Fast contingency filtering based on linear voltage drop estimates

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Abstract—This paper deals with the filtering of harmless contingencies in voltage stability and security analyses. In many systems, a post-contingency load flow allows to identify the contingencies with severe impact on voltage stability, as causing either divergence or large voltage drops. However, for filtering purposes, accurate voltage drops need not be computed; linear estimates obtained from sensitivity formulas are appropriate. For a given contingency, the method used in this paper solves a sparse linear system to update the phase angles, assuming constant voltage magnitudes. Then, assuming constant active power flows in the branches, a second sparse system is solved to update the voltage magnitudes. This yields better accuracy than one full load flow iteration while retaining the sparse structure of a decoupled formulation. For contingency analysis, the incidents are filtered by comparing the voltage drops to a threshold. For secure power margin computations, the same procedure is used after stressing the system in the specified direction. The method has been tested on a real system where it has been found to combine simplicity of implementation, quality of filtering and computational efficiency.

Index Terms—Voltage stability, voltage security assessment, contingency filtering, sensitivity analysis, electrical decoupling, CRIC method.

I. INTRODUCTION

Contingency filtering is an essential step of power system security analysis, needed to discard the numerous contingencies with little impact on the system, which would slow down the analysis. This step is even more needed in real-time applications, when numerous (e.g. N-2) contingencies are involved, or when time simulations are used to assess the system response.

A great part of publications on contingency filtering date back to the 80’s. At that time the emphasis was on contingency analysis within the context of static security, the objective being to cut down the computational effort of repeated load flow computations without losing accuracy.

In [1], [2], the DC load flow was used to compute performance indices in order to rank contingencies with respect to their impact on the system. While the DC approximation is often appropriate for identifying branch overloads, more refined methods are needed to deal with voltage magnitudes.

To this purpose, linear approximations of the AC load flow equations were considered in a simple contingency filtering technique which consists of performing a single P-θ followed by a single Q-V iteration of the fast decoupled load flow [3]. In [4], the Q-V iteration was replaced with a fast Q-V iteration, solved only for a subset of voltage sensitive buses, determined with a method inspired of the concentric relaxation [5]. A direct ranking method for voltage contingency selection was proposed in [6], using a second-order performance index which can be computed without determining post-contingency bus voltages.

Experience has shown that contingency ranking is heavily dependent on the performance index used. In particular, it may be prone to masking problems, such as ranking a contingency causing many small limit violations equally with one leading to few large limit violations. To reduce masking problems, it may be required to choose appropriate weighting factors in the index [4].

In the meantime, the computational power has increased dramatically, and dynamic security assessment can now be envisaged in real-time [7], [8]. In this context, the objective of contingency filtering has somewhat shifted to reducing the computational effort of repeated time domain simulations.

This paper focuses on Voltage Security Assessment (VSA). VSA methods fall in basically two categories [9], [10]:

- Contingency analysis consists of assessing the impact of credible disturbances on the system, the focus being on voltage drops experienced by transmission buses;
- Security margin computation consists of evaluating how far the system can be stressed (for instance, how much power can be consumed or transferred) before its response to a specified set of contingencies becomes unacceptable.

The rationale behind the method presented in this paper is as follows:

- in (not all but) many practical cases, a post-contingency load flow allow to identify contingencies with significant impact on long-term voltage stability. Indeed, load flow equations with constant power loads and enforcement of generator reactive power limits correspond to the long-term equilibrium that prevails after load voltages have been restored by Load Tap Changers (LTCs) and machine rotor (or stator) currents have been limited. Insofar as voltage instability results from the loss of such an equilibrium, the corresponding load flow equations no longer have a solution and the Newton-Raphson iterations diverge;
- on the other hand, divergence may result from purely numerical results. Furthermore, some dynamic controls helping stability cannot be taken into account in the static load flow calculation. Conversely, instability may
result from a dynamic behaviour that cannot either be accounted. To compensate for these limitations, when using a load flow computation, it is appropriate to label potentially harmful those contingencies causing some voltages to drop by more than some value, in addition to those causing divergence:

- to this purpose, accurate post-contingency voltages need not be computed; estimates obtained from the already mentioned linearized load flow equations may be appropriate to filter out the harmless contingencies. To the authors’ knowledge, however, few publications report on the performance of these simple linear methods in the context of voltage stability studies where voltages may experience large drops.

