A Global Understanding of Transient Surface Flows Applied to Hydrology and River Processes

M. Pirotton
Department of Hydraulic Constructions, University of Liège, 6 Quai Banning, B - 4000 Liège, Belgium

Abstract

Computations of transient surface flows is required for the whole analysis of flood waves generated by storms or dam failures, tidal oscillations and flows issuing from sluice gates operations or pump and turbine control. A global understanding of the physics of surface flows, in the fields of hydrology and hydrodynamics, leads to these very large prospects of applications, summarized in the scope of water resources management. However, a reliable numerical resolution has to be found, which applies for a wide range of hydrodynamic processes including problems of shocks.

Without considering the great range of the flow values, the article looks at a gradual approach in order to handle the common problem of potential presence of jumps growing in most hydrologic cycle flows :

- Upstream of the managed plant, the transformation of a rain recording to a flow signal at the catchment outlet. It is solved with a physically based model that reproduces the thin water layer flowing on the ground. The potential presence of discontinuities is demonstrated in computations working on three-dimensional digital terrain models, leading to an original numerical approach in accordance with the physical meaning.

- In hydraulic plants, the simulation of the resources management, with an accurate behaviour of the code in reproducing all the flows induced to maximise specific criteria.

- Downstream, the transformation of the hydraulic outputs from the managed plant, with the determination of all the outcomes in real conditions for people and goods near the sites.

Following the open air course of water, this chronological analysis of most of the hydrologic cycle flows is based on a global theoretical and numerical approach of the physics of transient surface flows. Applications assess the selective character obtained by modified finite element methods, with suitable shocks properly located and reproduced. They point out the benefits in considering first common physical features of the flows, then in discerning numerical shortcomings as well as cautions aimed to ensure the best numerical efficiency in the computations of rising floods, daily exploitation of power plants or dam break flood wave propagations.

1.Introduction

Today, the increasing water supply, arisen from the technical and social evolution, calls out for a fit management of the available resources. The forecasting, indispensable to anticipate regulating operations, prompts the manager to resort to numerical tools. Furthermore, the optimization of all planning projects has to handle the effects on water resources of land-use change issuing from industrial activity on the whole.

In this way, the general aim of this paper is to develop a global approach to reproduce most of the hydrologic cycle flows. The proposed hydraulic models must be able to predict the inputs and outputs of a managed plant; in other words, they have to reflect the impact of normal or hazardous exploitation in the vicinity of a plant, from the very low water level situations up to dam-break flood wave propagations.

2.Overland flow

The first fundamental stage to consider in the open air water course treats the runoff of the raindrop after its fall on the ground. In order to sum up the most complex processes acting upon the input variable (the rainfalls) to convert it into an ouptut variable (the flows they induce at the outlet of the catchment), we will refer to physical considerations in searching direct modelization of the thin water layer flowing on the ground.

With the physical interpretation of the parameters, the impact of any change of catchment features can be predicted with confidence by modifying their value. Sensitivity tests can be performed with data variation in realistic limits to quantify the corresponding error scope of the output hydrographs. Besides, physically based distributed models easily handle every topographic characteristics and soil types.

2.1. Theoretical model

We focus here on the thin water layer propagating on natural tridimensional slopes in order to compute the lateral discharge for each element river. This additional inflow that propagates in the main drainage path is then computed in a final stage with the specific software further described.

It is generally assumed that a dynamic equilibrium exists between friction and gravity components in the direction of flow. Resulting from the scale difference between the characteristic spatial dimension and water layer thickness, dynamic equations reduce to biunivoque relationships between the speed components and the water depth. They convert the continuity equation, expressed in terms of water layer thickness, into the following fully non-linear quasi-tridimensional system :

$$\frac{\partial h}{\partial t} + \frac{\partial (a_i h^{m+1})}{\partial x_i} + B(h) = 0 \qquad \text{(on the domain D)} \qquad (1)$$

with the following notations :

$h(x_i,t)$: the water depth
$B(h)$: the general function including the infiltration speed into the soil, the intensity of rainfalls and other effects of exchanges and losses
a_i : parameters including topographical data, surface and flow characteristics.

This kinematic wave approximation was proposed for the first time in the hydrologic field by Woolhizer and Ligett. They assessed the conditions of validity referring to two adimensional numbers and proved that overland flows conform to the fit range of these adimensional values.

Besides, several experiments confirmed that laminar and turbulent flows arise for each covering, depending on soil properties, fluctuations in depth and roughness and raindrop impact. A reasonable agreement can be assumed with the biunique relation between depth and discharge proposed by Manning.

