Three dimensional sediment transport model of the Belgian coastal zone

Application of the CART theory

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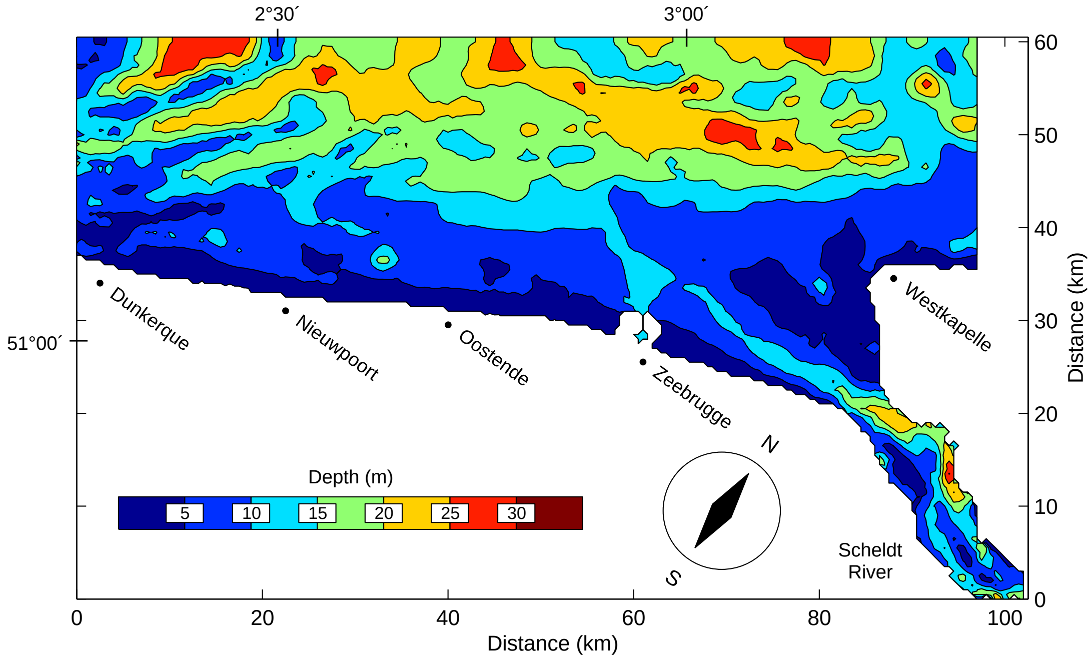
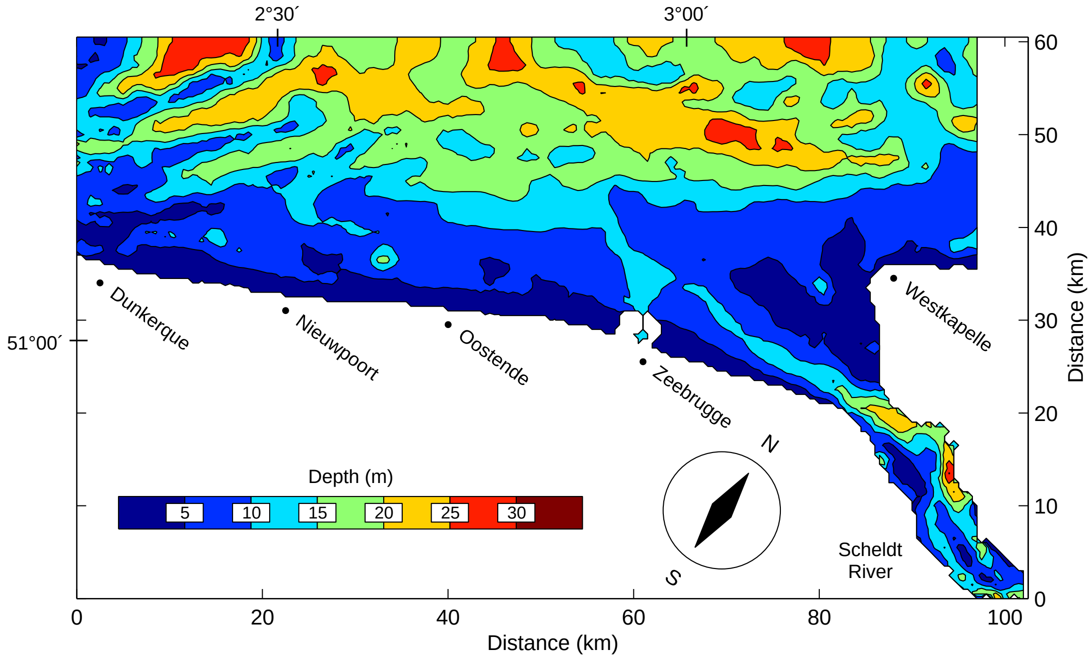
Study Area



