An elasto-viscoplastic model for chalk including suction effects

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ABSTRACT: During the six years long Pasachalk project devoted to the mechanical behaviour of high porosity chalks from North Sea oilfields, the constitutive model Pasachalk (Collin et al., 2002) was proposed based on the Barcelona Basic Model (BBM) (Alonso et al., 1990). The approach was based on the similarities found between the oil-water interactions (oil and water being the non wetting and wetting fluid respectively) in oil reservoir chalk and the air-water interactions in unsaturated soils. This approach appeared to be relevant to interpret the subsidence of the seafloor during waterflooding operations for enhanced oil recovery that has been observed in North Sea oilfields (e.g. Ekofisk oilfield). Another important component of subsidence was then related to the creep behaviour of the multiphase chalk (De Gennaro et al., 2003). A modified Pasachalk model was proposed to account for time effects using the framework of Perzyna's viscoplasticity (1964) but without considering suction effects. Based on available experimental results (Priol et al., 2007), a modified version of the viscoplastic Pasachalk model including suction effects is proposed in this paper.

1 INTRODUCTION

The mechanical behaviour of chalk has been extensively studied since the early eighties, in particular with regard to the behaviour of oil reservoir chalks in the North Sea (Ekofisk oilfield, see Hermansen et al., 2000, Nagel 2001). More recently, the risk assessment of the long-term stability of chalk pillars in mineworkings has been considered. In both situations, two poorly miscible pore fluids of different wettabilities are contained by chalk: water and oil in reservoir chalks and water and air in continental chalks from mines and quarries. In multiphase chalks, the partial saturation can change with time. The water saturation in oil reservoir chalks can increase due to reservoir enhanced exploitation by waterflooding (e.g. Ekofisk oilfield). Water saturation changes in mine chalks can be a consequence of the combined effects of changes in water table and in the hygrometry of the mine.

Like in unsaturated soils, it has been showed that changes in partial saturation have an influence on the mechanical response of chalk, the higher the degree of water saturation, the higher the compressibility and the lower the strength (water weakening effect). In oil reservoirs, mines and quarries, the overburden formations apply long term hydro-mechanical loadings, resulting in creep, a behaviour feature particularly pronounced in chalks.

Some recent experimental and theoretical approaches carried out on partially saturated chalks have confirmed the relevance of some concepts of the mechanics of unsaturated soils to multiphase chalk behaviour, as suggested by Delage et al. (1996). Collin et al. (2002) proposed a modified version of the Barcelona Basic Model (BBM, Alonso et al., 1990) called Pasachalk model to model the behaviour of oil reservoir chalks. This model was extended to include viscoplastic behaviour of reservoir chalk but this latter model did not take implicitly the suction effect into account (De Gennaro et al., 2003). The effects of the oil-water suction on the time dependent behaviour of reservoir chalks has only been partially explored up to now. In this regard, recent findings (Priol, 2005; Priol et al., 2007) from oedometer tests carried out on chalk samples seem to suggest that the delayed strain of chalk is well correlated with the over-stress ratio (i.e. the ratio between the actual stress and the yield stress).

Based on these findings and on other available results, a modified version of the viscoplastic Pasachalk model including oil-water suction effects is proposed in this paper.

2 EXPERIMENTAL EVIDENCE OF CREEP IN CHALK

Priol (2005) and Priol et al. (2007) reported results of oedometer compression tests carried out on Lixhe chalk (an outcrop chalk from Belgium) saturated with oil, with water, partially saturated and dry (Fig. 1). By analyzing the evolution of the creep curves obtained during multiple step loading tests, it was suggested to consider the following rheological law to fit the experimental data:

