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Scantling optimization based on convex linearizations and a dual approach—Part II

Philippe Rigo[a,*], Claude Fleury[b]

[a] *Department of Naval Architecture (ANAST), University of Liege, Chemin des Chevreuils No 1, B52/3, B-4000 Liege, Belgium*

[b] *Department of Aerospace (LTAS), University of Liege, Chemin des Chevreuils No 1, B52/3, B-4000 Liege, Belgium*

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Abstract

The development of the LBR-5 "Stiffened Panels Software" is included in the development of a new design methodology to ease and to improve preliminary studies of naval structures and floating hydraulic structures. It allows, as of the first draft, an optimization of the scantling of the structure's constituent elements. The ultimate target is to link standard design tools (steel structure CAD, hull form, hydrostatic curves, floating stability, weight estimation, etc.) with a rational optimization design module and a minimum construction cost (or minimum weight) objective function. It is developed to be a user-oriented tool. The optimization module is composed of three basic modules (OPTI, CONSTRAINT and COST) and a group of sub-modules (in external databases). Among these the user selects a set of relevant sub-modules (i.e. geometrical and structural constraints). Since the present optimization deals with least construction costs (as objective function), and uses an explicit objective function (not empirical), the user must specify labor costs (unitary material costs, welding, cutting, etc.). This paper is the second part of a series of two articles. The previous paper focused on the 'Module-Oriented Optimization' methodology and on the rational constraints (Rigo, Marine Structures, 2001). This paper presents the optimization algorithm based on convex linearization and a dual approach (OPTI module). It also includes the optimization of a FSO unit as a detailed example. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Optimization; Preliminary design; Stiffened structure; Construction cost; Design methodology; Convex linearization; Dual approach; Floating storage offloading

*Corresponding author. Tel.: +32-43-66-93-66; fax: +32-43-66-91-33.
E-mail address: ph.rigo@ulg.ac.be (P. Rigo).

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1. Introduction

Guidelines and major orientations of a ship structural design are always defined during the earliest phases of a project, i.e. the preliminary design stage or the first draft that corresponds in most cases to the offer. It is thus easy to understand why an optimization tool is attractive, especially one designed for use at the preliminary stage. This is precisely the way the LBR-5 optimization software for stiffened hydraulic and naval structures was conceptualised, created and developed [1–4]. 'LBR-5' is the French acronym of "Logiciel des Bordages Raidis", i.e. "Stiffened Panels Software", version 5.0.

The ultimate target is to link standard design tools (steel structure CAD, hull form, hydrostatic curves, floating stability, weight estimation, etc.) with a rational optimization design module and a minimum construction cost objective function. Rigo [1] discusses more extensively this important aspect. LBR-5 is this rational optimization module for structures composed of stiffened plates and stiffened cylindrical shells. It is an integrated model to analyse and optimize naval and hydraulic structures at their earliest stages: tendering and preliminary design. Initial scantling is not mandatory. Designers can start directly with an automatic search for optimum sizing (scantling). Design variables (plate thickness, stiffener dimensions and their spacing) are freely selected by the user.

LBR-5 (Fig. 1) is composed of three basic modules (OPTI, CONSTRAINT and COST). The user can select the relevant constraints (geometrical and structural constraints) in external databases. In addition, standard constraint sets are proposed to users (CONSTRAINT module). When the optimization deals with least construction costs, unitary material, welding, cutting and labor costs must be specified by the user to define an explicit objective function (not empirical). For least weight, these unitary costs are not required.

Presently, detailed information on these modules can be found in [1–4]. Using all these data (constraints, objective function and sensitivity analysis), the optimum solution is found using an optimization algorithm based on convex linearizations and a dual approach [5–7] (OPTI module). Independent of the number of design variables and constraints, the number of iterations requiring a complete structural re-analysis is limited to 10 or 15.

LBR-4 [8], the previous version of the "stiffened panel method" for elastic analysis of stiffened structures, was the starting point for the development of the LBR-5 optimization module. Its role is to provide in the CONSTRAINT module a fast and reliable assessment of the stress pattern existing in the 3D stiffened structure [1]. So, the LBR-5 software is the result of the integration inside the same package of the LBR-4 [8] and CONLIN [6] software and constitutes a new tool to achieve a structural optimization, i.e. to define the optimum scantling.

The development of the LBR-5 module is included in the development of a new design methodology [1]. The objective is to create a user-oriented optimization technique, in permanent evolution, i.e. that evolves with the user and his individual needs. We define these as a "Module-Oriented Optimization" (Fig. 1).

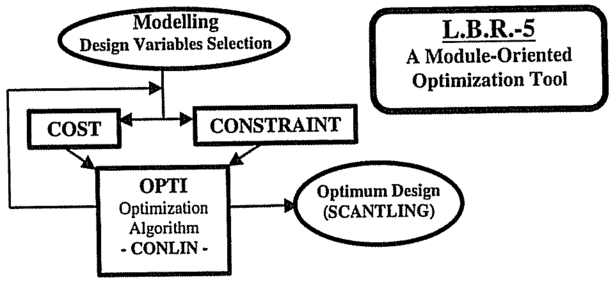

Fig. 1. Basic configuration of the LBR-5 model and database presentation.

This paper is the second part of a series of two articles. The first one describes the scientific aspects of the rational optimization procedure, the innovative concept and methodology [1], and the way they are implemented. It focuses on the "Module-Oriented Optimization" methodology and on the CONSTRAINT module. This last module performs a 3D analysis of the general behaviour of the structure and explicitly take into account all the relevant limit states of the structure (service limit states and ultimate limit states) thanks to a rational analysis of the structure. Due to lack of space, the important materials discussed in the introduction of [1] are not repeated here. For instance the state of art and a comprehensive comparison with previous works [9–15] are only presented in [1].

As a complement to this first paper [1], this article presents the relevant information on the mathematical algorithm of the OPTI module and a detailed application on the optimization of a floating storage offloading unit (FSO). Soon, a third paper will be published with advanced information on the COST module [16]. It will show the integration of construction and manufacturing costs in the optimization process (through the cost objective function).

2. The OPTI model and the optimization algorithm

The OPTI module is based on the CONLIN code developed by Fleury [5–7] using a convex linearization of the constraints and the objective function combined in a dual approach. With this algorithm, large constrained problems with implicit and non-linear constraints can be easily solved (Fig. 2). The main difficulty in solving a dual problem is dealing with the non-linear and implicit constraints. In order to avoid a large number of time-consuming re-assessments of these non-linear and implicit functions, Fleury suggests applying convex approximations. At each iteration all the functions (objective function and constraints) are replaced by an approximation called "convex". In a word, the complex initial optimization problem is decomposed in a sequence of more simple convex optimization problems (obtained through a convex linearization) that can be easily solved using a dual approach (Fig. 2).

