$
	Xanthan gum	<i>Aspergilli</i>	
	Polyhydroxyalkanoates	<i>Xanthomonas campestris</i>	400 million US\$
DNA	Vaccines	<i>Alcaligenes eutrophus</i>	
		<i>Escherichia coli</i>	

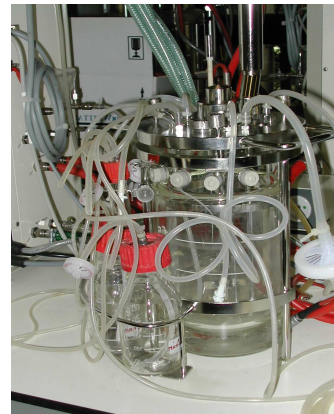


Flasks



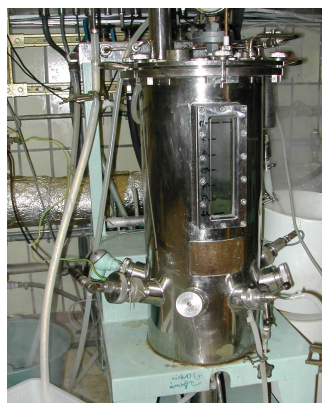
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2L bioreactor



8

20 L bioreactor



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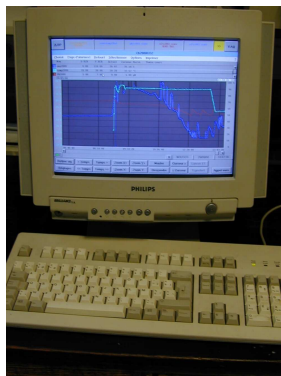
2000 L bioreactor



500 and 2000 L bioreactors

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Control system

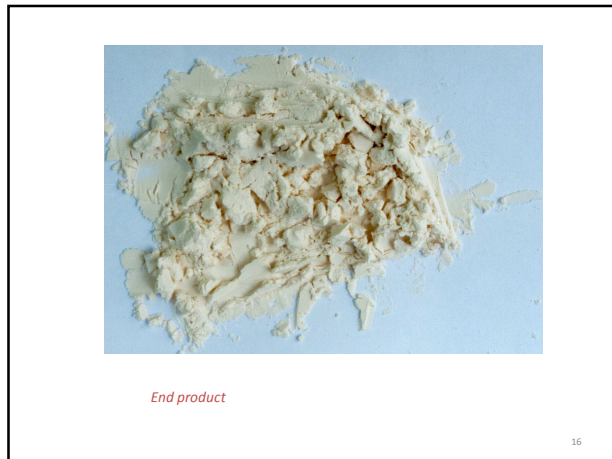


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Centrifugation



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**Section 2 : Microbial physiology in process conditions**

- Optimisation of microbial growth and product formation
- Importance of response and adaptation to stress conditions

**2.1. Microbial growth : basic techniques for the quantification of biomass**

- Plate count
  -
- Microscopy (direct count)
  -
- Dry matter

-Spectrophotometer

(1) (2)

- Indirect parameters extracted from sensors (direct parameter now possible)

2.2. Interface between cellular components and the extracellular environment

GRAM POSITIVE

GRAM NEGATIVE

EUBACTERIAL LIPID

ARCHAEBACTERIAL LIPIDS

Gram Positive

Gram Negative

Peptidoglycan

Teichoic acid

Lipoteichoic acid

Outer membrane layer

Periplasmic space

Cell membrane

Cell membrane

Peptidoglycan (thin layer)

Lipopolysaccharides

Porins

Phospholipids

Membrane protein

Lipoproteins

Membrane protein

Typical growth curve :

(A)

(B)

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Number of cells	1	2	4	8	16	32
Number of generations	1	2	3	4	5	
Exponential Value	2 <sup>1</sup>	2 <sup>2</sup>	2 <sup>3</sup>	2 <sup>4</sup>	2 <sup>5</sup>	

(a)

(b)

Traditional approach : growth curve

(A)

(B)

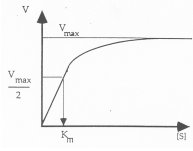
$$\frac{dX}{dt} = \mu \cdot X$$

$\mu$  : specific growth rate (h<sup>-1</sup>)

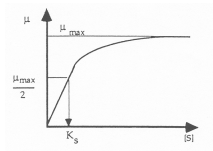


**Traditional approach : Monod**

Micro organisms are considered as a « pool of enzymes »  
Theory of a limiting substrate → Monod



Michaelis-Menten



Monod

$$\mu = \mu_{max} \frac{S}{K_s + S}$$

$$\frac{dX}{dt} = \mu_{max} \frac{S \cdot X}{K_s + S}$$

**First problem**

$\mu_{max}$  and  $K_s$  varies with pH, T°, nature of the substrate, exposure to stress,...

Can be resolved by one of the numerous Monod modified expression

Monod (1942)	$\mu(S) = \frac{\mu_{max} S}{K_s + S}$
Tessier (1942)	$\mu(S) = \mu_{max} \left( 1 - \exp\left(-\frac{S}{K_s}\right) \right)$
Moser (1958)	$\mu(S) = \frac{\mu_{max} S^2}{K_s + S^2}, \lambda > 0$
Contois (1959)	$\mu(S, C) = \frac{\mu_{max} S}{K_s + C + S}$
Powell (1967)	$\mu(S) = \frac{\mu_{max}}{2K_s} (K_s + S - \sqrt{K_s^2 + S^2 - 4K_s S})$
Peringer, Blachere, Corrieu and Laine (1972)	$\mu(S, A) = \mu_{max} \frac{S}{K_s + S} \left( \frac{A}{K_s + A} + \frac{1}{1 + K_s A} \right) + \mu_{max}^2$ with $2\mu_{max}^2 + \mu_{max}^2 = \mu_{max}$
Jackson and Edwards (1975)	$\mu(S, H^+) = \frac{\mu_{max} S}{\left(1 + \frac{K_s}{H^+} + \frac{H^+}{K_1}\right) (K_s + S + S^2/K_1 + K_s H^+)}$
Olsson (1976)	$\mu(S, A) = \mu_{max} \frac{S A}{(K_s + S)(K_s + A)}$
Dourado and Calvet (1983)	$\mu(S, P) = \mu_{max} \frac{S}{(K_s + S + S^2/K_1)(K_s + P)} \left(1 - \frac{P}{P_0}\right)$
Williams, Yousefpour and Swainik (1984)	$\mu(S, A, P) = \left(\frac{K_s S}{K_s + S} + \frac{K_s P}{K_s + P}\right) \left(\frac{A}{K_s + A} + K_s A - K_s\right)$

S = substrate concentration, C = cell concentration, P = product concentration, A = dissolved oxygen concentration, H<sup>+</sup> = hydrogen ion concentration, K<sub>1</sub> = constant, μ = specific growth rate, μ<sub>max</sub> = maximum specific growth rate.

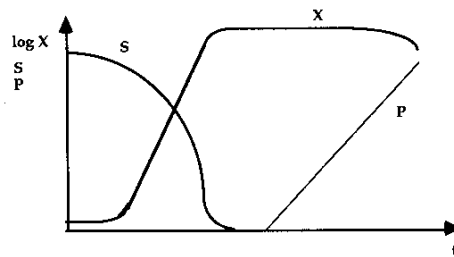
**Second problem : substrate depletion in batch system**

$\mu_{max}$  and  $K_s$  varies with substrate concentration (simple Monod expression)  
BUT ALSO

Substrate concentration can induce a shift from a metabolic pathway to another (homofermentative shift to a heterofermentative pathway)

Solution : fed-batch system (if vessel is considered as perfectly mixed) or structured kinetic model

**Growth, substrate consumption and metabolite production**

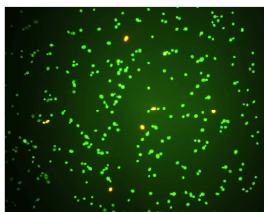


**Different modelling alternatives for microbial growth :**

-The simplest way to express microbial growth : Monod type equation (saturation)

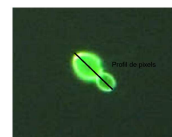
- Structured modelling : take into account the internal dynamics of the system to be studied

- Segregated modelling : take into account the heterogeneity of the microbial population

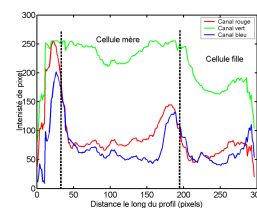


A

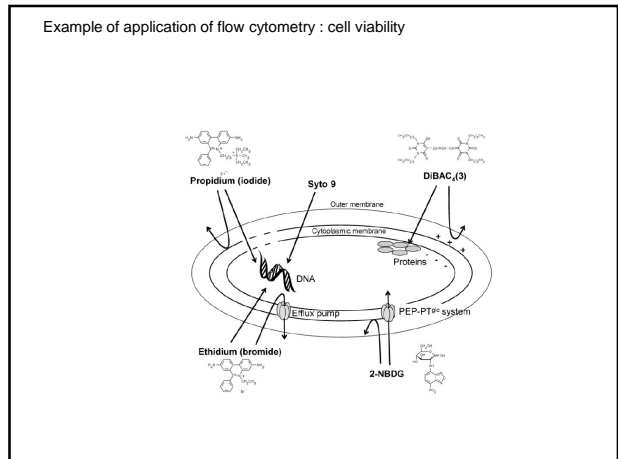
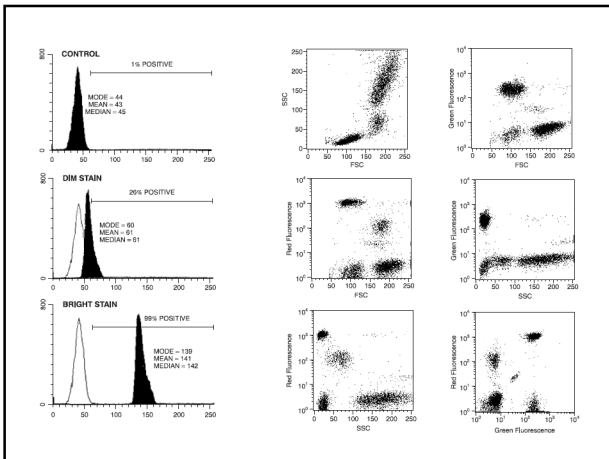
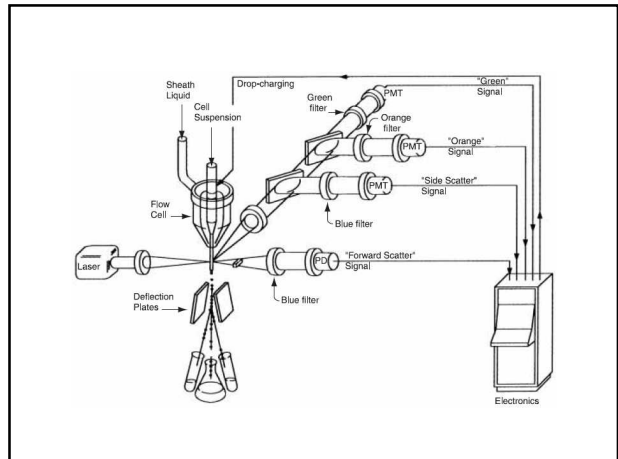
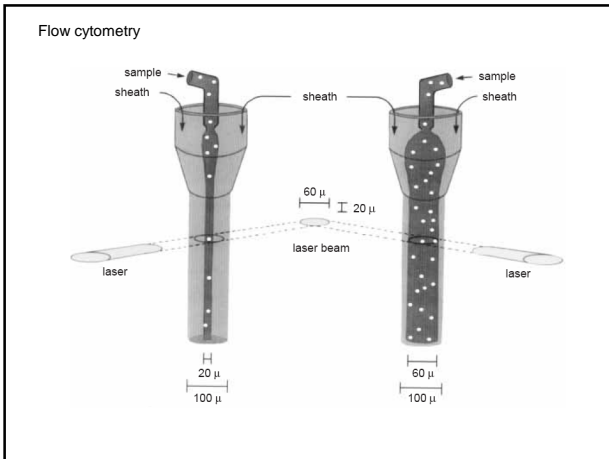
**Image analysis**



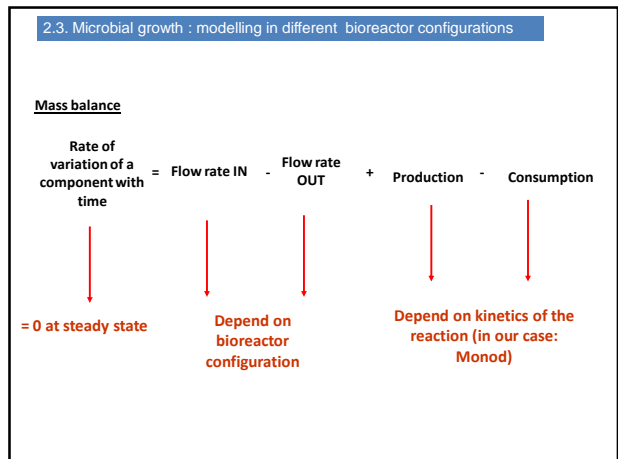
A



B



PROBE	DYE FLUORESCENCE SPECTRA	EXCITATION	EMISSION
	λ (nm)	λ (nm)	λ (nm)
FLUORESCEN (FIC)	350-450	450-550	500-600
RHODAMINE (RITC)	350-450	450-550	500-600
PHYCOERYTHRIN (PE)	350-450	450-550	500-600
PE-FITC-BED FANDEM	350-450	450-550	500-600
PE-FITC TANDDEM	350-450	450-550	500-600
PE-FITC	350-450	450-550	500-600
TEXAS RED / SE191	350-450	450-550	500-600
ALLOPHYCOXYANIN (APC)	350-450	450-550	500-600
AMCA	350-450	450-550	500-600
CASCADE BLUE	350-450	450-550	500-600
PROPIDIUM IODIDE (PI)	350-450	450-550	500-600
TOTO-1/THIAZOLE ORANGE	350-450	450-550	500-600
ACRIDINE ORANGE (AO)	350-450	450-550	500-600
MITHRAMYCIN	350-450	450-550	500-600
7-ACTINOMYCIN D (7-AAD)	350-450	450-550	500-600
DAPI	350-450	450-550	500-600
TOCOST 30342	350-450	450-550	500-600
PHYCEN Y	350-450	450-550	500-600
DIOCE131	350-450	450-550	500-600
DMC120 / CY3	350-450	450-550	500-600
INDO1 / NO Ca <sup>2+</sup>	350-450	450-550	500-600
INDO1 / Ca <sup>2+</sup>	350-450	450-550	500-600
SOURCE EMISSION			
ARGON ION LASER	350-450	450-550	500-600
KRYPTON ION LASER	350-450	450-550	500-600
HELIUM-NEON LASER	350-450	450-550	500-600
HELIUM-CADMIUM LASER	350-450	450-550	500-600
MERCURY ARC LAMP	350-450	450-550	500-600



Hypothesis:

- Perfectly mixed reactor
- No cellular death
- No metabolite synthesis
- The only limiting substrate is the carbon source
- Dissolved oxygen is not limiting

In these conditions, Monod equation can be applied

$$\mu = \mu_{\max} \frac{S}{S + K_s}$$

V	S	X
□	□	□
□	□	□

V = working volume (m<sup>3</sup>)  
 S = substrate concentration (g/l)  
 X = biomass concentration (g/l)

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The rate of biomass variation with time (g/l.h) :

$$\frac{dX}{dt} = r_x = \mu \cdot X$$

$$r_x = X \cdot \mu_{\max} \frac{S}{S + K_s}$$

X : biomass concentration (g de cellules/litre)  
 S : substrate concentration (g/l)  
 $\mu_{\max}$  : maximal growth rate (h<sup>-1</sup>)  
 $K_s$  : substrate affinity constant (g/l)

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For the substrate:

$$r_s = - \frac{dS}{dt} = - \frac{r_x}{Y_{x/s}} + m_s \cdot X$$

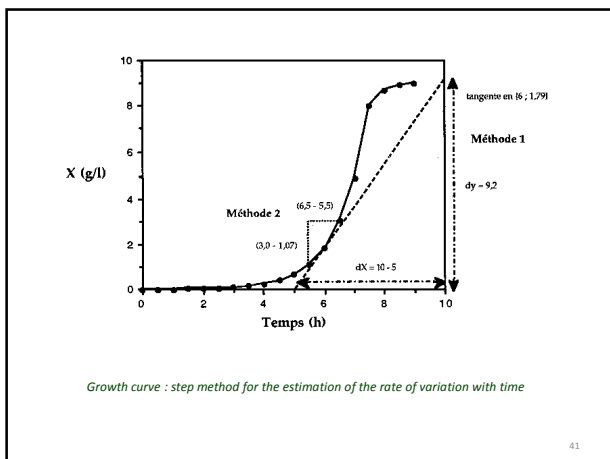
$r_s$  : substrate consumption rate (g/l.h)  
 $Y_{x/s}$  : yield coefficient (g biomass per g substrate)  
 $m_s$  : maintenance constant (g de substrat / g de cellules . H)

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*Experimental results : batch culture of Bacillus subtilis*

Durée (h)	$r_x$ (g/l.h)	$r_s$ (g/l.h)
0	-	-
0,5	0,003	0
1	0,008	0,02
1,5	0,011	0,04
2	0,019	0,04
2,5	0,044	0,09
3	0,083	0,20
3,5	0,142	0,27
4	0,242	0,54
4,5	0,412	0,96
5	0,698	1,43
5,5	1,158	2,34
6	1,940	4,44
6,5	3,089	7,41
7	5,00	11,10
7,5	3,811	8,89
8	0,87	0,087
8,5	0,27	0,012
9	-	-

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*Biomass and substrate variation rates : step method*

Durée (h)	$r_x$ (g/l.h)	$r_s$ (g/l.h)
0	-	-
0,5	0,003	0
1	0,008	0,02
1,5	0,011	0,04
2	0,019	0,04
2,5	0,044	0,09
3	0,083	0,20
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6,5	3,089	7,41
7	5,00	11,10
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8	0,87	0,087
8,5	0,27	0,012
9	-	-

