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# Influence of morphological characteristics of heterogeneous moraine formations on their mechanical behaviour using image and statistical analysis

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#### Abstract

The study of landslide stability on mountain slopes becomes more difficult when the sliding materials are heterogeneous. This is a current problem with the old glacial moraines now under study in the Aspe Valley, Pyrénées. Analysis of slope stability numerical models necessitates accurate data about mechanical and physical properties. Because moraines are very heterogeneous, a large sample is necessary, but, unfortunately, data acquisition costs a lot of time and money. Therefore, we would like to estimate mechanical properties from correlated variables that are easier to acquire (morphological variables using image analysis). Observations in the field and previous mechanical results in the laboratory have shown that differences between the behaviour of moraines seem to be related not only to their three-dimensional structure but also to the morphological and petrographical characteristics of their components. The moraines are classified based on textural characteristics at the sample scales based on the distributions of size and shape of their constitutive elements (blocks, matrix, etc.). Then, we study the statistical distribution of the variables to highlight the most significant variables. Next, we evaluate the results of the mechanical behaviour of the moraines, with the internal angle of friction and the effective cohesion. On seven specific moraines, we established relations between the effective internal angle of friction, the elongation factor and the roughness factor.

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### 1. Introduction

In this work, unstable moraine formations (Fig. 1) on mountain slopes occur in heterogeneous granular formations (Fig. 2), which highlight a relation be-

tween the internal angle of friction that characterises the mechanical behaviour and shape factors of the grains that build the skeleton of these formations. This study is based on a classical analysis of the mechanical behaviour and the results of image analysis to characterise and quantify the size and the shape of the components of granular formations. Frossard (1978) and De Jaeger (1991) showed, by

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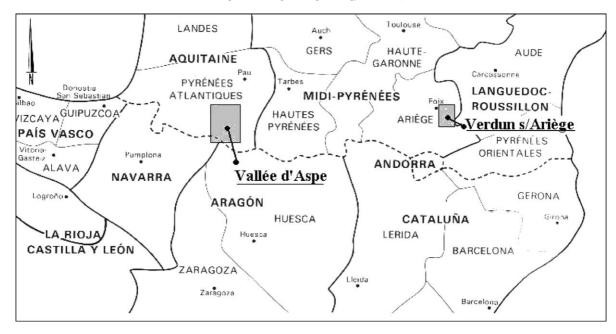


Fig. 1. Location of the studied area.

the measurements and transformations on images, relations connecting morphological variables to physical properties. These reliable scientific relations used shape variables defined on the shape chart (Krumbein, 1941, Rittenhouse, 1943, Krumbein and Sloss, 1963). To eliminate the visual, therefore, subjective estimation of shape variables based on the use of charts, we propose to work with image analysis. Moraine-forming materials are like a large mass of unsorted, heterogeneous glacial or periglacial material characterised by rock debris of all sizes, from angular blocks meters in size to very fine rock. In addition to the block sizes, lithology, petrography and the spatial distribution of the blocks are also heterogeneous. Therefore, it is difficult, if not impossible, to collect a large sample of mechanical and physical data from the moraine in order to execute good simulations to run in numerical programs. To overcome this difficulty correlations are determined between the physical and mechanical characteristics of the rock mass and their structural properties at the scale of the sample (from cm to mm). After correlations are derived, estimates of mechanical and physical data can be provided for the nodes of a grid and then numerical computation can be performed. This has the advantage of decreasing the costs both of the laboratory tests and the sampling in the field.

Experimental frameworks already exist in the literature (Morris, 1959; Frederick, 1960; Mackey, 1963; Zelasko, 1966; Frossard, 1978; Favre, 1980; Biarez and Hicher, 1989; De Jaeger, 1991; Lecomte and Mechling, 1999; Bourdeau, 1999) showing relations between the shape of particles and the mechanical properties, but only concerning sand or silt. Therefore, these relationships cannot be applied to geological formations characterised by a large granulometric distribution (Lebourg, 2000, Lebourg et al., 2000, 2002).

The variables taken into account in this study are mechanical properties obtained from triaxial tests, grain size distribution variables and textural variables (Pirard, 1993, Lebourg, 2000). This data set will permit us to estimate a mechanical property (internal angle of friction,  $\varphi'$ ) using shape variables, which describe the moraine components. As a conclusion, new developments are proposed.