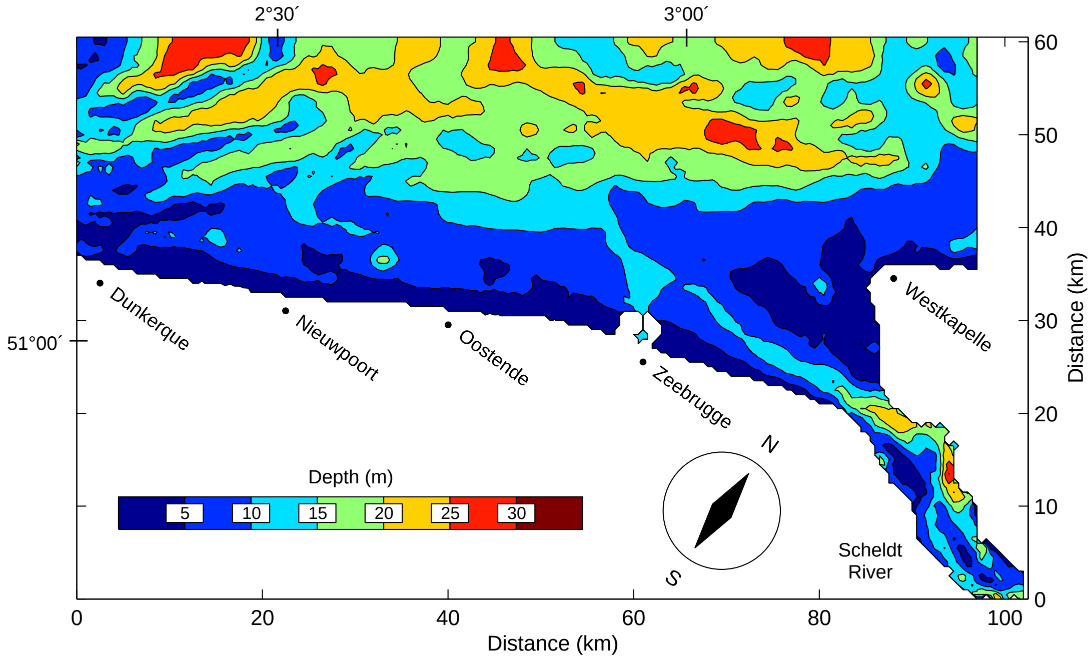
2°30´
3°00´
51°00´
Dunkerque
Nieuwpoort
Oostende
Zeebrugge
Westkapelle
Depth (m)
5
10
15
20
25
30
N
S
Scheldt
River
0
20
40
60
80
100
Distance (km)
0
10
20
30
40
50
60
Distance (km)

Study Area



- Shallow and irregular bathymetry.
- Hydrodynamic dominated by tides, winds and waves.
- Intense mixing of the water column during the entire year.
- Exchanges with offshore waters limited.

Study Area



- Sediment distribution influenced by the complex sand banks system.
- Sediments consist of fine to medium sand with a fining trend to the east.
- Large mud fields (concentration of SPM $> 400\ mg/l$) occur between Oostende and the Westerschelde estuary.

