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Calculation Method for Design of Reinforced Concrete Columns under Fire Conditions

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The determination of the fire resistance of concrete columns is essentially based on tabulated data containing the dimensions of the cross section and values of the concrete cover. However, more scientific approaches such as analytical formulations should be proposed to consulting engineers for a quick and efficient design. A large number of experimental results have been examined; they have been performed at the Universities of Ghent and Liège in Belgium, at the Technical University of Braunschweig, and at the Fire Research Station in Ottawa, Canada. A computer code SAFIR, developed at the University of Liège for the simulation of the structural behavior under fire conditions, has been used for the analysis of the experimental results and for the progressive development of the formulation. The design formula has been obtained in three steps. The first step consists of determining the plastic crushing load of the column at elevated temperature on the basis of numerical simulations. The second step is the determination of the buckling coefficient for centrically loaded columns. The third step is the development of a nonlinear amplification term for eccentrical loads. The formula has been calibrated to take into consideration the particular effects of the concrete cover and the additional amplification appearing for the high values of the slenderness ratio.

Keywords: columns; fire resistance; reinforced concrete.

INTRODUCTION

It is generally acknowledged that concrete elements behave satisfactorily under fire conditions. Concrete is a semi-insulating material; therefore, the concrete cover surrounding the longitudinal reinforcement provides some degree of thermal insulation. The temperature increase in the main reinforcement is rather slow, and consequently, the yield strength of the steel bar decreases slowly.

One must be aware, however, that high values of the fire resistance cannot always be reached with concrete structures. Concrete elements have to be designed appropriately to ensure an adequate fire endurance. To decrease the risk of spalling, a suitable concrete mix design has to be chosen and appropriate detailing has to be ensured.

The design of concrete constructions that can be submitted to fire conditions has been one of the concerns of international organizations such as FIP and CEB.[1-3] More recently the European Commission for Standardization (CEN) has published the document ENV 1992-1-2, Structural Fire Design.[4] In North America, ACI[5] has also published recommendations on this matter.

Simplified calculation methods have been proposed, but their applicability has mainly been checked for concrete elements in bending such as beams and slabs. It has not yet been proven that the extension of this type of method to columns is in sufficient agreement with experimental results.

The developments and recommendations published by FIP and CEB[1-3] are mainly based on research studies made in Europe. In North America, significant contributions must also be pointed out, such as Harmathy's[6] on development of fires and material properties, Abrams'[7] on material properties and experimental studies on concrete elements, Gustaferro's[8] on design of concrete and particularly prestressed concrete structures, and Bresler's[9] work with the Fire Research Group of the University of California at Berkeley on numerical simulation of the structural behavior of concrete frames.

In the case of simply supported beams and slabs, the failure mechanism is well-known; due to temperature increases in the tensile reinforcement, yielding strains lead to collapse of the element. In columns the failure mechanism is difficult to predict: not only the crushing of concrete may occur but also buckling of the column. The limit between these two failure modes cannot always be defined precisely. Furthermore, there is a wide scatter of experimental results due to observations of spalling concrete, which influences significantly the fire resistance of the column.

Experimental research studies on concrete columns submitted to fire have already been performed. Among these studies, test results from the Technical University of Braunschweig[10] and from the Fire Research Station in Ottawa[11] have been examined. To gain additional information, an extensive experimental research project has been realized in Belgium, tests on long columns have been performed at the University of Ghent, and on short columns at the University of Liège in view of studying the effect of buckling. The influence of the relevant parameters has been analyzed.[12]

Since 1975 the Department of Bridges and Structural Engineering of the University of Liège has been involved in the elaboration of numerical models for the determination of the fire resistance of structural elements.[13] A computer code called SAFIR has been developed by one of the authors for the simulation of the behavior of steel, concrete, and composite steel-concrete structures under fire conditions.[14] This code has been used for the analysis of the experimental results and for the progressive development of the formulation. The failure mechanisms have been analyzed, and temperature distributions on the cross sections have been determined, which is essential for the elaboration of a simplified procedure.

However, consulting engineers do not always have this kind of tool at their disposal and to proceed to a quick and efficient

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design, it is important to elaborate simplified methods based on an analytical formulation.

Very few contributions have been published on this matter. There is agreement with adopting a reduction of the efficient cross section at elevated temperatures, but there are differences in the way of doing it. In Anderberg's study[15] some parameters still have to be calibrated. Hertz's contribution[16] does not indicate the method to be applied to the reduced column; he also makes reference to methods that are no longer used. In both cases, it is necessary to have at one's disposal diagrams giving the temperature distribution on the cross sections at various times during the fire exposure.

This lack of analytical formulation is particular to concrete structures. Methods have been developed for steel columns and adopted in Eurocode 3, which is devoted to steel constructions.[17] They constitute an extension of the methods recommended at ordinary temperature.[18] They define a relation between the critical stress and the slenderness ratio, taking into account the temperature variation of the mechanical characteristics.

Therefore, the approach followed in the development of the method should take into consideration the following aspects:

1. The formulation should be such that it is not necessary to use diagrams giving the temperature on the cross sections of concrete elements.

2. The formulation should contain the parameters that have a significant influence, and this influence should be calibrated.

RESEARCH SIGNIFICANCE

Whereas simplified calculation methods are widely used in the case of steel, it is not current practice for concrete elements where the determination of fire resistance is essentially based on tabulated data containing the dimensions of the cross section and values of the concrete cover.

However, more scientific approaches such as analytical formulations should be proposed to consulting engineers for a quick and efficient design.

