1D NUMERICAL APPROACH TO MODEL THE FLOW OVER A PIANO KEY WEIR (PKW)

Approche numérique 1D pour la modélisation des écoulements sur un déversoir en touches de piano (PKW)

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KEY WORDS

Finite volume, flux vector splitting, multiblock, bottom slope discretization, lateral discharge

ABSTRACT

Piano Key Weir (PKW) is a new type of weir showing appealing hydraulic capacities, in particular for low heads. However, its complex geometry, with a large set of parameters, makes difficult the understanding of the flow features and consequently the definition of an optimal design. In this paper, we investigate how a 1D numerical approach can predict the flow over a PKW and thus help in identifying the most relevant geometric parameters. After a detailed description of the numerical model, a comparison of the numerical results with experimental data is realized, showing promising results.

RESUME

Le déversoir en touche de piano (Piano Key Weir – PKW) est un nouveau type de déversoir qui possède des capacités hydrauliques intéressantes, en particulier pour de faibles hauteurs de charge. Cependant, sa géométrie complexe, avec un grand nombre de paramètres, rend difficile la compréhension de son fonctionnement hydraulique et donc le dimensionnement d'une solution optimale. Dans cet article, on analyse comment une approche numérique 1D peut prédire l'écoulement sur un PKW et dés lors aider à identifier les paramètres géométriques les plus pertinents. Après une description détaillée du modèle numérique, une première comparaison des résultats numériques avec des résultats expérimentaux est réalisée, débouchant sur des résultats prometteurs.

1. INTRODUCTION

The Piano Key Weir (PKW) is an original type of weir developed by Lempérière and Ouamane [1, 13] to improve the design of a labyrinth weir. Using overhangs, they reduce the basis length of the structure and facilitate thus its location on dam crests (Fig. 1). The first scale model studies showed that this new type of weir can be four times more efficient than a conventional Creager at constant head and crest length on a dam [14].

The PKW shows geometric specificities such as up- and/or downstream overhangs with variable width, inlet and outlet bottom slopes,… involving a large set of parameters (Fig. 1). The basic hydraulic structure of a PKW, a "PKW-element", is composed of a lateral wall between half an inlet and half an outlet. This sequence or its symmetric complement are repeated *n* times to make the whole structure. The main geometric parameters of a PKW are the width *W* of a PKW-element, the weir height *P*, the lateral crest length *B*, the

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inlet and outlet widths *a* and *b*, the up- and downstream overhang lengths *c* and *d* and the wall thickness *e* (Fig. 1).

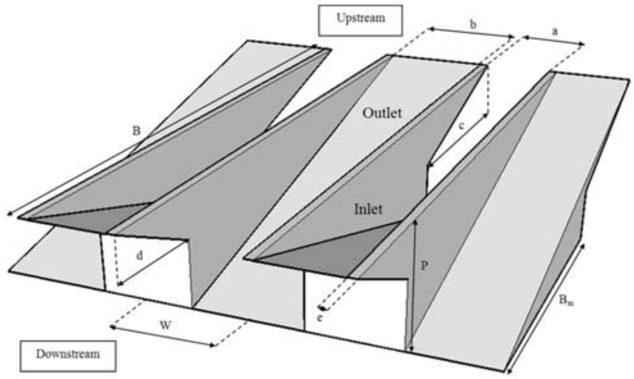

Figure 1: Sketch of a typical Piano Key Weir geometry and main geometrical parameters.

Experimental studies have been or are currently carried out in different laboratories to characterize the influence of a number of the PKW geometrical parameters on its discharge capacity [8, 10, 11, 15, 16]. 3D numerical modelling of the flow over the structure is also realized by EDF with promising results [9].

