Technical note

“Sectorial coordinates of thin-walled cold-formed sections”

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This document explains the calculation of the sectorial coordinates ω required for the calculation of the bimoment developing on a thin-walled cross-section. Figure 1 shows a thin-walled member of arbitrary cross-section shape with wall thickness considerably smaller than the other sectional dimensions. The global coordinate system is located at the center of gravity of the cross-section, with x-axis coinciding with the beam axis. In addition, the *s* coordinate is located at the centerline of the cross-section, and an arbitrary point P_o is chosen for its origin ($s_o = 0$).

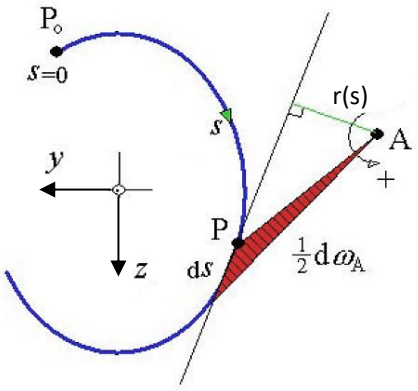


Figure 1: Notations of the sectorial coordinates calculation

The sectorial coordinate ω_A measured from the pole A is defined according to Figure 1 as a gradient $d\omega_A = \pm r(s)ds$ where the sine is determined by the assumed positive rotation around the pole. Therefore:

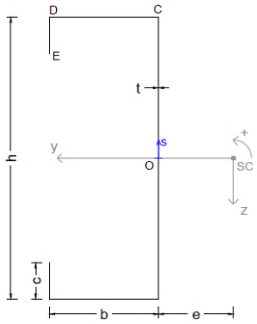
$$\omega_A = \int_{P_o}^{P} d\omega_A = \pm \int_{P_o}^{P} r(s)ds$$

Although for the commonly used sections the location of the pole – shear centre – is known, the sectorial coordinate ω_o at P_o is unknown. The general methodology is to calculate the sectorial coordinates from the shear center as a function of the unknown ω_o. In case that the position of the shear center is also unknown (complex cross-section shapes), then the pole is assumed at one random point (y_{sc}, z_{sc}).

Knowing that $S_{\omega y} = \int_A \omega y dA = 0$, $S_{\omega z} = \int_A \omega z dA = 0$ and $S_\omega = \int_A \omega dA = 0$ expressed on the principal axes, all the unknowns can be found.

In the following, the sectorial coordinates for a lipped-channel section, a sigma section of which the shear centre cannot be inscribed in the rectangle defined by the two flanges (type 1) and a sigma section of which the shear centre can be inscribed in the rectangle defined by the two flanges (type 2) are calculated; the computations are assisted by Figure 2 and all the results are summarized in Table 1. It should be noted that all computations are always referring to the centre line dimensions.

> Sectorial coordinates of a lipped-channel section:



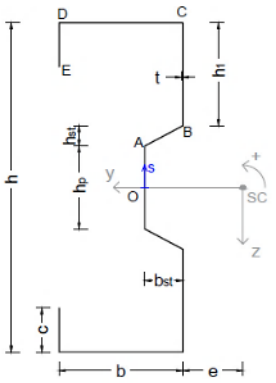
$$\omega_O = 0 \text{ (due to symmetry)}$$

$$\omega_C = -\int_O^C r_1(s)ds = -e[s_1]_0^{\frac{h}{2}} = -\frac{eh}{2}$$

$$\omega_D = \omega_C + \int_C^D r_2(s)ds = -\frac{eh}{2} + \frac{h}{2}[s_2]_0^b = -\frac{eh}{2} + \frac{h}{2}b = \frac{h}{2}(b-e)$$

$$\omega_E = \omega_D + \int_D^E r_3(s)ds = \frac{h}{2}(b-e) + (b+e)[s_3]_0^c = \frac{h}{2}(b-e) + (b+e)c$$

> Sectorial coordinates of a type 1 sigma section:



$$\omega_O = 0 \text{ (due to symmetry)}$$

$$\omega_A = -\int_O^A r_1(s)ds = -(b_{st}+e)[s_1]_0^{\frac{h_p}{2}} = -(b_{st}+e)\frac{h_p}{2}$$

$$\omega_B = \omega_A - \int_A^B r_2(s)ds = -(b_{st}+e)\frac{h_p}{2} - 2\hat{A}_{ABSC}$$

$$\hat{A}_{ABSC} = \frac{1}{2}\left|det\begin{vmatrix} e & -h_{st}-\frac{h_p}{2} \\ b_{st}+e & -\frac{h_p}{2} \end{vmatrix}\right| = \frac{1}{2}\left|-\frac{eh_p}{2} - (b_{st}+e)(-h_{st}-\frac{h_p}{2})\right|$$

$$2\hat{A}_{ABSC} = \left|-\frac{eh_p}{2} + b_{st}h_{st} + \frac{b_{st}h_p}{2} + eh_{st} + \frac{eh_p}{2}\right| = b_{st}h_{st} + \frac{b_{st}h_p}{2} + eh_{st}$$

$$\omega_B = -b_{st}\frac{h_p}{2} - \frac{eh_p}{2} - b_{st}h_{st} - \frac{b_{st}h_p}{2} - eh_{st} = -e\left(\frac{h_p}{2} + h_{st}\right) - b_{st}(h_p + h_{st})$$

$$\omega_C = \omega_B - \int_B^C r_3(s)ds = \omega_B - e[s_3]_0^{h_1} = \omega_B - eh_1$$

