Damage Diagnosis of Beam-like Structures Based on Sensitivities of Principal Component Analysis Results

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NOMENCLATURE

K, M, C	stiffness, mass and damping matrix
X	observation (snapshot) matrix
p	vector of parameters
$\mathbf{U}, \mathbf{V}, \mathbf{\Sigma}$	matrices of left and right singular vectors and of singular values
$\alpha_{ji}^{k}, \beta_{ji}^{k}$	projection coefficients
H	FRF matrix
ω	angular frequency
Δ	sensitivity variation
d^{I} , d^{II}	first and second derivatives of the vector Δ

ABSTRACT

This paper addresses the problem of damage detection and localization in linear-form structures. Principal Component Analysis (PCA) is a popular technique for dynamic system investigation. The aim of the paper is to present a damage diagnosis method based on sensitivities of PCA results in the frequency domain. Starting from Frequency Response Functions (FRFs) measured at different locations on the beam, PCA is performed to determine the main features of the signals. Sensitivities of principal directions obtained from PCA to beam parameters are then computed and inspected according to the location of sensors; their variation from the healthy state to the damaged state indicates damage locations. It is worth noting that damage localization is performed without the need of modal identification. Once the damage has been localized, its evaluation may be quantified if a structural model is available. This evaluation is based on a model updating procedure using previously estimated sensitivities. The efficiency and limitations of the proposed method are illustrated using numerical and experimental examples.

1. Introduction

A dynamic transformation, resulting from a variety of causes, e.g. structural damage or nonlinearity onset, may disturb or threaten the normal working conditions of a system. Hence, questions of the detection, localization and severity estimation of those events have attracted the attention of countless engineering researchers in recent times. The detection, localization and assessment of the damage allow to reduce maintenance costs and to ensure safety.

In the last decade, the problem of damage localization and assessment has been approached from many directions. Often based on monitoring modal features, these processes can be achieved by using an analytical model and/or promptly by measurement. Damage can cause change in structural parameters, involving the mass, damping and stiffness matrices of the structure. Thus many methods deal directly with these system matrices. The Finite Element Method (FEM) is an efficient tool in this process [1]. The problem of detection may be resolved by this method through model updating or sensitivity analysis. For damage localization and evaluation, model updating is utilized to reconstruct the stiffness perturbation matrix [2]. This may be combined with a genetic algorithm [3] or based on modal parameter sensitivity [4]. In these cases, a well fitted numerical model is essential to compare with the actual system.

Methods using measurement are also widely used because of their availability in practice. Yan and Golinval [5] achieved damage localization by analyzing flexibility and stiffness without system matrices, using time data measurements. Koo et al. [6] detected and localized low-level damage in beam-like structures using deflections obtained by modal flexibility matrices. Following localization, Kim and Stubbs [7] estimated damage severity based on the mode shape of a beam structure. Rucka and Wilde [8] decomposed measured FRFs by continuous wavelet transform in order to achieve damage localization. Cao and Qiao [9] recently used a novel Laplacian scheme for damage localization. Other authors have located damage by comparing identified mode shapes [10] or their second-order derivatives [11] in varying levels of damage. Sampaio et al. [12] extended the method proposed in [11] through the use of measured FRFs.

Natural frequency sensitivity has also been used extensively for the purposes of damage localization. Ray and Tian [10] discussed the sensitivity of natural frequencies with respect to the location of local damage. In that study, damage localization involved the consideration of mode shape change. Other authors [13-15] have located damage by measuring natural frequency changes both before and after the occurrence of damage. However, such methods, based on frequency sensitivity with respect to damage variables require an accurate analytical model. Jiang and Wang [16] extended the frequency sensitivity approach by eliminating that requirement. However, an optimization scheme is still needed to estimate the unknown system matrices through an identified model using input-output measurement data.

It is not only the issue of localization that has become the subject of recent study; the assessment of damage is also increasingly attracting the interest of researchers [2-4, 7, 14]. Yang et al. [17] estimated damage severity by computing the current stiffness of each element. They used Hilbert-Huang spectral analysis based only on acceleration measurements using a known mass matrix assumption. Taking an alternative approach, other authors have used methods involving the updating of a finite element model of the examined structure and have used sensitivity analysis to discover the effective parameter. For example, Messina et al. [13] estimated the size of defects in a structure based on the sensitivity of frequencies with respect to damage locations where all the structural elements were considered as potentially damaged sites. Teughels and De Roeck [18] identified damage in the highway bridge. They updated both Young's modulus and the shear modulus using an iterative sensitivity based FE model updating method.

This study focuses on the use of sensitivity analysis for resolving the problems of damage localization and evaluation. Natural frequencies are known to be successful in characterizing dynamical systems. Mode shapes, meanwhile, have been considered effective in model updating, since these shapes condense most of the deformation database of the structure. Here, we use not only sensitivity of frequency, but also of mode shape, a subject which appears less developed in the literature. A modal identification is not necessary for the objective of localization. In monitoring the distortion of a sensitivity vector, the localization may be carried out in the first step. An analytical model is then needed for model updating, and this enables the assessment of the damage.