$$\frac{e}{e_{oi}} = \beta_i t^{-\alpha_{cr}} \qquad (1)$$

where e is the void ratio, e_{oi} the initial void ratio, t the time, β_i a coefficient accounting for the instantaneous
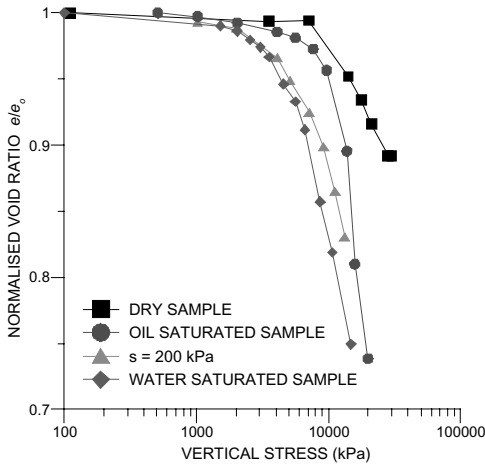

Figure 1. Oedometer compression tests on Lixhe chalk at various saturation states (Priol et al., 2007).
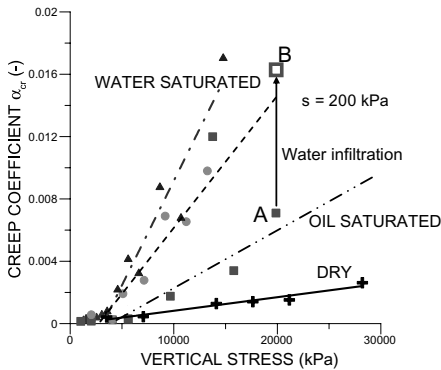

Figure 2. Influence of stress level and suction on the creep coefficient α_{cr} (Priol et al., 2007).

settlement and α_{cr} for the time dependent settlement. The evolution of α_{cr} is directly related to the creep behaviour of chalk (Fig. 2). One can observe that the amount of creep is both stress and suction dependent, the higher the wettability of chalk, the larger the amount of creep.

3 MODEL FORMULATION

Perzyna's viscoplastic approach (Perzyna, 1964) has been adopted because it is based on a framework similar to that of elastoplasticity, facilitating further time-dependent developments of the elastoplastic Pasachalk model. Various viscoplastic models have been formulated adopting the Bjerrum's notion of equivalent (or reference) time (e.g. Bjerrum, 1967; Borja & Kavazanjian, 1985; Hickman & Gutierrez, 2007). Other models have used the concept of the Non Stationary Flow Surface (NSFS) theory (e.g. Nova, 1982; Liingaard et al., 2004). A recent review of the literature is given by Liingaard et al., (2004). The Pasachalk model (Collin et al., 2002) is a cap model with a yield surface composed of three components:

(*i*) *Cam-Clay pore collapse model*
The Cam-Clay yield surface is adopted at low stress inclinations, with the following expression:

$$f_1 \equiv II_{\hat{\sigma}}^2 + m^2 \left(I_\sigma + \frac{3c(s)}{\tan \phi_C} \right) (I_\sigma - 3p_0) = 0 \qquad (2)$$

where I_σ and II_σ are the first and second stress invariants, c is the cohesion, ϕ_C is the friction angle in compression path, p_0 is the apparent pre-consolidation pressure that defines the size of the yield surface and m is a coefficient taking into account the effect of the Lode angle β. The coefficient m is defined by:

$$m = a (1 + b \sin 3\beta)^n \qquad (3)$$

where parameters a, b and n must verify some convexity conditions (Van Eekelen, 1980).

Assuming associated plastic flow, the apparent pre-consolidation pressure p_0 is related to the volumetric plastic strain $d\varepsilon^{vp}$ following the kinematic equation:

$$dp_0 = \frac{1 + e}{\lambda - \kappa} p_0 d\varepsilon_v^p \qquad (4)$$

where λ is the compression coefficient and κ is the elastic coefficient.

Expression (4) allows both hardening or softening behaviour to be accounted for according to the sign of the volumetric plastic strain. However the softening zone will not be considered here. It can also be

noted that the irreversible volumetric strain includes the coupled effect of mechanical and suction changes.

