NEW APPROACH IN MODELING FLOODS OVER STEEP RIVER BASINS

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The actual possibilities of numerical simulation allow a deterministic and physically based approach of natural phenomena. Hydrology is not getting away from it and develops models based on hydraulic equations to compute the flood generation and routing on watersheds. The flow behavior observed on surface runoff differs from the one growing in a river or a channel. Therefore the classical head loss formulations, empirically developed for turbulent and rough uniform flows in rivers, are not applicable to surface runoff. In order to find a more suitable friction law, laboratory experiments were conducted to define the relation between the mean velocity and the surface parameters.

A numerical model, named Faitou, was developed to simulate the generation and to propagate floods on steep river basins. It solves the 2D kinematic wave equation over the catchment topography with the finite volume method and is coupled with a hydrodynamic modeling of the river network. This software, along with the new head loss equation, demonstrates its potential and qualities when applied to some alpine watersheds.

1. INTRODUCTION

Regardless of all research efforts completed since long ago, floods are responsible for 37% of all dam breaks observed around the world. A diminution of the uncertainties of their prevision will contribute to a better risk analysis, and so forth to propose adequate preventive and constructive measures.

In the past, many engineers (Lauterburg 1887; Hofbauer 1916; Kürsteiner 1917; Melli 1924; Müller 1943; Heusser 1947) have suggested simple empirical equations to estimate the design discharge flowing through the reservoir and its associated spillway. These equations require principally the area of the watershed and some other local parameters. In the same time, statistical methods were developed to extrapolate an historical discharge record of some years to the 100 or 1000 years return period discharge. This approach is extensively used but requires some data, sometimes difficult to obtain in small alpine watersheds, precisely where large dams are projected or build.

The next decisive step was the development of global hydrologic models, like the unit hydrograph [1]. It became possible to predict the full hydrograph resulting from a rain. The major question concerning these models lies in the validity of their parameters for a wide range of flood amplitude. It is always possible to calibrate such a model for some observed floods. But how believe in the constancy of the calibrated parameters when computing the probable maximum flood of the watershed?

More recently, with the computer technology development, the trend is to compute the rainfall – runoff relation by means of hydraulic equations describing the water flow over the watershed surface and in the river network [2], [3]. These models are so called physically based because their parameters have all a physical meaning. The major is the roughness coefficient of the Manning or Chezy equation. It is the only adjustable parameter in a optimization process. For the Manning coefficient n, the experience shows that for some alpine watershed, an approximate value of

n = 10 is required to fit with the observed hydrograph. This value is of course completely out of the validity domain for which the Manning equation was developed. One more time, the question of the constancy of this parameter is of prime importance when computing extreme floods.

This contribution will propose a new head loss equation particularly adapted for the overland flow. For its practical application, a numerical model was developed, which solves the 2-D kinematic wave equation over the watershed topography described by a digital elevation model. This hydrological code is coupled with a hydrodynamic 1-D scheme which performs the flood routing in the river network.

2. HEAD LOSS COMPUTATION

2.1 Experimental investigation

In order to propose a new equation for head loss estimation, an experimental investigation was performed. The main objective was the determination of the relation between the mean velocity and the water depth over a surface. The real surface micro-topography of a soil is very complex. For simplicity reasons, the investigated surface was created with spheres geometrically arranged on a plane. This micro-topography model of a natural soil is particularly simple and fully described by two parameters, that is the sphere diameter D and the cover density of spheres p. The cover density is defined by the ratio between the total projected surface area of the spheres and the surface area of the plane. Geometrical considerations show a maximum cover density of p_{max} = 0.9069. The tested cover densities are shown on figure 1.