In the VSA context, attention has been paid more recently to contingency filtering for security margin computation. Here, the objective is to identify those contingencies which leave the system with small security margins in terms of power transfer rather than evaluating their impact in the base case situation (i.e. without power transfer).

The power margins considered in most publications refer to Post-Contingency Loadability Limits (PCLLs) [9], and indicate how much power can be transferred after the contingency has occurred.

In this context, two types of approaches may be distinguished: those in which the contingency is simulated explicitly and post-contingency system information is used [11], [12], [13] and those relying on first or second order sensitivity information [14], [15], [16]. Reference [14], for instance, proposed to estimate the PCLL through a well-known formula giving the sensitivity of load power margin to parameters. The latter involves the left eigenvector of the zero eigenvalue at the saddle-node bifurcation point. Nevertheless, the approximate character of this linearized formula for large changes in parameters may not justify the involved eigenvector computation.

In this paper, Secure Operation Limits (SOLs) are considered rather than PCLLs to estimate security margins. The former indicate how much power can be transferred in the pre-contingency configuration until one of the specified contingencies becomes harmful [9]. The information provided by SOLs is closer to the need of system operators. In addition, contingency filtering can be easily embedded in the binary search used to compute SOLs [8].

This paper proposes to use linear voltage drop estimates for filtering purposes in both contingency analysis and security margin determination. Furthermore, the so-called CRIC technique initially proposed by Carpentier in [17] has been considered to obtain the linearized changes with high computational efficiency.

The paper is organized as follows. The linear approximation is presented in Section II and its application to VSA in Section III. Results on a real-life system are reported in Section IV, while concluding remarks are offered in Section V.

II. LINEARIZED ANALYSIS OF CONTINGENCIES

A. Brief review of linear methods

Let the traditional power flow equations be written in compact form as:

\[ p^o - f^o(v^o, \theta^o) = 0 \]
\[ q^o - g^o(v^o, \theta^o) = 0 \]

where \( p^o \) and \( q^o \) are the active and reactive power injections, \( f^o \) and \( g^o \) are well-known functions, and superscript \( o \) refers to the base case situation. Let the corresponding post-contingency equations be written as:

\[ p - f(v, \theta) = 0 \] (1)
\[ q - g(v, \theta) = 0 \] (2)

where \( p \) and \( q \) account for generator trippings and \( f \) and \( g \) for branch trippings. We seek to obtain a good estimate \( \Delta \theta \) (resp. \( \Delta v \)) of the exact change in phase angles \( \theta - \theta^o \) (resp. voltage magnitudes \( v - v^o \)).

A simple approach consists of relying on a Taylor series expansion of \( f \) and \( g \) around \((v^o, \theta^o)\):

\[ p - f(v^o, \theta^o) - f_\theta \Delta \theta - f_v \Delta v = 0 \] (3)
\[ q - g(v^o, \theta^o) - g_\theta \Delta \theta - g_v \Delta v = 0 \] (4)

where \( f_\theta \) denotes the Jacobian matrix of \( f \) with respect to \( \theta \), and similarly for the other matrices.