In the framework of a coupled numerical – experimental research currently undertaken at the Laboratory of Engineering Hydraulics of the University of Liege, a simplified numerical model of the flow over a PKW has been developed. The research aims at improving the understanding of the flow behaviour on this new type of weir and at setting up efficient design rules to predict its discharge capacity. The numerical model, presented in detail in section 2, is based on a 1D modelling of the inlet and the outlet, taking into account the possible interaction between both flows by exchange of discharge along the lateral crest. The performance of the model, simple to apply and little time consuming, is assessed through the comparison of the numerical results with experimental data in section 3.

2. FLOW MODEL

2.1 Principles and objectives

The main goal of the numerical model is to help in identifying the most relevant geometric parameters of the PKW governing its discharge capacities and to assess their pertinent range of variation prior to undertaking experimental studies.

The numerical model considers the smallest hydraulic element of a PKW, made of a lateral wall, half an inlet and half an outlet. The inlet and the outlet are modelled as parallel 1D channels, possibly interacting by exchange of mass and momentum along the lateral crest, and linked by an upstream reservoir (Fig. 2). To avoid the need for downstream boundary condition and considering that PKW usually works as a free weir, i.e. without influence of the tailwater level, the inlet and the outlet are both extended by independent steep slope channels, ensuring supercritical flow downstream of the simulation (Fig. 2).

According to experimental observations [10, 11], the main flow direction in the outlet follows the bottom slope. Consequently, the *x*-axis has been locally inclined in the numerical model. It is not the case in the inlet where the main flow direction is rather horizontal. The *x*-axis has thus been directed horizontally along the inlet (Fig. 2).

The geometric paramaters needed to set up the numerical model are the weir height *P*, the lateral crest length *B*, the up- and downstream overhangs lengths *c* and *d* and the inlet and outlet widths *a* and *b*. In the model, the inlet channel width is *a/2* and the outlet channel one *b/2*.

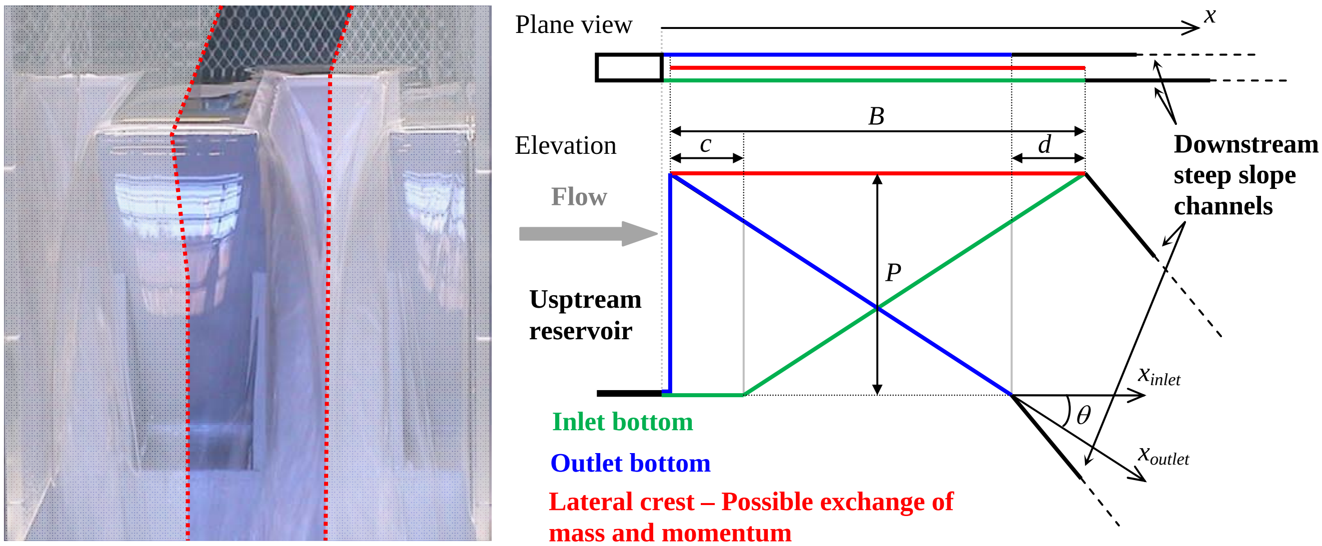

Figure 2: Basic element of a PKW (left) and numerical model layout with main geometric parameters (right)

2.2 Mathematical model

The flow model is based on the one-dimensional cross-section-averaged equations of mass and momentum conservation. In this standard 1D approach, it is basically assumed that velocities normal to the main flow direction are significantly smaller than those in this main flow direction. Consequently, the pressure field is almost hydrostatic everywhere and the free surface is horizontal along the transverse direction.

