StrucOpt manuscript No.
(will be inserted by the editor)

Note on topology optimization of continuum structures including self-weight

Michael Bruyneel, Pierre Duysinx

Abstract This paper proposes to investigate topology optimization with density dependent body forces and especially self-weight loading. Surprisingly the solution of such problems can not be based on a direct extension of the solution procedure used for minimum compliance topology optimization with fixed external loads. At first the particular difficulties arising in the considered topology problems are pointed out: non-monotonous behaviour of the compliance, possible unconstrained character of the optimum and parasitic effect for low densities when using the power model (SIMP). To get of rid of the last problem requires to modify the power law model for low densities. The other problems require to revisit the solution procedure and the selection of appropriate structural approximations. Numerical applications compare the efficiency of different approximation schemes of the MMA family. It is shown that important improvements are achieved when the solution is carried out when using the Gradient Based Method of Moving Asymptotes (GBMMA) approximations. Criteria for selecting the approximations are suggested. In addition, the applications are also the opportunity to illustrate the strong influence of the ratio between the applied loads and the structural weight onto the optimal structural topology.

Key words topology optimization, self-weight, convex approximations, MMA

Received: date / Revised version: date

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1 Introduction

Since the work by Bendsøe and Kikuchi (1988), much research in topology optimization has been devoted to extend the basic minimum compliance design problem for a given volume of material to various problems and design criteria (for a review, see Bendsøe and Sigmund (2003) and Eschenauer and Olhoff (2001)).

To solve such large scale problems the optimization methods based on the *sequential convex programming* as defined by Fleury (1993) proved to be a general and flexible, but also efficient tool (Duysinx (1997)). The approximation concept consists in replacing the original optimization problem:

$$\begin{aligned} \min_{\mathbf{X}} \quad & g_0(\mathbf{X}) \\ \text{s.t.:} \quad & g_j(\mathbf{X}) \ \leq \ g_j^{max} \qquad j = 1 \dots m \\ & \underline{x}_i \ \leq \ x_i \ \leq \overline{x}_i \qquad i = 1 \dots n \end{aligned} \tag{1}$$

which is implicit in terms of the design variables $\mathbf{X} = \{x_i, i = 1 \dots n\}$, by the solution of a sequence of explicit and convex approximated sub-problems that are built based on a variant of a Taylor series expansion $\tilde{g}_j(\mathbf{X})$ of the involved design functions $g_j(\mathbf{X})$:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \tilde{g}_0^{(k)}(\mathbf{X}) \\ \text{s.t.:} \quad & \tilde{g}_j^{(k)}(\mathbf{X}) \ \leq \ g_j^{max} \qquad j = 1 \dots m \\ & \underline{x}_i^{(k)} \ \leq \ x_i \ \leq \overline{x}_i^{(k)} \qquad i = 1 \dots n \end{aligned} \tag{2}$$

where k is the iteration index. The local approximation techniques generally require to perform a structural and a sensitivity analysis for the computation of the function values and their derivatives. Introducing such an approximation techniques allows to decrease the number of structural analyses required to reach the optimum of the problem (1).

Several publications (see for examples Duysinx et al. (1995), Duysinx and Bendsøe (1998), Pedersen (2000), Sigmund (2001)) reported successful and efficient solution of topology problems while resorting to classical approximation schemes, such as CONLIN (the convex linearization method by Fleury and Braibant (1986)) and

MMA (the Method of Moving Asymptotes (MMA) derived by Svanberg (1987)).

The problem of optimal design for self-weight was first discussed by Rozvany (1977) for plastic design and later extended for any body forces by the same author in his book (Rozvany (1989)). Optimal design of with self-weight loading has been treated successfully for beam, arch and plate problems (e.g. Rozvany et al. (1980); Wang and Rozvany (1983); Karihaloo and Kanagasundaram (1987); Rozvany et al. (1988)), composite structures (e.g. Kwak et al. (1997)) or shape optimisation (e.g. Imam (1998)). Nevertheless surprisingly, there is a very little amount of published work dealing with topology optimization using homogenization and density-dependent body forces like self-weight, centrifugal loads, inertia loads, etc. To the authors' knowledge the only published results in topology optimization are Turteltaub and Washabaugh (1999) and Park et al. (2003). However taking into account density dependent loads is extremely important for the preliminary design of many structures : the self-weight of large civil engineering structures and the body forces coming from the centrifugal acceleration in rotating machines are indeed dominant effects. Therefore it is wondering that so little attention has been paid to this kind of applications while efficient solution procedures are now available for the standard problem of topology optimization with external dead loads.

In a recent communication (Bruyneel and Duysinx (2001)), the authors pointed out that the classical approach based on these procedures can be much less efficient and even fail when dealing with the compliance minimization of structures subjected to their own weight. The present paper investigates the nature of these problems and shows that the standard topology optimization procedure diverges mainly when it is applied to problems including density-dependent body forces.

The paper is organized as following. At first the formulation of topology problems including density dependent body-forces is stated (section 2) and the sensitivity analysis is briefly reminded (section 3). The solution procedure based on different MMA approximations is summarized in section 4.

In section 5, a case study shows the particular nature of the topology optimization problem including self-weight: non monotonous behavior of the compliance, possible unconstrained character of the optimum and parasitic effect due to the incorrect modeling of effective mechanical and mass properties in the vicinity of zero-density with the classical SIMP law.

Section 6 investigates the selection of appropriate approximation schemes for topology optimization if monotonous or non monotonous responses are present. The answer that is proposed here relies on an evolution of the Gradient Based MMA (or GBMMA) approximation procedure originally developed in the context of composite structure optimization (Bruyneel et al. (2002)). This procedure is based on several approximation schemes of the MMA family and an automatic strategy to select the most suited scheme. This new procedure stabilizes the optimization process and reduces the number of iterations to come to a stationary solution. Numerical applications developed in the paper put also into the light the influence of the topology upon the ratio between the dead loads and the density dependent loads.

2
Formulation of a topology optimization problem with density dependent body loads

The basic formulation of a topology optimization problem (Bendsøe and Sigmund (2003)) consists in minimizing the energy of the applied loads, called compliance, for a given volume fraction of the material.

Here the local mechanical properties of the material are parameterized with a power law model called SIMP model (see for instance Bendsøe (1989) and Zhou and Rozvany (1991)). The effective density ρ and the material stiffness E are related to the base material properties ρ_o and E_o for $p > 1$ according to:

$$\rho = \mu\, \rho_o \tag{3}$$

$$E = \mu^p\, E_o \tag{4}$$

The continuum mechanics problem is dicretized with the Finite Element Method. In this approach, $\mathbf{g}$ and $\mathbf{q}$ are respectively the vectors of node loads and node displacements, related by the structural stiffness matrix $\mathbf{K}$ through the equilibrium equation $\mathbf{Kq} = \mathbf{g}$. The density field is also discretized using a usual element by element constant density function. Thus a variable μ_i is attached to each finite element "i" of the model.