Equations (3, 4) are nothing but the first iteration of the Newton-Raphson algorithm initialized from \((v^o, \theta^o)\). To gain computing time, it has been proposed to estimate \( \Delta v \) and \( \Delta \theta \) from the first two half-iterations of the fast decoupled version of this algorithm. Namely, in the first half-iteration, the sensitivity of active power to voltage is neglected (usual DC approximation). Simplifying and reorganizing (3) yields:

\[ f_\theta \Delta \theta = p - f(v^o, \theta^o) \] (5)

This linear system is solved with respect to \( \Delta \theta \) and the phase angles are updated accordingly:

\[ \theta^1 = \theta^o + \Delta \theta \] (6)

In the second half-iteration, the sensitivity of reactive power to phase angle is neglected, while the updated phase angles (6) are used. Thus, Eq. (4) is modified into:

\[ g_v \Delta v = q - g(v^o, \theta^1) \] (7)

which is solved to obtain \( \Delta v \).

While experience has shown that it is acceptable to neglect \( f_v \), \( \Delta v \) in (3), neglecting \( g_\theta \Delta \theta \) in (4) may be questionable, especially in the stressed system conditions considered in voltage stability studies, or in lower voltage networks where the decoupling assumption does not apply very well (low \( X/R \) ratios).

Remark. It has been further proposed to use constant \( f_\theta \) and \( g_v \) matrices, computed for \( v = 1 \) pu and \( \theta = 0 \) [3]. This approximation is valid as long as phase angle differences remain small and voltages close to 1 pu, which is even more questionable in voltage stability studies.

1CRIC stands for “Calcul des Réseaux Implicitement Couplés” (Computation of Implicitly Coupled Networks)
B. The CRIC method

The CRIC method [17] is able to provide estimates of the voltage variations that are more accurate than those based on the linearization (3,4) of the full load flow equations, while retaining the computational efficiency of the fast decoupled method.

As indicated above, reliable estimates of the phase angles are obtained from (5) and the CRIC method also relies on this simplification to obtain the updated phase angles (6).

While the fast decoupled approach keeps the phase angles constant when evaluating Δv, on the contrary, the CRIC method keeps the active power injections constant at the value obtained after updating the phase angles, i.e. f(v^θ, θ^1).

This way of doing matches more closely the original set of equations (1,2).

Thus, the equations to be solved are:

\[
\begin{align*}
    f(v^θ, θ^1) - f(v, θ) &= 0 \\
    q - g(v, θ) &= 0
\end{align*}
\]

Replacing the second term in (8) by its Taylor series expansion around (v^θ, θ^1) yields:

\[
f(v^θ, θ^1) - f(v^θ, θ^1) - f_θ Δθ - f_v Δv = 0
\]
or:

\[
f_θ Δθ + f_v Δv = 0
\]

Similarly, Eq. (9) can be expanded into:

\[
q - g(v^θ, θ^1) - g_θ Δθ - g_v Δv = 0
\]
or:

\[g_θ Δθ + g_v Δv = q - g(v^θ, θ^1)
\]
in which the updated phase angles θ^1 are used to compute the Jacobian matrices and the right-hand side of (11).

Solving (10) for Δθ and replacing in (11) one obtains:

\[
J_{qv}(g_v - g_θ f_v^{-1} f_θ) Δv = q - g(v^θ, θ^1)
\]

The matrix J_{qv} is well-known in voltage stability analysis [18]. This matrix, however, is not sparse. To preserve sparsity, one possibility is to solve the unreduced system (10, 11), which is larger but sparse.

Instead, the second idea underlying the CRIC method consists in computing a good sparse approximation of J_{qv}. To this purpose, it is assumed that active power flows in branches are constant rather than active power injections at buses.

The active power flow in the i – j branch can be written symbolically as:

\[P_{ij} = f(V_i, V_j, θ_i - θ_j)
\]

where V_i, θ_i (resp. V_j, θ_j) is the voltage at bus i (resp. j). The phase difference can be obtained from (13):

\[θ_i - θ_j = φ(V_i, V_j, P_{ij})
\]

and replaced into the corresponding reactive power flow equation, which takes on the form:

\[Q_{ij} = g(V_i, V_j, θ_i - θ_j) = g(V_i, V_j, φ(V_i, V_j, P_{ij}))
\]

P_{ij} being a fixed parameter, (14) involves voltage magnitudes only, and can be rewritten formally as:

\[Q_{ij} = g(V_i, V_j, P_{ij})
\]

The corresponding computations are detailed in the Appendix.