In view of developing such a formula, a large number of experimental results performed in four fire research stations have been examined. On the other hand, a computer code developed at the University of Liège has been used for the analysis of the test results and calibration.

The design formula is related to the plastic crushing load of the column at elevated temperatures, which is reduced by the buckling coefficient and a nonlinear amplification term taking into account the effect of eccentrical loads.

The formula has been calibrated and is easy to use in design for the determination of the ultimate load capacity in case of a prescribed fire resistance, as well as for the determinatior the fire resistance of a column in a building.

ANALYSIS OF EXPERIMENTAL RESEARCH STUDIES

It is important to examine experimental results from vari fire research stations as some parameters vary from one lal ratory to another including: end conditions, length of the c umn, and eccentricity of the load. While the temperatı conditions are very similar, since the ISO 834 standard temp ature-time curve is applied in Europe and the ULC - S 101 Canada, the thermal transfer conditions in the furnaces may different.

Three series of tests will be examined:

1. 39 tests realized at the Technical University of Brauns weig in Germany;[10]

2. 23 tests realized at the Fire Research Station in Ottawa Canada;[11]

3. 21 tests realized in Belgium;[12] 16 at the University of Gh on long columns and five at the University of Liège on sh columns.

The series of 39 tests performed at the Technical Univers of Braunschweig are presented in Appendix 1.* Most of t columns are hinged at both ends; some are hinged at one e and clamped at the other end. The load has been maintain constant up to the failure of the column.

The series of 23 tests performed at the Fire Research Stati in Ottawa are presented in Appendix 2.* Most of the colum tested are clamped at both ends. Some tests have been realiz with other end conditions (clamped - hinged and hingec hinged). All tests, except one, have been realized with the sa concrete cover, and all except two with no eccentricity of t load. The load has remained constant up to the failure of t column. Compared with the Braunschweig series, the fire ı sistances obtained are very high, which shows the influence the tests' conditions (for most of the tests, the column w clamped at both ends with no eccentricity of the load).

The series of 21 tests performed in Belgium are presented Appendix 3;* 16 tests have been made at the University of Ghe on slender columns ($l = 3.90$ m) and five at the University Liège on short columns ($l = 2.10$ m). All the columns tested Belgium have free rotation at both ends. In these series of tes the load has been applied in a different way. The applied lo N_{appl} has been maintained constant up to the expected fire r sistance time (1 or 2 hr). If at that moment failure has not o curred, the load is increased within a few minutes up to tl value N_{max} corresponding to the failure of the column.

This study has been realized mainly to examine the influen of the relevant parameters. The most significant results[12] a the following:

1. in Liège and in Ghent, tests realized using longitudin bars with a large diameter (Ø25 mm) have led to low values fire resistance;

2. the influence of the concrete cover seems less determina than what results from CEB/FIP Recommendations[1-3] or E rocode 2;[4]

3. the end conditions influence significantly the fire resi tance (compare the results of the tests of Ottawa with those Braunschweig, Liège, and Ghent).

*The Appendixes are available in xerographic or similar form from ACI headqua ters, where they will be kept permanently on file, at a charge equal to the cost of re duction plus handling at the time of request.

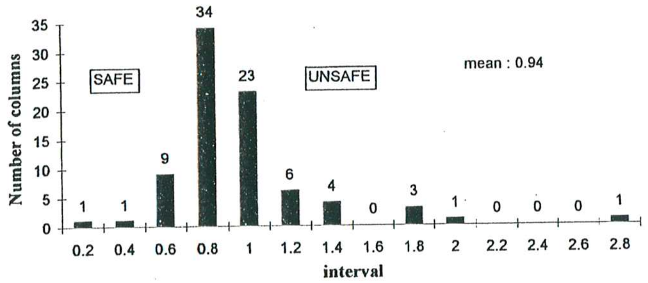

Fig. 1—Distribution of ratio $R_{f\,SAFIR}/R_{f\,test}$ *for all tests.*

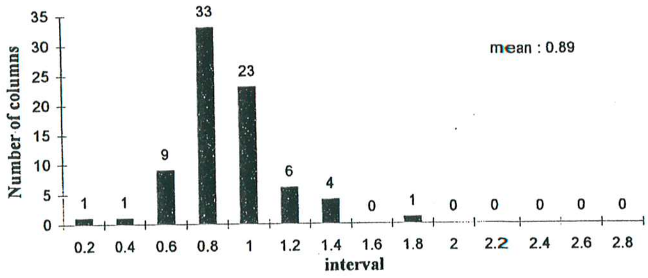

Fig. 2— Distribution of ratio $R_{f\,SAFIR}/R_{f\,test}$ *for all tests except those with reinforcement of* ∅*25 mm.*

NUMERICAL SIMULATIONS PERFORMED AT UNIVERSITY OF LIÈGE

A computer code called SAFIR has been developed by one of the authors at the University of Liège for the simulation of the behavior of steel, concrete, and composite steel-concrete structures under fire conditions.[14] This code accounting for large displacements, nonlinear material laws,and nonuniform temperature distribution (but not spalling) can be considered as a general calculation model in the sense of Eurocode 2, Part 1-2. SAFIR contains both a thermal and a mechanical part. The nonuniform temperature on the cross section is calculated by means of the finite element method (FEM). This method is also used for the evaluation of the structural behavior. In case of plane structures, the frame is subdivided in beam elements with 7 degrees of freedom that can take geometrical nonlinearities into account. The variation of the mechanical properties on the cross section is evaluated by means of a subdivision in meshes corresponding to the subdivision in finite elements in the thermal part. In this way the temperature and the mechanical characteristics are evaluated at the same points.