In order to consider non-linear implicit constraints ($C(X_i)$), Fleury proposes replacing these constraints with approximated explicit linear constraints by using

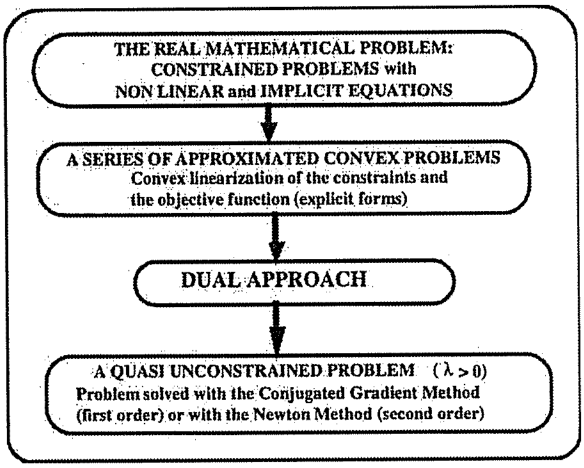

Fig. 2. The CONLIN model: convex approximations and dual approach.

convex linerization. He suggests using the first term of the Taylor series expansion. Three linear alternatives are possible:

- Linearization with standard design variables $(X_i, i=1,N)$:

$$C(X_i) = \tilde{C}(X_i) = C(X_i(0)) + \sum [X_i - X_i(0)]\partial C(X_i(0))/\partial X_i. \quad (1)$$

- Linearization with reciprocal design variables $(1/X_i, i=1,N)$:

$$C(X_i) = \tilde{C}(X_i) = C(X_i(0)) + \sum [1/X_i - 1/X_i(0)]\partial C(X_i(0))/\partial(1/X_i). \quad (2)$$

- Convex linearization with mixed variables $(X_k, k=1,L)$ and $(1/X_j, j=L+1,N)$

$$C(X_i) = \tilde{C}(X_i) = C(X_i(0)) + \sum_{k=1}^{L} [X_k - X_k(0)]\partial C(X_k(0))/\partial X_k$$
$$+ \sum_{j=L+1}^{N} [1/X_j - 1/X_j(0)]\partial C(X_j(0))/\partial(1/X_j). \quad (3)$$

As design variables refer to dimensions such as plate thickness, web height, etc., it is not suitable to use X_i to linearize constraints related to stress, strength and displacement. It is better to use reciprocal linearization $(1/X_i)$. On the other hand, geometrical constraints (for instance: "$w-h\leqslant 0$", with h, the web height and w, the flange width) must be linearized with standard design variables (X_i) instead of reciprocal ones $(1/X_i)$. Therefore, for a general case, it is obvious that mixed linearization is the better way. But the problem remains how to determine which linerization is the most suitable (reciprocal variable $1/X$ or direct variable X) for each design variable. Fleury has responded to this, proposing to make this selection in a way that replaces the actual design space (feasible domain for the design variables) by a smaller domain, included in the actual one, but convex. One can summarise in this way: since the substitution design space is conservative, this leads to a solution that is still admissible, but that could be "slightly" different from the real optimum. Step by step, this conservatism is released as one comes closer to the real optimum.

The convexity of the design space and conservation allow a safe and fast convergence. The convergence is safe because, at each iteration, the updated solution has a tendency to still remain in the feasible domain. Fleury has demonstrated that an efficient convex linearization can be achieved by selecting the group of variables (X_i) and the group of reciprocal variables ($1/X_i$) according to the sign of the first derivative of the function to linearize, that is $\partial C(X_i(0))/\partial X_i$.

For a given design variable X_i:

- a linearization with standard variable X_i is achieved if $\partial C(X_i(0))/\partial X_i > 0$;
- a linearization with reciprocal variable $1/X_i$ is performed if $\partial C(X_i(0))/\partial X_i < 0$.

Therefore Eq. (3) becomes:

$$C(X_i) = \tilde{C}(X_i) = C(X_i(0)) + \sum_{+} [X_k - X_k(0)]\partial C(X_k(0))/\partial X_k - \sum_{-} [1/X_j - 1/X_j(0)](X_j(0))^2 \partial C(X_j(0))/\partial X_j \quad (4)$$

with

$\partial C(X_k(0))/\partial X_k > 0 \quad (1 \leqslant k \leqslant N); \quad \partial C(X_j(0))/\partial X_j < 0 \quad (1 \leqslant j \leqslant N),$

for $i = 1, N$.

The proposed convex linearization is very "user friendly" as only the values of $C(X_1(0))$ and $\partial C(X_k(0))/\partial X_k$ are required. The linearization is done automatically at each step (iteration) and the convergence order is 2. In addition, the main advantage of the proposed convex linearization is the conservatism of the approximated function. Let's note however that conservatism is only guaranteed with regards to initial linear functions in X_k and in $1/X_j$. To avoid numerical problem the equations have to be normalised before starting the convex linearization. Fleury explains this procedure in [6,7].

As an example, consider the $C1$ and $C2$ constraints (Eqs. (5a) and (5b)) and some feasible linearizations (Eqs. (5c)–(5f)):

$$C1(X) = 5X_2 - X_1^2 - 10 \leqslant 0, \quad (5a)$$

$$C2(X) = 5/4X_2^2 + 16/X_1^2 - 13 \leqslant 0. \quad (5b)$$

The considered initial point of the design variables is $X(0) = (X_1, X_2) = (2, 2)$. At this point, the two constraints and their first derivatives are the same as:

$C1(2,2) = C2(2,2) = -4$

and

$$\partial C1(2,2)/\partial X = \partial C2(2,2)/\partial X = (-4,5).$$

Based on these equalities, all the linearized equations of $C1$ and $C2$ (Eqs. (5c)–(5f)) are the same. According to the linearization models, Eqs. (1)–(4), we obtain for both $C1$ and $C2$:

- Standard linearization, X, Eq. (1): $5X_2 - 4X_1 - 6 \leqslant 0.$ (5c)
- Reciprocal linearization, $1/X$, Eq. (2): $-20/X_2 + 16X_1 - 2 \leqslant 0.$ (5d)
- Convex linearization, Eq. (4): $5X_2 + 16/X_1 - 22 \leqslant 0.$ (5e)
  The variables are: X_2 as $\partial C/\partial X_2 = 5 > 0$, $1/X_1$ as $\partial C/\partial X_1 = -4 < 0$.
- Concave linearization, with X_1 and $1/X_2$: $5X_2 + 16/X_1 - 22 \leqslant 0.$ (5f)

Fig. 3 shows that the convex linearization (Eq. (5e)), is the best one and the most conservative way, even if it fails with regards to $C2$, which is initially more convex (a quadratic function in X_2 and $1/X_1$, Eq. (5b)) than the approximated convex function (a linear function in X_2 and $1/X_1$, Eq. (5e)).

2.1. *A dual algorithm for optimization problem*

At each iteration, the nomalised problem to solve is the following:

MIN $[\Sigma F_j/X_j - \Sigma F_i X_i]$ with N design variables X submitted to M constraint :
$\Sigma C_j/X_j - \Sigma C_i X_i \leqslant CM$ and $X_{i\min} \leqslant X_i \leqslant X_{i\max}$ the lower-upper bounds.

