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rac. **59** (1993), 23 - 40.

nical report no. 90-3. Center for the mathe-
re, USA.

27 - 32.

ary Elements XII, vol 2 (1990), pp 425 - 434.
nics.

numer. Meth. Engng. **32** (1991), 1371 - 1387.

Math J. **38** (1989), 527 - 556.

9), 2316 - 2319.

991), 127 - 145.

R. Soc. Lond. A. **445** (1994), 637 - 652.

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A GLOBAL APPROACH OF UNSTEADY SURFACE FLOWS COMPUTATIONS INCLUDING SHOCKS, BY FINITE ELEMENTS

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Abstract

A wide range of hydrodynamic fields is concerned with the common problem of shocks, and a reliable resolution by a global understanding of suitable numerical modifications to introduce leads to very large prospects of applications, especially in the scope of water resources management.

Without considering the great range of the flow values, the paper presents a gradual approach in order to handle this fundamental question. A chronological analysis of most hydrologic cycle flows is proposed in several specific stages of modelization.

Successful results in quasi two- and three-dimensional simulations in real conditions are obtained by a global approach in the theoretical and numerical study of the physics of the unsteady free surface flows. Applications including shocks properly reproduced point out the benefits in considering first common physical features of the flows, then in discerning numerical shortcomings as well as cautions aimed to ensure the best numerical efficiency.

1. INTRODUCTION

Water represents for Man such a fundamental concern for his life and his well-being that it gives rise to creative talents in accordance with science and technology knowledge of each age. Multiple traces of know-how in the field of hydraulic engineering inspire respect and prove the dynamism developed right from the start to approach most of the hydraulic problems we face today.

But priority aims for the management of hydraulic plants are changing with the technical and social evolution of our industrial societies, and other influences have to be considered.

Today, the increasing water supply, arisen from the technical and social evolution, calls out for a fit management of the available resources. In the same way, the optimisation of all planning projects today have to handle the effects on water resources of land-use change issuing from industrial activity on the whole.

The general aim of this paper is to develop a global approach for setting up hydraulic models and working out efficient resolutions. They must be able to predict the inputs and outputs of a plant optimally managed; in other words, to model the impact of normal or hazardous exploitation in the vicinity of a plant, from the very low water level situations up to dam-break flood wave propagations.

2. HYDROLOGICAL PHYSICALLY BASED MODEL

In the analysis of the hydrologic cycle flows, the first fundamental stage to consider in the open air water course consists of the transformation processes arising after the fall of the raindrop on the ground up to the flows it induces at the outlet of the catchment.

Recently, the modelling of catchment hydrology has experienced a renewal of interest, particularly in searching direct modelization of the thin water layer flowing on the ground. With the physical interpretation of the parameters, the impact of any change of catchment features can be predicted with confidence by modifying their value. Sensitivity tests can be performed with data variation in realistic limits to quantify the corresponding error scope of the output hydrographs. Besides, physically based distributed models easily handle every topographic characteristics and soil types.

From the theoretical point of view, we will show on a simple example that these models are confined to work on "equivalent" catchment elements of simple shape, instead of the original quasi-tridimensional topography, if the presence of potential discontinuities is neglected. Modifying the classical formulation, we will propose a robust finite element approach to handle the kinematic hypothesis applied to overland flow processes on a general digital terrain model.

2.1. Theoretical model

We focus here on the thin water layer propagating on natural tridimensional slopes in order to compute the lateral discharge for each element river. This additional inflow that propagates in the main drainage path is then computed in a final stage with the specific software further described.

