Combinatorial Optimization Approaches to the Design of Load Shedding Schemes against Voltage Instability

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Abstract. This paper proposes a methodology for the design of automatic load shedding against long-term voltage instability. In a first step, a set of training scenarios is set up, corresponding to various operating conditions and disturbances. Each scenario is analyzed to determine the minimal load shedding which stabilizes the system, with due consideration for the shedding location and delay. In a second step, the parameters of a closed-loop undervoltage load shedding scheme are determined so as to: (i) approach as closely as possible the optimal sheddings computed in the first step, over the whole set of scenarios; (ii) stabilize the system in all the unstable scenarios and (iii) shed no load in the stable ones. The corresponding optimization problem is solved using three methods : genetic algorithms, branch and bound approach and the so-called "sequential design". A detailed example is given on the Hydro-Québec system in which load shedding is presently planned.

Keywords. Voltage stability, emergency control, load shedding, combinatorial optimization.

I. INTRODUCTION

There are two lines of defence against incidents which jeopardize the stability of power systems:

• *preventively*: analyze the system security margins with respect to credible contingencies, i.e. incidents with a reasonable probability of occurrence, and take appropriate preventive actions to restore sufficient margins when needed;

• *correctively*: implement automatic corrective actions, through System Protection Schemes (SPS)[1] to face the more severe, but less likely incidents [1].

The preventive security criteria usually require that the system remains stable after any credible contingency, without the help of corrective actions. The main reason is that these actions usually affect the system generation and/or load, which is acceptable only in the presence of severe disturbances.

The present paper concentrates on long-term voltage instability, driven by Load Tap Changers (LTCs), generator OverExcitation Limiters (OELs), switched shunt compensation, restorative loads, and possibly secondary voltage control. This type of instability has become a major threat in many systems [2, 3].

[1]also referred to as Special Protection Schemes

Since long-term voltage instability is triggered mainly by the loss of generation or transmission facilities, "N-1" contingencies corresponding to the loss of a single equipment are usually considered in preventive security analysis. On the other hand, N-2 and more severe disturbances should be counteracted by an SPS. While it must be used in the last resort and to the least extent, automatic load shedding is very effective in this respect.

A few undervoltage load shedding schemes have been implemented throughout the world (e.g. [4]). Beside time-domain simulation, methods have been proposed to identify the best location, time and amount of shedding in a given unstable scenario (e.g [5, 6]).

This paper proposes a methodology to help planners in designing this type of SPS. The latter consists of two steps:

• in the first step, a set of *training scenarios* is set up, corresponding to various operating conditions and various disturbances. Each unstable scenario is analyzed to determine the minimal load shedding which stabilizes the system, with due consideration for the shedding delay;

• in the second step, the parameters of a closed-loop protection are determined in order to approach as closely as possible the optimal sheddings computed in the first step, over the whole set of scenarios. A "clever" enumerative optimization method is used to this purpose.

The paper is organized as follows. Section II describes how the minimal load shedding is determined when analyzing the unstable scenarios in the first step. Sections III to VI deals with the second step of the procedure. Section VII provides a rather complex example taken from the Hydro-Québec system, in which an undervoltage load shedding scheme is planned. The paper ends up with some concluding remarks.

II. DETERMINATION OF OPTIMAL LOAD SHEDDING

Location, amount and delay are the three main characteristics of load shedding. Obviously the amount of load shedding should be minimal.

For a given shedding delay and location, the minimal amount of shedding can be simply determined by binary or incremental search, resorting to time-domain simulations to check the system behaviour. As far as long-term voltage stability is concerned, the computing times can be dramatically reduced by using the Quasi Steady-State (QSS) simulation technique. This well-documented approach is based on time decomposition and consists of replacing the short-term dynamics by equilibrium equations, while focusing on the long-term dynamics [3, 7] .

The remaining of this section briefly adresses the delay and location issues.

A. Optimizing with respect to the shedding delay

The first motivation for delaying load shedding is to ascertain that the system is indeed voltage unstable, and hence to avoid shedding load unduly.

The second motivation is that it may be advantageous to let other post-contingency controls act first. An example is provided in Fig. 1, relative to the Hydro-Québec system considered in Section VII [8]. In this system, automatic shunt reactor tripping significantly contributes to stabilizing the system in its post-contingency configuration. The figure shows the minimal amount of load shedding P^{min} as a function of the shedding delay τ. As can be seen, 280 MW load are saved when the shedding is delayed by 16 seconds, allowing 2970 Mvar to be tripped before load is shed. In the design of a load shedding protection, we will use the minimum $P^{\star}$ of the P^{min} vs. τ characteristic as a target value.

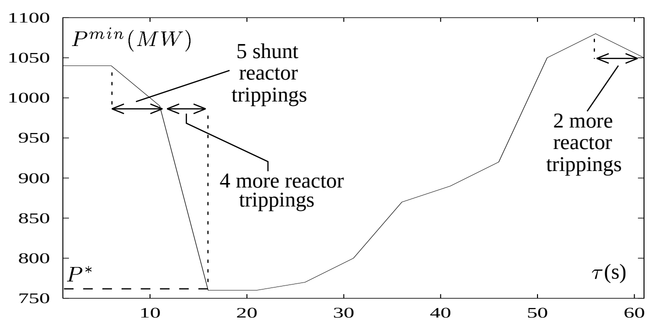

Figure 1: A shedding characteristic of the Hydro-Québec system [8]

B. Optimizing with respect to the shedding location.

The identification of optimal location can be obtained through a small-disturbance analysis coupled with time-domain simulation [6]. Along the unstable trajectory provided by a time-domain method, sensitivity analysis is used to identify the critical point, at which the eigenvector corresponding to the (almost) zero eigenvalue is computed. This information allows to rank load buses by decreasing efficiency of load shedding. A given amount of load shedding is distributed over the buses according to this ranking, taking into account the interruptible fraction of each load.