This problem is called a primal problem with reference to the X_i design variables, called primary or primal design variables. It is a constrained problem with N design variables and M constraints. This problem cannot be solved easily with classic

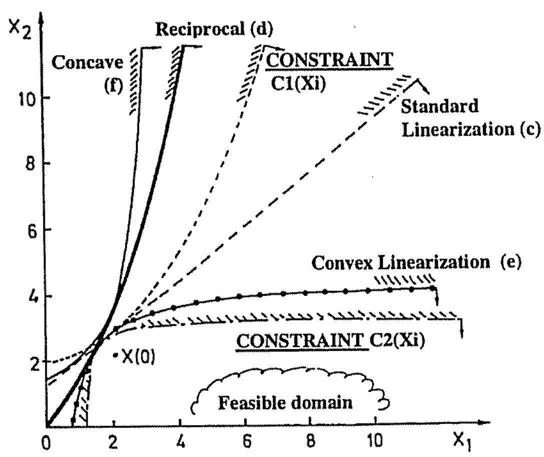

Fig. 3. Comparison between different linearizations.

methods such as, for example, the conjugated gradient. A dual approach will be used here to replace the primal *constrained problem* with N unknowns by an *unconstrained problem* with M unknowns (called the dual problem). This technique is especially advantageous when $M \ll N$. Unfortunately, with regard to the applications considered in this paper, this last advantage is not relevant since M and N are the same order of size.

To the primal problem (convex with separable variables), one can associate the dual problem:

$$\mathrm{MAX}(\lambda)\,[\mathrm{Min}(X) \text{ de } L(X,\lambda)] \text{ and } X_{i\,\min} \leqslant X_i \leqslant X_{i\,\max}$$

with $L(X,\lambda)$ the Lagrangien, λ_k the M multipliers of Lagrange (*dual variables*)

$$L(X,\lambda) = \sum_j \frac{F_j}{X_j} - \sum_i F_i X_i + \sum_{k=1}^{M} \lambda_k \left(\sum_j \frac{C_{jk}}{X_j} - \sum_i C_{ik} X_i - CM_k \right)_k, \tag{6}$$

which is also a function with separable variables.

Because the Lagrangien function is separable (Eq. (6)), the single dual problem with N design variables (N dimensions) is replaced by a series of N problems with a single dimension:

$$1(\lambda) = \mathrm{MIN}(X) \text{ of } L(X) = \sum_{i=1}^{N} \mathrm{MIN}\, L_i(X_i) \quad \text{with} \quad X_{i\,\min} \leqslant X_i \leqslant X_{i\,\max},$$

as $L(X) = \sum_{i=1}^{N} L_i(X_i)$ (as function with separable variables).

Each term of the $L(X)$ minimisation (Eq. (6)) can be written in an explicit form: $\mathrm{Min}\, L_i(X_i) = A_i X_i + B_i/X_i$, where A_i and B_i are defined as: $\sum_{K=1}^{N} C_i \lambda_k + F_i = \mathrm{fct}(\lambda_k)$.

Note that the minimisation related to each X_i variable requires that $\partial L_i/\partial X_i = 0$. Then,

$$X_{i\,\min} \leqslant X_i = \sqrt{B_i/A_i} \leqslant X_{i\,\max} \quad \text{and} \quad X_i = \mathrm{fct}(\lambda_k).$$

3. Optimization of a FSO barge

This least cost optimization example concerns the optimization of a FSO barge of 336 m with a capacity of 370,000 t, designed to serve as floating reservoir (provisory storage area) in view to receiving crude oil before being transferred on board tankers FSO. It is a moored barge, without its own propulsion system with a 2,500,000 barrel capacity. The anchorage, independent of the barge, permits an almost free motion (Figs. 4 and 5). The barge is filled using a pipeline connected to the shore. The small discharge of the pipeline induces uniform and slow loading. On the other hand, the discharge of the FSO unit that corresponds to the filling of a 2,000,000 barrels very large crude carrier (VLCC), is very fast and not uniform. The main characteristics of the barge are given in Table 1.

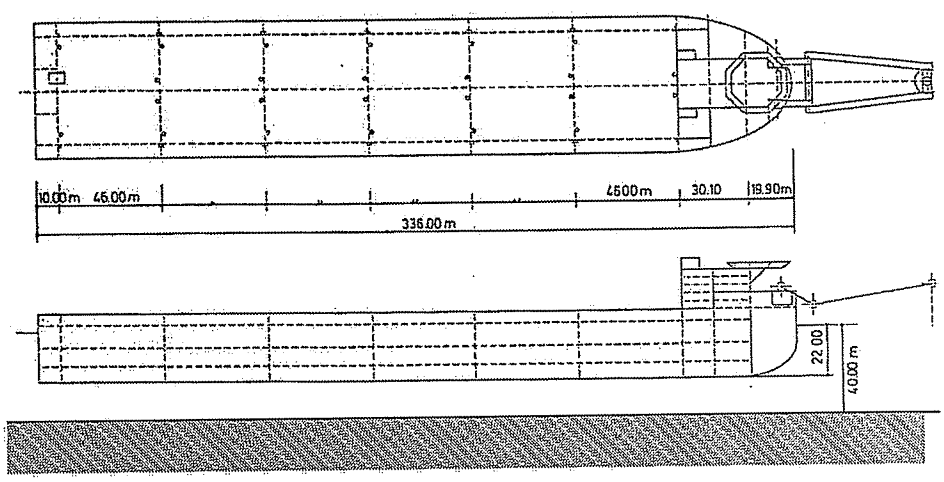

Fig. 4. General view of the FSO barge.

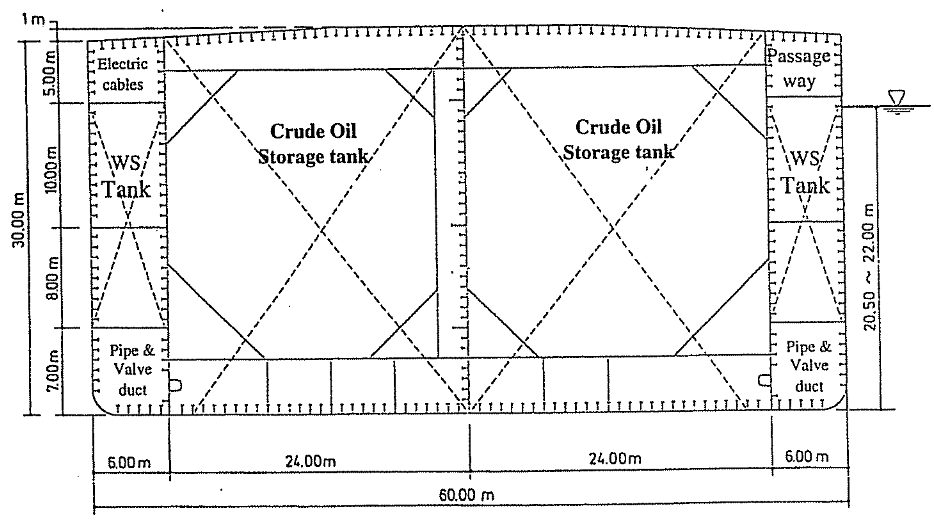

Fig. 5. Midship section of the FSO barge.