It is generally assumed that a dynamic equilibrium exists between friction and gravity components in the direction of flow. Resulting from the scale difference between the characteristic spatial dimension and water layer thickness, biunivoque relations can be assumed between the speed components and the water depth. Therefore, the continuity equation, expressed in terms of water layer thickness, reduces to the following fully non-linear quasi-tridimensional system :

$$\frac{\partial h}{\partial t} + \frac{\partial \left(a_i h^{m+1} \right)}{\partial x_i} + B(h) = 0 \qquad \text{(on the domain D)} \qquad (1)$$

with the following notations :

$h(x_i,t)$: the water depth
$B(h)$: the general function including the infiltration speed into the soil, the intensity of rainfalls and other effects of exchanges and losses
a_i : the parameters including topographical data, surface and flow characteristics

This equation has been entitled kinematic wave approximation and was proposed for the first time in the hydrology field by Woolhizer and Ligett. They assessed the conditions of validity referring to two adimensional numbers (Froude number and kinematic wave number) and proved that overland flows conform to the fit range of these adimensional values.

Besides, several experiments confirmed that laminar and turbulent flows arise for each covering, depending on soil properties, fluctuations in depth and roughness and raindrop impact. A reasonable agreement can be assumed with the biunique relation between depth and discharge proposed by Manning ($m = 2/3$).

BASED MODEL

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2.2. Kinematic jumps in overland flows

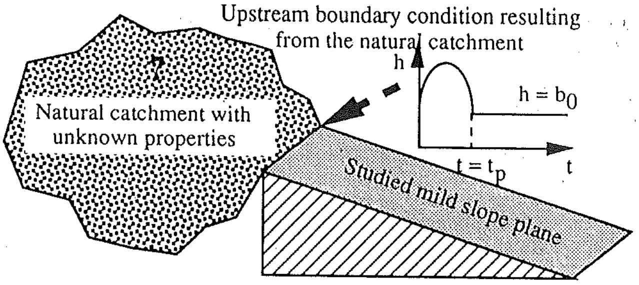

Fig 1. - Symbolic description of the studied catchment

A natural catchment ends downstream in a constant slope plane, as shown on figure 1. No information is provided on physical properties and topographical features concerning the natural upstream part of the studied basin. Nevertheless, we know that at the upstream boundary of the slope, the temporal evolution of the water depth resulting from the runoff on the unknown catchment has the shape of half a period of sinusoid, followed by a constant value. We leave the system evolve freely in the time and we investigate the impervious plane submitted to a constant rainfall intensity.

Since the celerity is an increasing function of water depth, the higher points of the wave will overtake and crowd the lower portions in front of them. Thus, the wave becomes steep and the analytical free surface flow takes the breaking profile illustrated on figure 2.

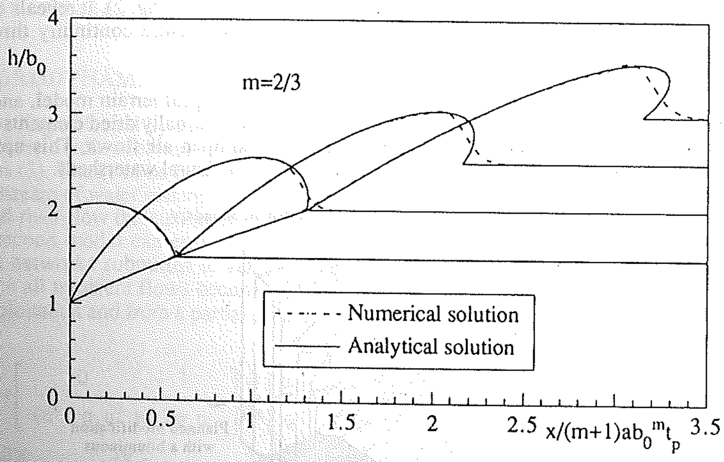

Fig 2. - Free surface flow surface on the mild slope plane at $t/t_p = 0.5, 1, 1.5, 2$

However, this solution is not in accordance with the fundamental assumption of the kinematic hypothesis, which ensures a biunivoque relation between depth and discharge. A theoretical analysis of the overland flow process demonstrates that the motion can be approximated, at this scale of modelization, with moving bores satisfying the continuity requirement. In the continuous domains of the solution, the kinematic wave approximation is proved to remain a suitable model.