## 2. Geological setting and location

We worked on two sectors, one composed of a gneissic, Paleozoic, fractured substratum (Ariége, central Pyrenees, moraine T<sub>7</sub>) and the other composed of a

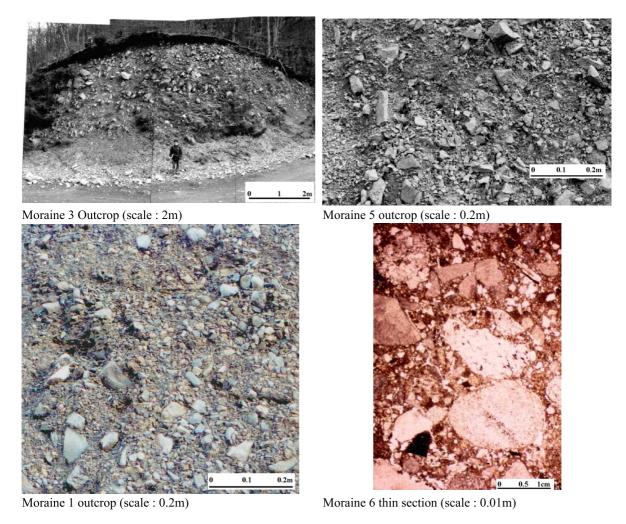


Fig. 2. Photos of moraines outcrop.

sedimentary, Paleozoic, substratum that was also fractured and folded (Aspe Valley, Atlantic Pyrenees, moraines  $T_1$  to  $T_6$ ) (Fig. 1). The study of glacial till deposits and local geomorphology show that moraine deposits, which are currently moving, are deep and contained in glacial basins. The dynamic erosion of different substrata can be either guided by the formations fracturing and the differential granular disintegration in the case of gneiss (Ariégeoise Pyrenees), or guided by tectonic structures or by variations of petrographical facies in the case of a sedimentary substratum (Aspe Valley, Pyrenees). The existence of several glacial erosion phases generated two different types of erosion: the first one, very abrasive is conditioned by

lithological discontinuities and mechanical weaknesses, while the other glacial phases depend directly on geological superstructures. After glacial retreat, various types of moraines, such as weathered moraine, subglacial moraine and frontal moraine are represented. Moraines studied here are lateral moraines.

# 3. Textural and mechanical properties

### 3.1. Size and shape of particles

The large granulometric variability of moraines is one of their heterogeneity characteristics. We have decided to classify the particles into three sets depending on their size (see below) (scale levels; the outcrop and the samples of large size and standard size (Lebourg, 2000)). We studied seven moraines from two mountainous areas: Vallée d'Aspe (Pyrénées Atlantiques, France) and Vallée de Verdun (Ariège, France).

### 3.1.1. Moraine: a heterogeneous formation

Moraine deposits result from the erosion of the substratum. The physical disintegration of materials varies according to the schistous, sandy or calcareous environment, and conditions the petrographic distribution of the moraines. The greater the distance they are located from the origin of the glacial system, the more the moraines are a product of a detrital mixture between blocks, rollers and gravels eroded by the glaciers. Thus, this mixture results from a very complex transport on, within, and under the ice. We studied the moraines on three different scales by considering that each scale of the moraine consists of a granular skeleton in a fine matrix. For the three levels of scale, the higher and lower granulometric limits are defined (Fig. 3).

Table 1 Granulometric limits

Limits	$N_1$	$N_2$	$N_3$	$N_4$	N <sub>5</sub>
Upper limit (mm)	100	20	5	2	1
Lower limit (mm)	20	5	2	1	0.4

In this paper, particles size limits are determined from a practical point of view; in the first, values of the limits are determined by the higher particle size that should be contained in samples prepared for mechanical testing (triaxial compression test). In the second case, the lower limit is obtained in a more intuitive way. We call 'matrix' all the grains of size less than this threshold. Indeed, the skeleton consists of grains forming the soil composition which takes into account the efforts and conditions of shear strength (Frossard, 1978, De Jaeger, 1991).

The scale levels are materialised by the sizes of the test sample tubes and correspond to associations of granulometric levels (Table 1, Fig. 4.).

Image analysis of isolated grains allows size variables to be measured, followed by the calculation of

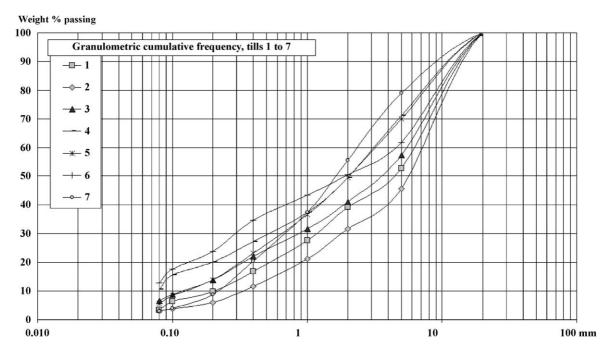


Fig. 3. Granulometric cumulative frequency, moraines 1 to 7.

