Multivariate discrimination of sands using elongation and wear indices.

Eric Pirard[1], Nicolas Vergara[1]

[1] Université de Liège, GeomaC, Géoressources Minérales, Sart Tilman B52/3, 4000 Liège, BELGIUM

ABSTRACT

Identical sieve fractions of crushed and river sands have been examined through optical image analysis using shape parameters derived from operations based on Euclidean mathematical morphology. Results on populations of 5.000 to 10.000 particles have been examined in the multivariate space defined by the sieving diameter, the inertial elongation and a specific wear index measure. The use of multivariate analysis helps to discriminate between different populations of sand particles. This kind of analysis can be further used to correlate with the abrasive character of the sands and their behavior in compactness tests.

1 INTRODUCTION

Sand particles are ubiquitous in nature and in the mineral's industry. The definition of sand relates merely to the size range of the particles. It is common practice in civil engineering to qualify as "sand fraction" all particles between 80 µm and 2 mm. Sand can thus be produced by natural processes (erosion, sedimentation) and by industrial processes (crushing, sieving). As a result, sands may vary significantly in terms of mineralogical composition, shape distribution and size distribution.

The effect of size distributions on the mechanical behavior of sands and the relative performance of mixing sands with different size distributions have been investigated by many authors [1],[2]. On the contrary, only a few systematic investigations have been performed on the influence of shape distribution [3]. But, due to the lack of reliable shape analysis methods, these investigations were based on a visual appreciation of shape characteristics by using morphoscopic charts such as those proposed by Krumbein [4], Rittenhouse [5] or Lees [6].

In order to be able to correlate mechanical performance of sand heaps or sand-based products (concrete,etc.) with their intrinsic characteristics, it is essential to develop a precise quantification of size and shape attributes and to make sure that the methodology is sensitive enough to differentiate between sands from different origins. It is the purpose of this paper to test the capabilities of accurate digital image analysis algorithms for discriminating between various sand fractions.

2 MATERIALS AND METHODS

Four different river sands and six varieties of crushed limestone sands have been selected for a research project on the formulation of high durability and high performance concrete. Each sand has been split into six fractions by sieving [80 - 125 µm]; [125 - 250 µm]; [250 - 500 µm]; [500 - 1000 µm]; [1000 - 2000 µm] and [> 2000 µm].

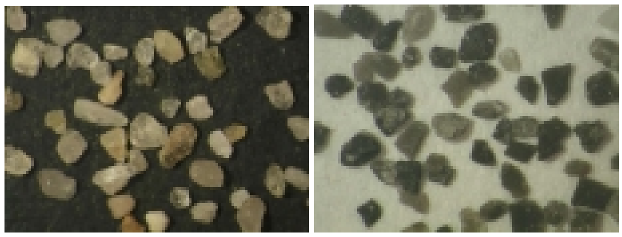


Figure 1 Micrographs of [250-500µm] fractions from a river sand (left) and a crushed sand (right).

The different size fractions were fed into the Alpaga optical imaging instrument [7] operating at a resolution between 4,37 µm and 12,27 µm per pixel. Such a variable resolution is not mandatory for classical size distribution analysis. But, in order to allow for comparative shape analysis between very different size fractions, it is good practice to maintain a magnification ratio such that the number of pixels per particle stays sufficiently high and within a reasonable interval (e.g. 2000 – 8000 pixels).

The Alpaga instrument combines a vibrating feeder and a glass-slides conveyor belt running at adjustable speed in order to achieve optimal dispersion of individual particles. Glass slides are guided within the depth of focus of a video camera and particles are imaged with an optimal shutter speed when passing above a homogeneous backlight panel so as to maintain the risk of blur within the pixel resolution.

Because of their controlled position it can be shown that 2D image analysis of individual particles leads to perfect correlation with sieves without any artificial correction or calibration procedure [8].

For each size fraction a random selection of 5000 to 10000 particles were submitted to image analysis (Callisto software) and all results were stored in files allowing for morphostatistical processing and interactive retrieving of any particle outline (Callistat software). For sake of efficient data handling on a conventional PC, multivariate graphics and statistics involving all ten sands were computed on random subsets of 300 grains. Because of the proven

Gaussian distribution of all parameters involved in this study, it is acceptable to consider that this selection does not significantly affect the computed confidence intervals.