The reactive power injection at bus i is given by:

\[Q_i = Q_{si} + \sum Q_{ij} = Q_{si} + \sum g(V_i, V_j, P_{ij})
\]

where Q_{si} accounts for shunt compensation and the sums extend over all branches incident to bus i. Hence, the Jacobian matrix defined by:

\[
J_{qv}(i, j) = \frac{∂Q_i}{∂V_j} \quad i, j = 1, \ldots, n
\]

has the same sparse structure as the g_v matrix in (7).

To summarize, the method consists of solving (5) with respect to Δθ, updating θ according to (6), and solving

\[
\begin{align*}
    J_{qv} Δv &= q - g(v^θ, θ^1) \\
    \text{with respect to } Δv
\end{align*}
\]

The generator reactive power limits are checked and if some of them are exceeded, the status of the buses are changed as usual and Eqs. (5,18) are solved again.

III. APPLICATION TO VOLTAGE SECURITY ASSESSMENT

A. Contingency analysis

For VSA purposes, the emphasis is on voltage drops at transmission buses. A simple filtering technique consists of computing the (linear approximation of) voltage changes Δv for each contingency, and checking either the post-contingency voltages:

\[V_i^v + ΔV_i \geq V_{min} \quad i = 1, \ldots, N
\]
or the changes themselves:

\[ΔV_i \geq -ΔV_i \iff |ΔV_i| > ΔV \quad i = 1, \ldots, N
\]

where V_i^v is the base case voltage at the i-th bus, ΔV_i is the corresponding component of Δv and ΔV is a positive threshold.

Clearly, the first test is more related to the “quality” of post-contingency voltages. When dealing with voltage stability, we found the second test more appropriate. Indeed, some voltages may be already low in the base case without a risk of voltage instability, in which case the first test leads to false alarms.

Expectedly, ΔV has to be chosen carefully to reach a compromise between false alarms and non identification of harmful contingencies. Practical experience is reported in Section IV.

B. Security margin computation

As mentioned in the Introduction, for a given direction of stress, the SOL corresponds to the most stressed operating point such that the system can withstand any contingency of a specified list. In the SOL computation, the stressed system states are obtained from a (pre-contingency) load flow
computation, while the contingency impact is assessed using tools ranging from load flow to detailed dynamic simulation.

The SOLs can be determined by binary search [8], [13], [9]. This simple and robust method consists of building smaller and smaller intervals \([S_l, S_u]\) of stress values such that \(S_l\) corresponds to an acceptable post-contingency evolution and \(S_u\) to an unacceptable one. At each step, the interval is divided in two equal parts; if the midpoint is found acceptable (resp. unacceptable) it is taken as the new lower (resp. upper) bound. The procedure is repeated until \(S_u - S_l\) is smaller than a specified tolerance \(\Delta\).

To deal with multiple contingencies, a Simultaneous Binary Search (SBS) is preferable. At a given step of this procedure, only the unacceptable contingencies remaining from the previous step are simulated. If at least one of them is unacceptable, the acceptable ones are discarded (since their limits are higher than the current stress) and the search proceeds. The process converges towards the lowest limit. A simple example with 4 contingencies is shown in Fig. 1. The sequence of tested stresses is \(S_0, S_1, S_2, S_3, S_4\). The SOL corresponds to \(S_4\) and relates to contingency no. 1. Contingency no. 4 is discarded at stress \(S_0\), contingency no. 3 at \(S_1\) and contingency no. 2 at \(S_3\).

\[
\begin{array}{cccc}
0 & S_2 & S_4 & S_1 & S_0 \\
cont. no 1 & A & A & R & R \\
cont. no 2 & A & A & R & R \\
cont. no 3 & A & A & R & R \\
cont. no 4 & A & A & R & R \\
\end{array}
\]

Fig. 1. Simultaneous binary search [8]

A form of filtering takes place at the first step of the SBS (at stress \(S_0\) in Fig. 1), when discarding the contingencies with an acceptable system response. However, in spite of the QSS simulation speed, it may take too long to simulate the system response to each contingency of a long list. Hence the idea of filtering them on the basis of the (linearly approximated) voltage drops \(\Delta v\).