2. Sensitivity analysis for Principal Component Analysis

The behavior of a dynamical system depends on many parameters related to material, geometry and dimensions. The sensitivity of a quantity to a parameter is described by the first and higher orders of its partial derivatives with respect to the parameter. Sensitivity analysis of modal parameters may be a useful tool for uncovering and locating damaged or changed components of a structure. On one hand, we know that the dynamic behavior of a system is fully characterized by its modal parameters which result from the resolution of an eigenvalue problem based on the system matrices (when a model is available). On the other hand, principal component analysis (PCA) of the response matrix of the system is also a way to extract modal features (i.e. principal directions) which span the same subspace as the eigenmodes of the system [19]). The second approach based on PCA is used in this study to examine modal parameter sensitivities.

Let us consider the observation matrix $\mathbf{X}^{m \times N}$ which contains the dynamic responses (snapshots) of the system where m is the number of measured co-ordinates and N is the number of time instants. We will assume that it depends on a vector of parameters **p**. The observation matrix **X** can be decomposed using Singular Value Decomposition (SVD):

$$\mathbf{X} = \mathbf{X}(\mathbf{p}) = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \tag{1}$$

where **U** and **V** are two orthogonal matrices, whose columns represent respectively left and right singular vectors; $\boldsymbol{\Sigma}$ contains singular values of descending importance: $\sigma_1 > \sigma_2 > ... > \sigma_m$.

A sensitivity analysis is performed here by taking the derivative of the observation matrix with respect to **p**:

$$\frac{\partial \mathbf{X}}{\partial \mathbf{p}} = \frac{\partial \mathbf{U}}{\partial \mathbf{p}}\boldsymbol{\Sigma}\mathbf{V}^T + \mathbf{U}\frac{\partial \boldsymbol{\Sigma}}{\partial \mathbf{p}}\mathbf{V}^T + \mathbf{U}\boldsymbol{\Sigma}\frac{\partial \mathbf{V}^T}{\partial \mathbf{p}} \tag{2}$$

Through this equation, the sensitivity of the system dynamic response shows its dependence on the sensitivity of each SVD term. So, the determination of $\frac{\partial \mathbf{U}}{\partial \mathbf{p}}$, $\frac{\partial \boldsymbol{\Sigma}}{\partial \mathbf{p}}$ and $\frac{\partial \mathbf{V}}{\partial \mathbf{p}}$ is necessary. Junkins and Kim [20] developed a method to compute the partial derivatives of SVD factors. The singular value sensitivity and the left and right singular vector sensitivity are simply given by the following equations:

$$\frac{\partial \sigma_i}{\partial p_k} = \mathbf{U}_i^T \frac{\partial \mathbf{X}}{\partial p_k}\mathbf{V}_i \; ; \quad \frac{\partial \mathbf{U}_i}{\partial p_k} = \sum_{j=1}^{m} \alpha_{ji}^k \mathbf{U}_j \; ; \quad \frac{\partial \mathbf{V}_i}{\partial p_k} = \sum_{j=1}^{m} \beta_{ji}^k \mathbf{V}_j \tag{3}$$

The partial derivatives of the singular vectors are computed through multiplying them by projection coefficients. These coefficients are given by equation (4) for the off-diagonal case and by equation (5) for the diagonal elements.

$$\alpha_{ji}^k = \frac{1}{\sigma_i^2 - \sigma_j^2}\left[\sigma_i\left(\mathbf{U}_j^T \frac{\partial \mathbf{X}}{\partial p_k}\mathbf{V}_i\right) + \sigma_j\left(\mathbf{U}_i^T \frac{\partial \mathbf{X}}{\partial p_k}\mathbf{V}_j\right)^T\right]; \quad \beta_{ji}^k = \frac{1}{\sigma_i^2 - \sigma_j^2}\left[\sigma_j\left(\mathbf{U}_j^T \frac{\partial \mathbf{X}}{\partial p_k}\mathbf{V}_i\right) + \sigma_i\left(\mathbf{U}_i^T \frac{\partial \mathbf{X}}{\partial p_k}\mathbf{V}_j\right)^T\right], \qquad j \neq i \tag{4}$$

$$\alpha_{ii}^k - \beta_{ii}^k = \frac{1}{\sigma_i}\left(\mathbf{U}_i^T \frac{\partial \mathbf{X}}{\partial p_k}\mathbf{V}_i - \frac{\partial \sigma_i}{\partial p_k}\right); \quad \alpha_{ii}^k - \beta_{ii}^k = \frac{1}{\sigma_i}\left(-\mathbf{V}_i^T \frac{\partial \mathbf{X}^T}{\partial p_k}\mathbf{U}_i + \frac{\partial \sigma_i}{\partial p_k}\right), \qquad j = i \tag{5}$$

The sensitivity analysis for PCA may also be developed in the frequency domain, e.g. by considering frequency response functions (FRFs) [19]. As the dynamical system matrices depend on a vector of parameters **p**, the FRF matrix takes the form:

$$\mathbf{H}(\omega, \mathbf{p}) = \left[-\omega^2 \, \mathbf{M}(\mathbf{p}) + i\omega \, \mathbf{C}(\mathbf{p}) + \mathbf{K}(\mathbf{p})\right]^{-1} \tag{6}$$

where ω represents the circular frequency. With regard to sensitivity analysis, the partial derivative of equation (6) with respect to one parameter p_k may be written [19], [24]:

$$\frac{\partial \mathbf{H}}{\partial p_k} = -\mathbf{H}(\omega,\mathbf{p})\frac{\partial\left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)}{\partial p_k}\mathbf{H}(\omega,\mathbf{p}) = \mathbf{H}(\omega,\mathbf{p})\left(\omega^2\frac{\partial \mathbf{M}}{\partial p_k} - i\omega\frac{\partial \mathbf{C}}{\partial p_k} - \frac{\partial \mathbf{K}}{\partial p_k}\right)\mathbf{H}(\omega,\mathbf{p}) \tag{7}$$

Equation (7) provides a way of determining the derivative of the FRF matrix needed for the sensitivity analysis by means of the partial derivative of the system matrices.