The conservative form of the governing equations can be written as follows, using vector notations and assuming a rectangular cross-section of constant width:

$$\frac{\partial \mathbf{s}}{\partial t}+\frac{\partial \mathbf{f}}{\partial x}=\mathbf{S}_0-\mathbf{S}_\mathrm{f}+\mathbf{S}_\mathrm{l} \tag{1}$$

where $\mathbf{s}=\left[\Omega \quad Q\right]^\mathrm{T}$ is the vector of the conservative unknowns. $\mathbf{f}$ represents the advective and pressure fluxes:

$$\mathbf{f}=\begin{pmatrix} Q \\ uQ+\frac{1}{2}g\cos\theta L h^2 \end{pmatrix} \tag{2}$$

$\mathbf{S}_0$, $\mathbf{S}_\mathrm{f}$ and $\mathbf{S}_\mathrm{l}$ designate respectively the bottom slope, the friction terms and the lateral exchange terms:

$$\mathbf{S}_0=-g\cos\theta\,\Omega\left[0 \quad \partial z_b/\partial x\right]^\mathrm{T}+g\sin\theta\,\Omega\left[0 \quad 1\right]^\mathrm{T} \tag{3}$$

$$\mathbf{S}_\mathrm{f}=\left[0 \quad \tau_\mathrm{bx}/\rho\right]^\mathrm{T} \tag{4}$$

$$\mathbf{S}_\mathrm{f}=\left[q_\mathrm{l} \quad \alpha u q_\mathrm{l}\right]^\mathrm{T} \tag{5}$$

In Eq. (1) to (5), t is the time, x the space coordinate, Ω the cross-section, Q the discharge, h the water depth, u the cross-section-averaged velocity, L the section width, z_b the bottom elevation, g the gravity acceleration, θ the inclination of x-axis, ρ the density of water, τ_{bx} the bottom shear stress, q_l the lateral unit discharge and α a coefficient [0,1] quantifying the change in momentum because of the lateral discharge.

2.3 Grid and numerical scheme

The space discretization step in both channels is constant to take advantage of the lower computation time and the gain in accuracy provided by this type of regular grids. In addition, this enables a direct computation of the lateral exchanges mesh by mesh without interpolation of the flow variables.

Eq. (1) is discretized in space with a finite volume scheme. This ensures a correct mass and momentum conservation, which is needed for handling properly discontinuous solutions such as moving hydraulic jumps. As a consequence, no assumption is required regarding the smoothness of the solution. Reconstruction at cells interfaces is performed with a first order constant approach.

The fluxes $\mathbf{f}$ are computed by a Flux Vector Splitting (FVS) method, where the upwinding direction of each term of the fluxes is simply dictated by the sign of the flow velocity reconstructed at the cells interfaces [3, 6, 7]. It can be formally expressed as:

$$\mathbf{f}^{+} = \begin{pmatrix} Q \\ uQ \end{pmatrix} \quad ; \quad \mathbf{f}^{-} = \begin{pmatrix} 0 \\ g\cos\theta L h^2/2 \end{pmatrix} \tag{6}$$

where the exponents + and - refer to, respectively, an upstream and a downstream evaluation of the corresponding terms. A Von Neumann stability analysis has shown that this FVS ensures a stable spatial discretization of the terms $\partial\mathbf{f}/\partial x$ [2]. Besides low computation costs, this FVS has the advantages of being completely Froude independent and of facilitating the adequacy of discretization of the bottom slope term [3, 6].

