MODEL REDUCTION TECHNIQUES IN NONLINEAR DYNAMICS

USING PROPER ORTHOGONAL DECOMPOSITION

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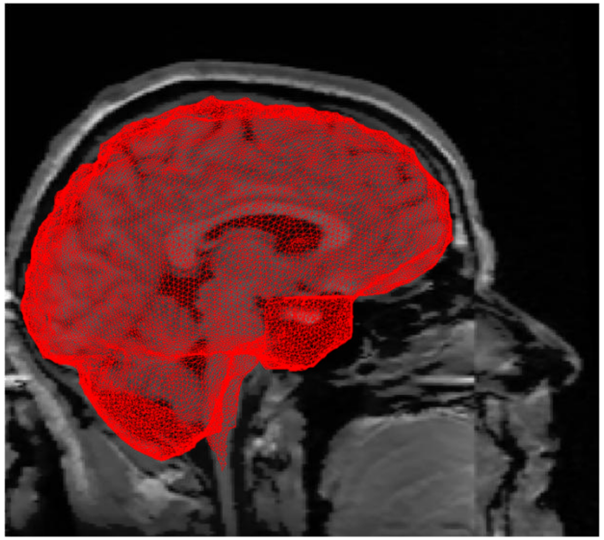
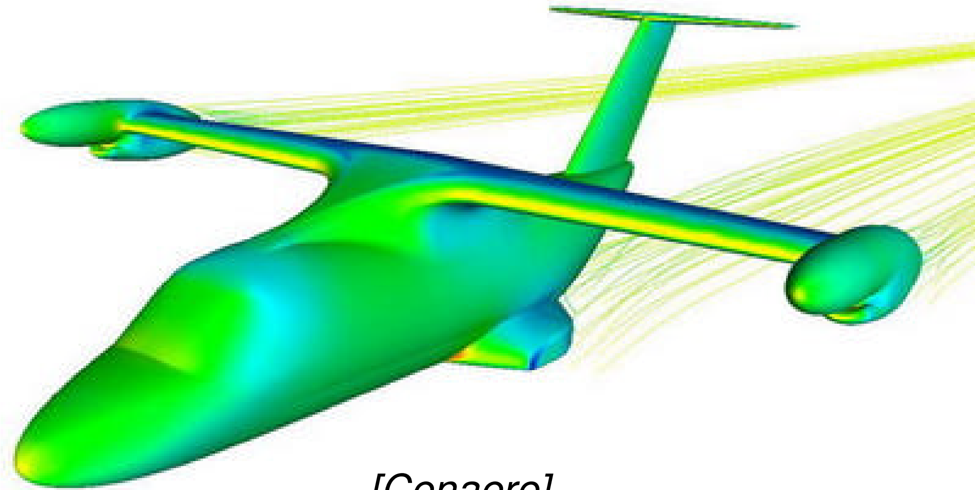
Finite element simulations are increasingly large



[D'Otreppe,2009]



[Cenaero]

Accurate and detailed modelling

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High number of DOFs

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High computational time

The size of the model and the computational time have to be reduced

From the detailed mathematical model, the goal is to create a reduced model which

- represents well the dynamics
- is robust w.r.t. parameter changes
- reduces the computational time

Outline

1 Galerkin projection

2 Non-linear beam formulation

3 Proper orthogonal decomposition

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The accuracy is closely related to the choice of the projection basis

Full Space $\mathbb{R}^N$

$$\mathbf{R}\left(\dot{\mathbf{x}}, \mathbf{x}, t\right) = 0$$

$$\mathbf{x}\left(t = 0\right) = \mathbf{x}^0$$

Reduced Space $\mathbb{V}^k$

$$\tilde{\mathbf{x}} = \Phi \mathbf{x}_r$$

Reduced Transform

$$\mathbf{x} \approx \tilde{\mathbf{x}} = \Phi \mathbf{x}_r$$

Reduced Problem

$$\Phi^T \mathbf{R}\left(\Phi \dot{\mathbf{x}}_r, \Phi \mathbf{x}_r, t\right) = 0$$