SAFIR has been used to analyze the results of the 83 tests and for the progressive development of the formulation. To assess the results of the computer code when applied to the particular case of concrete columns, a comparison has been made between the numerical results given by SAFIR and the experimental results.

For this comparison, the ratios $R_{f\ SAFIR}/R_{f\ test}$ have been evaluated, $R_{f SAFIR}$ being the numerical fire resistance given by SAFIR and $R_{f test}$ the fire resistance given by the test. A graphical representation of the distribution of the ratios is illustrated in Fig. 1.

The distribution of Fig. 1 appears quite satisfactory since the mean is equal to 0.94, and 57 tests (out of 83, i.e., 69 percent) are situated between 0.6 and 1. However, there are a few very high values observed.

Most of these high values correspond to columns containing longitudinal bars with ∅ = 25mm and tested at the University of Ghent. As already mentioned, premature failure has been observed for these columns due to spalling phenomenons. If these tests are disregarded, the distribution concerning the remaining 78 tests presented in Fig. 2 is more favorable: the mean is equal to 0.89 and 56 tests (i.e., 72 percent) are situated between 0.6 and 1. All the tests except three are situated between 0.4 and 1.4.

It can be concluded that by using the computer code SAFIR a good agreement is obtained between numerical and experimental results. It seems, therefore, quite permissible to use SAFIR as a support for the simulation of additional fire resistance tests and for the development of the formulation.

METHODS AND FORMULATIONS EXISTING AT ORDINARY AND ELEVATED TEMPERATURES

Since the early applications of reinforced concrete, several types of methods have been developed for the determination of the ultimate load of concrete columns submitted to short duration loading at ordinary temperature.

In the beginning of the 20th century and even during the two world wars, elastic methods were used. Among them Rankine's method[19] was the most popular and appeared in many codes devoted to design of reinforced concrete structures. For eccentrical loads, amplification terms calculated on an elastic basis were used. Books on reinforced concrete[20] published after 1960 still refer to Rankine's method.

When concrete columns are submitted to compression and bending, not only geometrical nonlinearities, but also material nonlinearities, such as nonlinear stress-strain curve and cracking, are observed. Therefore, many people were convinced that elastic methods could not describe satisfactorily the buckling of concrete columns. Under the influence of CEB, attempts were made to develop more appropriate methods. Contributions published in 1964 in CEB bulletins[21] show several of these attempts presented by research workers in Europe and North America. Some time later most of the developments of a new procedure called the model column were published by CEB.[22] This method was finalized later on and published in 1977 in the *Manual on Buckling and Instability*.[23]

Though the method of the model column can solve the problem correctly, the amount of calculation needed requires the use of a computer, or of tables or graphs obtained by computer calculations. Simplifications such as assuming that the shape of the deflection curve is sinusoidal have been proposed, but this does not lead to an analytical formulation at the end.

In some formulations the ultimate load can still be calculated in function of the slenderness ratio, but the relations are no longer derived from elastic methods; they have been calibrated on experimental results. This is the case in the French BAEL Recommendations.[24] However, this method is only valid for centrically loaded columns; for eccentrical loads it is recommended to adopt the method of the model column.

As already mentioned, very few contributions have been published in cases of concrete structures submitted to fire, and their applicability has mainly been checked for elements in bending. The formulation developed at the University of Liège and presented hereafter is a new type of approach that avoids the use of diagrams by giving the temperature distribution on the cross section.

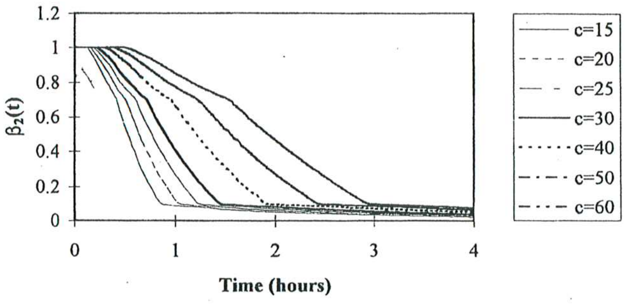

Fig. 3—Variation of factor $\beta_1(t)$ *in function of time for cross sections 300 x 300 mm and 500 x 500 mm.*

METHOD FOR DETERMINATION OF ULTIMATE LOAD AND OF FIRE RESISTANCE

Plastic crushing load at elevated temperatures

For very short columns, no buckling exists and for a centrical load, the ultimate load is the plastic crushing load. When the column is submitted to fire conditions, this ultimate load varies continuously with time. The crushing load at a definite moment is a function of the temperature in steel and concrete. The temperature in the reinforcing bar can be well-defined, but this is not the case in concrete where the temperature varies from one point to another, and therefore, not with the concrete compressive strength, which is temperature dependent. The plastic crushing load of the column at a particular time t corresponding to a specific thermal program (in this case the ISO 834 standard temperature-time curve) is given by

$$N_p(t) = N_{pc}(t)+N_{ps}(t)=$$

$$\int_{A_c} f_c[\theta(t)] \cdot dA_c + \sum_{j=1}^{nb} f_y[\theta_j(t)]A_{sj} \qquad (1)$$

with

$N_p(t)$ = plastic crushing load of the column
$N_{pc}(t)$ = crushing load of the concrete section
$N_{ps}(t)$ = yeild strength of the reinforcing bars
$f_c[\theta(t)]$ = concrete compressive strength corresponding to the temperature reached at time t
nb = number of longitudinal bars
$f_y[\theta_j(t)]$ = yeild strength in reinforcing bar j corresponding to the temperature reached at time t.
A_c = concrete cross section
A_{sj} = cross section of bar j
θ = temperature