The optimization of a 46 m hold composed of two center tanks of $24\,m \times 30\,m \times 46\,m$ and two lateral ballast tanks of 6 m in width was performed. The two considered loading cases are presented on Fig. 6 and the modelling is shown in Fig. 7. The maximal hull girder bending moment (without waves) has been valued at 670,000 t m (6.57×10^6 kN m) and the shear force at 25,000 t (245,200 kN).

Table 1
Main characteristics of the FSO barge

Lpp (length between perpendiculars)	336 m (10 + 6 × 46 + 50 m)
B (width)	60 m (6 + 24 + 24 + 6 m)
H (depth)	30 m
T (draft)	20.5 m
C_b (block coefficient)	0.95
Light weight (hull + propulsion devices)	32,740 t
Number of crude oil tanks	12 × 33,782 m^3
Unit length of crude oil tanks	46 m
Unit breadth of crude oil tanks	24 m
Unit volume of crude oil tanks	405,389 m^3
Number of barrels (bbl)	2,549,819 bbl (1 bbl = 0.1589873 m^3)
Crude oil density	0.93 t/m^3 (9.3 kN/m^3)
Water ballast: side hull	59,600 m^3
Water ballast: aft deep-tanks	9,500 m^3
Water ballast: fore deep-tanks	20,000 m^3
Tanks (fresh water and diesel-oil)	1000 m^3 and 2000 m^3
Pumps	4 × 1800 m^3/h and 977 kW
Total installed power	8880 kW
Thrusters (transversal thrust only)	2 × 2500 kW et 300 kN/unit
Accommodation	50 persons

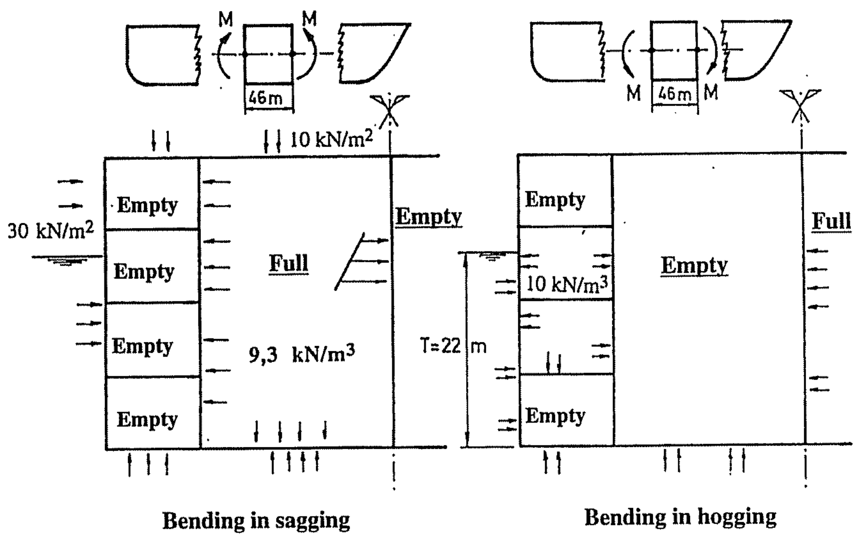

Fig. 6. Considered loading cases.

Optimum costs are calculated using the following cost and productivity data:

Reference plate thickness: 10 mm.
Unitary labor cost (Euro/m h)/material cost (Euro/t): $k = 0.08$.

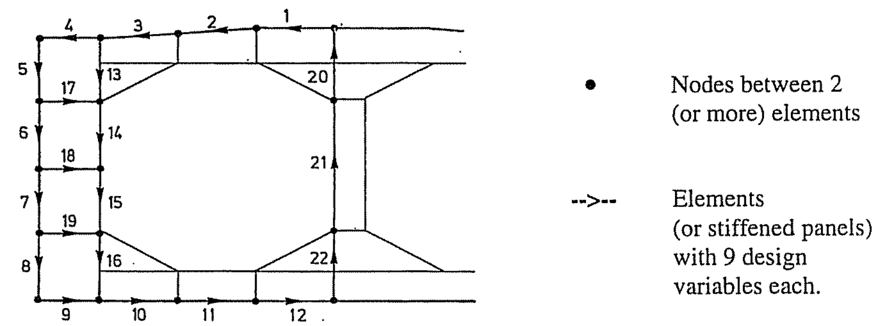

Fig. 7. Mesh modelling used for LBR-5 for the FSO midship section.

Unitary price of steel:	$C1 = 0.57$ Euro/kg, $\Delta C1 = -0.6\%$ (if AE235),
	$C1 = 0.65$ Euro/kg, $\Delta C1 = -0.6\%$ (if AE355).
Price of welding (materials only):	$C8 = 1.00$ Euro/m, $\Delta C8 = 15\%$
Manpower:	
plate:	$P10 = 0.5$ m h/m^2, $\Delta P10 = 7\%$
frames (assembling with plate):	$P4 = P5 = 1$ m h/m, $\Delta P4 = \Delta P5 = 10\%$,
frames (if built on site):	$P9 = 0.5$ m h/m, $\Delta P9 = 1\%$.

The mesh model of the FSO unit includes:

- 22 stiffened panels with 9 design variables each;
- 2 additional panels to simulate the symmetry axis (or boundary conditions);
- 198 design variables (9 × 22 panels);
- 48 equality constraints between design variables are used, e.g., to impose uniform frame spacing for the deck, bottom and central bulkhead in the center tanks and another one in the side ballast tanks.
- 198 geometrical constraints (9 × 22 panels). Since the web heights of longitudinal and transversal members are quite important, no geometrical constraints were selected for web slenderness. Web buckling stability and possibly their bracketing are then verified afterwards (post-optimization);
- 396 structural constraints (198 by load case):
  - σ_c frame (web/plate junction—web/flange junction and flange),σ_c stiffener (web/plate—web/flange and flange) and σ_c plate, which should be verified that $\sigma_c \leqslant s\sigma_o$ (with $s = 0.65$ and $\sigma_s = 355\,\text{N/mm}^2$);
  - local plate buckling: $\delta_{\text{MIN}} \leqslant \delta$ (with δ_{MIN} the minimum plate thickness to avoid buckling);
  - ultimate strength of stiffened panel: $\sigma/\sigma_{\text{ULT}} \leqslant s$ with $s = 0.55$;
- Two constraints on the ultimate hull girder/box girder strength: $M/M_{\text{ULT}} \leqslant s$ ($s = 0.55$).

In order to define optimal scantlings (least cost and least weight), side constraints are imposed on the design variables (XI_{MAX}, XI_{MIN}). For instance, the upper limit for the (δ) plate thickness is fixed at 40 mm.