Classical image analysis [9] is based on an equivalent disk diameter (D_O), a shape factor (Φ) and a roundness factor (F) most often defined as :

$$D_O = \sqrt{\frac{4.A}{\pi}} \qquad (1)$$

$$\Phi = \frac{D_{Max}}{D_O} \qquad (2)$$

$$F = \frac{4.\pi.A}{P^2} \qquad (3)$$

where A designates the projected area, D_{Max} the maximum Feret diameter and P the perimeter.

However, it is not recommended to compare results from different instruments because the way a perimeter or a maximum Feret diameter is computed may vary significantly.

The parameters used in this study do not rely on area or perimeter measurements. Elongation measurements are computed from the major (D_A) and minor (D_B) axes of an equivalent inertia ellipse [10] :

$$El = \frac{D_A - D_B}{D_A + D_B} \qquad [4]$$

The sieving diameter (D_{IN}) and the wear index (W) are respectively the maximum and the negative first order moment of the set of all maximum inscribable discs [11]. Table 1 illustrates the results on four selected shapes with distinct roundness.

Table 1. Comparison of image analysis parameters on selected synthetic diamond particles.

				
D_O (µm)	385,8	363,2	433,5	354,5
F (%)	74	74	61	61
D_{IN} (µm)	455	418,6	409,5	391,3
W (%)	82,55	65,71	64,14	46,18
El (%)	5,94	13,15	43,56	5,76

3 RESULTS AND DISCUSSION

Because of the large amount of results, graphics and statistics this paper will be restricted to comparing the [250-500µm] fractions. Fig. 2 shows the sub-sieve size distributions obtained from the apparent volumic fraction of particles whose sieving diameter passes a virtual AFNOR NFX11-501 series. The quality of correlation between sieves and image analysis is discussed elsewhere [8]. The crushed sands diameters are typically more spread than the river sands and a significant amount of particles (15 %) exceed the 500 µm barrier. A logical interpretation of this is that crushed sands have been able to pass through a 500 µm side mesh in a diagonal position (500 µm x 1,41 =705 µm). This strongly supports the idea that many crushed particles are flat chips, whereas river particles are much more isometric. Obviously a flatness index cannot be measured from a projection of particles lying at rest on a glass slide, but it can be estimated by analyzing the difference between mechanical sieving and image analysis !

Fig. 3 summarizes statistical results obtained on elongation and wear indices by plotting side by side the mean (m) and standard error around the mean as well as the 68 % confidence intervals (m±s) for the different sand fractions. Noticeable differences appear between river and crushed sands, with the last ones being more elongated and less well rounded but hypothesis testing was not performed at this stage, considering that both size and shape information had to be considered in a multivariate analysis.

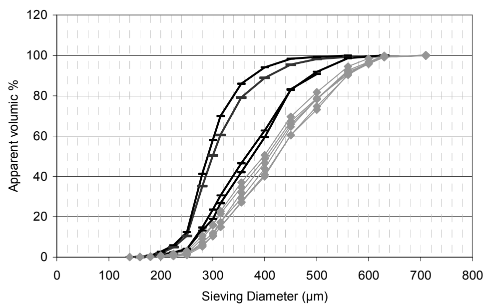


Figure 2. Size distribution of the 250-500 µm fraction of four river sands (black -) and six crushed sands (grey ♦).

Correlations between shape parameters are very often misinterpreted as being characteristic of the product under study. But, many shape parameters are simply correlated because of the redundancy in the geometrical concepts they measure. A typical example of this is the negative correlation between the shape factor (Φ) and the roundness factor (F) as defined in equations (2) and (3). This is because F also includes a measure of elongation. The more elongated a particle, the larger its perimeter with respect to its projected area, hence F diminishes whereas Φ increases.