Hydrodynamic model

- 3D, baroclinic (T,S), k turbulence closure.
- Horizontal resolution $500 \times 500\ m$, 10 unequally spaced σ-levels.
- Forcings: 21 tidal components and NCEP meteorological data.
- Finite volume method, Arakawa C grid, mode-splitting method.
- Semi-implicit vertical advection and turbulent diffusion.
- TVD advection scheme with superbee flux limiter used for advection of scalar quantities.
- Grid parallel to the coast.
- Flooding and drying algorithm.
- Coupled with a large scale model presenting the same characteristics and covering the whole North-Western European Continental Shelf.

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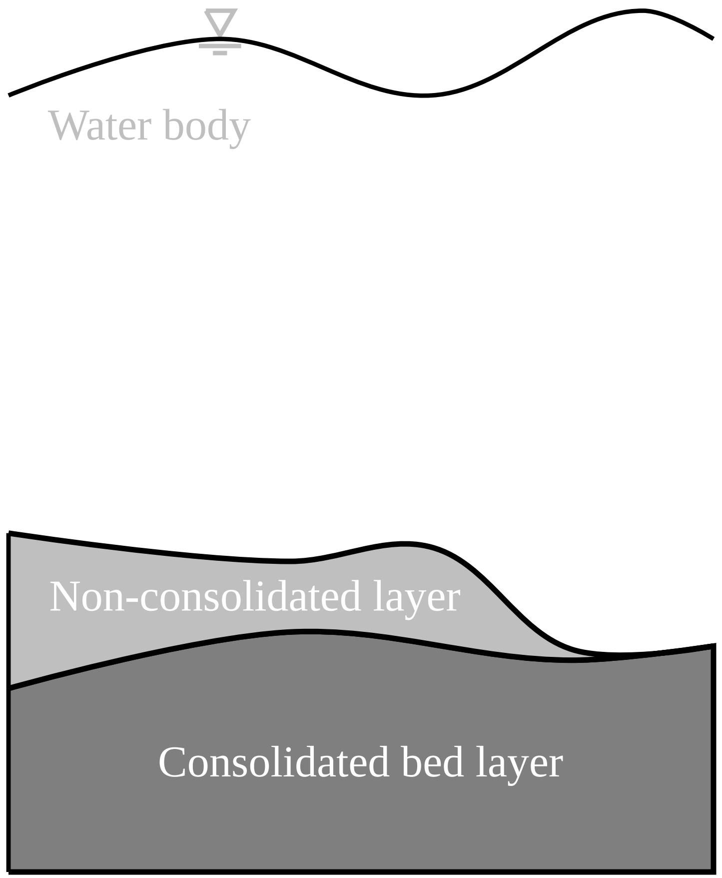
- 4 classes of suspended sediments to take in account the variability of their dynamic properties.
- Sediment processes:

Neglected	Considered
Bedload tranport	Sedimentation
Sand-mud interactions	Erosion of the seabed
Fluidization	Deposition
Liquefaction	Consolidation (partly)
Changing in material properties	
Biological Processes	
Flocculation	

Sediment transport model (2)