2.4 Source terms

The discretization of the topography gradients is always a challenging task when setting up a numerical flow solver based on the depth- or cross-section-averaged equations. The bed slope appears as a source term in the momentum equations. As a driving force of the flow, it has however to be discretized carefully, in particular regarding the treatment of the advective terms leading to the water movement, such as pressure and momentum.

The first step to assess topography gradients discretization is to analyze the situation of still water on an irregular bottom. In this case, momentum equation simplify and there are only two remaining terms: the advective term of hydrostatic pressure variation and the topography gradient. In this case, according to the FVS characteristics, a suitable treatment of the topography gradient source term is a downstream discretization of the bottom slope and a mean evaluation of the corresponding water depths [4, 6]. For a cell i and considering a constant reconstruction of the variables, the bottom slope discretization writes:

$$-g\cos\theta\,\Omega \left.\frac{\partial z_b}{\partial x}\right|_i \rightarrow -g\cos\theta \frac{\left(\Omega|_{i+1} + \Omega_i\right)}{2} \frac{\left(z_b|_{i+1} - z_b|_i\right)}{\Delta x} \tag{7}$$

where subscript $i+1$ refers to the downstream cell along *x*-axis.

This approach fulfils the numerical compatibility conditions defined by Nujic [12] regarding the stability of water at rest. The formulation is suited to be used in both 1- and 2D models, along *x*- and *y*- axis [6]. Its very light expression benefits directly from the simplicity of the original spatial discretization scheme. Nevertheless, this formulation constitutes only a first step towards an adequate form of the topography gradient as it is not entirely suited regarding water in movement over an irregular bed. The effect of kinetic terms is not taken into account and, consequently, poor evaluation of the flow energy evolution can occur when modeling flow, even stationary, over an irregular topography [4]. To overcome this problem on the upstream side of the outlet channel, i.e. where the topography gradient is locally the most important, the momentum equation has been locally replaced by the energy equation, using an approach depicted by Erpicum [4]. This technique has not been applied on the whole inlet and outlet lengths as it is not suited to compute correctly shocks such as hydraulic jumps [4], which can occur in the outlet.

The bottom friction is conventionally modeled with the Manning formula, where the Manning coefficient n characterizes the surface roughness and R is the hydraulic radius, specifically defined by Eq. 9, where z_s is the lateral weir elevation, to take into account the reduced width of the inlet and the outlet.

$$\frac{\tau_{bx}}{\rho} = \frac{g n^2 \Omega\, u |u|}{R^{4/3}} \tag{8}$$

$$R = \frac{\Omega}{L + \min(h, z_s - z_b)} \tag{9}$$

Finally, the lateral unit discharge in the lateral exchange terms is computed on each point of the lateral weir depending on the head difference ΔH between the inlet and the outlet, without considering the kinetic terms along the inlet and outlet axis:

$$\Delta H = \max\left(0, z_{b,in} + h_{in} - z_s\right) - \max\left(0, z_{b,out} + h_{out} - z_s\right) \tag{10}$$

$$q_l = \mu\sqrt{2g\left|\Delta H\right|^3}\,\mathrm{sgn}\left(\Delta H\right) \tag{11}$$

$$q_{l,in} = -q_l \text{ and } q_{l,out} = q_l \tag{12}$$

where μ is the lateral weir discharge coefficient and subscripts *in* and *out* refer respectively to the inlet and the outlet channel.

2.5 Upstream reservoir

The upstream reservoir is an important part of the numerical model as it distributes the discharge between the inlet and the outlet channel. It is also in the upstream reservoir that the value of the head on the PKW will be measured to define the release efficiency of the structure.