Let us consider the FRFs for a single input at location *s*, and build a subset of the FRF matrix (6):

$$\mathbf{H}^s(\omega) = \begin{bmatrix} h_1(\omega_1) & h_1(\omega_2) & ... & h_1(\omega_N) \\ h_2(\omega_1) & h_2(\omega_2) & ... & h_2(\omega_N) \\ ... & ... & ... & ... \\ h_m(\omega_1) & h_m(\omega_2) & ... & h_m(\omega_N) \end{bmatrix} \tag{8}$$

where m is the number of measured co-ordinates and N is the number of frequency lines.
This matrix is the frequency domain analog of the observation matrix **X**. The rows in (8) represent the responses at the measured degrees of freedom (DOFs), while the columns are “snapshots” of the FRFs at different frequencies. We consider that this matrix depends on a given set of parameters. We can assess its principal

components through SVD by (1) where the left singular vectors give spatial information, the diagonal matrix of singular values shows scaling parameters and the right singular vectors represent modulation functions depending on frequency. In other words, this SVD separates information depending on space and on frequency.

The sensitivity of the *i* th principal components can be computed by (3)-(5). First, we compute the SVD of the FRF matrix in (8) for the set of responses and the chosen input location. Then, the partial derivatives of (8) are determined using equation (7). For a particular input, only a subset of the derivatives in (7) is needed.

3. Damage localization based on sensitivity analysis of the FRF matrix

In the following, sensitivity analysis is used to resolve the problem of damage localization and evaluation. We present now some simplifications that may be carried out in experimental practice.

Giving the FRF matrix $\mathbf{H}^s$ for a single input at location *s* of the system and its SVD, the sensitivity computation of the principal components (PCs) requires the partial derivatives $\partial \mathbf{H}^s / \partial p_k$ which are a subset of $\partial \mathbf{H} / \partial p_k$. This quantity may be assessed by (7) requiring the partial derivative of the system matrices with respect to system parameters. If the parameter concerned is a coefficient k_e of the stiffness matrix **K**, the partial derivatives of the system matrices are such that $\partial \mathbf{M} / \partial p_k$ and $\partial \mathbf{C} / \partial p_k$ equal zero and $\partial \mathbf{K} / \partial p_k = \partial \mathbf{K} / \partial k_e$.

Although only a subset of $\partial \mathbf{H} / \partial p_k$ is needed for a particular input *s*, i.e. $\partial \mathbf{H}^s / \partial p_k$, which corresponds to the *s* th column of $\partial \mathbf{H} / \partial p_k$, the calculation of (7) demands the whole matrix $\mathbf{H}$, which turns out to be costly. However, we can compute $\partial \mathbf{H}^s / \partial p_k$ by measuring only some columns of $\mathbf{H}$, as explained below.

We recall that our parameter of interest is a coefficient k_e of the stiffness matrix **K**. Equation (6) shows that FRF matrices are symmetric if system matrices are symmetric. In experiment, the number of degrees of freedom (DOF) equals the number of response sensors. So, the FRF matrix has the same size as the number of sensors. Let us consider for instance a structure instrumented with 4 sensors. The FRF matrix takes the symmetrical form:

$$\mathbf{H}(\omega) = \begin{bmatrix} a & b & c & d \\ b & e & f & g \\ c & f & h & i \\ d & g & i & k \end{bmatrix} \tag{9}$$

Assuming that k_e accords to the second DOF only, we have: $\partial \mathbf{M} / \partial p_k = 0$; $\partial \mathbf{C} / \partial p_k = 0$ and:

$$\frac{\partial \mathbf{K}}{\partial p_k} = \frac{\partial \mathbf{K}}{\partial k_e} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Equation (7) allows us to deduce the partial derivative of the FRF matrix:

$$\frac{\partial \mathbf{H}}{\partial p_k} = -\mathbf{H}(\omega, \mathbf{p}) \frac{\partial \mathbf{K}}{\partial p_k} \mathbf{H}(\omega, \mathbf{p}) = -\begin{bmatrix} \begin{bmatrix} b & e & f & g \end{bmatrix} b \\ \begin{bmatrix} b & e & f & g \end{bmatrix} e \\ \begin{bmatrix} b & e & f & g \end{bmatrix} f \\ \begin{bmatrix} b & e & f & g \end{bmatrix} g \end{bmatrix} \tag{11}$$

To compute the sensitivity of $\mathbf{H}^s$, only the *s* th column of $\partial \mathbf{H} / \partial p_k$ is needed, which is written in (12) in setting $\mathbf{H}_{k_e} = \begin{bmatrix} b & e & f & g \end{bmatrix}^T$. This relies entirely upon the column corresponding to k_e in the FRF matrix in equation (9).

$$\frac{\partial \mathbf{H}^s}{\partial p_k} = -\mathbf{H}_{k_e} \, \mathbf{H}_{k_e, s} \tag{12}$$

$\mathbf{H}_{k_e,s}$ is the s th element of the vector $\mathbf{H}_{k_e}$. Thus, the sensitivity of $\mathbf{H}^s$ with respect to k_e does not involve the entire matrix $\mathbf{H}$; only the column relating to k_e is needed.

