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EFFICIENCY MEASUREMENT, MULTIPLE-OUTPUT TECHNOLOGIES AND DISTANCE FUNCTIONS: WITH APPLICATION TO EUROPEAN RAILWAYS*

by

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ABSTRACT

The single-output production function has long been regarded as one of the principle limitations of the econometric approach to technical efficiency measurement. If one wished to investigate efficiency in a multiple-output industry using econometric methods one would usually either: (a) aggregate outputs into a single index of output (e.g., total revenue or a multilateral Tornqvist output index); or (b) attempt to model the technology using a dual cost function. The first of these methods require that output prices be observable (and reflect revenue maximising behaviour), while the latter approach requires an assumption of cost-minimising behaviour. There are a number of instances, however, when neither of these requirements are met (the public sector contains many examples). In this study we outline the recently developed distance function solution to the multi-output problem. The method is illustrated using data on European railways. Output-orientated, input-orientated and constant returns to scale distance functions are estimated using corrected ordinary least squares. The distance function estimates are also compared with production function estimates involving aggregate output measures. These comparisons indicate that, for the case of European railways, a production function involving a multilateral Tornqvist output index exhibits substantially less aggregation bias relative to a production function that uses total revenue as a measure of aggregate output.

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1. Introduction

The transformation of inputs into outputs is the primary purpose of the firm. The functional relationship between inputs and outputs is generally described as the production function. Economists have been attempting to obtain estimates of production functions by fitting mathematical functions to sample data for many decades. Most of these analyses have involved the estimation of parametric functions using least squares methods. A growing literature has, however, developed which observes that the production function should theoretically represent the *maximum* output levels obtained from given inputs and that least squares methods do not properly accommodate this notion because they permit observations to lie both below and above the estimated function. Thus a number of authors have suggested alternative methods of estimating production functions (or *production frontiers* as they are often termed in this literature) which attempt to fit functions which bound the data from above (to varying degrees). The distance an observed point, for a particular firm, lies below the estimated production frontier is then often interpreted as the *technical inefficiency* of the firm. An excellent introduction to this body of literature, which has become known as the efficiency measurement literature, is provided by Lovell (1993).

The efficiency measurement literature may be roughly organised into two groups according to the methodology that is used to construct the reference technology. Namely, parametric methods [including the stochastic frontier approach of Aigner, Lovell and Schmidt (1977) and the deterministic approach of Aigner and Chu (1968)] and non-parametric methods, such as data envelopment analysis (DEA) described in Charnes, Cooper and Rhodes (1978) and the Free Disposable Hull (FDH) approach used by Deprins, Simar and Tulkens (1984). The relative merits of the alternative approaches are often listed as being that the (stochastic) parametric approach can account for noise and allow conventional hypothesis tests to be conducted, while the non-parametric approach has the advantage of not requiring the arbitrary selection of a functional form for the production structure and distributional forms for the error terms and that it can easily account for multiple outputs.

Much effort has been devoted to attempts to adjust these various methods to correct their shortcomings [see Lovell (1993) for further discussion]. In this study we look at suitable methods of addressing the single-output nature of the traditional parametric production function. The majority of econometric studies which have attempted to model a multiple-output technology have either: (a) aggregated the multiple outputs into a single index of output (this index may be simply aggregate revenue or perhaps a multi-lateral superlative index such as a Tornqvist[1] or Fisher index); or (b) modelled the technology using a dual cost function.[2] These approaches, however, require certain assumptions to be made. The first of these methods require that output prices be observable (and reflect revenue maximising behaviour), while the latter approach requires an assumption of cost-minimising behaviour. There are a number of instances, however, when neither of these requirements are met. The public sector contains many examples. In this paper we consider the case of European Railways where the vast majority of organisations are both government-owned and highly regulated. The railways are a multiple-output industry where the primary two output

[1] See Caves, Christensen and Diewert (1982a).

[2] For example, see Schmidt and Lovell (1979) or Ferrier and Lovell (1990). Also note that a dual profit or revenue function could alternatively be considered.

groups are freight and passengers. It is an industry where both input and output prices are observable (although imperfectly) but where the behavioural assumptions of cost minimisation and/or revenue maximisation are unlikely to be valid. Other examples of industries in which these assumptions are also unlikely to be appropriate include public hospitals and schools, where multiple outputs are produced and, furthermore, where output prices are very difficult, if not impossible, to identify.

Some recent parametric frontier papers have attempted to solve the multiple output problem by estimating the production technology using either: (a) an input requirements function [e.g., Bjurek, Hjalmarsson and Forsund (1990), Kumbhakar and Hjalmarsson (1991) and Gathon and Perelman (1992)] in which a single (possibly aggregate) input is expressed as a function of a number of outputs; or (b) an output- or input-orientated distance function [e.g., Lovell et al (1994), and Grosskopf et al (1996)] which can accommodate both multiple inputs and multiple outputs.

The input requirements function approach has the advantage of permitting multiple outputs but at the cost of restricting the production technology to a single input. This restriction may be accommodated by constructing an aggregate index of inputs or by assuming that inputs are used in fixed proportions and selecting the most important input as the dependent variable [Gathon and Perelman (1992)]. The distance function approach, however, requires no such restriction. It appears to be an ideal solution to our problem, yet at the same time it is a concept which can be quite difficult to visualise since it involves a function where the dependant variable (the distance) is not observable, and for the simplest example of a multi-input, multi-output technology we must think in a minimum of four dimensions.

The present paper has four primary aims.

1. To discuss the various methods that one may use to estimate parametric distance functions. In particular to explore the linkages between the various estimation methods and to illustrate their relationship with ordinary least squares (OLS) estimation of production functions and input requirement functions.
2. To use the distance function methodology to measure technical efficiency in European Railways.
3. To assess the impact of output aggregation upon technical efficiency measures.
4. To investigate the sensitivity of the technical efficiency estimates to the selection of model orientation.

This paper is organised into sections. The following section provides an introduction to distance functions and surveys estimation methods. In Section 3 technical efficiency in European railways is investigated using a variety of estimated distance functions, and in the final section some conclusions and suggestions for future work are made.

2. Distance Functions

We begin by defining the production technology of the firm using the output set, P(x), which represents the set of all output vectors, $y \in R_+^M$, which can be produced using the input vector, $x \in R_+^K$. That is,

$$P(x) = \left\{ y \in R_{+}^{M} : x \text{ can produce } y \right\}. \tag{1}$$

We assume that the technology satisfies the axioms listed in Fare (1988).

The output distance function, introduced by Shephard (1970), is defined on the output set, P(x), as:

$$D_O(x, y) = \min\left\{ \theta : (y / \theta) \in P(x) \right\}. \tag{2}$$

As noted in Lovell et al (1994), $D_O(x,y)$ is non-decreasing, positively linearly homogeneous and convex in y, and decreasing in x. The distance function, $D_O(x,y)$, will take a value which is less than or equal to one if the output vector, y, is an element of the feasible production set, P(x). That is, $D_O(x,y) \leq 1$ if $y \in P(x)$. Furthermore, the distance function will take a value of unity if y is located on the outer boundary of the production possibility set. That is, $D_O(x,y)=1$ if $y \in \text{Isoq } P(x) = \left\{ y : y \in P(x), \omega y \notin P(x), \omega > 1 \right\}$, using similar notation to that used by Lovell et al (1994).

Note that a distance function may be specified with either an input orientation or an output orientation. In this paper we begin by focusing upon an output distance function primarily because we wish to make comparisons between technical efficiency measures made relative to a production frontier (with an aggregate output measure) and technical efficiency measures obtained from a distance function.

A 2-output, 1-input Example

It is useful to illustrate the concept of an output distance function using an example where two outputs, y_1 and y_2, are produced using a single input, x. Now for a given quantity of the input, x, we can represent the production technology on the two dimensional diagram in Figure 1. Here the production possibility set, P(x), is the area bounded by the production possibility frontier, Isoq P(x), and the y_1 and y_2 axes. The value of the distance function for the point, $A=(y_{1a}, y_{2a})$, which defines the production point where firm A produces y_{1a} of output 1 and y_{2a} of output 2, using the given quantity of the input, is equal to the ratio θ=0A/0B.

