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Toward robust parameterized reduced-order model of non-linear structures using POD

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This work addresses the development of robust parameterized reduced-order model (ROM) for non-linear structures. The Proper Orthogonal Decomposition (POD) approach as well as two methods attempting to make it more robust are studied. Their advantages and drawbacks are highlighted on a simple test-case and their applicability for more complex systems is assessed.

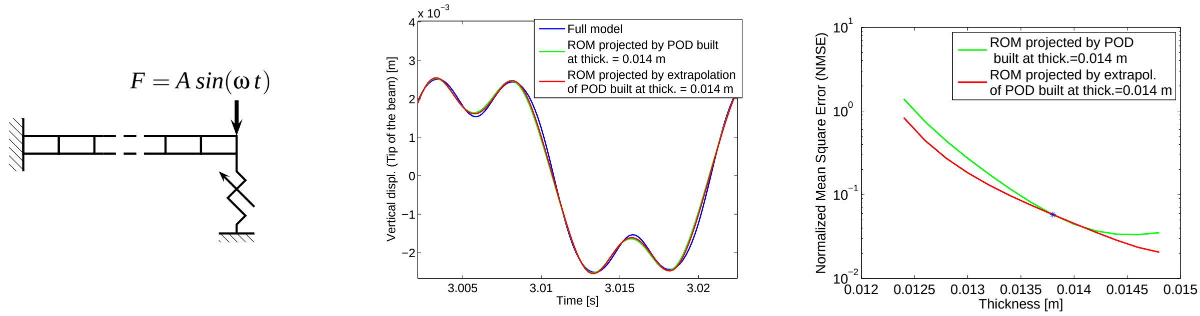
Methods for the reduction of linear structures have been intensively studied in the past and are now well-established (*e.g.* component mode technique, balanced truncation, moment matching). Some extensions of these linear methods have been proposed in order to account for changes in the model parameters. However, these reduction methods failed to give acceptable results when applied to non-linear structures.

Several model reduction techniques have been recently developed for non-linear dynamic systems. Most of them involve the selection of modes that capture the dominant behavior of the system to be simulated. The system dynamics is then projected on a low-dimensional space spanned by the modes via a Galerkin process. A popular approach is the Proper Orthogonal Decomposition [1] (a.k.a. the Karhunen-Loève expansion (KLE), principal component analysis (PCA) and empirical orthogonal function (EOF)). In order to construct the POD modes, the full model is first simulated and some relevant snapshots of the numerical results are considered [2]. The POD provides an orthonormal basis that allows to represent the given data in a least-square optimal sense. The POD method is thus a data-driven method. Unfortunately, the POD-based ROM is sensitive to parameter variations. Indeed, any significant parameter change requires to rebuild the modal basis, thereby involving a computationally expensive simulation of the full model. The development of more robust ROM with respect to parameter changes is now an important research field.

The method proposed by Amsallen et al. [3] allows to adapt the pre-computed POD-based ROMs to modifications in the model parameters. A set of pre-computed ROMs corresponding to different operating points in the parameter space are interpolated consistently in a particular tangent space. The method is decomposed in four steps. Firstly, a point in the Grassmann manifold is chosen as the reference point for the interpolation. Secondly, each pre-computed basis sufficiently close to the reference point is projected onto the tangent space at the reference point. Thirdly, interpolation is made in this tangent space in order to determine the new geodesic corresponding to the adapted ROM. Finally, the interpolated result is transported back to the originating tangent space.

A different way to make the POD more robust is proposed by Hay et al. [4]. The proposed approach relies on the use of the sensitivities of the POD basis with respect to parameter changes. The POD basis is either extrapolated in the parameter space, either expanded using the mode sensitivities. This second approach allows to include in the basis the preferred directions in phase space where parameter changes occur. The POD-modes sensitivities are derived by a semi-analytical method [5]. Since the POD modes are defined as the eigenvectors of the time response covariance matrix, the eigenvalue problem is first differentiated leading to a relation between the POD sensitivities and the sensitivities of the dynamic response. According to the semi-analytical method [6], these latest are computed at the same time than the simulation of the full model, with a limited computational cost.

In the present study, the efficiency of the two approaches are studied on a simple system consisting of a beam with a cubic stiffness located at the beam end (Fig 1(a)). The system is excited by a sinusoidal force. The Finite Element method is used to discretize the model and the time response is computed using a Newmark integration scheme [7]. The robustness of the POD-based ROM is assessed with respect to the thickness of the beam. The first studies show good promise. For example, Figure 1 (b) and (c) presents the results reached with the extrapolation method. The modal basis is defined for a thickness of 0.014 m and the ROM is used to compute the dynamic response in a range of thicknesses. Figure 1(b) illustrates the high fidelity of the ROM and Figure 1(c) shows that the extrapolation method leads to a reduction in the normalized mean square error with respect to the thickness parameter. However, several issues still need to be explored. Among them, an important problem is to determine a suitable discretization of the parameter space in which the interpolation method takes place. This discretization should be performed without exploring systematically all the operating points, which would involve a computational cost growing exponentially with the number of parameters. The use of an error estimate based on the ROM itself with the dual-weighted-residual methods (DWR) [8] seems to be well-suited to decide whether an operating point in the parameter space should be retained.



(a) Non-linear beam model (b) Time response - Thick.= 12.4 mm (c) NMSE for different thicknesses

Figure 1: Extrapolation method results

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