4. Localization indicators

When $\partial\mathbf{H}^s/\partial p_k$ has been computed, the sensitivity of principal components can be determined next using equations (3)-(5). The sensitivities of the left singular vectors are good candidates for resolving localization problems of linear-form structures, e.g. chain-like or beam-like structures. In each working condition of the system, we can compute the sensitivity $\partial\mathbf{U}_i/\partial p_k$. The reference state is denoted by $\partial\mathbf{U}_i^R/\partial p_k$, and the deviation of the current condition may be assessed as follows:

$$\Delta = \Delta\left(\partial\mathbf{U}_i/\partial p_k\right) = \partial\mathbf{U}_i/\partial p_k - \partial\mathbf{U}_i^R/\partial p_k \tag{13}$$

Other indicators may be utilized to better locate dynamic change, such as:

$$d_j^{\mathrm{I}} = \frac{1}{r}\left(\Delta\big|_j - \Delta\big|_{j-1}\right) \text{ and } d_j^{\mathrm{II}} = \frac{1}{r^2}\left(\Delta\big|_{j+1} - 2\Delta\big|_j + \Delta\big|_{j-1}\right) \tag{14}$$

where r is the average distance between measurement points. The indicators d^{I} and d^{II}, effectively comparable with the first and second derivatives of vector Δ, may allow the maximization of useful information for damage localization.

The formula d^{II} is widely identified in the literature of damage localization, e.g. in [11, 12]. However, the previous methods compared mode shape vectors or FRF data. In this study, the sensitivity of singular vectors is the subject under examination.

5. Damage evaluation

Once the damage has been localized, it may then be assessed by a technique of model parameter updating. We first assemble the modal features of the system coming from the SVD of the FRF matrix (8) into a vector **v** called the model vector. Principal components (PCs) in **U** or their energies in $\mathbf{\Sigma}$ may be considered to construct the model vector. In the literature, PC vectors have been often considered as more convenient for damage detection so they will be used here. The model vector **v** of a system is usually a nonlinear function of the parameters $\mathbf{p} = [p_1 \ldots p_{n_p}]^T$ where n_p is the number of parameters. The Taylor series expansion (limited to the first two terms) of this vector in terms of the parameters is given by:

$$\mathbf{v}(\mathbf{p}) = \mathbf{v}_a + \sum_{k=1}^{n_p}\frac{\partial\mathbf{v}}{\partial p_k}p_k = \mathbf{v}_a + \mathbf{S}\,\Delta\mathbf{p} \tag{15}$$

where $\mathbf{v}_a = \mathbf{v}\big|_{\mathbf{p}=\mathbf{p}_a}$ represents the model vector evaluated at the linearization point $\mathbf{p} = \mathbf{p}_a$. **S** is the sensitivity matrix of which the columns are the sensitivity vectors. The changes in parameter are represented by $\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_a$.

The residual vector measures the difference between analytical and measured structural behaviors [21]:

$$\mathbf{r}_w = \mathbf{W}_\mathbf{v}\mathbf{r} = \mathbf{W}_\mathbf{v}(\mathbf{v}_m - \mathbf{v}(\mathbf{p})) \tag{16}$$

where $\mathbf{v}_m$ represents measured quantities. The weighting matrix $\mathbf{W}_\mathbf{v}$ (according to a weighted least squares approach) takes care of the relative importance of each term in the residual vector **r**. Substituting (15) into (16) and setting $\mathbf{r}_a = \mathbf{v}_m - \mathbf{v}_a$ leads to the linearized residual vector:

$$\mathbf{r}_w = \mathbf{W}_\mathbf{v}\mathbf{r} = \mathbf{W}_\mathbf{v}(\mathbf{v}_m - \mathbf{v}_a - \mathbf{S}\,\Delta\mathbf{p}) = \mathbf{W}_\mathbf{v}(\mathbf{r}_a - \mathbf{S}\,\Delta\mathbf{p}) \tag{17}$$

Let us define the penalty (objective) function to be minimized as the weighted squared sum of the residual vector:

$$\mathbf{J} = \min\left(\mathbf{r}_w^{\mathrm{T}}\mathbf{r}_w\right) = \min\left(\mathbf{r}^{\mathrm{T}}\mathbf{W}\mathbf{r}\right), \quad \mathbf{W} = \mathbf{W}_\mathbf{v}^{\mathrm{T}}\mathbf{W}_\mathbf{v} \tag{18}$$

Equation (17) may be solved from the objective function derivative $\partial\mathbf{J}/\partial\Delta\mathbf{p} = 0$, which produces the linear equation: $\mathbf{W}_\mathbf{v}\mathbf{S}\,\Delta\mathbf{p} = \mathbf{W}_\mathbf{v}\mathbf{r}_a$, for which the solution is:

$$\Delta\mathbf{p} = (\mathbf{S}^{\mathrm{T}}\,\mathbf{W}\,\mathbf{S})^{-1}\mathbf{S}^{\mathrm{T}}\,\mathbf{W}\,\mathbf{r}_a \tag{19}$$

In our approach, we will assume that the number of measurements is larger than the number of updating parameters, which yields an overdetermined system of equations. Note that the conditioning of the sensitivity matrix **S** plays an important role in the accuracy and the uniqueness of the solution. The solution may be implemented using QR decomposition or SVD, which allows to check the conditioning of **S** [21].