If one notes by $\mu = \{\mu_i, i = 1, ..., n\}$ the vector of continuous design variables and by V_i the volume of the i_{th} finite element, the formulation of the topology optimization including self-weight (or more generally density dependent body forces) can be stated as follows:

$$\begin{aligned} &\min_{\mu} \ \mathbf{C} = \mathbf{g}^T \mathbf{q} \\ &\text{s.t.:}\ \underline{V} \ \leq \ \textstyle\sum_{i=1}^{n} \mu_i\, V_i \ \leq \ \overline{V} \\ &\qquad \underline{\mu}_i \ \leq \ \mu_i \ \leq \overline{\mu}_i \qquad\qquad i = 1 \ldots n \end{aligned} \tag{5}$$

It is important to remark that in (5), a maximum $\overline{V}$ and a minimum $\underline{V} > 0$ bound on the volume of material are introduced. The minimum bound on the volume of material $0 < \underline{V}$ is necessary to reject the trivial solution $\mu_i = 0, \forall\, i$, which is feasible when a pure self-weight loading is considered, but which is non sense from an engineering point of view. In other words, well -posed engineering problems requires the presence of a non-design dependent load or of a non-structural mass (which may be due to the presence of a non zero minimum density).

Classically, the stiffness matrix $\mathbf{K}$ and the displacement vector $\mathbf{q}$ depend explicitly or implicitly on the density variables, while the load vector $\mathbf{g}$, which are point loads or surface tractions is a constant. The particularity of the present paper is to consider also body forces that depend on the density, that is here the structural self-weight. In the numerical applications, we consider 4-nodes quadrangular finite elements and a gravity load applied along the vertical Y direction so that one fourth of the weight of each finite element i_{th} is on each of its 4 nodes.

$$g_{i,X} = 0 \quad g_{i,Y} = -\mu_i \, \rho_o \, a_g \, V_i/4 \tag{6}$$

where a_g is the absolute value of the gravitational acceleration. The contributions of adjacent elements are summed at common nodes.

Finally to avoid well known numerical instabilities, i.e. mesh dependency and checkerboard patterns, that often happen in the solution of topology optimization problems, a filtering technique proposed by Sigmund (1997) is used for solving (5).

3 Sensitivity analysis

By considering the derivative of the equilibrium equation, it is comes that the sensitivity of the compliance C writes:

$$\frac{\partial C}{\partial \mu_i} \; = \; 2\, \mathbf{q}^T \frac{\partial \mathbf{g}}{\partial \mu_i} \; - \; \mathbf{q}^T \frac{\partial \mathbf{K}}{\partial \mu_i} \mathbf{q} \tag{7}$$

When $\partial \mathbf{g}/\partial \mu_i$ vanishes, the derivative (7) is always negative, and the structural behaviour of the compliance is then monotonous. This fact was exploited to build efficient update strategies based on optimality conditions in the first works devoted to topology optimization (see for example Bendsøe and Sigmund (2003)).

When density dependent loads are considered, the first term of (7) doesn't vanish anymore and it can be seen that the derivatives of the compliance (7) can be either positive or negative and even change sign when changing the value of the design variables. In this case, the compliance C can experience a non-monotonous character with respect to the considered design variable μ_i. As it will be shown later, this property raises big difficulties in the standard solution procedure.

4 Approximations of the MMA family

Because of their general character, the approximations based on the concept of moving asymptotes (Svanberg (1987) and Bruyneel et al. (2002)) are considered here to approximate the structural responses $g_j(\mu)$ involved in (5).

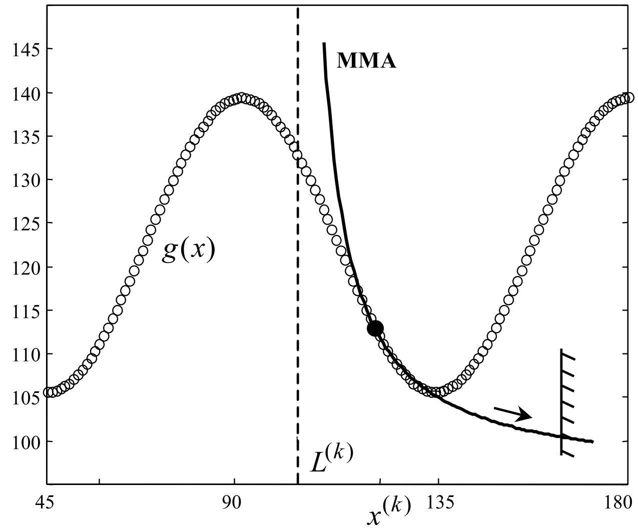

Fig. 1 Monotonous approximation around $\mathbf{x}^{(k)}$

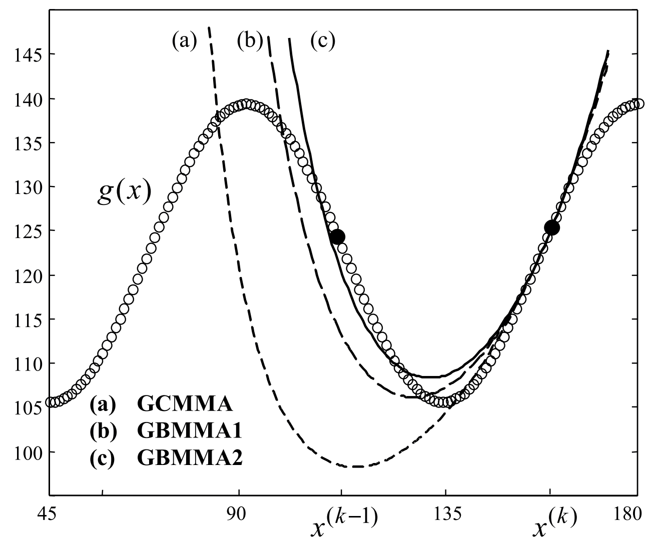

Fig. 2 Non monotonous approximations around $\mathbf{x}^{(k)}$

4.1 Non monotonous approximations

In the Globally Convergent version of MMA proposed by Svanberg (1995), each function $g_j(\mu)$ is approximated according to the following general expansion

$$\tilde{g}_j(\mu) = g_j(\mu^{(k)}) + \sum_{i=1}^{n} p_{ij}^{(k)} \left(\frac{1}{U_i^{(k)} - \mu_i} - \frac{1}{U_i^{(k)} - \mu_i^{(k)}}\right) + \sum_{i=1}^{n} q_{ij}^{(k)} \left(\frac{1}{\mu_i - L_i^{(k)}} - \frac{1}{\mu_i^{(k)} - L_i^{(k)}}\right) \tag{8}$$

$g_j(\mu^{(k)})$ is the function value at the current iteration k, whereas the parameters $p_{ij}^{(k)}$ and $q_{ij}^{(k)}$ are computed based on the first order derivatives, on the asymptotes $L_i^{(k)}$ and $U_i^{(k)}$, and on a non monotonic parameter $\rho_j^{(k)}$. At each iteration k, the asymptotes $L_i^{(k)}$ and $U_i^{(k)}$ are updated according to a heuristic rule that is the same as for the classical MMA, while the parameter $\rho_j^{(k)}$ is updated on the basis of a rule proposed by Svanberg to ensure the globally convergent character of the approximation. If the parameters $p_{ij}^{(k)}$ and $q_{ij}^{(k)}$ in (8) are positive, the approximation is convex. Because of the presence of parameter $\rho_j^{(k)}$ the approximation is non monotonous as illustrated in Fig. 2.