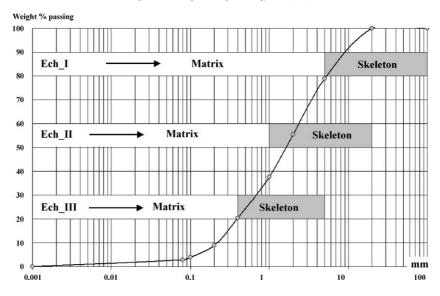


Fig. 4. Scale levels.

shape variables for each grain. This provides for a cumulative frequency for each variable and for each observation level.

### 3.1.2. Image analysis and textural variables

The objective of the textural variables characterisation is to find a criterion of classification for the moraine. The image of each individual grain resting on a white sheet of paper has been photographed under a binocular magnifying lens equipped with a black and white CCD camera (Fig. 5). Because of

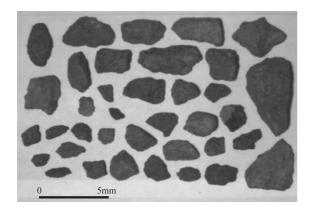


Fig. 5. Moraine elements from the middle size sampling scale level  $(N_3)$ .

good contrast, a simple threshold has proven to be sufficient for a reliable extraction of the grain outline and further image analysis. Derived size and shape variables are obviously two-dimensional. However, it is important to note that the inner circle diameter of a grain resting on a plane parallel to the image plane is strongly correlated to the sieve size. Elongation factors are clearly two-dimensional and cannot be considered as correlated to a flatness index (ratio between the smallest and largest diameter in three-dimensional space) whereas textural variables are reasonably indicative of a three dimensional texture. In this research, we have studied seven moraines  $(T_1 \text{ to } T_7)$  with five different observation levels or granulometric levels (N1 to N<sub>5</sub>). Table 2 compares the five granulometric levels

Table 2
Definition of the scale levels and localisation of the limits on the granulometric distribution

Scale levels	Outcrop	Sample large size	Sample standard size	
	Ech_I	Ech_II	Ech_III	
Scale	meter	decimeter	centimeter	
Granulometric levels	$N_1$ to $N_3$	$N_2$ to $N_4$	$N_3$ to $N_5$	
Upper limit (mm)	100	20	5	
Lower limit (mm)	2	1	0.4	

Table 3 Numbers of individuals studied by levels of class

	$T_1$	$T_2$	T <sub>3</sub>	$T_4$	T <sub>5</sub>	$T_6$	T <sub>7</sub>
Class N <sub>5</sub>	163	192	217	200	163	230	217
Class N <sub>4</sub>	103	132	113	108	102	136	109
Class N <sub>3</sub>	88	91	81	80	81	69	42
Class N <sub>2</sub>	245	204	216	196	219	151	246
Class N <sub>1</sub>	101	91	93	97	112	98	91

(N<sub>1</sub> to N<sub>5</sub>) and the three scale levels (Ech\_I, outcrop; Ech\_II, large sample size; Ech\_III) sample standard size. Table 3 gives the number of particles studied within each class.

The following parametric groups have been considered:

Size variables	
A	projected area of each grain (mm <sup>2</sup> )
$D_{\mathrm{A}}$	as above but expressed as the diameter of a circle of equivalent area (mm)
$D_{\mathrm{M}}$	the diameter of the maximum inscribed disc (mm)
$D_{\rm N}$	the diameter of the maximum circumscribed disc (mm)
$D_{ m Ell~M}$ and $D_{ m Ell~N}$	major and minor diameters of an ellipse with equivalent inertia moments (mm) (Madella, 1970)

Elongation	variables (non-dimensional) (Figs. 6 and 7)
$I_{\rm NM}$	$D_{ m N}\!/D_{ m M}$
$I_{ m Ell}$	$D_{ m Ell~N}/D_{ m Ell~M}$

Texture variables (non-dimensional) (Fig. 8) (Pirard, 1993, 1994)						
$W_{ m V}$	equivalent roundness					
$R_{\rm g}$	global roughness					
$R_{\rm gc}$	corrected global roughness					
$R_{\rm mor}$	morphological roughness					

The roundness index expresses the maturity of a particle in an abrasion process. The final state of abrasion is a perfect disc whose equivalent roundness is 100%. An almost perfect correlation exists with the visual rendering in Krumbein's chart (Krumbein, 1941) of a sphericity factor previously proposed by Wadell (1933).

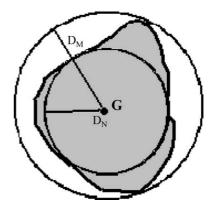


Fig. 6. D<sub>M</sub> and D<sub>N</sub> (size factors).