The updating process is shown in Figure 1 where the i th PC is considered as the model vector. The two separated branches correspond to: 1) the analytical model which will be updated; 2) experimental damage responses, input position and correction parameters. The choice of correction parameters may be based on the damage localization results.

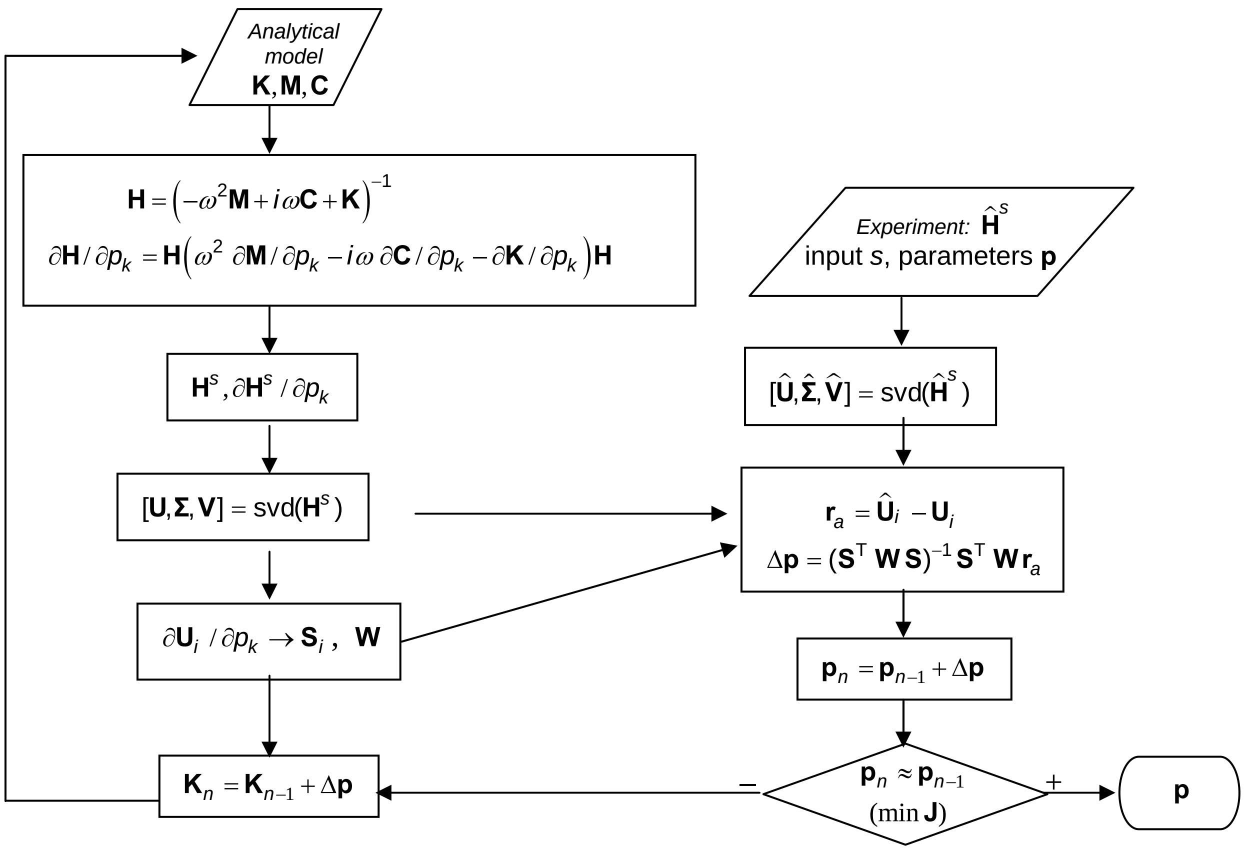

Figure 1: Updating diagram

5.1. Choice of weighting matrix

The positive definite weighting matrix is usually a diagonal matrix whose elements are given by the reciprocals of the variance of the corresponding measurements [22]. This matrix may be based on estimated standard deviations to take into account the relative uncertainty in the parameters and measurements:

$$\mathbf{W}=\mathrm{diag}(w_1,w_2,\ldots,w_i,\ldots,w_m)\,;\quad \mathbf{W}=\mathbf{Var}^{-1}\ \text{with}\ \mathbf{Var}=\mathrm{diag}(\sigma_1^2,\sigma_2^2,\ldots,\sigma_i^2,\ldots,\sigma_m^2) \tag{20}$$

where m is the number of measurements and σ_i is the standard deviation of the i th measurement. The relationship between **W** and the variance matrix **Var** is assumed to be reciprocal because a correct data has a small variance but presents a significant weight in the estimate. Similarly, in our problem of damage evaluation, the weight may be intensified according to damaged locations in order to improve the efficiency of the technique, i.e. to accelerate the convergence and raise the accuracy.

6. Application to damage localization

6.1. Numerical example of a cantilever beam

Let us examine a steel cantilever beam with a length of 700 *mm*, and a square section of dimension 14 *mm*. The beam is modeled by twenty finite elements as illustrated in Figure 2. The input location is chosen at node 7 and the snapshot matrix is assembled from FRFs corresponding to the vertical displacements at nodes 1 to 20.