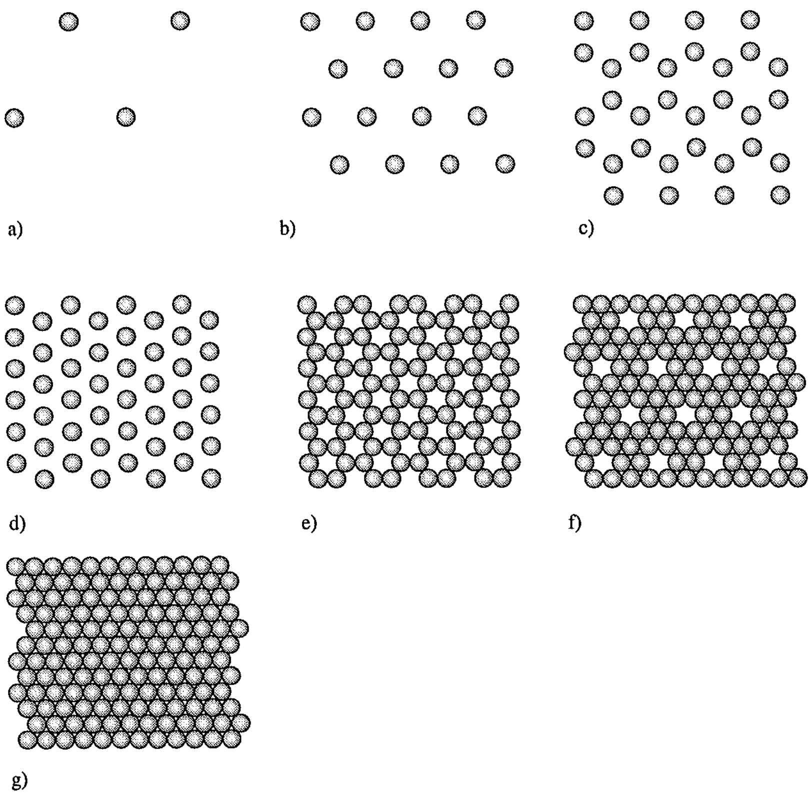

Figure 1 Geometrical layout of the spheres corresponding to cover density of a) 2.53%, b) 10.1%, c) 20.3%, d) 30.4%, e) 50.2%, f) 70.0% and g) 90.7% (maximum cover density p_{max}).

In this sphere model, the different hydraulic parameters can be computed analytically. In order to work with adimensional parameters, let's define the cross-section porosity as :

$$\eta = \frac{A_m}{A_{tot}} \tag{1}$$

with A_m the wetted surface area. A_{tot}, the total surface area, is the product of the width B of the plane and the water depth y. It can be shown that :

$$\begin{aligned} \eta &= 1 - \sqrt{\frac{p}{p_{\max}} \frac{D}{8y}} (\theta - \sin\theta) \qquad & y \le D \\ \eta &= 1 - \sqrt{\frac{p}{p_{\max}} \frac{\pi}{4} \frac{D}{y}} \qquad & y \ge D \end{aligned} \tag{2}$$

In the same way, a non-dimensional wetted perimeter Ω is defined as the ratio between the wetted perimeter P and the width B of the plane. For the sphere model and after some geometric calculation, one can obtain :

$$\begin{aligned} \Omega &= 1 + \sqrt{\frac{p}{p_{\max}} \frac{\theta}{2}} \qquad & y \le D \\ \Omega &= 1 + \sqrt{\frac{p}{p_{\max}} \pi} \qquad & y \ge D \end{aligned} \tag{3}$$

In equations (2) and (3), the angle θ is only defined for $y \le D$ by :

$$\cos\left(\frac{\theta}{2}\right) = 1 - \frac{2y}{D} \tag{4}$$

With the above definitions of the cross section porosity η and the non-dimensional wetted perimeter Ω, the hydraulic radius R_h can be expressed as :

$$R_h = \frac{\eta}{\Omega} y \tag{5}$$

By considering the presence of macro-roughness elements over the surface, the hydraulic radius differs from the water depth usually adopted in all past studies of flow over a plane.

This sphere model is at the root of the elaboration of the head loss relation. For this purpose, the experimental facility showed on figure 2 was built in the Laboratory of Hydraulic Constructions of the Swiss Federal Institute of Technology in Lausanne. Different steady flow conditions can be imposed over a 2 m long and 1 m width plane covered with spheres. This plane can be installed with any slope between the horizontal and 45°.

A direct measure of the water depth or the mean velocity is not workable due to the thickness of the flowing water layer as well as due to the spatial variation of the discrete values. As a matter of fact, only the mean values of the flow depth and the velocity are of interest and for this reason, an indirect measurement of water depths was elaborated. When steady state flow conditions are installed, the flowing water layer is suddenly cut upstream and downstream and all the imprisoned water is diverted into an electronic weighting system. With the knowledge of the discharge and this measure of the water volume, a geometrical computation gives the mean water depth and the mean velocity.