2.3. Numerical resolution

From the numerical point of view, the physical relevance of those kinematic shocks suggests the unique formulation of a weak solution of the problem to be resolved. Nevertheless, the classical Galerkin formulation used with the finite element method is unsuitable for predicting and tracking shocks that occur in the correct solution. It induces multi-peaked solutions highlighted by significant errors in the global volume balance.

The proposed shock capturing approach is obtained by introducing discontinuous weighting functions in order to selectively dissipate the parasitic waves. The application of the Petrov-Galerkin technique to the basic equations, along with the use of polynominal functions to represent the unknowns, consists of an orthogonal projection of the residual error due to discretization approximation to a set of linearly independent complete functions included in a vector P. The specific context of application suggests the following test function components for P :

$$P_i = N_i + \alpha'_{ux}\frac{\partial N_i}{\partial x} + \alpha'_{uy}\frac{\partial N_i}{\partial y} \qquad (2)$$

with
n : the number of discretization points
$h = N^T . H$: with H the approximate unknowns vector
$N^T = < N_1, N_2, \ldots, N_n >$: with N_i the classical interpolation functions
$P^T = < P_1, P_2, \ldots, P_n >$: with P_i the original shape functions
α'_{ux} , α'_{uy} : the parameters setting the degree of dissipation

The numerical simulation of the previously described example shows the complete agreement of the numerical solution with the theoretical approach (fig. 2). It reveals the highly dissipative character of the method along with the way to ensure continuity through any kinematic jump.

The proposed non-linear package works on a general digital terrain model, and handles spatial and temporal variations of rainfall and soil properties. Partially dried elements deal with the effects of infiltration, acting as a withdrawal for the open air flows. This approach is actually comforted by comparison with experimental data on natural watersheds.

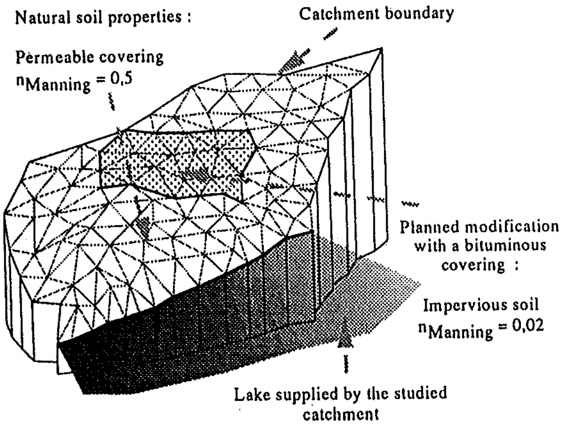

Fig 3. - Discretisation of the studied geometry

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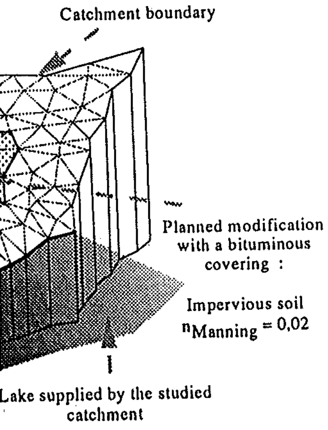

ation of the studied geometry

The physical meaning of each parameter of the modelization and the much more realistic mathematical approach in terms of understanding the physics of overland flow processes allow to reflect easily the impact of any change of catchment features, as highlighted by the following example on a small 34-ha natural catchment of submitted to a constant rainfall of 15.10^{-6} m/s during 1200 s.

It analyses the effects on the hydrograph poured in a lake resulting from a covering modification. The first simulation is carried out with natural soil parameters while the curve of modified inputs reflects the effects of a planned bituminous surface (fig. 4).