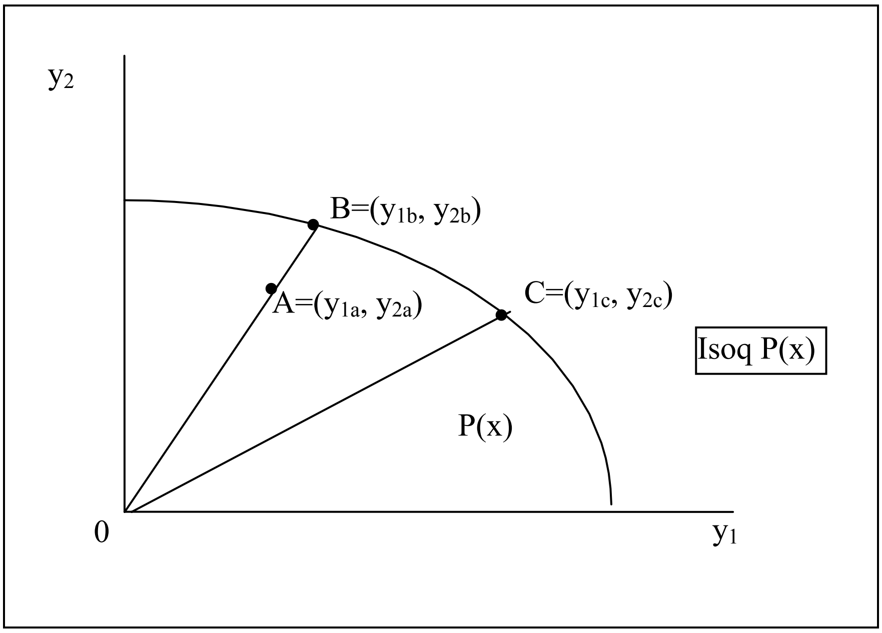
This distance is the inverse of the proportion by which the production of all output quantities could be increased while still remaining within the feasible production possibility set for the given input level. Observe that this distance measure is the inverse of the Farrell-type output-orientated measure of technical inefficiency (see Fare, Grosskopf and Lovell 1994). Geometrically, the distance measure, θ, is equal to the ratio 0A/0B. The distances 0A and 0B are radial distances and hence are equal to the Euclidean distances

$$0A = \left\| y_a \right\| = \sqrt{y_{1a}^2 + y_{2a}^2} \tag{3}$$

and

FIGURE 1

Output Distance Function and Production Possibility Set



$$0B = \|y_b\| = \sqrt{y_{1b}^2 + y_{2b}^2}\ , \tag{4}$$

respectively.

We also observe that the points B and C are on the production possibility surface, denoted by Isoq P(x), and hence would have distance function values equal to 1.

A 1-output, 1-input Example

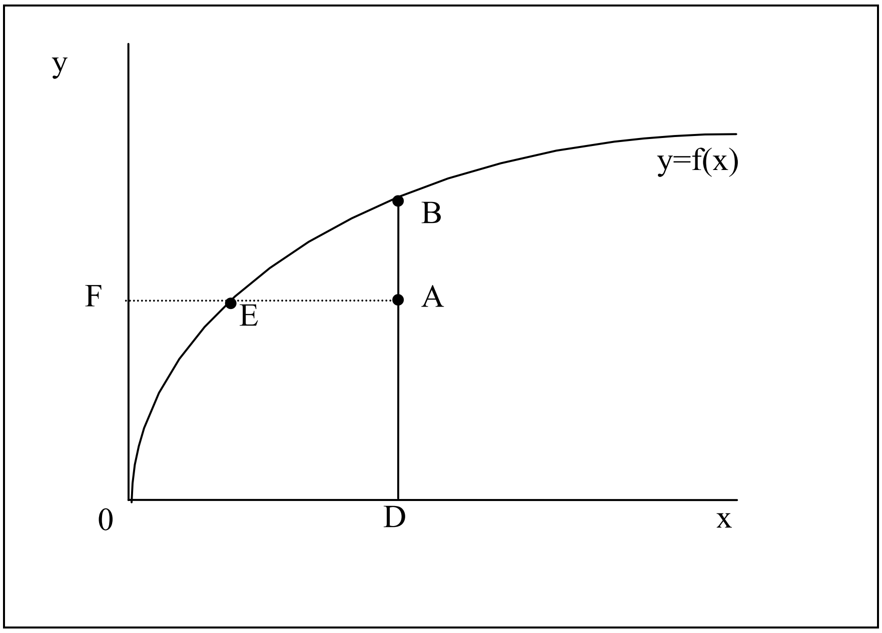
To assist with understanding the concept of a distance function it is useful to consider the simplest possible example where one output is produced using one input. The value of the output distance function is the distance each production point lies below the production possibility curve. Since we have only one output and one input, we can represent all production possibility curves (which are in fact points not curves in this simple case) for all levels of the input, x, using a two dimensional diagram. An example of which is depicted in Figure 2, where we have drawn a decreasing returns to scale production function, y=f(x).

In Figure 2 the value of the output distance function for the production point, A, will be the distance θ=DA/DB. This is the inverse of the proportion by which this firm may expand output while still using its given level of input. This measure is obtained in the same way as the measure described earlier for the two-output case. It is a radial output measure with input level held constant.

It can also be noted that in the diagram in Figure 2 we can define an input distance function by looking at proportional reductions in input quantities with output levels held constant. In this instance the *input* distance function measure for the point, A, would be equal to FA/FE. More will be said on input distance functions later in this paper. We shall now consider the issue of estimation of output distance functions.

FIGURE 2

Distance Functions for a 1-output, 1-input Example



Estimation Methods

In the above discussion we have assumed that the production possibility frontier is known and hence that the required distances may be calculated. In reality the production surface is unknown and must be estimated in some way. Given sample data on N firms, there are a number of alternative ways in which the frontier could be calculated. Five alternative methods of estimating distance function technologies (i.e., frontiers) have been used in recent years. Namely,

1. construction of a non-parametric piece-wise linear frontier using linear programming (DEA) [e.g., Fare, et al (1989), Fare et al (1994)];
2. construction of a non-parametric frontier using FDH [see Deprins, Simar and Tulkens (1984)];
3. construction of a parametric deterministic frontier using linear programming [e.g., Forsund and Hjalmarsson (1987), Fare et al (1993)];
4. estimation of a parametric deterministic frontier using corrected ordinary least squares [e.g., Lovell et al (1994), Grosskopf et al (1996)]; and
5. estimation of a parametric stochastic frontier using maximum likelihood estimation [e.g., Hetemaki (1996)].

The two methods used in this paper will be the parametric methods denoted as methods 4 and 5, above.[3]

[3] The sensitivity of results to alternative estimation methods is considered in the working paper: Coelli and Perelman (1996).

Functional Form

One of the first decisions that must be made in a parametric empirical analysis is the selection of an appropriate functional form. The functional form for the distance function would ideally be:

1. flexible;
2. easy to calculate; and
3. permit the imposition of homogeneity.

The translog form has been selected by the majority of authors [e.g. Lovell et al (1994), Grosskopf et al (1996)] since it is able to satisfy these three requirements. The Cobb-Douglas form, which has been a popular choice in production analyses for a number of decades, does satisfy points 2 and 3 but falls down under point 1 because of its restrictive elasticity of substitution and scale properties. Furthermore, as noted by Klein (1953, p227), the Cobb-Douglas transformation function is not an acceptable model of a firm in a purely competitive industry because it is not concave in the output dimensions. For example, this would imply that the output transformation curve in Figure 1 would have an "isoquant-like" shape which is convex to the origin rather than the concave curve depicted there.[4]

The translog distance function for the case of M outputs and K inputs is specified as[5]

$$\begin{aligned}\ln D_{Oi} = \; & \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} \ln y_{mi} \ln y_{ni} + \sum_{k=1}^{K} \beta_k \ln x_{ki} \\ & + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \ln x_{ki} \ln x_{li} + \sum_{k=1}^{K}\sum_{m=1}^{M} \delta_{km} \ln x_{ki} \ln y_{mi}, \quad i=1,2,...,N, \end{aligned} \tag{5}$$

where i denotes the i-th firm in the sample. Note that to obtain the frontier surface (i.e., the transformation function) one would set $D_{Oi}=1$, which implies the left hand side of equation (5) is equal to zero.

The restrictions required for homogeneity of degree +1 in outputs are

$$\sum_{m=1}^{M} \alpha_m = 1 \tag{6a}$$

and

$$\sum_{n=1}^{M} \alpha_{mn} = 0, \quad m=1,2,...,M, \quad \text{and} \quad \sum_{m=1}^{M} \delta_{km} = 0, \quad k=1,2,...,K, \tag{6b}$$

and those required for symmetry are

$$\alpha_{mn} = \alpha_{nm}, \quad m,n=1,2,...,M, \quad \text{and} \quad \beta_{kl} = \beta_{kl}, \quad k,l=1,2,...,K. \tag{7}$$

We also note in passing that the restrictions required for separability between inputs and outputs are

[4] This is not such a serious problem, however, when optimising behaviour is not an issue. For example, when the primary interest is in obtaining technical measures.

[5] Note that the terms *natural logarithm*, *logarithm, log* and *ln* will be used interchangeably to represent the natural logarithm in this paper.

$$\delta_{km} = 0, \quad k=1,2,...,K, \ \ m=1,2,...,M. \tag{8}$$

These last restrictions will be used when we test for separability in the following section.

A convenient method of imposing the homogeneity constraint upon equation (5) is to follow Lovell et al (1994) and observe that homogeneity implies that

$$D_O(x,\omega y) = \omega D_O(x,y), \text{ for any } \omega>0. \tag{9}$$

Hence if we arbitrarily choose one of the outputs, such as the M-th output, and set $\omega=1/y_M$ we obtain

$$D_O(x,y/y_M) = D_O(x,y)/y_M. \tag{10}$$

For the translog form this provides:

$$\ln(D_{Oi}/y_{Mi}) = \alpha_0 + \sum_{m=1}^{M-1}\alpha_m \ln y^*_{mi} + \frac{1}{2}\sum_{m=1}^{M-1}\sum_{n=1}^{M-1}\alpha_{mn} \ln y^*_{mi} \ln y^*_{ni} + \sum_{k=1}^{K}\beta_k \ln x_{ki}$$
$$+\frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K}\beta_{kl} \ln x_{ki} \ln x_{li} + \sum_{k=1}^{K}\sum_{m=1}^{M-1}\delta_{km} \ln x_{ki} \ln y^*_{mi}, \quad i=1,2,...,N. \tag{11}$$

where $y^*_m = y_m/y_M$. Observe that when $y_m = y_M$ the ratio, y^*_m, is equal to one and hence the log of the ratio is zero. Thus all terms involving the m-th output also become zero. This is why the summations involving y^*_m in the above expression are over M-1 and not over M.