The global roughness is a measure of the area of a particle that cannot be covered by discs of a diameter larger than 80% of the maximum inscribed diameter  $D_{\rm M}$ . The value is expressed as a percentage of the total area of the particle ( $R_{\rm g}$ ) or as a percentage of ( $D_{\rm M}$ )<sup>2</sup>( $R_{\rm gc}$ ). A global roughness of 0% indicates a very smooth particle.

The morphological roughness is the negative first order moment of the opening function as suggested by Serra (1982) and normalised by the size of the particle. Although it is based on a different mathematical formula than  $R_{\rm g}$ , it appears for most shapes that  $R_{\rm g}$  and  $R_{\rm mor}$  are strongly correlated.

### 3.1.3. Size variables and scale levels

We present here the results of size variables for the moraines  $T_1$  to  $T_7$ . The classification of moraines by area, Table 4, shows that it is impossible to distinguish the moraine at different scale levels (I to III).

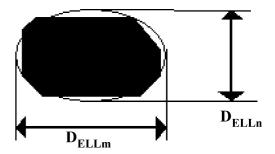


Fig. 7.  $D_{ELLN}$  and  $D_{ELLM}$  (size factors).

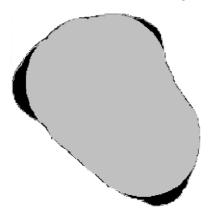


Fig. 8. The global roughness is the area fraction of the black region as compared to the total area of the particle (Rg).

The same conclusion holds for any size variables as is commonly the case in the literature Dickin (1971), Pike (1973), Frossard (1978, 1979), Biarez et al. (1989) and De Jaeger (1991). Section 3.1.4 is an attempt to use shape variables as a means to classify granular formations.

### 3.1.4. Shapes variables

As with the size variables, we have chosen to develop one variable: the elongation factor:  $I_{\rm NM}$  ratio. The elongation factor indicates grain shape, the closer this factor is to 0, the more elongated is the grain. Conversely, an elongation factor close to 1 indicates that the grain is more round and stocky. The  $I_{\rm NM}$  cumulative frequency distribution is similar for the moraines up to a translation, more over it can be seen (Table 5) that the moraine formations can be classified with regard to the mean  $I_{\rm NM}$  value. Increasing values of the mean  $I_{\rm NM}$  are associated with the same hierarchical set  $\{T_1, T_5, T_3, T_4, T_2, T_7\}$  regardless of the scale level. It should be noticed that for moraines  $T_3$  and  $T_4$  the two scale levels I and II seem to be reversed.

We observe that values given in Table 5 change from one moraine to another and that their classifications are, at all scale levels, of the same order. Only some values of the elongation factor for moraines  $T_3$  and  $T_4$  change in the hierarchy. This is explained by the proximity of the two moraines  $T_3$  and  $T_4$  in their genetic and petrographical origin, as well as the geographical location (Lebourg et al., 2000, 2001). The simple classification of moraines by the elongation factor shows the existence of quantifiable differences be-

tween moraines. The independence of shape variables is realised by comparing the size variables relative frequencies (area) in function of classes of a shape variable (elongation factor).

To justify the choice of the average value to characterise a shape factor, we must check the statistical distribution law. The adjustment tests of the elongation factor distribution to a Laplace Gauss law show that the hypothesis cannot be rejected based on the level of the  $\chi^2$  mean test (for risks of 0.001, 0.01 and 0.05).

Results for all shape variables (scale level III) are presented in Table 6. This table has average values of: global roughness ( $R_{\rm g}$ ), morphological roughness ( $R_{\rm mor}$ ), corrected global roughness ( $R_{\rm gc}$ ), blunted factor ( $I_{\rm E}$  and  $W_{\rm V}$ ) and elongation factors ( $I_{\rm NM}$  and  $I_{\rm Ell}$ ).

Values presented in Table 6 show that the different moraines represented here at scale level III are well discriminated. We find, for values of blunted factor and elongation factor, that the grains are characterised by a more roundish aspect. This correlation between blunted factor and elongation factor is also equivalent to roughness, morphological and global roughness factors. Thus, it is possible to establish a classification of the moraines according to the elongation factor as well as roughness factors.

It is therefore possible to discriminate moraines from each other by a shape variable (Table 6), and thus for a scale level. This is especially so for the elongation factor (Table 5) that allows the differentiation and the characterisation of moraines independent of the scale levels (I, II or III). Below, we consider only the values for the sample scale (Ech\_III) because they were characterised better by the mechanical test.

