EXPERIMENTAL IDENTIFICATION OF A NON-LINEAR BEAM, PROPER ORTHOGONAL DECOMPOSITION

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SUMMARY: The aim of the paper is twofold. On one hand, the Proper Orthogonal Decomposition (POD), also known as Karhunen-Loeve Transform is described. The POD is a means of extracting spatial information from a set of time series data. This decomposition can be used as an orthogonal basis for efficient representation of the behaviour of the structure. The Proper Orthogonal Modes (POMs) have been interpreted mainly as empirical system modes and the application of POD to measured displacements leads to an estimation of the normal modes if the structural mass matrix is known. On the other hand, an application of this method is presented. The use of the proper orthogonal modes for the identification of parameters of non-linear dynamical structures is investigated using an optimization procedure based on the difference between the experimental and simulated POMs. To illustrate the method, an experimental example with a local asymmetric non-linearity is described.

KEYWORDS: non-linear system identification - proper orthogonal decomposition - geometrical non-linearity - experimental application.

INTRODUCTION

For the purpose of dynamic analysis and design, most structures are usually approximated by a linear model. Modal testing and analysis are the most widely used linear techniques for modeling, prediction and control of the system dynamic behaviour, as well as for solving updating or damage detection problems. However, a lot of real structures exhibit non-linear behaviour which can be caused by a number of different reasons. Non-linear behaviour is observed even in rather simple structures like plates and beams, as a result of buckling or large deformation related effects. The non-linear behaviour of a structure may also be due to a local (friction, joint and link flexibility, backlash and clearance, non-linear contact) or a global non-linearity (geometric non-linearities, nonlinear material behaviour). The basic principles which apply for a linear system and form the basis for modal analysis are no longer valid for nonlinear systems. The superposition and homogeneity as well as the Maxwell reciprocity principles do not apply for a non-linear system. A non-linear mechanical system shows a tendency to redistribute the energy of the input spectrum. This results in modulation, super- and sub-harmonics, broadband spectra in some areas. The generation of harmonics depends on the excitation. The frequency response functions are also excitation dependent, which makes impossible their further application for modal analysis. Modal models are quite inapt to predict the behaviour of non-linear systems.

Accordingly, new tools for the detection, quantification and modeling of non-linearities in dynamical systems are necessary. A lot of effort was spent into developing methods to detect the presence of non-linearities in a system. Some procedures rely on characteristic features for non-linear systems, like the distortion of the FRF plots. Others suggest testing the validity of basic linear principles. Most procedures compare the responses of the linear system and the system under test. During the last years, a lot of work was done on modeling the non-linear behaviour of dynamic systems. Different tools are suggested in this direction. One of the first approach to the identification of SDOF systems began with Masri and Caughey [1]. The procedure of the identification is based on Newton's second law. The method analyzes non-linear systems in terms of their internal restoring forces. It was generalized to multi-degree-of-freedom (MDOF) systems but suffered from the necessary knowledge of the displacement, velocity and acceleration data for each DOF. A lot of researchers worked on the method in the following years and some experimental studies were published on the identification of non-linear automotive components [2, 3]. The use of Volterra series as a way to describe non-linear systems is one of the most widely accepted one. The identification procedure for non-linear dynamic systems uses the higher order FRF's [4]. Other techniques include spectral analysis (Conditioned Reverse Path Method [5]) and different NARMAX models [6, 7], which also makes a useful link with Volterra's series. In this paper we suggest the application of the Karhunen-Loeve transform or Proper Orthogonal Decomposition (POD) for the identification of non-linear MDOF systems.

THE PROPER ORTHOGONAL DECOMPOSITION

Proper Orthogonal Decomposition (POD), also known as Karhunen-Loeve (K-L) decomposition, is based on a statistical formulation, although it facilitates modal projections of partial differential equations into reduced-order deterministic models [8]. The K-L method has been applied successfully in the fields of fluid dynamic [9], thermics [10] and signal processing [11].