<u>A Digression on Production Functions and Input Requirement Functions</u>

This is an ideal point to make a quick digression to look at some special cases of distance functions. We firstly observe that a single-output production function is equivalent to an output distance function when production only involves one output. Hence, if we set M=1 and also set $D_{0i}=1$, so as to trace out the production surface, equation (11) becomes

$$-\ln(y_{Mi}) = \alpha_0 + \sum_{k=1}^{K}\beta_k \ln x_{ki} + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K}\beta_{kl} \ln x_{ki} \ln x_{li}, \quad i=1,2,...,N. \tag{12}$$

This is the (negative of the) very familiar translog production function.

Although we have not yet presented a formal definition of input distance functions we have informally stated that they involve a proportional reduction in input usage for a given output vector. A translog *input* distance function is therefore obtained by imposing homogeneity of degree +1 in inputs (instead of in outputs) upon the transformation function. Thus instead of obtaining equation (11) we will obtain

$$\ln(D_{Ii}/x_{Ki}) = \alpha_0 + \sum_{m=1}^{M}\alpha_m \ln y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M}\alpha_{mn} \ln y_{mi} \ln y_{ni} + \sum_{k=1}^{K-1}\beta_k \ln x^*_{ki}$$
$$+\frac{1}{2}\sum_{k=1}^{K-1}\sum_{l=1}^{K-1}\beta_{kl} \ln x^*_{ki} \ln x^*_{ki} + \sum_{k=1}^{K-1}\sum_{m=1}^{M}\delta_{km} \ln x^*_{ki} \ln y_{mi}, \quad i=1,2,...,N. \tag{13}$$

where $x^*_k = x_k/x_K$ and D_{Ii} denotes the input-orientated distance measure. Now in an analogous manner to the relationship we have noted between production functions and output distance functions in the single output case, we also observe that an input distance function will be equivalent to an input requirement function when a single input is used in the production process. Hence, if we set K=1 and also set D_{Ii}=1, so as to trace out the production surface, equation (13) becomes

$$-\ln(x_{Ki}) = \alpha_0 + \sum_{m=1}^{M} \alpha_k \ln y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} \ln y_{mi} \ln y_{ni} \qquad i=1,2,...,N. \tag{14}$$

This is the (negative of the) translog input requirements function.

Estimation of a Parametric Form

With the selection of a suitable functional form for our output distance function completed we must now select an appropriate method of obtaining estimates of the unknown parameters of the function. That is, we must obtain estimates of the parameters of the function (which is the outer surface of the production possibility set) such that the function is "a good fit" to the data.

This task may be described using simple algebra by rewriting equation (11) as

$$\ln(D_{Oi}/y_{Mi}) = TL(x_i,y_i/y_{Mi},\alpha,\beta), \qquad i=1,2,...,N, \tag{15}$$

or

$$\ln(D_{Oi})-\ln(y_{Mi}) = TL(x_i,y_i/y_{Mi},\alpha,\beta), \qquad i=1,2,...,N, \tag{16}$$

and hence

$$-\ln(y_{Mi}) = TL(x_i,y_i/y_{Mi},\alpha,\beta) - \ln(D_{Oi}), \qquad i=1,2,...,N. \tag{17}$$

What is required here is the selection of parameter values for the translog function which ensure the function fits the observed data "as closely as possible" while maintaining the requirement that $0<D_{Oi}\leq 1$, which implies that $-\infty<\ln(D_{Oi})\leq 0$.

The three parametric methods mentioned earlier use three different "best fit" criteria. Fare et al (1993), use a variant of the Aigner and Chu (1968) parametric linear programming methodology to "estimate" their distance function. They clearly define their criterion as the maximisation of the sum of the natural logarithms of the output distances, D_{Oi}. Since the output distances are bounded by zero and one, their logarithms must then be zero or negative. Thus this is equivalent to minimising the sum of the "deviations" of the observations below the frontier, where the "deviations" are defined as the logarithms of the inverses of the distances.[6]

COLS Estimation of a Distance Function

Lovell et al (1994), use the corrected ordinary least squares (COLS) method[7] to estimate an output distance function. The function is fitted in two steps. The first step involves interpreting the unobservable term "$-\ln(D_{Oi})$" in equation (17) as a random error term and estimating the translog distance function using OLS. In the second step the OLS estimate of the intercept parameter, α_0, is adjusted (by adding the largest negative OLS residual to it) so that the function no longer passes through the centre of the observed points but bounds them from above. The distance measure for the i-th firm is then calculated as the exponent of the (corrected) OLS residual.

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[6] Schmidt (1976) also observes that the linear programming approach of Aigner and Chu (1968) is equivalent to estimating the frontier using maximum likelihood under the assumption that the logs of the distances are exponentially distributed.

[7] This method is described in Greene (1980) for the case of production and cost functions.

The statistical properties of the COLS estimator is discussed in Greene (1980). He observes that, given the assumptions that: (i) the regressors and the error term are independent; (ii) the error term is iid with finite mean and finite, positive variance; and (iii) the regressors are well behaved (see Greene (1980, p31) for details), then OLS provides best linear unbiased and consistent estimates of the parameters, with the exception of the intercept parameter, α_0, which will be biased and inconsistent. He also establishes that, given the above assumptions, a consistent estimator of the intercept can be obtained by adding the largest OLS residual to the OLS estimate of the intercept.

One question which is often asked by persons when first exposed to the OLS estimation of a multi-output distance function, such as equation (17), relates to the appearance of outputs as regressors and hence to the possibility of simultaneous equation bias. It can be argued, however, that only ratios of the outputs appear as regressors and that these ratios may be assumed to be exogenous, since the distance function is defined for radial (proportional) expansion of all outputs, given the input levels, and hence by definition the output ratios are held constant for each firm.

At first glance OLS estimation of the output distance function in equation (17) would appear to be quite similar to OLS estimation of a production function formulation, with $-\ln y_M$ as the dependent variable. It is important to observe, however, that OLS applied to this equation does not fit that translog transformation function which minimises the sum of squares of the deviations between observed and predicted values of $\ln y_M$. OLS will, in fact, fit that function which minimises the sum of the squares of the (radial) deviations between observed and predicted values of the natural logarithm of the norm of the y vectors. That is, where the norm of the y vector is the Euclidean distance:

$$\|y\| = \sqrt{y_1^2 + y_2^2 + \ldots + y_M^2}\ . \tag{18}$$

This distance may be visualised as the distance along a ray from the point (y,x) to the point (0,x). For the case of a two-output, single-input technology, this distance is equal to

$$\begin{aligned}\|(y,x)-(0,x)\| &= \sqrt{(y_1-0)^2 + (y_2-0)^2 + (x_1-x_1)^2} \\ &= \sqrt{y_1^2 + y_2^2} = \|y\|\,. \end{aligned} \tag{19}$$

Here we observe the relationship between the estimation method and the radial Farrell-type output-orientated technical efficiency measures described in Fare, Grosskopf and Lovell (1994).[8]

Lovell et al (1994) reports R-squared measures to provide an indication of the goodness-of-fit of their fitted OLS distance functions. They do not explicitly specify how the R-squared measures are obtained, hence we assume they have reported the conventional OLS R-squared measure that is routinely produced by econometrics software, such as the SHAZAM program used in this paper [see White (1993)]. The conventional R-squared measure that is reported when OLS is used to estimate the distance function in equation (17) is equal to:

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[8] The fitting of a function by minimisation of the sum of squares of (logged) radial deviations does not appear to be given a particular name by any of the authors who have used the method. We suggest that the term *Euclidean Least Squares (ELS)* could be used to describe the methodology.

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_{i=1}^{N} (\ln y_{Mi} - \ln \hat{y}_{Mi})^2}{\sum_{i=1}^{N} (\ln y_{Mi} - \overline{\ln y_M})^2}. \tag{20}$$

This measure will provide a different value depending upon which of the outputs is chosen as the normalising output because the denominator in equation (20) is the total sum of squares in the logs of that output. However, it is important to stress that the parameter estimates that are obtained are not affected by the choice of the normalising output.

Given this discussion, we suggest that a more appropriate R-squared measure would appear to be:

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_{i=1}^{N} \left(\ln\|y_i\| - \ln\|\hat{y}_i\|\right)^2}{\sum_{i=1}^{N} \left(\ln\|y_i\| - \overline{\ln\|y\|}\right)^2}, \tag{21}$$

which is the proportion of the total sum of squares (from the sample mean) in the logs of the radial distances explained by the fitted function. If the R-squared value is equal to one then all observed points would lie upon the fitted function. If the value is zero then this implies that the input quantities do not explain any of the observed variation in the $\ln\|y_i\|$ around the sample mean value, $\overline{\ln\|y\|}$.