### 3.2. Mechanical properties

The mechanical properties usually used in soil mechanics and, for the slope stability prediction, are the effective internal friction angle  $(\phi')$  and the

Table 4
Average grain area for the level scales I, II and III (A: area in mm<sup>2</sup>)

	$T_1$	$T_2$	$T_3$	$T_4$	$T_5$	$T_6$	$T_7$
A Ech_III	5.6	5.6	6.8	7.2	6.5	6.1	5.6
A Ech_II	123	89	202	67	97	95	46
A Ech_I	4026	3890	6521	3298	4626	4699	4941

Table 5 Average elongation factor for the scale levels I, II and III  $\,$ 

	$T_1$		T <sub>2</sub>		T <sub>3</sub>		T <sub>4</sub>		T <sub>5</sub>		$T_6$		$T_7$	
	Value	Rk	Value	Rk	Value	Rk	Value	Rk	Value	Rk	Value	Rk	Value	Rk
$I_{\rm NM}$ _III	0.505	1	0.562	5	0.550	3	0.558	4	0.534	2	0.566	6	0.590	7
$\sigma$	0.09		0.09		0.06		0.06		0.05		0.07		0.03	
$I_{\rm NM}$ _II	0.521	1	0.566	5	0.560	4	0.555	3	0.538	2	0.575	6	0.592	7
σ	0.08		0.07		0.07		0.06		0.07		0.07		0.05	
$I_{\rm NM-}I$	0.498	1	0.573	5	0.570	4	0.565	3	0.550	2	0.576	6	0.593	7
σ	0.06		0.06		0.06		0.06		0.06		0.06		0.04	

Value and rank (Rk).

effective cohesion (C'). These properties are used in the equation of the Mohr–Coulomb failure criterion:  $\tau = C' + \sigma_N \cdot \tan \varphi'$ , where  $\tau$  is the shear stress and  $\sigma_N$  the normal stress (Costet and Sanglerat, 1981).

### 3.2.1. Triaxial tests

As previously stated the moraine slope stability depends on the values of the internal angle of friction  $(\varphi')$  and the effective cohesion (C'). We used triaxial compression tests (CD) obtained at the scale of the samples. The use of the Lambe representation  $(s=(\sigma_1+\sigma_3)/2,\ t=(\sigma_1-\sigma_3)/2)$  enables us to calculate the values of the internal angle of friction  $(\varphi')$  and the effective cohesion (C') by reducing the estimation variance. The calculation of  $\varphi'$  is accomplished by linear regression, according to the least rectangles method (Dagnelie, 1998). This allows us to define the confidence interval of  $\varphi'$  and C'.

For most of the moraines, average values of the internal angle of friction lie between  $20^{\circ}$  and  $36^{\circ}$ , which confirms the great variability of the mechanical behaviour. For the cohesion, we observe also a great variation from 0 to 80 kPa (Table 7).

Table 6
Shape variables for the scale level of the sample Ech\_III

Shape variables	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>	T <sub>6</sub>	T <sub>7</sub>
$W_{ m V}$	0.516	0.579	0.567	0.512	0.579	0.581	um.
$R_{\rm g}$	0.120	0.098	0.103	0.102	0.111	0.084	um.
$R_{\rm mor}$	0.212	0.170	0.183	0.184	0.194	0.146	um.
$R_{\rm gc}$	0.701	0.525	0.561	0.533	0.657	0.417	um.
$I_{ m ell}$	0.644	0.694	0.704	0.711	0.605	0.724	um.
$I_{\rm NM}$	0.505	0.562	0.550	0.558	0.534	0.566	0.590

For T7, um. mean unmeasured.

# 4. Influence of the morphological variables on the mechanical behaviour

### 4.1. Previous studies

Studies accomplished in the past 50 years on granular materials, and more particularly on their behaviour, made it possible to highlight the influence of the grain shape factors on their mechanical behaviour. The evolution of this entire work and identification of the morphological variables can be credited to the evolution of tools allowing the measurement of the morphological variables.

From the work of Chen (1948) and Terzaghi (1967), we note that a lower friction angle is allotted to round sands than to angular sands. The angle of friction is measured by testing, whereas the estimate of the shape factors is given by much more subjective criteria. The thesis by Morris (1959) is among the early work undertaken regarding the influence of the grain morphology on their mechanical behaviour. Morris (1959) suggested the principle indicating that granular stability of particles is a function of their shape and texture. It highlights the influence of roughness on shear strength, which increases according to roughness until an optimum threshold is research.