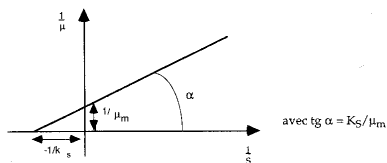
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Computation of  $\mu_m$  and  $K_s$

$$\mu = \mu_m \cdot \frac{S}{K_s + S}$$

$$\frac{1}{\mu} = \frac{1}{\mu_m} \cdot \frac{K_s + S}{S}$$

$$\frac{1}{\mu} = \frac{K_s}{\mu_m} \cdot \frac{1}{S} + \frac{1}{\mu_m}$$

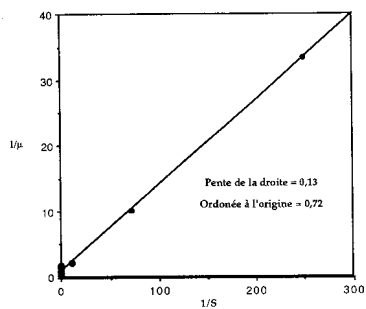


Lineweaver-Burke

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Durée (h)	1/S (g/l)	$\mu$ (h <sup>-1</sup> )	1/ $\mu$ (h)
0	0,05		
0,5	0,05	0,15	6,67
1	0,05	0,35	2,86
1,5	0,05	0,39	2,56
2	0,05	0,56	1,79
2,5	0,05	0,94	1,06
3	0,05	1,06	0,94
3,5	0,051	1,09	0,91
4	0,051	1,09	0,91
4,5	0,052	1,11	0,90
5	0,054	1,10	0,91
5,5	0,056	1,08	0,93
6	0,061	1,08	0,93
6,5	0,075	1,03	0,97
7	0,112	1,02	0,98
7,5	10,90	0,48	2,08
8	71,20	0,10	10
8,5	249,0	0,03	33,33
9	626,5		

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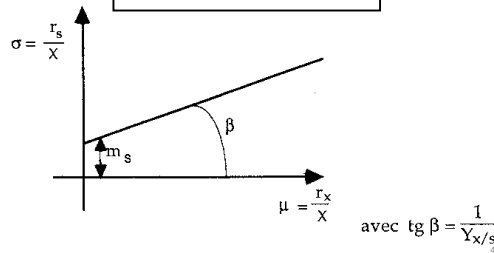


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$$r_s = \frac{r_x}{Y_{x/s}} + X \cdot m_s$$

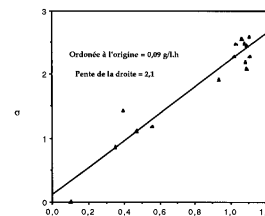
$$\frac{r_s}{X} = \frac{1}{Y_{x/s}} \cdot \frac{r_x}{X} + m_s$$

$$\frac{r_s}{X} = \frac{1}{Y_{x/s}} \cdot \mu + m_s \quad \sigma = \frac{r_s}{X}$$



Durée (h)	$\mu = r_x/X$ (h <sup>-1</sup> )	$\sigma = r_s/X$ (g de glucose / g de cellules . h)
0	-	-
0,5	0,15	-
1	0,35	-
1,5	0,39	0,87
2	0,56	1,43
2,5	0,94	1,18
3	1,06	1,91
3,5	1,09	2,56
4	1,09	2,08
4,5	1,11	2,45
5	1,10	2,59
5,5	1,08	2,27
6	1,08	2,19
6,5	1,03	2,48
7	1,02	2,47
7,5	0,48	1,11
8	0,10	0,01
8,5	0,03	-
9	-	-

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$$\frac{1}{Y_{x/s}} = \text{slope} = 2,1$$

$$Y_{x/s} = 0,48 \text{ g of cells / g de glucose}$$

$$M_s = y(x=0) = 0,09 \text{ g de glucose / g of cells.h}$$

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**Productivity associated with batch reactors:**

!!! Cultivation timelength + time associated with industrial operations

**Total time is calculated by :**

$$t_{total} = T_{exp} + t_0 = \frac{1}{\mu_{max}} \ln \frac{X_f}{X_0} + t_0 \quad \text{WITH} \quad \ln \frac{X_f}{X_0} = \mu_{max} \cdot T_{exp}$$

$$t_0 = t_V + t_N + t_R + t_S + T_L + T_F$$

**Amount of biomass produced :**  $X_f - X_0 = Y_{X/S} \cdot S_0$

**Productivity (batch) :**

$$Pr oductivity(batch) = \frac{\mu_{max} \cdot Y_{X/S} \cdot S_0}{\ln \frac{X_f}{X_0} + \mu_{max} \cdot t_0}$$

**Fed-batch culture : mass balance**

**Monod hypothesis:**

- Perfectly mixed reactor
- No cellular death
- The only limiting substrate is the carbon source
- Dissolved oxygen is provided in excess

**First case : metabolites synthesis**

1. **Growth period: equations identical to those used for batch**

$$\frac{dX}{dt} = r_x = \mu_{max} \frac{S \cdot X}{K_s + S} \quad \frac{dS}{dt} = -r_s = -\frac{r_x}{Y_{x/s}} - m_s \cdot X \quad \frac{dV}{dt} = 0$$

2. **Metabolite synthesis period (fed-batch) :**

$\frac{dV}{dt} > 0$

Q, S<sub>a</sub>

S, V, P, X

→ **BIOMASS :** cellular concentration is kept constant and  $r_x = 0$ .  
Variation at the level of the biomass concentration can only be attributed to dilution effect :

$$\frac{dX}{dt} = -\frac{dV}{dt} \frac{X}{V}$$

→ **SUBSTRATE :**  $\frac{dS}{dt} = -r_s - \frac{dV}{dt} \frac{S}{V} + Q \frac{S_a}{V}$

**Hypothesis : added substrate is immediately consumed (S = 0)**

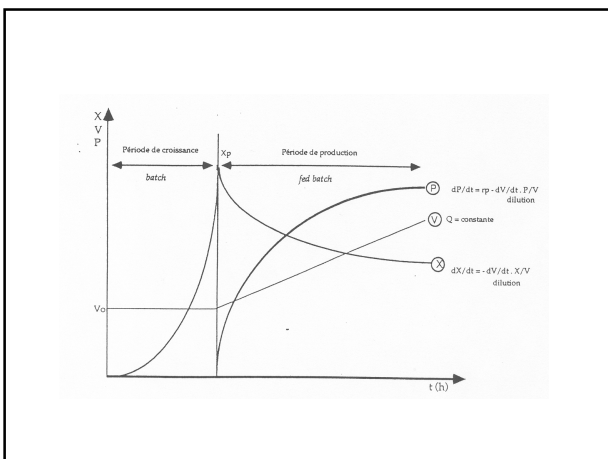
$$\frac{dS}{dt} = -r_s + Q \frac{S_a}{V} = 0$$

$$r_s = Q \cdot \frac{S_a}{V} = m_s \cdot X + \frac{r_p}{Y_{p/s}}$$

→ **METABOLITE :**  $\frac{dP}{dt} = r_p - \frac{dV}{dt} \frac{P}{V}$

$$r_p = \chi \cdot X$$

→ **VOLUME :**  $\frac{dV}{dt} = Q$



**2<sup>nd</sup> case : biomass production.**

1. **Growth period: see batch .**

2. **Fed-batch period:**

→ **BIOMASS :**  $\frac{dX}{dt} = r_x - \frac{dV}{dt} \frac{X}{V} = \mu \cdot X - \frac{Q}{V} \cdot X$  with  $\mu = cst$

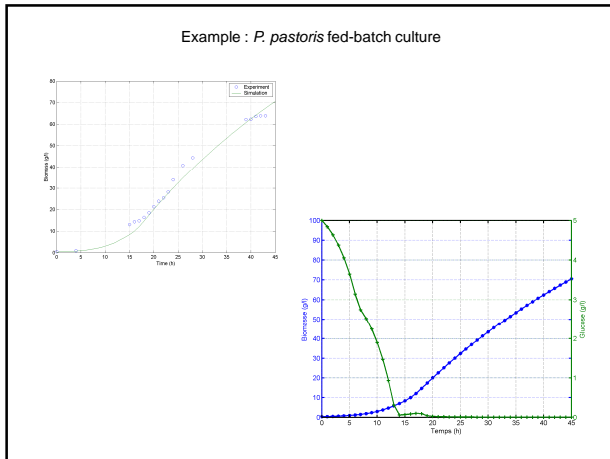
→ **SUBSTRATE :**  $\frac{dS}{dt} = -r_s - \frac{dV}{dt} \frac{S}{V} + \frac{dV}{dt} \frac{S_a}{V}$

$$\frac{dS}{dt} = -r_s - Q \frac{S}{V} + Q \frac{S_a}{V} = -r_s + \frac{Q}{V} (S_a - S)$$

$$r_s = \frac{Q}{V} (S_a - S) = \frac{r_x}{Y_{x/s}} + m_s \cdot X \quad \text{If } S = cste \text{ and } dS/dt = 0$$

→ **VOLUME :**  $\frac{dV}{dt} = Q$  with Q being comprised between Q<sub>min</sub> et Q<sub>max</sub>

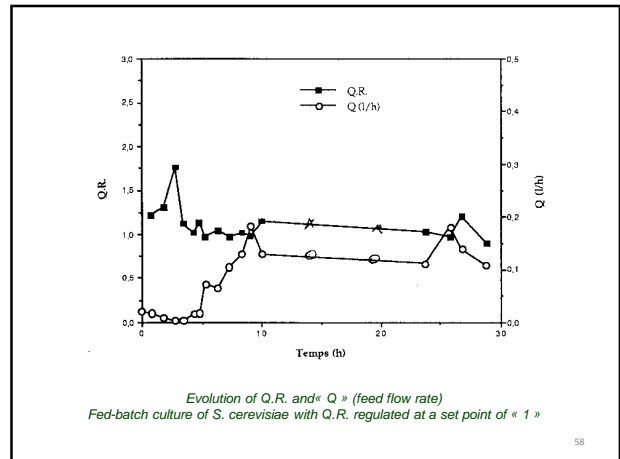
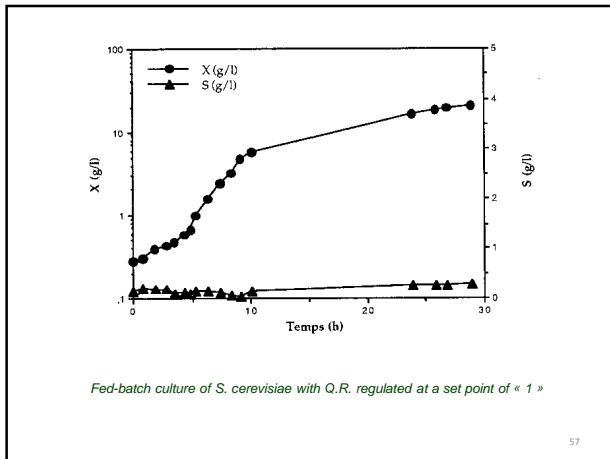




Fed-batch culture of *S. cerevisiae* with Q.R. regulated at a set point of « 1 »  
( $S_a = 200\text{g/l}$ ;  $V_0 = 10$  liters;  $V_f = 12,88$  liters)

Durées (h)	X (g/l)	S (g/l)	Q.R.	Q (l/h)	V (litre)	$S_a \cdot (V-V_0)$ (g)*
0	0,28	0,15	-	0,02	10,00	1,6
0,83	0,3	0,21	1,22	0,016	10,013	2,6
1,83	0,39	0,19	1,3	0,009	10,022	4,1
2,83	0,42	0,18	1,76	0,003	10,025	5
3,5	0,47	0,11	1,12	0,003	10,027	5,4
4,33	0,58	0,12	1,02	0,016	10,040	8
4,83	0,66	0,11	1,13	0,016	10,048	9,6
5,33	0,97	0,14	0,97	0,072	10,084	16,8
6,33	1,52	0,14	1,04	0,063	10,147	29,4
7,33	2,42	0,13	0,97	0,104	10,251	50,2
8,33	3,12	0,06	1,02	0,129	10,38	76
9,08	4,68	0,04	0,98	0,181	10,516	103,2
10,08	5,66	0,14	1,15	0,128	10,644	128,8
23,83	16,17	0,27	1,03	0,11	12,164	432,8
25,83	18,33	0,24	0,97	0,18	12,527	505,4
26,83	19,11	0,25	1,2	0,139	12,666	533,2
28,83	20,28	0,28	0,9	0,107	12,88	576,0

\*  $S_a \cdot (V-V_0)$  = Apport cumulé en glucose (g)



**Control of glucose effect positive strains**

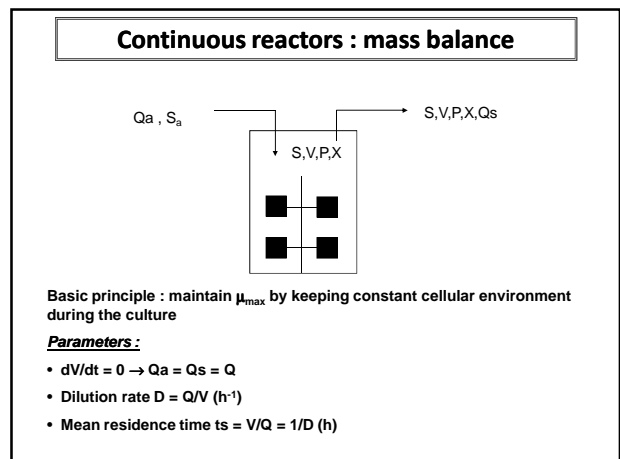
Software sensor: respiratory ratio

- $QR > 1 \rightarrow$  alcohol production (overflow)
- $QR = 1 \rightarrow$  growth
- $QR < 1 \rightarrow$  alcohol reassimilation

QR can be measured by gas balance analysis (derived method can also be used to monitor  $K_p a$ )

Other control procedures:

- Ethanol, glucose sensors
- pHstat, DOstat
- Exponential feeding algorithm (if kinetics parameters are well known)



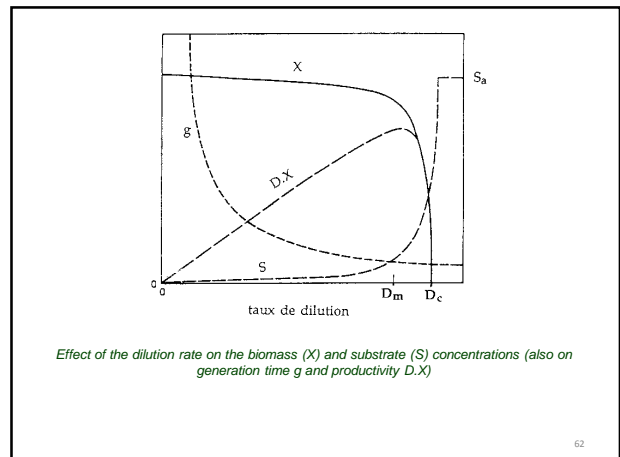
→ BIOMASS :  $\frac{dX}{dt} = \mu \cdot X - D \cdot X = X(\mu - D)$

→ SUBSTRATE :  $\frac{dS}{dt} = D \cdot S_a - R_s - D \cdot S = D(S_a - S) - R_s$

→ METABOLITE :  $\frac{dP}{dt} = r_p - D \cdot P$

**dX/dt, dS/dt and dP/dt depend on the value of D:**

- Critical dilution rate  $D_c$
- Maximal dilution rate  $D_m$



**1st CASE :  $D > D_c$**   
 Bioreactor wash out →  $dX/dt < 0$

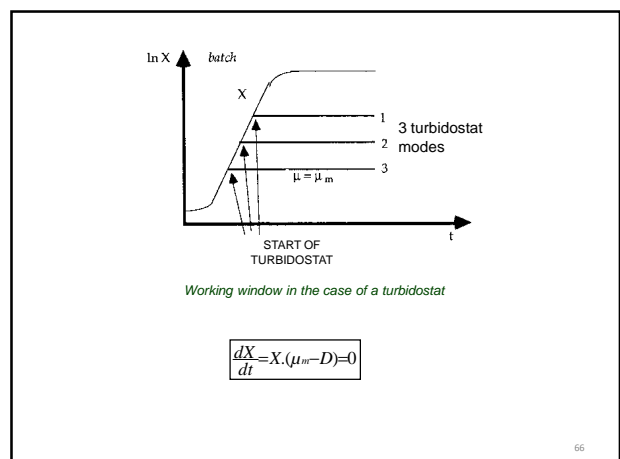
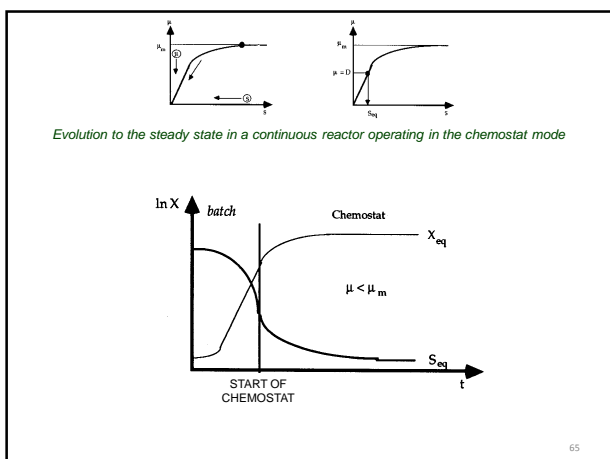
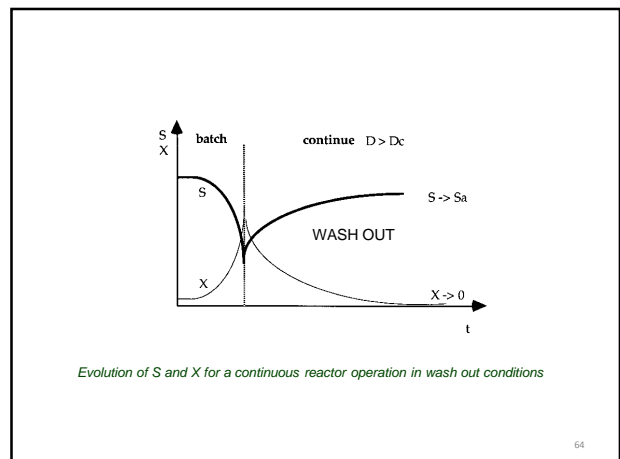
**2nd CASE :  $0 < D < D_c$**   
 Evolution toward a steady state :