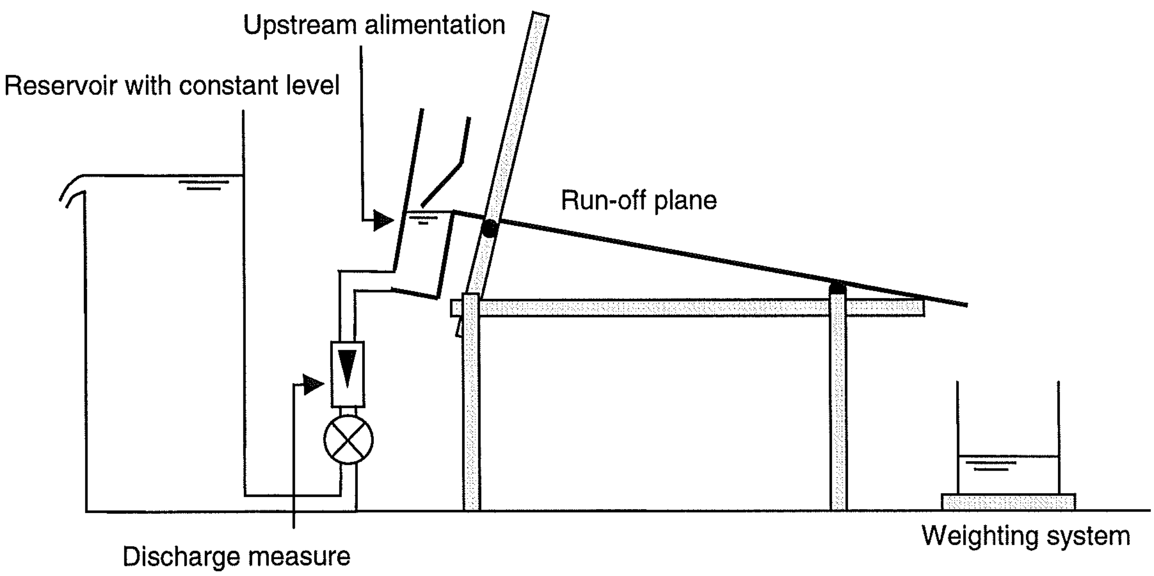

Figure 2 Experimental setup for the study of the relation between the mean velocity and the water depth for a sheet flow in a macro-rougness.

By varying the discharge, the bottom slope of the plane and the cover density of spheres, 273 tests has been carried out, giving the same number of points in the relation water depth – mean velocity.

2.2 New head loss equation

The detailed setting up of the new head loss equation is given in [4]. Only the major result is presented here. For simplicity, let's define :

$$A = \frac{0.345\,p^{0.545}\,\Omega}{8\,g\,\eta^3\,y}$$
$$B = \frac{3\nu\left(3+\dfrac{\Omega^2}{\eta^3}\right)}{4\,g\,y^2} \qquad (6)$$
$$C = -S_0$$

where g = gravitational acceleration; ν = kinematic viscosity and S_0 = bottom slope of the plane. For $y \leq D$, the mean velocity V writes :

$$V = \frac{-B+\sqrt{B^2-4AC}}{2A} \qquad (7)$$

For $y > D$, the equation becomes :

$$V = V_0\frac{D}{y} + \sqrt{\frac{8\,g\,S_0}{f}}(y-D+\Delta y)^{1/2}\left(1-\frac{D}{y}\right) \qquad (8)$$

with V_0 = reference velocity computed with equation (7) for y = D. The friction coefficient f is calculated by :

$$\sqrt{\frac{8}{f}} = 5.62\log\left(\frac{y-D+\Delta y}{D}\right) + 3.13\,p^{-0.613} \qquad (9)$$

In the above expressions, Δy is the distance between the origin of the new water depth reference system and the top of the spheres. This distance can be found from the implicit relation :

$$\sqrt{g\,S_0}\left(5.62\log\left(\frac{\Delta y}{D}\right)+3.13\,p^{-0.613}\right)\Delta y^{1/2}=V_0 \tag{10}$$

This new head loss relation (equation 7 and 8) well suited for overland flow includes many advantages. It guaranties a smooth and continuous transition between laminar and turbulent flow. The term noted *B* in equation (6) is the laminar term. For a smooth plane without any roughness element, the cross section porosity η and the non-dimensional wetted perimeter Ω tend both towards 1 and *B* reduces exactly to the analytical solution of a laminar flow over a plane. The parameter *A* in equation (6) is the turbulent term, proved to be identical to the Chezy formulation. When the flow overtops the roughness elements, the mean velocity grows very quickly. Equation (10) provides a smooth transition between equation (7) and (8). The friction equation (9) is very similar to those of Bathurst [5] for large-scale roughness flows.