$$\mathbf{x}_r\left(t = 0\right) = \Phi^T \mathbf{x}^0$$

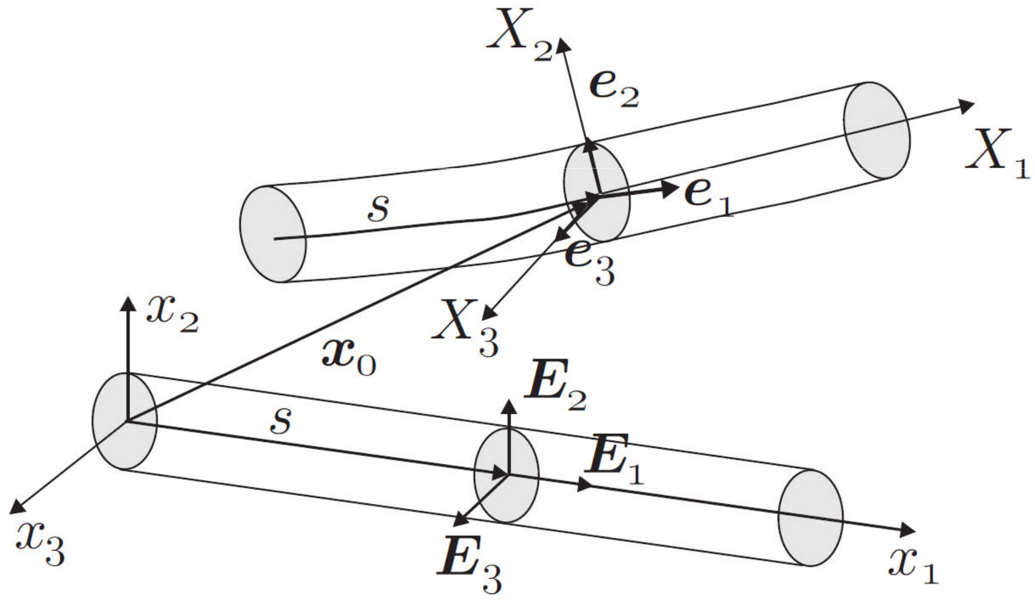
The computational time is not reduced as much as expected in non-linear dynamics

$$\Phi^T M \Phi \, \mathbf{x}_r + \underbrace{\Phi^T}_{k \times \color{red}{N}} \underbrace{F_{NL} \left(\Phi \dot{\mathbf{x}}_r , \Phi \mathbf{x}_r , t \right)}_{\color{red}{N} \times 1} = \Phi^T F_{ext}$$

Computational complexity of non-linear term still depends on N

Outline

The beam formulation takes into account the large displacements



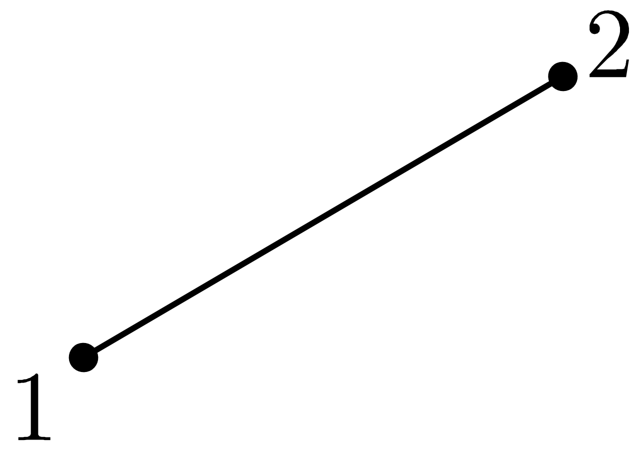
[Cardona & Lens, 2008]

Kinematic assumptions

- Initially straight
- Beam cross-sections remain plane
- Deformation of the neutral axis allowed
- Rotational kinetic energy of cross-sections

Non-linear beam discretization

$$\mathbf{q} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{\psi}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{\psi}_2 \end{bmatrix}$$