$X = X_{eq} \rightarrow r_x = \mu \cdot X = D \cdot X$

$S = S_{eq} \rightarrow r_s = D \cdot (S_a - S) \rightarrow$  CHEMOSTAT

$P = P_{eq} \rightarrow r_p = D \cdot P$

**3rd CASE :  $D = D_c = \mu_{max}$**   
 $dX/dt = X \cdot (\mu_{max} - D) = 0 \rightarrow$  TURBIDOSTAT



**Chemostat : modelling**

X and S mass balance equations with Monod kinetics. At steady state,  $\mu = D$  :

$$S_{eq} = K_s \frac{D}{\mu_{max} - D}$$

$$X_{eq} = \frac{Y_{x/s} D}{D + m_s Y_{x/s}} (S_0 - S_{eq})$$

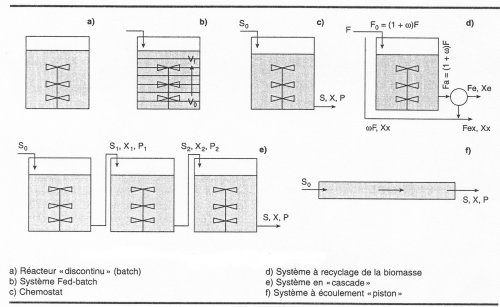
If we focus our attention on generation time and productivity :

$$g = \frac{\ln 2}{\mu_{eq}} = \frac{\ln 2}{D}$$

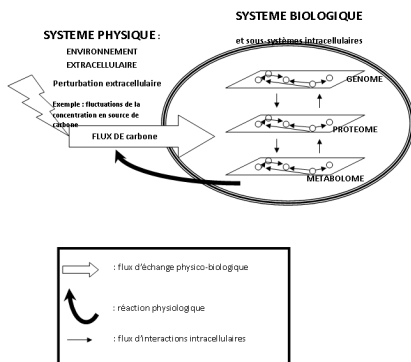
$$productivité = X_{eq} D = Y_{x/s} (S_0 - S_{eq}) D$$

$$D = D_m = \mu_m \left[ 1 - \sqrt{\frac{K_s}{K_s + S_0}} \right]$$

**Bioreactor operating mode : synopsis**



**2.4. Impact of process-related stresses on microbial physiology**



**Process-related stresses:**

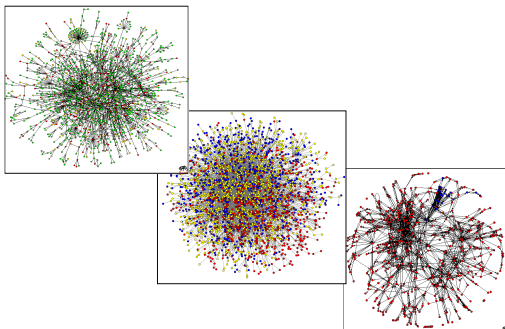
- Thermal shock
- Hypoxia
- pH shock
- Carbon starvation
- Carbon excess
- Nitrogen starvation
- Choc osmotique (*fed-batch*)
- Haute densité cellulaire (*fed-batch*)
- Turbopobiosis

**Physiological impact:**

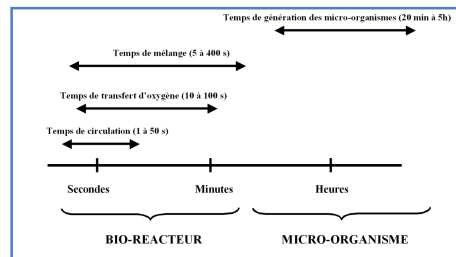
- Short-term : metabolic shift
  - Long-term : gene induction/repression
- Depending also on the intensity and the frequency of stress

**System biology**

Les différents composants des systèmes (métabolome, protéome, fluxome) sont organisés en réseaux  
 Existence d'interactions complexes au sein de ces différents réseaux

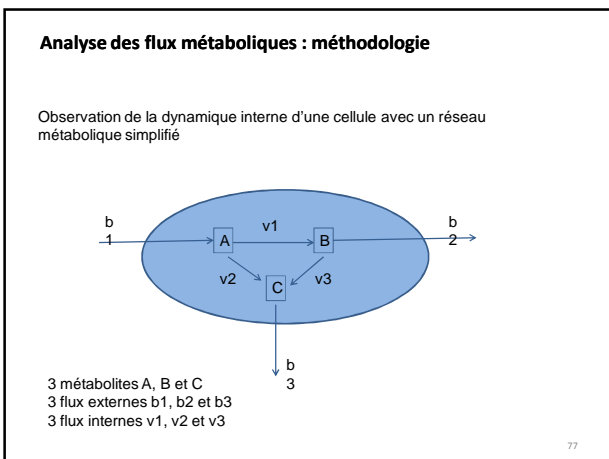
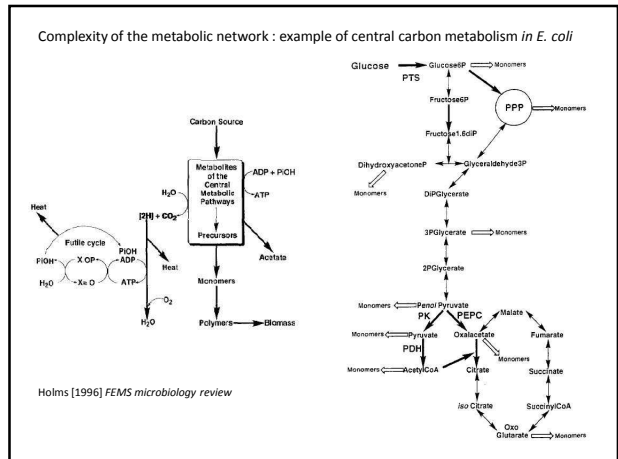
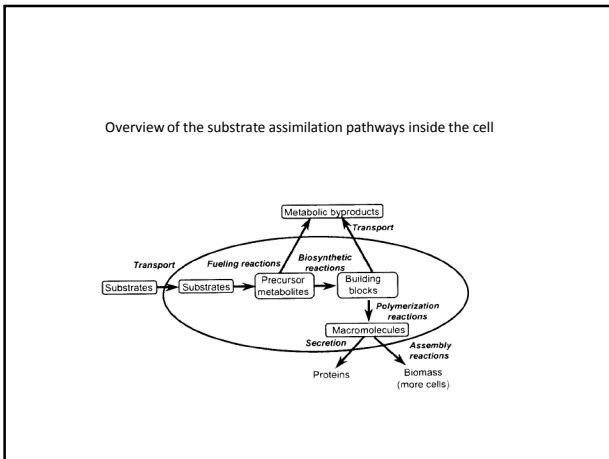
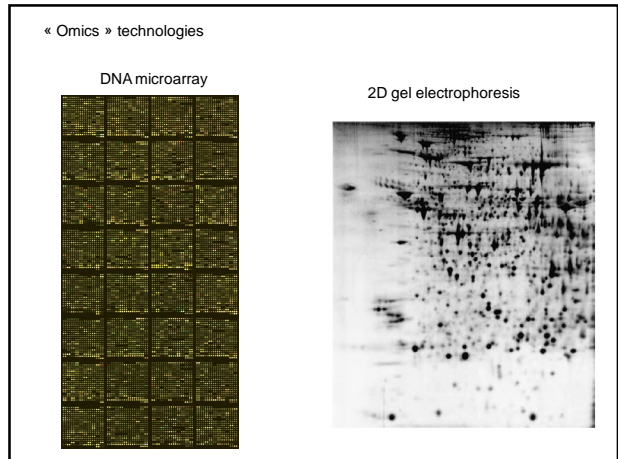
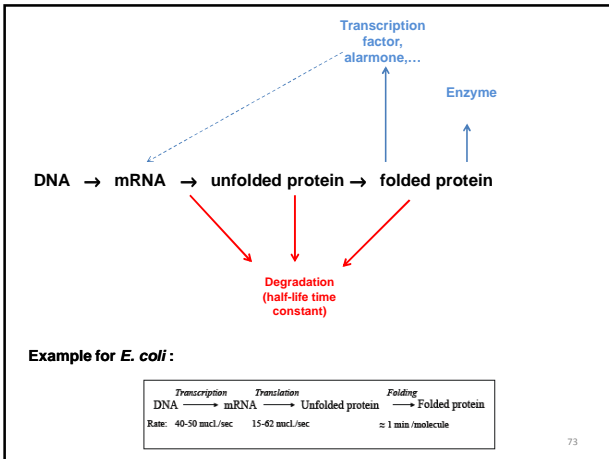


**Etude des temps caractéristiques du procédé :**



Temps caractéristiques pour les réactions enzymatiques du métabolisme : peuvent être inférieurs à la seconde (dépendent de la constante cinétique de réaction)

Temps caractéristiques pour la synthèse des ARN<sub>m</sub> : quelques dizaines de secondes (dépendent de la longueur de l'ARN messenger)



Balance de masse dynamique :

$$\frac{dA}{dt} = -v1 - v2 + b1$$

$$\frac{dB}{dt} = v1 + v3 - b2$$

$$\frac{dC}{dt} = v2 - v3 - b3$$

Sous forme matricielle :

$$\frac{d}{dt} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v1 \\ v2 \\ v3 \\ b1 \\ b2 \\ b3 \end{bmatrix}$$

$$\frac{dX}{dt} = S \cdot v \quad \begin{matrix} S : \text{matrice stoechiométrique} \\ v : \text{vecteur de flux} \end{matrix}$$

Balance de masse à l'équilibre :

$$S \cdot v = 0$$

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Réseau métabolique à l'état stationnaire (turnover des intermédiaires) sous forme matricielle :

$$S \cdot v = 0$$

On a K métabolites et J réactions

La matrice stoechiométrique S a donc les dimensions (K,J) et le vecteur de flux s a la dimension (J,1)

Le modèle revient à un système d'équations linéaires dont le nombre de degré de liberté  $F = J - K$

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3 cas de figure :

- Si  $F = 0$ , le système est déterminé et la solution est unique
- Si  $F < 0$ , le système est sur-déterminé
- Si  $F > 0$ , le système est sous-déterminé et il existe plusieurs solutions

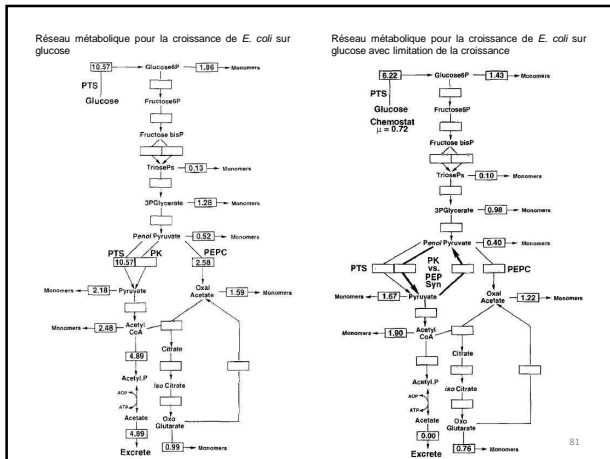
Dans la pratique, pour les réseaux métaboliques les plus complexes, le système est toujours sous-déterminé. On diminue le nombre de degré de liberté en mesurant certains flux. Le système d'équations linéaires s'écrit alors :

$$S \cdot v = S_m \cdot v_m + S_{nm} \cdot v_{nm} = 0$$

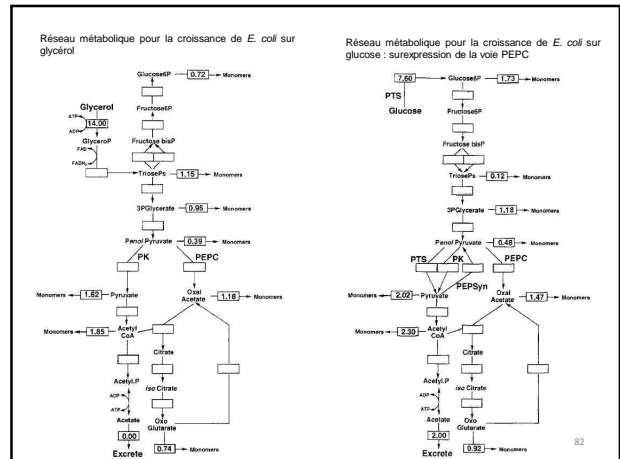
La solution est alors obtenue par :

$$v_{nm} = -(S_{nm})^{-1} \cdot S_m \cdot v_m$$

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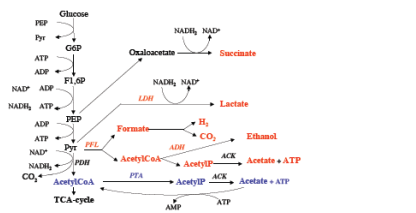


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Bascule métabolique déclenchée par :

- Excès de glucose
- Manque d'oxygène (voie des acides mixtes chez certains procaryotes)

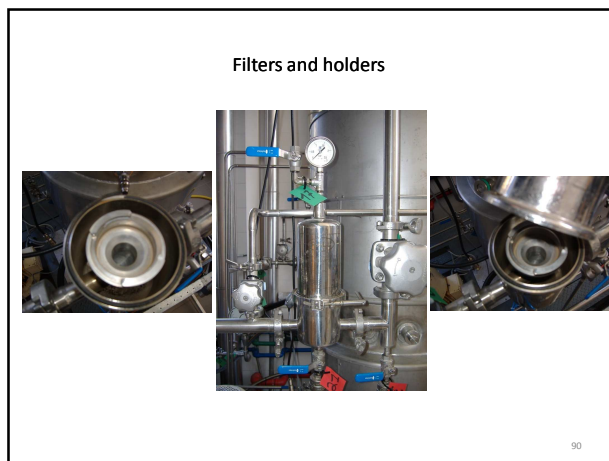
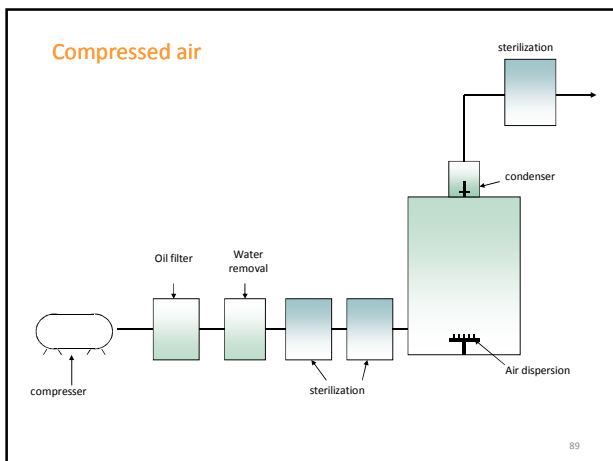
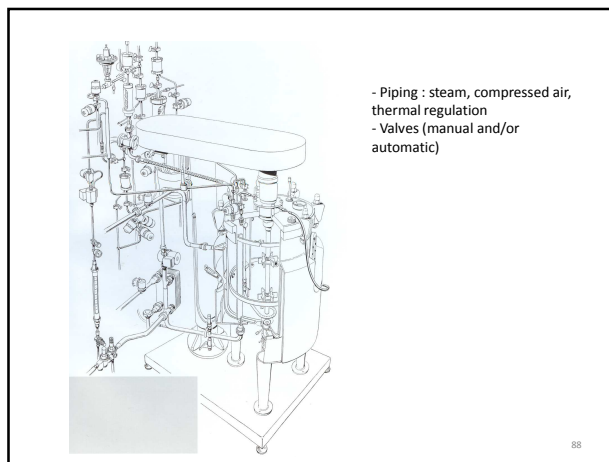
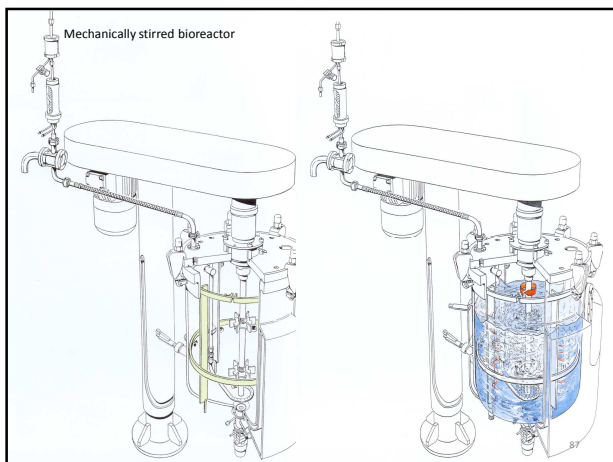
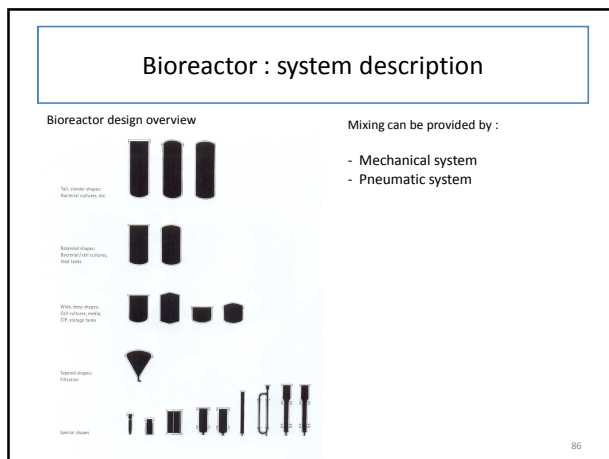
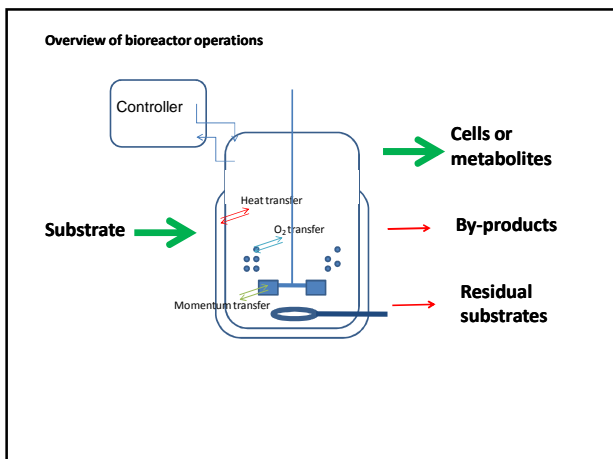
Exemples : *E. coli*



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Section 3 : Basic bioreactor design

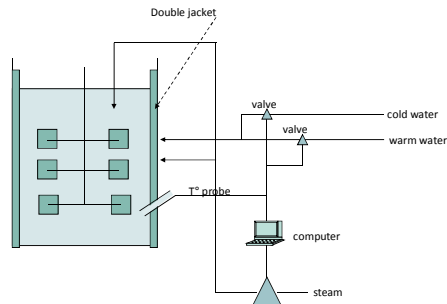




**Air filter**



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*Scheme of a typical temperature regulation loop*

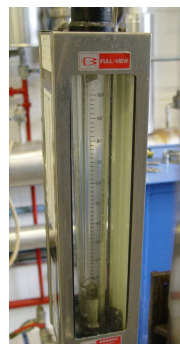
92

**Pressure**

Increase the pressure in the vessel in order to :  
 -Avoid contaminations  
 -Increase oxygen solubility in water (or aqueous media)



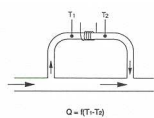
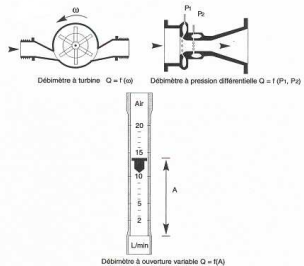
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*Classical flowmeter (ball)*