As shown in Bruyneel et al. (2002), the original Svanberg's GCMMA scheme can be much improved when exploiting the information at previous iteration points. In the Gradient Based MMA approximation schemes (or GBMMA), the gradients information from the previous iteration $k-1$ is used in place of $\rho_j^{(k)}$ to build (8).

For GBMMA1, $p_{ij}^{(k)}$ and $q_{ij}^{(k)}$ in (8) are determined by matching the first partial derivatives at the current and previous design points. They are analytically computed from the following set of equations:

$$\frac{\partial g_j(\mu^{(k)})}{\partial \mu_i} = \frac{p_{ij}^{(k)}}{(U_i^{(k)} - \mu_i^{(k)})^2} - \frac{q_{ij}^{(k)}}{(\mu_i^{(k)} - L_i^{(k)})^2} \tag{9}$$
$$\frac{\partial g_j(\mu^{(k-1)})}{\partial \mu_i} = \frac{p_{ij}^{(k)}}{(U_i^{(k)} - \mu_i^{(k-1)})^2} - \frac{q_{ij}^{(k)}}{(\mu_i^{(k-1)} - L_i^{(k)})^2}$$

In GBMMA2, the quality of the approximation (8) is improved by using an estimation of the diagonal second order derivatives (10) introduced for the first time in Duysinx et al. (1995). Determining parameters $p_{ij}^{(k)}$ and $q_{ij}^{(k)}$ of the scheme then relies on the first partial derivatives at the current design points and on the estimated second order diagonal derivatives (10).

$$\frac{\partial^2 g_j(\mu^{(k)})}{\partial \mu_i^2} \simeq \frac{\dfrac{\partial g_j(\mu^{(k)})}{\partial \mu_i} - \dfrac{\partial g_j(\mu^{(k-1)})}{\partial \mu_i}}{\mu_i^{(k)} - \mu_i^{(k-1)}} \tag{10}$$

It was observed on numerical tests that it is interesting to use GBMMA2 when the current design point is in the vicinity of the optimum, that is at the end of the optimization process. Indeed, it makes sense that in the final convergence stages, the use of second order information, even if estimated, improves the convergence speed. Based on this observation, the contribution of a given design variable μ_i in a given design function $g_j(\mu)$ can be approximated by GBMMA2 when the criterion (11) is verified:

$$\frac{|\mu_i^{(k)} - \mu_i^{(k-1)}|}{\overline{\mu}_i - \underline{\mu}_i} \leq \; SWITCH \tag{11}$$

Otherwise, GBMMA1 is used. This leads to consider the mixed non monotonous GBMMA1-GBMMA2 approximation, for $SWITCH \in]0,1[$.

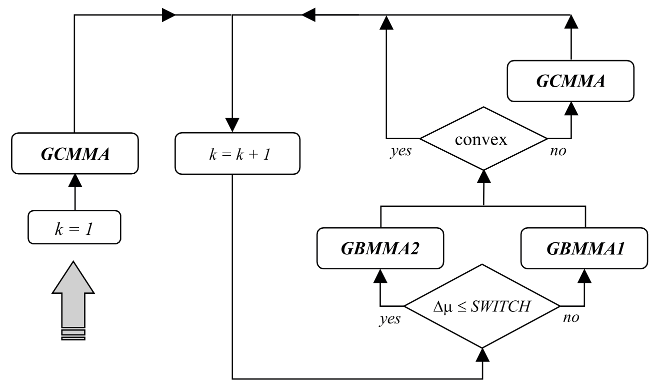

Fig. 3 Selection of the non monotonous approximation based on GCMMA, GBMMA1 and GBMMA2

When $p_{ij}^{(k)}$ and $q_{ij}^{(k)}$ computed by GBMMA1 or GBMMA2 are not positive, the approximation procedure switches automatically back to a classical GCMMA to keep a convex approximation. The automatic selection of the non monotonous convex approximation based on (11) is summarized in Fig. 3.

4.2 Monotonous approximations

Monotonous approximations like MMA or CONLIN can also been recovered as special cases of the more general non monotonous approximation GCMMA. For these approximations, only one asymptote is used at a time, which means that depending on the sign of the first derivatives, either $p_{ij}^{(k)}$ or $q_{ij}^{(k)}$ is set to zero.

The classic Method of Moving Asymptotes (Svanberg (1987)), which is illustrated in Fig. 1, is given by

$$\tilde{g}_j(\mu) = g_j(\mu^{(k)}) + \sum_{+,i}^{n} p_{ij}^{(k)} \left(\frac{1}{U_i^{(k)} - \mu_i} - \frac{1}{U_i^{(k)} - \mu_i^{(k)}}\right) + \sum_{-,i}^{n} q_{ij}^{(k)} \left(\frac{1}{\mu_i - L_i^{(k)}} - \frac{1}{\mu_i^{(k)} - L_i^{(k)}}\right) \tag{12}$$

In addition, a move-limits strategy proposed by Svanberg (1987), is necessary to restrict the range of variation of the design variables during the optimization process.