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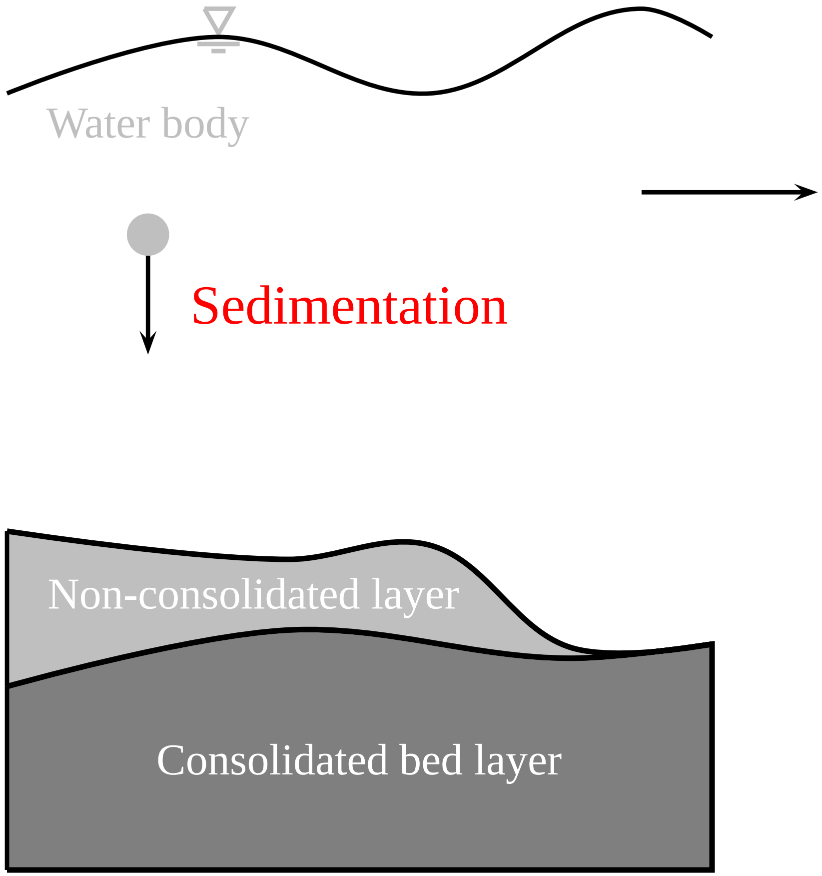
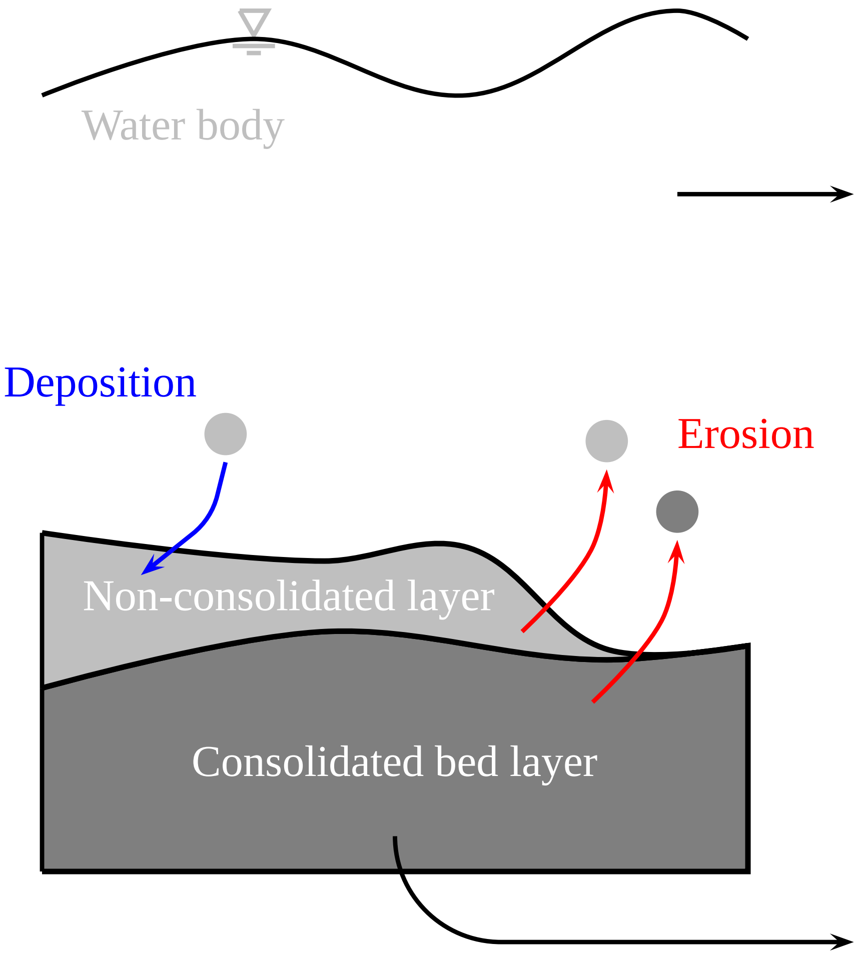


$$\frac{\partial C^i}{\partial t} + \nabla \cdot \left[\left(\mathbf{v} + \mathbf{w}_s^i \right) C^i - \mathbf{K} \cdot \nabla C^i \right] = 0$$