The Proper Orthogonal Decomposition is a means of extracting spatial information from a set of time series data available on a domain. The use of Karhunen-Loeve (K-L) transform is of great help in non-linear settings where traditional linear techniques such as modal testing and power spectrum analyses cannot be applied. The advantage of this method lies in the fact that the proper orthogonal modes (POMs) obtained from the K-L decomposition for a given set of parameters, can be used to reconstruct the response of a system whose parameters take different values from the original system. The additional advantage of the K-L analysis is that it can be applied, not only to conservative systems, but also to dissipative ones and that it provides information about the spatial structure of the system dynamics as well as the energy contained in the system.

The method was first applied to turbulence problems by Lumley [9]. The POD allows to quantify spatial coherence in turbulence and structures [12, 13]. A recent work [14] has shown that the application of POD to measured displacements of a discrete structure with a known mass matrix leads to an estimation of the normal modes. In reference [8] the K-L method is applied to vibroimpacting beams and rotors to create low dimensional models, via a Galerkin projection.

Mathematical formulation of the POD

The POMs are shown here to be the eigenfunctions of the space correlation tensor. Define a random field $u(x,t)$ on some domain Ω. This field is first decomposed into mean ($U(x) =< u >$) and time varying ($v(x,t)$) parts. This is represented as :

$$u(x,t) = U(x) + v(x,t) \tag{1}$$

This fields are sampled at finite number of points in time. Hence at a fixed time t_n, the system displays an instantaneous snapshot $v_n(x)$, that is a continuous function of x, with $x \in \Omega$. Now we are looking for a representative structure $\phi(x)$ of the ensemble of N snapshots. This coherent structure is computed by minimizing the objective function λ:

$$Minimize \quad \{\lambda = \sum_{n=1}^{N} (\phi(x) - v_n(x))^2\} \quad \forall x \in \Omega \tag{2}$$

Eqn. (2) can be compactly expressed as the maximization problem :

$$Maximize \quad \{\lambda = \frac{< (\phi, v_n)^2 >}{(\phi, \phi)}\} \quad \forall x \in \Omega \tag{3}$$

with the following notations :

$$(f,g) \equiv \int_\Omega f(x)\, g(x)\, d\Omega \quad \text{inner product of f and g}$$
$$< v_n > \equiv \frac{1}{N} \sum_{n=1}^{N} v_n(x) \qquad \text{average of snapshots}$$

This equation is equivalent to the following integral eigenvalue problem :

$$\int_\Omega K(x,x')\, \phi(x')\, dx' = \lambda\, \phi(x') \tag{4}$$

where the two point correlation function (K) is defined as

$$K(x,x') = \frac{1}{N} \sum_{n=1}^{N} v_n(x)\, v_n(x') \tag{5}$$

Eqn. (4) has a finite number of orthogonal solutions $\phi^i(x)$ with corresponding real and positive eigenvalues λ^i. The eigenvalue with the largest magnitude is the maximum which is achieved in the maximization problem (3). The second largest eigenvalue is the maximum of the same problem restricted to the space orthogonal to the first eigenfunction, and so on. In order to make the computation unique, the eigenfunctions are normalized. Therefore we can use it as a basis for the decomposition of the field :

$$v(x,t) = \sum_{i=1}^{N} a^i(t)\, \phi^i(x) \tag{6}$$

It should be noted that time-dependent part in Eqn. (6) forms orthogonal modes. Thus the POD can be viewed as a bi-orthogonal decomposition because of the space-time symmetry of the decomposition.

Discrete formulation

Suppose S linear snapshots v_i of size M obtained for instance by measurements of the acceleration on a beam at M locations. The $M \times M$ covariance matrix C [11] is defined as:

$$C = \frac{1}{S} \sum_{i=0}^{S-1} v_i v_i^T \tag{7}$$

Its eigenvectors ϕ_k satisfying

$$C\phi_k = \lambda_k \phi_k, \qquad k = 0, \ldots, S-1 \tag{8}$$

with

$$\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{S-1} > 0 \tag{9}$$

form the proper orthogonal decomposition. Each λ_k corresponds to the vector ϕ_k and represents the relative importance of that vector in the data. This decomposition is the best basis in term of de-correlation.