The upstream reservoir is modeled as two special twin 1D finite volumes, with distinct discharges $Q_{R,out}$ and $Q_{R,in}$ but a single cross-section value Ω_R, as depicted in figure 3. The reservoir width is *a/2 + b/2* and it is only one space step in length. All the source terms are neglected to compute the time evolution of the three reservoir variables. Eq. 1 results merely in one mass balance equation

$$\frac{d\Omega}{dt} + \frac{Q_{R/out} + Q_{R/in} - Q_{Up}}{\Delta x} = 0 \tag{13}$$

and two momentum equations

$$\frac{dQ_{R,i}}{dt} + \frac{u_{R/i}Q_{R/i} - \delta_i Q_{Up}^2 / \Omega_R}{\Delta x} + \frac{1}{2} g L_i \frac{h_{R/i}^2 - h_R^2}{\Delta x} = 0 \text{ with } i=in, out \tag{14}$$

where the subscript *R* refers to the reservoir, *R,in* to the part of the reservoir upstream of the inlet, *R,out* to the part of the reservoir upstream of the outlet, *Up* to the upstream discharge boundary condition and *R/i* and *R/out* to the boundary between the reservoir cell and the first inlet and outlet cell respectively. δ_i is the ratio between the reservoir width and the inlet or the outlet width:

$$\delta_{in} = \frac{a}{a+b}, \delta_{out} = \frac{b}{a+b} = 1 - \delta_{in} \tag{15}$$

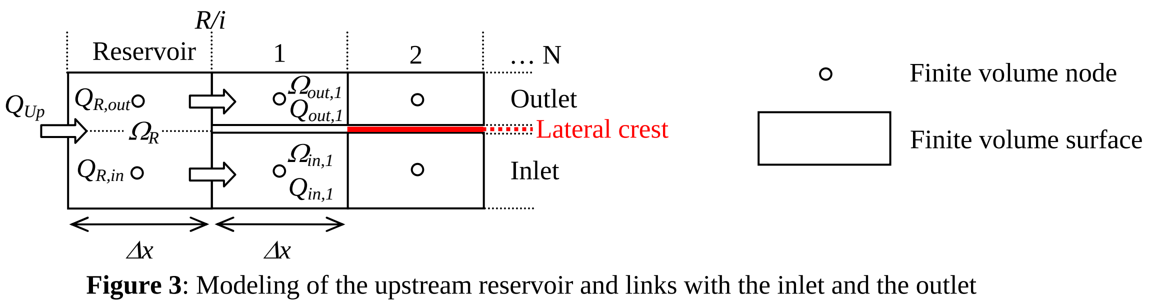

Figure 3: Modeling of the upstream reservoir and links with the inlet and the outlet

The procedure to model the reservoir is very close to the ones developed by the authors to link 1D and 2D models or to perform 2D multiblock – multimodel modeling [3, 5, 6].

2.6 Boundary conditions

The value of the upstream discharge is the only value to be prescribed as a boundary condition. The steep slope of both channels in the downstream part of the model leads to supercritical flow and no outflow boundary condition is thus needed.

2.7 Time discretization

Since the model is applied to compute steady-state solutions, the time integration is performed by means of a 3-step first order accurate Runge-Kutta algorithm, providing adequate dissipation in time. For stability reasons, the time step is constrained by the Courant-Friedrichs-Levy condition based on gravity waves. A semi-implicit treatment of the friction term is used, without requiring additional computational costs.

Slight changes in the Runge-Kutta algorithm coefficients allow modifying its dissipation properties and make it suitable for accurate transient computations.

2.8 Other features

The solver has been written in Visual Basic using VB-Application in the software Microsoft Excel. With a typical space step of 5 mm to model standard experimental PKW 50 cm long and 10 cm wide, the computation of the flow over the structure takes less than 2 min on a desktop computer.

A convergence criteria has been defined on the basis of the discharge evolution in the reservoir ($Q_{R,in}+Q_{R,out}$) compared to the upstream discharge boundary condition. When the difference between both values is lower than a given tolerance during a fixed time, the computation is assumed to be converged.