III. STATEMENT OF THE PROBLEM

A. Scenario analysis

The first step of our approach consists in setting up a set $\mathcal{S}$ of training scenarios, corresponding to various topologies, load levels, generation schemes, contingencies, etc.

In principle all the scenarios to be dealt with by a single protection should involve the same weak area of the system. In other words, the instability modes and hence the optimal shedding locations should be rather close for all the unstable scenarios of the set. Therefore, we assume that a common bus ranking can be set up for all of them. Once this ranking has been identified, the minimal amount of load shedding $P_i^{\star}$ is determined for each unstable scenario, according to the procedure outlined in section II.A.

B. Logic of the load shedding protection

The protection relies on a measured signal which is typically the average voltage V over several transmission buses in the load area of concern. Other measurements could also enter the logic, such as the reactive reserve of neighbouring generators, etc.

We consider a protection based on k rules of the type:

if V is smaller than V_i^{min} during d_i seconds, shed ΔP_i MW

The number k of rules is decided a priori; in practice it is typically equal to 2 or 3.

It must be emphasized that such a protection operates in closed loop since V is continuously measured and the same rule may trigger several successive load sheddings. An example from the Hydro-Québec system is again given in Fig. 3, in which a star indicates a shunt compensation switching and Rx a load shedding due to rule Rx. As can be seen, the total shedding results from two firings of R_2, followed by one firing of R_1.

Note also that the above defined rules are ”concurrent”. In the case of Fig. 2, for instance, both rules have their ”if clause” satisfied just after the disturbance. Due to its much larger timing, R_1 is not fired before the signal is reset under the effect of R_2. One could also think of a protection relying on ”exclusive” rules, i.e. a single rule can be activated at a time. In the above example, this would consist of activating R_1 only when V is in between 0.93 and 0.95 pu, and R_2 when it is below 0.93 pu. Although exclusive rules are an interesting alternative, we will focus in this paper on the concurrent rules.

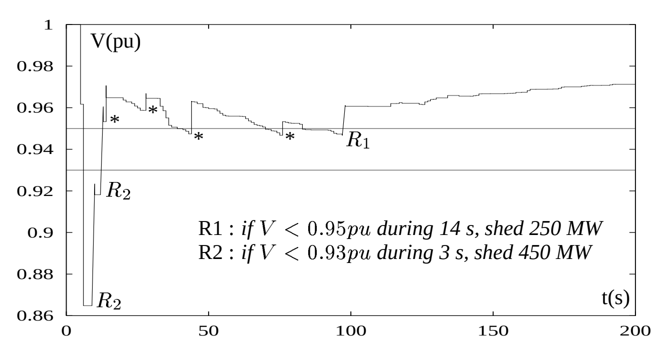

Figure 2: 2-rule protection example from the Hydro-Québec system

C. Statement of the design problem

Given the s training scenarios, the problem is to determine the $3k$-dimensional vector of unknowns:

$$\begin{aligned} \mathbf{x} &= [(V_1^{min}, d_1, \Delta P_1), \ldots, (V_k^{min}, d_k, \Delta P_k)] && (1) \\ &= [\mathbf{x_1}, \ldots, \mathbf{x_k}] && (2) \end{aligned}$$

such that the following requirements are met:
1. the amount of load shedding must be as close as possible to the minimum $P_i^\star$ determined in the first step;
2. all unstable scenarios must be saved (SPS dependability);
3. no load must be shed in a stable scenario (SPS security);
4. optionally, some other constraints can be imposed. For instance, the distribution voltages should not stay below some threshold for more than some time.

This can be translated into an optimization problem: minimize the discrepancies $P_i^{sh}(\mathbf{x_1}, ..., \mathbf{x_k}) - P_i^\star$, where $P_i^{sh}(\mathbf{x_1}, ..., \mathbf{x_k})$ is the total load power shed in the i-th scenario, for a given protection setting $\mathbf{x}$. Among the possible objective functions, let us quote the "sum" objective :

$$\min_{\mathbf{x}} \quad \textstyle\sum_i [P_i^{sh}(\mathbf{x}) - P_i^\star + p_i(\mathbf{x})]$$

and the "minmax" objective :

$$\min_{\mathbf{x}} \quad \max_i [P_i^{sh}(\mathbf{x}) - P_i^\star + p_i(\mathbf{x})] \qquad (3)$$

where the sum and the $\max$ extend over the unstable scenarios and $p_i(\mathbf{x})$ is a penalty term accounting for the violation of the above requirements. The sum objective was treated in [8] and will be no longer considered in this paper, where we concentrate on the minmax objective (3).

The penalties $p_i(\mathbf{x})$ are chosen as follows.