Computation of the POD using Singular Value Decomposition (SVD)

The complete bi-orthogonal decomposition of the data may be obtained by use of the SVD. For instance the SVD which is related to Principal Component Analysis, is used in reference [15] to compute modal metrics to solve model updating problems in an optimization procedure. Let $x(t)$ denote a response time-history, where x is a vector containing the displacement, velocity or acceleration at M discrete locations. The discrete matrix X is formed:

$$X = \begin{bmatrix} x_1(t_1) & \cdots & x_1(t_N) \\ \vdots & \ddots & \vdots \\ x_M(t_1) & \cdots & x_M(t_N) \end{bmatrix} \tag{10}$$

So, each row corresponds to a time history at one location and each column corresponds to a snapshot of the system at a specific time. Now the singular value decomposition of matrix X can be written as:

$$X = U\Sigma V^T \tag{11}$$

with U an orthonormal matrix (size $M \times M$) of eigenvectors of XX^T that represents the POMs and V an orthonormal matrix (size $N \times N$) of eigenvectors of X^TX that are the time modulations of the POMs. The size of the matrix Σ is $M \times N$ but only the main diagonal has non-zero elements that are the singular values of X, sorted in descending order. If the matrix X is rank deficient, i.e. some rows (or columns) can be generated by a linear superposition of the others, some of the singular values will be zero. The SVD can be used to estimate the rank of a matrix and filter out the measurement noise by discarding the modes associated with singular values smaller than a threshold value related to the presence of noise. In this paper the SVD is used to compute the POMs and the normalized basic shapes including the response time-histories [16].

Application to parameter identification

The identification of the non-linear parameters of a structure is based on the solution of an optimization problem which consists in minimizing the difference between the bi-orthogonal decompositions of the measured and simulated data respectively. Let us define the objective function F as :

$$F = \sum_i \sum_j (\Delta U_{ij})^2 + \sum_j (\Delta \Sigma_{jj})^2 + \sum_j \sum_k (\Delta V_{jk})^2 \qquad (12)$$

where ΔU_{ij}, $\Delta \Sigma_{jj}$ and ΔV_{jk} are the differences between the matrices containing the bi-orthogonal decompositions i.e. the differences between the SVD of the measured and simulated data (Eqn. (11)). It must be stated that the full decomposition is not retained in the objective function. Only the terms corresponding to the higher singular values are considered, i.e. the POMs that contain the greatest amount of energy in the signal. Let be $\Sigma_{11} > \Sigma_{22} > \ldots > \Sigma_{MM}$ the decreasing singular values of the experimental data. If we define $E = \sum_{j=1}^{M} \Sigma_{jj}$ the total energy in the data, only the p modes corresponding to a given percentage (e.g. 90%) of this energy are retained in the objective function i.e.:

$$\sum_{j=1}^{p} \frac{\Sigma_{jj}}{E} > 90\% \qquad (13)$$

In order to retain the time information but without the drawback of its oscillatory nature, the Wavelet Transform or the windowed discrete-time Fourier transform of the time decomposition associated with each POM is computed, giving a time-frequency representation of the energy contained in the signal. The frequencies that correspond to the maximum of the transform are extracted. This information, which is time dependent, is included into the objective function (12) instead of the right-singular vectors. It allows to transform the oscillatory nature of the time information into a more suitable one.

Then the objective function F may be minimized using standard optimization algorithms.

DESCRIPTION OF THE BENCHMARK

The benchmark is similar to the one proposed by the Ecole Centrale de Lyon (France) in the framework of COST Action F3 working group on "Identification of non-linear systems" [17]. This experimental application involves a clamped beam with a thin part at the end of the beam (see Fig. 1).

Seven accelerometers which span regularly the beam are used to measure the response. The structure is excited with a hammer at coordinate number 6.