Other selected limits (side constraints) are:

2.87 m	$\leqslant \Delta_{\text{Frames}}$	$\leqslant$ 7.66 m,
0.50 m	$\leqslant \Delta_{\text{Stiffeners}}$	$\leqslant$ 1.00 m,
1.20 m	$\leqslant h_{\text{web}}$ frames (center tanks)	$\leqslant$ 6.00 m,
0.50 m	$\leqslant h_{\text{web}}$ frames (side tanks)	$\leqslant$ 2.50 m (except in panels 13, 16, 18),
8.0 mm	$\leqslant$ Web thickness	$\leqslant$ 30.0 mm (or 40 mm),
..... etc.		

Since the first results showed the importance of the "$\delta \leqslant 40$ mm" side constraints, a second analysis was performed, imposing $\delta \leqslant 30$ mm. In addition, the frame spacing in the center tanks (Δc (center tanks)) and those in the side tanks (Δc (side tanks)) are considered to be independent. However, it is imposed that:

Δc (side tanks) = Δc (center tanks)/α with α an integer number lower than 3 ($\alpha \leqslant 3$).

Table 1 compares optima for six different configurations ($C1$–$C6$):

Optimum for δ (plating) $\leqslant$ 40 mm:

Least cost:

$C1$: Δ_{Frames} (side tanks) = Δ_{Frames} (center tanks)
$C2$: Δ_{Frames} (side tanks) = $\frac{1}{2}$ Δ_{Frames} (center tanks)

Least weight:

$C3$: Δ_{Frames} (side tanks) = Δ_{Frames} (center tanks)
$C4$: Δ_{Frames} (side tanks) = $\frac{1}{2}$ Δ_{Frames} (center tanks)

Optimum of δ (plating) $\leqslant$ 30 mm:

Least cost:

$C5$: Δ_{Frames} (side tanks) = Δ_{Frames} (center tanks)

Least weight:

$C6$: Δ_{Frames} (side tanks) = Δ_{Frames} (center tanks)

Note that costs and weights refer to a half-structure (30 m wide) and that stiffening and bracketing (transverse members, webs, etc.) are not included in the weight.

Detailed optimal scantlings are presented in Figs. 8, 9 and 10a and b, respectively, $\delta \leqslant 40$ mm (least cost), $\delta \leqslant 40$ mm (least weight) and $\delta \leqslant 30$ mm (least cost). The "raw" scantlings presented in these figures are not "*ready to use*". They require minor changes such as rounded brackets in the corners, slow variation of the web height, etc. So, to establish execution plans and for practical and constructive reasons, greater standardization is advisable (examples: uniform thickness for the deck plate, side shell and bottom plate, uniform frame height, etc.).

Such standardization could have been selected as requirements for the optimization process, but were not intentionally, in order to amplify optimization process potentialities and to better differentiate optimum weight and optimum cost.

Analysis of the comparative table and of the scantling shows that (see Table 2):

- The maximal plate thickness (30 or 40 mm) is an active constraint that strongly conditions the optimum (active constraints). Thus, there is more than a 30% increase in weight and cost when selected ($\delta \leqslant 30$ mm) as a side constraint.

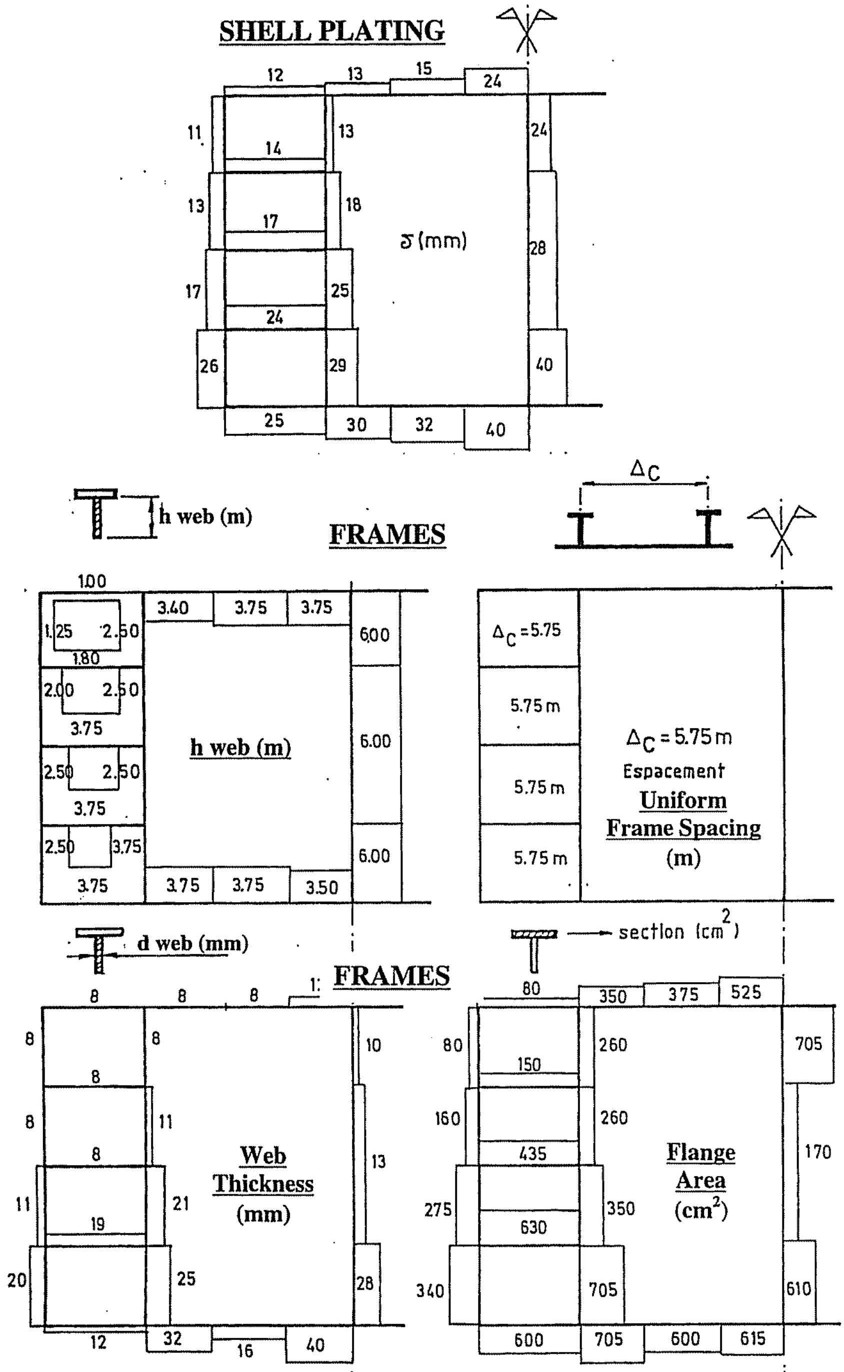

Fig. 8. Optimal scantling of the FSO barge (least cost—$\delta \leqslant 40$ mm, $\Delta = 5.75$ m).