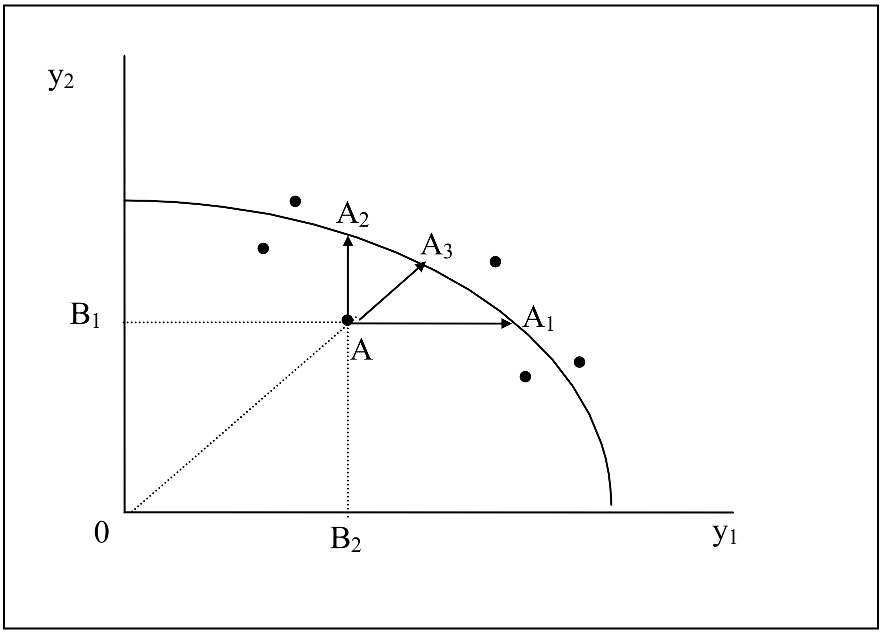
In Figure 3 we illustrate the difference between the estimation of a two-output technology using OLS applied to an output distance function versus OLS estimation of the production technology using a production function where one output is expressed as a function of the other output and the inputs. If we obtain OLS estimates of a production function with $\ln y_1$ as the dependent variable then it is evident that we will be fitting that function which minimises the sum of the squares of the (horizontal) deviations of $\ln y_1$ from $\ln \hat{y}_1$ [or equivalently the sum of squares of $\ln(y_1 / \hat{y}_1)$]. For the case of point A in Figure 3 (the exponent of) this deviation is equivalent to the ratio B_1A/B_1A_1. If $\ln y_2$ was used as the dependent variable the relevant ratio would be B_2A/B_2A_2, and in the case of the estimation of the output distance function the relevant ratio would be the radial measure $0A/0A_3$.

The three alternative methods are likely to identify three different estimates of the "true" production surface. The "best" estimates to use depend upon ones intentions. The distance function estimates may be preferred simply because the method avoids the necessity to have to arbitrarily select one of the outputs to be used as the dependent variable. They may also be preferred because the distance function approach implies the *radial* projection of observed points onto the frontier surface. Any *non-radial* projection (such as that resulting if one output is arbitrarily selected as the dependant variable) will imply a change in the output mix between the observed and the projected point. This would imply a rather confusing situation where, if one were to assume the observed points were allocatively efficient, then a systematic degree of allocative inefficiency would be introduced into the projected points.

One instance, however, when one may be able to argue for the selection of one output (or a subset of outputs in the M>2 case) as the appropriate curve fitting dimension(s) is when one is considered to only have discretionary control over a subset of the outputs, and hence that a projected point that involves the expansion of the non-discretionary output(s) is not feasible.

FIGURE 3

OLS Estimation of a Multi-output Technology



Input Distance Functions

The above discussion considers various methods of fitting a curve in one or more output dimensions. It is important to note that the transformation function can also be fitted from an input perspective. The input distance function may be defined on the input set, L(y), as:

$$D_I(x,y) = \max\{\rho : (x/\rho) \in L(y)\}, \tag{22}$$

where the input set L(y) represents the set of all input vectors, $x \in R_+^K$, which can produce the output vector, $y \in R_+^M$. That is,

$$L(y) = \{x \in R_+^K : x \text{ can produce } y\}. \tag{23}$$

$D_I(x,y)$ is non-decreasing, positively linearly homogeneous and concave in x, and increasing in y. The distance function, $D_I(x,y)$, will take a value which is greater than or equal to one if the input vector, x, is an element of the feasible input set, L(y). That is, $D_I(x,y) \geq 1$ if $x \in L(y)$. Furthermore, the distance function will take a value of unity if x is located on the inner boundary of the input set.

Estimation of a translog input distance function by COLS closely follows the approach used for output distance functions. The two main differences are that homogeneity is imposed in the inputs (instead of the outputs)[9] and, after OLS estimates are obtained, the OLS estimate of the intercept is adjusted by adding the largest positive residual (instead of the largest negative residual).

It is of interest to note that, under constant returns to scale (CRS), the input distance function is equivalent to the inverse of the output distance function (i.e., $D_O=1/D_I$) (Fare et al 1994). That is, the proportion by which one is able to radially expand output (with input held fixed), will be exactly equal to the proportion by which one is able to radially reduce input usage (with output held constant). In the case of the translog output distance function, CRS is imposed by imposing homogeneity of degree -1 in inputs. The resulting function will obviously be exactly equal to the negative of the input distance function in which homogeneity of degree -1 has been imposed in outputs.[10]

<u>ML Estimation of a Stochastic Distance Function</u>

The above two methods of fitting a parametric distance function explicitly assume that all deviations between observed production points and the production surface are due to technical inefficiency. The main criticism of these deterministic frontier methods is that they do not account for the possible influence of data noise (e.g., as a result of measurement error or model miss-specification) upon the shape and positioning of the frontier and hence that the methods are sensitive to the influence of outliers.

One method that can be used to attempt to account for the influence of noise upon an estimated frontier is to apply the stochastic frontier approach proposed by Aigner, Lovell and Schmidt (1977), which involves the specification of a frontier function with an error term with two components: a symmetric error to account for noise and an asymmetric error to account for inefficiency. We begin by appending a symmetric error term, v_i, to equation (17) to account for noise, and also change the notation "$-\ln(D_{Oi})$" to "u_i". We thus obtain a stochastic output distance function

$$-\ln(y_{Mi}) = TL(x_i,y_i/y_{Mi},\alpha,\beta) + v_i + u_i, \qquad i=1,2,...,N. \tag{24}$$

Given appropriate distributional assumptions for v_i and u_i, the parameters of this stochastic translog distance function can be estimated using maximum likelihood. We follow the suggestion of Aigner et al (1977) in this paper and assume that the v_i are iid $N(0,\sigma_v^2)$, and distributed independently of the u_i which are assumed to be iid $|N(0,\sigma_u^2)|$.

The predicted value of the output distance for the i-th firm, $D_{Oi}=\exp(-u_i)$, is not directly observable because u_i only appears as part of the composed error term, $e_i=vi+u_i$. Predictions may, however, be obtained using a modification of the

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[9] See equation (13).

[10] It is interesting to note that when using DEA under variable returns to scale (VRS), the output- and input-orientated models will estimate exactly the same frontier surface and therefore, by definition, identify the same set of firms as being efficient. The DEA efficiency measures, however, may differ between the two orientations. Parametric distance functions, however, differ from this in that both the efficiency measures *and* the estimated frontier may differ with the orientation under VRS. However we note that under constant returns to scale the estimated frontiers and the efficiency measures are unaffected by orientation when using either parametric or non-parametric methods.

conditional expectation formulae presented in Jondrow et al (1982) and Battese and Coelli (1988). The output distance function value for the i-th firm may be obtained using the conditional expectation

$$D_{Oi} = E[\exp(-u_i)|e_i]$$
$$= \frac{1-\Phi(\sigma_A - \gamma e_i/\sigma_A)}{1-\Phi(\gamma e_i/\sigma_A)} \exp(\gamma e_i + \sigma_A^2/2) \quad (25)$$

where $\sigma_A=\sqrt{\gamma(1-\gamma)\sigma^2}$, $\sigma^2=\sigma_u^2+\sigma_v^2$, $\gamma=\sigma_u^2/\sigma^2$, and $\Phi(.)$ represents the distribution function of a standard normal random variable.[11] An operational predictor of D_{Oi} is obtained by replacing the unknown parameters in equation (22) with their ML estimates. The maximum likelihood estimates of the unknown parameters and the distance function predictions obtained in this paper are calculated using the computer program, FRONTIER, Version 4.1 (see Coelli (1994)).

The stochastic frontier method may also be applied in a similar manner to a translog input distance function. The primary difference to note is that non-positive error term will now be subtracted from the equation rather than added. That is, the stochastic input distance function would appear as

$$-\ln(x_{Ki}) = TL(x_i/x_{Ki},y_i,\alpha,\beta) + v_i - u_i, \quad (26)$$

and the input distances would be predicted as

$$D_{Ii} = E[\exp(u_i)|e_i] \quad (27)$$

where $e_i = v_i - u_i$.

3. Application to European Railways

There is a long tradition in the estimation of production characteristics and performances in railways. From the Klein's (1953) seminal econometric study on US railways to the recent studies using frontier analysis techniques [Perelman and Pestieau (1988), Deprins and Simar (1988), Gathon and Perelman (1992)]. The majority of this research is devoted to detailed partial productivity analysis [British Railways Board and University of Leeds (1979), Nash (1985)] and to total factor productivity (TFP) comparisons based on the estimation of multi-output cost functions [Caves et al. (1980, 1981)][12].