Thus, in this application, contingencies are filtered based on their impact at stress \(S_0\) (instead of base case, as in Section III.A.). The threshold \(\delta V\) has to be adjusted accordingly, as discussed in the next section.

IV. RESULTS

A. Test system

The method has been extensively tested on a model of the EHV (400 and 225 kV) system operated by RTE, the French transmission system operator. The tests have concentrated on a region of the RTE system where security is on some occasions constrained by voltage stability.

The model includes 1203 buses at the EHV level. The HV subtransmission and MV distribution systems are represented in a simplified way by attaching an EHV-HV transformer in cascade with an HV-MV transformer, at each EHV bus of concern. The former corresponds to an existing equipment while the latter is an equivalent. Both are controlled by LTCs. This brings 1024 additional buses in the model.

B. Simulation tools and criteria

RTE uses QSS simulation for voltage security analysis. In our tests, a post-contingency evolution has been refused if the voltages of some EHV transmission buses reach the low value of 0.85 pu. For instance, Fig. 2 shows a marginally accepted situation where the lowest transmission voltage approaches this lower limit. The system subsequently recovers under the effect of secondary voltage control [19]. The latter is taken into account in the QSS time simulation but not in the linear voltage drop estimates, in order to preserve computational efficiency.

![Fig. 2. Post-contingency voltage evolution](image)

C. Accuracy with respect to full load flow

The accuracy of the proposed linearized method has been checked with respect to a full AC load flow, by comparing the voltage magnitudes computed by both methods on a set of 180 single and double contingencies. The full load flow converges for all of them.

For instance, Fig. 3 compares the voltage drops provided by both approaches, for a mild and a severe contingency, respectively. Similarly, Fig. 4 compares the increases in generator reactive power productions. Expectedly, the discrepancies between both approaches increase with the severity of the contingency. However, the accuracy of the proposed method is quite satisfactory. In any case, it is good enough for filtering purposes in VSA. It can even be a substitute to full load flow in static security analysis [17].

For the most severe contingency, Fig. 5 shows the voltage drops sorted by increasing order of magnitude. The error introduced by the linear approximation decreases with the magnitude of the voltage drop itself. In fact, the relative error on the voltage drop is rather constant from one bus to another.

Figure 6 is similar to the right barchart in Fig. 3 but shows additionally the voltage drops obtained with a full load flow incorporating secondary voltage control. Expectedly, the voltage drops are smaller. In fact, the pilot node voltages
Fig. 3. Largest voltage drops for a mild (left part) and a severe (right part) contingency

Fig. 4. Largest increases in reactive power productions for a mild (left part) and a severe (right part) contingency

are not affected by contingencies (provided the controlling generators do not meet limits). This figure is given for comparison purposes but, as indicated in the previous section, secondary voltage control is not taken into account in the filtering procedure, to keep it simple and fast. The voltage drops are thus larger than in reality, but the threshold $\delta_V$ is accordingly set to a higher value.

D. Filtering for contingency analysis

The filtering capabilities of the linear voltage drop estimates have been checked, on the same set of contingencies as well as on large set of 16,000 double contingencies. In the latter, two single contingencies are applied at the same time.

The threshold $\delta_V$ has been chosen as follows. QSS simulations have been run to identify the unacceptable contingencies. Then the linear voltage drop estimates have been computed at all buses for all contingencies. $\delta_V$ should be as large possible to minimize the number of false alarms, but small enough to have all unacceptable contingencies correctly identified. Based on the above set of results, a value $\delta_V = 0.09$ pu was found to be a good compromise.

