Early Prediction of Isolas during Frequency Sweeps: An Analytical, Numerical, and Experimental Study

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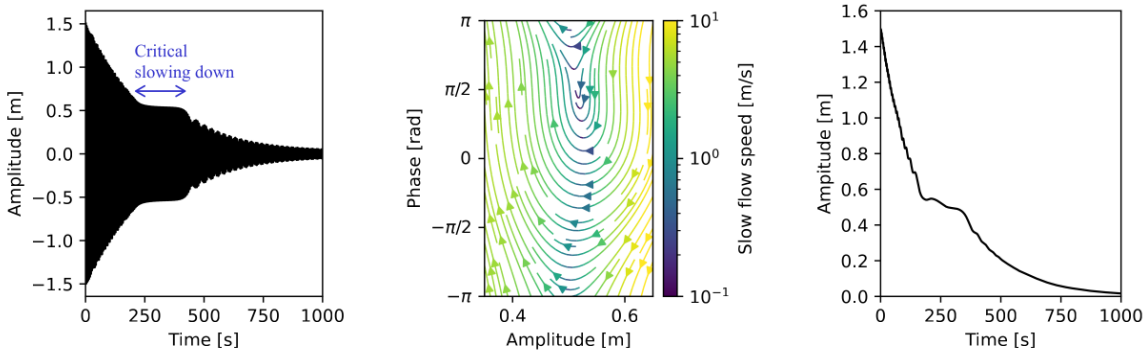
Abstract. Isolated response branches occurring in nonlinear dynamical systems remain undetected by standard vibration testing procedures. These branches, commonly referred to as isolas, can induce large-amplitude oscillations and thereby pose a significant threat to structural integrity. Predicting their onset before they become directly observable is an open challenge. In this work, we show that critical slowing down provides a robust early-warning signature of isola emergence, identified via recovery time indicators. This is established analytically using the slow flow equations of the Duffing oscillator, and validated numerically and experimentally.

Introduction

Isolas are disconnected from the main frequency response branch and therefore cannot be captured through classical sine sweep testing, leading to a potential risk to structural integrity. Despite recent advances in nonlinear system identification, predicting their onset before direct observation remains an open challenge. Critical slowing down offers a promising route to fill this gap. This well-established phenomenon, documented across a wide range of complex systems [1], manifests as a progressive increase in recovery time as a system approaches a bifurcation. Its application to nonlinear dynamical systems has recently begun to attract attention [2], though its connection to isola emergence, governed by a pair of saddle-node bifurcations on a detached branch, has not been established. The present work addresses this challenge, deriving early-warning indicators from the transient response of the Duffing oscillator, and validating them numerically and experimentally on an electronic oscillator mimicking a Duffing oscillator with multiple subharmonic isolas.

Results and Discussion

The 1:3 subharmonic resonance of the Duffing oscillator is studied as a representative case. After the two subharmonic solution branches have merged and the isola has disappeared, the system retains a memory of the vanished bifurcation in the form of a saddle-node ghost. When perturbed at an energy level commensurate with the former subharmonic amplitude, the transient response exhibits a pronounced slowing down near the ghost saddle-node, accompanied by a temporary phase locking between the forcing and the free-response component. As shown in Figure 1a, this slowing down manifests as a marked deceleration in the transient envelope at an amplitude consistent with the subharmonic branch. The underlying mechanism is clarified analytically through the slow flow of the Duffing oscillator derived with the method of multiple scales. When the system trajectory passes in the vicinity of the ghost saddle-node, where the slow-flow velocity reaches a well-defined minimum, a slowing down in the transient response is observed (Figures 1b–1c). The analytical prediction is in excellent agreement with numerical simulations, confirming that recovery time provides a robust and quantifiable indicator of isola proximity, detectable before the isola itself becomes observable.



(a) System response. (b) State space of the slow flow. (c) Envelope of the slow flow.

Figure 1: Numerical (a) and analytical (b-c) observations of the critical slowing down during the transient response of the Duffing oscillator in the vicinity of the 1:3 subharmonic resonance.

References

[1] Scheffer M. (2009) Early-warning signals for critical transitions.. Nature, **461**:53-59.

[2] Habib G. (2023) Predicting saddle-node bifurcations using transient dynamics: a model-free approach. Nonlinear Dyn, **111**:20579-20596.