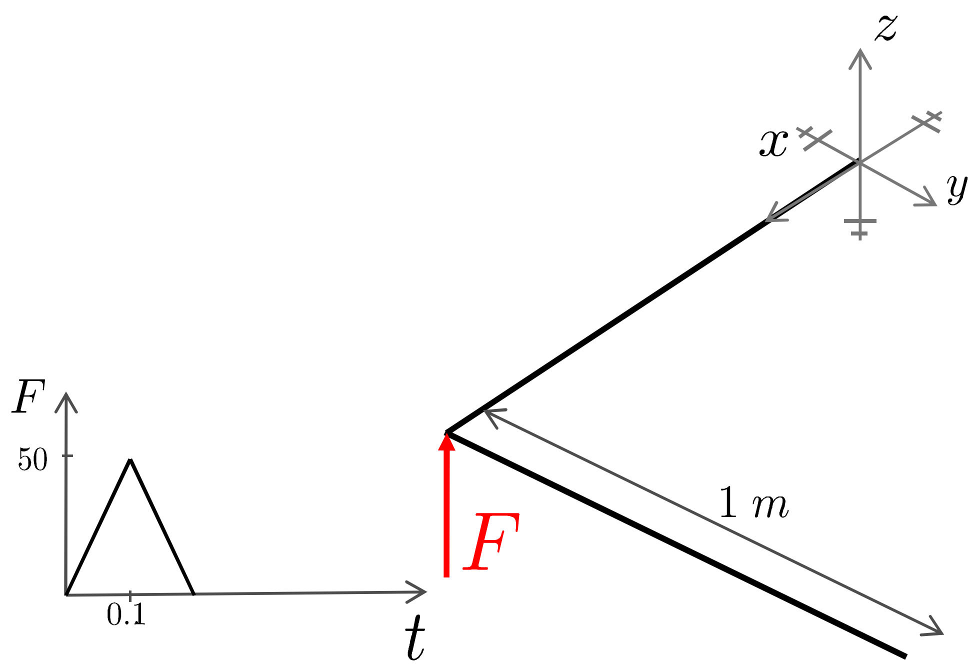


$$\mathbf{x}(s) \ = \ N_1(s)\,\mathbf{x}_1 \ + \ N_2(s)\,\mathbf{x}_2$$

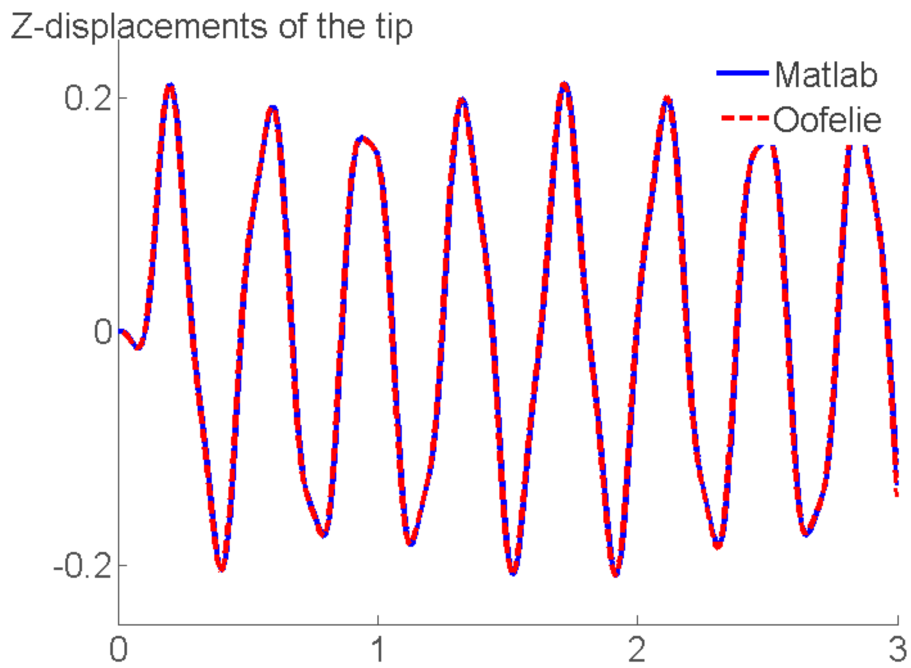
$$\boldsymbol{\psi}(s) \ = \ N_1(s)\,\boldsymbol{\psi}_1 \ + \ N_2(s)\,\boldsymbol{\psi}_2$$

$$\mathbf{M}(\mathbf{q})\ \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{g}^{int}(\mathbf{q}) = \mathbf{g}^{e}$$

The home-made Matlab results agree with Oofelie results



[Simo et al 1988, Cardona & Lens 2007]



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The Proper Orthogonal Decomposition provides optimal projection basis