(ii) Internal friction model

In order to formulate a friction model based on a Mohr-Coulomb type failure criterion with a smoothed plastic surface, Van Eekelen's (1980) formulation has been adopted. It is based on a modification of Drucker-Prager's failure cone by introducing a dependence on Lode's angle β, leading to the following expression of the failure criterion:

$$f_2 \equiv II_{\hat{\sigma}} - m \left(I_\sigma + \frac{3c(s)}{\tan \phi_C} \right) = 0 \tag{5}$$

An associated plasticity is considered also for the friction mechanism.

(iii) Suction effect on yield surface (BBM model)

Several phenomena are typical of unsaturated soils:

- The yield stress p_0 and the material stiffness increase with suction. In BBM this is described by the LC curve, the formulation of which has been adapted for chalk as follows:

$$p_0(s) = p_0(0) + \Delta p_0 \frac{s}{s + s^*} \tag{6}$$

where $p_0(0)$ is the yield stress for $s = 0$, Δp_0 is the variation of p_0 between water and oil saturated sample and s^* is a parameter controlling the shape of the *LC* curve.
- Cohesion increases with suction. This is modelled using Eq. (7).

$$c(s) = c(0) + k \, s \tag{7}$$

where k is a material constant, $c(0)$ is the cohesion at water saturated state. Note that in chalk, experiments showed that the friction angle is independent of the saturating fluid.

Mechanical elastoviscoplastic model

Viscous effects in chalk may be observed in triaxial tests performed at various stress rates and/or involving creep stages (Pasachalk2, 2004). The time-dependent behaviour of chalk is introduced here based on the elastoviscoplastic approach proposed by Perzyna (1964). Hence, strains are divided into reversible and irreversible parts (related mechanical and suction loading):

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{m,e} + \dot{\varepsilon}_{ij}^{s,e} + \dot{\varepsilon}_{ij}^{m,vp} + \dot{\varepsilon}_{ij}^{s,p} \tag{8}$$

It has been observed that suction variations do not evolve permanent strains. Moreover, it is assumed that reversible strains related to suction are not time-dependent. The following relationship is taken [Alonso et al., 1990]:

$$\dot{\varepsilon}_{ij}^{s,e} = \frac{\kappa_s}{(1 + e)} \frac{\dot{s}}{(s + p_{at})} \delta_{ij} = h_{ij}^e \dot{s} \tag{9}$$

The stress increment can thus be expressed as follows:

$$\underline{\dot{\sigma}} = \underline{\underline{C}}^e(s) \, (\underline{\dot{\varepsilon}} - \underline{\dot{\varepsilon}}^{s,e} - \underline{\dot{\varepsilon}}^{m,vp}) \tag{10}$$

Since only the irreversible behaviour is time-dependent, the elastic moduli of the Pasachalk model can be kept. The values of the moduli defining Hooke's law are recalled in Table 1. The elastic parameters are suction dependent. The following linear expressions (Pasachalk 2, 2004) have been chosen for the volumetric and shear moduli as a function of suction:

$$K(s) = K(0) + k_s \cdot s \tag{11}$$

$$G(s) = G(0) + g_s \cdot s \tag{12}$$

where $K(0)$ and $G(0)$ are the elastic moduli for a nul suction (water saturated condition), k_s and g_s are equal to 38 and 66.7 respectively to model the increase of stiffness with the suction.

The irreversible strain may be described as normal to some potential g:

$$\underline{\dot{\varepsilon}}^{m,vp} = \gamma \, \langle \phi(f) \rangle \, \frac{\partial g}{\partial \underline{\sigma}} \tag{13}$$

This formulation is similar to the elastoplastic one, but it is not based on the consistency condition. The amount of strain rate is described with respect to a reference surface f, similar to the yield surface. Then, one may define two irreversible mechanisms, one dedicated to the pore collapse mechanism named f_c, the second one to friction failure named f_d.

The reference surface f_c has the same equation as f_1 in the Pasachalk model. The reference surface actually represents the elastoplastic yield surface defined based on a hypothetical experiment with an infinitely low strain rate. The function f_c may help to define the overstress, as a measure of the amount of the stress state going outside the reference surface.