The work of Zelasko (1966) also treats the relation between size, shape of the grains and shear strength. The variables used are similar to those, which have been defined for moraines. Lundgren (1960, in De Jaeger, 1991) suggested an empirical relation with a corrective constant and four variables, which it defined in a similar way, the shear strength of granular materials.

$$\varphi = 36^{\circ} + \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4$$

Table 7 Internal angle of friction ( $\varphi'$  in degrees) and effective cohesion (C' in kPa), with the values of the confidence interval (0.05) for the seven moraines

	$T_1$	$T_2$	$T_3$	$T_4$	T <sub>5</sub>	$T_6$	$T_7$
$\varphi'$	$20.1 \pm 0.5$	$32.8 \pm 0.5$	$30.6 \pm 0.2$	$29.5 \pm 0.2$	$27.2 \pm 0.8$	$33 \pm 0.4$	$36.1 \pm 0.4$
c'	$34 \pm 30$	$0 \pm 40$	$38 \pm 13$	$79 \pm 8$	$35 \pm 60$	$8 \pm 28$	$0 \pm 42$

With  $\varphi_1$  grain shape  $[-6^{\circ}$  to  $1^{\circ}]$ ,  $\varphi_3$  uniformity correction  $[-3^{\circ}$  to  $3^{\circ}]$ ,  $\varphi_2$  grain size  $[0^{\circ}$  to  $2^{\circ}]$  and  $\varphi_4$  compaction correction  $[-6^{\circ}$  to  $6^{\circ}]$ .

Similar to the shape charts (Krumbein, 1941; Rittenhouse, 1943; Krumbein and Sloss, 1963), the variables used here are of a subjective nature and the shape is not clearly defined. We found, more recently, in studies by Favre (1980) and Biarez and Hicher (1989), the relation between the angle  $\varphi_{\rm pp}$  (angle of perfect plasticity) and shape, size, granulometric and petrographic factors. Thus, they defined the relation.

$$\varphi_{pp} = 31.5 + \varphi_D + \varphi_F + \varphi_M + \varphi_{Cu}$$

where  $\phi_D$  grain size [  $-5^{\circ}$  to  $5^{\circ}$ ],  $\phi_F$  grain shape [  $-1^{\circ}$  to  $1^{\circ}$ ],  $\phi_M$  petrography [ $1^{\circ}$  for each 10% of carbonate] and  $\phi_{Cu}$  granulometric distribution [ $2^{\circ}$  to  $6^{\circ}$ ].

Generally, the entire study shows that shear strength, and more largely the mechanical behaviour, is influenced by grain morphology.

In Section 4.2, we present the analysis of the variables studied here and the possible relation existing between the mechanical and shape variables. To quantify the relations between them, we choose to use multidimensional statistical analysis.

# 4.2. Multidimensional statistical analysis of variables

The approach by the various scale levels was a means of identifying the heterogeneity of the moraines. It was noted, subject to some assumptions, that the analysis of the variables on the Ech\_III scale was representative of the entire solid mass (Lebourg, 2000).

The statistical analysis proposed here made it possible to observe and quantify the relationships between the variables for the individuals (moraines). To accomplish this we use the multidimensional analysis and more particularly the principal component analysis (PCA). This analysis allows us to

represent all the variables studied in a new way using the integration of the correlations factors (Dagnelie, 1998).

# 4.2.1. Test of correlation between mechanical properties and shape variables

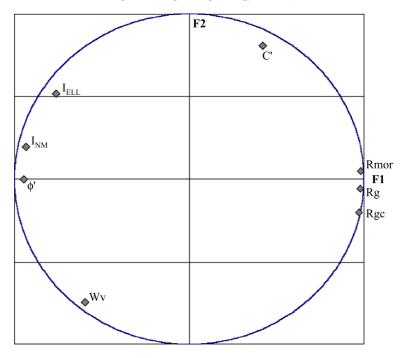
Tests and measures undertaken on the seven moraines have allowed characterisation and quantification of different variables, but also their classification regarding to the type of measurement. As observed in Tables 6 and 7, there is a similarity in the moraine classification according to the mechanical property (effective internal friction angle) and shapes variables (elongations factor and roughness factor). To study the correlations of all variables, it is necessary to calculate correlation coefficients of all measured and calculated variables.

Variables used in the analysis are:

- Mechanical properties: the effective internal friction angle  $\varphi'(\circ)$ , the effective cohesion C'(kPa).
- Shape variables: the global roughness  $(R_{\rm g})$ , the morphological roughness  $(R_{\rm mor})$ , the corrected global roughness  $(R_{\rm gc})$ , the blunted factor  $(I_{\rm E}$  and  $W_{\rm V})$  and the elongation factors  $(I_{\rm NM}$  and  $I_{\rm EII})$ .

Fig. 9 shows the variables projection in the space F1F2 representing 92% of the total variance. Variables contribute to the creation of the space F1F2 and some of these variables show a strong positive relationship:  $I_{\rm NM}$  and  $\varphi'$ , and a strong negative relationship;  $R_{\rm g}$  and  $\varphi'$ . This variable projection in the F1F2 space allows us to visualize the concentrations and distances between variables.