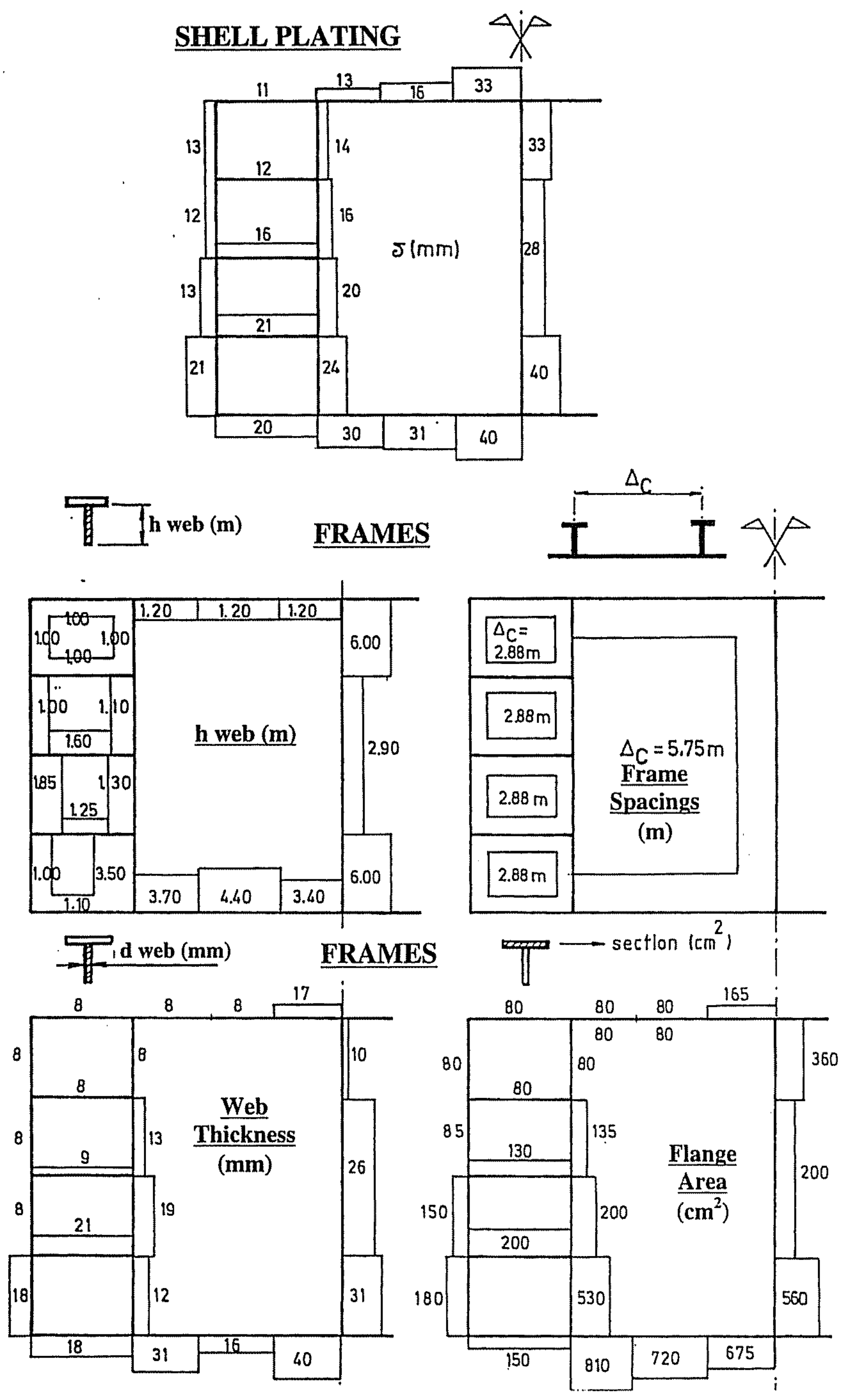

Fig. 9. Optimal scantling of the FSO barge (least weight—$\delta \leqslant 40$ mm, $\Delta = 5.75$ and 2.875 m).