When the system is unstable (requirement 2 violated), transmission voltages eventually become smaller than some threshold V_{low}. Denoting by t_{low} the time at which this occurs, the penalty takes on the form:

$$p_i = \frac{C_1}{t_{low} + C_2} \qquad C_1 \gg 0 \quad C_2 > 0 \qquad (4)$$

Let t_{rec} be the recovery time, i.e. the time at which voltages are again larger than a specified value V_{min}. Requirement 4 consists in specifying that t_{rec} is smaller than a given value t_{rec}^{max}. If this does not hold, the penalty is taken as:

$$p_i = C_3(t_{rec} - t_{rec}^{max}) \qquad C_3 \gg 0 \qquad (5)$$

Note that with the above penalties, the more dangerous a situation (i.e. the shorter t_{low} or the larger t_{rec}), the higher the penalty. This is expected to provide the optimization method with information on how to improve the parameters.

D. Choice of the V_i^{min} thresholds

We assume that the values V_i^{min} are chosen by the designer based on a preliminary analysis of the scenarios as well as on his knowledge of the system. Attention must be paid to the following aspects:
• according to requirement 3, the controller must be prevented from shedding load in stable scenarios. To this purpose, the V_i^{min} threshold should be set below the values taken by the voltage signal in the stable scenarios;
• when possible, the system must be given a chance to stabilize using only "normal" post-contingency controls. Coming back to our previous example, Fig. 1 shows that in order to reduce the amount of load shedding the controller should act after the first 9 shunt reactors have been tripped. To ensure such a good synchronization, the V_i^{min} parameters should be adjusted below the threshold of these devices;
• on the other hand, Fig. 1 suggests that the V_i^{min} parameters should be high enough so that the controller acts before the minimal load to shed becomes prohibitive;
• another reason for setting V_i^{min} not too low is the quality of customer voltage. Very depressed situations should be corrected quickly enough in order to minimize customers'trouble.

Once the V_i^{min} parameters have been carefully chosen, we are sure the controller will not act when facing the stable scenarios of $\mathcal{S}$. Therefore, we only consider in the sequel the subset $\mathcal{S}'$ of unstable scenarios contained into $\mathcal{S}$.

E. Parameter space discretization

The optimization problem (3-5) is complex. Indeed, both P_i^{sh} and p_i must be determined from time-domain simulations and hence, explicit analytical expressions cannot be established. Moreover, they vary with $\mathbf{x_1}, ..., \mathbf{x_k}$ in a discontinuous manner. This prevents from using analytical optimization methods. Also, multiple local minima are expected.

Combinatorial optimization methods seem better suited to this purpose. The latter require to discretize the parameter space in order to obtain a finite number of candidates for the global optimum. This discretization must meet practical requirements. For example, the lower bounds on time delays d_i must be chosen so that the controller does not act on temporary voltage drops. Also, the ΔP_i discretization step should be close to the smallest block of load one expects to shed.

A "pure" enumerative approach would consist in evaluating the objective function for each possible vector of unknowns (2) and selecting the best one as solution of the problem. However, it is very time consuming and cannot be envisaged in real-life problems. The next three sections are devoted to describing three "clever" enumerative approaches.

IV. GENETIC ALGORITHMS

Genetic algorithms (GAs) are optimization techniques inspired by the theory of evolution. They combine survival of the fittest among string structures with a structured yet randomized information exchange to form a search algorithm with some of the innovative flair of human search. They allow to find near-optimum solutions of multimodal objective functions. However, there exist no objective criteria indicating how far the proposed design is from the optimal one.

This type of method was the first we applied to our optimization problem, as reported in [8]. The results were encouraging, but the other methods proposed in this paper are more effective while requiring less computing time.

V. BRANCH AND BOUND METHOD

From simple physical observations, it is possible to formulate our optimization problem (3-5) as a tree exploration problem, which will be solved elegantly by means of a branch and bound type of approach.

A. Protection design as a tree exploration formulation

Consider Fig. 3 showing the time evolution of the V signal after disturbances in the presence of a 2-rule load shedding protection. Under the effect of the first contingency (curves 1 and 2) the voltage drops under the V_1^{min} threshold of the first rule. Let us assume that by setting the $\mathbf{x_1}$ parameters of the load shedding protection to the value $\mathbf{x_1}^{(i)}$, the system voltage recovers as shown by curve 1. V never falls under V_2^{min}, and the contingency is counteracted by rule R_1 alone. Let us furthermore assume that by setting $\mathbf{x_1}$ to $\mathbf{x_1}^{(j)}$, rule R_1 is not able to bring the voltage back above V_1^{min}, as shown by curve 2. As a result, V falls below V_2^{min}, which means that both rule 1 and rule 2 could be fired. The same can happen under the effect of the second contingency, corresponding to curve 3, after which V falls right away below V_2^{min}.

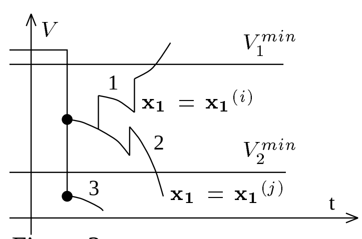

Figure 3: mechanism of rule triggering

This simple example shows that, for a given setting of $\mathbf{x_1}$, the $\mathcal{S}'$ set can be divided into two subsets : $\mathcal{S}_1$, which contains the scenarios that only trigger the first rule, and $\mathcal{S}_2$, made up of the remaining scenarios. As shown in the example, which scenarios go into which subset depends on the particular choice of $\mathbf{x_1}$.