2.2. Spurious oscillations in overland flows computations

The first theoretical example shows that models built on the kinematic hypothesis are confined to work on "equivalent" catchment elements of simple shape, instead of the original quasi-tridimensional topography, whenever the presence of potential discontinuities is neglected.

We suppose that a natural catchment ends downstream in a constant slope plane. We provide no information about physical, topographical and morphological properties of the natural upstream part of the studied basin. Nevertheless, we describe the temporal evolution of the water depth at the upstream boundary of the ending slope. Due to the transformation processes on the unknown catchment, it takes the shape of half a sinusoïdal period, followed by a constant value. We leave the system evolve freely in the time and we investigate the impervious plane submitted to a constant rainfall intensity.
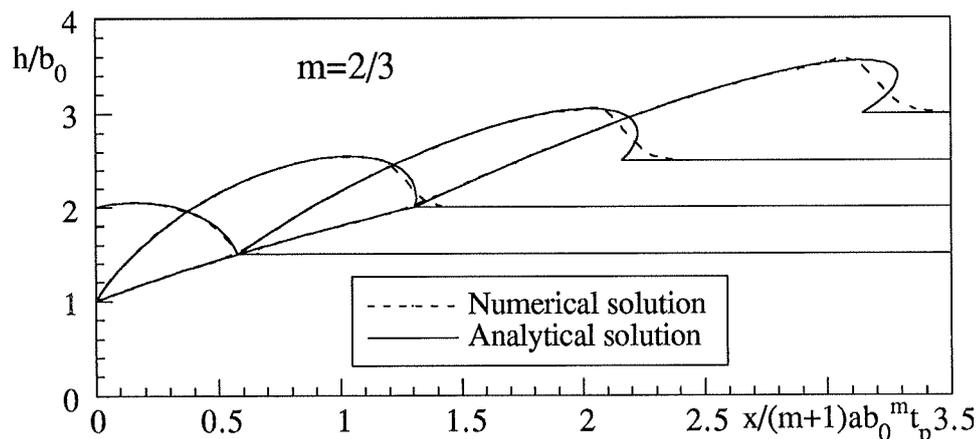

Fig. 1. - Free surface flow on the mild slope plane for different times

Since the celerity is an increasing function of water depth, the higher points of the wave will overtake and crowd the lower portions in front of them. The wave becomes steep and the analytical free surface flow takes the breaking profile

illustrated on fig. 1. This solution brings fundamental contradictions referring to kinematic hypothesis, which ensures a biunivoque relation between depth and discharge. A theoretical approach of overland flow dynamics proves that a fit approximation consists, at this scale of modelization, to introduce moving bores satisfying the continuity requirement. Despite these shocks that are physically relevant, the kinematic wave approximation is theoretically confirmed to describe the continuous part of the solution.

2.3. Numerical considerations

It can be stressed that the classical Galerkin formulation used with the finite element method fails to predict and track with confidence shocks that occur in the correct solution. It induces parasitic effects highlighted by significant errors in the global volume balance.

The proposed shock capturing approach introduces discontinuous weighting functions in order to selectively dissipate the parasitic waves. The application of the Petrov-Galerkin technique to the basic equations leads to an orthogonal projection of the residual error due to discretization approximation to a set of linearly independent functions included in a vector P. We suggest the following test function components for P :

$$P_i = N_i + \alpha'_{ux} \frac{\partial N_i}{\partial x} + \alpha'_{uy} \frac{\partial N_i}{\partial y} \qquad (2)$$

with

n	:	the number of discretization points
$h = N^T . H$	:	with H the approximate unknowns vector
$N^T = < N_1, N_2,..., N_n >$	:	with N_i the classical interpolation functions
$P^T = < P_1, P_2,..., P_n >$	:	with P_i the original shape functions
$\alpha'_{ux} , \alpha'_{uy}$	:	the dissipation parameters