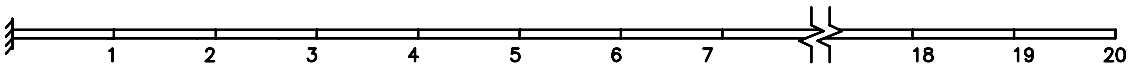

Figure 2: Cantilever beam

We model the damage by a stiffness reduction of a beam element. Four states are examined: the reference (healthy) state, and 3 levels of damage (L_1, L_2, L_3) induced by a reduction of stiffness of respectively 10%, 20% and 40%. The damage is assumed to occur in element 12. Note that the maximum deviations on the first three frequencies from the reference state are 0.70%, 1.41% and 2.81% for the 3 levels respectively.

For illustration, sensitivity analysis results are shown in Figure 3 according to the parameter $p_k = k_{15}$. Note that similar results were obtained for other stiffness parameter in various positions. The FRFs were considered in the frequency range from 0 Hz to 165Hz at intervals of 1 Hz.
Figure 3 shows the sensitivity difference $\Delta(\partial \mathbf{U}_1 / \partial k_{15})$ of the first left singular vector with respect to the coefficient associated to the 15th DOF in the stiffness matrix and its derivatives d^{I}, d^{II}.

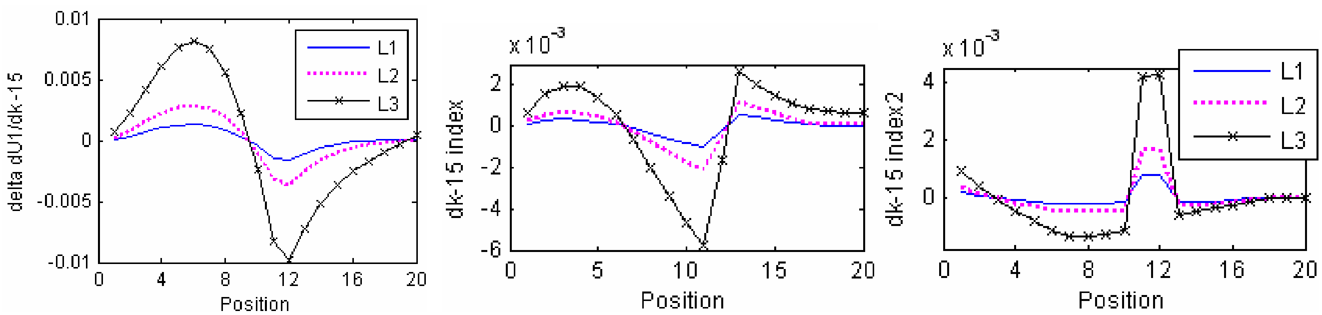

Figure 3: $\Delta(\partial \mathbf{U}_1 / \partial k_{15})$, d^{I}, d^{II} for 3 levels with $p_k = k_{15}$

It is observed that the $\Delta(\partial \mathbf{U}_1 / \partial k_{15})$ curves are discontinuous at DOFs 11 and 12; index d^{I} shows a discontinuity with large variations around element 12 and finally, index d^{II} allows us to discover explicitly the position of the damaged element.

The method reveals itself robust when damage develops in several elements. Localization results are shown in Figure 4 in two different cases of damage. We note that index d^{II} does not indicate the same level of damage in the damaged elements. However, the damage locations are accurately indicated. The difference in magnitude is due to unequal sensitivities for various damage locations, as discussed in [10].

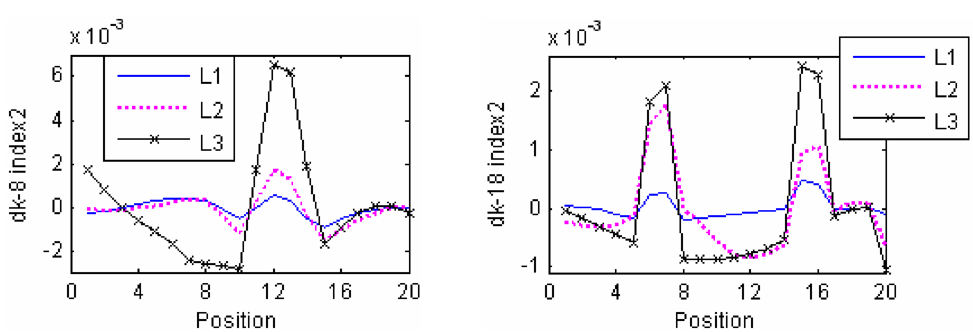

Figure 4: d^{II} for 3 levels, damage at elements 12, 13, 14 (left, $p_k = k_8$); 7 and 16 (right, $p_k = k_{18}$)

Damage evaluation

In this section, the beam is examined with the model of 10 elements. We consider firstly the damage occurring in a single element. For example, the stiffness of element 5 is subjected to reduction. FRF matrices are considered for an excitation at node 9. Frequency lines are selected from 0 to 200 rad/sec. Because the damage localization is readily identified, the first PC and its derivative with respect to the damaged element are considered. For the sake of efficiency and accuracy, only the stiffness of the damaged element is taken into account in the parameter vector **p**.

The diagonal weighting matrix defined by (20), as discussed above, is intensified in entries according to damaged elements in order to increase the calculation efficiency. For this purpose, the standard deviation of the principal component (PC) vector elements is preliminarily assumed to be 10% of the corresponding element in the considered PC. And in particular, to take advantage of sensitivity vector, the entries matched to the damaged elements are assigned a lower variance so as to give a larger weighting in the algorithm. A fitting variance for measurement in the damaged elements can improve significantly the convergence speed. In the tests below, the standard deviation values according to the damaged elements were multiplied by a factor between 1% and 3%. The evaluation of some of the damage is shown in Figure 5. Several levels are considered so that the stiffness of element 5 is reduced of 5%, 10%, 20%, 30% and 50%. The data has also been perturbed with 5% of noise, but it still shows satisfactory results.