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**Mass flowmeter**



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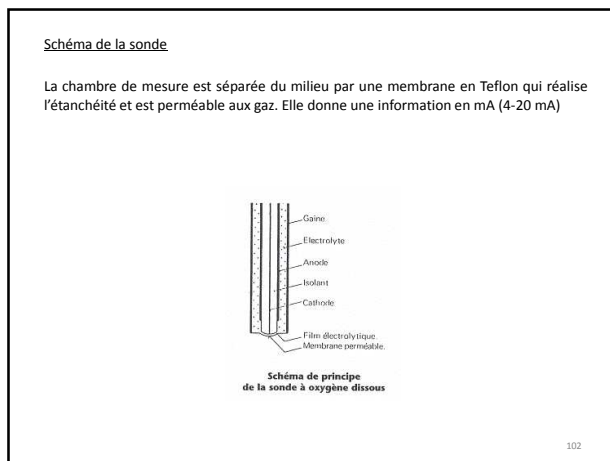
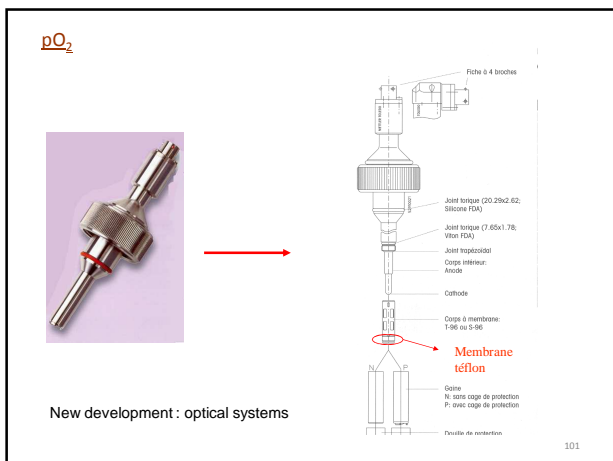
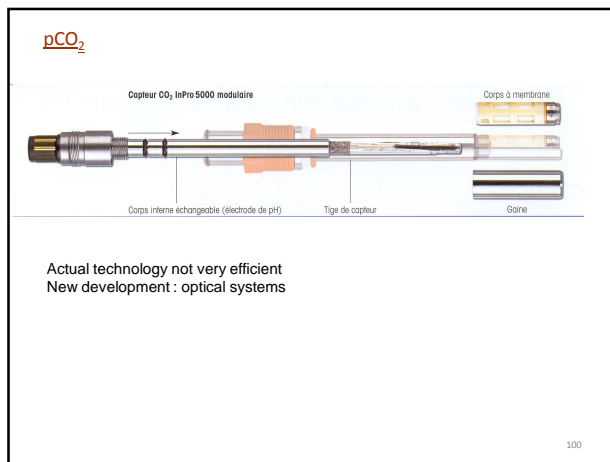
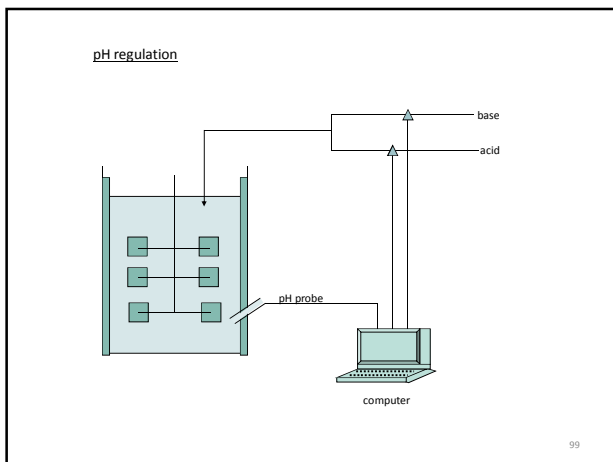
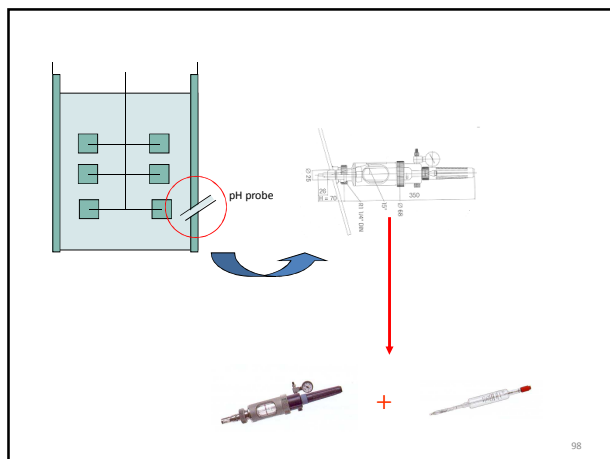
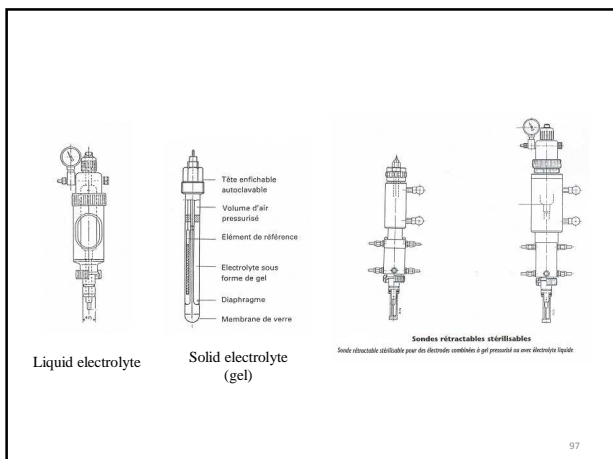
**pH management**

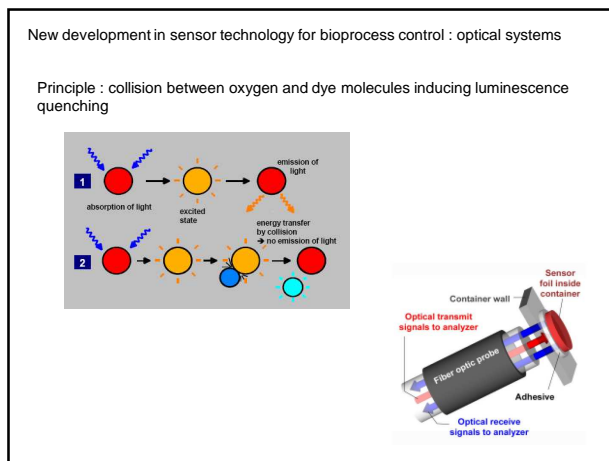
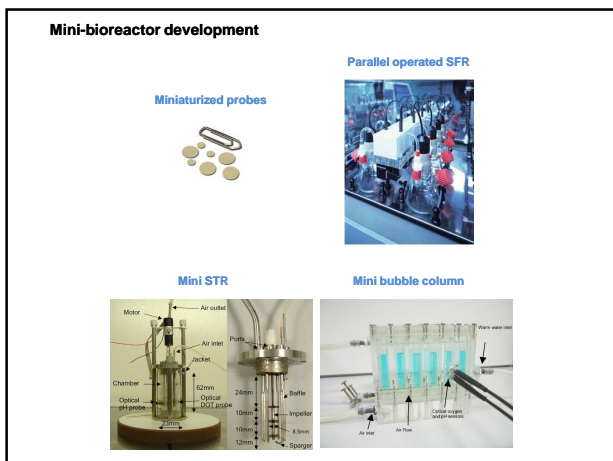
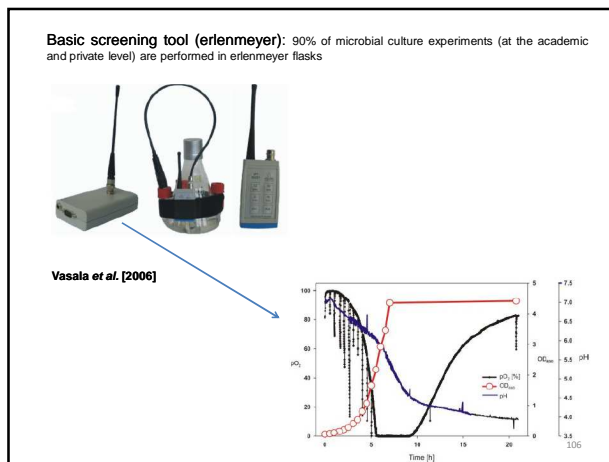
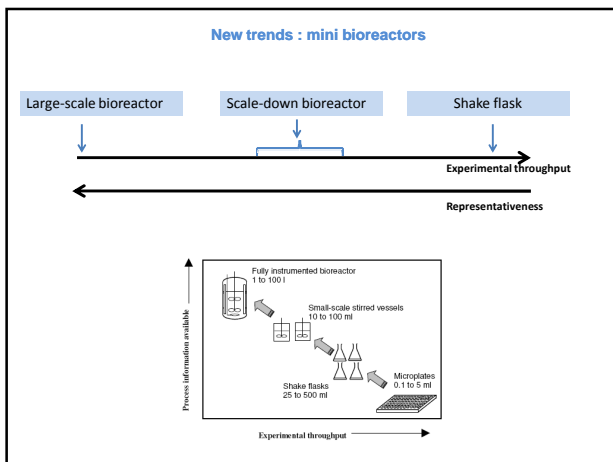
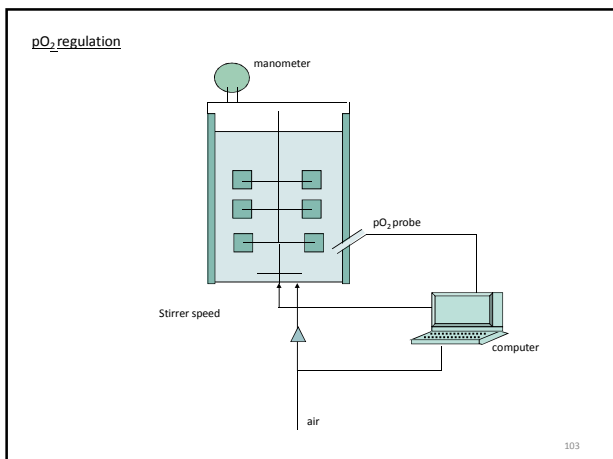
Relation between voltage and pH:  
 $pH = a + bU$   
 Where b depends on  $T^\circ$

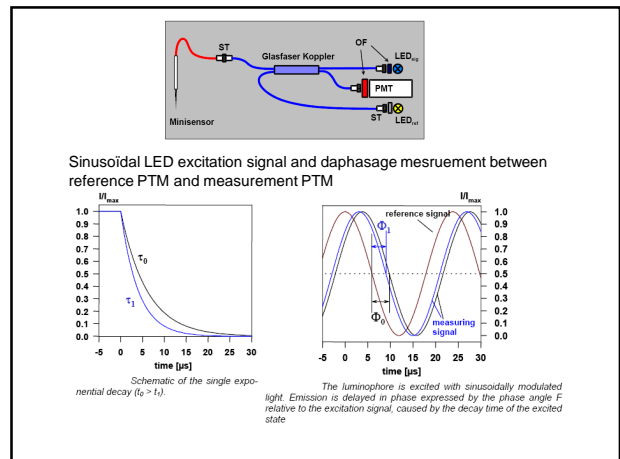
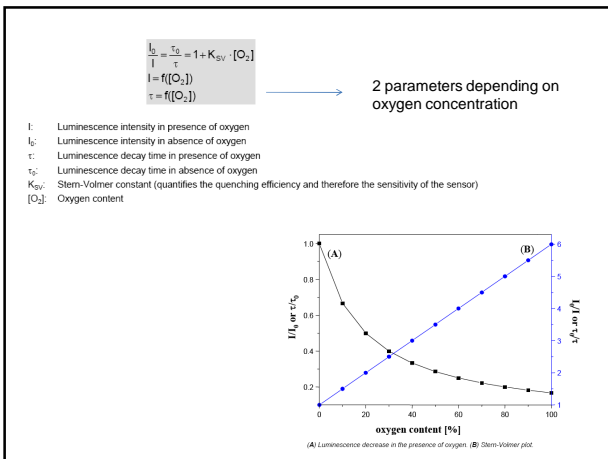


**New development : optical system**

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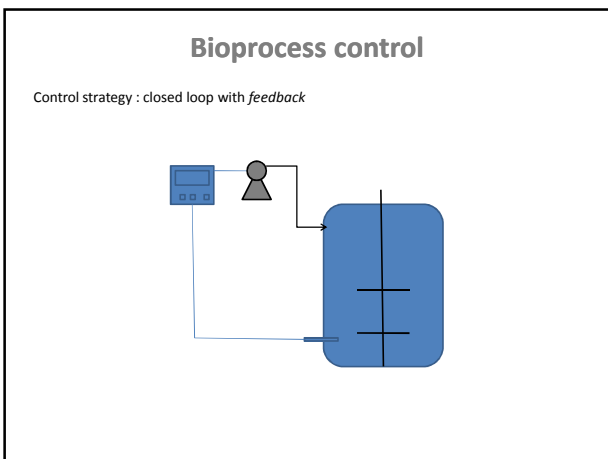
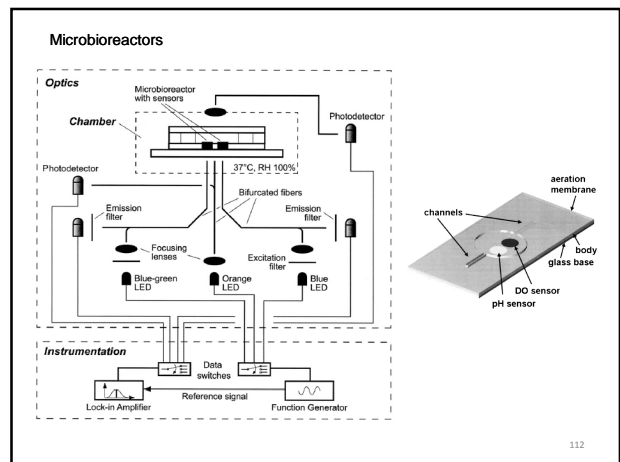






Comparison with polarographic probes:

- No polarization lag time
- No oxygen consumption
- No contamination with other dissolved species
- Non invasive measurement
- No membrane, no electrolyte required
- Insensitive to electrical interferences

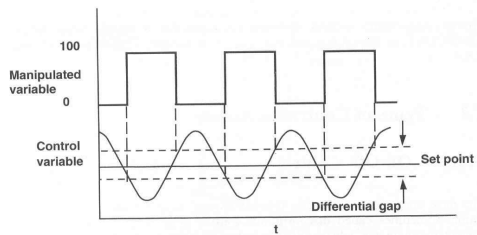


## Control algorithms

- ON-OFF control
- Proportionnel (P) control
- Proportionnel-integral (PI)
- Proportionnel-différentiel
- Proportionnel-integral-différentiel (PID)

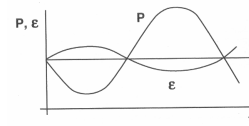


### ON-OFF control



Possibility to introduce a dead zone where controller is never activated  
Simple system, but tends to lead to oscillations of the controlled variable

### Proportionnal control



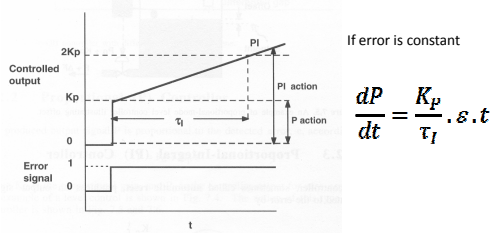
$$P = P_0 + K_p \cdot \varepsilon$$

P : signal output (signal envoyé par le contrôleur)  
ε : error (difference between the output signal, i.e. controller impulse, and the input signal, i.e. the probe signal)  
K<sub>p</sub> : proportionnal gain

### Proportional-integral control

$$P = P_0 + K_p \cdot \varepsilon + \frac{K_p}{\tau_I} \cdot \int_0^t \varepsilon \cdot dt$$

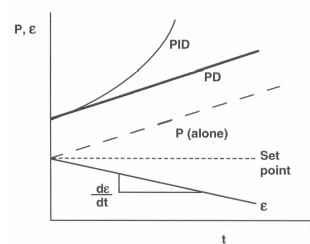
τ<sub>I</sub> : integration time constant



### Proportional-differential control

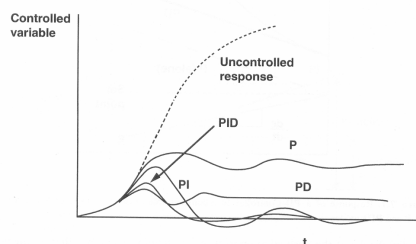
$$P = P_0 + K_p \cdot \varepsilon + K_p \cdot \tau_D \cdot \frac{d\varepsilon}{dt}$$

τ<sub>D</sub> : differential time constant



### Proportional-integral-differential control (PID)

$$P = P_0 + K_p \cdot \varepsilon + \frac{K_p}{\tau_I} \cdot \int_0^t \varepsilon \cdot dt + K_p \cdot \tau_D \cdot \frac{d\varepsilon}{dt}$$

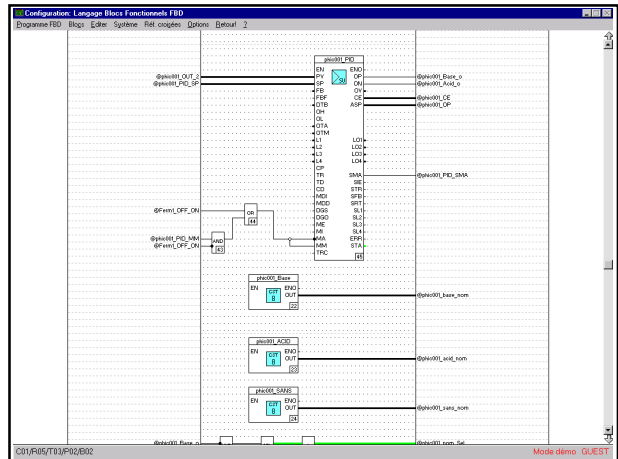


### Practical implementation of PID controller :

- At the level of transmitter



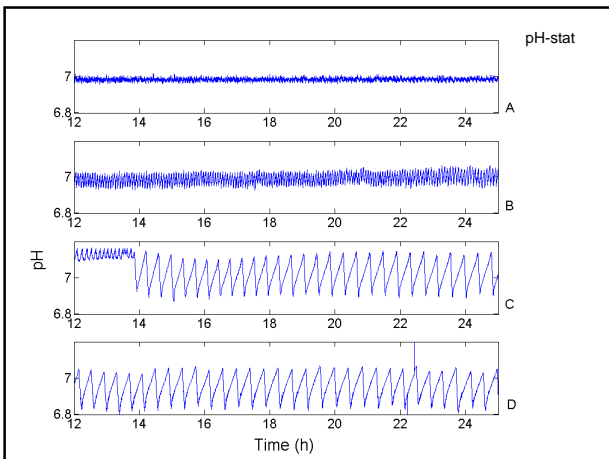
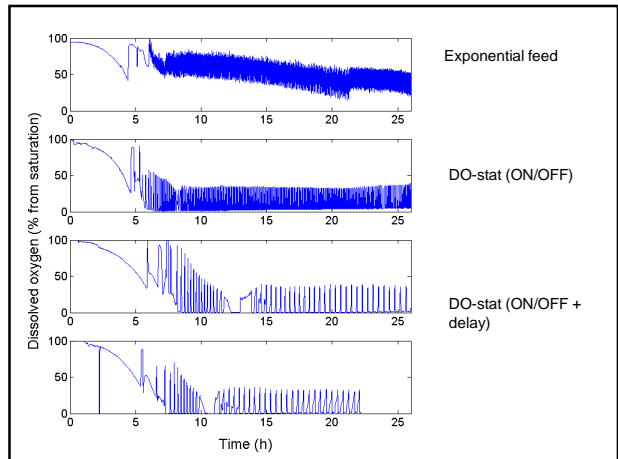
- At the level of central controller