3. NUMERICAL MODELING

In order to apply the above new head loss equation, a numerical model named Faitou has been developed. A second objective of the model is to refine as much as possible the geometry definition of the watershed and the river network. For this purpose, it works on a digital elevation model of the topography and provides the needed tools for an automatic basin delineation coupled with the generation of the entire river network.

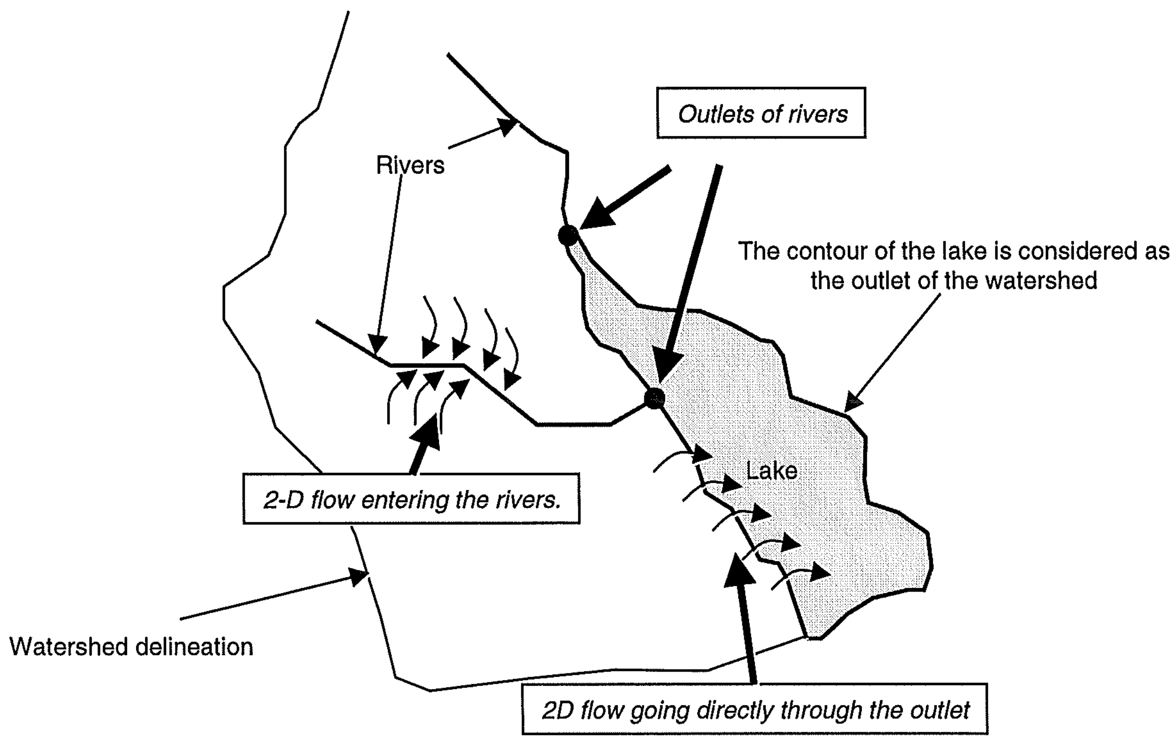

Figure 3 Schematic view of some different flows handled by the numerical model.

3.1 Mathematical formulation

The kinematic wave assumption was demonstrated to be valid for the overland flow in [3]. For a 2-D surface with a steep slope covered with macro roughness elements, the kinematic wave equation writes :

$$\frac{\partial(\phi\, y)}{\partial t}+\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}=i\cos(S_0) \tag{11}$$

with ϕ = volume porosity of the macro-roughness elements; t = time; q_x = flux in the x direction; q_y = flux in the y direction and i = rain intensity. The fluxes are calculated with the new head loss equation presented in chapter 2.2.