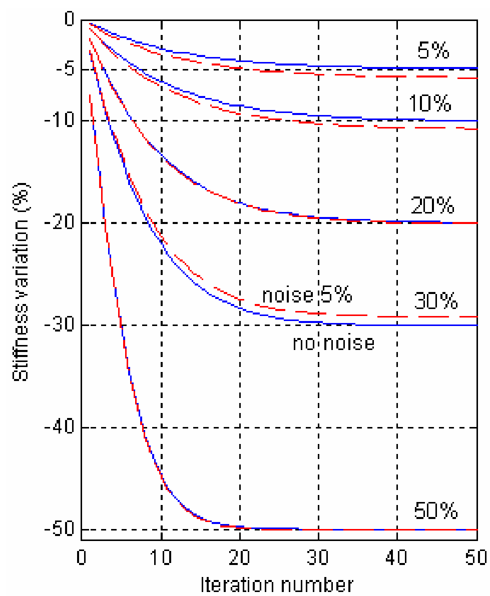

Figure 5: Evaluation of damage, noise free (_____) and noise at 5% (__ __ __)

Now let us examine the problem of some elements being damaged; those elements may be side by side or distant from each other. The results of various occurrences of damage are given in Figure 6 for the same or for different levels, respectively. 5% of noise was taken into account.

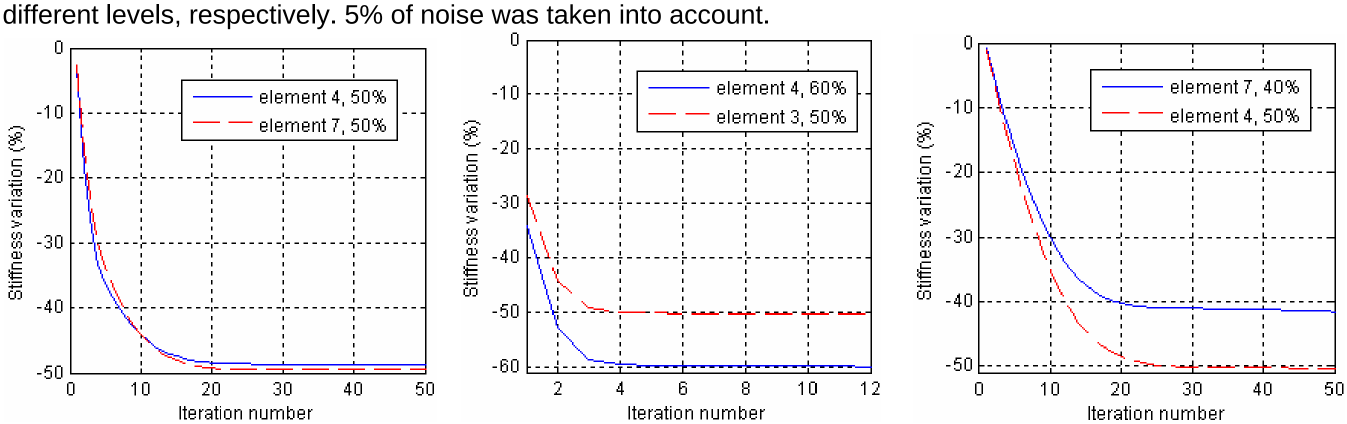

Figure 6: Evaluation of damage when stiffness reduction occurred in some elements

It is worth noting that the accuracy of the result depends on several factors when many elements are damaged, namely, the relative position of the damaged elements and the difference between their levels of damage. Input position also plays an important role on the number of iterations of the updating process. Generally, the estimate is more effective if the degrees of damage of different elements are not too different. Data recorded from an excitation close to the damaged location often accelerates the convergence speed.

6.2. Experiments involving a mass-spring system

The next example involves the system of eight DOFs shown in Figure 7 and for which data are available in [23]. The system comprises 8 translating masses connected by springs. In the undamaged configuration, all the springs have the same constant: 56.7 kN/m. Each mass weighs 419.5 grams; the weight is 559.3 grams for the mass located at the end which is attached to the shaker.

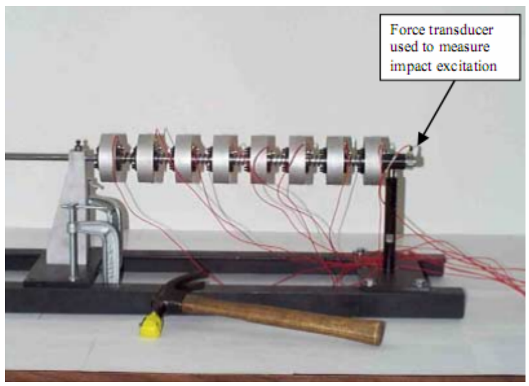

Figure 7: Eight degrees of freedom system

The acceleration responses and also the FRFs of all the masses are measured with the excitation force applied to mass 1 – the first mass at the right-hand end (Figure 7). The FRFs are assembled so as to localize the damage by the proposed method. Frequency lines are selected from 0 to 55 Hz at intervals of 0.1562 Hz. Two types of excitation are produced: hammer impact and random excitation using a shaker.