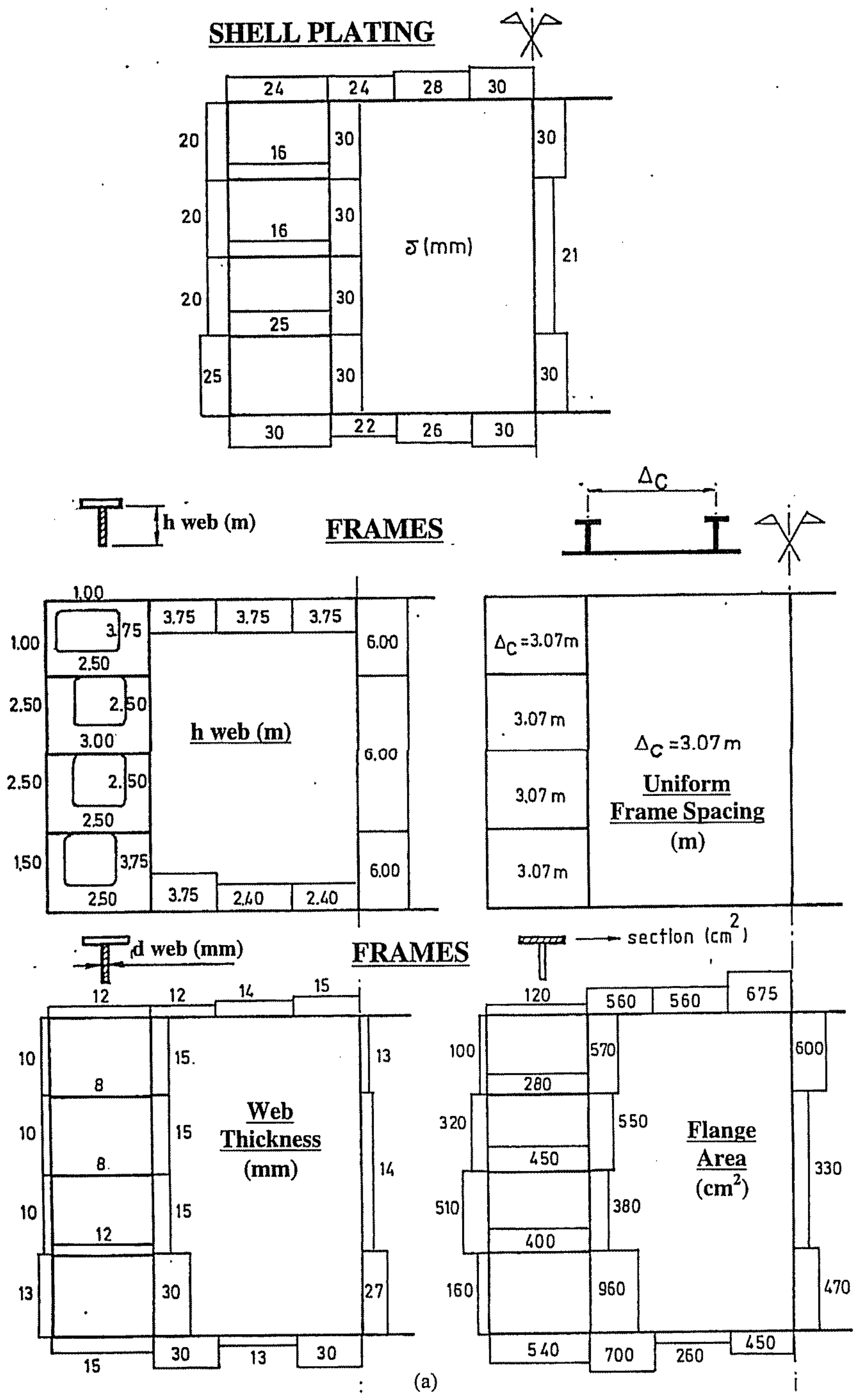

Fig. 10. (a) and (b). Optimal scantling of the FSO barge (least cost—$\delta \leqslant 30$ mm, $\Delta = 3.07$ m).

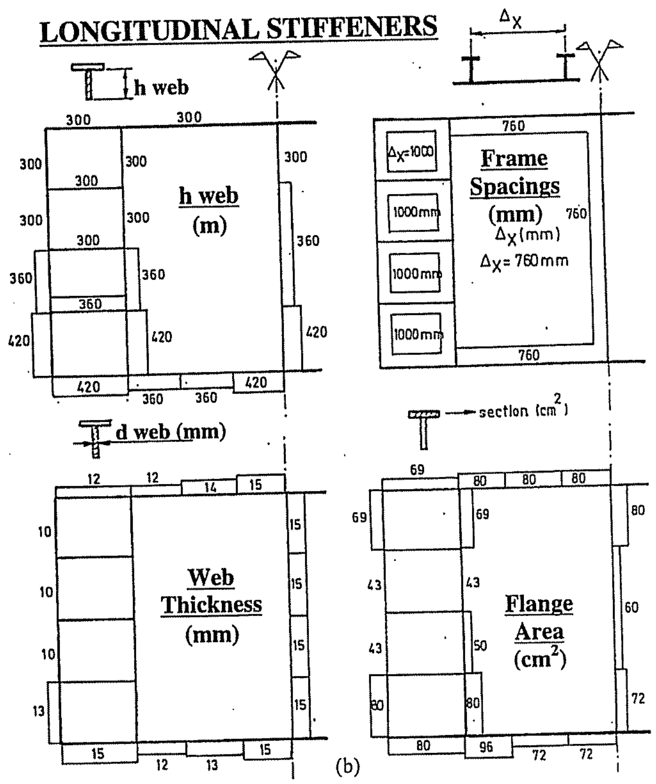

Figure 10. (*Continued*).

- If one selects $\delta \leqslant 40$ mm, the optimum scantling varies considerably depending on whether one searches for optimum weight or optimum cost. On the other hand, with a maximal plate thickness of 30 mm, the feasible design space (that is, the space of the design variables) is so reduced that the optimum cost and weight are nearly identical.
- Optimization of the frame scantling in the large tanks generally involves tall webs at mid span (large bending moment) and thick webs near their extremities (important shear forces).
- Doubling the number of frames in the side tanks ($\Delta_{\text{side tanks}} = 0.5\Delta_{\text{center tanks}}$) allows, in some cases, to reduce the weight. Unfortunately, this is also always synonymous with higher costs. Therefore, it doesn't seem feasible to envision this solution.
- Least weight scantlings are in general not economic solutions. Thus, the cost variation between least weight and least cost is 5% for $\delta \leqslant 40$ mm and 18% for $\delta \leqslant 30$ mm. On the other hand, for weight, the least cost scantlings leads to feasible structures: weights in the least cost solution are only 1% or 2% higher than in the

Table 2
Comparison between the different optimum (after 10 iterations)

Configurations	Weight kN (%)	Cost 10^6 Euro (%)	Cost per kg Euro/kg	Δ(side tanks) and N[a]	Δ(center tanks) and N[a]
$\delta \leqslant 40$ mm					
Least cost					
$C1$: $\Delta_{\text{side tank}} = \Delta_{\text{center tanks}}$	29,280 (109%)	6.34 (100%)	2.17	5.75 m $N=7$	5.75 m $N=7$
$C2$: $\Delta_{\text{side tank}} = \frac{1}{2}\Delta_{\text{center tanks}}$	29,740 (111%)	6.63 (105%)	2.23	6.57 m $N=6$	3.285 m $N=13$
Least weight					
$C3$: $\Delta_{\text{side tank}} = \Delta_{\text{center tanks}}$	27,150 (101%)	6.70 (106%)	2.42	5.11 m $N=8$	5.11 m $N=8$
$C4$: $\Delta_{\text{side tank}} = \frac{1}{2}\Delta_{\text{center tanks}}$	26,850 (100%)	7.13 (113%)	2.61	5.75 m $N=7$	2.875 m $N=15$
$\delta \leqslant 30$ mm					
Least cost $C5$: $\Delta_{\text{side tank}} = \Delta_{\text{center tanks}}$	38,870 (145%)	8.52 (134%)	2.19	3.07 m $N=14$	3.07 m $N=14$
Least weight $C6$: $\Delta_{\text{side tank}} = \Delta_{\text{center tanks}}$	38,500 (143%)	9.64 (152%)	2.50	3.07 m $N=14$	3.07 m $N=14$
Initial scantling (Start of the opt. process)	39,370 (147%)	9.74 (154%)	2.47	7.66 m $N=5$	7.66 m $N=5$

[a] N=Number of frames for a 46 m long hold $N=(46/\Delta)-1$.

least weight one. This demonstrates the attractiveness of least cost optimization, compared to standard least weight optimization.

- Finally, the recommended scantlings are:
  - for least cost ($C=100\%$, $P=109\%$):
    - $\delta \leqslant 40$ mm with 7 frames ($\Delta = 5.75$ m),
    - cost per kilo: 2.17 Euro.
  - for least weight ($C=106\%$, $P=101\%$):
    - $\delta \leqslant 40$ mm with 8 frames ($\Delta = 5.11$ m),
    - cost per kilo: 2.42 Euro.
- Concerning the cost per kilo or unitary cost (Euro/kg), least cost optimization leads to unitary costs 10%–15% lower than for least weight optimization (2.17 Euro/kg instead of 2.42 Euro/kg).

Table 3 gives an example of the convergence process observed at the time of optimization of this FSO barge. This analysis concerns a least weight optimization with $\delta \leqslant 40$ mm.

Fig. 11 presents three different frame optimum designs for the FSO midship section. These designs are the result of optimizations performed with the LBR5 model, followed by a standardization of the frame size and by adding brackets.