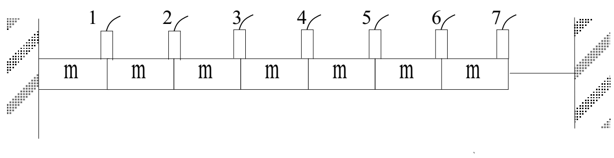

Fig. 1: Tested structure.

RESULTS OF THE IDENTIFICATION

Linear part

Prior to the non-linear study, an updating of the structure considered as linear is performed at a very low level of excitation. The main steel beam is modeled with seven beam finite elements and the thin part with four beam finite elements. A supplementary element is added to model the junction between the two beams. The updating variables are the rotational and translation stiffnesses at the two clamped ends and at the junction.

Non-linear part

In this section, the underlying linear system is assumed to be known. To model the non-linear effect, grounded quadratic and cubic springs are introduced at the junction. The cubic spring takes the stiffening effect of the thin part into account while the quadratic one models the presence of the second harmonic of the first mode in the measured data. The parameters to identify are the cubic and quadratic stiffnesses as well as the damping ratio of first two modes for the underlying linear system.

During the optimization process, two sets of time histories have to be compared:

- the first set represents the reference case and has to be generated only once;
- the second set corresponds to the model and has to be generated at each iteration of the optimization process.

Matrices U, wavelet transform of V, and Σ values are obtained from Eqn. (11) for both cases and introduced in the objective function (12). The objective function is written in terms of the first four POMs, i.e. with the highest singular values, which represents 99% of the energy. Then the optimization process yields the solution of the parameter identification.

The simulation is performed over a time period of 2.34 seconds. The data is sampled at 2560 Hz.

After updating, the values of the quadratic and cubic stiffnesses are respectively $7.9\,10^6 Nm^{-2}$ and $6.1\,10^9 Nm^{-3}$, which is of the same order of magnitude than the values found using the conditioned reverse path method [18]. Fig. 2 represents the comparison between the first four reconstructed POMs of the structure considered as linear and the experimental non-linear ones, excited by the same impulsion. At Fig. 3, the comparison after updating of the non-linear parameters shows that the reconstructed POMs match the original ones.

Fig. 4 and 6 show the wavelet transform of the first and second POM time modulations. In these figures, the maximum of the wavelet transform is superimposed. The instantaneous frequencies and envelopes corresponding to these maxima are plotted for the first and second POMs in Fig. 5 and 7. It can be seen that the reconstructed instantaneous frequency decreases as in the experimental case. However, the envelope of the second reconstructed POM decreases faster than the experimental one. Further work still has to be conducted in order to improve this result.

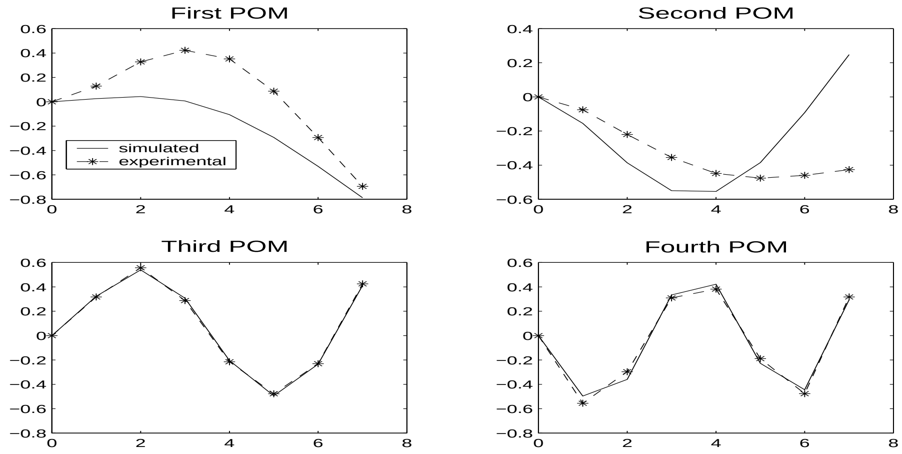

Fig. 2: First four POM of the experimental and underlying linear sysrem.