The computation of the previously described example was carried out in order to reveal the highly dissipative character of the method. This comparison (see fig. 2) conforts the choice of the weak formulation of the problem, which ensures continuity through any kinematic jump.
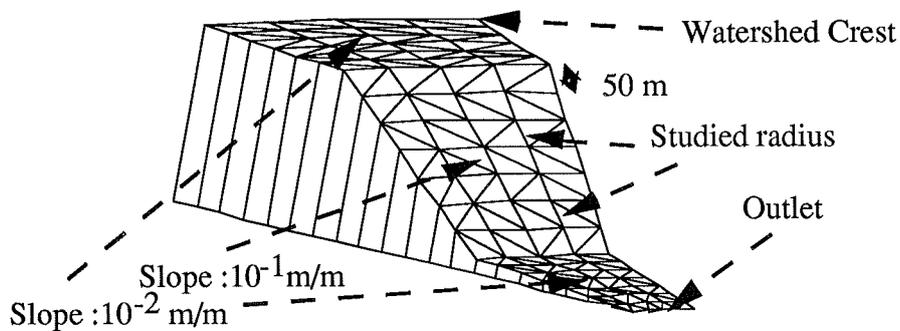

Watershed Crest

50 m

Studied radius

Outlet

Slope :10^{-1}m/m

Slope :10^{-2} m/m

Fig 2. - Discretization of the studied catchment

The following example suggests in practice the occurence of this kind of shocks when we are dealing with a succession of natural slopes. The converging watershed area described in fig. 3 is impervious but its morphological parameters values correspond to natural properties.

Theoretical considerations assess that if the first upstream transition of slopes introduces no hydraulic singularity, on the other hand the downstream one generates a transient shock because of the sudden decrease of discharge capabilities for a given water thickness. The code first produces a rising undisturbed shock, then a steady stage exhibiting the equilibrium for constant rainfalls of 5.10^{-6} m/s and finally the falling stage of the thin water layer after the end of this rainfall event.
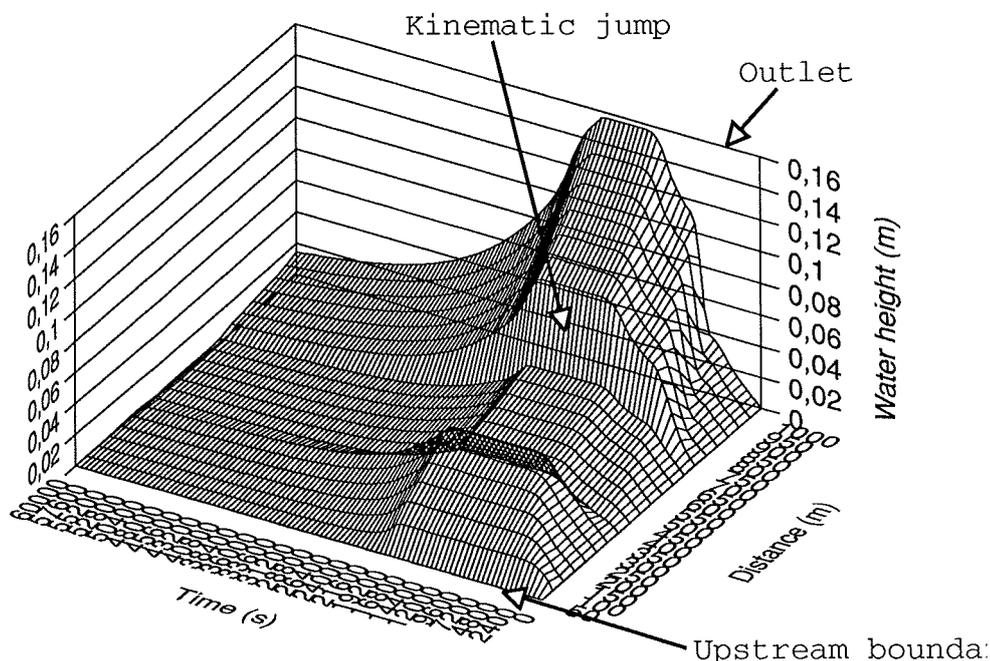
Fig. 3. - Temporal evolution of water height on the studied radius

The proposed non-linear package deals with the effects of infiltration (partially dried elements), works on general digital terrain models and handles spatial and temporal variations of rainfall and soil properties. This approach is actually comforted by comparison with experimental data on natural watersheds. The physical meaning of each parameter of the modelization and the much more realistic mathematical approach in terms of understanding the physics of overland flow processes allow to reflect easily the impact of any change of catchment features.