If the cross section is discretized in meshes like in the SAFIR computer code, $N_p(t)$ can be written as follows

$$N_p(t) = \sum_{i=1}^{nm} A_{ci} f_c[\theta_i(t)] + \sum_{j=1}^{nb} A_{sj} f_y[\theta_j(t)] \qquad (2)$$

where

nm = number of meshes on the concrete cross section
A_{ci} = area of concrete mesh i

Eq.(2) will be simply written

$$N_p(t) = \sum_i A_{ci} f_{ci}(t) + \sum_j A_{sj} f_{yj}(t) \qquad (3)$$

The reduction of the concrete compressive strength can be expressed as follows

$$f_c(\theta) = k_c(\theta) \cdot f_c \qquad (4)$$

f_c = concrete compressive strength at ordinary temperature
$k_c(\theta)$ = here defined according to the simplified relation proposed in Chapter 3 of Eurocode 2, Part 1-2.[4]

Let us express $N_{pc}(t)$ in the following form

$$N_{pc}(t) = \beta_1(t) A_c \cdot f_c$$

in which

$$\beta_1(t) = \frac{\sum_i A_{ci}\, f_{ci}(t)}{A_c f_c} \qquad (5)$$

$\beta_1(t)$ will obviously depend on the type and size of the cross section. In this work, only rectangular sections have been considered but the formulation can be extended easily to circular columns. In practice, no section is smaller than 150 x 150 mm, and very few are larger than 500 x 500-mm, except for high-rise buildings. On the other hand, if b and h are the dimensions of the cross section, it is very unusual to find in practice $h/b > 2$.

Numerical simulations have been performed using the computer code SAFIR and taking into account the preceding remarks. The variation of $\beta_1(t)$ is presented in Fig. 3 for columns with cross sections 300 x 300 mm and 500 x 500 mm, submitted to the ISO 834 standard temperature-time curve.

A satisfactory representation of $\beta_1(t)$ is given by the following relation

$$\beta_1(t) = \frac{1}{\sqrt{1 + (a_1 t)^{a_2}}} \qquad (6)$$

with t in hours.

Variables a_1 and a_2 have been determined to fit the preceding curves, and they are thus functions of the column section. Simulations have shown that, provided the dimensions of the cross sections are limited as defined previously, it is sufficient to consider the area A_c of the cross section, and that it is not necessary to differentiate the dimensions. This can be explained as follows: considering the massivity factor that is defined (as with steel elements) by the ratio between the perimeter and the cross-section, the variation is only 5 percent for the same cross section when going from $b = h$ to $b = 2h$. After calibration the following values are proposed

$$a_1 = 0.3A_c^{-0.5}$$
$$a_2 = A_c^{-0.25} \qquad (7)$$

with A_c in m^2. Fig. 3 shows the adequacy of the curve fitting.

Considering now the factor $N_{ps}(t)$, it can be determined in the same way as $N_{pc}(t)$ by numerical simulation. The expression for $N_{ps}(t)$ is

$$N_{ps}(t) = \sum_j A_{sj}\, f_{yj}(t) \qquad (8)$$

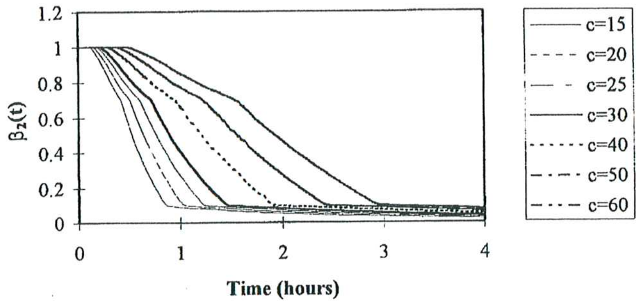

Fig. 4—Variation of factor β_2(t) in function of time for different values of concrete cover.

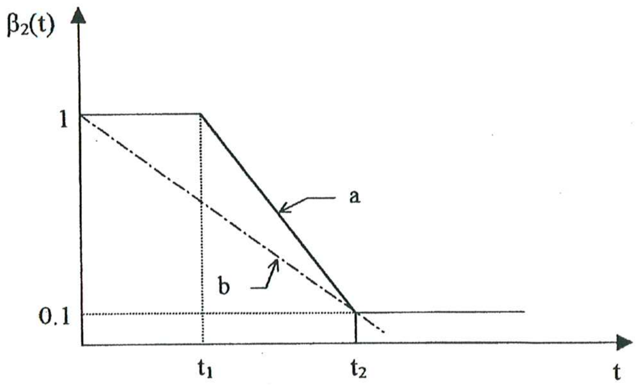

Fig. 5—Schematic representation of β_2(t).

The reduction of steel yield strength can be expressed in the following way

$$f_y(\theta) = k_y(\theta) \cdot f_y \quad (9)$$

where f_y = steel yield strength at ordinary temperature.

$k_y(\theta)$ is defined according to the simplified relation proposed in Chapter 3 of Eurocode 2, Part 1-2. In this section of EC2, two equations are presented depending on the strains existing at the ultimate limit state. For reinforcement in compression, the strain will most often remain moderate, while for reinforcement in tension, strains up to 2 percent can be reached. In this case, the relation to be considered is the one corresponding to steel in compression.

$N_{ps}(t)$ will be expressed in the same way as $N_{pc}(t)$

$$N_{ps}(t) = \beta_2(t) \cdot A_s \cdot f_y \quad (10)$$

in which

$$\beta_2(t) = \frac{\sum_j A_{sj} f_{yj}(t)}{A_s \cdot f_y}$$

It is assumed that all reinforcements are concentrated in the corners, which is on the safe side due to more extreme thermal exposure.