3. APPLICATIONS AND COMPARISON WITH EXPERIMENTAL DATA

To assess the model performance and accuracy, the numerical model results have been compared with experimental data measured on a large scale model of a PKW built in the Laboratory of Engineering Hydraulics of the University of Liege.

The physical model is extensively detailed by Machiels [10, 11]. Its geometric characterictics are P=0.525m, B=0.63m, $c=d$=0.18m, $a=b$=0.18m. Its release capacities have been extensively measured for unit discharges ranging from 0.013 m³/s/m up to 0.47 m³/s/m.

The numerical model has been built with a space step of 5 mm. The lateral discharge coefficient μ in Eq. 11 is equal to 0.385 (thick crest). The Manning roughness coefficient n is 0.011 (PVC). α coefficient in Eq. 5 is assumed to be equal to 1 (full exchange of momentum between the inlet and the outlet). Inclination of the outlet axis is 49.7°. The solver has been used to compute the flow over the PKW, and thus to compute the upstream head, for unit discharges ranging from 0.055 m³/s/m up to .0555 m³/s/m.

The results are summarized in figure 4 with the experimental and the numerical release efficiency curve of the PKW. Abscissa of the graph are the non dimensional heads *H/P* on the PKW where *H* is the head in the upstream reservoir. Ordinates are the discharge coefficients C_w of the PKW, calculated as

$$C_w = \frac{Q}{W_T\sqrt{2gH^3}} \qquad (15)$$

where W_T is the width of the PKW in the upstream reservoir, i.e. the length of the weir on the dam crest, equal to $n\frac{a+b}{2}$. This approach, suggested by Ouamane [14], enables a direct comparison of the PKW efficiency with the one of a conventional weir of same length on a dam crest.

The numerical results are in satisfactory agreement with the experimental ones in a large part of the C_w-*H/P* curve, especially for moderate head ratios (Fig. 4). The numerical curve shape is similar to the experimental one. For large and very small head ratios, the efficiency of the PKW is systematically over-estimated by the numerical model.

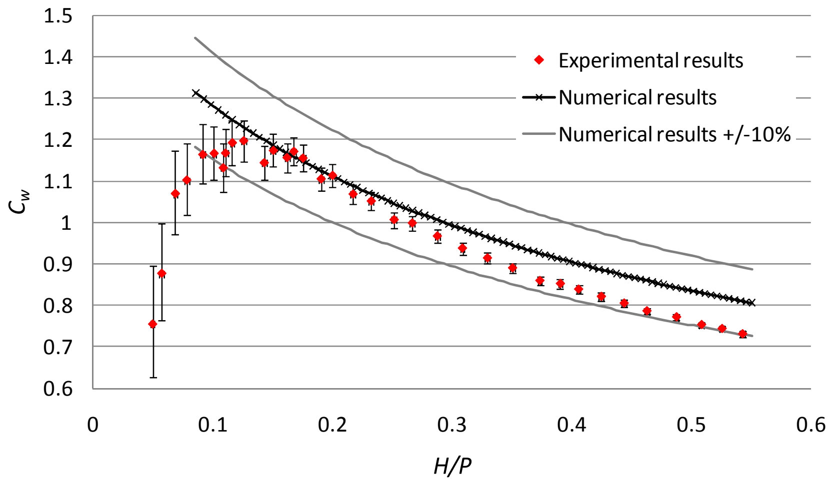

Figure 4: Comparison of experimental and numerical results - Release capacity of the PKW

In a next step, the detailed numerical results (Fig. 5 and Fig. 6) will be careffuly compared to the experimental measures in order to more precisely assess the solver relevance, and eventually to help in improving the conceptual approach and the mathematical model. It will also be possible to investigate the influence of the model parameters such as μ and α.