The analytical framework used in many of the above studies is influenced by the three characteristics which are common to almost all railways companies. First, multi-output production: passenger and freight services are provided simultaneously and share mainly the same inputs. Second, all railways companies benefit from some degree of (natural) monopoly, even if other transportation modes indirectly compete with them. Third, railroad passenger transportation, and to a lesser extent freight transportation, are public services which are often submitted to high regulation.

The three characteristics described above are common to all 17 European railways companies considered in this study (see Table 1 for a list of these companies). All of

[11] The variance parameters σ_u^2 and σ_v^2 are replaced with γ and σ^2 for the purposes of estimation. For further detail see Coelli (1994).
[12] For a survey of these studies, see Dodgson (1985).

the companies produce both passenger and freight services and, with the one exception of the privately owned Swiss company, BLS, they are state-owned (during the sample period). All companies hold a natural monopoly position on the rail transportation, but in return, their activity is constrained to varying degrees by public authorities.

The multi-output dual cost function approach that has been applied by many authors to the North American industry is likely to be an even less appropriate method of analysis in the state-owned European industry, where cost-minimisation is unlikely to be an objective which has a high priority with the various bureaucrats who manage the industry. In fact, in terms of performance measurement, Pestieau and Tulkens (1994) argue that technical efficiency measurement is probably the only appropriate way to compare the performance of enterprises operating in such environments. They observe that the technical efficiency objective, that is, the maximisation of physical outputs for a given combination of physical inputs,[13] is, in fact, the only objective that is compatible with all other objectives fixed by the control authorities and, for this reason, appears to be an unavoidable goal.

Data

We estimate the multi-output/multi-input technology of European railways using annual data on 17 companies observed over the five year period from 1979 to 1983. The physical data used is derived from data published by the International Union of Railways (UIC, 1979-1983). Passenger service output and freight service output are measured using the sum of distances travelled by each passenger and the sum of distances travelled by each tonne of freight, respectively.

The primary inputs used in rail transportation are labour, energy and capital. Labour is measured by the annual mean of monthly data on staff levels, after taking into account the number of workers supplied by private contractors and after making the necessary adjustments for staff paid on an hourly basis. The railways transportation staff includes operating and traffic employees as well as those workers in charge of the traction, the rolling stock and the permanent way maintenance and supervision.

Three different sources of energy were used by rail locomotives during the sample period, namely, coal, diesel and electricity. Calorific equivalence's among these energy sources are often used to estimate the total consumption by railways, even if these equivalence's are not representative of their traction power equivalence's. In this study we adopt a different point of view. We apply the traction power equivalence's calculated by Gathon (1991) using econometric estimation.[14] These equivalence scales are as follows: 1 tonne of coal = 0.73 kW/h; and 1 tonne of diesel = 4.48 kW/h. Compared with calorific equivalence's, these values imply that only 10% and 35% of coal and diesel, respectively, are actually transformed into traction power.

As always, the selection of a capital variable was a difficult task. The total length of lines is the capital variable that is used in this study. Other possible alternatives include: the length of tracks, the number of locomotives and the number of vehicles (coaches, railcars and wagons), however these variables were all found to be closely

[13] Or alternatively minimising the inputs required to produce given outputs.

[14] The estimation relies on the same sample of European railways analysed here but covers the period 1961-1984.

related to the chosen variable (all correlation coefficients in excess of 0.90) and hence it was considered that the length of lines would be a reasonable proxy for total capital.

Summing up, the five variables used in our analysis are defined as follows:

Outputs : y_1 = passenger km,

y_2 = tonnes km of freight,

Inputs : x_1 = annual average staff in railways transportation,[15]

x_2 = energy consumption in kW/h (traction power equivalence's),

x_3 = km of lines.

One of the principle aims of this study is to assess the impact of measuring technical efficiency relative to a production frontier involving a single aggregate output measure versus using a multiple-output distance function. To this end we define two commonly used aggregate output measures. The first is the total revenue of railways transportation obtained by adding the revenues from passenger and freight services together.[16] The second aggregate output measure calculated is a multilateral Tornqvist output index (also known as the CCD index after Caves, Christensen and Diewert 1982a) which uses revenue shares to weight passenger and freight activities.

The sample means of all variables are presented in Table 1 for each of the 17 companies over the period 1979-1983. The panel consists of European national railways that are for a large part interconnected. The data exhibits large variation both in the scale of operations and the input and output ratios. For example, the largest firms, BR, DB and SNCF are more than one hundred times bigger (in terms of lines, labour or outputs) than the smallest companies, BLS and CFL. Also, in terms of output composition, some companies such as SNCF and VR, display an even balance between passengers and freight transportation, while other railroads, such as NS and DSB, are primarily interested in passenger services, leaving most freight traffic to other transportation modes.

Results and Discussion

Two alternative estimation methods are considered in this study. Namely, COLS and ML. The parameter estimates are presented in Table 2. Results for six different model formulations are presented:

1. a production function with total revenue used as a measure of aggregate output;
2. a production function with aggregate output constructed using a multilateral Tornqvist index;
3. an output distance function with separability between inputs and outputs imposed;
4. an (unrestricted) output distance function;
5. an input distance function; and

[15] It should be noted that many of the rail companies in our sample are also involved in other activities, such as bus and ship transportation. The data used in this analysis is, however, confined to the inputs and outputs pertaining to the railways transportation activities of these companies.

[16] These revenues are obtained by converting the nominal revenue figures to 1980 values using the relevant GDP deflator in each country. These figures are then deflated using OECD PPP GDP deflators to obtain our final revenue values expressed in 1980 ECUs.

6. a CRS distance function.

COLS Results

We will begin by discussing the COLS results which are listed in the first six columns of Table 2. We firstly observe that the R-squared measures[17] and t-ratios indicate these estimated models appear to be a reasonable fit to the observed data. All R-squared values are in excess of 97% and the t-ratios on all first-order coefficients and the majority of second-order coefficients exceed 2 in absolute value. All first-order terms are also observed to have correct signs, with the exception of the results for the model involving the revenue measure where the coefficient of the log of capital is both negative and significant. In fact, the results for the revenue model are vastly different to the other five sets of results, which as a group appear fairly similar, at least in the first-order terms.

The disparity between the revenue function results and the results obtained from the other specifications is also reflected in the technical efficiency predictions obtained from the various models. The means of these predictions for each rail company for each model are presented in Table 3 and correlations between the various sets of technical efficiency predictions are presented in Table 4. The most striking result is seen in Table 4, where the largest correlation between the technical efficiency predictions of the revenue model and the other five sets of COLS results is 0.178, while the correlations among the remaining five sets of results range from 0.469 to 0.967. We also observe from the bottom row of Table 3 that the mean technical efficiency for the revenue model is 0.536 while the means obtained from the other five models are of the order of 0.8 to 0.9. Thus we conclude that, assuming that the distance function estimates are closest to the true parameter values, the use of total revenue as a measure of aggregate output in this empirical analysis appears less than satisfactory. This is not a big surprise given that few publicly owned rail organisations set output prices with market conditions or cost recovery notions in mind. Furthermore, issues are further complicated by a variety of government subsidies that are paid to many of these companies. These payments are not included in the present analysis. Future work could involve an investigation of the influence of the inclusion of these subsidies into the revenue figures. The hypothesis being that the revenue model results may then become more similar to the other results.

The production function results obtained using the multilateral Tornqvist index appear much better than the revenue model results. The parameter estimates have the expected signs and are much closer to the distance function estimates and the correlations between the technical efficiency predictions are positive and mostly of the order of 0.5 or more. The improved performance is probably due to a number of factors. First, the Tornqvist index relies upon revenue shares rather than actual price levels and hence will be less susceptible to inter-country price differentials as long as any differentials or subsidies are fairly evenly distributed across the two output groups. A second possible reason for a better performance is that the (deflated) revenue measure may be interpreted as an implicit quantity index and hence that if the price index used in deflation is a Laspeyres or Paasche index then the implicit technology is a linear technology. The Tornqvist index, on the other hand, is a superlative index because it is exact for the translog form which is a flexible

[17] The R-squared measures refer to the OLS estimates, not to the COLS estimates.

functional form (i.e., a second order approximation to an arbitrary functional form) and hence is likely to provide a better measure of aggregate output.

The output distance function results presented in column 4 of Table 2 appear well behaved and well estimated. The second-order output cross-product term, α_{12}, has the correct sign so as to *encourage* the transformation curve to have a concave shape (rather than the convex Cobb-Douglas shape that would result if this term was zero). We also observe that the first-order input coefficients sum to a value greater than one indicating the presence of increasing returns to scale at the mean. This observation conforms with results obtained in the majority of empirical railways analyses.

Separability restricted output distance function results are also presented in column 3 of Table 2 for comparative purposes. This model was included because it is observed that the Tornqvist production function differs from the output distance function in two respects: (1) it is separable and (2) output aggregation is achieved using revenue share information rather than by estimated coefficients. The separability restricted function was estimated in an attempt to shed some light on the relative importance of these two factors. The results show that the separability restricted distance function is not noticeably similar or dissimilar to either the unrestricted output distance function or the Tornqvist production function, suggesting the two factors contribute to the observed differences in a fairly even manner. The correlations between efficiency predictions in Table 4 also appear to support this observation. It is interesting to note, however, the null hypothesis of separability is rejected by a generalised likelihood-ratio test at a 1% level of significance.