$\beta_2(t)$ depends on the concrete cover and on the cross section. Numerical simulations, not reproduced herein, show that the influence of the cross section is small compared to that of the concrete cover; it has therefore been neglected. The variation of $\beta_2(t)$ is presented in Fig. 4 for columns 300 x 300 mm and concrete cover varying between 15 and 60 mm.

A schematic representation of $\beta_2(t)$ is given by Curve *a* in Fig. 5. This curve is characterized by points P_1 (t_1, 1) and P_2 (t_2, 0.1). As the contribution of $N_{ps}(t)$ to $N_p(t)$ is noticeably less important than that of $N_{pc}(t)$, one can use a more simplified representation for $\beta_2(t)$ (Curve *b*), which is on the safe side. In this way, $\beta_2(t)$ is only a function of t_2, which itself is a function of the concrete cover *c*. One gets

$$\beta_2(t) = 1 - \frac{0.9t}{t_2} > 0 \quad (11)$$

$$t_2 = 0.046c + 0.11$$

with t_2 and t in hours and c in mm.

The formula has to be calibrated, mainly to take account of the effect of spalling. With almost all concrete columns $R_f >$ 30 minutes can be obtained, but during this first half hour, most of them will be affected by concrete spalling, leading to a decrease of the effective cross section. Comparisons with experimental results have shown that a reduction factor of 0.85 should be applied. Spalling appears suddenly between 0 and 30 min, but it is impossible to predict at which moment. Therefore, it has been assumed that the effective cross section decreases progressively during the first half hour. The relation giving the failure load $N_{u1}(t)$ for centrically loaded short columns can thus be written in the following way

$$N_{u1}(t) = \gamma(t) \cdot N_p(t)$$

$$\gamma(t) = 1 - 0.3t \qquad \text{for } t < 0.5 \text{ hr}; \quad (12)$$

$$\gamma(t) = 0.85 \qquad \text{for } t > 0.5 \text{ hr}$$

The spalling effects on the increase in steel temperature will be taken into account by further calibration.

Ultimate load for centrically loaded slender columns

When the length of the column increases, buckling phenomenons appear. Failure is no longer due to crushing of concrete, and in the particular case of centrically loaded columns, the ultimate load is the buckling load.

As already mentioned, the method of the model column is currently used for eccentrically loaded columns. For centrically loaded columns, some approaches still use the buckling reduction factor depending on the slenderness ratio. This is the case in the French BAEL Recommendations,[24] where the ultimate load is given by the following relation

$$N_u = \chi(\lambda) \cdot N_p \quad (13)$$

where

χ = buckling reduction factor or coefficient
λ = slenderness ratio

At elevated temperatures, it is assumed that Eq. (13) is still valid, but the buckling coefficient should be different from the

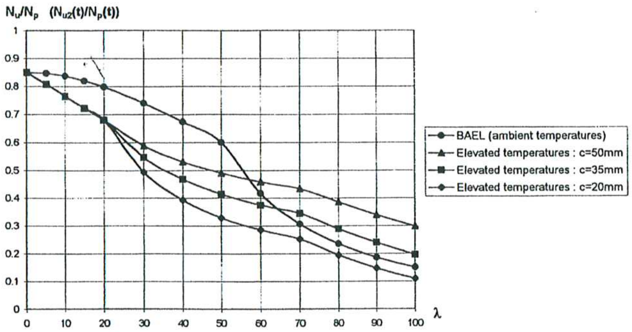

Fig. 6—Variation of N_u/N_p *for different values of concrete cover.*

one defined in the French Recommendations[24] for ambient temperatures. In fact, the influence of buckling should be more important, because of the progressive decrease of the effective stiffness. At elevated temperatures, Eq. (13) becomes

$$N_{u2}(t) = \chi(\lambda) \cdot N_{u1}(t) \tag{14}$$

where $N_{u2}(t)$= failure load for centrically loaded slender columns.

By performing numerical simulations and comparing with experimental results, the following relations have been derived

$$\chi(\lambda) = 1 - \frac{\lambda}{100} \quad \text{for } \lambda \le 20$$

$$\chi(\lambda) = 0.80\left(\frac{20}{\lambda}\right)^{0.7\left(\frac{225-c}{200}\right)^5} \quad \text{for } 20 < \lambda \le 70 \tag{15}$$

$$\chi(\lambda) = 0.80\left(\frac{20}{\lambda}\right)^{0.7\left(\frac{\lambda}{70}\right)\left(\frac{225-c}{200}\right)^5} \quad \text{for } \lambda > 70$$

with c in mm.

Fig. 6 presents a comparison between these curves and those proposed in the BAEL Recommendations. At elevated temperatures, $R_f > 30$ min will always be obtained and the coefficient γ defined in Eq. (12) will be equal to 0.85. Therefore, it is better to include the coefficient 0.85 and make the comparison for N_u/N_p.

It can be seen that the influence of the concrete cover is taken into account in Eq. (15). This effect can be explained physically. Due to various imperfections, the column will deflect laterally. Cracking appears and develops more, particularly in the central part of the element. In the cracked zone the tensile reinforcement plays an essential role that explains the importance at elevated temperatures of the concrete cover controlling the heating of the reinforcement.

Ultimate load for eccentrically loaded columns

This problem is appreciably more complicated, as almost no analytical formulation exists presently at ordinary temperature.