Furthermore, by forcing $L_i^{(k)} = 0$ and $U_i^{(k)} \to \infty$, MMA is reduced to Convex Linearization scheme CONLIN proposed by Fleury and Braibant (1986):

$$\tilde{g}_j(\mu) = g_j(\mu^{(k)}) + \sum_{+,i} \frac{\partial g_j(\mu^{(k)})}{\partial \mu_i}(\mu_i - \mu_i^{(k)}) - \sum_{-,i} (\mu_i^{(k)})^2 \frac{\partial g_j(\mu^{(k)})}{\partial \mu_i}\left(\frac{1}{\mu_i} - \frac{1}{\mu_i^{(k)}}\right) \tag{13}$$

For CONLIN approximation, a move-limit strategy (14) is also used:

$$\underline{\mu}_i \leq \; \mu_i^{(k-1)} - \Delta\mu \leq \; \mu_i \leq \; \mu_i^{(k-1)} + \Delta\mu \leq \; \overline{\mu}_i \tag{14}$$

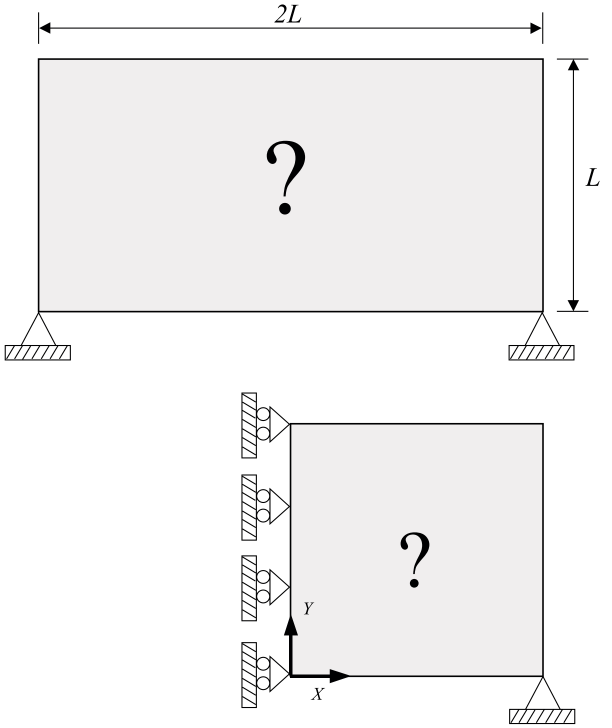

Fig. 4 Design domain and supports. Minimum density of 0.2

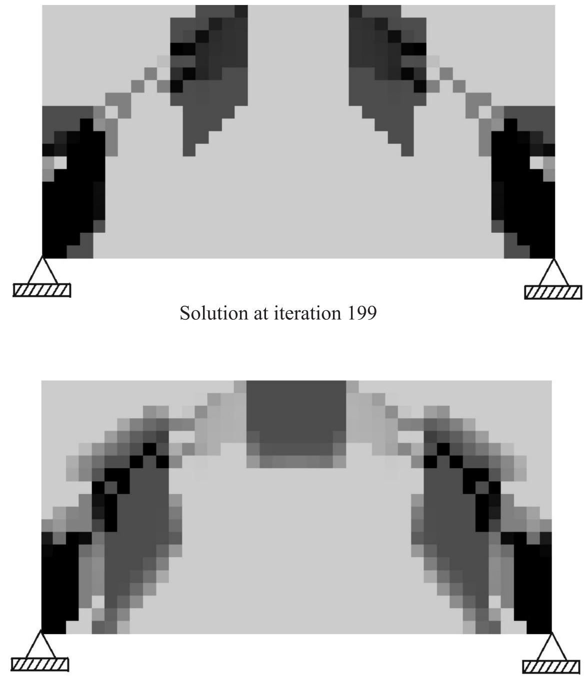

Fig. 5 Solutions obtained with CONLIN at iterations 199 and 200

5
A case study including the self-weight

In order to distinguish more clearly the difficulties arising with topology optimization including self-weight, we consider two versions of the same test case, in which the different features are successively introduced.

In the first version, the minimum allowable pseudo-density is arbitrarily set to 0.2 (reinforcement problem), in order to prevent the appearance of problems related to the non consistent behaviour of the power law model for low density. This problem allows to point out the non monotonous character of the compliance and to show that this property is the origin of failure of optimization procedures based on monotonous approximations.

Then in the second version, the minimum density is set to the usual value of 0.01 to push forward the problem related to the numerical "artifact" of the SIMP modeling of mechanical properties in the vicinity of zero-density.

The test case illustrated in Fig. 4 consists in designing a structure that relies on two supports, while supporting its own weight. Intuitively an arch type structure is expected. The reference length L is $L = 1m$. Due to symmetry conditions, only one half of the design domain is studied and is discretized with 20×20 4-node quadrangular finite elements of 8 degrees of freedom. The mechanical properties of the base material to be distributed in the domain are: $E_o = 1N/m^2$, $\nu = 0.3$ and $\rho_o = 1kg/m^3$, while the gravitational acceleration a_g is $9.81kgm/s^2$. The exponent p in (4) is equal to 2. The maximum available amount of material $\overline{V}$ at the solution is 80%, while the minimum amount of material $\underline{V}$ is set to 1%.

The stopping criteria adopted is based on the maximum variation of the design variables over two design steps where $TOL = 0.0001$:

$$\max_{i=1\ldots n} |\mu_i^{(k)} - \mu_i^{(k-1)}| \leq TOL \tag{15}$$

5.1
Non monotonous behaviour of the compliance

The particular behaviour of the compliance when self-weight is considered is illustrated on the reinforcement problem with a minimum density of ($\underline{\mu} = 0.2$).

At first CONLIN, a monotonous approximation is used to solve the problem. The parameter $\Delta\mu = 0.5$ is used in the move-limit strategy of (14). After a large number of iterations (200), the optimization process is still not convergent and the values of the compliance and of the volume oscillate from one iteration to another (see Fig. 6). As suggested by Fig. 5, which gives the material distribution at iterations 199 and 200, the problem stems from the oscillation of several density design variables. (A grey scale is used for representing the emerging structure: black is solid ($\mu = 1$) and white is the void ($\mu = 0.$)).

Although CONLIN and MMA have been used successfully many times for the solution of topology problems (e.g. Duysinx and Bendsøe (1998); Pedersen (2000); Sigmund (2001)), monotonous approximations are not

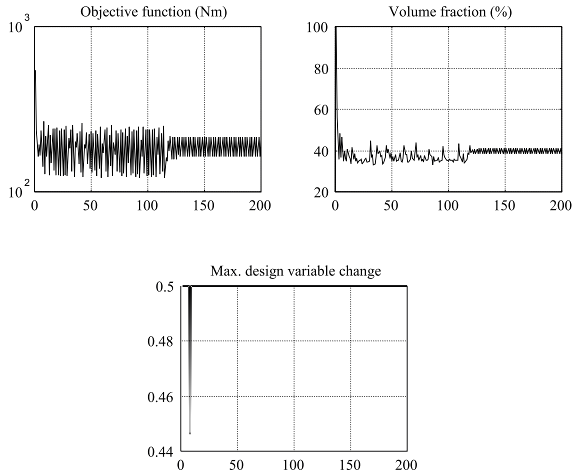

Fig. 6 Iteration history with CONLIN limited to 200 iterations

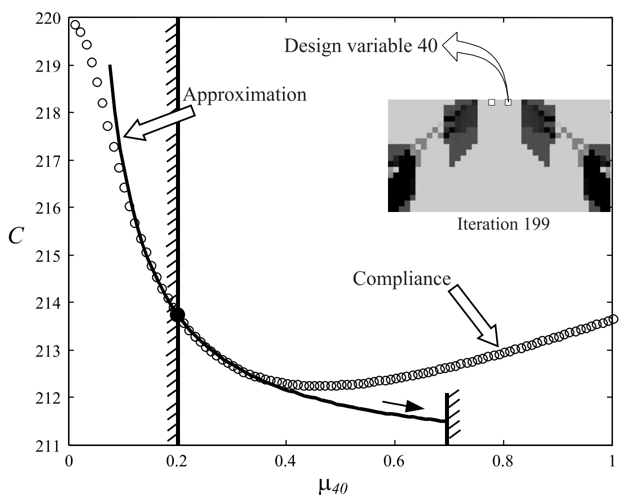

Fig. 7 Evolution of the compliance according to the design variable 40. Monotonous approximation at iteration 199 and bounds on the design variable