The fifth set of COLS estimates presented in Table 2 are for an input distance function (column 5). The input distance function results are included partly for purely comparative purposes but also because one could argue that an input orientation may be more appropriate in railways because the managers are likely to have more discretionary control over inputs rather than outputs.[18] This argument for endogenous input quantities and exogenous output quantities has been presented by a number of authors to justify the use of dual cost functions to investigate the multi-output railways technology.

The input distance function results are reassuringly similar to the output distance function results. The first-order parameters do not differ greatly, other than by the expected degree due to the imposition of the homogeneity constraint upon the inputs instead of the outputs. The sum of the first-order output coefficients is less than one in absolute value, indicating the presence of increasing returns to scale. Finally a value of 0.967 is reported in Table 4 for the correlation between the technical efficiency predictions from the two models. Thus, all indicators suggest that the choice of orientation is not terribly crucial in this particular industry, especially if one's primary interest is in performance measurement.

The final set of COLS estimates listed in column 6 of Table 2 are for the CRS distance function. These are included to allow us to test the hypothesis of constant returns to scale and also to view the impact of this restriction upon parameter estimates and technical efficiency predictions. The main thing we observe is a dramatic decline in the log-likelihood function (LLF) value from 86.4 for the output distance function and 96.3 for the input distance function to 45.0 for the CRS distance

[18] Recall that the main reason we began with the output orientation was because it was a natural progression from a single output production function.

function. Given that the CRS model estimates five fewer free parameters than the unrestricted distance functions, a generalised likelihood-ratio test would reject the null hypothesis of constant returns to scale at the 5% or even 1% levels.

A final point of interest is the comparison between the mean technical efficiencies of the single-output Tornqvist production function and the two-output output distance function, where the mean efficiency level rises from 0.776 for the former to 0.862 for the latter. This reduction in inefficiency when an extra dimension is added to the model appears to be similar to the dimensionality dilemma which is a well documented problem in DEA analyses.

ML Results

Columns 7 and 8 of Table 2 contain the ML estimates for the unrestricted output and input distance functions, respectively. The results for the other four model formulations are not reported because they were found to be essentially no different to the OLS results. The γ estimates obtained for these four models were found to be equal to zero and the LLF values were thus no different to the values obtained for the OLS estimates. The two unrestricted distance functions, on the other hand, were found to have γ estimates in excess of 0.99 in value and furthermore obtained LLF values which indicated that the γ parameter was a significant addition to the model at a 1% level of significance using the one-sided likelihood-ratio test recommended in Coelli (1995).

This unusual behaviour of the ML estimator, selecting a γ value from either extreme of the sensible range, causes us to treat the results with some caution. An examination of the other parameter estimates does not reveal any dramatic differences between the COLS and ML estimates, however the correlations between the technical efficiency predictions from the two methods are uniformly bad, being no larger than 0.1 in absolute value (though the correlation between the output and input ML results is equal to 0.907). The reason(s) for these unusual results are not apparent to us at present. One possibility is that the wide range in scales of operation of the railways companies considered, along with the second-order flexibility of the translog functional form, have resulted in the ML method adjusting the second order coefficients so that the function "bends" at the extremities of the data range to have the extreme observations placed quite close to the frontier which results in an adjustment to the distribution of the residuals so as to closely resemble a half-normal distribution. Our suspicions are further supported by the observation that when our analysis was repeated using the simpler Cobb-Douglas form the same problems did not occur. However, further experimentation using other data sets and perhaps simulated data are required for us further investigate this "hunch".[19]

Preferred Results

Finally, we must address the question of which of our various sets of results do we wish to identify as our preferred results for the purpose of discussing the relative performance of European railways. The COLS estimates of the (unrestricted) output distance function is selected as the preferred estimates following a process of elimination. The ML estimates are rejected for the reasons outlined earlier. The

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[19] Hetemaki (1996), reports similar problems to those discussed here, even though his stochastic distance function involves a restricted translog form in which all second order parameters associated with inputs are set to zero.

production function estimates are rejected because they involve output aggregation. The separability and CRS restricted models are rejected on the basis of the likelihood ratio tests. Finally the output distance function is selected over the input distance function because it is believed that it would be easier for a railway to expand market share rather than to reduce the usage of capital and labour in the short run. The technical efficiency predictions for the output distance function are tabulated in column 4 of Table 3. We observe a mean technical efficiency level of 0.862 and mean values for individual companies which range from 0.708 for the United Kingdom to 0.957 for Italy.

It should be stressed, however, that these figures are raw technical efficiency estimates which are not adjusted for environmental conditions. Thus any observed differences may be due to either environmental differences or management factors. For example, the good performance of Italy in this analysis conflicts with observations made in previous analyses.[20] Further analysis is obviously required for us to be able to disentangle the relative contribution of the above two factors.

4. Conclusions

The first observation that we must reiterate is that we are not confident in our ML translog distance function estimates. All estimated functions either collapsed to an average function, where all deviations from the frontier are assumed due to noise, or to a full frontier, where all deviations are assumed due to inefficiency. The reason for this problem is suspected to be due to the combined effects of large variability in the sample data, along with the flexibility of the translog functional form, but this is yet to be confirmed. We thus confine the remainder of our conclusions to observations concerning the COLS results.

A key conclusion of this paper is that the use of total revenue as a measure of aggregate output in an empirical analysis of European railways, even after careful deflation, appears fraught with danger. This is not a terribly surprising result given that few publicly owned rail organisations set output prices with market conditions or cost recovery notions in mind. The production function parameter estimates obtained using the revenue measure differ substantially from those obtained using the multilateral Tornqvist index and the distance function approach, to the extent that the first order term associated with capital becomes (significantly) negative in the analysis involving the revenue measure. The differences between the revenue model results and those from the other models are further reflected in technical efficiency predictions which are found to be essentially orthogonal to the technical efficiency predictions obtained from the alternative models.[21]

A more positive conclusion that may be derived from our analysis is that the multilateral Tornqvist index appears to be a suitable method of aggregating output. The results obtained using a single-output production function with aggregate output calculated using a multilateral Tornqvist index did not differ substantially from the

[20] This may also be due in part to the use of different variables, time periods and estimation methods in previous studies.

[21] It should be noted that the problems associated with the use of a revenue measure of aggregate output are complicated by exchange rate issues in studies which cross national borders. A study involving data on firms from a single country would not be concerned by such issues but may still be influenced by regional price variations and also by the possibility that the output price ratio may not be an accurate guide to the slope of the product transformation curve when revenue maximising behaviour is not present.

output distance function results. Both the parameter estimates and the technical efficiency predictions appear quite similar. The one obvious difference, however, is that the production function results do not provide information on the shape of the product transformation curve, since this information is implicitly included in the construction of the multilateral Tornqvist output index using the revenue shares.

Another conclusion that may be derived from our analysis is that parameter estimates and technical efficiency predictions are found to be quite insensitive to model orientation. The similarity of results between model orientations tends to suggest that the issue of endogeneity is not a serious one in this industry. Whether this observation can be extended to other industries remains to be answered. Our naive view is that we are attempting to find a method of fitting a production technology to a scatter of points in R^{+}_{M+K} space and that the selection of dimension(s) in which to minimise deviations of observed points from the fitted function should not have a great influence upon results when the data is not distributed in an abnormal manner.

A number of tasks are yet to be completed in this paper. One is to trace out the family of production possibility curves implied by our estimated functions to investigate their curvature. Another important task is to evaluate scale economies, curvature and monotonicity at each data point. Furthermore, we must also extend the analysis to account for environmental factors, such as network density and average trip length. This may be done by either including such variables directly into the various models or alternatively by regressing the efficiency predictions upon these factors in a second-stage analysis.

One area of future work that could also of interest would be to obtain input price data that would permit us to conduct an analysis of these data using a multi-output cost function. Given that cost-minimising behaviour is not prevalent in most European railways, this may permit one to investigate the robustness of the dual cost function approach to violations of the required behavioural assumptions.

References

Aigner, D.J. and S.F. Chu (1968), "On Estimating the Industry Production Function", *American Economic Review*, 58, 826-839.

Aigner, D.J., C.A.K Lovell, and P. Schmidt (1977), "Formulation and Estimation of Stochastic Frontier Production Function Models", *Journal of Econometrics*, 6, 21-37.

Battese, G.E. and T.J. Coelli (1988), "Prediction of Firm-Level Technical Efficiencies With a Generalised Frontier Production Function and Panel Data", *Journal of Econometrics*, 38, 387-399.

British Railways Board and University of Leeds (1979), *A Comparative Study of European Rail Performance*, BRB, London.

Caves, D.W. and L.R. Christensen (1980), "The Relative Efficiency of Public and Private Firms in a Competitive Environment : The Case of Canadian Railroads", *Journal of Political Economy*, 80, 958-976.

Caves, D.W., L.R. Christensen and J.A. Swanson (1981), "Productivity Growth, Scale Economies and Capacity Utilization in U.S. Railroads, 1955-74", *The American Economic Review*, 71, 994-1002.

Caves, D.W., L.R. Christensen and W.E. Diewert (1982a), "Multilateral Comparisons of Output, Input and Productivity Using Superlative Index Numbers", *Economic Journal*, 92: 73-86.