First, several experiments were implemented with the system in the healthy state (denoted “H”) and in the damage state (denoted “D”). Then damage was simulated by a 14% stiffness reduction in spring 5 (between masses 5 and 6). As the excitation was applied only on mass 1, the partial derivative was taken with respect to the first DOF (equations (9)-(12)). The damage index d^{I} is shown in Figure 8, where the healthy states are denoted “H” and the damaged states “D”.

The damage index marks a clear distinction between the two groups – healthy and damaged. Healthy states show regular indexes in all positions, so they do not display any abnormality. By contrast, all the “damaged” curves reveal a high peak in point 6 or 5 where the slope is the most noticeable.

Damage evaluation

In order to evaluate the damage, it is necessary to build an analytical model. The first PC and its sensitivity are assessed in all conditions for the damage evaluation. So, for the sake of consistent data, a frequency line is selected from 6 to 29 Hz, which covers only the first physical mode and removes noisy low frequencies. The variations in stiffness of spring 5 are shown in Figure 9 for impact and random excitations respectively. In both types of excitation, the results are very satisfactory. All false-positives indicate small stiffness variations in the element concerned, and they show any detection of damage. By contrast, all damaged states give an evaluation very close to the exact damage – a stiffness reduction of 14%.

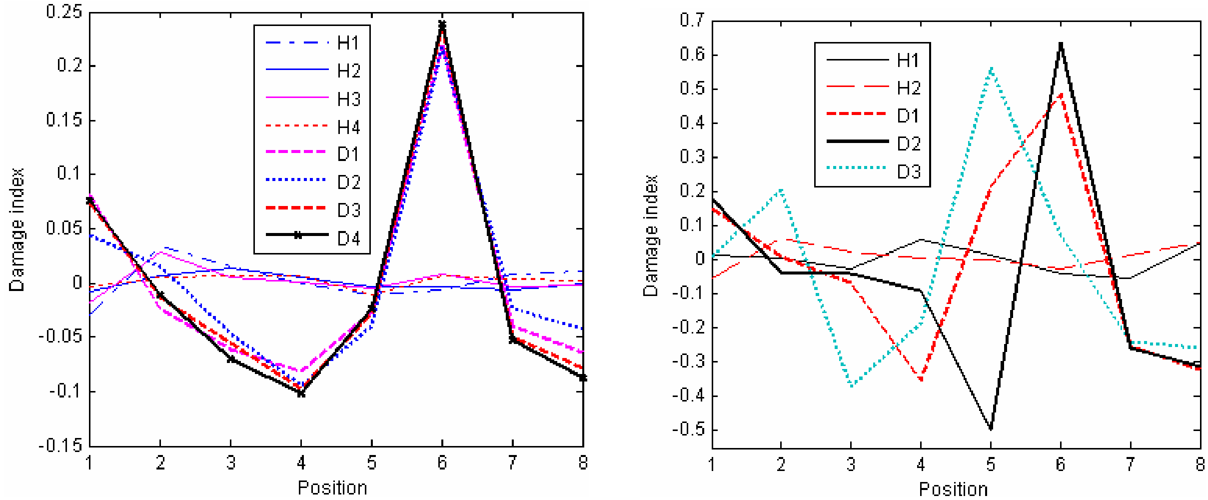

Figure 8: d^{I} by impact (left) and random excitation (right)

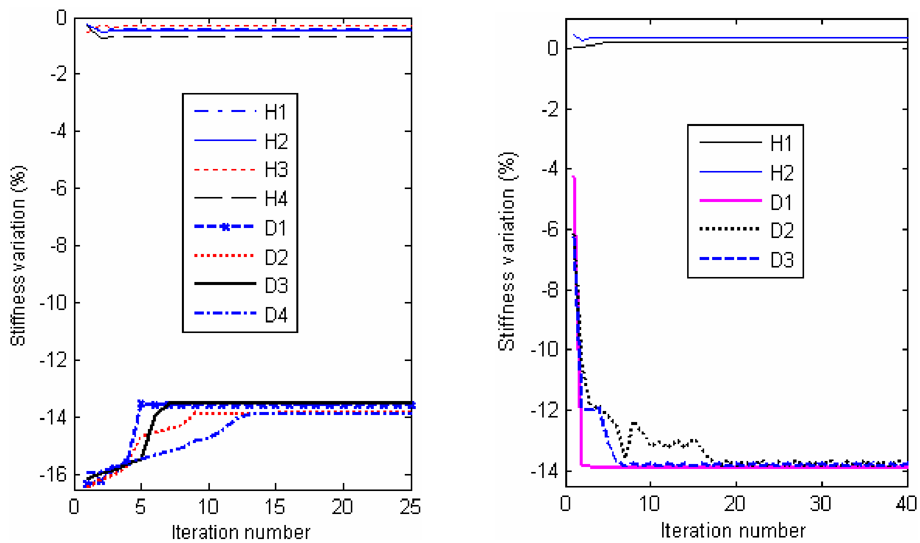

Figure 9: Evaluation of damage in cases of impact (left) and random (right) excitation

7. Conclusions

The sensitivity computation of principal components by analytical methods has been verified in [19] in both the time domain and the frequency domain. The contribution of the present study is its application of sensitivity analysis in the frequency domain to the problem of damage localization and evaluation. Damage localization is achieved in the first step as a result of the difference in principal component sensitivity between the reference and the damaged states. The method has proved efficient in damage localization in circumstances where either only one or where several elements are involved.

As sensitivity computation from FRFs is easy, the technique should be suitable for online monitoring. The analytical model is not required in damage localization but it is necessary in carrying out damage evaluation.

8. References

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