### Advanced bioprocess control

**Fed-batch control :**

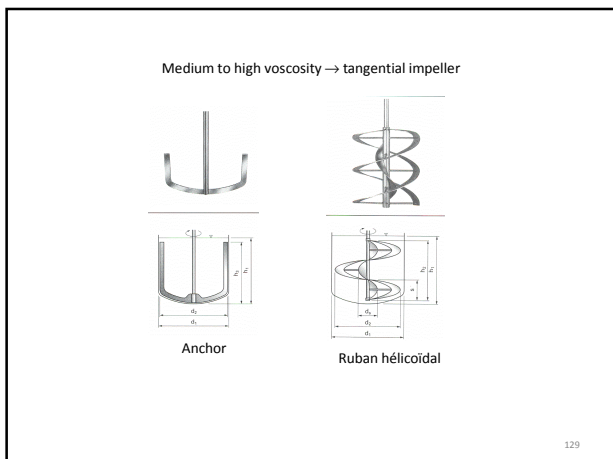
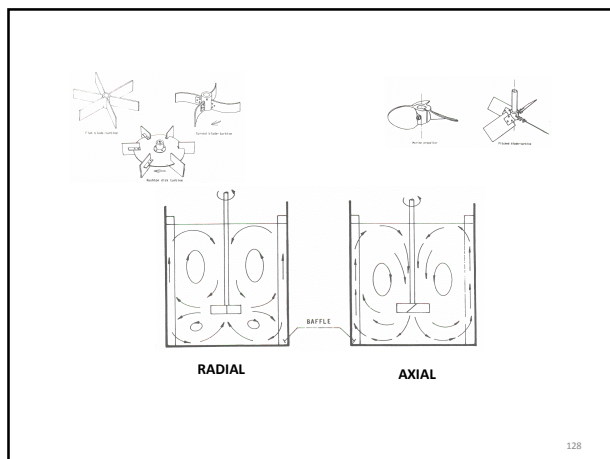
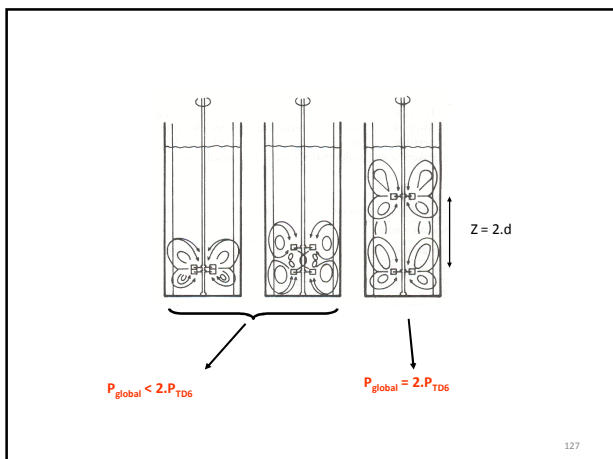
- Exponential feed  $Q = Q_0 \cdot \exp(\mu \cdot t)$
- RQ (respiratory quotient)
- Feedback loop (biomass, ethanol or glucose probe)
- pH-stat
- DO-stat



### 1. Basic bioreactor design : standard geometry

The diagram shows a standard bioreactor geometry. It is a cylindrical vessel with diameter  $D$  and height  $H$ . The impeller diameter is  $d$ . The distance from the bottom of the vessel to the center of the impeller is  $Y$ . The following parameters are listed:

- $H = D$
- $Y = D/3$
- $e = D/10$
- $0.5D < d < 0.8D$



**Calculation of power dissipated**

**Three kinds of variables :**

- Geometry
- Nature of the fluid
- Mixing related

**Vaschy-Buckingham theorem :** from 16 to 13 dimensionless variables

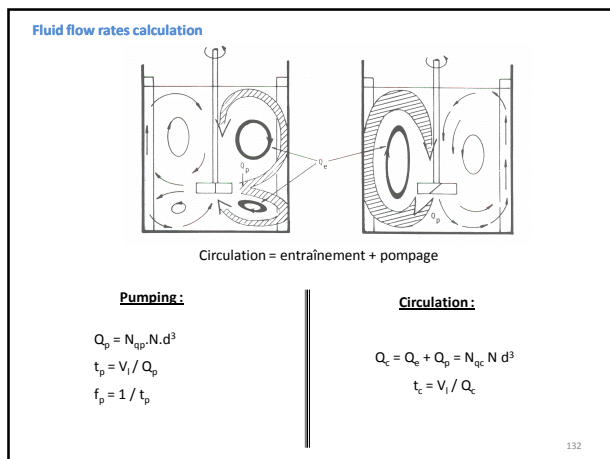
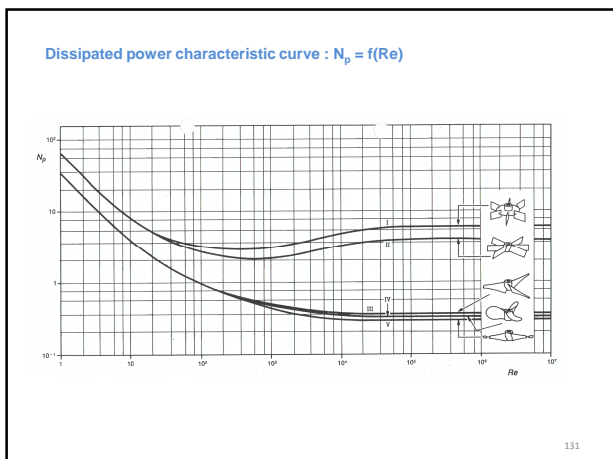
$$N_p = k \cdot Re^\alpha \cdot Fr^\gamma \cdot We^\epsilon \cdot (H/d)^{\alpha_1} \cdot (D/d)^{\alpha_2} \dots$$

Reynolds number (Re) =  $\rho N d^2 / \mu$

Power or Newton number ( $N_p$ ) =  $P / \rho N^3 d^5$

Weber number (We) =  $\rho N^2 d^3 / \sigma$

Froude number (Fr) =  $N^2 d / g$



**Application :**

Considering a standard stirred vessel (D = 0,3 m) filled with water. Mixing is promoted by a TD6 impeller at a stirring rate 400 min<sup>-1</sup>.

TD6 dimensionless numbers:

$N_p = 5,5$

$N_{qp} = 0,85$

$N_{qc} = 1,51$

Calculate the power dissipated, peripheral speed, pumping and circulation flow rates, as well as pumping and circulation times

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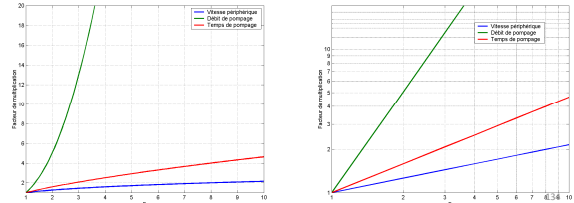
**2. Scaling-up basic principle : similarities principle**

$$N_{\text{industriel}} = N_{\text{lab}} \cdot (D_{\text{industriel}}/D_{\text{lab}})^{a/\beta}$$

$$\text{Avec } F = D_{\text{industriel}}/D_{\text{lab}}$$

Example : P/V as scaling-up criteria

$$\frac{P}{V} = \frac{N_p \cdot \rho \cdot N^3 \cdot d^5}{D^3} = \frac{N^3 \cdot D^5}{D^3} = N^3 \cdot D^2$$



**Application :**

The culture of an anaerobic microorganism has been set up in a pilot-scale bioreactor with a working volume of 785 liters (standard vessel : D = 1m). Mixing is ensured by a TD6 impeller at a stirring rate of 180 min<sup>-1</sup> in a water-like medium.

Calculate the peripheral speed, pumping and circulation flow rate, and pumping and circulation times

The culture has to be up-scaled to a 50 m<sup>3</sup> bioreactor. How does the above mentioned mixing parameters vary if volumetric dissipated power is the scaling factor (i.e., is kept constant during the up-scaling).

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**Answer :**

Volume (m <sup>3</sup> )	0,79	50,00
d (m)	0,33	1,33
N (s <sup>-1</sup> )	3,00	1,16
P(W)	610,60	38891,45
<b>P/V (W/m<sup>3</sup>)</b>	<b>777,83</b>	<b>777,83</b>
Vitesse périphérique (m/s)	3,14	4,84
Débit de pompage (m <sup>3</sup> /s)	0,09	2,27
Temps de pompage (s)	8,52	22,07

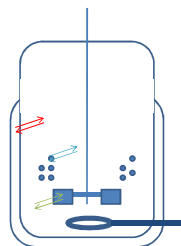
1,54  
24,58  
2,59

Multiplicative factor = f(F)

136

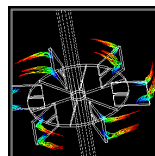
**3. Transport operations**

- Mass transfer
- Heat transfer
- Momentum transfer

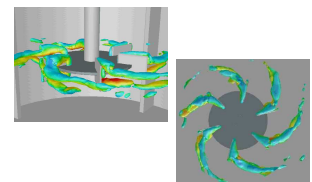


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**3.1. Gas-liquid dispersion and oxygen transfer**



Phase L



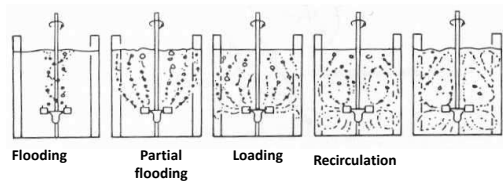
Phase L + phase G

Bakker [2000]

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**Transition between different gas-liquid flow regimes :**

From left to right, the air flow rate is kept constant and stirrer speed is progressively increased

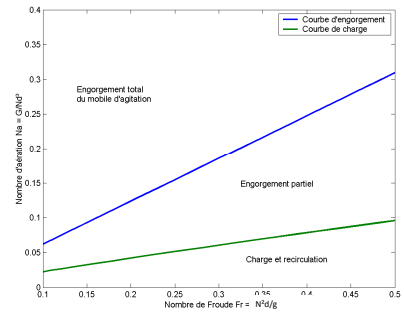


Nienow [1998]

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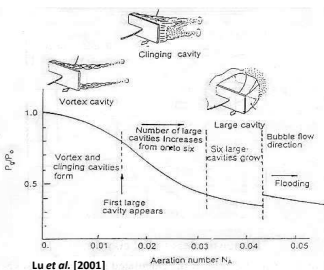
**Flow diagram (dimensionless) :**

**Aeration number (Na) in function of the Froude number (Fr)**  
**Propensity of air flow rate in relation with mechanical stirring intensity**



140

**Power dissipated in G-L system :  $P_g$  decrease when the air flow rate is increased**



Lu et al. [2001]

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Michel et Miller :

$$P_g = m \cdot (P_0)^2 \cdot Nd^3 / G^{0.56} n$$

For a TD6 :  $m = 0,78$   $n = 0,45$

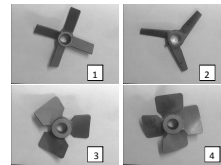
Proportional effect on liquid flow rates:

$$Q_{D,a\acute{e}r\acute{e}} = (P_{a\acute{e}r\acute{e}}/P_0) \cdot Q_0$$

$$Q_{C,a\acute{e}r\acute{e}} = (P_{a\acute{e}r\acute{e}}/P_0) \cdot Q_C$$

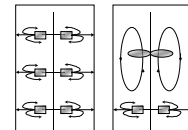
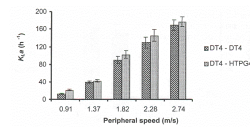
**Optimization of the impeller geometry**

**1. Profiled propellers :**



- 1 : Pitched blade turbine
- 2 : Profiled propeller A310 (solidity ratio : 22%)
- 3 : Profiled propeller A340 (solidity ratio : 67%)
- 4 : Profiled propeller A315 (solidity ratio : 87%)

**2. Hybrid multi-impeller system :**



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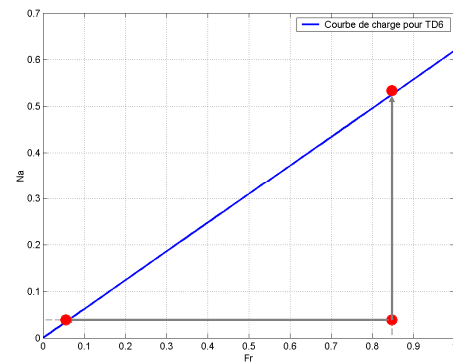
**Application :**

Consider a standard stirred vessel ( $D = 0.1m$ ). Mixing is ensured by a TD6 impeller ( $N = 300 \text{ min}^{-1}$ ) in a water-like medium. Air is sparged at a flow rate of  $0.5 \text{vvm}$ .

The loading curve for the TD6 is :

$$Na = 30 \cdot (d/D)^{3.5} \cdot Fr$$

What is the maximum flow rate acceptable in order to keep an efficient G-L dispersion (and by keeping a constant stirrer rate) ?





**Application :**

We consider two kinds of impeller : a TD6 and a profiled propeller (d = 0.5 m). The equations for the loading curve are the following:

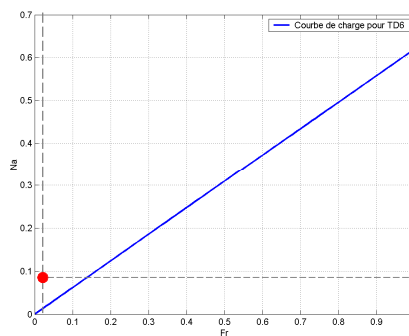
TD6 :  $Na = 30 \cdot (d/D)^{3.5} \cdot Fr$   
 Propeller :  $Na = 6000 \cdot (d/D)^{1.55} \cdot Fr^{2.7}$

Considering the constraints presented in the table, determine the gas-liquid flow regime for each impeller

	N maximum for shear (s <sup>-1</sup> )	G minimum for Oxygen transfer (vvm)
TD6	0,51	0,13
Profiled propeller	0,83	0,19

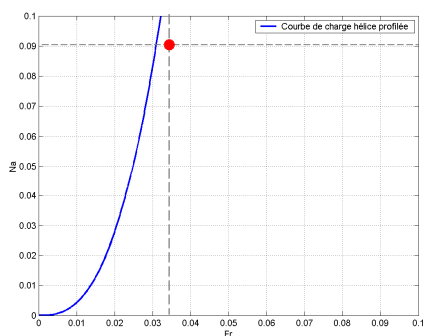
145

**Résolution :**



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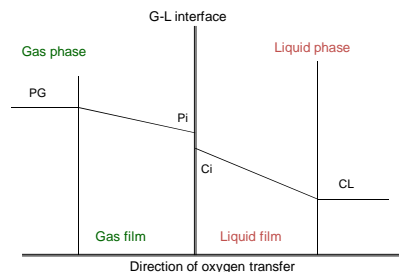
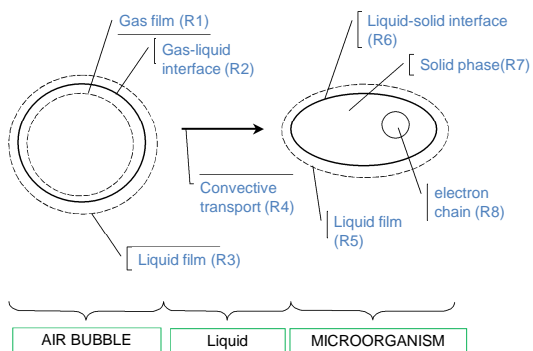
**Résolution (suite) :**



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**Oxygen transfer**

Substrate	Concentration in liquid phase (ppm)	Critical concentration (ppm)	Consumption rate (mmoles/g biomass.h)
Glucose	10.000	100	2.6
Oxygen	7	0.8	7.7



$P_i = He \cdot C_i$

$q_g = k_g \cdot (p_g - p_i)$   
 $q_L = k_L \cdot (C_i - C_L)$   
 $q_g = q_L$

Concentrations at the interface are approximated : global exchange coefficients

$q_g = K_g \cdot (p_g - p^*)$   
 $q_L = K_L \cdot (CL^0 - C_L)$

$1/K_L = 1/k_L + 1/H \cdot e \cdot k_G$

Solubility of oxygen is very low in water, i.e. He is very

$q = k_L S(C^0_L - C_L)$

By dividing by volume :

$Q = q/V_L = k_L S/V_L (C^0_L - C_L)$

Amount of oxygen transferred per time :

$Q = k_L a(C^0_L - C_L)$

General form :

$dC_L/dt = k_L a(C^0_L - C_L) - Q_0$

$C_L$  : dissolved oxygen concentration in liquid phase (mg/L)  
 $C^0_L$  : maximum dissolved oxygen concentration in liquid phase  
 $K_L a$  : oxygen transfer coefficient (s<sup>-1</sup>)  
 $Q_0$  : oxygen uptake rate by the microorganisms (mg/L.h)

**Optimization of the oxygen transfer rate**

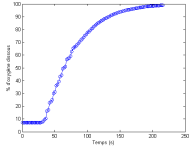
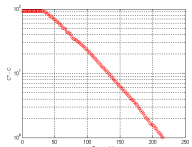
- Increase  $K_L a$  : nature of te medium
- Increase  $a$  : play on mixing performances
- Increase  $(C^0_L - C_L)$  : play on pressure/temperature to modify the G/L equilibrium

**Methods for estimating  $K_L a$  : oxygen probe (indirect) static gassing-in gassing-out**

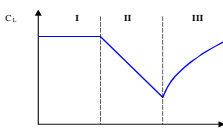
$$\frac{dC}{dt} = k_L a \cdot (C^* - C_L)$$

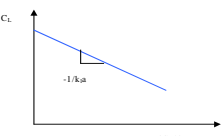
$$\int \frac{dC}{C^* - C_L} = k_L a \cdot t$$

$$\ln(C^* - C_L) = k_L a \cdot t$$

**Methods for estimating  $K_L a$  : oxygen probe (direct) dynamic gassing-in gassing out**

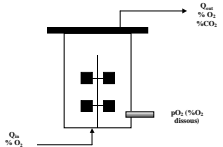




**Part II :**  $\frac{dC_L}{dt} = OTR - OUR = k_L a \cdot (C^* - C_L) - q_0$   
 $k_L a \cdot (C^* - C_L) = 0$   
 $\frac{dC_L}{dt} = OUR = -q_0 \cdot X$

**Part III :**  $C_L = C^* - \frac{1}{k_L a} \cdot \left( \frac{dC_L}{dt} + q_0 \right)$

**Methods for estimating  $K_L a$  : gas balance (direct)**



At steady-state:

$$\frac{dC_L}{dt} = 0$$

$$q_0 = k_L a \cdot (C^* - C_L)$$

$$k_L a = \frac{q_0}{C^* - C_L}$$

$$k_L a = \frac{Q_{INO2} - Q_{OUTO2}}{C^* - C_L}$$

Biological demand for oxygen ( $q_0$ ) determined by gas-balance analysis:

$$Q_{INO2} = Q_{IN} \cdot 0.2094$$

$$Q_{INN2} = Q_{IN} \cdot 0.79$$

$$\% N_{2OUT} = 100 - \% O_2 - \% CO_2$$

$$Q_{OUT} = \frac{100}{\% N_{2OUT}} \cdot Q_{INN2}$$

$$Q_{OUTO2} = \frac{\% O_2}{100} \cdot Q_{OUT}$$

**Application :**

Consider a culture of *Penicillium* sp. The reactor has a volume of 50 L and aeration flow rate is of 0,16 vvm (inlet air: 20,94% d'O<sub>2</sub> et 79% de N<sub>2</sub>). Culture is performed at 30°C and at atmospheric pre ssure (C<sup>0</sup><sub>L</sub> oxygen = 7,6 mg/l).