The approach followed in this section is based on developments made for steel columns at ordinary temperature for which interaction formulas are available. Adaptations and calibrations will then be realized to come to a good agreement with experimental results on concrete columns at elevated temperatures.

In Eurocode 3,[18] members subjected to combined bending and axial compression must satisfy the following relation, in which only one plane of bending is considered and lateral torsional buckling is not relevant

$$\frac{N}{\chi \cdot N_p} + \frac{kM}{M_P} < 1 \tag{16}$$

N_p = plastic axial load
M_p = plastic moment
$\chi N_p = N_u$ = ultimate load for the same column centrically loaded

At the ultimate limit state this relation becomes

$$\frac{N_u^*}{N_u} + \frac{kM_u^*}{M_p} = 1 \tag{17}$$

where N_u^*, M_u^* = load and bending moment at the ultimate limit state for the eccentrically loaded column.

k is the nonlinear amplification term for which many relations have been proposed, such as the one presented in Eurocode 3. In this study, the authors prefer to come back to one of the first formulations proposed by Campus and Massonnet,[25] and transformed by Austin[26] a few years later.

The formula was first derived for design with allowable stresses. Taking into account the ultimate limit state and the extension of yielding at failure, the formula can be written as

$$\frac{N_u^*}{N_u} + \frac{K_M}{1-(N_u^* \bullet N_E)(N_u \bullet N_p)} \frac{M_{u,\,equ}^*}{M_p} = 1 \tag{18}$$

with

N_E = Euler critical load = $\dfrac{\pi^2 EI}{l_{fl}^2}$

$M_{u,\,equ}^*$ = equivalent moment given by

$M_{u,\,equ}^*$ = $(0.6 + 0.4\, M_1/M_2)M_{max}$

M_1 and M_2 are the moments at the ends of the column and M_{max} is the maximum of M_1 and M_2.

K_M = correction coefficient taking into account the extension of yielding at failure.

Comparing Eq. (17) and (18) it can be noticed that

$$kM_u^* = \frac{K_M \cdot M_{u,\,equ}^*}{1-(N_u^*/N_E)\cdot(N_u/N_p)} \tag{19}$$

By defining $\eta(\lambda)$ the buckling coefficient for eccentrically loaded columns, the following relation can be written

$$N_u^* = \eta(\lambda) \cdot N_p \tag{20}$$

Noticing that

$$N_p/N_E = \chi(\lambda) \cdot \bar{\lambda}^2$$

with

$$\bar{\lambda} = \lambda/\lambda_E$$

λ_E : Eulerian slenderness ratio $= \pi(E/f_y)^{1/2}$

$$M^*_{u,equ} = N^*_u \cdot e_{equ} = N^*_u \cdot e$$

K_M close to 1, will be assumed here = 1.

Eq. (18) can thus be written in the following way

$$\frac{\eta(\lambda)}{\chi(\lambda)}\left[1 + \frac{\chi(\lambda) \cdot N_p \cdot e}{1 - \eta(\lambda) \cdot \chi(\lambda) \cdot \bar{\lambda}^2 M_p}\right] = 1 \qquad (21)$$

Therefore

$$\eta(\lambda) = \frac{\chi(\lambda)}{1 + \dfrac{eN_p}{\dfrac{1}{\chi(\lambda)} - \eta(\lambda)\bar{\lambda}^2 M_p}} \qquad (22)$$

in which the nonlinear amplification term appears clearly.

This formula is not applicable directly to concrete structures at elevated temperatures. In this case, the formula will be written as

$$N_{u3}(t) = \eta(\lambda) \cdot N_p(t)$$

To derive $\eta(\lambda)$ for concrete structures at elevated temperatures, the following transformations have been made.

- By examining the plastic moment M_p and the plastic axial load N_p of rectangular cross sections, it has been assumed that

$$M_p/N_p = h/k_1 \qquad (23)$$

with h = smaller dimension of the cross section; $k_1 = 10$.
On the other hand

$$\bar{\lambda} = (\lambda/\lambda_E) \quad \text{with } \lambda_E = \pi(E/f_c)^{1/2}$$

close to 100 for an ordinary concrete. Therefore, it has been assumed that

$$\bar{\lambda}^2 = k_2\lambda^2 \qquad (24)$$

with $k_2 = 10^{-4}$.

- To avoid the complication arising from the nonlinear character of the amplification term, $\eta(\lambda)$ has been calculated for all the columns tested and its mean value (0.3) determined. The term $\eta(\lambda)$ on the right hand side has then been replaced by a constant value $k_3 = 0.3$.

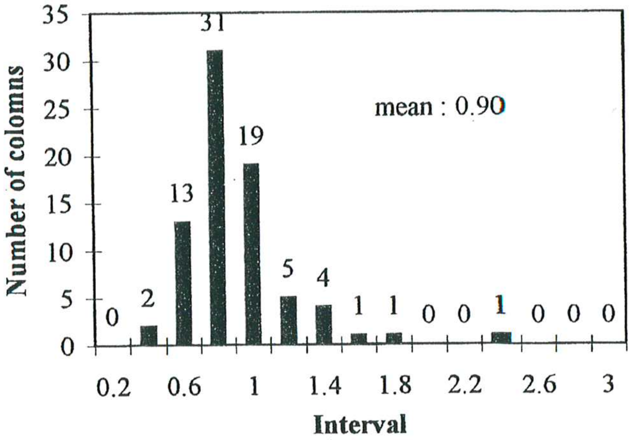

Fig. 7—Distribution of N_{u3th}/N_{u3test}.