The finite volume method [4] solves the integral form of the equation (11), called weak formulation because its solution is only exact in mean over the control volume Ω_i :

$$\iint_{\Omega_i}\frac{\partial(\phi\, y)}{\partial t}d\Omega_i+\iint_{\Omega_i}\frac{\partial q_x}{\partial x}d\Omega_i+\iint_{\Omega_i}\frac{\partial q_y}{\partial y}d\Omega_i=\iint_{\Omega_i} i\cos(S_0)\,d\Omega_i \tag{12}$$

By denoting :

$$\mathbf{F}=\begin{bmatrix} q_x \\ q_y \end{bmatrix} \tag{13}$$

and :

$$S=i\cos(S_0) \tag{14}$$

Equation (12) can be expressed for the finite volume i as :

$$\frac{d(\phi\, y_i)}{dt}+\frac{1}{\Omega_i}\oint_{\partial\Omega_i}\mathbf{F}\,\mathbf{n}\,d\partial\Omega_i=S_i \tag{15}$$

The second and third terms of equation (12) have been converted in contour integral thanks to the divergence theorem of Green. Other parameters are presented in figure 4.

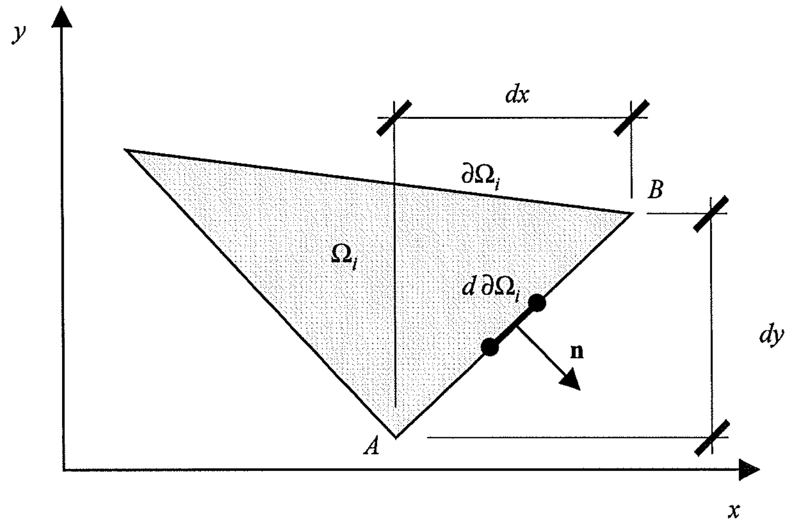

Figure 4 Definition of the parameters used in the finite volume method for the computation of fluxes across the boundary.

3.2 Model generation

A digital elevation model of an alpine watershed can include many hundred of thousand points. So, tools are needed to automatically generate the computation model. This can be achived in three steps beginning with the computation of a convergence matrix. The local convergence value is computed by counting all the upstream cells. On this basis, and after some slight altitude modifications to remove local depressions, the river network is generated. Finally, the 2-D network of triangular finite elements is created over the points belonging to the watershed and according to the existing rivers. A result example of this preprocessing is showed in figure 5.

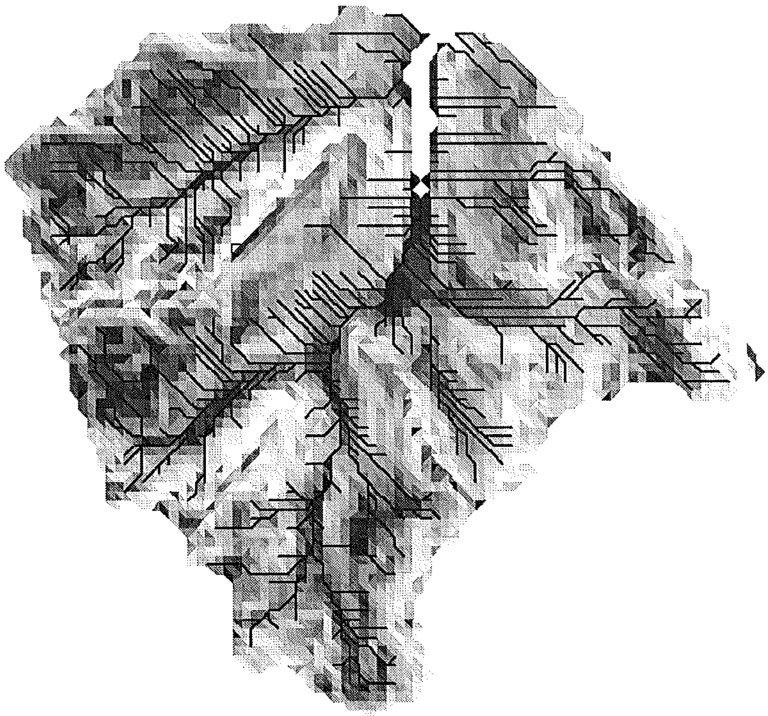

Figure 5 2-D and 1-D model of an alpine watershed generated automatically by Faitou.