Average particle sizes $[\mu m]$	Settling velocities $[mm/s]$
2	0.005
6	0.05
10	0.1
35	1

Sediment transport model (2)



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$$\frac{\partial C_s^i}{\partial t} = F_{dep}^i - F_{ero}^i$$

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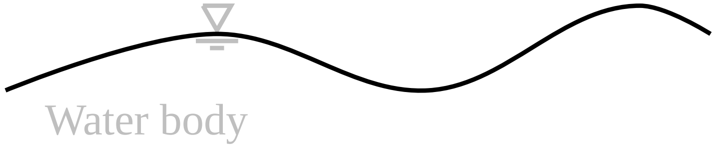

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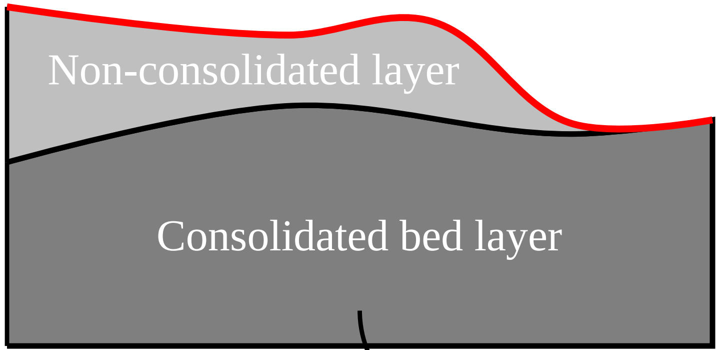

$$\overbrace{\left[\left(\mathbf{v} + \mathbf{w}_s^i \right) C^i - \mathbf{K} \cdot \nabla C^i \right]} \cdot \mathbf{n} = \underbrace{F_{dep}^i - F_{ero}^i}$$

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$$P^i_{sed} = \begin{cases} \left(1 - \frac{\tau_b}{\tau^i_{crd}}\right) & \text{if} \quad \tau_b < \tau^i_{crd} \\ 0 & \text{if} \quad \tau_b \geq \tau^i_{crd} \end{cases}$$

- τ_{crd} is the critical bottom shear stress for deposition $= 0.5\ Pa$.
- τ_b is the bottom shear stress calculated under the combined effects of wave and currents.

Erosion

- Ariathurai (1974):

$$F^i_{ero} = \underbrace{P^i_{ero}}_{\text{erosion threshold}} M^i f^i \quad \text{where} \quad f^i = \frac{C^i_s}{\sum_j C^j_s}$$

Erosion

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$$P^i_{ero} = \begin{cases} \left(\dfrac{\tau_b}{\tau^i_{cre}} - 1\right) & \text{if} \quad \tau_b > \tau^i_{cre} \\ 0 & \text{if} \quad \tau_b \leq \tau^i_{cre} \end{cases}$$