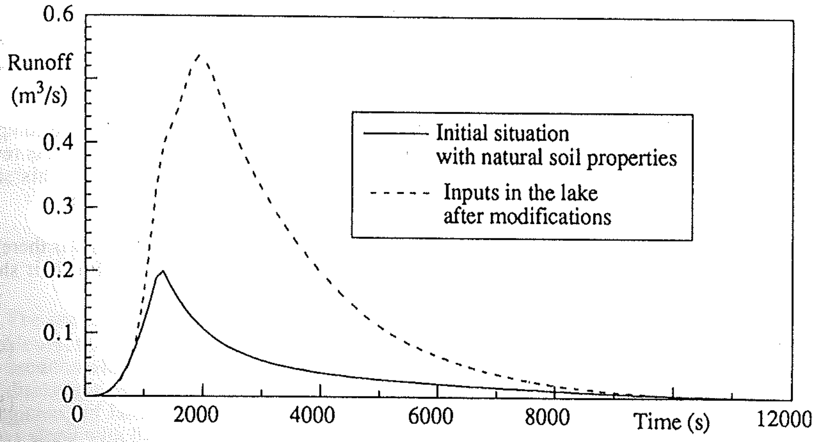

Fig 4. - Flood hydrographs poured in the lake before and after soil covering changes

3. HYDRODYNAMICS OF RIVER NETWORKS WITH MIXED FLOW RATES

The spreading of lateral hydrographs computed by the suggested hydrological code has now to be treated by unidimensional computations of unsteady free surface flows based on the set of equations (3). Thus can be followed and explained the whole story of a flood formation, from the thin stream of water rushing down the hillside to the temporal and spatial evolution of water depth and river flow threatening riverside residents. However, the main assumptions at the root of this second model can ensure suitable modelization of water ressources management of complex networks submitted to variable conditions, such as power plants. Finally, it should reproduce all transient flows occuring downstream of the dam, with outputs resulting from a daily exploitation and from a partial or complete, sudden or gradual, collapse of a dam.

$$\frac{\partial}{\partial t}\begin{bmatrix}\omega\\ q\end{bmatrix}+\begin{bmatrix}0 & 1\\ (c^2-\rho_\omega u^2) & 2\rho_\omega u\end{bmatrix}\frac{\partial}{dx}\begin{bmatrix}\omega\\ q\end{bmatrix}+\begin{bmatrix} & -q_L & \\ g\omega\frac{\partial Z}{\partial x}_{|\Omega}+ & F & +2\frac{\partial}{\partial x}\left(\nu\,\omega\frac{\partial u}{\partial x}\right)\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\equiv\frac{\partial X}{\partial t}+A(X)\frac{\partial X}{\partial x}+D(X)=\frac{\partial X}{\partial t}+\frac{\partial B(X)}{\partial x}+D'(X)=0 \qquad (3)$$

with

F : the friction term
ω : the wet cross-section

u, q, q_L : the average speed in the section, the flow and the lateral inflow
Z : the elevation of free surface
ν : the kinematic viscosity
ρ_ω : the parameter of unequal distribution of the axial speed in the section

The coexistence of several flow rates with shocks and bores in ramified nets of variable cross section arms requires the development of suitable capturing methods. They have to simulate sharp transitions without excessive smearing on several meshes or excessive growing of dissipative processes.

Gaining from the whole experience of the hydrological package, a deep knowledge of factors affecting the intrinsic behaviours of the discretizations was at the root of an original finite element method approach.

Modifying the Galerkin method, the earlier formulation is first generalized to the complete quasi-bidimensional set of equations (3). Numerical experimentations including shocks reveal an uneven improvement depending more particularly of the Froude number. This stage suggests that the dissipation has to take the flow conditions themselves into account.

The following formulation, based on dissymetric continuous test functions P is therefore proposed, which, in simplified conditions, reproduces schemes well known for their shock capturing capabilities :

$$[\omega\ q]^T = N'^T \,.\, Y \tag{4}$$

$$P^T = N'^T + \alpha_{N+1}\, A^T\, W^T \tag{5}$$

with

α_{N+1} : a parameter to optimize, setting the degree of dissipation.
W : the matrix including one-dimensional functions with degree (N+1), by reference to the classic interpolation functions expressed in N'

3.1. Computation of recent floods on the river Meuse

The following simulation is inspired from the confluence of the Meuse and the Ourthe rivers, the Meuse derivation and the King Albert canal in Liège, Belgium. This network lies between the dams of Ivoz-Ramet, Chênée and Monsin and the locks of Monsin. Figure 5 illustrates the studied geometry.