This observation can be generalized to any number k of rules: the values assigned to the first $k-1$ vectors $\mathbf{x_1}, ..., \mathbf{x_{k-1}}$ define a partition of $\mathcal{S}'$ into k subsets $\mathcal{S}_1, ..., \mathcal{S}_k$, such that the scenarios in $\mathcal{S}_i$ only trigger the *i* first rules of the protection.

Based on these considerations, we can now represent the choice of the various $\mathbf{x_1}, ..., \mathbf{x_k}$ parameters as the exploration of a search tree. This is illustrated in Fig. 4 on a 2-rule example. We start from the root node R. Each branch binding R to one of the first-level nodes corresponds to one of the *m* possible instances of $\mathbf{x_1}$. Note that each of these instances define a different $\mathcal{S}_1$ subset. In the same way, any branch binding the first and second-level nodes corresponds to one of the *n* possible instances of $\mathbf{x_2}$ and defines a different $\mathcal{S}_2$ subset. Clearly, each protection setting corresponds to one path between R and a leaf node.

Furthermore, we can assign to any such path a "length" which is the value of the objective function (6) for the particular setting of concern. Consequently, optimizing a *k*-rule protection amounts to finding the shortest path between root and leaf nodes in the corresponding *k*-level tree.

The length of each path is given by :

$$F_{\mathcal{S}'}(\mathbf{x}_1, ..., \mathbf{x}_k) \triangleq \max_{i \in \mathcal{S}'} [P_i^{sh}(\mathbf{x_1}, ..., \mathbf{x_k}) - P_i^\star + p_i(\mathbf{x_1}, ..., \mathbf{x_k})] \tag{6}$$

Using the above defined partition, this can be rewritten as:

$$F_{\mathcal{S}'}(\mathbf{x}_1, ..., \mathbf{x}_k) = \max(F_{\mathcal{S}_1}(\mathbf{x_1}), ..., F_{\mathcal{S}_k}(\mathbf{x_1}, ..., \mathbf{x_k})) \tag{7}$$

where

$$F_{\mathcal{S}_j}(\mathbf{x}_1, ..., \mathbf{x}_j) \triangleq \max_{i \in \mathcal{S}_j} [P_i^{sh}(\mathbf{x_1}, ..., \mathbf{x_j}) - P_i^\star + p_i(\mathbf{x_1}, ..., \mathbf{x_j})]$$

Similarly, we can assign to each node *C* of the tree a value *F(C)* which is the length between this node and the root node. If *C* is a *j*th-level node directly linked to a *(j-1)*th-level node *D*, we have from (7):

$$F(C) = \max[F(D), F_{\mathcal{S}_j}(\mathbf{x_1}, ..., \mathbf{x_j})] \tag{8}$$

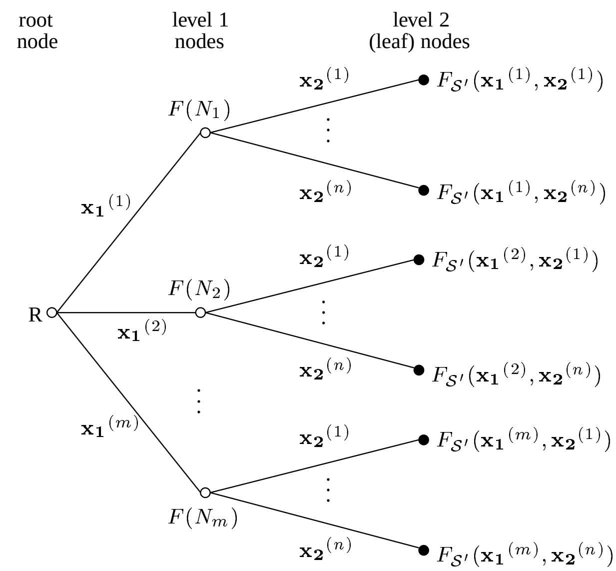

Figure 4: search tree corresponding to a 2-rule protection

B. Branch and bound approach

The branch and bound method [9] applies to any tree enumeration problem in which the objective function can only increase when moving from one level to the next in the corresponding search tree. The idea of the method is basically to keep track of the best value of the objective function reached so far, which is used as an upper bound B on the sought global minimum. This bound is used in order to identify those sub-trees that need not be explored, because their leaves correspond to values of the objective function higher than B. Skipping those sub-trees allows to (hopefully drastically) reduce the size of the space to explore.

The algorithm can be written in recursive form as follows:

Branch and bound (original algorithm)

```
branch_and_bound (R)

procedure branch_and_bound(D)
  for each branch below node D:
     let C be the node linked to D through this branch
     evaluate F(C)
     if F(C) < B:
       then if C is a leaf node:
              then B := F(C);                        (*)
              else branch_and_bound(C);
            endif
       else skip                                     (**)
     endif
  endfor
endprocedure
```

C. On the partition of the training set

As explained in section V.A, the branch and bound approach relies on the partition of $\mathcal{S}'$ into *k* subsets of scenarios. Each subset $\mathcal{S}_j$ contains scenarios which could only trigger the *j* first rules of the protection, and its composition is different for each *j*th-level node.