well suited to the problem considered here. The reason is the non monotonous behaviour of the compliance with respect to some design variables when density dependent loads are considered. In Fig. 7, for instance one plots the behaviour of the compliance with respect to design variable 40, which oscillates between its lower and upper bounds. Its is clear that the compliance is non monotonous with respect to this variable. Using monotonous approximations, the minimum of the subproblem with respect to this variable is only governed by alternatively the upper and lower bounds limiting the range of variation of the variable. Oscillations appear and the optimization process does not converge.

This kind of trouble can be avoided with non monotonous approximations like GCMMA (Svanberg (1995)). Fig. 8 shows the solution of the problem under study when using GCMMA. The convergence becomes smooth and a nice optimum distribution is reached. However

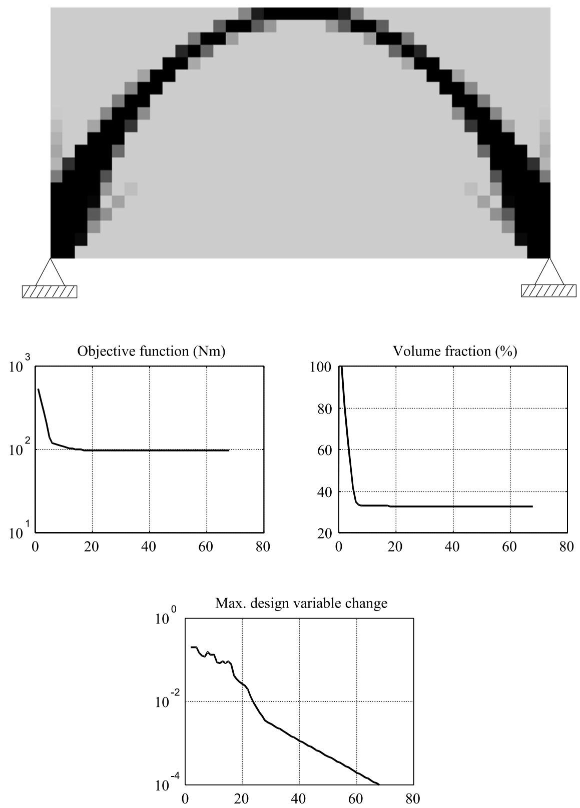

Fig. 8 Solution obtained with GCMMA

the solution might be very slow. It will be shown later that convergence speed can be improved by using Gradient Based MMA approximations, that are also non monotonous.

5.2 Remark on the volume restriction

It is also interesting to remark that optimal solutions of problems with design dependent loads can be unconstrained. Here, the volume fraction of material used in the optimal solution remains stuck at 32.5% of the available design domain even if it is allowed to take a larger value, e.g. 80% as illustrated in Fig. 9. This is another particularity of the topology optimization including density dependent loads already observed in optimal topology problems of rotating bodies by Turteltaub and Washabaugh (1999). This property is possible because of the non monotonous character of the compliance.

5.3 Modeling of material properties for low densities

The problem related to the solution of non monotonous problems being fixed by resorting to GCMMA or any non monotonous approximation, one can detect another difficulty of self-weight topology problems. It is related

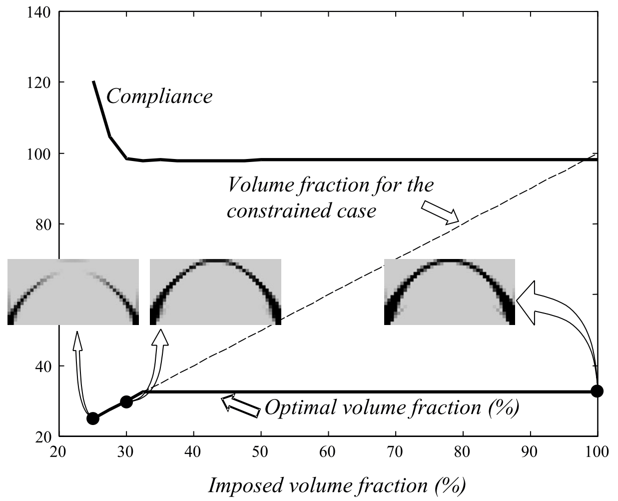

Fig. 9 Evolution of the volume fraction at the solution

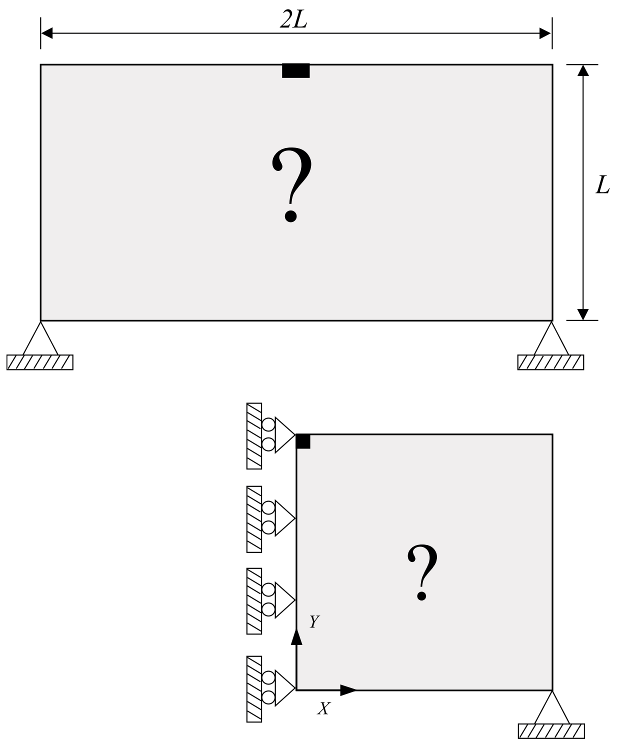

Fig. 10 Design domain, supports and non structural mass at the top. Minimum density of 0.01

to the material model based on a power law when densities are close to zero. This is illustrated on the second version of the test-case in which the lower bound of design variables is very small ($\underline{\mu} = 0.01$). The test case is no longer a reinforcement one, so a non structural mass, which is placed at the top of the structure is necessary to ensure that the problem is well-posed (Fig. 10).