3 . Open-channel flow with mixed flow rates

The spreading of lateral hydrographs computed by the suggested hydrological code has now to be achieved with unidimensional computations of unsteady free surface flows induced in nets of natural rivers. Thus is followed and explained the whole story of a flood formation, from the thin stream of water rushing

down the hillside to the temporal and spatial evolution of water depth and river flow threatening riverside residents. However, the main assumptions that lead to the set of equations (3) do not prevent the second software from helping to manage water ressources of complex networks submitted to variable conditions, such as power plants. Finally, it should reproduce all transient flows occuring downstream of the dam, with outputs resulting from a daily exploitation and from a partial or complete, sudden or gradual, dam collapse.

$$\frac{\partial}{\partial t}\begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ (c^2 - \rho_\omega u^2) & 2\rho_\omega u \end{bmatrix} \frac{\partial}{\partial x}\begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} -q_L \\ g\omega \frac{\partial Z}{\partial x}\Big|_\Omega + F + 2\frac{\partial}{\partial x}\left(\nu\,\omega\frac{\partial u}{\partial x}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\equiv \frac{\partial X}{\partial t} + A(X)\frac{\partial X}{\partial x} + D(X) = \frac{\partial X}{\partial t} + \frac{\partial B(X)}{\partial x} + D'(X) = 0 \qquad (3)$$

with
F   :   the friction term
w   :   the wet cross-section
u, q, q_L :   the average speed in the section, the flow and the lateral inflow
Z   :   the elevation of free surface
ν :   the kinematic viscosity
ρ_ω :   the parameter of uneven distribution of the axial speed in the section

The coexistence of several flow rates with shocks and bores in ramified nets of variable cross section arms requires the development of suitable capturing methods. They have to simulate sharp transitions without excessive smearing on several meshes or excessive growing of dissipative processes. Gaining from the whole experience of the hydrological package, the knowledge of factors affecting the intrinsic behaviours of the discretizations was devoted to the development of an original finite element method approach .

In a first stage, the earlier formulation was fitted to the complete quasi-bidimensional set of equations (3). Nevertheless, numerical experimentations including shocks revealed an uneven improvement depending more particularly on the Froude number. This essential stage suggested that the dissipation has to take the flow conditions themselves into account.

The following formulation, based on dissymetric continuous test functions P was therefore investigated, which, in simplified conditions, agrees with schemes well known for their shock capturing capabilities :

$$[\omega\ q]^T = N'^T \cdot Y \qquad (4)$$

$$P^T = N'^T + \alpha_{N+1}\, A^T\, W^T \qquad (5)$$

with

α_{N+1} :   a parameter to optimize, setting the degree of dissipation.
W   :   the matrix including one-dimensional functions with degree (N+1), by reference to the classic interpolation functions expressed in N'

In the field of the hydraulic resources management, the excellent fit between computed water depths and gauged elevations obtained assessed the adequacy of the code to help the manager in gaining a clear view of the hydraulic processes involved in the system, by simulations of transient flows.

3.2. Transient modifications of flow rates

Flood wave propagations occuring after a gradual growing of discharge conditions upstream the studied geometry can induce the worst conditions to assess the accuracy of a code in handling subcritical and supercritical flows to numerically coexist at any moment in different locations of the discretization.
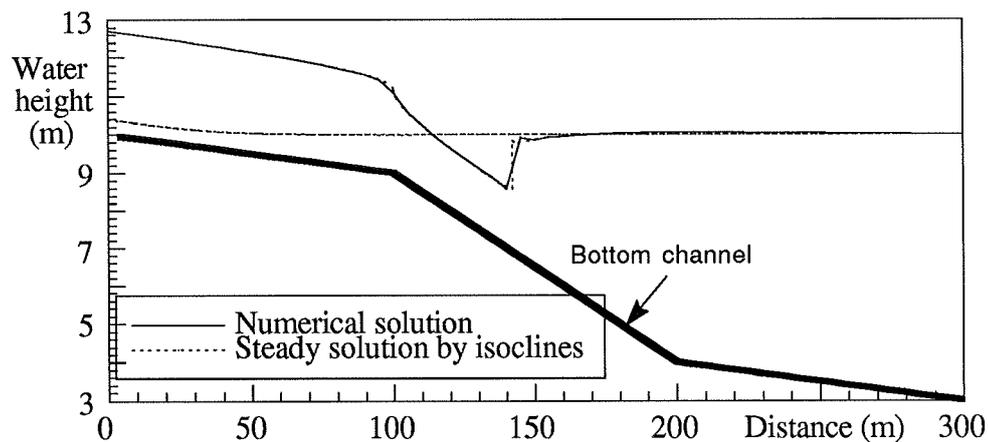

Fig. 4. - Comparison of initial and final steady state

The bottom of the studied rectangular channel is composed with two slope planes. The initial steady state illustrated in fig. 4 exhibits an subcritical flow rate on the whole domain which corresponds to an imposed upstream discharge of 5 m^3/s. The downstream water elevation does not evolve during the simulation.