Fig. 10 shows the totality of the moraines in the main plane F1F2, in which there is an obvious classification of the moraines (without the moraine  $T_7$  because not all variables were measured for it). Moraine  $T_3$  lies near the origin and can be considered as a mean value moraine. Moraine  $T_1$  and moraines  $T_6$  and  $T_2$  lie on opposite sides of the F1 axis



	F1	F2
φ'	-0.95	-0.00
C'	0.42	0.81
Wv	-0.60	-0.75
Rg	0.98	-0.06
Rmor	0.98	0.05
Rgc	0.97	-0.20
$I_{ELL}$	-0.76	0.52
$I_{NM}$	-0.94	0.20

F1 and F2 data for each variables

Fig. 9. Factor loadings plot, 92% of the total variance is explained by the first two components (the circle is the representation of the maximum limit of representation).

because of their petrographic composition (a large proportion of schist for  $T_1$  in contrast with a large proportion of sandstone for  $T_6$  and  $T_2$ ). Moraines  $T_4$  and  $T_5$  are separated mainly because of their quartz composition.

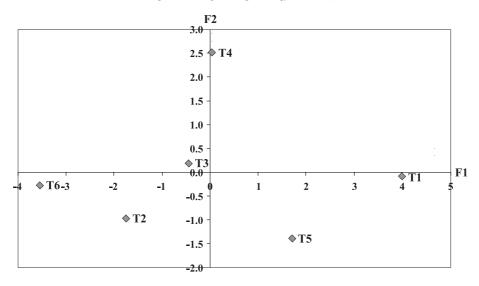
# 4.3. Analysis of the results

# 4.3.1. Correlation of $\varphi'$ with the roughness factor

The correlation coefficient between the variable  $\varphi'$  and the roughness factor  $(R_g)$  is strong (r = -0.88,

Table 8). This coefficient shows a high negative relationship between internal angle of friction and the roughness factor. It is thus possible, on the basis of this relation, to propose a function of  $R_g$  and  $\varphi'(Fig. 11)$ .

This result shows that effective internal friction angle, which characterises the shear strength, is dependent on the roughness of soil grains where shearing surfaces are initiated. However, the roughness factor is not strong enough for use in estimating the internal angle of friction. Instead, we use the roughness factor to quantify the shear stress that must



	F1	F2
T1	3.99	-0.08
T2	-1.75	-0.97
Т3	-0.45	0.19
T4	0.04	2.52
T5	1.71	-1.39
Т6	-3.53	-0.27

F1 and F2 data for each moraine T1 to T6.

Fig. 10. Plot of the first two coordinate axes for PCA.

be developed. Next, we look at the results of the correlation between the elongation factor and the angle  $\varphi'$ .

4.3.2. Correlation of  $\varphi'$  with the elongation factor  $I_{NM}$  We define the experimental law for the influence of the elongation factor on the angle  $\varphi'$  for the six moraines used in the PCA (r = -0.98, Table 8).

Table 8
Matrix correlation for parameters considered

	$\varphi'$	C'	$W_{\rm V}$	$R_{\rm g}$	$R_{\rm mor}$	$R_{\rm gc}$	$I_{\mathrm{ell}}$	$I_{\rm NM}$
$\varphi'$	1.00	-0.31	0.63	-0.88	-0.89	-0.88	0.67	0.98
C'		1.00	-0.74	0.37	0.47	0.27	-0.04	-0.16
$W_{\rm V}$			1.00	-0.52	-0.60	-0.40	0.01	0.46
$R_{\rm g}$				1.00	0.99	0.99	-0.76	-0.90
$R_{\rm mor}$					1.00	0.97	-0.72	-0.88
$R_{\rm gc}$						1.00	-0.86	-0.91
$I_{\mathrm{ell}}$							1.00	0.75
$I_{\rm NM}$								1.00

This result shows that effective internal friction angle, which characterises the shear strength, is dependent on the shape of soil grains where shearing surfaces are initiated.

As stated previously, it is difficult to estimate the effective internal friction angle with shape variables because of the subjective nature of such variables. With the proposed relation, it is possible to estimate the internal friction angle using the elongation factor  $I_{\rm NM}$ . Based on previous results, we assume a linear relationship between  $I_{\rm NM}$  and  $\varphi'$ ; the regression gives the following equation:

$$\varphi' = 203I_{\text{NM}} - 82 \text{ with } R^2 = 0.952$$

Fig. 12 shows the various measurements of  $I_{\rm NM}$  and  $\varphi'$ , the straight linear regression and the local estimates with their confidence limits (risk 5%).

