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Note on singular optima in laminates design problems

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Abstract This paper studies the design of laminates subject to restrictions on the plies strength. The minimum weight design is considered. It is shown that this formulation includes singular optima, which are similar to the ones observed in topology optimization including local stress constraints. In laminates design, these singular optima are linked to the removal of 'zero thickness' plies from the stacking sequence. It is shown how the fibers orientations variables can circumvent the singularity by relaxing the strength constraints related to such vanishing plies. This demonstrates the key role of fibers orientations in the optimization of laminates, and the need for their efficient treatment as design variables.

Key words Laminates, fibers orientations, singular optima

1 The considered laminates design problem

Schmit and Farshi (1973) early showed the weakness of the formulation that consists in finding the minimum thickness of a laminate under strength criteria, for fixed fibers orientations.

Let us consider a symmetric $[0/90]_S$ laminate subject to an in-plane axial load of $N_1 = 1000\ N/mm$ and $N_2 = N_6 = 0$ (Fig. 1). The base material is the graphite-epoxy T300/5208 (Tsai and Hahn (1980)).

The considered design problem consists in minimizing the laminate's total thickness under strength restrictions

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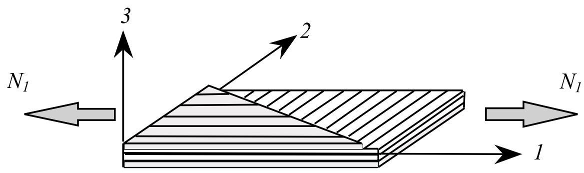

Fig. 1 Laminate under consideration

expressed by the Tsai-Wu criterion computed in each ply. The plies thicknesses are allowed to vary continuously between a lower and an upper bound (1).

$$\begin{array}{lll} \min\limits_{\mathbf{t}} & 2(t_{0^\circ} + t_{90^\circ}) & \\ \text{s.t.:} & TW(t_i) \ \leq \ 1 & i = 0^\circ, 90^\circ \\ & 0.01mm \leq \ t_i \ \leq 5mm & i = 0^\circ, 90^\circ \end{array} \tag{1}$$

The considered design problem (1) has a somewhat academic character and is in some sense non-practical. This problem is well known to have a simple degenerated solution, which leads to the so-called singularity phenomenon of stress constraints in the case of composite structures. This problem serves here to illustrate the authors' conjecture that ply orientation variables have a key role to relax the design problem.

In Fig. 2, the optimal solution $[0/90]^*_S$ obtained by using the approximation concepts approach (Bruyneel et al. (2002)) is characterized by 90°-ply thicknesses equal to the lower bound 0.01mm (point 'O' in Fig. 3). It can be concluded that the vanishing 90°-plies are not necessary to characterize the optimum. This is indeed evident according to the laminate' stacking sequence and the direction of the applied load (Fig. 1).

As illustrated in Fig. 2, if the optimization is restarted with only 0° plies, that is from an initial $[0/0]_S$ laminate, the obtained optimal solution $[0]^*_S$ is surprisingly better with regards to the total thickness of the laminate (point '+' in Fig. 3). One concludes that the first obtained solution is not optimal, because the weight still can be decreased.

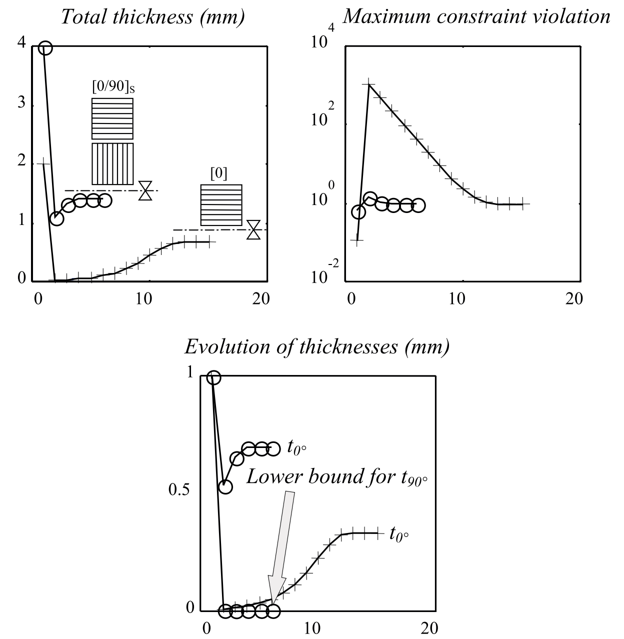

Fig. 2 Iteration history for the solution of problem (1)