An initial guess of the partition is made by considering the value of the signal V “just after disturbance”, i.e. before any load shedding. This is best seen from the 2-rule example of Fig. 3. Under the effect of the first disturbance (curves 1 and 2), the voltage drops in between V_1^{min} and V_2^{min}. It is reasonable to initially assume that the first rule will be enough to stabilize the system (this is merely a first guess, questioned in the sequel); this scenario is thus initially put into $\mathcal{S}_1$. On the other hand, the second disturbance (curve 3) makes the voltage fall directly below V_2^{min}, which might trigger the two rules; this scenario is thus set directly into $\mathcal{S}_2$.

The partition of $\mathcal{S}'$ is modified as follows in the course of exploring the search tree. With reference to curve 2 of the same example, when testing the instance $\mathbf{x_1}^{(j)}$ of $\mathbf{x_1}$, it is detected that V falls below the V_2^{min} threshold. Consequently, for that particular choice of $\mathbf{x_1}$, the scenario is moved from subset $\mathcal{S}_1$ to subset $\mathcal{S}_2$ in the remaining to the branch and bound procedure. Note that the maximum in (8) is determined over the remaining $\mathcal{S}_1$ subset.

The same principle applies to any number k of rules.

Note that the lower the voltage just after disturbance, the faster the processing of the corresponding scenario.

D. Speeding up the branch and bound algorithm

In order to speed up computations, two improvements have been brought to the algorithm given in the previous section.

Improvement 1. At step (**) of the algorithm, a saving in computing time is obtained by skipping the exploration of the subtree below node C. Larger savings are possible by evaluating $F(C)$ at all nodes C directly linked to D, and processing these branches by increasing order of $F(C)$ instead of the arbitrary order used in the above algorithm. Indeed, if we suppose that D is a *j*th-level node corresponding to $\mathbf{x_1} = \mathbf{x_1^{(i)}}, ..., \mathbf{x_j} = \mathbf{x_j^{(q)}}$, we have :

$$\forall\, l > p\ :\ F_{\mathcal{S}_{j+1}}(\mathbf{x_1}^{(i)}, ..., \mathbf{x_{j+1}}^{(p)}) \leq F_{\mathcal{S}_{j+1}}(\mathbf{x_1}^{(i)}, ..., \mathbf{x_{j+1}}^{(l)}) \quad (9)$$

Indeed, with the instances of $\mathbf{x_{j+1}}$ so sorted, at step (**) of the the algorithm, if we have

$$F_{\mathcal{S}_{j+1}}(\mathbf{x_1}^{(i)}, ..., \mathbf{x_{j+1}}^{(p)}) > B$$

by virtue of (9) we also have

$$F_{\mathcal{S}_{j+1}}(\mathbf{x_1}^{(i)}, ..., \mathbf{x_{j+1}}^{(l)}) > B \quad \forall\, l > p$$

and hence all the subtrees below the D (not only the one below node C in the above algorithm) can be skipped.

Improvement 2. Furthermore, the above branch sorting allows to test a sufficient condition for the lower bound to be also the sought minimum objective function (and hence the search to stop). Namely, if the temporary minimum $[\mathbf{x_1}^{(i)}, ..., \mathbf{x_k}^{(p)}]$ found after step (*) of the algorithm is such that :

$$F_{\mathcal{S}_1}(\mathbf{x_1}^{(i)}) \geq F_{\mathcal{S}_j}(\mathbf{x_1}^{(i)}, ..., \mathbf{x_j}^{(p)}) \quad \forall j \in \{2, .., k\} \quad (10)$$

then the temporary minimum is also the global one. Indeed,

$$\forall\, u > j,\ \forall\, v : F_{\mathcal{S}'}(\mathbf{x_1}^{(u)}, ..., \mathbf{x_k}^{(v)}) \geq F_{\mathcal{S}_1}(\mathbf{x_1}^{(u)}) \geq F_{\mathcal{S}_1}(\mathbf{x_1}^{(j)})$$

and hence

$$F_{\mathcal{S}'}(\mathbf{x_1}^{(u)}, ..., \mathbf{x_k}^{(v)}) \geq F_{\mathcal{S}'}(\mathbf{x_1}^{(j)}, ..., \mathbf{x_k}^{(p)})$$

Clearly, if the above sufficient condition (10) is early satisfied, the search is early stopped. Therefrom the idea of exploring the branches of the tree in an order that maximizes the *probability* of having this condition satisfied.

Accordingly, the paths from root node to leaves are classified into two sets. The $\mathcal{I}$ set contains *a priori* interesting paths satisfying the first of the *k-1* conditions (10), i.e.

$$F_{\mathcal{S}_1}(\mathbf{x_1}^{(j)}) \geq F_{\mathcal{S}_2}(\mathbf{x_1}^{(j)}, \mathbf{x_2}^{(p)})$$

Their total length is clearly greater or equal to $F_{\mathcal{S}_1}(\mathbf{x_1}^{(j)})$.

On the other hand, the $\mathcal{U}$ set contains the *a priori* uninteresting paths that do not satisfy the first of conditions (10). Their total length is greater or equal to $F_{\mathcal{S}_2}(\mathbf{x_1}^{(j)}, \mathbf{x_2}^{(p)})$.

In order to maximize the probability of satisfying as soon as possible the *k-1* conditions (10), we first explore the paths contained into $\mathcal{I}$. However, if the lower bound of any path contained in $\mathcal{U}$ is lower than the next $\mathcal{I}$ path to explore, we decide to first consider the corresponding $\mathcal{U}$ path.