With the three preceding transformations, Eq. (22) can thus be written

$$\eta(\lambda) = \frac{\chi(\lambda)}{1 + \dfrac{10e/h}{\dfrac{1}{\chi(\lambda)} - 3.10^{-5}\lambda^2}} \qquad (25)$$

It is now possible to compare the theoretical results given by the preceding relation with the experimental results. This has been done in the same way as previously and presented in Fig. 7 for all columns except those tested in Belgium with reinforcing bars Ø = 25 mm and Column III 6 tested in Ottawa for which the condition $h/b > ½$ is not verified. The distribution of the ratio N_{u3th}/N_{u3test} appears quite satisfactory since the mean is equal to 0.90 and 72 tests (out of 77, i.e., 93.5 percent) are situated between 0.4 and 1.4.

To show the adequacy of Eq. (22) with respect to parameters c and λ, diagrams giving the ratio N_{u3th}/N_{u3test} in function of c and λ for all the columns are presented in Fig. 8 and 9. A linear regression line obtained by the least square method is plotted. Ideally this line should be horizontal, and for safety purposes with values somewhat smaller than 1. Fig. 8 and 9 show that the parameters c and λ are correctly taken into account.

HOW TO USE FORMULATION

The basic formula is

$$N_u(t) = N_{u3}(t) = \gamma(t) \cdot \eta(\lambda) \cdot N_p(t) \qquad (26)$$

$$\gamma(t) = 1 - 0.3t \quad \text{for } t < 0.5\ hr$$
$$\gamma(t) = 0.85 \quad \text{for } t > 0.5\ hr \qquad (12)$$

$$\eta(\lambda) = \frac{\chi(\lambda)}{1 + \dfrac{10e/h}{\dfrac{1}{\chi(\lambda)} - 3.10^{-5}\lambda^2}} \qquad (25)$$

$$\chi(\lambda) = 1 - \frac{\lambda}{100} \quad \text{for } \lambda \leq 20$$

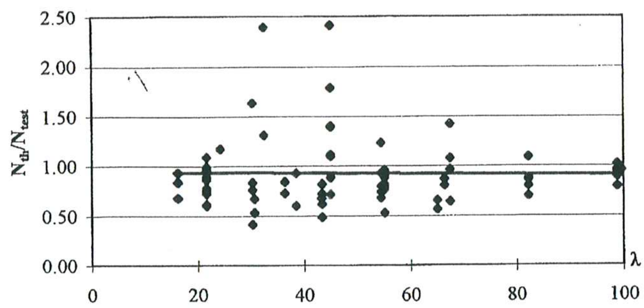

Fig. 8—Variation of N_{u3th}/N_{u3test} *in function of* λ.

$$\chi(\lambda) = 0.80\left(\frac{20}{\lambda}\right)^{0.7\left(\frac{225-c}{200}\right)^5} \quad \text{for } 20 < \lambda \leq 70 \qquad (15)$$

$$\chi(\lambda) = 0.80\left(\frac{20}{\lambda}\right)^{0.7\left(\frac{\lambda}{70}\right)\left(\frac{225-c}{200}\right)^5} \quad \text{for } \lambda > 70$$

$$N_p(t) = \beta_1(t) \cdot A_c \cdot f_c + \beta_2(t) \cdot A_s \cdot f_y \qquad (27)$$

$$\beta_1(t) = \frac{1}{\sqrt{1+(a_1 \cdot t)^{a_2}}} \qquad (6)$$

$$a_1 = 0.3 \cdot A_c^{-0.5} \qquad (7)$$

$$a_2 = A_c^{-0.25}$$

$$\beta_2(t) = 1 - \frac{0.9t}{0.046c + 0.111} > 0$$

where

$N_u(t)(=N_{u3}(t))$ = ultimate load for eccentrically loaded columns
e = load eccentricity (mm)
h = smaller dimension of the cross section (mm)
λ = slenderness ratio
c = concrete cover (mm)
A_c = concrete area (m^2)
A_s = steel area (m^2)
f_c = compressive strength of concrete (N/mm^2)
f_y = yielding strength of steel (N/mm^2)
t = time or fire resistance (hr)

In this research, the mean values f_{cm} and f_{ym} have been considered in the comparison with experimental results. For design purposes, the same procedure as the one recommended in the Eurocodes should be adopted. Therefore, the characteristic values f_{ck} and f_{yk} should be considered.

Examples

1. Calculation of the ultimate load N_u for a given fire resistance R_f The procedure is applied to Column 21 B with the following data (see Appendix 3*)

A_c = 200 x 300 mm	6 ϕ 12 mm	L = 3.90 m
c = 25 mm	e = 20 mm	hinged at both ends
f_c = 35.7 N/mm^2	f_y = 493 N/mm^2	R_f = 120 min

*The Appendixes are available in xerographic or similar form from ACI headquarters, where they will be kept permanently on file, at a charge equal to the cost of reproduction plus handling at the time of request.

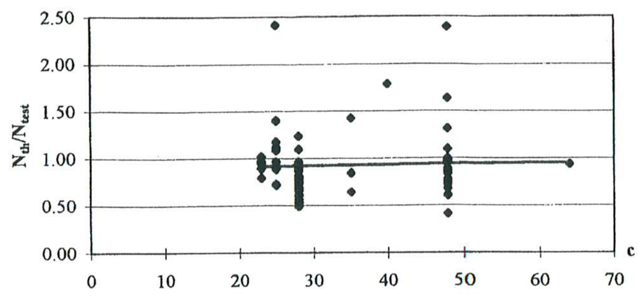

Fig. 9—Variation of N_{u3th}/N_{u3test} *in function of* c.