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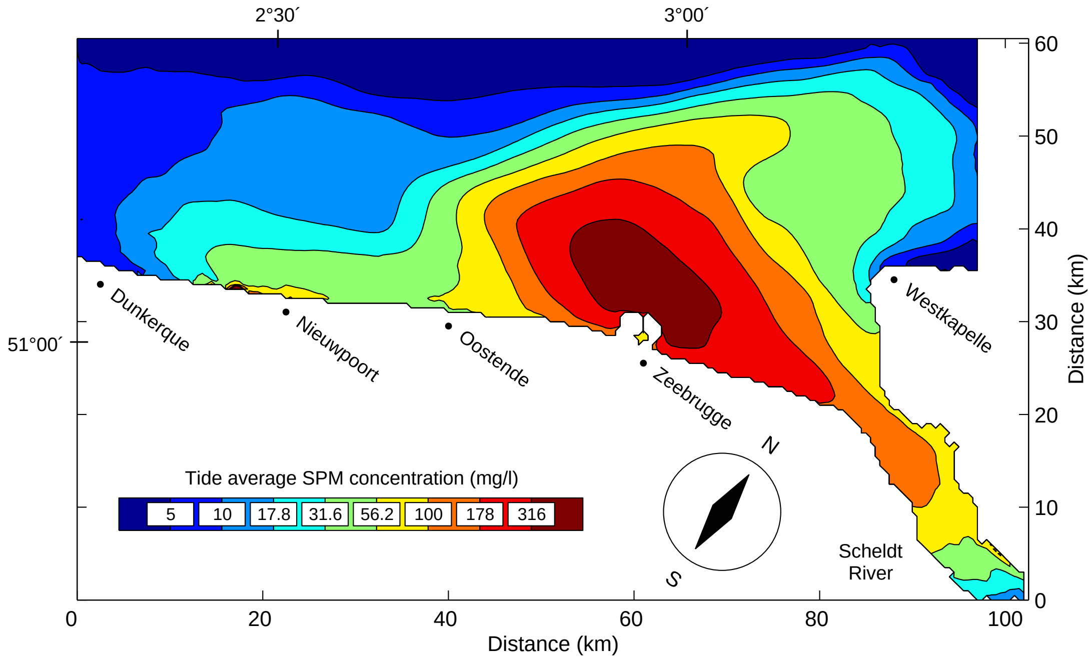
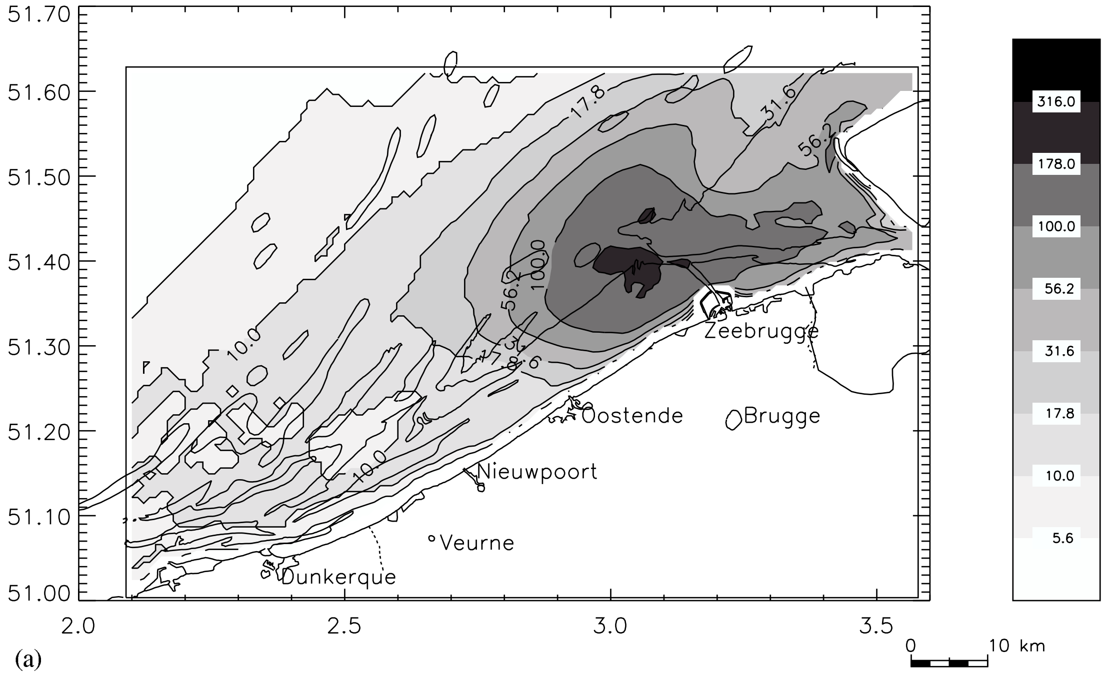
- τ_{cre} is the critical bottom shear stress for erosion. It is set to 0.5 *Pa* for freshly deposed mud (in the non-consolidated layer) and to 2 *Pa* for erosion in the parent bed layer.
- Erosion of the parent bed only occurs when the upper non-consolidated layer is completely eroded.