The so improved branch and bound algorithm can be applied to any *k*-rule optimization problem and allows to find the best design for the corresponding controller. It is much faster than a pure enumerative approach, but time savings depend on the quality of the upper bound B found during the first iterations.

VI. SEQUENTIAL DESIGN

Altough the above branch and bound approach yields dramatic improvements in processing time, it can still remain heavy, particularly when the training set is large. This motivated the development of a suboptimal but faster method, referred to here as the sequential approach.

This approach consists in determining $\mathbf{x_1}, ..., \mathbf{x_k}$ sequentially, each $\mathbf{x_j}$ being computed from the corresponding training subset $\mathcal{S}_j$. More precisely, $\mathbf{x_j}$ minimizes $F_{\mathcal{S}_j}(\mathbf{x_1}, ..., \mathbf{x_j})$ where the $\mathbf{x_1}, ..., \mathbf{x_{j-1}}$ parameters are fixed at their previously determined values. This yields higher values of the objective function (3) than with the branch and bound approach since each $\mathbf{x_j}$ is now only determined from the corresponding $\mathcal{S}_j$ subset without taking into account the influence of these parameters on other scenarios. Our simulation results have shown that a good compromise between accuracy and computing time can be obtained by adjusting the procedure of section V.C in the following manner. The initial partition of $\mathcal{S}'$ is still obtained by considering the value of the signal V just after disturbance. However, each $F_{\mathcal{S}_j}(\mathbf{x_1}, ..., \mathbf{x_j})$ is now determined from all scenarios initially contained into $\mathcal{S}_j$, before moving to $\mathcal{S}_{j+1}$ those whose voltage falls below V_{j+1}^{min} for the current choice of $\mathbf{x_j}$. This allows to choose $\mathbf{x_j}$ taking into account scenarios from subsets $\mathcal{S}_l$ ($l > j$), which are influenced by $\mathbf{x_j}$. In the case of the branch and bound approach, this influence was implicitly taken into account by exploring the various sub-trees of the search tree.

Note that the protection obtained corresponds to the first leaf node reached by the branch and bound algorithm with improvement 1 (sorted branches), and hence the tree exploration is merely skipped. Note also that the first rule is optimized with respect to all the scenarios of $\mathcal{S}_1$. Hence, the controller is the solution of another optimization problem whose objective is to find the best-suited protection for the less severe disturbances.

VII. RESULTS ON THE HYDRO-QUEBEC SYSTEM

A. Voltage stability of the Hydro-Québec system

The Hydro-Québec system is characterized by great distances (more than 1000 km) between the large hydro generation areas (James Bay, Churchill Falls and Manic-Outardes) and the main load center (around Montréal and Québec City). Accordingly, the company has developed an extensive 735-kV transmission system, whose lines are located along two main axes. This system is angle stability limited in the North, voltage stability limited in the South (near the load center). Frequency stability is also a concern due to the system interconnection through DC links only, as well as the sensitivity of loads to voltage.

In the recent years, Hydro-Québec has undertaken a major program to upgrade the reliability of its transmission system. In particular a defence plan is being deployed against extreme contingencies [10]. This includes generation rejection and remote load shedding, automatic shunt reactor switching, underfrequency load shedding and in a near future, undervoltage load shedding.

Beside static var compensators and synchronous condensers, the automatic shunt reactor switching devices, known under the French acronym MAIS, play an important rôle in voltage control [11]. These devices, in operation since early 1997, are now available in 22 735-kV substations and control a large part of the total 25,500 Mvar shunt compensation. Each MAIS relies on the local voltage, the coordination between substations being performed through the switching delays. While fast-acting MAIS can improve transient (angle) stability, slower MAIS significantly contribute to voltage stability.

Preventive security assessment is also a major concern. Presently, secure operation limits are determined in operational planning, with the help of ASTRE, the QSS simulation program developed at the University of Liège [3, 7]. This fast tool has been used in the present study. The corresponding model includes around 550 buses, 100 generators and 230 LTCs. The total load is around 33,000 MW.

B. Training scenarios and main protection parameters

The study reported in this paper involves 8 system configurations, summarized in Table 1.

Table 1. System configurations considered in the training scenarios

configuration	number of: 735-kV lines out of service	synchronous condensers	MAIS devices
A	0	6	5
B	1	6	9
C	1	6	8
D	1	8	16
E	1	8	14
F	4	8	16
G	3	8	14
H	3	8	14

Table 2 details the 36 scenarios finally selected. They involve N-1, N-2 and N-3 contingencies, respectively. In accordance with the standard operating rules, the system is stable following any N-1 incident. The MAIS devices can be used to this purpose. In each unstable scenario, the best load shedding location has been identified. Therefrom, a common ranking of load buses has been set up. For simplicity, each load is assumed fully interruptible. Using this bus ranking, the minimal amount of load shedding $P_i^\star$ required to stabilize the system has been determined in the 19 unstable scenarios. The values, computed with an accuracy of 10 MW, are given in Table 2. The most severe incident requires to shed load at 8 buses.

The scenarios have been chosen according to the following guidelines :
- the training set includes 17 scenarios with $P_i^\star = 0$ in order to adjust V_1^{min} and lower bounds on d_i parameters ;
- on the other hand, the nonzero values of $P_i^\star$ range rather uniformly in the [0 1790] MW interval, between the marginally and the severely unstable cases.