Basis definition $$\min_{\Phi} \sum_i \left\| \mathbf{x}\left(t = t_i\right) - \tilde{\mathbf{x}}\left(t = t_i\right) \right\|_{\mathcal{K}}^2$$

subject to $$\left(\Phi, \Phi\right)_{\mathcal{K}} = \mathbb{I}_{k \times k}$$

with the $\mathcal{K}$-inner product $(u, v)_{\mathcal{K}} = \vec{u}^T \mathcal{K} \vec{v}$

Properties

- data-driven method
- minimization of the error between $\mathbf{x}$ and $\tilde{\mathbf{x}}$

The quality of the basis depends on the snaphots matrix

Simulation

↓

Pre-process

$$\mathbf{x}\left(t_i\right) = \mathbf{x}\left(t_i\right) - \bar{x} \quad i = 1,\ ..\ , T$$

↓

Eigenvalue problem

$$\frac{1}{T}\mathbf{X}^T\ \mathcal{K}\ \mathbf{X}\ \Psi = \lambda\ \Psi$$

↓

Orthonormalization

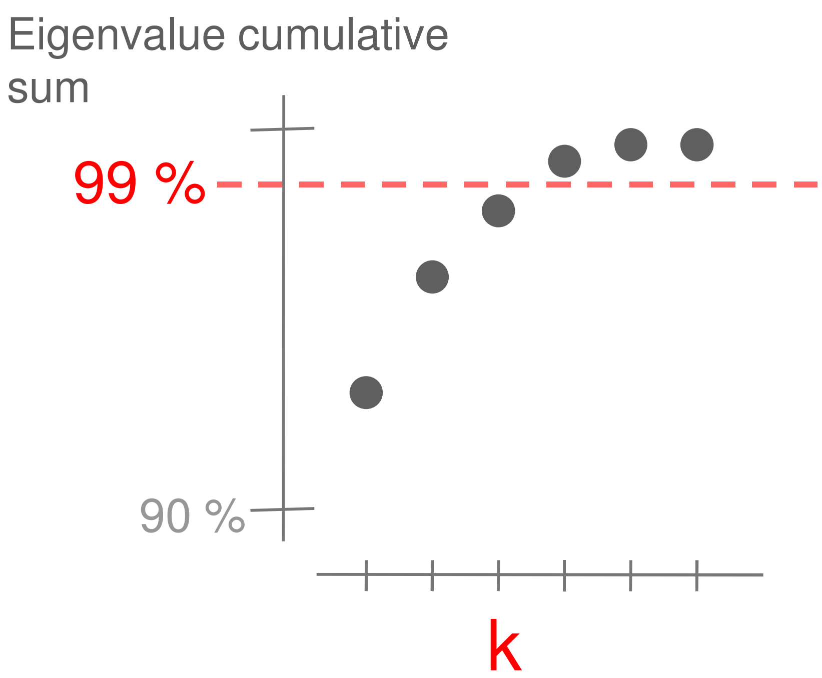
$$\Psi \rightarrow \Phi\ :\ \Phi^T\Phi = \mathbb{I},\ span(\Phi) = span(\Psi)$$

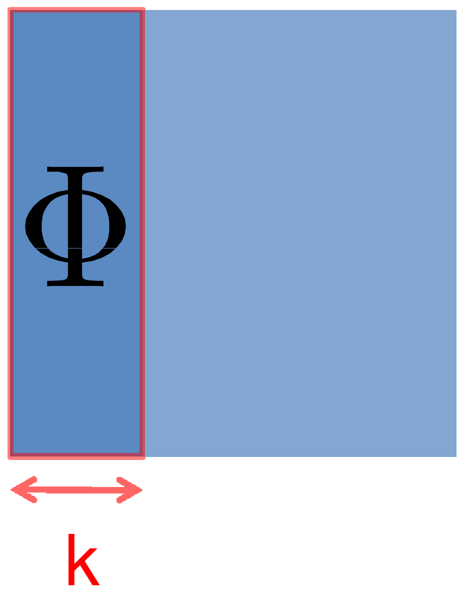
↓

$$\tilde{\mathbf{x}} = \bar{\mathbf{x}} + \Phi\mathbf{x}_r$$

The Proper Orthogonal values give information about the size of the basis

Truncation error connected with the eigenvalues and the amount of « energy » included in the basis span