Although GCMMA is used, it is seen in Fig. 11 that an undesirable effect appears in the solution of the problem: erratic intermediate density patterns alter the final topology. The explanation is the following. The ratio between the weight $\mathbf{g} \div \mu$ and the stiffness $\mathbf{K} \div \mu^p$ becomes infinite when the effective pseudo-density tends to zero, which means that the displacements and the compliance become unbounded in low density regions. The algorithm tries to fix the problem by letting some material to reduce the uncontrolled node displacements. This is total artificial from an engineering point of view.

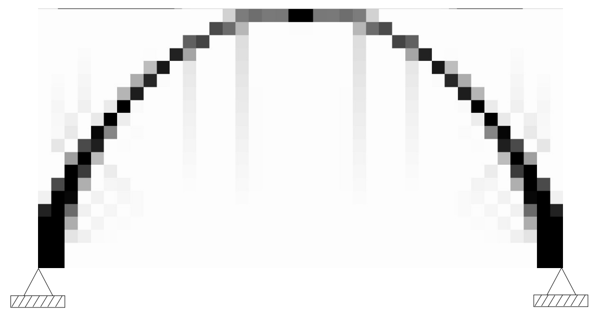

Fig. 11 Solution obtained with GCMMA for a minimum density of 0.01

Following the work done by Pedersen (2000, 2001) for eigenvalues and prestressed problems, the parameterization (4) can easily be modified to avoid this undesirable effect. A linear profile is selected under a given pseudo-density μ_C, as illustrated in Fig. 12. A threshold of $\mu_C = 0.25$ proved to be efficient in our numerical applications. The modified model then takes the form (16).

$$
\begin{aligned}
\rho_i &= \mu_i \, \rho_0 \qquad 0 < \underline{\mu}_i \leq \mu_i \leq 1 \\
E_i &= \begin{cases} \mu_i^p \, E_0 & \mu_C < \mu_i \leq 1 \\ \mu_i \, (\mu_C^{p-1} \, E_0) & 0 < \underline{\mu}_i \leq \mu_i \leq \mu_C \end{cases}
\end{aligned} \tag{16}
$$

The modified interpolation law limits the ratio between the weight load and the stiffness to a given finite value for low densities, and stabilizes the optimization process.

One may be worried about the fact that the relationship (16) is now non-differentiable whereas our solution algorithm is gradient based. However while theoretically this is a problem, practically we experienced non problem during numerical applications. Indeed, the non-smooth point is not a solution point, and there is no change of sign of the derivative, so the algorithm just goes through the non-smooth points. A more elegant solution could be found by adopting the alternative interpolation model proposed by Stolpe and Svanberg (2001), which is smooth everywhere and which has always a positive (non zero) slope at zero density. The particular choice of the interpolation law has no influence upon the conclusions of this paper. The chosen modification of the SIMP model (16) has just the advantage to keep on working with the power law that is very popular.

The solution of the arch problem with the modified SIMP law (16) and GCMMA is proposed in Fig. 13. When compared to the material distribution of Fig. 11,

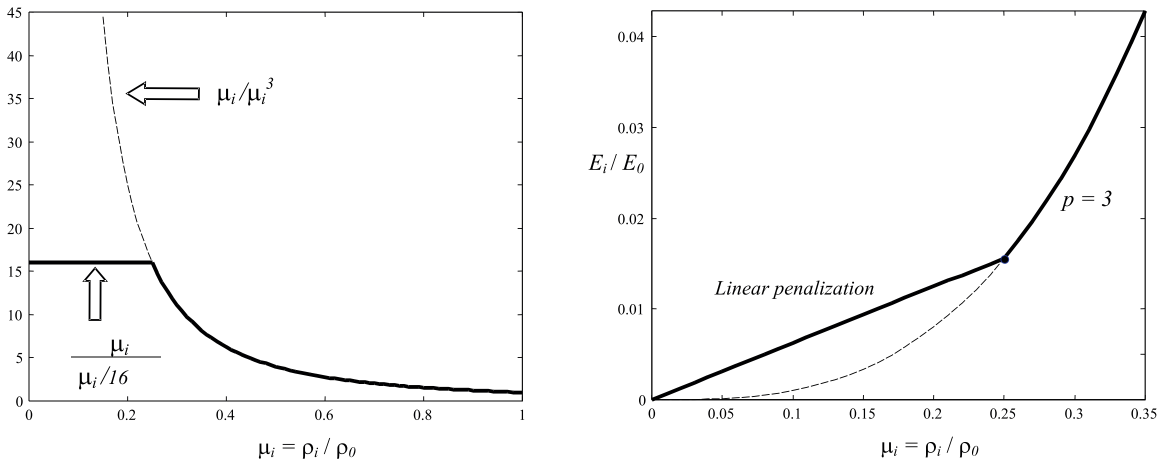

Fig. 12 Penalizations of the intermediate densities. $\mu_C = 0.25$ for the modified material parameterization

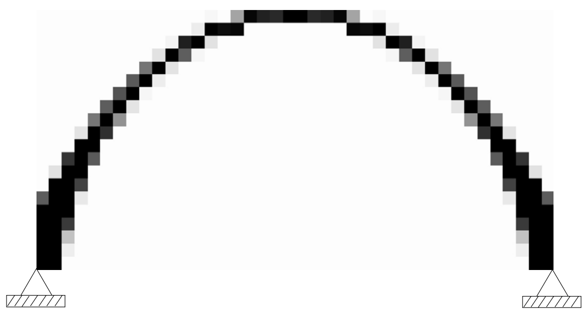

Fig. 13 Optimal topology for the arch problem when using GCMMA and the modified SIMP law

Fig. 14 Topology obtained by CONLIN at iteration 199 (modified SIMP law)

the optimal material distribution is free of parasitic appendices in the low density regions.

Finally it is interesting to verify with Fig. 14 and 15 that the optimization process still diverges with CONLIN even if the modified SIMP law is used. Oscillations of the design variables appear during the optimization process: some of them take their values successively at the upper bound and at the lower bound defined in (13), where $\Delta\mu = 0.3$. Obviously the modified SIMP law doesn't remove the solution problem discussed previously and the solution still requires the use of non monotonous approximations. Thence the two difficulties pointed out here are definitively two independent problems and must be treated separately.