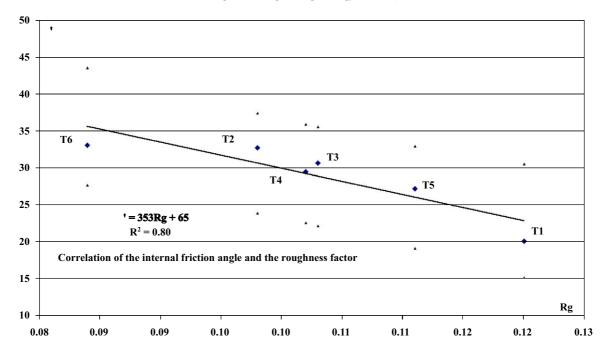


Fig. 11. Correlation of  $\varphi'$  with the roughness factor  $R_{\rm g}$ .

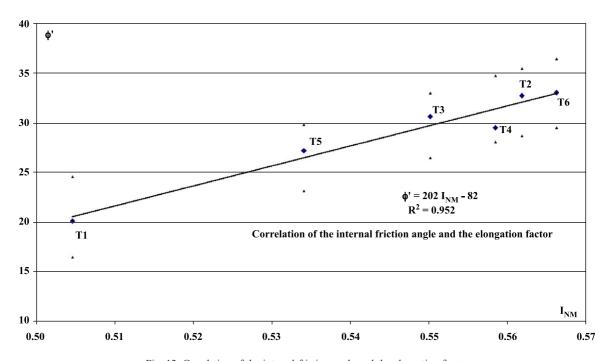


Fig. 12. Correlation of the internal friction angle and the elongation factor.

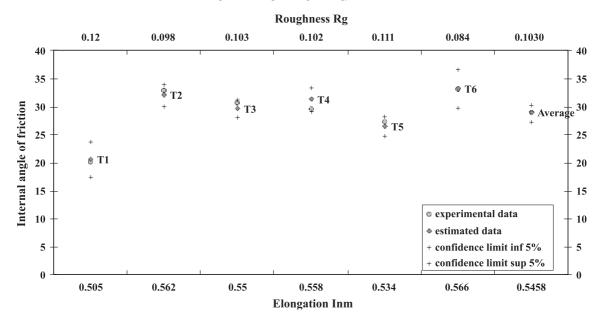


Fig. 13. Multiregression of the internal friction angle, the elongation factor and the roughness factor.

# 4.3.3. Multiregression of $\varphi'$ with $I_{NM}$ and $R_g$ factors

The linear regression proposed for the estimation of the internal friction angle with the elongation factor  $I_{\rm NM}$ , and the roughness factor  $R_{\rm g}$ , can be developed using linear multiregression (Dagnelie, 1982; Grolier and Riss, 1997).

Looking at the previous results and the multiregression of  $\varphi'$  explained by  $I_{\rm NM}$  and  $R_{\rm g}$ , we assume a relationship between  $I_{\rm NM}$ ,  $R_{\rm g}$  and  $\varphi'$ ; the regression analysis provides the following equation:

$$\varphi' = -74 - 21.5R_{\rm g} + 192I_{\rm NM}$$
 with  $R^2 = 0.908$ 

Fig. 13 shows the various measurements of  $R_{\rm g}$ ,  $I_{\rm NM}$  and  $\varphi'$ , the multiregression and the local estimates with their confidence limits (risk 5%).

### 5. Conclusion

The results presented in this paper indicate a way to evaluate the moraines using physical and mechanical variables and to integrate their heterogeneity based on scale levels. In the paper using the multidimensional analysis, we quantified correlations between physical, mechanical and morphological variables of the moraines at the scale level of the sample (Ech\_III).

Analysis of the matrix of correlations highlighted the strong correlations between internal angle of friction and the elongation and roughness factors. This nonfortuitous correlation, between the shape of the grains and the internal angle of friction, had already been observed by other authors on sands, but by using more subjective shape variables. The results and relations defined for sands were observed and applied to the moraines. Thus, a strong correlation was highlighted between the  $I_{\rm NM}$  factor and the angle  $\varphi'$ . The quantification of this correlation led to the development of an experimental law generalized for the moraines located in two different glacial valleys. This relation makes it possible to estimate the effective angle of friction, by using simple and economic elongation factor measurements. Multiregression results suggested another experimental law with  $R_{\rm g}$  and  $I_{\rm NM}$  reducing the error of the estimate of the internal friction angle  $\varphi'$ . Relations, which make it possible to consider the angle  $\varphi'$ , can also be used in making geotechnical maps.

### 6. Uncited references

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