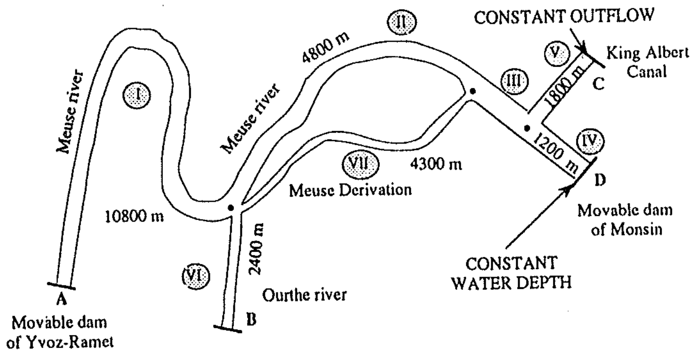

Fig 5. - Discretisation of the studied geometry

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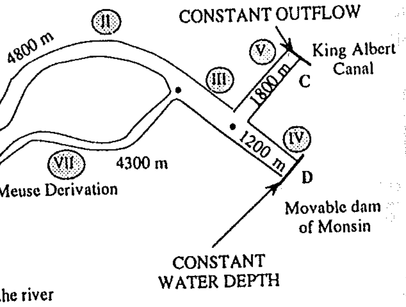

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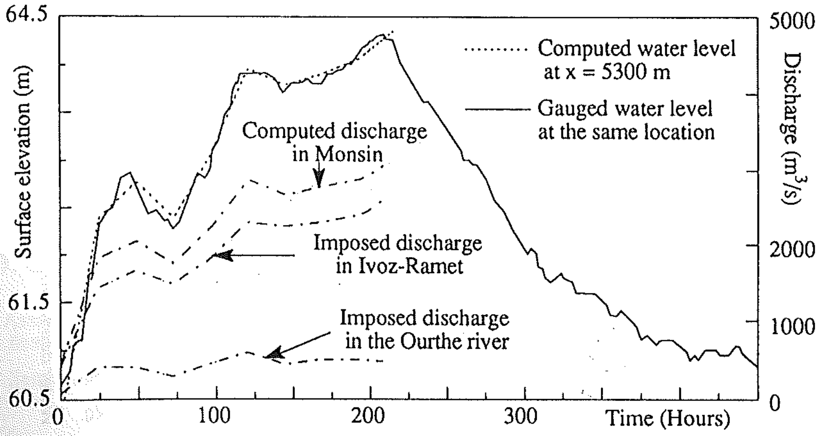

Fig 6. - Comparison between the computed and measured surface water level for the illustrated imposed discharge curves

The sections are nearly trapezoïdal with an approximate average width of 120 m for the river Meuse, 60 m for the Meuse Derivation (by-pass canal). The simulation handles the real cross-sections while the Chezy coefficients are issued from preliminary fitting computations for usual discharges. With suitable limit conditions and initial data corresponding to the situation at the beginning of January 1995, we intend to find the magnitude of the flood that occured during 15 days along with the peak curve 9 days after the beginning.

The figure 6 shows the proposed daily flow curve. The excellent fit between the corresponding water depth curve in the arm I, 5300 m downstream the movable dam of Ivoz-Ramet and the gauged elevation during the floodings assesses the proposed discharge diagrams and the peak values of 2800 m^3/s at point A as well as 3200 m^3/s at point D.

3.2. Computation of dam-break flood wave propagations

Flood wave propagations occuring after a sudden and complete dam-break create the worst conditions to assess the accuracy of a code in handling subcritical and supercritical flows to numerically coexist at any moment in different locations of the discretization.