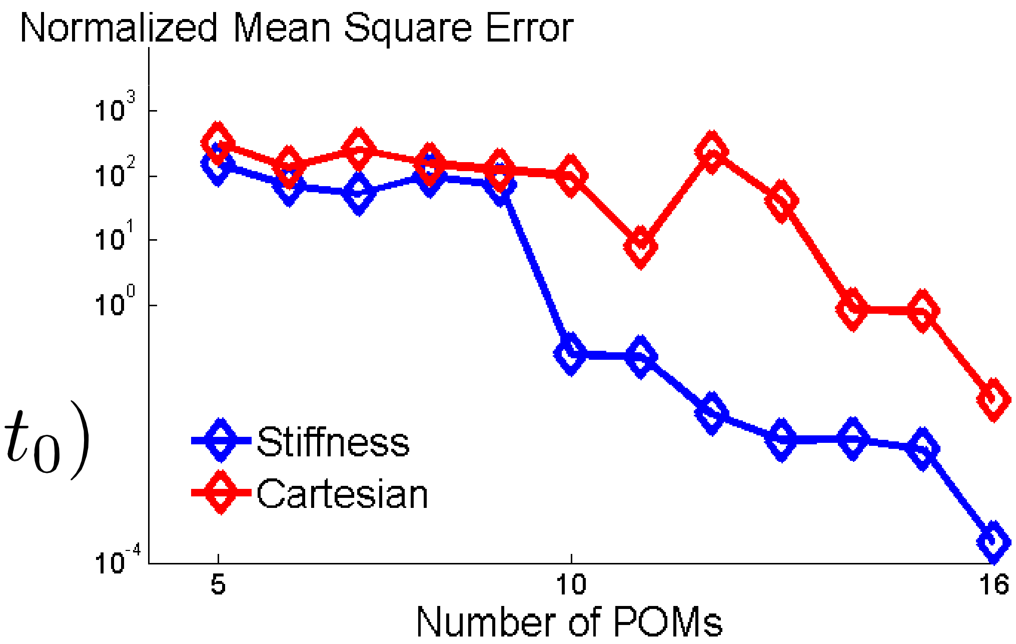




The weighted POD improves the reduced model

$$\text{Cartesian} : \mathcal{K} = \mathbb{I}$$

$$\text{Stiffness} : \quad \mathcal{K} = \mathbf{K}_{tg}\,(t = t_0)$$

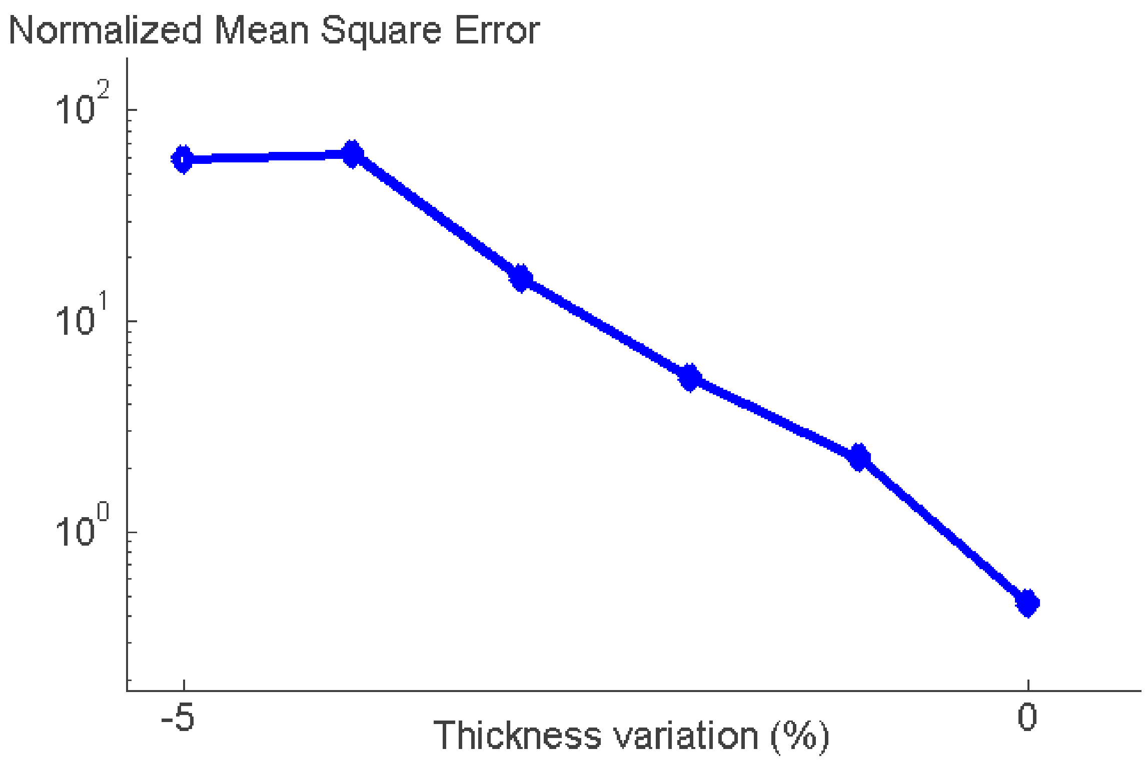


« Energy » related to the linear potential energy $\mathcal{V} = \frac{1}{2}\mathbf{x}^T\mathbf{K}_{tg}\mathbf{x}$

The POD-based reduced model is not robust w.r.t parameter changes

Parameter : geometry, excitation, initial conditions,…

Range of validity of POD-based reduced model is small

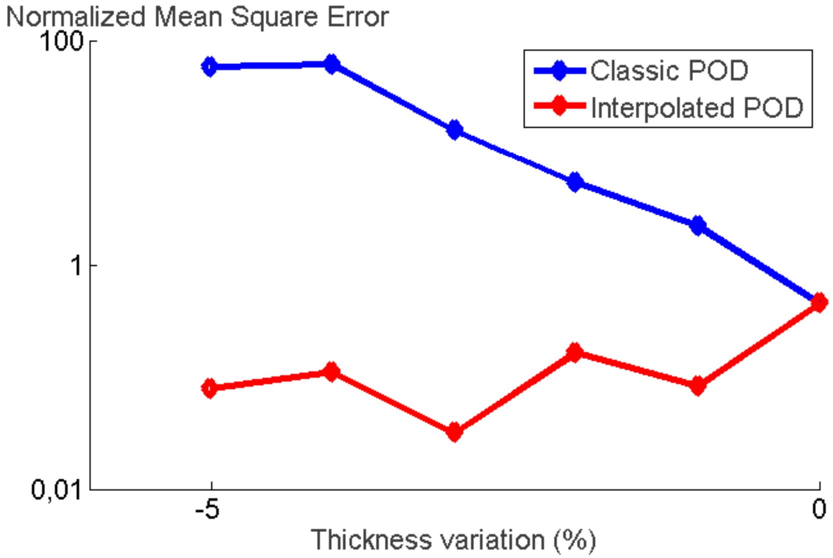


The robustness can be increased by interpolation or extrapolation/expansion

[Amsallem & Farhat 2008 , Hay et al 2009]

Interpolation : differential geometry principle, interpolation in the tangent space of the Grassmann manifold