Table 1. Elastic parameters (Collin et al., 2002).

	Water	Oil
K [MPa]	612	726
G [MPa]	500	700
E [MPa]	1180	1590
ν [−]	0.18	0.14

Concerning the pore collapse mechanism, the creep potential is based on the following equations:

$$\langle \phi_c(f_c) \rangle = \left(\frac{p_0^d}{p_0^{vp}} - 1 \right)^{\alpha_c} \qquad (14)$$

and (Shao et al., 1993):

$$\gamma = \omega \left(\frac{I_\sigma}{p_a} \right)^{\iota} \qquad (15)$$

where the viscous parameters are: γ, the fluidity parameter, ω, p_a and ι, the parameters defining the influence of stress on the fluidity parameter and α_c, the exponent of the visco-plastic strain relation (14).

The parameters defining the yield surface of the elastoplastic model for a stress rate of 10^{-3} MPa/s are given in the Table 2. The viscous parameters concern mainly the pore collapse mechanism because the failure criterion is assumed to be time-independent. Hence, only the viscous parameters γ (fluidity parameter), the reference surface f_c and the exponent α_c of visco-plastic strain relation have to be determined.

As shear failure is assumed to be time independent, the reference surface f_c related to pore collapse only depends on the apparent viscoplastic pre-consolidation pressure p_0^{vp}. Experiments have shown that the pre-consolidation pressure depended directly on the stress-rate. This relation is not defined directly in the model: the effect of rate dependence comes as a result of the chosen visco-plastic formulation.

The p_0^{vp} value and the other viscous parameters have been determined by trial and error process in order to fit isotropic compression tests on saturated chalk (oil and water), with loading rate ranging between 5×10^{-5} and 10^{-2} MPa/s (Pasachalk 2, 2004). Within the assumed loading rate range the final values of all parameters are given in Table 3. Note that, in agreement with the notion of overstress, it is not necessary to chose different values of viscous parameters (α, ω, ι) for oil or water saturated samples, as the influence of suction is taken into account through the apparent pre-consolidation pressure.

For intermediate degrees of saturation, the LC curve adopted is similar to that used in the elastoplastic model, using Eq. (6) with $\Delta p_0^{vp} = 3$ MPa, $p_0^{vp}(0) = 2$

Table 2. Yield surface parameters of the elastoplastic model.

	Water	Oil
Friction angle ϕ [°]	22	22
Cohesion c [MPa]	1.5	2.0
Pre-consolidation p_0 [MPa]	10	21
Compressibility index λ	0.195	0.195

Table 3. Viscous parameters of the model.

	Water	Oil
Pre-consolidation p_0^{vp} [MPa]	2	5
Parameter α_c	5	5
Parameter ω	$5.1\ 10^{-9}$	$5.1\ 10^{-9}$
Parameter ι	0,0	0,0

MPa et $s^* = 0.2$ MPa. It is important to notice that the same value of the compressibility index λ has been used for the definition of the hardening law of the viscoplastic model.

4 ASSESSMENT OF THE VISCOUS PARAMETERS

One of the major shortcomings associated with Perzyna's approach is the definition of the viscous parameters and of the reference surfaces, which are usually found by a trial and error process and not directly experimentally determined. In order to link more directly the parameters to experimental measurement the results of CRS (Constant Rate of Strain) oedometer compression tests at different strain rates and suction (water or oil saturated, 200 kPa suction and dry samples) are first analysed (Priol, 2005; Priol et al., 2007).