The data collected from outlet gas analysis and dissolved oxygen probe:

Temps (h)	%O2	%CO2	pO2
2	20,26	0,54	71,7
20	20,1	0,84	0,3

Calculate  $K_L a$  for the two culture times

**$K_L a$  calculation :**

**Van't Riet correlations :**

- For distilled water (coalescing):  
 $k_L a = 0.026 \cdot (P/V)^{0.4} \cdot (G/S)^{0.5}$
- For medium containing electrolytes (non coalescing):  
 $k_L a = 0.002 \cdot (P/V)^{0.7} \cdot (G/S)^{0.2}$

General form for pneumatic reactor (without mechanical stirring):  
 $k_L a = C \cdot (G/S)^{\alpha}$

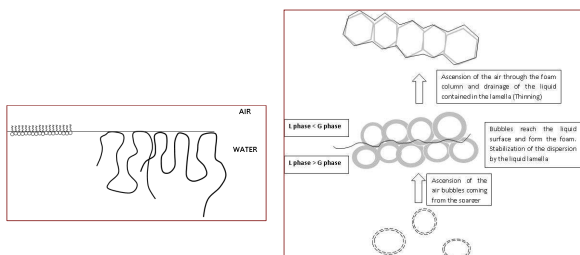
157

**Application :**

Consider a standard stirred vessel ( $D = 0.1\text{m}$ ). Mixing is ensured by a TD6 impeller ( $N = 300\text{ min}^{-1}$ ) in a water-like medium. Air is sparged at a flow rate of 0.5 vvm.

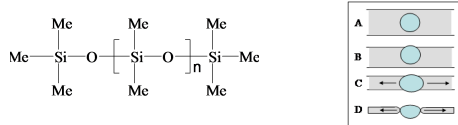
Calculate the power dissipated and the  $K_L a$  if we consider a non-coalescing medium.

**OTR limiting phenomena : foam**



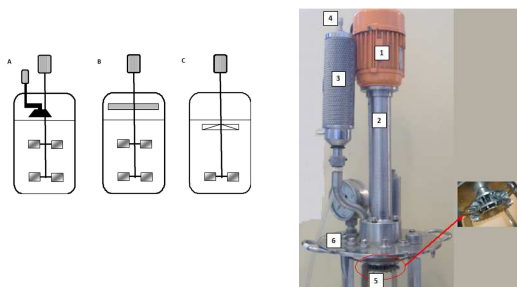
159

**Chemical antifoam : silicon backbone**



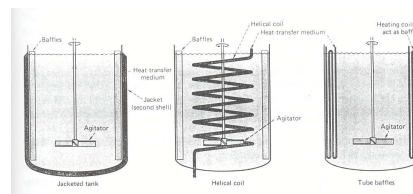
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**Mechanical defoamer**



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**3.2. Heat transfer**



$$Q = U \cdot A \cdot (T_o - T_i)$$

$$\frac{1}{U} = \frac{1}{h_i} + \frac{e}{\lambda_e} + \frac{1}{h_e}$$

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**Dimensional analysis :**

Variables to be considered :  $D, \lambda, \mu, \rho, u, C_p$  et h.  
 Dimensions to be considered : M, L, T, et t.

→Nusselt number :  $Nu = \frac{hD}{\lambda}$

→Reynolds number :  $Re = \frac{\rho \cdot v \cdot D}{\mu}$

→ Prandtl number :  $Pr = \frac{C_p \mu}{\lambda}$

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Dimensional analysis:  $Nu = C \cdot Re^a \cdot Pr^b \cdot Vi^c$

-TD6 with heat jacket:

$$Nu = 0,74 \cdot Re^{0,66} \cdot Pr^{0,33} \cdot Vi^{0,14}$$

- Pitched blade turbine with serpentine:

$$Nu_{serp} = 0,00752 \cdot Re^{0,722} \cdot Pr^{0,4} \cdot Vi^{0,2}$$

- Propeller with serpentine :

$$Nu_{serp} = 0,011 \cdot Re^{0,67} \cdot Pr^{0,37} \cdot Vi^{0,2}$$

- TD6 with serpentine :

$$Nu_{serp} = 0,0205 \cdot Re^{0,67} \cdot Pr^{0,37} \cdot Vi^{0,2}$$

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Relation between heat transfer and microbial growth

In intensive aerobic processes, heat released is proportional to the amount of oxygen consumed :

$$Q_{exo} = V_{H_2O} \cdot OTR$$

Heat extracted by heat exchanger:

$$Q_{ech} = k \cdot A \cdot \Delta t$$

At steady-state, heat generated by microbial growth is proportional to heat removed at the level of the exchanger:

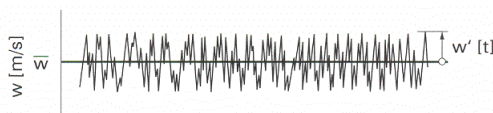
$$k \cdot A \cdot \Delta t = V_{H_2O} \cdot OTR$$

But, scale-up problem, as  $A \sim V_c^{2/3}$

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3.3. Momentum transfer

**Characteristics of urbulence :**



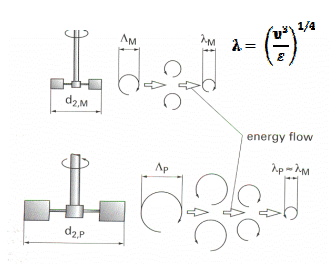
$$w_i = w + w'$$

$$k = \frac{1}{2} (w_x'^2 + w_y'^2 + w_z'^2)$$

$$w_x'^2 = w_y'^2 = w_z'^2$$

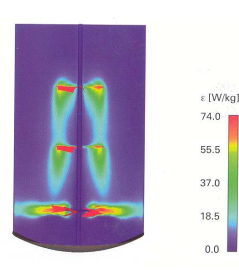
166

**Kolmogoroff : eddies scale**



$$\lambda = \left( \frac{\nu^3}{\epsilon} \right)^{1/4}$$


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Limite de Kolmogoroff → en réalité, on a une répartition du taux de dissipation d'énergie.  
 La plupart de la dissipation prend place dans l'environnement direct du mobile d'agitation.

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### Mixing time measurement

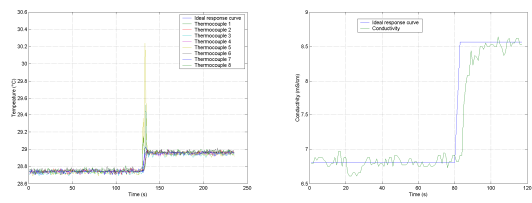


2 phenomena:  
- Circulation  
- Turbulence

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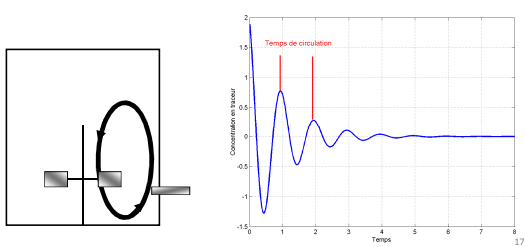
### Methods involving probes:

Traceur	Sonde
Acide-base	pH
Solution saline	Conductivité
Solution chauffée	Thermocouple



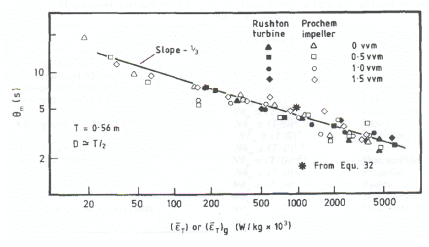
170

### Equations based on circulation

$$t_m \cdot N = 3,9 \cdot (d/D)^3 \cdot N_{gc}^{-1}$$


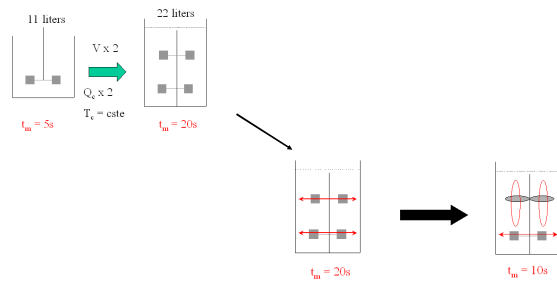
171

### Correlation based on turbulence

$$t_m = 5,9 \cdot D^{2/3} \cdot \epsilon_T^{-1/3} \cdot (d/D)^{-1/3}$$


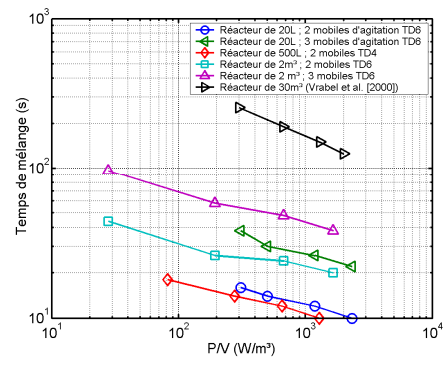
172

### Mixing time in function of the number of agitation stages:



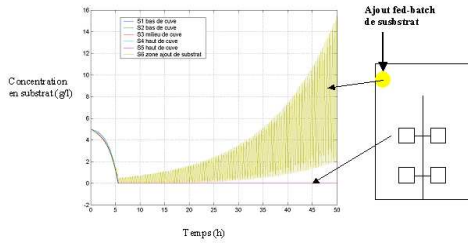
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### Evolution of mixing time during scale-up :



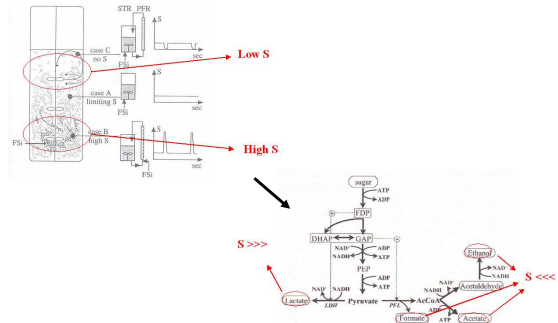
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Example of the impact of mixing time on process efficiency : concentration gradient during a fed-batch process



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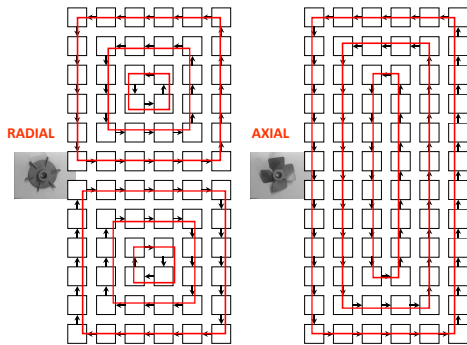
Effect of concentration gradient on cell physiology : example of *Lactococcus lactis*



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**Structured model for bioreactor hydrodynamics**

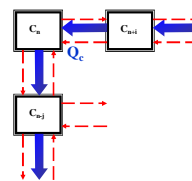
- Compartment model
- Network-of-zones



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**Mathematical basis**

Evolution of concentration with time for each compartment :



The evolution of  $C_n$  is described by an ordinary differential equation (ODE):

$$V \cdot \frac{dC_n}{dt} = Q_c \cdot (C_{n+1} - C_n) + Q_c \cdot (C_{n+1} + C_{n-1} - 2C_n)$$

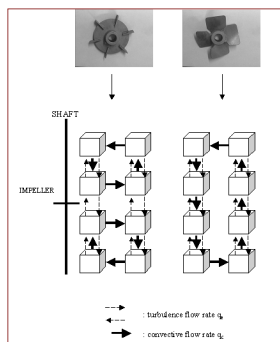
Convective fluxes ( $Q_c$ ) are calculated (circulation flow rate) and the turbulence flow rates are estimated on the basis of mixing time experiments

System of ODEs is numerically resolved by a Runge-Kutta routine (e.g., ode45 routine in MatLab)

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**Application:**

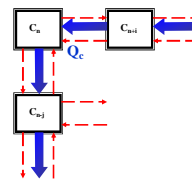
A stirred bioreactor ( $V = 500L$ ) is equipped with a TD6 or a profiled propeller. The hydrodynamics can be modelled by the compartment principle, leading to the model structure shown on the figure. Write the ODEs system for the evolution of the concentration of a specie C in the compartments



179

**Mathematical basis**

Evolution of concentration with time for each compartment :



The evolution of  $C_n$  is described by an ordinary differential equation (ODE):

$$V \cdot \frac{dC_n}{dt} = Q_c \cdot (C_{n+1} - C_n) + Q_c \cdot (C_{n+1} + C_{n-1} - 2C_n)$$

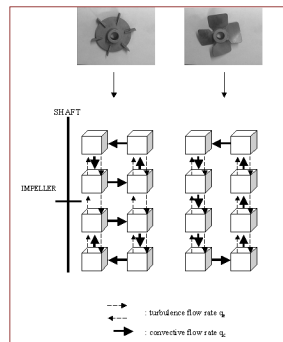
Convective fluxes ( $Q_c$ ) are calculated (circulation flow rate) and the turbulence flow rates are estimated on the basis of mixing time experiments

System of ODEs is numerically resolved by a Runge-Kutta routine (e.g., ode45 routine in MatLab)

180

**Application:**

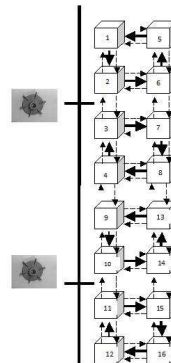
A stirred bioreactor ( $V = 500L$ ) is equipped with a TD6 or a profiled propeller. The hydrodynamics can be modelled by the compartment principle, leading to the model structure shown on the figure. Write the ODEs system for the evolution of the concentration of a specie C in the compartments



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**Application:**

Consider a two-staged stirred bioreactor (TD6-TD6) as shown on the figure. If  $q_c = 0.1 \text{ m}^3/\text{s}$  et  $q_t = 0.2 \text{ m}^3/\text{s}$ . Model and simulate the mixing time when a pulse is added at the level of the first compartment



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**Resolving the ODEs system with MatLab :**

First file.m :

- function  $y = f(t,y)$
- Constant
- Algebraic equations
- Ordinary differential equations, ODEs (in matrix form)

Second file.m :

- Solver `ode`
- function `plot`

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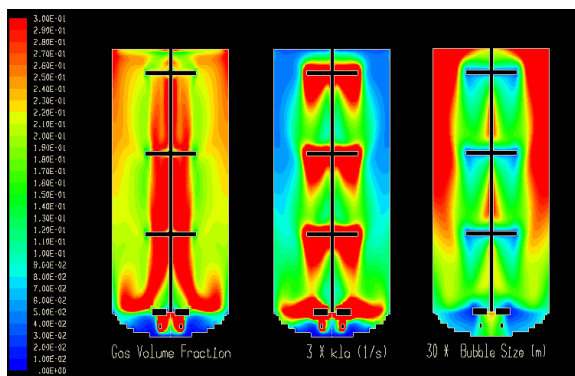
**Application of structured modelling procedure to oxygen transfer**

2 scaling-up criteria:

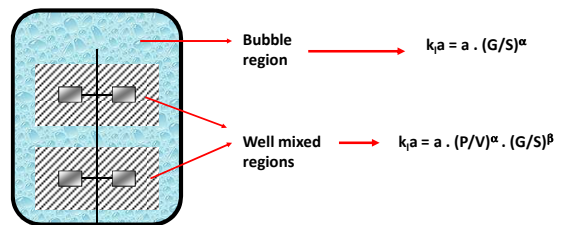
- Air flow rate:  $G/V$  (en v.v.m.) ou  $G/S$  (en  $\text{m}^3/\text{s}$ )
- Agitation rate:  $\pi N d$  ou  $P/V$

But doesn't necessarily lead to reliable results, considering the spatial distribution of basic hydrodynamic parameter inside the vessel

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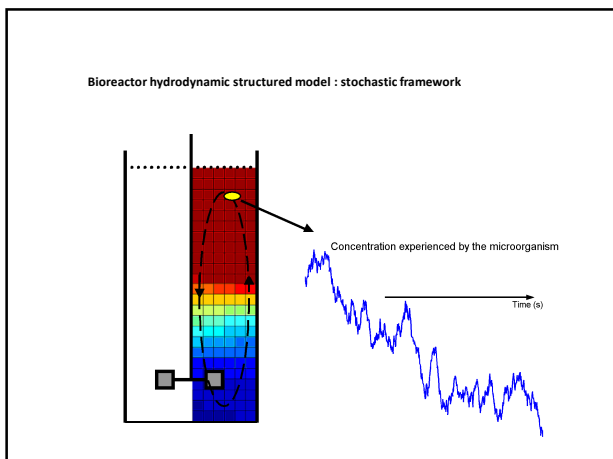


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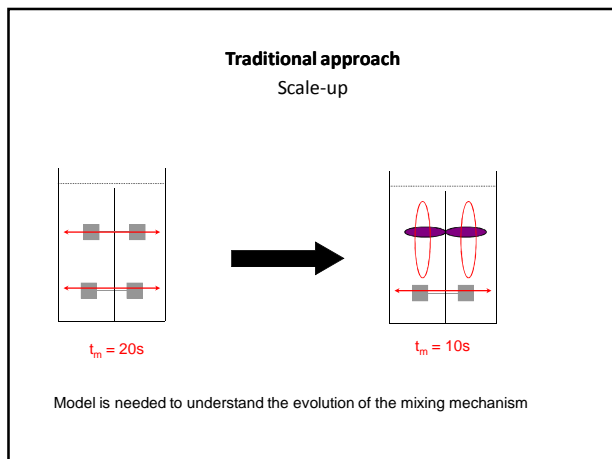
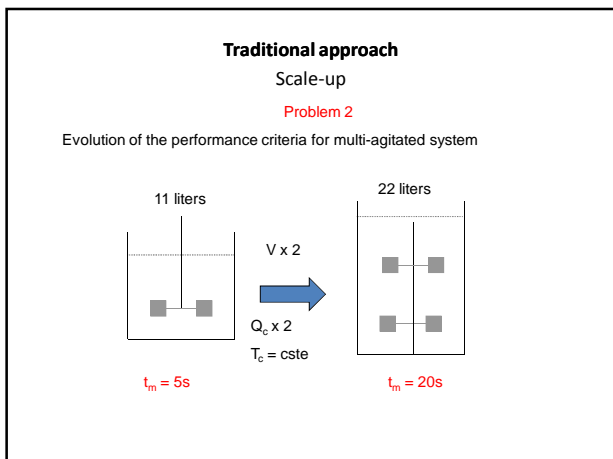


Oosterhuis N.M.G., Kossen N.W.F. [1993]

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Case study 1 : modelling approaches for bioreactor hydrodynamics



**4. Bioreactor hydrodynamics : local approach**

**FIRST : homogenisation process**

Mechanical constraints  $\rightarrow$  P/V limitation  $\rightarrow$  increase of the mixing time

Consequences : gradients establishment (substrate, pH,  $O_2$ , ...)