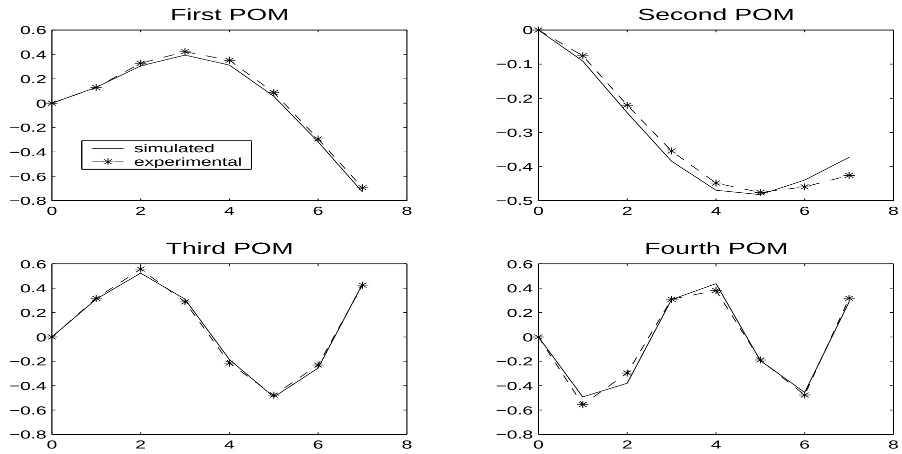

Fig. 3: First four POMs after updating.

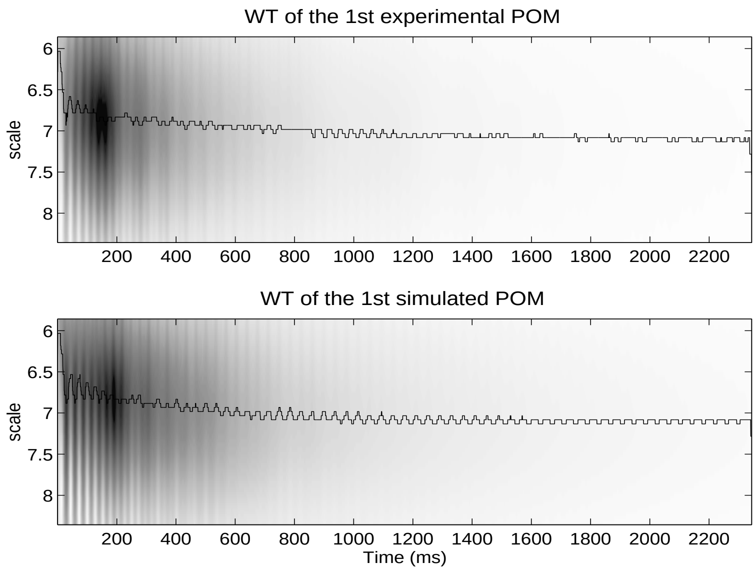

Fig. 4: Wavelet Transform for the first POM.

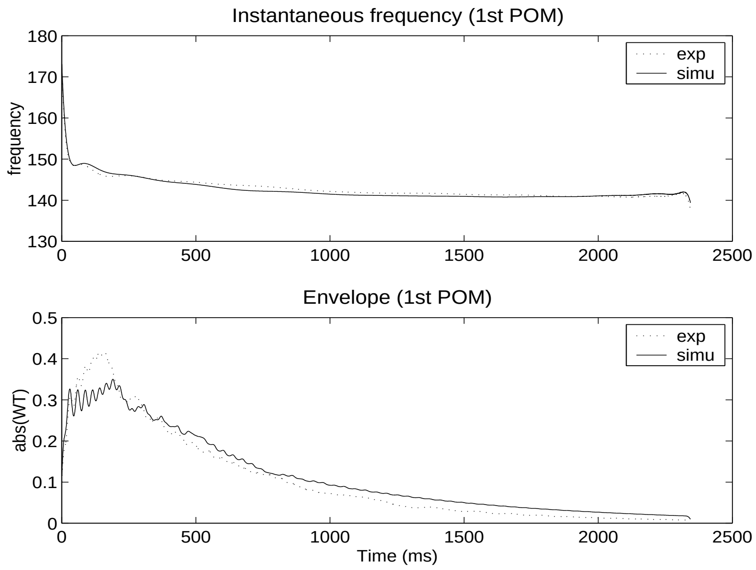

Fig. 5: Instantaneous frequency and envelope for the first POM.