Extrapolation/expansion : use of the POD basis sensitivities



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Conclusions and further work

Reduction of non-linear problems by Galerkin projection

Non-linear beams benchmark

Projection basis computed by proper orthogonal decomposition

Influence of the metric : cartesian/stiffness

Increase of the robustness by interpolation/expansion

Efficient discretization of the parameter space

Reduction of the complexity of non-linear terms

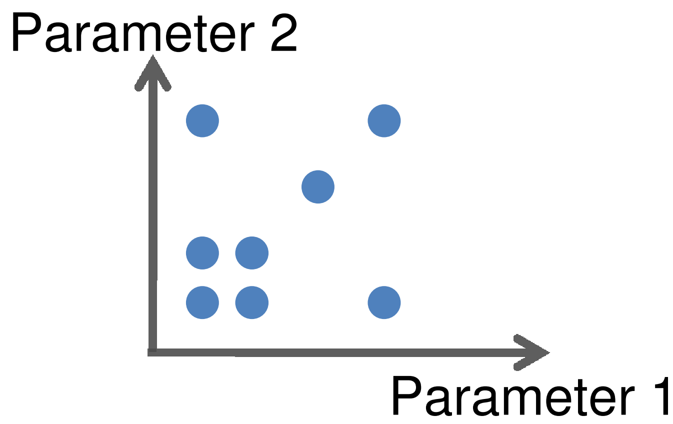
Further research is still needed

Reduction in the complexity of the non-linear term :

Empirical Interpolation method

Discretization of the parameter space :

Greedy method



Error estimator without the full response:

Dual-Weigthed-Residual