Fig. 15 Topology obtained by CONLIN at iteration 200 (modified SIMP law)

6
Improving solution performances with GBMMA

We now show that there is great interest in using the recent Gradient Based MMA approximations to solve delicate topology optimization problems like self-weight loaded problems. Three numerical applications are proposed and solved with different approximation schemes from the MMA family. The performances of the different approximations are compared on three applications: the arch problem already explored previously, the beam structure (that is a variant of the so called MBB beam) and a bridge design problem. The main comparison criterion here is the number of iterations since all the optimization techniques under study require the some the same number of F.E. and sensitivity analyses at each iteration. In the following applications, the stopping criteria is satisfied when the maximum variation of the design variables is lower than a user defined precision TOL in (15). This parameter will be varied between 0.01 and 0.0001. For all applications, the gravity acts from

Table 1 Number of iterations needed for solving the arch problem for different values of TOL in (15). GBMMA1-GBMMA2 is related to Fig. 3 ($SWITCH = 0.2$)

Approximations	0.01	0.001	0.0001
MMA	130	402	438
GCMMA	80	200	253
GBMMA1	51	98	130
GBMMA2	73	109	139
GBMMA1-GBMMA2	54	91	112

top to bottom and the gravitational acceleration a_g is $9.81kgm/s^2$.

6.1 Arch structure

The problem of Fig. 10 is considered, where a non structural mass is placed at the top to load the structure. The data of the problem are the same as in the previous sections. $L = 1m$ is a reference length. The mechanical properties of the base material to be distributed in the domain are: $E_o = 1N/m^2$, $\nu = 0.3$ and $\rho_o = 1kg/m^3$ while an an exponent $p = 2$ is selected in the SIMP law.

At first we remind the reader that when CONLIN is used, no optimal topology can be obtained. The topology changes from one iteration to an other (Figs. 14 and 15) and there are oscillations of the design variables during the optimization process. Such a monotonous approximation is definitively not efficient for solving this non monotonous problem.

For the other approximations described in this paper: MMA, GCMMA, GBMMA1, GBMMA2 and GBMMA1-GBMMA2, a solution can always be reached (similar to Fig. 13). Although MMA gives rise to monotonous approximations of the design functions, it is able to come to an optimal topology, thanks to a robust move-limits strategy suggested by Svanberg (1987). However, as reported in Table 1 for different values of the precision TOL in (15), MMA requires a lot of iterations: twice more than GCMMA and nearly four times more than the best GBMMA. For self-weight problems, the non monotonous approximations are obviously much more efficient, especially when gradients from the previous iteration are used as in GBMMA approximations. According to the results of Table 1, GBMMA is always faster than GCMMA. The best results are obtained with the automatic strategy combining GBMMA1 and GBMMA2. In this case, the mixed GBMMA1-GBMMA2 scheme is nearly twice faster than GCMMA.

6.2 Beam structure

The second application given in Fig. 16 is a variant of the classic MBB beam. Due to symmetry conditions, one

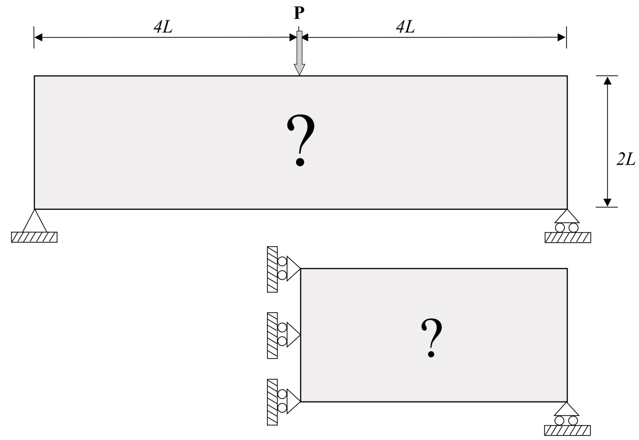

Fig. 16 MBB beam: design domain, supports and applied load

half of the design domain is discretized with 40×20 quadrangular finite elements of 8 degrees of freedom. The mechanical properties of the base material to be distributed in the domain is characterized by a Young modulus of $E_o = 100N/m^2$, $\nu = 0.3$ and $\rho_o = 1kg/m^3$. A penalization $p = 3$ is chosen in the SIMP model. The obtained volume fraction of material at the optimal solution is 24% for the problem including only self-weight. For an objective comparison, this value is taken as the bound $\overline{V}$ for the volume constraint (5) assigned to the problems including external loads.

When self-weight is not taken into account, the amplitude of the load P has no influence on the resulting optimal topology, shown in Fig. 17. In this case, 103 iterations are needed to reach the solution with the monotonous MMA approximation, while 229 design steps are required for GCMMA (with $TOL = 0.01$). This can be attributed to the too conservative character of the non monotonous GCMMA in solving such a classical topology optimization problem without self-weight loads. In the case of fixed loads (i.e. non design dependent), we recover the usual conclusion that MMA works very well.

Conversely when only self-weight is considered in the design problem, MMA, GCMMA, GBMMA1 and GBMMA2 take respectively 121, 112, 76 and 55 iterations to find the optimal topology (with $TOL = 0.01$) and it is clear that there is a strong advantage in using non monotonous schemes.

Let's now investigate situations in which the self-weight and the applied load P are considered simultaneously in the design problem. As illustrated in Fig. 17 a first conclusion is that the resulting topology depends on the ratio between the applied load and the structural weight at the solution. It is seen that when the structural weight becomes preponderant in comparison to the applied load, the stiffeners under the load disappear and the shape of the structure tends to be an arch, which makes sense from an engineering point of view.