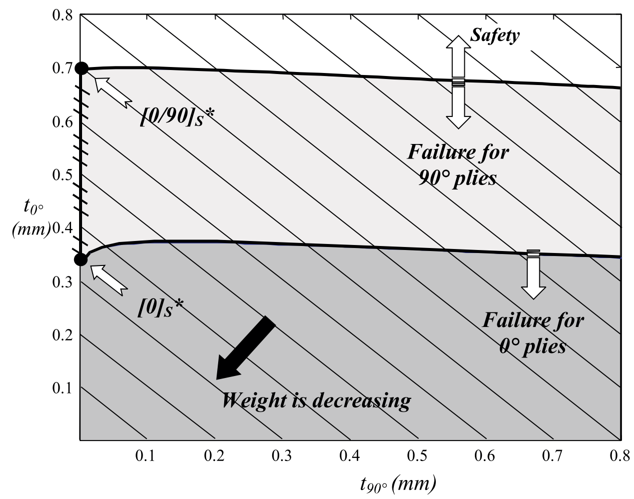

Fig. 3 Design domain of problem (1)

2 The existence of singular optima in laminates design problem

The reason of this difference results from a bad formulation of the problem, which is related to the removal of a structural member at the optimum design (here, the plies oriented at 90°), that is a change in the topology of the laminate' stacking sequence. This singularity, which is well known in topology optimization including stress constraints (Kirsch (1990), Cheng and Jiang (1992), Rozvany and Birker (1994), Rozvany (1996) Cheng and Guo (1997), Duysinx and Bendsøe (1998)), can be explained by plotting the design domain of problem (1), illustrated in Fig. 3. The iso-values of the objective function are parallel lines representing the total weight, which is decreasing in the indicated direction. The Tsai-Wu limiting values of each ply are also plotted. They divide the design domain into regions of safety and failure.

Because of the simple loading, 0° plies are mainly stressed along the fibers while 90° plies are stressed perpendicularly to the fibers. Tsai-Wu criterion predicts the ply failure respectively along the fibers or perpendicularly to the fibers with the appropriate stress limits in tension or compression. In that way, the failure criterion is here similar to stress criterion for bars. Thus it is easy to imagine the similarity between the stress singularity phenomenon for truss and for composite problems in the present application. More generally, for a non-simple loading, stress singularity can also appear with quadratic failure criteria like the Tsai-Wu one, as demonstrated by Duysinx and Bendsøe (1998) with von Mises criterion for continuum isotropic structures.

Both optimum laminates $[0/90]_S^*$ and $[0]_S^*$ are points that minimize the total thickness while satisfying the strength restrictions. The singular optimum $[0]_S^*$ lies at the end of a line segment attached to the feasible domain. Because of the presence of the strength constraint related to 90° plies, classical optimization algorithms can not reach the true optimal solution $[0]_S^*$, which is located in a degenerated part of the design space (see Fig. 3)

In order to eliminate this singular optimum, and to reach the optimum of the design problem, ϵ-relaxation techniques may be used (Cheng and Guo (1997) and Duysinx and Bendsøe (1998)). However in the frame of fibers reinforced composite materials, another approach can be derived from a reformulation of the design problem.

3 Eliminating the singular optima in laminates design problem

As illustrated in Figs. 4, 5 and 6, the optimal solution corresponding to a $[0]_S^*$ laminate can be easily found if the fiber orientations can vary in the problem (1).

The fact that having ply orientation variables can circumvent the degeneracy problem has been suggested by a couple of authors before. For instance, in (Haftka and Gürdal 1992,p425), the singularity of Schmit and Farshi's solution is mentioned as a rationale for introducing ply orientation design variables, but it is not demonstrated. This note clearly shows this conjecture.

Indeed, when the fibers orientations are taken into account in the design process, the design domain changes as the initial 90° orientations tend to their optimal values of 0°. At the final stage, strength constraints related to each ply are identical (Fig. 6) and the true optimal solution is easily found.

The problem is completely relaxed when the optimal fibers orientations are determined (Fig. 6). Anyway, great savings in the structural weight can be achieved for orientations close to the optimal ones (Figs. 4 and 5).