4. APPLICATION EXAMPLE

For illustration purpose, the simulation of a historical flood is presented here to demonstrate the capabilities of the new approach in modeling floods in steep river basins. The chosen watershed lies upstream of the artificial reservoir of the Mattmark dam in Switzerland. During September 1993, a major flood occured in this alpine area, inducing severe damages in the downstream valley. This 37 km^2 watershed is situated between the altitudes of 2200 and 3900 ma.m. with a mean slope of about 21%. It is essentially composed with non-productive soils and rocks.

The model has been generated from a digital elevation model with a resolution of 50 m. It consists of 28'229 finite volumes, 46'472 boundary where fluxes are calculated at each time step, 4723 cross sections and 385 junctions of rivers. The gauging hydrograph resulted from water level records and management data's. With the two calculation methods used, the peak discharge entering the reservoir has been estimated between 134 and 152 m^3/s.

The figure 6 highlights the very good agreement between the simulated and measured hydrographs. This example illustrates the potential of a spatially distributed and unsteady numerical simulation for the rainfall – runoff computation. Other applications confirm the reliability of this type of modeling [4], [6].

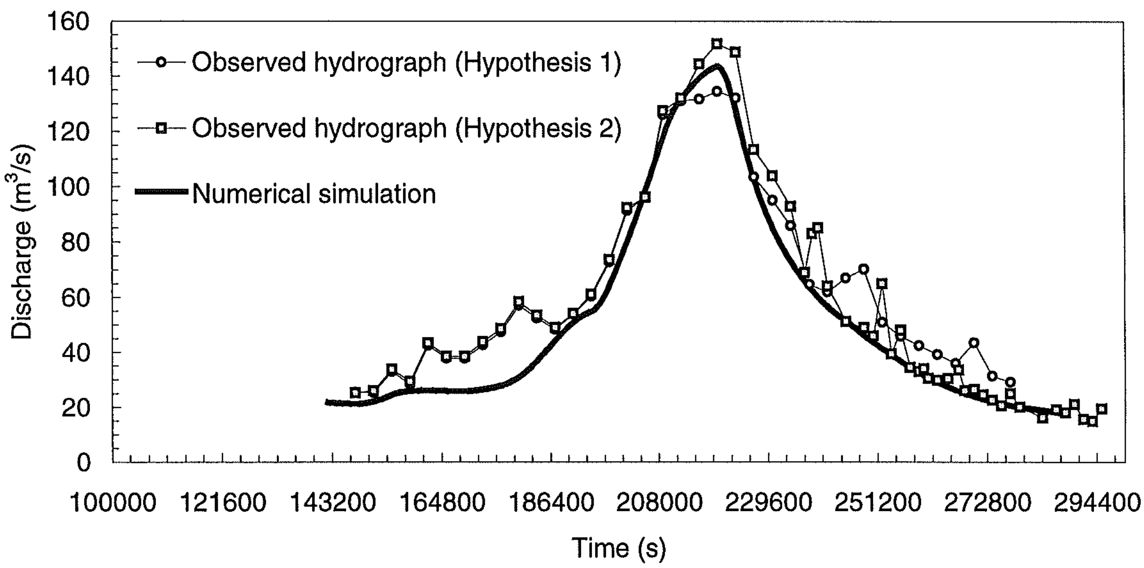

Figure 6 Observed and simulated hydrographs for the 1993 flood into the Mattmark reservoir.

5. CONCLUSIONS

This contribution proposes a new head loss equation particularly valuable for a physically based and deterministic hydrologic model. This mathematical formulation is valid for laminar as well as for turbulent flows. It converges also towards classical expressions when the water depth becomes large compared with the roughness elements. This behavior is particularly useful when extrapolating a calibrated model for the computation of extreme floods.

Based upon the kinematic wave equation, a numerical simulation program has been developed. It solves the overland flow with a finite volume method and then routes the flood in the river network. This powerful hydrological package opens up large prospects as a flood and landuse management tool.

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