As recalled in the Introduction, severe (e.g. N-2) contingencies may lead to divergence of the Newton-Raphson iterations. This does not occur with the proposed method, which is non iterative. Instead, large voltage drops $\Delta V_i$ are expected. As an illustration, Fig. 7 shows the voltage drops obtained for such a severe contingency. Several buses exhibit a voltage drop much larger than the 0.09 pu threshold, thereby clearly identifying this contingency as dangerous.

The filtering results obtained on the set of 16,000 contingencies are summarized in Table I. As can be seen, many harmless contingencies are eliminated. The proposed method leads to 34-11 = 23 false alarms, i.e. slightly less than the full load flow (39-11=28) because the same threshold $\delta_V$ has been taken for both methods and the linearly estimated voltage drops are a little smaller, as shown by Fig. 3. All
the dangerous contingencies are correctly included in the "potentially dangerous" set.

**TABLE I**

**FILTERING PERFORMANCES**

<table>
<thead>
<tr>
<th>Total Nb. of contingencies</th>
<th>16,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis by QSS simulation</td>
<td>11 dangerous</td>
</tr>
<tr>
<td>Filtering by full load flow</td>
<td>39 potentially dangerous</td>
</tr>
<tr>
<td>Filtering by proposed method</td>
<td>34 potentially dangerous</td>
</tr>
</tbody>
</table>

**E. Filtering for security margin determination**

Security margins are assessed in terms of maximum pre-contingency load power increase, the loads being increased at the national level.

For this VSA aspect, the threshold $\delta V$ can be chosen as indicated hereafter.

Figure 8 shows, for three constraining N-1 contingencies, how the maximum post-contingency voltage drop provided by the proposed method evolves with the pre-contingency stress. On each curve, the secure operation limit of the corresponding contingency is marked with a dot.

Thus, if the initial stress $S_o$ of the SBS (see Fig. 1) is set to - say - 4000 MW, only contingency # 1 has an SOL lower than $S_o$ and $\delta V$ can be set as high as 0.13 pu to identify this constraining contingency. The other contingencies can be discarded since their SOL is larger than $S_o$, the maximum stress of interest. Similarly, if $S_o$ is set to 9000 MW, $\delta V$ has to be decreased around 0.11 pu in order to have contingencies # 2 and # 3 flagged as potentially dangerous and included in the SBS. If a single threshold $\delta V$ is sought, whatever the pre-contingency stress $S_o$, the smaller 0.11 pu value has to be taken (at the expense of possibly more false alarms).

**F. Computational efficiency**

Table II compares the computing times taken by the full load flow and the proposed method, for various contingencies.
A 2.2-GHz PC running Windows 2000 has been used. Both methods start from the solved pre-contingency base case. As can be seen, the linear method take 0.01 s on the average. The severe N-2 contingency, however, takes more time because the linear systems (5,18) have to be solved a second time after some generators are switched under reactive power limit. For the same contingency, the full load flow diverges and the maximum number of iterations is reached.

### Table II

**Computing times (in seconds) per contingency**

<table>
<thead>
<tr>
<th></th>
<th>N-1 mild</th>
<th>N-1 severe</th>
<th>N-2 mild</th>
<th>N-2 severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>full load flow</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>0.26</td>
</tr>
<tr>
<td>proposed method</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The computational efficiency in the context of security margin determination is illustrated in Table III, which compares the computing times without filtering, with filtering by load flow and with filtering by the proposed method. The test has been performed on the previously mentioned set of 16,000 contingencies. When filtering is performed, the SBS is run over the set of contingencies declared potentially harmful, and each combination of stress and contingency is simulated with the QSS time-domain method. The contingencies unduly flagged dangerous at the filtering step (false alarms) are immediately discarded.