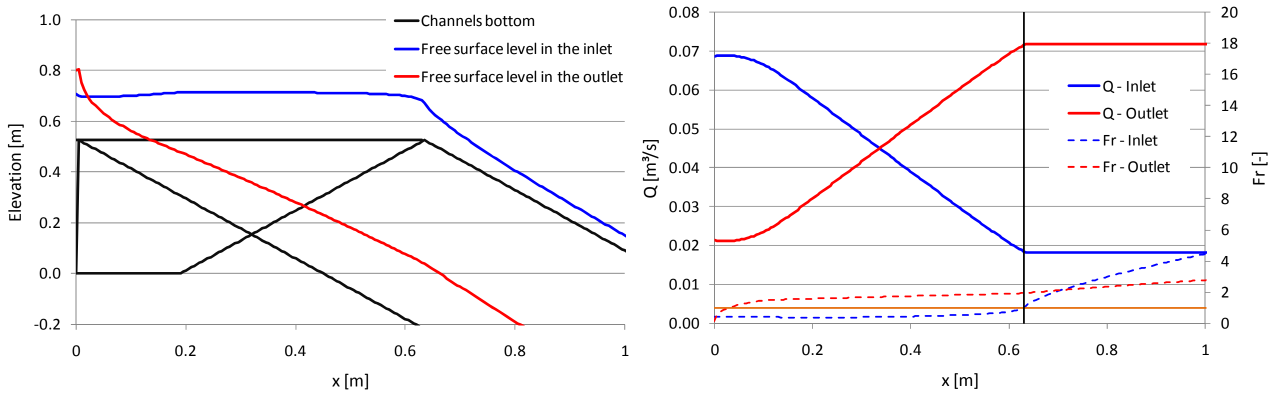

Figure 5: Numerical results for a unit discharge of 0.50 m³/s/m

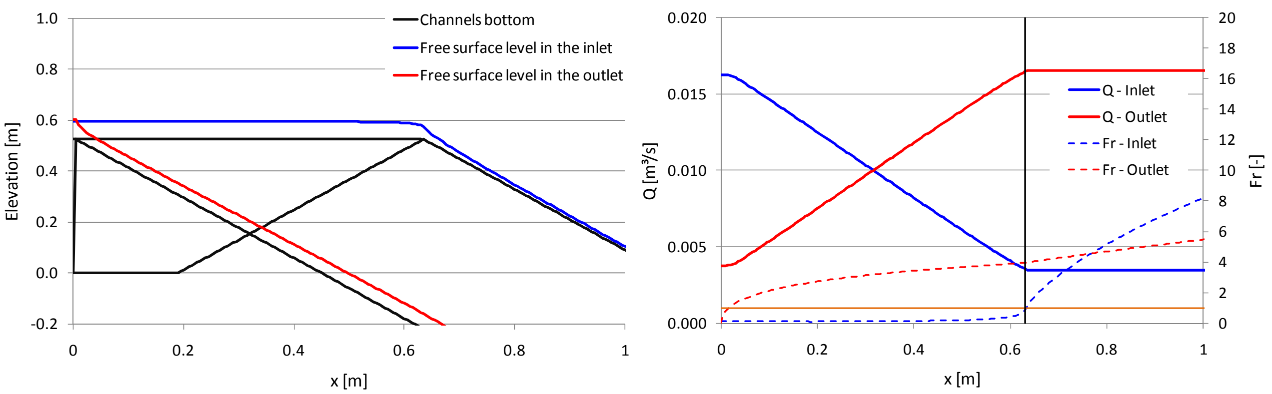

Figure 6: Numerical results for a discharge of 0.11 m³/s/m

4. CONCLUSIONS

A simplified numerical model of the flow over a PKW has been set up. It is based on a 1D modelling of the inlet and the outlet separately, with a common upstream reservoir and possible interaction between both flows by exchange of discharge along the lateral crest.

The comparison of the first numerical results with experimental data appears promising in terms of capacity of the numerical model to predict the release capacity of the structure. The solver could thus help in identifying the most relevant geometric parameters to be investigated on scale model studies. Further investigations and comparisons with experimental data will show how the detailed flow characteristics on the structure are correctly predicted by the numerical model.

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