**Fed-batch process**

**Oxygen transfer**

Enfors et al. [2001] Journal of biotechnology

Bakker [2003]

**SECOND : circulation process**

Increase of the broth volume  $\rightarrow$  increase of the variability (the randomness) of the circulation paths taken by the microorganisms

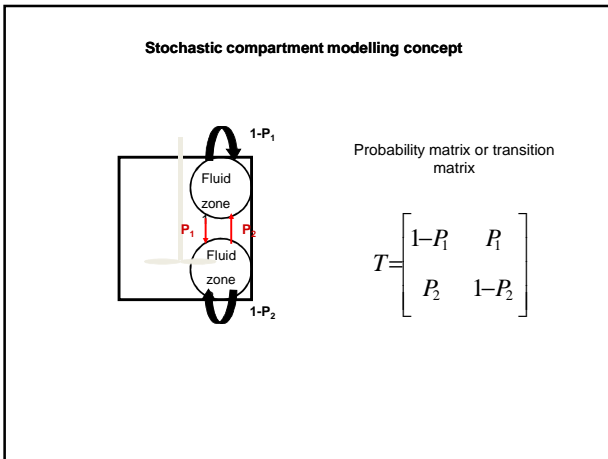
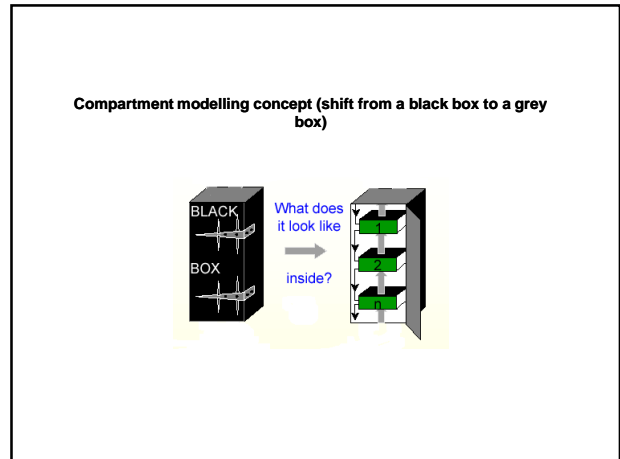
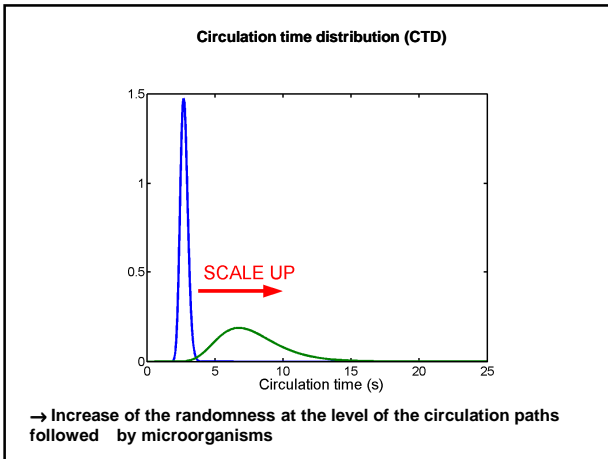
$\rightarrow$  this phenomena is not often considered

If we observe three distincts circulation paths followed by a microorganism :

There is not a single value of circulation time, but a CTD

Nandev et al. [1992] Biotechnology and bioengineering





**Mathematical implementation of stochastic models :  
Markov chain**

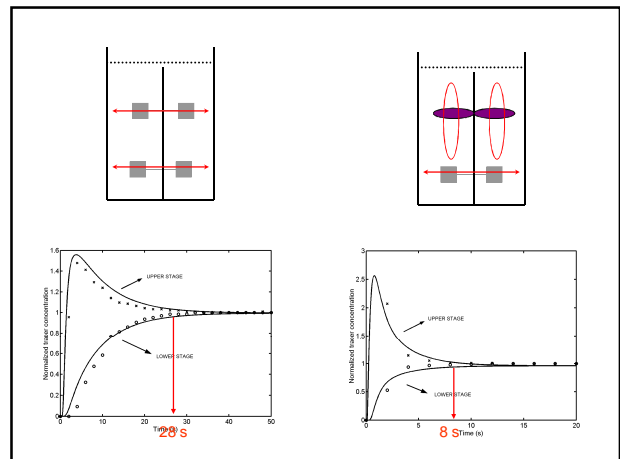
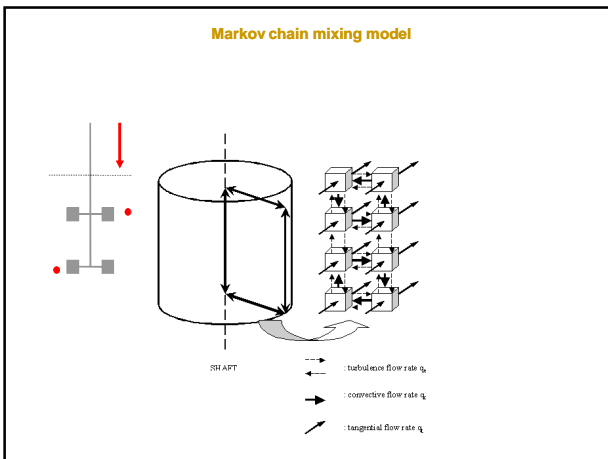
Let  $S$  be the discrete state vector of the previous system, the following equation allow us to calculate the discrete time evolution of the system:

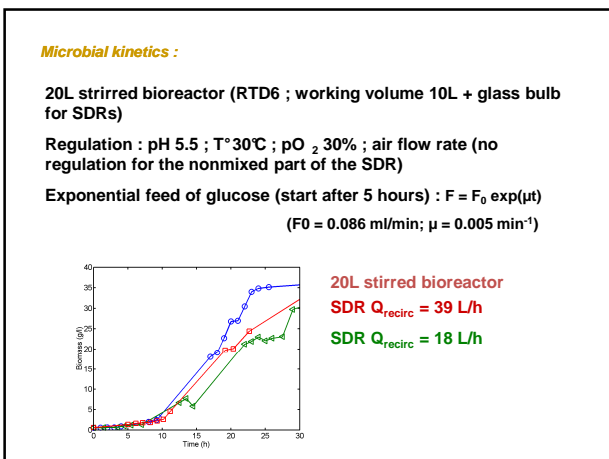
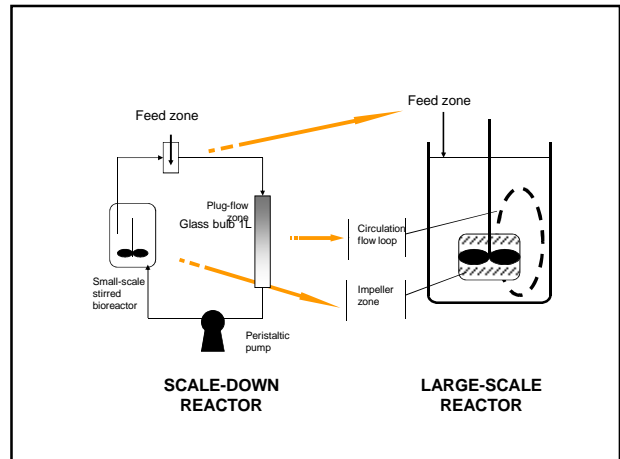
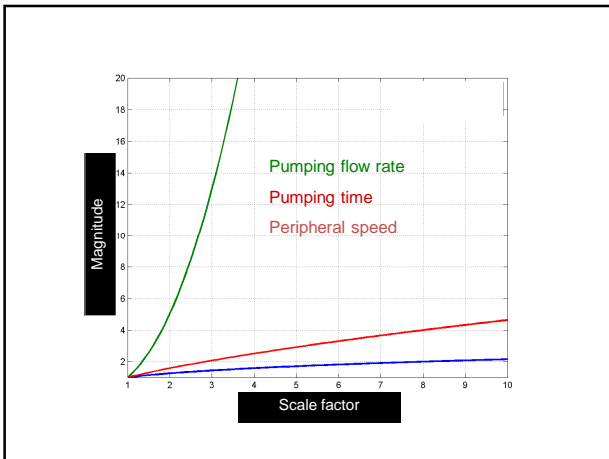
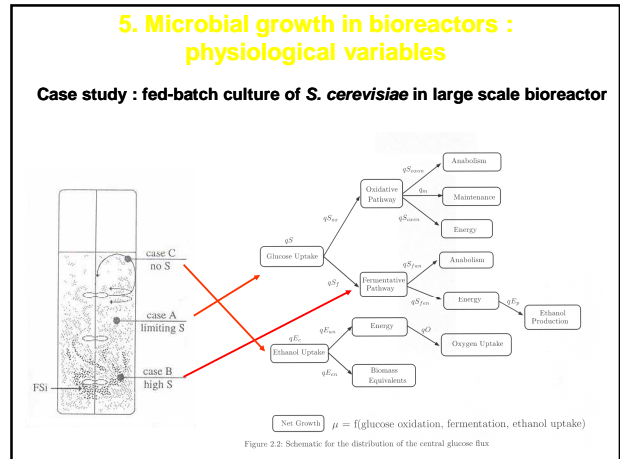
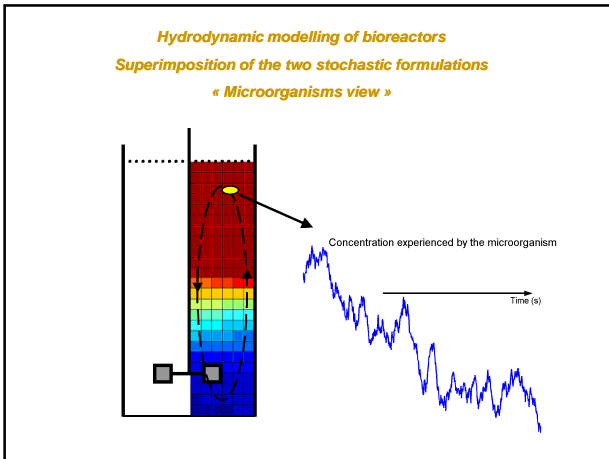
$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix}_t \times \begin{bmatrix} 1-P_1 & P_1 \\ P_2 & 1-P_2 \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}_{t+1}$$

This equation can be generalised :

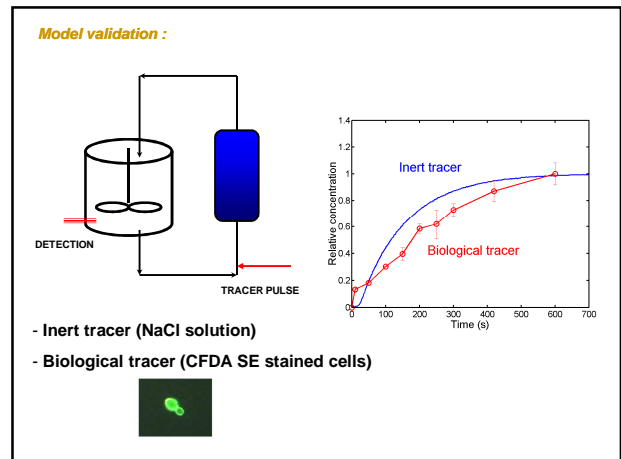
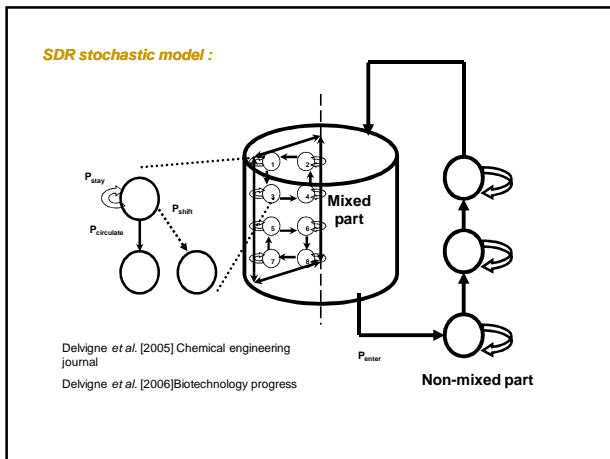
$$S_t = S_0 \cdot T^t$$

Don't keep track of the particle history, but computationally light  
→ Well suited for the simulation of the **homogenisation of a solute** (large amount of molecules, particles)





Reactor	Biomass yield $Y_{xs}$
Classical bioreactor	0.48
SDR recirculation flow rate $Q = 18 \text{ l/h}$	0.36
SDR recirculation flow rate $Q = 39 \text{ l/h}$	0.45



**6. Perspective : modelling the microbial response to bioreactor environmental fluctuations**

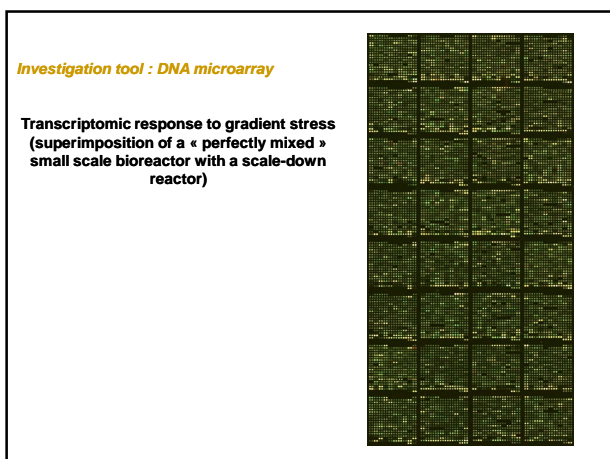
We have described the physical parameters  
 BUT  
 What about the biologicals implications

What is the microbial response in front of these environmental fluctuations ?  
 TO MAKE THE LINK  
 What about the environmental sensing capabilities of microorganisms ?