The computation is performed after the arrival of a flood that requires the discharge to increase linearly up to 100 m^3/s. This variation implies no change of upstream flow regime. Thus no information is provided, through limit conditions, about potential shocks occurrences into the domain.

The previously described code brings some selective dissipation in the system, in order to obtain the solution illustrated on the fig. 6. The solution is perfectly reproduced by comparison with the steady solution issued from isoclines calculations, showing sharp transition correctly situated and exhibiting neither excessive smearing nor excessive growing of dissipative processes. The fig. 7 gives the whole history of the rise of this hydraulic jump.

With the assurance of reliable simulations of unsteady appearances, movement and disappearances of discontinuities in the most various conditions of hydraulics, the code was applied successfully to real networks to accurately reproduce the flood waves propagation resulting from a sudden complete collapse of a large Belgian gravity dam.
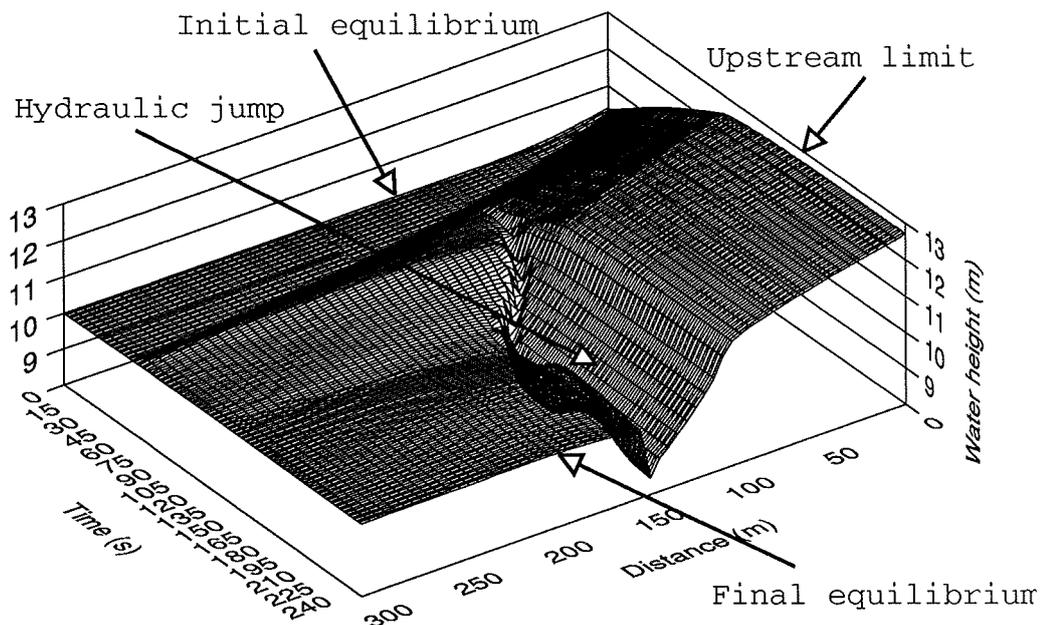
Fig. 6. - Temporal evolution of the surface profile with a rising hydraulic jump

4. Conclusions

The global approach in the theoretical and numerical study of the physics of the unsteady free surface flows passes beyond the characteristic scale differences.

This paper points out the benefits of this point of view in considering first common physical features of the flows, then in discerning numerical shortcomings also as cautions aimed to ensure the best numerical efficiency.

Seen in this light, a promising scope of research points to success for the setting up of a uniformity in theoretical and numerical coupled models based on a mutual experience, covering a wide range of applications.

References

[1]   WOOLHISER D.A. and LIGGETT J.A., *Unsteady one-dimensional flow over a plane - the rising hydrograph*, Water Resources Research, Vol 3, nr 3, 1967, pp 753-771.

[2]   MORGALI J.R., *Laminar and turbulent overland flow hydrographs*, Journal of the Hydraulics Division, ASCE, No HY2, February 1970, pp 441-460.

[3]   PIROTTON M., *Modélisation des discontinuités en écoulement instationnaire à surface libre (Du ruissellement hydrologique en fine lame à la propagation d'ondes consécutives aux ruptures de barrages)*, Ph.D. Thesis, University of Liège, 479 pages, April 1994.