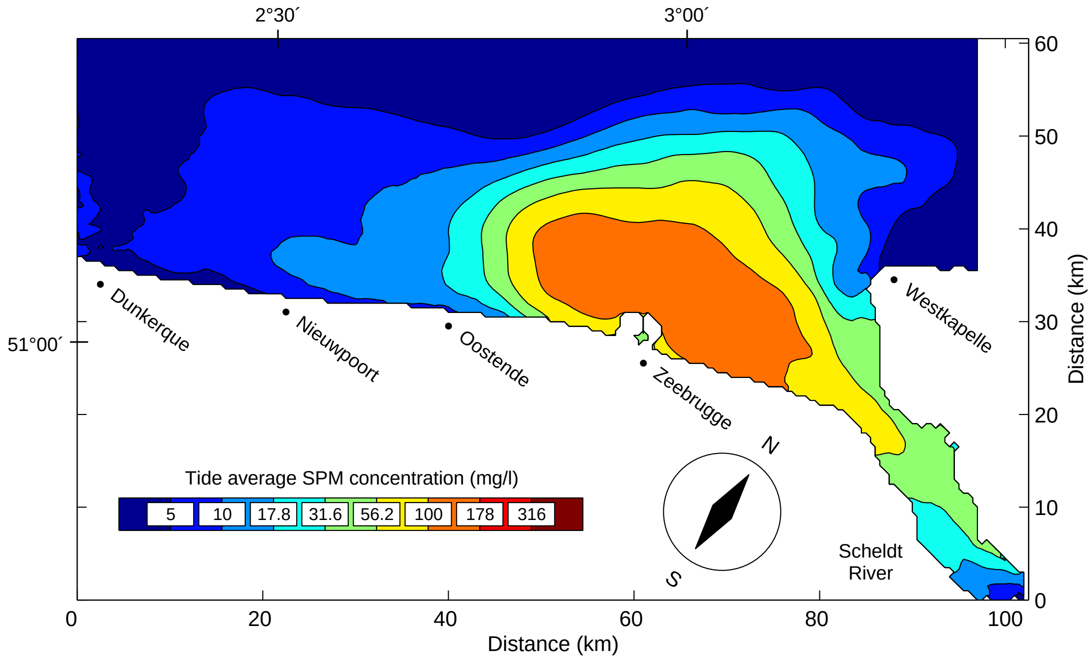
Validation

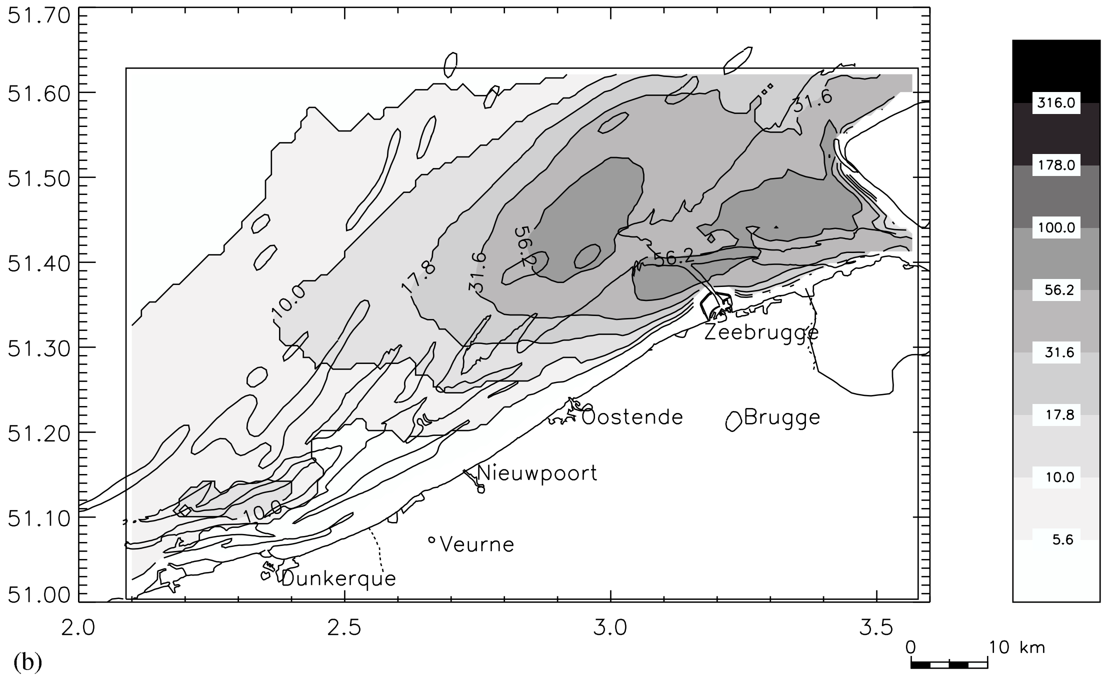




(a)

Validation





(b)

CART: sediment (1)

- New state variables: $\begin{cases} \alpha^i = C^i a^i \\ \alpha_s^i = C_s^i a_s^i \end{cases}$

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- Differential equations:

$$\frac{\partial \alpha^i}{\partial t} + \nabla \cdot \left[\left(\mathbf{v} + \mathbf{w}^i_s \right) \alpha^i \right] = C^i + \nabla \cdot \left(\mathbf{K} \cdot \nabla \alpha^i \right)$$

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- Sediments are eroded and deposed *with their ages*:

$$F^i_{\alpha,ero} = a^i_s F^i_{ero} \quad \text{and} \quad F^i_{\alpha,dep} = a^i F^i_{dep}$$

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- Equations coupled through the boundary condition:

$$\left[\left(\mathbf{v} + \mathbf{w}^i_s \right) \alpha^i - \mathbf{K} \nabla \alpha^i \right] \cdot \mathbf{n} = F^i_{\alpha,dep} - F^i_{\alpha,ero}$$

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- Quantification of the transport rate of mud across the BCZ

  $\Rightarrow$ age of a particle = the time elapsed since that particle passed through a source region S

  $\Rightarrow$ Advantages : circumvents the difficulties of diffusion process and implementation of a Lagrangian approach

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   - New variables $\tilde{C}^i$ and $\tilde{C}_s^i$