It was observed that for a given suction the yield limit (i.e. apparent pre-consolidation pressure) is a function of the imposed strain rate, as already shown in clays by Leroueil et al. (1985). The following relationship coupling the yield limit and the strain rate proposed by Leroueil appeared to fit reasonably data obtained on Lixhe chalk:

$$\log_{10}(\sigma_p) = A + \frac{1}{m'} \log_{10}(\dot{\varepsilon}_1) \qquad (16)$$

where σ_p is the yield limit, $\dot{\varepsilon}_1$ is the strain rate and A and m' two material parameters. Table 4 summarizes the values of A and m' obtained for Lixhe chalk. Equation (16) describes a linear relationship between yield limit and strain rate in a $\log_{10}(\sigma_p) : \log_{10}(\dot{\varepsilon}_1)$ plot. It is worth noting that values of m' depends now also on suction (Tab. 4). In other words the slope of the linear relationship (16) increases when suction decreases. This is a new further coupling which extends the original Leroueil's relationship. Equation (16) gives the opportunity to define the size of the reference surface defined as the elastoplastic yield surface based on a hypothetical experiment with an infinitely low strain rate. Considering an extremely low strain rate (10–13 s^{-1}), the yield stress of the reference surface

Table 4. Material parameters of Leroueil's law.

	A	m'
Water	4,462	9,25
s = 200 kPa	4,516	10,9
Oil	4,451	16,66
Dry	4,499	22,2
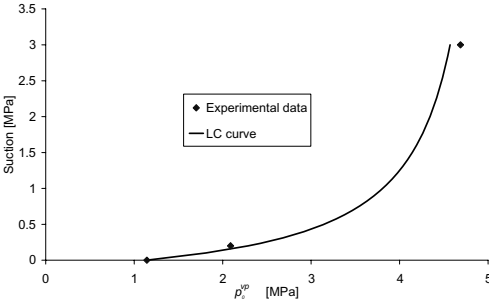
Figure 3. LC curve of the reference surface.

Figure 4. Influence of "normalized" stress level and suction on the creep coefficient α_{cr} (Priol et al. 2007).

Table 5. Viscous parameters of the model for unsaturated conditions.

	Water
Pre-consolidation p_0^{vp} [MPa] at s = 0 MPa	1,143
Parameter Δp_0^{vp} [MPa]	4
Parameter s^* [MPa]	0.5
Parameter α_c	5
Parameter ω	$5{,}1\ 10^{-9}$

for the different suction conditions could be defined, together with the LC curve of the viscous reference surface (Fig. 3).

Viscous parameters should now to be linked to the β_i and α_{cr} parameters of equation (1). Figure 2 shows a first discrepancy between the evolution of α_{cr} and the viscous parameters of the model. Indeed Figure 2 does not show a unique relation between α_{cr} and the stress state for the different saturation conditions. On the other hand, it is not necessary to chose different values of viscous parameters for oil or water saturated samples. The influence of suction is only taken into account through the LC curve.

The apparent contradiction can be explained by inspecting Figure 2. One can observe that the creep parameter α_{cr} remains very low up to a threshold that depends on the saturation conditions. Above the threshold, parameters follow a more or less linear relationship with slopes also depending on the saturation conditions. However, if the stress value is normalized with the apparent pre-consolidation pressure of each test as presented in Figure 4, the observed behaviour becomes reasonably independent of the saturation conditions.

It would be interesting to find a direct relationship between the α_{cr} and β_i parameters of equation (1) and the ω and α_c parameters of the viscous model (Equation 13). Unfortunately, it was not possible to find such a relationship analytically. The main reason is that equation (1) defines the total creep strain and the viscous strain rate is modelled through equation (13). Moreover, the analytical integration of the
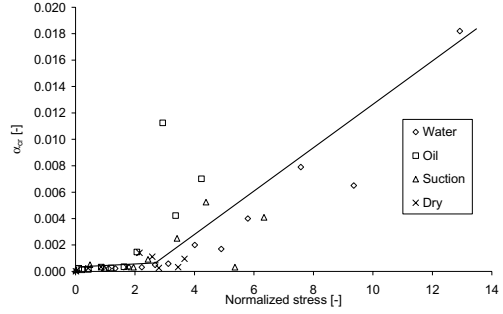
viscous model is not possible for any values of the material parameters.

It was thus decided to keep the first estimation of the two viscous parameters for the modelling of the multi-stage loading tests.