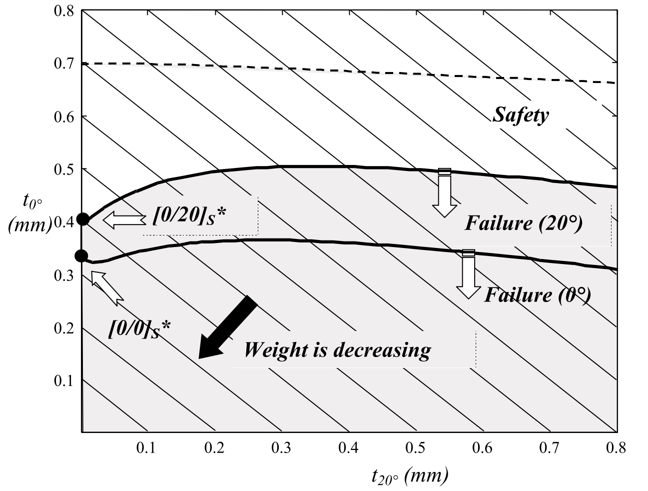

Fig. 4 Design domain of problem (1) for a $[0/20]_S$ laminate

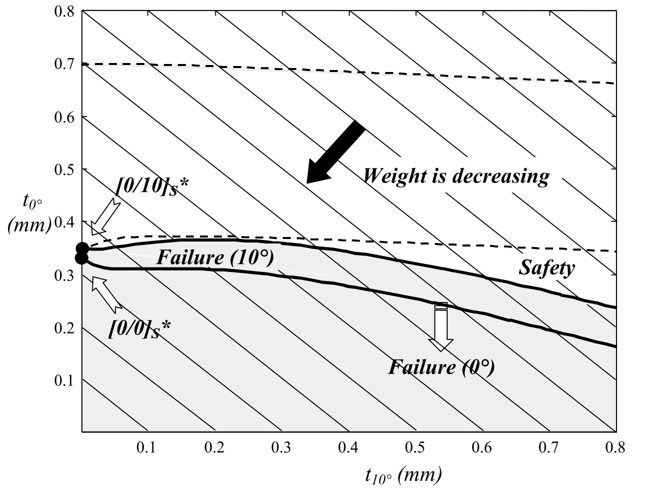

Fig. 5 Design domain of problem (1) for a $[0/10]_S$ laminate

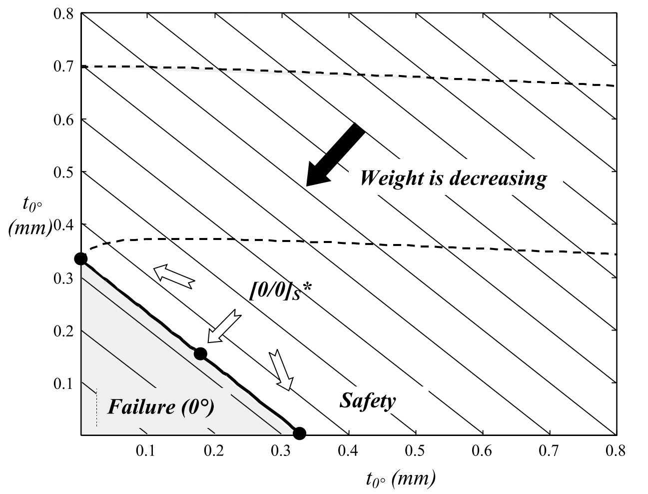

Fig. 6 Design domain of problem (1) for the optimal [0] laminate

4 Conclusions

Based on this simple example, it is clear that the design problem formulation strongly influences the presence or the absence of singular optima. The introduction of additional geometric variables allows to relax the problem and to circumvent the singular character of the optimum. In particular for laminate optimization with stress constraints, it is necessary to include both ply thickness and fiber orientation as design variables.

Then solving such problems requires efficient optimization methods able to deal efficiently with fibers orientations and plies thickness design variables. Efficient optimality criteria procedures leading to global optimum in terms of orientations can be found for example in Pedersen (1989) for 2D membrane composites, and in Krog (1996) for out-of-plane loadings, where dedicated optimality criterion are developed. However, these approaches are based on an energetic formulation (global criteria) and not on local criteria as strength constraints. A more general issue, better adapted to solve complex industrial problems, but may be not able to guarantee the global character of the optimum, is to use mathematical programming approaches such as GBMMA (Bruyneel et al. (2002)) for finding local optimal values in terms of fibers orientations in a formulation including plies stiffness and strength (see for example Bruyneel and Fleury (2002) and Bruyneel et al. (2001)).

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