Table 3
Convergence of the optimization process of the FSO barge (least cost and $\delta \leqslant 40$ mm)[a,b]

Iteration no	Weight $10^6 N$	Δ_{Frames} Tank frame spacing (m)	δ Panel 1 (mm)	δ Panel 12 (mm)	$\Delta_{Stiffeners}$ Panel 1 (m)	$\Delta_{Stiffeners}$ Panel 4 (m)
Start	39.37 (145%)	7.660	15.00	15.00	0.900	0.900
1	28.04	7.660	15.21	22.90	1.000	0.954
2	29.95	5.794	29.18	37.36	Max. value side constraints	1.000
3	28.47	5.874	31.06	40.00	—	1.000
4	27.82	5.589	30.46	Max. value side constraints	—	0.950
5	27.50	5.346	29.97	—	—	0.913
6	27.32	5.279	30.04	—	—	0.884
7	27.24	5.230	39.92	—	—	0.860
8	27.20	5.190	29.90	—	—	0.843
9	27.17	5.166	29.83	—	—	0.832
10	27.15 (100%)	5.138	29.95	—	—	0.825

[a] Final cost (after optimization): 6.70×10^6 Euro for a 46 m long hold (a half-structure is considered, without bulkheads).
[b] Cost per kg: 2.42 Euro/kg.

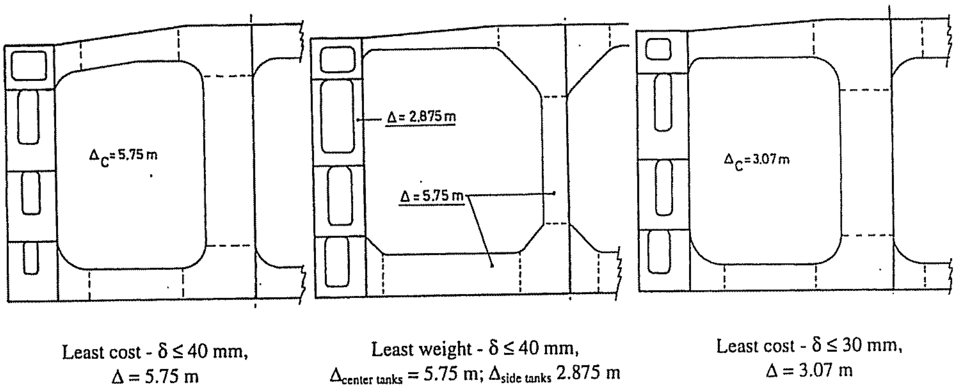

Fig. 11. Optimum midship sections for the FSO barge.

4. Summary and conclusions

With the framework of the new "Module-Oriented Optimization" concept, the multi purpose LBR-5 optimization model is presented in this paper. The COST, CONSTRAINT and OPTI modules are the 3 basic modules. The global optimization process is presented including an emphasis on the OPTI module. The CONSTRAINT module is detailed in a previous paper [1]. The COST module concerns the objective function: least weight and/or the least construction costs for which unitary material, welding, cutting and labor costs must be specified by the user.

Optimum analysis of a FSO barge is presented as application of the LBR-5 least cost optimization model. It shows that it is feasible to perform scantling optimization of large structures with a minimum cost objective function at the preliminary design stage. Alternative designs like reduced frame spacing in lateral tanks and impact of the maximum plate thickness on the optimum solution have been assessed. Least cost and least weight optimum are compared. It shows that starting from a least weight initial design, the cost is reduced by at least 15% without significant penalty on the weight.

With regards to the OPTI module, advantages and main characteristics of the LBR-5 are:

- Efficient and reliable optimizer (only 10–15 iterations are necessary to get the optimum);
- Large structures can be studied (100 panels, 900 design variables and 5000 constraints to cover up to 10 loading cases), and in addition initial scantling must not be feasible;
- Frame and stiffener spacing are design variables (topological design variables).

Future developments concerns: (i) Integration of the LBR-5 tool with industrial CAD packages and preliminary design tools (such interfaces are now under development); (ii) Hull shape optimization including fluid structure interaction; (iii) Multi-criteria analysis (construction cost, weight, generalized cost including operational costs, etc.) will be available using updated version of CONLIN; (iv) Reliability-based optimization (based on the ship life cycle); (v) To introduce fatigue limit state in the rational constraints.

References

[1] Rigo Ph. A module-oriented tool for optimum design of stiffened structures—Part I. Mar Struct 2001; 14(6): 611–29.

[2] Rigo Ph. Dévelopment d'un modèle intégré d'optimisation des structures navales et hydrauliques. Thèse d'Agrégration de l'Enseignement Supérieur, University of Liege, Belgium, 1998. 378pp.

[3] Rigo Ph. Optimization of ship structure—minimum cost, Second International Shipbuilding Conference, ISC'98, Krylov Shipbuilding Research Institute, St. Peterburg, Russia, Section C, 1998, p. 304–11.

[4] Rigo Ph. Least cost optimum design of stiffened hydraulic and floating structures. P.I.A.N.C. 1999;101:33–45.

[5] Fleury C, Braibant V. Structural optimization: a new dual method using mixed variables. Int J Numer Meth Eng 1986;23:409–28.

[6] Fleury C. CONLIN, an efficient dual optimizer based on convex approximation concepts. Struct Optim. 1989;1:81–9.

[7] Fleury C. Mathematical programming methods for constrained optimization: dual methods. In: Kamat MP, editor. Structural optimization: status and promise, progress in Astronautics and Aeronautics, Vol. 150. Washington, DC: AIAA, 1993. p.123–50.

[8] Rigo Ph. The computation of prismatic structures, applied to naval architecture. Mar Struct 1992;(3):313–32.

[9] Parsons M. Foundations of ship design (Course Packs NA470), Bin No. 1193 NA & ME Dept. University of Michigan, Ann Arbor, USA: DollarBill-Ulrich's, 2000. 450pp.

[10] Moe J, Lund S. Cost and weight minimization of structures with special emphasis on longitudinal strength members of tankers. Trans RINA 1968;110:43–70.

[11] Nowacki H, Brusis F, Swift PM. Tanker preliminary design—an optimization problem with constraints. Trans SNAME 1970;78:357–90.

[12] Hughes OF. Ship structural design. SNAME, New Jersey, 1988, 566pp.

[13] Mistree F, Hughes OF, Bras B. Compromise decision support problem and the adaptive linear programming algorithm, in structural optimization: status and promise. In: Kamat MP, editor. Progress in Astronautics and Aeronautics. Washington, DC: AIAA, 1992. p. 251–90.

[14] Rahman MK, Caldwell JB. Rule-based optimization of midship structures. Int J Mar Struct. 1992:(6):467–90.

[15] Sen P, Shi WB, Caldwell JB. Efficient design of panel structures by a general multiple criteria utility. Eng Opt 1989;14:287–310.

[16] Rigo Ph. Least cost optimization oriented preliminary design. Proceedings of the 2001 Ship Production Symposium, SNAME, Ypsilanti, Michigan, 2001, 19pp.