We consider here this kind of front propagation on the initial wet bottom of a horizontal channel. The analytic solution of Dressler gives the speed of the wave front, as well as the water depth of the constant state region just behind the front, versus the characteristic ratio h_1/h_0. The computation of the flow resulting from the sudden collapse of a 100 m high dam (h_1) is performed with the previously described code, bringing some selective dissipation in the system, in order to obtain the solution illustrated on the figure 6. The wave speed is perfectly reproduced by comparison with the analytic solution, showing sharp transitions exhibing neither excessive smearing on several meshes nor excessive growing of dissipative processes. The figure confirms the usual presence, at the dam site, of a constant water depth point, where critical conditions exist.

With the assurance of reliable simulations of unsteady appearances, movement and disappearances of discontinuities in the most various conditions of hydraulics, the code was applied to real networks to accurately reproduce the flood waves propagation resulting from a sudden complete collapse of a large Belgian gravity dam. The results showed secondary sharp waves propagating in the net of tributaries, reducing the disastrous effects of the floodings. It highlighted the influence of topography and roughness conditions on the formation and displacement of jumps, with sharp transitions up to 20 m on a sole 50 m long discretization mesh, without any numerical noise.

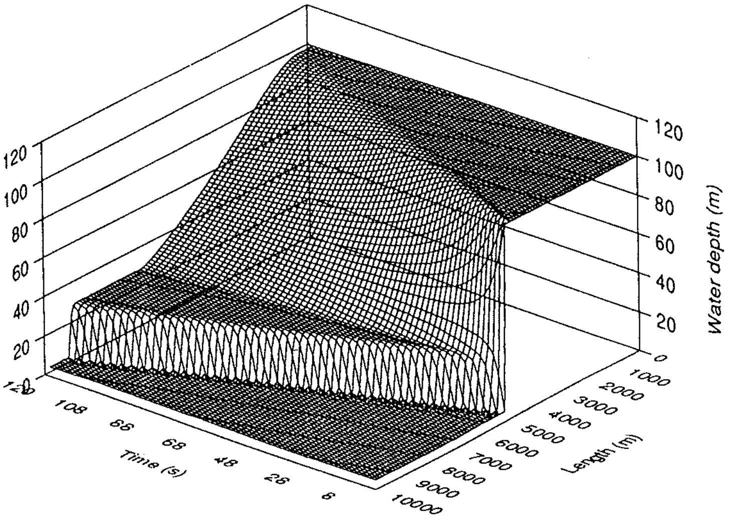

Fig 6. - Temporal evolution of the surface profile after a dam-breaking, for $h_0/h_1 = 20$

This kind of modelization gave valuable data for the working out of height and delay flooding maps, source of emergency plans for the downstream populations in danger.

4. CONCLUSIONS

The global approach in the theoretical and numerical study of the physics of the unsteady free surface flows passes beyond the characteristic scale differences. This paper points out the benefits of this point of view in considering first common physical features of the flows, then in discerning numerical shortcomings also as cautions aimed to ensure the best numerical efficiency.

Seen in this light, a promising scope of research points to success for the setting up of a uniformity in theoretical and numerical coupled models based on a mutual experience sustained by a clear understanding of the factors affecting the accuracy. It suggests an extensive use of reliable software covering a wide range of applications, for the working out of an optimal management of hydraulic resources, in a technological manner.

REFERENCES

[1] WOOLHISER D.A. and LIGGETT J.A., *Unsteady one-dimensional flow over a plane - the rising hydrograph*, Water Resources Research, Vol 3, n° 3, 1967, pp 753-771.

[2] MORGALI J.R., *Laminar and turbulent overland flow hydrographs*, Journal of the Hydraulics Division, ASCE, No HY2, February 1970, pp 441-460.

[3] PIROTTON M., *Modélisation des discontinuités en écoulement instationnaire à surface libre (Du ruissellement hydrologique en fine lame à la propagation d'ondes consécutives aux ruptures de barrages)*, Thèse de doctorat, Université de Liège, 211 figures, 479 pages, Avril 1994.