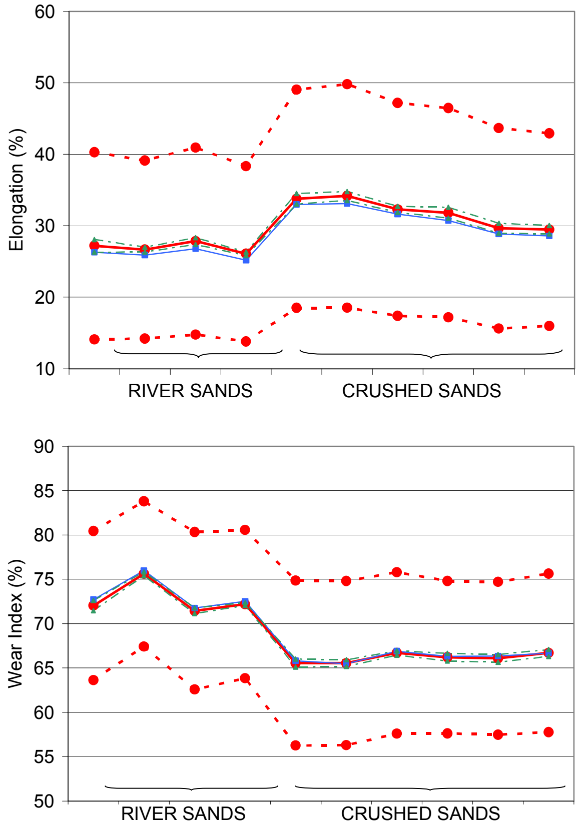
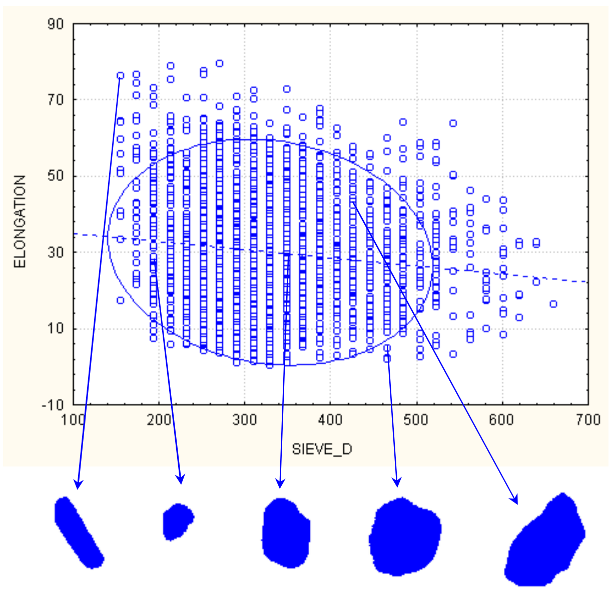


Figure 3 Side by side plot of mean elongation and wear index values with their standard errors and one standard deviation (68 %) confidence intervals (dotted lines) for all ten sands (rows 1 to 4 : river sands; rows 5 to 10 : crushed sands)

Bivariate scatterplots of elongation vs. sieving diameter and elongation vs. roundness are shown in figure 4 for two river sands. Additional insight into the results is provided by the interactive particle outline retrieval capabilities of the software.



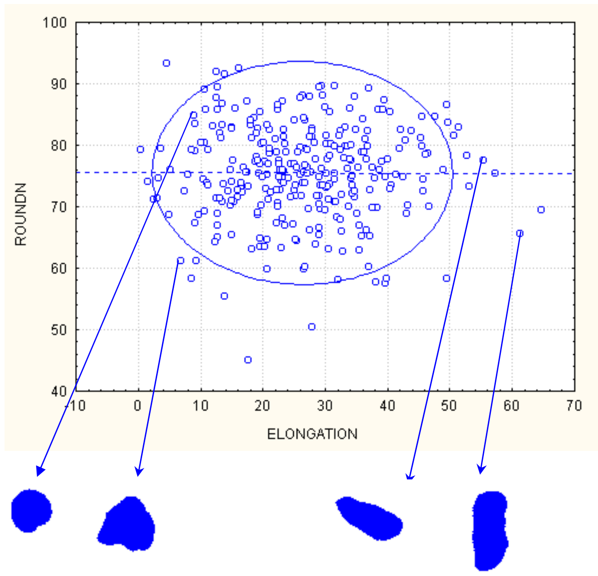


Figure 4 Scatterplot of sieving diameter vs. elongation and elongation vs. wear index for two different river sands.