5 NUMERICAL MODELLING

Some experimental results obtained by Priol (2005) by running creep oedometer tests under different suction conditions (water saturated, oil saturated and suction equal to 200 kPa) are reported in Figure 5 in terms of strain versus time curves. One can clearly see in the figure the various loading steps and the creep deformations under different applied stresses.

With the single set of parameters and the proposed visco-plastic model, the three tests have been modelled. Figures 6–8 show satisfactory agreement between experimental data and numerical predictions.

It should be emphasised that, besides creep tests, the collapse experiment can also be modelled by the proposed constitutive law. Indeed, during water-flooding, the suction is decreasing as well as the pre-consolidation pressure following the LC curve. This means that the overstress is growing during water-flooding, leading to an increase of the viscous creep deformation.
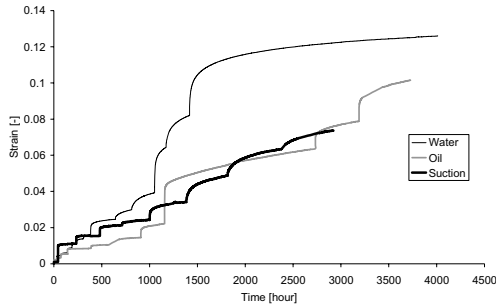
Figure 5. Multiple stage loading tests for different saturation conditions (Priol, 2005).
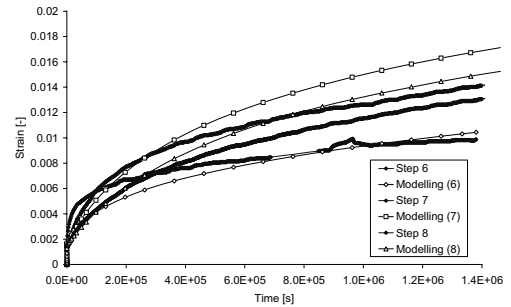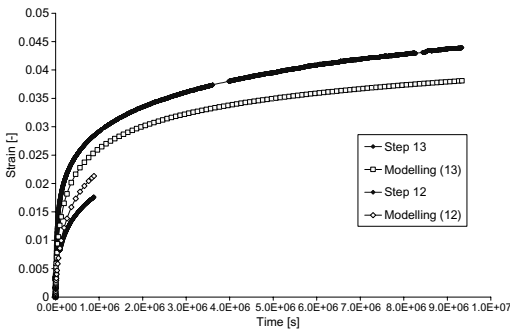

Figure 6. Numerical modeling of creep phase for water saturated chalk samples.

Figure 7. Numerical modeling of creep phase for oil saturated chalk samples.

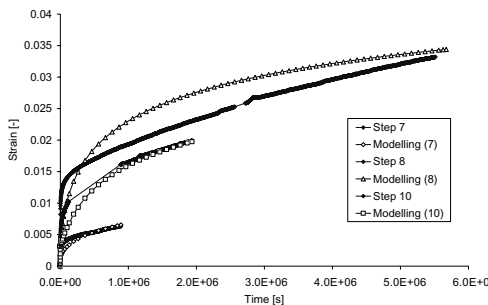Figure 8. Numerical modeling of creep phase for unsaturated chalk samples (s = 200 kPa).

significant creep deformations. The Pasachalk elasto-plastic model (Collin et al., 2002) derived from the Barcelona Basic Model for multiphase reservoir chalk has been extended to account for time effects and creep behaviour as a function of suction.

Some experimental results of multiple stage loading tests carried out on Lixhe chalk under different suction conditions showed that the results obtained under various suctions could be summarized into a single normalized curve.

The relevant viscous parameters of the model were determined based on these experimental results, without using a trial and error method. This has only been possible for the definition of the viscous reference surface. We did not succeed to find an analytical relationship between the parameters of the viscous model and the constitutive law.

By using one single set of parameters, different creep experiments under various suction conditions were simulated, with a satisfactory agreement between experimental data and numerical predictions.

ACKNOWLEDGMENTS

6 CONCLUSIONS

High porosity chalks have a complex mechanical behaviour that depends on chalk porosity, mineralogy, pore fluids, of temperature and of time with

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