The ultimate load can be obtained as follows

$\lambda = L/i = 67.5$
$A_c = 0.06 m^2 \qquad A_s = 6.8.10^{-4} m^2$
$a_1 = 0.3 A_c^{-0.5} = 1.22$
$a_2 = A_c^{-0.25} = 2.02$

$$\beta_1 = \frac{1}{\sqrt{1+(a_1 t)^{a_2}}} = 0.38$$

$$\beta_2 = 1 - \frac{0.9t}{0.046c + 0.111} = -0.42$$

$$\text{with} \quad \beta_2 \geq 0 \Rightarrow \beta_2 = 0$$

$$20 < \lambda < 70 \Rightarrow \alpha = 0.8\left(\frac{20}{\lambda}\right)^{0.7\left(\frac{225-c}{\lambda}\right)^5} = 0.34$$

$$\eta = \frac{\alpha}{1 + \frac{10e}{h\left(\frac{1}{a} - 3.10^{-5}\lambda^2\right)}} = 0.25$$

$$\gamma(t) = 0.85 \quad \text{since } t > 0.5 \text{ hr}$$

$$N_u = \gamma \cdot \eta \cdot (\beta_1 A_c f_c + \beta_2 A_s f_y) = 173\text{kN}$$

The load applied during this test was 178 kN

$$\Rightarrow \frac{N_u}{N_{exp}} = \frac{173}{178} = 0.97$$

2. Calculation of the fire resistance R_f for a given applied load N. The procedure is applied to Column 25 with the following data (see Appendix 3*)

A_c = 200 x 200 mm	4 ϕ 20 mm	L = 5.76 m
c = 28 mm	e = 10 mm	hinged at both ends
f_c = 39 N/mm^2	f_y = 443 N/mm^2	N = 208 kN

The fire resistance can be obtained as follows

$\lambda = L/i = 99.8$
$A_c = 0.04 \text{ m}^2 A_s = 1.26 . 10^{-3} m^2$
$a_1 = 0.3 A_c^{-0.5} = 1.5$
$a_2 = A_c^{-0.25} = 2.24$

$$\lambda > 70 \Rightarrow \alpha = 0.8\left(\frac{20}{\lambda}\right)^{0.7\left(\frac{\lambda}{70}\right)\left(\frac{225-c}{200}\right)^5} = 0.18$$

$$\eta = \frac{\alpha}{1 + \frac{10e}{h\left(\frac{1}{\alpha} - 3.10^{-5}\lambda^2\right)}} = 0.16$$

For $R_f = 30$ min

$$\beta_1 = \frac{1}{\sqrt{1 + (a_1 t)^{a_2}}} = 0.81$$

$$\beta_2 = 1 - \frac{0.9t}{0.046c + 0.111} = 0.68$$

$$\gamma = 0.85 \quad \text{since } t = 0.5 \text{ hr}$$

$$N_u = \gamma \cdot \eta \cdot (\beta_1 A_c f_c + \beta_2 A_s f_y) = 224kN > 208kN$$

For $R_f = 60$ min it can be shown that $N_u = 142$ kN < 208 kN.

For $R_f = 45$ min it can be shown that $N_u = 178.5$ kN < 208 kN.

Using a quadratic interpolation, one can find that $N_u \cong 208$ kN for $R_f = 35$ min.

The experimental fire resistance is 40 min

$$\Rightarrow \frac{R_f}{R_{f,exp}} = \frac{35}{40} = 0.875$$

LIMITATIONS OF MODEL

As the method has been calibrated with respect to experimental results, it is preferable to design columns with parameters remaining in the frame of all the tests examined. Therefore, the limitations of the model, covering most of the columns encountered in practice, are the following.

- Only columns submitted to ISO 834, ASTM E 119, and ULC S 101 standard temperature-time curves may be considered;
- $\lambda \leq 100$
- $0.04 \text{ m}^2 \leq A_c \leq 0.2 \text{ m}^2$;
- $h/b \geq 1/2$ (with $h \leq b$);
- 20 mm $\leq c \leq$ 50 mm;
- $e \leq h/2$;
- Even if the column is axially loaded, an eccentricity $e = 10$ mm should be adopted.
- As premature failures have been observed at the University of Ghent, additional research should be performed to see if a reduction of the fire resistance should be adopted in the case of longitudinal reinforcing bars with large diameters.

CONCLUSIONS

The main conclusions that can be drawn from this research are the following.

- In this study, test results from four different fire research stations have been analyzed for which significant variations of the determinant parameters are observed. The experimental sample is therefore quite representative.
- Numerical simulations have been performed using the computer code SAFIR developed in the Laboratory of Bridges and Structural Engineering of the University of Liège.
- The results have been compared with experimental results. A good agreement is observed, and most of the results given by SAFIR are on the safe side.
- SAFIR therefore has been used for the simulation of additional fire resistance tests and for the progressive development of the formulation.
- The design method proposed is easy to use. It is based on a formulation initially developed for steel columns, but which has been adapted for concrete columns at elevated temperatures and calibrated with respect to tests.
- The formula has been obtained in three steps: determination of the plastic crushing load; of the buckling coefficient; and of the nonlinear amplification term.
- With this method the ultimate load at elevated temperatures can be determined directly.
- The fire resistance has to be determined by an iterative procedure.
- To determine the ultimate load or the fire resistance, it is not necessary to examine tables or graphs giving the temperature distribution on the cross section at well-defined moments.

CONVERSION FACTORS

1 mm = 0.0394 in.
1 kg = 0.4536 lb
1 MPa = 145 psi

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