**In the case of *S. cerevisiae* :**

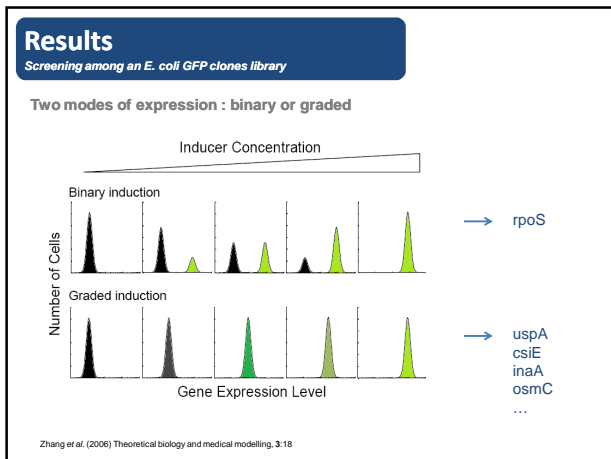
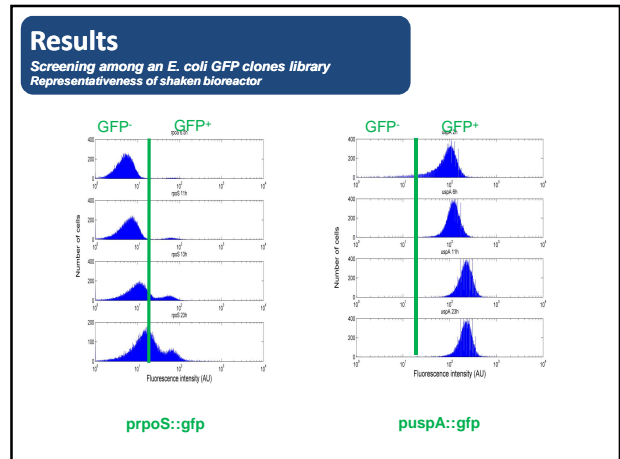
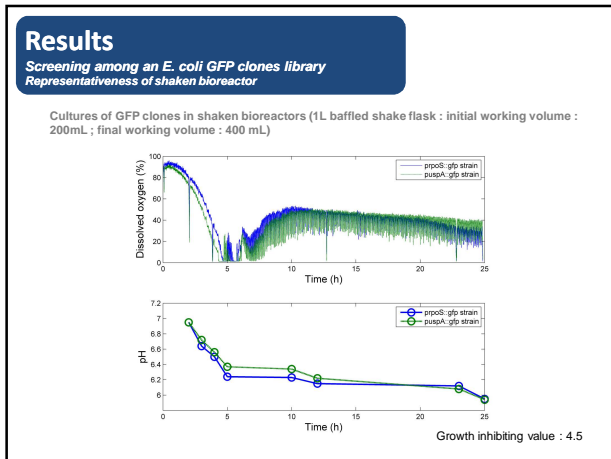
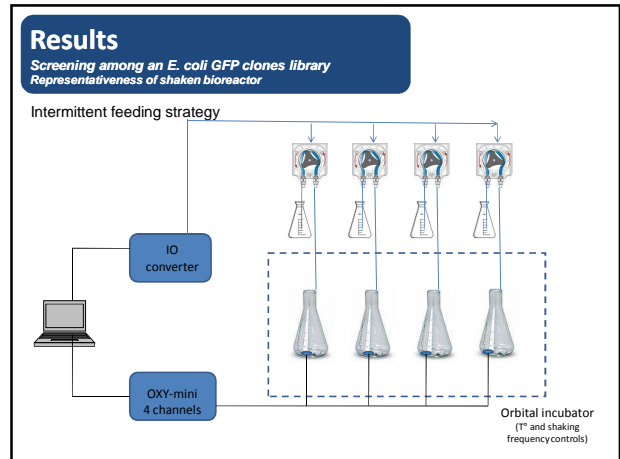
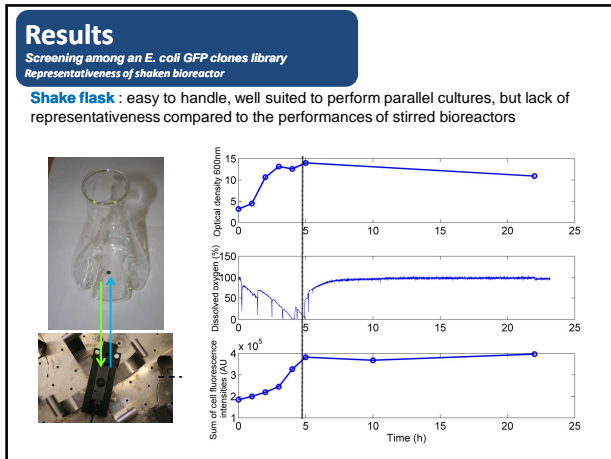
- Cells react very rapidly in front of glucose fluctuations
- Glucose sensing mechanisms are well documented but not quantified (Ozcan *et al.* [1999] Microbiology and molecular biology reviews)
- The cell don't sense all the environmental fluctuations (Sensors and some specific metabolic pathways act as intracellular homeostasis devices) c (Wolff *et al.* [2005] Journal of theoretical biology, Rao *et al.* [2002] Nature)

Important in order to make the link between the physical and the biological parameters



**Case study 2 : Whole cell biosensors for the detection of mixing imperfections**





### Results

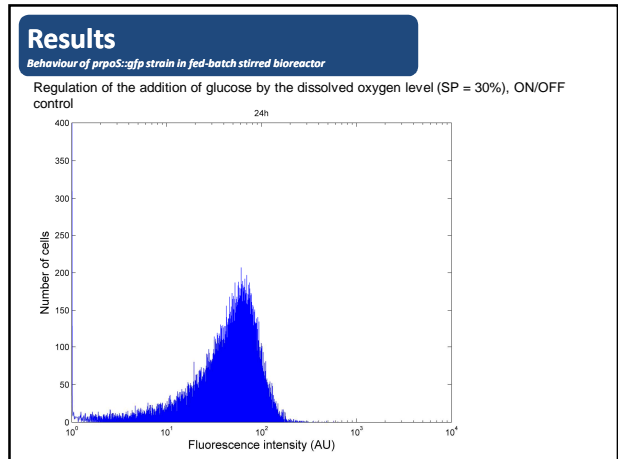
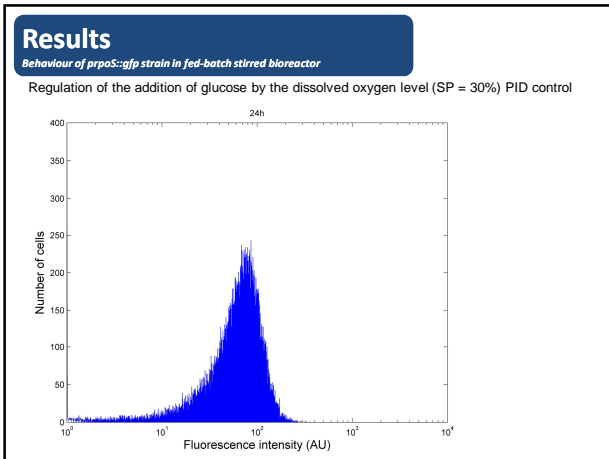
Screening among an *E. coli* GFP clones library

Binary mode of gene expression → sources :

- Short mRNA and protein half-lives
- High sensitivity for the detection of the reporter protein

Generally not observed for GFP reporter system considering the high protein stability of this system compared with β-galactosidase and luciferase reporters

This mechanism of gene induction give rise to differentially expressed phenotypes at the protein level. Can potentially be used to gain more sensitivity about the impact of extracellular fluctuations

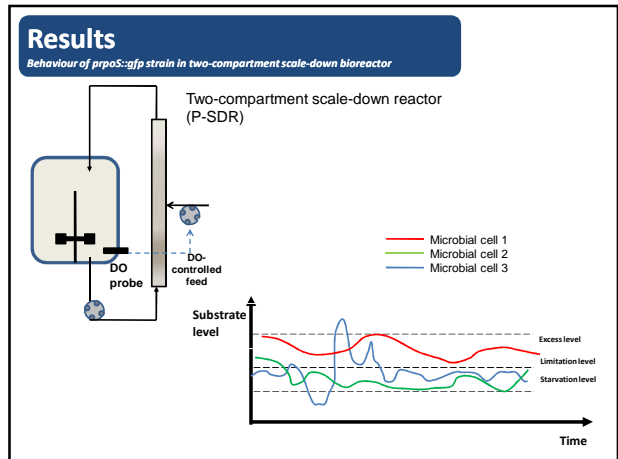


### Results

Behaviour of *prpoS::gfp* strain in fed-batch stirred bioreactor

**Basic observations :**

- Binary mode for GFP expression at the end of the batch phase and during the transition from batch to fed-batch phase
- After the induction of the major part of the population (all the cells are in the GFP+ state), graded mode of GFP expression is observed
- Successive glucose excess tends to slow down the binary expression phase



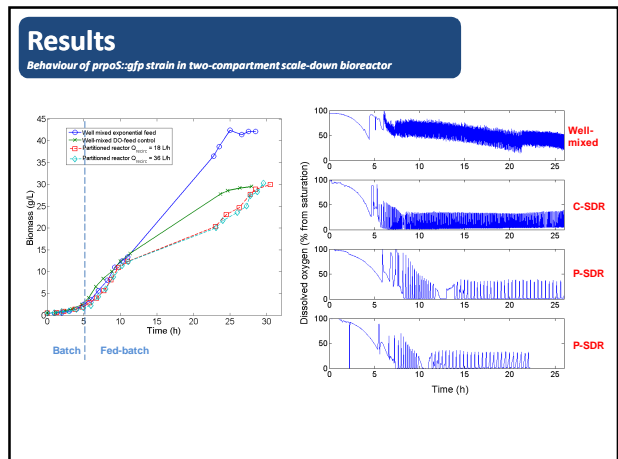
### Results

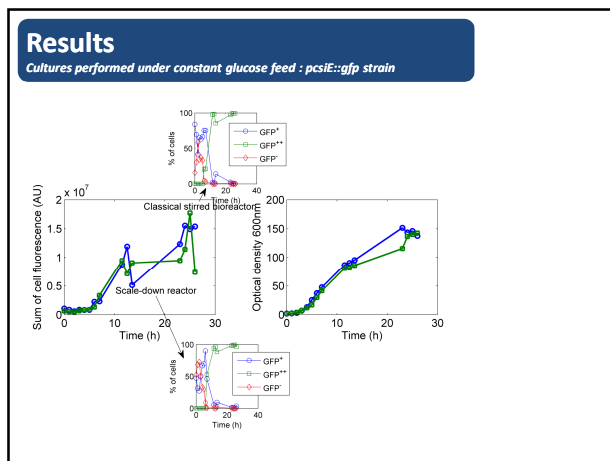
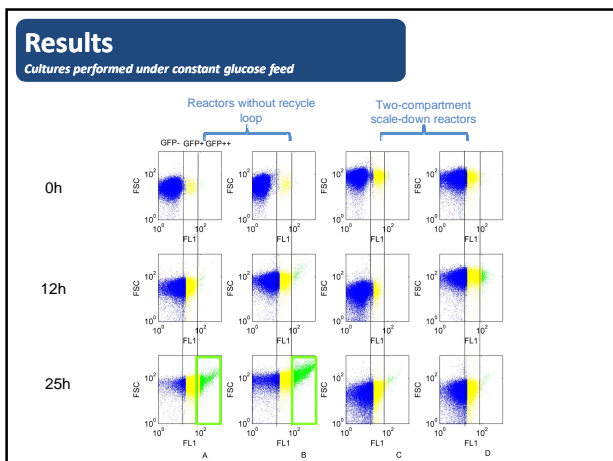
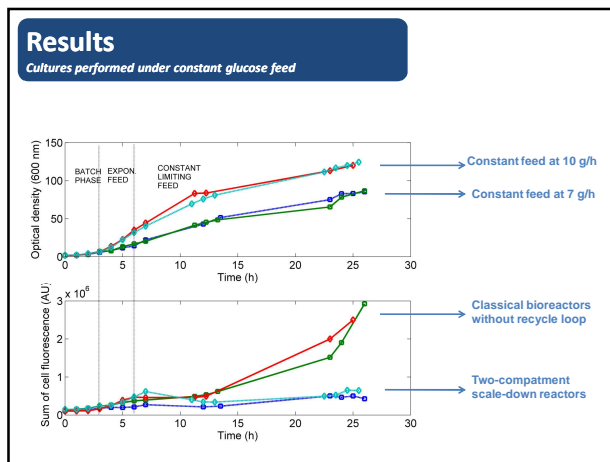
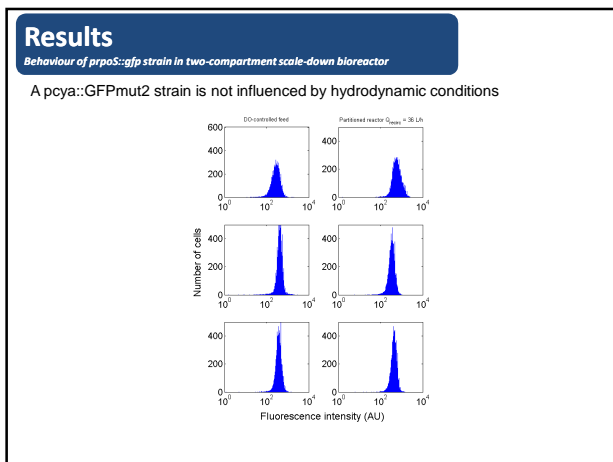
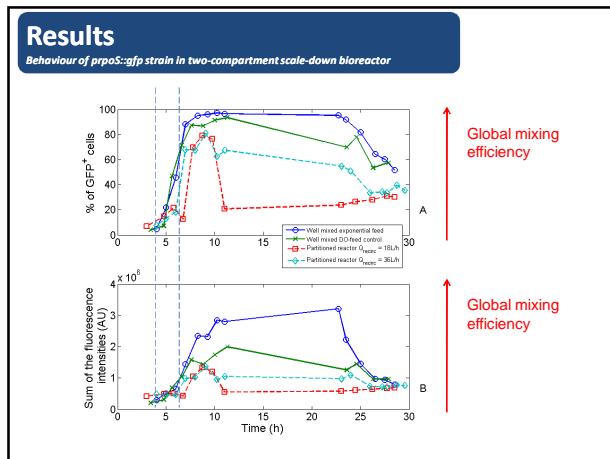
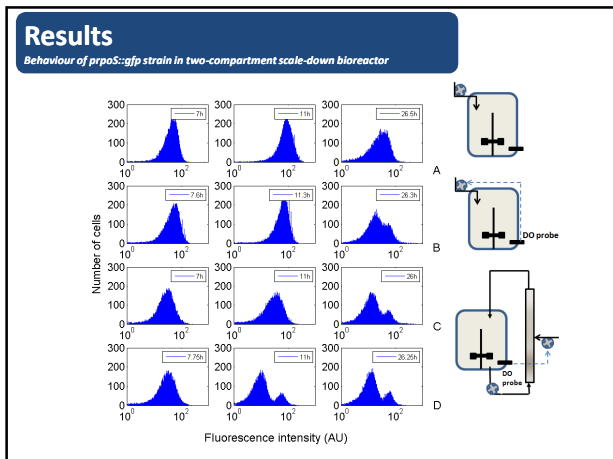
Behaviour of *prpoS::gfp* strain in two-compartment scale-down bioreactor

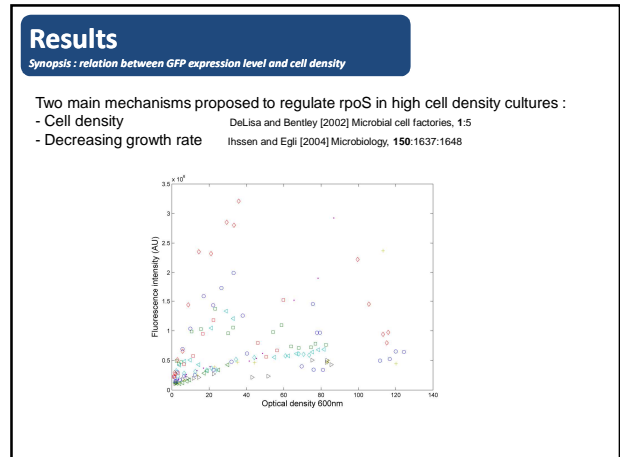
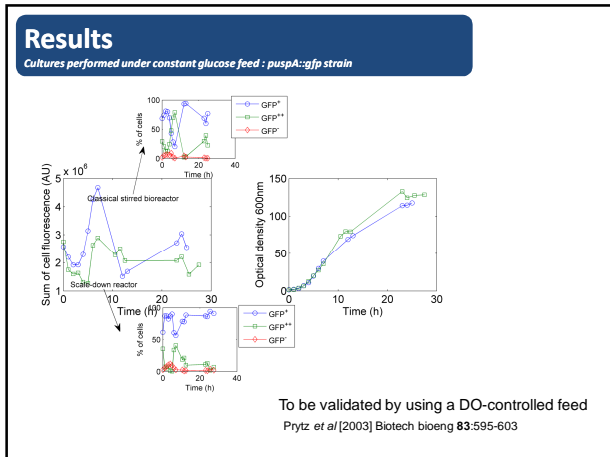
**Operating conditions :**

- Stirred bioreactor, working volume 10L
- Mineral medium, glucose as carbon source
- Fed-batch with exponential feed algorithm
- Scale-down approaches with DO-controlled fed-batch and partitioned reactor

Delvigne F. et al. [2009] *Microbial cell factories*, 8:15



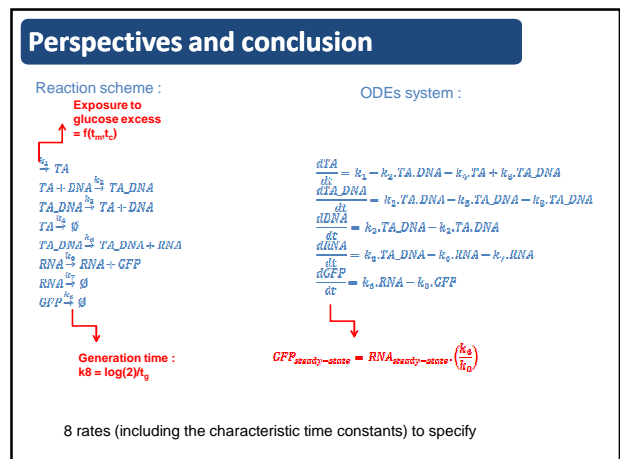
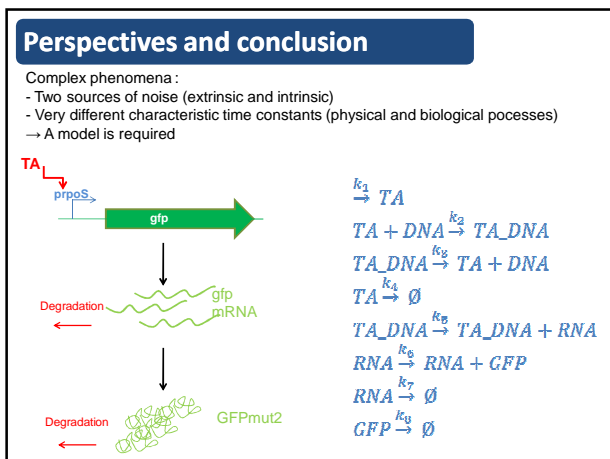
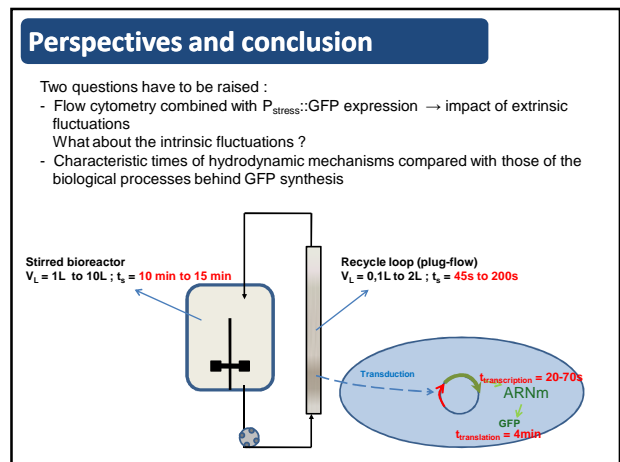




### Perspectives and conclusion

rpoS::GFP strains seems to react to the degree of homogeneity inside the bioreactor :

- Homogenous reactor : GFP+
- Inhomogenous reactor : GFP-



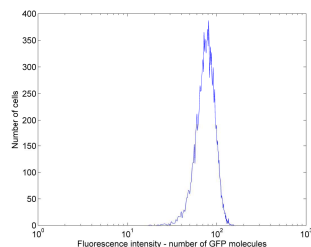


## Perspectives and conclusion

These equations can be used in the classical deterministic formalism (ODEs solver), but more interestingly in the stochastic formalism :

Probability that reaction  $\mu$  occurs at time  $\tau$  (Gillespie algorithm)

Gillespie [1977] J. of physical chemistry, 81:2340-2361



Example : simulation of 30,000 cells after 6 hours of induction