### Table III

**Computing time for security margin determination**

<table>
<thead>
<tr>
<th></th>
<th>none</th>
<th>by full load flow</th>
<th>by proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>filtering</td>
<td>step</td>
<td></td>
<td></td>
</tr>
<tr>
<td>filtering</td>
<td>4 h 19 min</td>
<td>4 min 43 s</td>
<td>2 min 4 s</td>
</tr>
<tr>
<td>SBS</td>
<td>4 h 19 min / 7 s</td>
<td>31 s</td>
<td>25 s</td>
</tr>
<tr>
<td>total</td>
<td>4 h 19 min / s</td>
<td>5 min 14 s</td>
<td>2 min 29 s</td>
</tr>
</tbody>
</table>

As can be seen, such a large problem could not be dealt with in real-time without contingency filtering. Furthermore, the proposed method significantly reduces the filtering time and leads to saving almost 50 % of the total computing time, thereby contributing to the feasibility of real-time VSA.

### V. Conclusion

Voltage drops obtained from sensitivity formulae can be used for filtering purposes in voltage security assessment. The method used in paper involves the same computational effort as one iteration of a fast decoupled load flow but is more accurate that a single iteration of a full load flow. The contingencies with little impact on voltages as well as those with large power margins are filtered out by comparing the voltage drops to a threshold \( \delta V \). The potentially harmful contingencies are subsequently processed with more advanced tools based on time simulation.

The method has been successfully tested on a real system. The choice of \( \delta V \) in order to minimize false alarms while identifying all dangerous contingencies has been illustrated. On that system, the computing time of the proposed filtering is 50 % smaller than the one relying on full load flows, and negligible compared to the time required by an exhaustive simulation of all contingencies.

When the objective is to identify the contingencies with smallest power margins, the proposed procedure is much simpler than those based on eigenvector or singular vectors.

The results confirm that, although simple, the proposed method meets the practical requirements of contingency filtering in a reliable and efficient way. It is thus a good candidate for real-time applications and for the analysis of numerous (e.g. N-2) contingencies.

### Appendix

Let all the network branches be represented by the circuit shown in Fig. 9, which encompasses the usual line, cable and transformer equivalents.

![Network branch model](image)

Fig. 9. Network branch model

With the notation shown in Fig. 9, the active and reactive power flows in branch \( i \rightarrow j \) are given by:

\[
P_{ij} = G_{ij} V_i^2 - Y_{ij} V_i \frac{V_j}{n_{ij}} \cos \alpha_{ij} \tag{19}
\]

\[
Q_{ij} = -(B_{sij} + B_{ij}) V_i^2 - Y_{ij} V_i \frac{V_j}{n_{ij}} \sin \alpha_{ij} \tag{20}
\]

where \( \alpha_{ij} = \phi_i - \phi_j - \theta_i - \theta_j \) and \( \phi_{ij} \) is flagging the dangerous at the filtering step (false alarms) are immediately discarded.

From (19) one easily obtains:

\[
\cos \alpha_{ij} = \frac{G_{ij} V_i^2 - P_{ij}}{Y_{ij} V_i (V_j / n_{ij})}
\]

and hence:

\[
\sin \alpha_{ij} = \sqrt{1 - \cos^2 \alpha_{ij}}
\]

\[
= \pm \sqrt{Y_{ij}^2 V_i^2 (V_j / n_{ij})^2 - (G_{ij} V_i^2 - P_{ij})^2} \tag{21}
\]

In most cases \( \eta_{ij} \approx -\pi / 2, |\theta_i - \theta_j| \ll \pi / 2 \) and \( \phi_{ij} = 0 \), hence \( \alpha_{ij} > 0 \) and the + sign must be chosen in (21). However, in some cases the opposite holds, in particular for negative series reactances (e.g. series capacitors, equivalent scheme of 3-winding transformers). In the sequel only the + sign variant is considered.

Introducing (21) into (20) gives the equivalent expression of the reactive power flow:

\[
Q_{ij} = -(B_{sij} + B_{ij}) V_i^2
\]

\[
- \sqrt{Y_{ij}^2 V_i^2 (V_j / n_{ij})^2 - (G_{ij} V_i^2 - P_{ij})^2} \tag{22}
\]

which takes on the form (15).
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REFERENCES


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