As regards the protection, two and three rules ($k = 2$ or 3) have been considered. The measured signal V is the average voltage over five 735-kV buses in the Montréal area. Requirements 1, 2 and 3 of Section III.C have been taken into account. However, in accordance with Hydro-Québec planning rules, the

Table 2. Description of the 36 training scenarios

No	conf.	incid. type	$P_i^\star$ (MW)	No	conf.	incid. type	$P_i^\star$ (MW)
1	A	N-1	0	19	E	N-2	890
2	A	N-1	0	20	E	N-2	890
3	A	N-2	1090	21	F	N-1	0
4	A	N-2	460	22	F	N-2	0
5	A	N-2	110	23	F	N-2	1110
6	A	N-3	1520	24	F	N-2	860
7	B	N-1	0	25	F	N-2	620
8	B	N-2	200	26	G	N-1	0
9	C	N-1	0	27	G	N-1	0
10	D	N-1	0	28	G	N-2	40
11	D	N-1	0	29	G	N-2	1790
12	D	N-2	0	30	G	N-2	880
13	D	N-2	740	31	G	N-2	760
14	D	N-2	350	32	H	N-1	0
15	E	N-1	0	33	H	N-1	0
16	E	N-1	0	34	H	N-2	310
17	E	N-2	0	35	H	N-2	730
18	E	N-2	0	36	H	N-2	600

3rd requirement has been amended by allowing some (hopefully small) load shedding to take place after a stable but severe incident. The N-2 scenarios Nb. 12, 17, 18 and 22 are concerned. The latter are handled as unstable scenarios with $P_i^\star = 0$ in (3).

Voltage thresholds were chosen as indicated in Section III.D. In order to obtain a good synchronization with MAIS whose settings are typically in the range [0.965 0.97 pu], the V_1^{min} parameter was adjusted to 0.95 pu. Moreover, this value guarantees that the controller will not act in the stable situations of Table 2, except for scenario 10 whose voltage falls below 0.93 pu at the beginning of the simulation. The thresholds relative to rules 2 and 3 were adjusted to 0.93 and 0.91 pu respectively, in order to act before the minimal amount of load shedding becomes prohibitive and also to minimize customers'trouble.

The remaining parameters were discretized as follows:

• delay d_1 : 13 values in the range [3 15] s. The lower bound allows to distinguish a voltage instability from a temporary undervoltage. Note that this value is lower than the minimal delay (12 s) required to ensure the controller will not act in scenario 10. The reason is that we cannot penalyze all the unstable cases, for which a short time delay could be more effective, because of a single stable scenario;

• shedding steps ΔP_1 : 6 values in the range [50 300] MW. The lower bound is the minimum amount that can be tripped by opening distribution feeders, while the upper bound has been limited to avoid excessive load shedding steps;

• delay $d_i,\ i > 1$: 8 values in the range [3 10] s. The upper bound is lower than the one used for rule R1 since we expect that the lower the voltage threshold of a rule, the lower the corresponding time delay;

• shedding steps $\Delta P_i,\ i > 1$: 11 values in the range [300 800] MW. Both the lower and the upper bound are greater than those of rule R1 since the lower the voltage threshold of a rule, the greater the amount of load to be shed by such rule.

The initial partition of $\mathcal{S}'$ is shown in Table 3. It is obtained as indicated in section IV.D. Note that scenario 28, whose JAD voltage is greater than 0.95 pu, is classified into $\mathcal{S}_1$ since the corresponding amount of load to shed is greater than 0 MW.

Table 3. Initial partition of $\mathcal{S}'$

2 rules	3 rules	incid. No	V_{JAD} (pu)	incid. No	V_{JAD} (pu)
$\mathcal{S}_1$	$\mathcal{S}_1$	5	0.9419		
		28	0.9505		
		35	0.9411		
$\mathcal{S}_2$	$\mathcal{S}_2$	4	0.9122		
		30	0.9249		
		34	0.9295		
	$\mathcal{S}_3$	3	0.8045	23	0.7767
		6	0.7950	24	0.8648
		8	0.8975	25	0.8728
		13	0.8101	29	0.7529
		14	0.9037	31	0.8813
		19	0.8648	36	0.8976
		20	0.8612		

C. Results and discussion

2-rule protection. The optimal rules obtained using the branch and bound approach are :

R1: *if* $V < 0.95$ *pu during 14 seconds, shed 250 MW*
R2: *if* $V < 0.93$ *pu during 3 seconds, shed 450 MW*
Minmax objective : 320 MW
Computing time : 4 hours

while those provided by the sequential design are :

R1: *if* $V < 0.95$ *pu during 12 seconds, shed 200 MW*
R2: *if* $V < 0.93$ *pu during 3 seconds, shed 400 MW*
Minmax objective : 340 MW
Computing time : 32 minutes

The structure of both protections is quite similar. Rule R1 is used in the less severe unstable scenarios; its rather long delay and moderate shedding yield a good coordination with the MAIS, which are given time to act. Rule R2 is used in more severe scenarios, leading to a more pronounced voltage drop. The structure of the above rules is "classical" in the sense that the larger the voltage drop, the greater the action and the smaller the delay to take this action. Figure 5 shows the performances of both protections in terms of "over-shedding" with respect to the optimal values P_i^*. No load is shed in any stable case.