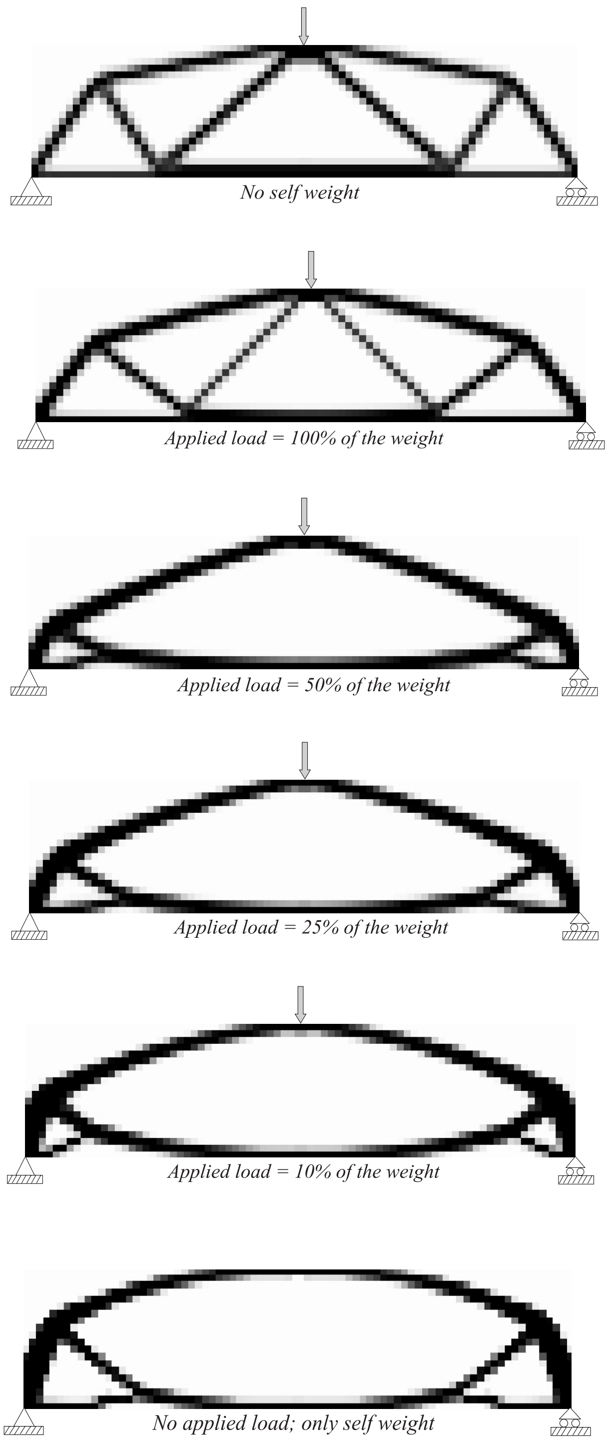

Fig. 17 Optimal topologies for different ratios between the applied load and the self-weight

It is then interesting to look at the solution effort required by the different algorithms. The number of iterations needed to reach the solutions is given in Table 2 in function of the ratio between the fixed load P and the self-weight of the structure (50% means that the applied load is 50% of the total structural weight at the solution). Although GCMMA gives rise to non monotonous approximations of the structural functions, it is sometimes not able to reach the optimum within a small number of iterations. This behaviour can be related to a sometimes too conservative character of the GCMMA approximation as admitted by its author (Svanberg (1995)). This behaviour can not be predicted: it depends on the 'good' choice of initial values of internal parameters of the algorithms in regards to the problem characteristics. This is a disadvantage of this scheme.

Table 2 Beam problem: Number of iterations required to solve the problem with $TOL = 0.01$ in (15). GBMMA1-GBMMA2 with $SWITCH = 0.2$

Load/weight	MMA	GCMMA	GBMMA1-GBMMA2
200%	90	176	101
100%	171	109	80
50%	123	273	106
25%	182	357	117
10%	176	180	120

In every situation, the number of structural analyses required to get the optimum is reduced when resorting to GBMMA approximations and using the information from the previous iteration. The performances of the mixed GBMMA1-GBMMA2 approximation procedure (with the parameter $SWITCH = 0.2$) is outstanding for all problems. Furthermore the number of iterations remains stable independently of problem characteristics, which is even better from a practical point of view for industrial applications.

6.3
Bridge structure

The definition of last application concerning the design of a bridge structure is given in Fig. 18. A non structural mass is placed at the top to load the structure to take into account the weight of the road surface. Because of symmetry, only one half of the design domain is studied and discretized with 20×20 quadrangular finite elements of 8 degrees of freedom. The mechanical properties of the base material to be distributed in the domain are: $E_o = 1N/m^2$, $\nu = 0.3$ and $\rho_o = 1kg/m^3$. The optimal topology is presented in Fig. 19. It is composed of an arch reinforced by additional vertical pillars.

A comparison of the performances of the different approximation schemes MMA, GCMMA and GBMMA1-GBMMA2 is reported in Table 3. In these numerical applications, the performances of GBMMA1-GBMMA2 are still extremely good, even if the superiority of GBMMA compared to MMA and GCMMA is not as large as in the other applications. Nonetheless, the most important thing is that GBMMA always provides one of the best solutions, which demonstrates the reliable character of this solution procedure.

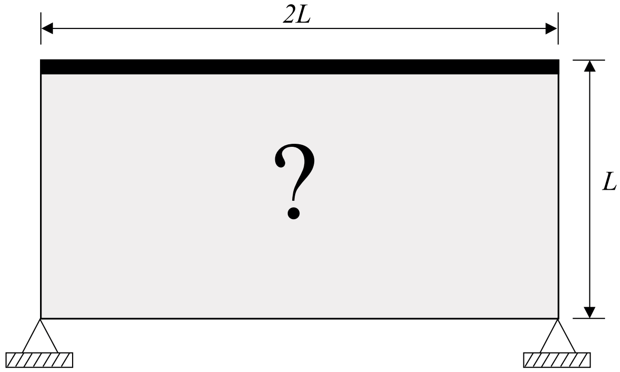

Fig. 18 Bridge structure: design domain, supports and non structural mass at the top

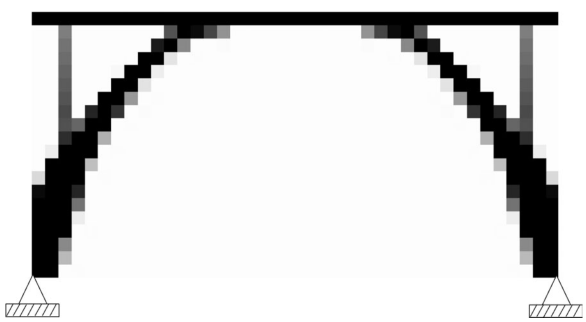

Fig. 19 Optimal topology for the bridge problem

Table 3 Bridge problem: number of iterations required to solve the problem. GBMMA1-GBMMA2 with $SWITCH = 0.2$

TOL	MMA	GCMMA	GBMMA1-GBMMA2
0.01	71	67	58
0.001	87	83	65
0.0001	109	97	101

7 Conclusions

The solution of topology optimization including the self-weight, and more generally of density dependent body loads, is not a direct extension of the classical design problems. The particularities of topology optimization including the self-weight and the difficulties in the solution of the related compliance minimization problem were presented: possible unconstrained character of the optimum, parasitic effect for low densities and non monotonous behaviour of the compliance. As the power law model is not appropriate for self-weight loading and density dependent body forces, a modification of the SIMP model in low density part was proposed and validated on numerical applications. But the major contribution of this work concernes some proposals for an efficient solution procedure. A comparison of different approximation schemes of the MMA family has been performed. When the self-weight of the structure is predominant in the problem, CONLIN and MMA approximations can diverge or converge very slowly, and a non monotonous approximation like GCMMA is advised. For classical topology optimization (with fixed external loads), monotonous approximations (e.g. MMA) remain reliable and generally faster for solving the design problem. However, in all cases, the recent GBMMA schemes using the gradient information at previous iteration points has been constantly superior to both MMA and GCMMA in terms of number of iterations and reliability. This scheme should therefore be preferred to the other ones. Finally, numerical applications has shown that considering the self-weight in the optimization process can strongly influence the optimal topologies.

Acknowledgements Professor Claude Fleury (University of Liège) is gratefully acknowledged for making the optimizer CONLIN available

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