Caves, D.W., L.R. Christensen and W.E. Diewert (1982b),"The Economic Theory of Index Numbers and the Measurement of Input, Output and Productivity", *Econometrica,* 50: 1393-1414.

Charnes, A., W.W. Cooper and E. Rhodes (1978), "Measuring the Efficiency of Decision Making Units", *European Journal of Operations Research*, 2, 429-444.

Coelli, T.J. (1994), *A Guide to FRONTIER Version 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation*, mimeo, Department of Econometrics, University of New England, Armidale.

Coelli, T.J. (1995), "Estimators and Hypothesis Tests for a Stochastic Frontier Function: A Monte Carlo Analysis", *Journal of Productivity Analysis*, 6, 247-268.

Coelli, T.J. and Perelman, S. (1996), "A Comparison of Parametric and Non-parametric Distance Functions: With Application to European Railways", mimeo, CREPP, University of Liege.

Deprins, D. and L. Simar (1988), "Mesure d'efficacité des ré seaux de chemins de fer", in *Gestion de l'é conomie et de l'entreprise: l'approche quantitative*, CORE (ed), Bruxelles, De Boek-Wesmael, 321-44.

Deprins, D., Simar, L. and H. Tulkens (1984), "Measuring Labor-Efficiency in Post Services", in M. Marchand, P. Pestieau and H. Tulkens (eds.), *The Performance of Public Enterprises. Concepts and Measurement*, North Holland, Amsterdam.

Dodgson, J. S. (1985), "A Survey of Recent Developments in the Measurement of Rail Total Factor Productivity", in K.J. Bulton and D.E. Pitfield (eds.), *International Railway Economics*, Gower House, London, 13-48.

Fare, R. (1980), *Fundamentals of Production Theory*, Lecture Notes in Economics and Mathematical Systems, Berlin, Springer-Verlag.

Fare, R., S. Grosskopf and C.A.K. Lovell (1994), *Production Frontiers*, Cambridge University Press.

Fare, R., S. Grosskopf, C.A.K. Lovell and C. Pasurka (1989), "Multilateral Productivity Comparisons when Some Outputs are Undesirable: A Nonparametric Approach", *Review of Economics and Statistics*, 71, 90-98.

Fare, R., S. Grosskopf, C.A.K. Lovell and S. Yaisawarng (1993), "Derivation of Shadow Prices for Undesirable Outputs: A Distance Function Approach", *Review of Economics and Statistics*, 75, 374-380.

Fare, R., S. Grosskopf, M. Norris and Z. Zhang (1994), "Productivity Growth, Technical Progress, and Efficiency Changes in Industrialised Countries", *American Economic Review*, 84, 66-83.

Farrell, M.J. (1957), "The Measurement of Productive Efficiency", *Journal of the Royal Statistical Society*, A CXX, Part 3, 253-290.

Ferrier, G.D. and C.A.K. Lovell (1990), "Measuring Cost Efficiency in Banking: Econometric and Linear Programming Evidence", *Journal of Econometrics*, 46, 229-245.

Gathon, H.-J. and P. Pestieau (1995), "Decomposing Efficiency Into its Managerial and its Regulatory Components : The Case of European Railways", *European Journal of Operational Research*, 80, 500-507.

Gathon, H.-J. and S. Perelman (1992), "Measuring Technical Efficiency in European Railways: A Panel Data Approach", *The Journal of Productivity Analysis*, 3, 131-151.

Gathon, H.J. (1991), "La performance des chemins de fer europé ens: Gestion et autonomie", Ph.D thesis, Department of Economics, University of Liège.

Greene, W. (1980), "Maximum Likelihood Estimation of Econometric Frontier Functions", *Journal of Econometrics*, 13, 27-56.

Grosskopf, S., K. Hayes, L. Taylor and W. Weber (1996), "Budget Constrained Frontier Measures of Fiscal equality and Efficiency in Schooling", *Review of Economics and Statistics*, forthcoming.

Hetemaki, L. (1996), "Environmental Regulation and Production Efficiency: Evidence from Pulp Industry", mimeo, Finnish Forest Research Institute, Helsinki.

Jondrow, J., C.A.K. Lovell, I.S. Materov and P. Schmidt (1982), "On Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model", *Journal of Econometrics*, 19, 233-238.

Klein, L.R. (1953), *A Textbook of Econometrics*, Row Peterson, New-York.

Lovell, C.A.K. (1993), “Production Frontiers and Productive Efficiency”, in Fried, H.O., C.A.K. Lovell and S.S. Schmidt (Eds*), The Measurement of Productive Efficiency*, Oxford University Press, New York, 3-67.

Lovell, C.A.K., S. Richardson, P. Travers and L.L. Wood (1994), “Resources and Functionings: A New View of Inequality in Australia”, W. Eichhorn (ed.), *Models and Measurement of Welfare and Inequality*, Berlin, Springer-Verlag.

Nash, C. (1985), “European Railways Comparisons - What Can We Learn?” in K.J. Bulton and D.E. Pitfield (eds.), *International Railway Economics*, Gower House, London, 237-270.

Perelman, S. and P. Pestieau (1988), “Technical Performance in Public Enterprises: A Comparative Study of Railways and Postal Services”, *European Economic Review*, 32, 432-441.

Pestieau, P. and H. Tulkens (1993), “Assessing and Explaining the Performance of Public Enterprises : Some Recent Evidence From the Productive Efficiency Viewpoint”, *Finanz Archiv*, 50, 293-323.

Schmidt, P. (1976), “On the Statistical Estimation of Parametric Frontier Production Functions”, The *Review of Economics and Statistics*, 58, 238-239.

Shephard, R.W. (1970), *Theory of Cost and Production Functions*, Princeton, Princeton University Press.

UIC (1961-1988), *International Railway Statistics*, Statistics of Individual Railways, Paris.

White, K.J. (1993), *SHAZAM User’s Reference Manual Version 7.0*, McGraw-Hill, New York.

TABLE 1
Sample descriptive statistics (1)
Mean values over the period 1979-1983

Railways	Country	Physical outputs: Passenger-km	Physical outputs: Tonnes-km of freight	Shares in total revenu: Passenger	Shares in total revenu: Freight	Tornqvist output	Total revenue index	Physical inputs: Staff (3)	Physical inputs: Energy	Physical inputs: Lines
		(10^6km)	(10^6km)	(%)	(%)	(2)	(10^6 ECU)	(10^3)	(10^3KM/h)	(km)
BLS	Switzerland	320	261	59.0	41.0	99.7	74.5	1.9	108.0	235
BR	United-Kingdom	30386	17612	61.6	38.4	8196.6	4031.6	202.4	5308.6	17401
CFF	Switzerland	11504	6877	48.4	51.6	2071.5	1378.0	37.4	1573.5	2905
CFL	Luxembourg	226	603	14.3	85.7	159.4	55.2	3.8	70.4	270
CH	Greece	1946	721	45.8	54.2	360.1	107.3	312.3	184.6	2461
CIE	Ireland	1236	642	66.1	33.9	279.1	89.2	7.4	154.1	1969
CP	Portugal	5731	996	85.1	14.9	1098.3	470.6	24.0	409.4	3608
DB	Germany	44244	60319	46.2	53.8	17014.7	8989.3	312.2	8925.7	28349
DSB	Denmark	4716	1615	62.6	37.4	906.2	325.1	18.4	605.7	2102
FS	Italy	45865	17378	68.6	31.4	9660.5	4015.2	217.4(	3797.4	16460
NS	Netherlands	12	3177	73.3	26.7	2068.7	615.9	27.2	1037.1	2884
NSB	Norway	2301	2748	43.4	56.6	873.0	151.5	15.7	432.0	4241
OBB	Austria	7180	10470	39.1	60.9	3057.7	2444.2	69.4	1633.6	5761
SJ	Sweden	6492	15151	38.6	61.4	3574.6	908.4	32.8	1536.7	11444
SNCB	Belgium	6900	7577	65.8	34.2	3434.2	1436.1	58.9	1360.4	4257
SNCF	France	55555	64372	50.2	49.8	20530.4	7353.0	243.7	7340.0	34503
VR	Finland	3235	8035	26.7	73.2	1964.4	450.3	24.4	583.5	6072
Mean		15053	12856	52.7	47.3	4410.5	1934.8	77.0	2062.9	8525

(1) The variables are defined in the Appendix [Source: UIC (1961-1988)]. (2) BLS (1979) = 100.0.
(3) In 1980 GDP constant prices and PPP (1 ECU = 1.392 U.S. dollar).