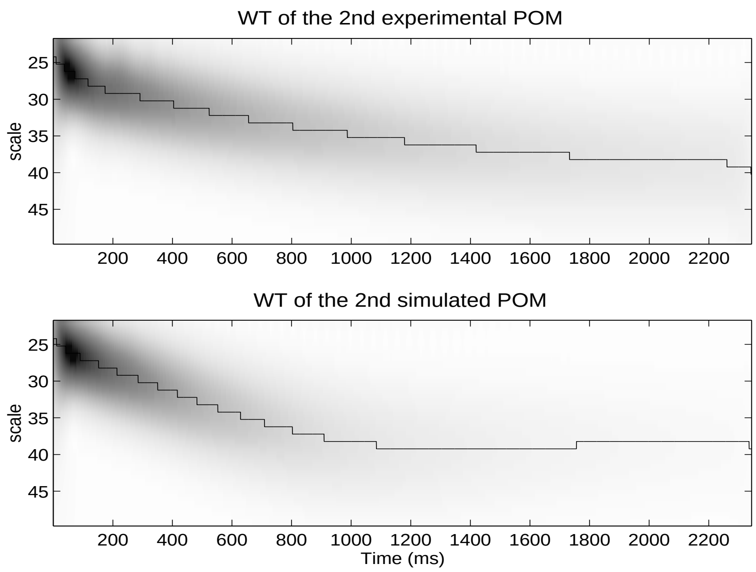

Fig. 6: Wavelet Transform for the second POM.

CONCLUSION

In this paper, proper orthogonal decomposition has been used to identify parameters of an experimental non-linear structure. The advantage of the method compared to other identification techniques is that it can be applied to MDOF systems, and that it provides an estimation of the modeling quality through the POMs and instantaneous frequencies. However, the results presented here still have to be improved, especially in the modeling of the junction between the two beams, but are very promising regarding to the difficult challenge of non-linear system identification in structural dynamics .

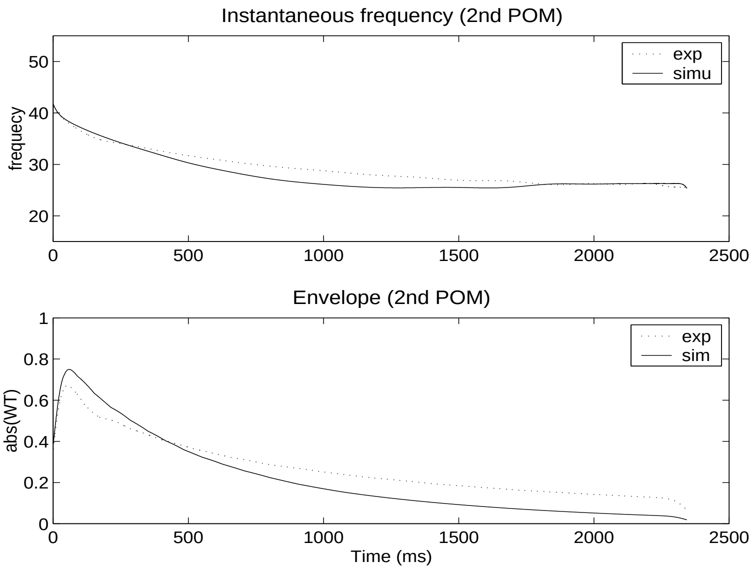

Fig. 7: Instantaneous frequency and envelope for the second POM.

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