From the computing time view point, the sequential design is much more effective while providing quite a good controller (the objective is only 20 MW worse than with the branch and bound approach).

3-rule protection. The optimal rules obtained using the branch and bound approach are :

R1: *if* $V < 0.95$ *pu during 5 seconds, shed 100 MW*
R2: *if* $V < 0.93$ *pu during 4 seconds, shed 450 MW*
R3: *if* $V < 0.91$ *pu during 3 seconds, shed 350 MW*
Minmax objective : 290 MW
Computing time : 24 hours

while those provided by the sequential design are :

R1: *if* $V < 0.95$ *pu during 12 seconds, shed 200 MW*
R2: *if* $V < 0.93$ *pu during 9 seconds, shed 500 MW*
R3: *if* $V < 0.91$ *pu during 5 seconds, shed 350 MW*

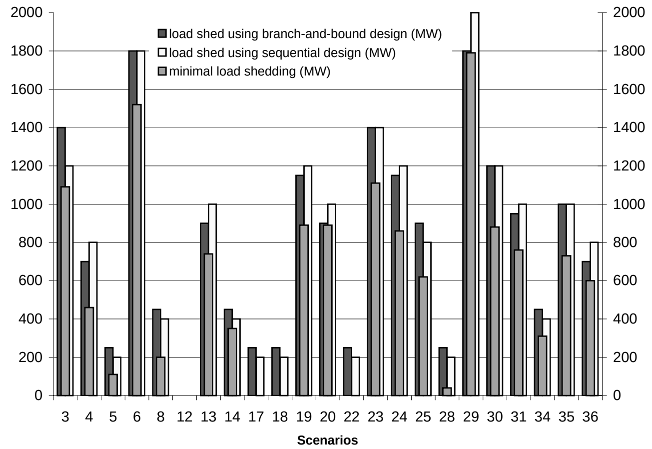

Figure 5: Performances of branch and bound and sequential designs

Minmax objective : 430 MW
Computing time : 51 minutes

In both cases, the structure of the protection is almost the "classical" one, except for the amount of load shed by rule R3. Figure 6 shows the performances of both protections. No load is shed in a stable situation with the sequential design, whereas the optimal protection sheds 100 MW when facing scenario 10. This exception to requirement 3 allows however to obtain a much better suited design in unstable situations.

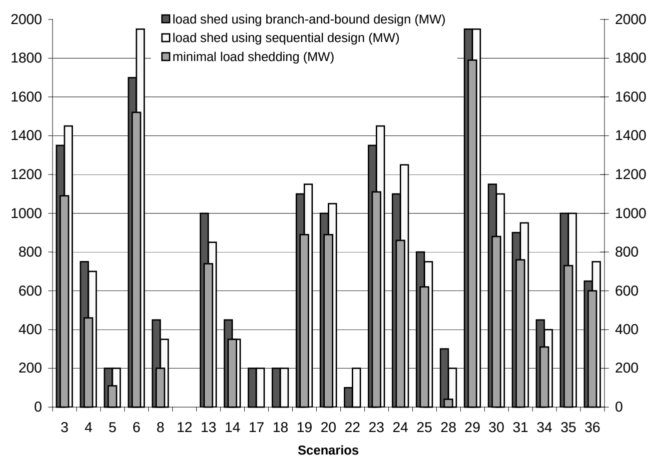

Figure 6: Performances of branch and bound and sequential designs

Again, the sequential design is much less time consuming. However, its performances are in this case less satisfactory than those obtained with the branch and bound approach.

Comparison. As expected, the branch and bound design is obviously better in the case of the 3-rule protection, since there is an additional rule allowing to meet requirement 2 more easily. On the other hand, the sequential approach seems to work better with a 2-rule controller. This could be due to the initial partition of $\mathcal{S}'$ into subsets $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$. Since there is no scenario from $\mathcal{S}_3$ initially classified into $\mathcal{S}_1$, $\mathbf{x_1}$ has been optimized without taking into account scenarios from $\mathcal{S}_3$. This could be improved by initially moving some scenarios from $\mathcal{S}_3$ to $\mathcal{S}_1$ in order they contribute to the choice of $\mathbf{x_1}$ before bringing them back to $\mathcal{S}_3$.

VIII. CONCLUDING REMARKS

In this paper the design of automatic load shedding schemes is formulated as a combinatorial optimization problem solved by means of both branch and bound and sequential approaches. This yields optimized rules which can be easily implemented and interpreted.

Obviously, many aspects remain to be investigated. The branch and bound approach is faster when a good upper bound B on the global optimum can be quickly determined. In this respect, the heuristic procedure described in Section V.D gave good results. A better approach could however be sought.

Although the branch and bound method is much less time consuming than a pure enumerative approach, the computing time could become prohibitive when considering large training sets. Results from section VII.C show that the sequential design could be in such cases a good compromise between the effectiveness of the obtained controller and the time required to find it. Further investigations are however needed before drawing a general conclusion.

It can now be envisaged to handle a larger number of scenarios (e.g. more system configurations and more incidents), a wider range of possible load behaviours (to take into account the uncertainty in their modelling) and possibly more detailed time simulations (in order to handle, for instance, short-term voltage instability situations, or to coordinate load shedding with other, fast countermeasures).

Nevertheless, the approach has already given very satisfactory results. In the Hydro-Québec system, for instance, it is already helping planners in the complex task of designing a robust system protection scheme.

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