TABLE 2

Estimated parameters for alternative models (1) (2) (3)

	COLS						ML	
	Total revenue	Tornqvist output aggregation	Distance functions				Distance functions	
			Output oriented		Input distance	CRS	Output distance	Input distance
			Separable	Not separable				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α_0	- 0.088 (1.2)	0.013 (0.3)	0.040 (2.8)	0.172 (2.8)	- 0.179 (3.3)	0.066 (1.5)	0.080 (1.2)	- 0.506 (1.1)
α_1	-	-	- 0.537 (11.1)	- 0.472 (11.1)	- 0.324 (6.9)	- 0.681 (19.6)	- 0.398 (17.1)	- 0.340 (8.2)
α_2	-	-	- 0.463	- 0.528	- 0.432 (12.2)	- 0.319	- 0.602	- 0.428 (12.2)
α_{11}	-	-	0.015 (2.2)	0.136 (2.2)	0.137 (2.2)	0.114 (1.8)	0.081 (5.7)	0.102 (1.9)
α_{22}	-	-	0.015	0.136	0.126 (2.1)	0.114	0.081	0.052 (1.0)
α_{12}	-	-	- 0.015	- 0.136	- 0.139 (2.5)	- 0.114	- 0.081	- 0.085 (1.8)
β_1	0.910 (6.9)	0.497 (7.0)	0.421 (4.2)	0.497 (4.2)	0.274 (3.0)	0.407 (5.2)	0.460 (6.3)	0.229 (3.3)
β_2	0.522 (4.8)	0.396 (6.8)	0.512 (9.4)	0.410 (5.1)	0.356 (5.7)	0.580 (9.0)	0.472 (8.1)	0.418 (8.2)
β_3	- 0.231 (3.2)	0.313 (8.1)	0.275 (7.8)	0.360 (4.9)	0.370	0.013	0.315 (2.4)	0.353
β_{11}	0.287 (0.5)	- 0.987 (3.2)	- 0.828 (3.1)	0.602 (1.2)	1.138 (2.6)	- 1.709 (4.7)	0.363 (1.7)	1.129 (2.8)
β_{22}	0.750 (1.6)	- 0.309 (1.2)	- 0.160 (0.6)	0.753 (2.0)	1.208 (4.0)	- 0.947 (3.8)	0.540 (1.1)	1.340 (4.0)
β_{33}	0.889 (1.8)	1.159 (4.3)	1.226 (3.7)	1.687 (3.7)	1.382	- 0.468	1.707 (4.8)	0.993
β_{12}	- 0.429 (0.8)	0.688 (2.5)	0.671 (2.2)	- 0.283 (0.7)	- 0.482 (1.3)	1.094 (3.9)	- 0.017 (3.3)	- 0.720 (1.9)
β_{13}	0.062 (0.2)	- 0.033 (0.2)	- 0.098 (0.6)	- 0.558 (1.9)	- 0.656	0.615	- 0.611 (3.1)	- 0.409
β_{23}	0.600 (1.9)	0.601 (3.6)	- 0.701 (4.4)	- 0.687 (4.5)	- 0.726	- 0.147	- 0.667 (5.9)	- 0.584
δ_{11}	-	-	-	- 0.520 (2.7)	- 0.734 (4.6)	0.590 (5.5)	- 0.381 (2.4)	- 0.633 (4.7)
δ_{12}	-	-	-	0.520	0.577 (3.4)	- 0.590	0.381	0.485 (3.4)
δ_{21}	-	-	-	0.478 (3.9)	0.398 (3.9)	- 0.197 (2.1)	0.475 (19.0)	0.395 (5.1)
δ_{22}	-	-	-	- 0.478	- 0.545 (4.6)	0.197	- 0.475	- 0.478 (4.9)
δ_{31}	-	-	-	0.014 (0.1)	0.336	- 0.393	- 0.113 (0.7)	0.238
δ_{32}	-	-	-	- 0.014	0.032	0.393	0.113	- 0.007
LLF (d.f.)	5.22 (75)	58.2 (75)	74.5 (73)	86.4 (70)	96.3 (70)	45.0 (75)	96.2 (68)	114.9 (68)
R^2	0.977	0.993	0.996	0.997	0.957	0.957	-	-
max(ε)	0.650	0.261	0.241	0.119	0.328	0.328	-	-
σ^2	0.059	0.017	0.012	0.009	0.023	0.023	0.018 (9.1)	0.016 (8.2)
γ	-	-	-		-	-	0.996 (24.4)	0.999 (7941)

(1) T-tests are presented in brackets. (2) All output distance functions parameters have been multiplied by - 1 in order to be comparable with the other results. (3) Underlined parameters are calculated by homogeneity conditions.

TABLE 3

Technical efficiency for alternative models

		COLS						ML	
		Total	Tornqvist	Distance functions				Distance functions	
Rail-	Country	revenue	output	Output oriented		Input	CRS	Output	Input
ways			aggregation	Separable	Not separable	distance		distance	distance
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
BLS	Switzerland	0.566	0.739	0.760	0.845	0.876	0.609	0.905	0.902
BR	United-Kingdom	0.425	0.640	0.643	0.708	0.727	0.559	0.929	0.946
CFF	Switzerland	0.522	0.803	0.805	0.917	0.946	0.852	0.963	0.968
CFL	Luxembourg	0.488	0.804	0.807	0.866	0.893	0.752	0.895	0.911
CH	Greece	0.417	0.659	0.736	0.796	0.836	0.741	0.935	0.922
CIE	Ireland	0.688	0.852	0.854	0.933	0.948	0.711	0.952	0.960
CP	Portugal	0.700	0.823	0.743	0.868	0.894	0.714	0.941	0.960
DB	Germany	0.589	0.826	0.794	0.869	0.879	0.689	0.884	0.898
DSB	Denmark	0.433	0.700	0.751	0.834	0.896	0.620	0.908	0.934
FS	Italy	0.417	0.904	0.917	0.957	0.973	0.787	0.921	0.921
NS	Netherlands	0.450	0.968	0.948	0.915	0.923	0.875	0.860	0.892
NSB	Norway	0.344	0.711	0.757	0.848	0.879	0.693	0.873	0.895
OBB	Austria	0.834	0.710	0.744	0.834	0.874	0.663	0.869	0.889
SJ	Sweden	0.570	0.752	0.760	0.841	0.872	0.748	0.839	0.864
SNCB	Belgium	0.559	0.684	0.754	0.849	0.898	0.702	0.887	0.922
SNCF	France	0.542	0.751	0.827	0.916	0.938	0.986	0.863	0.886
VR	Finland	0.564	0.865	0.835	0.859	0.890	0.674	0.934	0.935
	Mean	0.536	0.776	0.790	0.862	0.891	0.728	0.903	0.918

TABLE 4
Correlation table of alternative technical efficiency measures

	COLS						ML	
	Total revenue	Tornqvist output aggregation	Distance functions				Distance functions	
			Output oriented		Input distance	CRS	Output distance	Input distance
			Separable	Not separable				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
COLS: Corrected Ordinary Least Squares								
(1) Total revenue	1.000	0.136	0.004	0.178	0.170	- 0.043	0.427	0.302
(2) Tornqvist output aggregation		1.000	0.892	0.747	0.646	0.469	0.596	0.614
(3) Output distance - Separable			1.000	0.859	0.788	0.633	0.679	0.681
(4) Output distance – Not separable				1.000	0.967	0.591	0.873	0.874
(5) Input distance					1.000	0.523	0.864	0.924
(6) CRS						1.000	0.493	0.455
MLE: Maximum Likelihood Estimation								
(7) Output distance							1.000	0.908
(8) Input distance								1.000

Data Appendix

The data used in this paper was build on the basis of the *International Railways Statistics* published each year, since1925, by the International Union of Railways (Union International de Chemins de fer, UIC). For each input and output variable we indicate the corresponding table containing the annual statistics of individual railways. Also we give a summary of the UIC description for each of the selected statistics (UIC, 1979-1983)[22].

Inputs

Staff — *Operating and traffic staff* (Table 31)
Corresponds to the annual mean staff bound to the Railway by an employment contract and working in the following activities :
- central and regional operating and traffic departments;
- stations, halts, stopping points, town offices and signalling installations;
- train accompanying and inspection.

Energy — *Specific costs and revenue. General operating results for the period* (Table 72)
Traction power equivalence's calculated by Gathon (1991) using econometric estimation. These equivalence scales are as follows: 1 tonne of coal = 0.73 kW/h; and 1 tonne of diesel = 4.48 kW/h. Compared with calorific equivalence's, these values imply that only 10% and 35% of coal and diesel, respectively, are actually transformed into traction power.

Lines — *Lines* (Table 11)
Total length (in km) of lines worked at end of the including electrified and non electrified lines and broad and narrow gauge lines. Sections permanently out of use are excluded.

Outputs

Passenger — *Revenue-earning passenger traffic* (Table 51)
Number of passenger kilometres conveyed by rail calculated in accordance with the number of tickets sold multiplied by the kilometric distance for each journey of by a mean kilometric distance.

Freight — *Freight traffic* (Table 61)
Tonnes kilometres of revenue-earning traffic carried by rail obtained by multiplying the chargeable weight by the charging distance. This variable includes essentiallly full wagonloads as well as express parcels and small traffic (included postal packages).

Total revenue — *General operating results for the period* (Table 72)
Traffic revenue calculated in accordance with the UIC accountancy system.

Shares in total revenue — *Specific costs and revenue. General operating results for the period* (Table 72)

The railway companies included in the study are as follows :
Chemin de fer Bern-Loetshberg-Simplon (BLS, Switzerland); British Railways (BR); *Swiss Federal Railways* (CFF); *Luxembourg National Railway Company* (CFL); *Hellenic Railways Organisation* (CH); *Irish Transport Company* (CIE); *Portuguese Railways* (CP); *German Federal Railways* (DB); *Danish State Railways* (DSB); *Italian State Railways* (FS); *Netherlands Railways* (NS); *Norwegian State Railways* (NSB); *Austrial Federal Railways* (OBB); *Swedish State Railways* (SJ); *Belgian National Railway Company* (SNCB/NMBS);

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French National Railway Company (SNCF) and *Finnish State Railways* (VR).