Clearly, the slight negative correlation between sieving diameter and elongation points towards the lower probability for an elongated particle to pass through the lower 250µm sieve. Following [12] this probability is estimated as being proportional to the cube of the aspect ratio. The absence of correlation between elongation and wear index as measured here is indicative of the real independency of both measures. In other words, both parameters are not redundant and bring additional information to the characterization of the sand particles. Ellipses in both scatterplots of fig. 4 are computed on the basis of a 75 % confidence interval.

Discriminant analysis based on Mahalanobis distances was performed on the [250-500µm] sand fractions using a subset of 300 particles per sand. Table 2 indicates that hypothesis testing of a difference in the multivariate mean gave significant differences for all couples of river vs. crushed sands. The river sands group also display a higher variety as compared to crushed sands that appear to me more similar to each other.

A step by step discriminant analysis reveals that the wear index (W) is the most discriminant parameter when all sands are compared. If we restrict the analysis to crushed sands only, then elongation (El) is the most discriminant, whereas the wear index (W) and to a lesser extent the sieve diameter (D_{IN}) help to discriminate between the river sands.

Table 1 Squared Mahalanobis distances between river (Rx) and crushed (Cx) sands in the Elongation-Wear-Sieving diameter space. Grey shading is indicative of the absolute distance value. Blank cells indicate that hypothesis testing could not confirm a significant difference with a 5 % risk.

	R1	*C2*	*R3*	*R5*	*R8*
R1	0				
C2	1.00	0			
R3	0.33	2.28	0		
R5	0.01	0.89	0.37	0	
R8	0.38	1.92	0.18	0.42	0

	R1	*C2*	*R3*	*R5*	*R8*
C9	1.09	0.02	2.46	0.99	2.18
C10	0.65	0.14	1.67	0.61	1.29
C11	0.85	0.08	2.11	0.79	1.71
C12	0.61	0.14	1.75	0.56	1.39
C13	0.59	0.19	1.68	0.56	1.34

	C9	*C10*	*C11*	*C12*	*C13*
C9	0				
C10	0.19	0			
C11	0.11	0.06	0		
C12	0.19	0.09	0.04	0	
C13	0.22	0.05	0.04	0.03	0

DISCUSSION

Subtle morphological differences between sands can be evidenced through image analysis if care is taken to separate similar size fractions and if sensitive and robust shape parameters are used.

Simple univariate statistics are a poor tool for analyzing the morphometric differences. Because no universal shape parameter does exist it is much more powerful to rely on independent shape parameters and to use multivariate statistical tools such as multigaussian discriminant analysis.

Representative analyses based on thousands of individual particles are no longer a problem with the last generation of image analysis instruments.

REFERENCES

[1] Feret M. : Compacité des mortiers hydrauliques, Ann. Ponts et Chaussées, Vol. 7 IV (1892), 5-162.

[2] de Larrard F., Buil M. : Granularité et compacité dans les matériaux de genie civil. Materials and structures, Vol. 20 (1987), 117-126.

[3] de Jaeger J. : Influence de la morphologie des sables sur leur comportement mécanique, Ph.D. Thesis, Fac. Appl. Sc., Univ. Cath. Louvain (1991)

[4] Krumbein W. : The effects of abrasion on the size, shape and roundness ofrock fragments, Journal of Geology, Vol. 49, 1941, 482-519

[5] Rittenhouse G. : A visual method for estimating two-dimensional sphericity, J. of Sedimentary Petrology, Vol.13/2, 1943, 79-81

[6] Lees G. : A new method for determining the angularity of particles, Sedimentology, Vol.3, 1964, 2-21.

[7] http://www.occhio.be

[8] Pirard E. et al. : Direct estimation of sieve size distributions from 2-D image analysis of sand particles, Proc. Partec 2004 (this issue)

[9] Rideal G. et al., The development of new reference standards for particle sizing instrument calibration, World Congress on Particle Technology 3, 1998, Brighton, UK.

[10] Medalia A. : Dynamic shape factors of particles, Powder Technology, Vol.4, 1970, 117-138.

[11] Pirard E. : Shape processing and analysis using the calypter, Journal of Microscopy, Vol. 175/ 3, 1994, 214-221.

[12] Meloy T.P. and Durney T.E., Particle shape chromatography – the sieve cascadograph, Int. J. Mineral Processing, Vol. 11, 1983, 101-113

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the help of Anne Darimont and Xavier Willem from the Building Materials Research lab in providing the samples and having fruitful discussions about the results.