   - Additional conditions: $\tilde{C}^i = C^i$ and $\tilde{C}_s^i = C_s^i$ in S
   - At inflow open boundaries: $\tilde{C}^i = 0$ and $\tilde{C}_s^i = 0$
   - $\tilde{F}_{ero}^i = \frac{\tilde{C}_s^i}{C_s^i} F_{ero}^i$ and $\tilde{F}_{dep}^i = \frac{\tilde{C}^i}{C^i} F_{dep}^i$

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3. Compute the age contents of marked sediments $\tilde{\alpha}^i_s$ and $\tilde{\alpha}^i$ where

$$\tilde{F}^i_{\alpha,ero} = \frac{\tilde{\alpha}^i_s}{C^i_s} F^i_{ero} \quad \text{and} \quad \tilde{F}^i_{\alpha,dep} = \frac{\tilde{\alpha}^i}{C^i} F^i_{dep}$$

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4. Compute the age using the relations $\tilde{\alpha}^i = \tilde{C}^i \tilde{a}^i$ and $\tilde{\alpha}^i_s = \tilde{C}^i_s \tilde{a}^i_s$

transport resuspension

Conclusion

- The model was applied successfully to reproduce the mud behaviour in the Belgian coastal zone.
- The CART theory is a useful diagnostic tool to investigate the different behaviours of mud during deposition-resuspension events and transport.

Future developments:

- Pollutants and nutrients are transported preferentially in an absorbed state and tend to bind to the sediments

  ⇒ Their transport outside the coastal zone highly dependent on the sediments' dynamic.

  ⇒ Use this tool to quantify the transfer rate of contaminants through the behaviour of the suspended matter.

Contents