$$\omega_C = -e\left(\frac{h_p}{2} + h_{st} + h_1\right) - b_{st}(h_p + h_{st}) = -\frac{eh}{2} - b_{st}(h_p + h_{st})$$

or

$$\omega_C = -\int_O^C r_4(s)ds = 2\hat{A}_{OABCSC} = -2\left[\frac{1}{2}e\frac{h}{2} + \frac{1}{2}b_{st}\left(\frac{h_p}{2} + \frac{h_p}{2} + h_{st}\right)\right] = -e\frac{h}{2} - b_{st}(h_p + h_{st})$$

$$\omega_D = \omega_C + \int_C^D r_5(s)ds = -e\frac{h}{2} - b_{st}(h_p + h_{st}) + \frac{h}{2}[s_5]_0^b = -e\frac{h}{2} - b_{st}(h_p + h_{st}) + \frac{h}{2}b$$

$$\omega_D = \frac{h}{2}(b-e) - b_{st}(h_p + h_{st})$$

$$\omega_E = \omega_D + \int_D^E r_6(s)ds = \frac{h}{2}(b-e) - b_{st}(h_p + h_{st}) + (b+e)[s_6]_0^c$$

$$\omega_E = \frac{h}{2}(b-e) - b_{st}(h_p + h_{st}) + (b+e)c$$

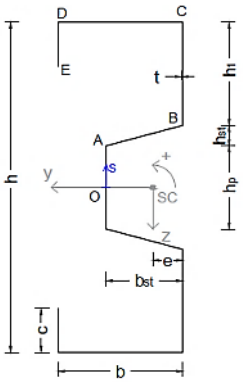
> Sectorial coordinates of a type 2 sigma section:

$\omega_O = 0$ (due to symmetry)

$$\omega_A = -\int_O^A r_1(s)ds = -(b_{st}-e)[s_1]_0^{\frac{h_p}{2}} = -(b_{st}-e)\frac{h_p}{2}$$

$$\omega_B = \omega_A - \int_A^B r_2(s)ds = -(b_{st}-e)\frac{h_p}{2} - 2\hat{A}_{ABSC}$$

$$\hat{A}_{ABSC} = \frac{1}{2}\left|det\begin{vmatrix} -e & -h_{st}-\frac{h_p}{2} \\ b_{st}-e & -\frac{h_p}{2} \end{vmatrix}\right| = \frac{1}{2}\left|\frac{eh_p}{2} - (b_{st}-e)(-h_{st}-\frac{h_p}{2})\right|$$



$$2\hat{A}_{ABSC} = \left|\frac{eh_p}{2} + b_{st}h_{st} + \frac{b_{st}h_p}{2} - eh_{st} - \frac{eh_p}{2}\right| = b_{st}h_{st} + \frac{b_{st}h_p}{2} - eh_{st}$$

$$\omega_B = -b_{st}\frac{h_p}{2} + \frac{eh_p}{2} - b_{st}h_{st} - \frac{b_{st}h_p}{2} + eh_{st}$$

$$\omega_B = e\left(\frac{h_p}{2} + h_{st}\right) - b_{st}(h_p + h_{st})$$

$$\omega_C = \omega_B + \int_B^C r_3(s)ds = \omega_B + e[s_3]_0^{h_1} = \omega_B + eh_1$$

$$\omega_C = e\left(\frac{h_p}{2} + h_{st} + h_1\right) - b_{st}(h_p + h_{st}) = \frac{eh}{2} - b_{st}(h_p + h_{st})$$

$$\omega_D = \omega_C + \int_C^D r_4(s)ds = e\frac{h}{2} - b_{st}(h_p + h_{st}) + \frac{h}{2}[s_4]_0^b = e\frac{h}{2} - b_{st}(h_p + h_{st}) + \frac{h}{2}b$$

$$\omega_D = \frac{h}{2}(b+e) - b_{st}(h_p + h_{st})$$

$$\omega_E = \omega_D + \int_D^E r_5(s)ds = \frac{h}{2}(b+e) - b_{st}(h_p + h_{st}) + (b-e)[s_5]_0^c$$

$$\omega_E = \frac{h}{2}(b+e) - b_{st}(h_p + h_{st}) + (b-e)c$$

Table 1 presents absolute vales and the relevant sine should be obtained from Figure 2 for the examined point.

Table 1: Sectorial coordinates shown in Figure 2

ω / section	Channel section	Sigma (type 1)	Sigma (type 2)
ω_0	0	0	0
ω_1	---	$\frac{h_p}{2}(b_{st}+e)$	$\frac{h_p}{2}(b_{st}-e)$
ω_2	---	$e\left(\frac{h_p}{2}+h_{st}\right)+b_{st}(h_p+h_{st})$	$e\left(\frac{h_p}{2}+h_{st}\right)-b_{st}(h_p+h_{st})$
ω_3	$\frac{eh}{2}$	$\frac{eh}{2}+b_{st}(h_p+h_{st})$	$\frac{eh}{2}-b_{st}(h_p+h_{st})$
ω_4	$\frac{h}{2}(b-e)$	$\frac{h}{2}(b-e)-b_{st}(h_p+h_{st})$	$\frac{h}{2}(b+e)-b_{st}(h_p+h_{st})$
ω_5	$\frac{h}{2}(b-e)+c(b+e)$	$\frac{h}{2}(b-e)-b_{st}(h_p+h_{st})+c(b+e)$	$\frac{h}{2}(b+e)-b_{st}(h_p+h_{st})+c(b-e)$

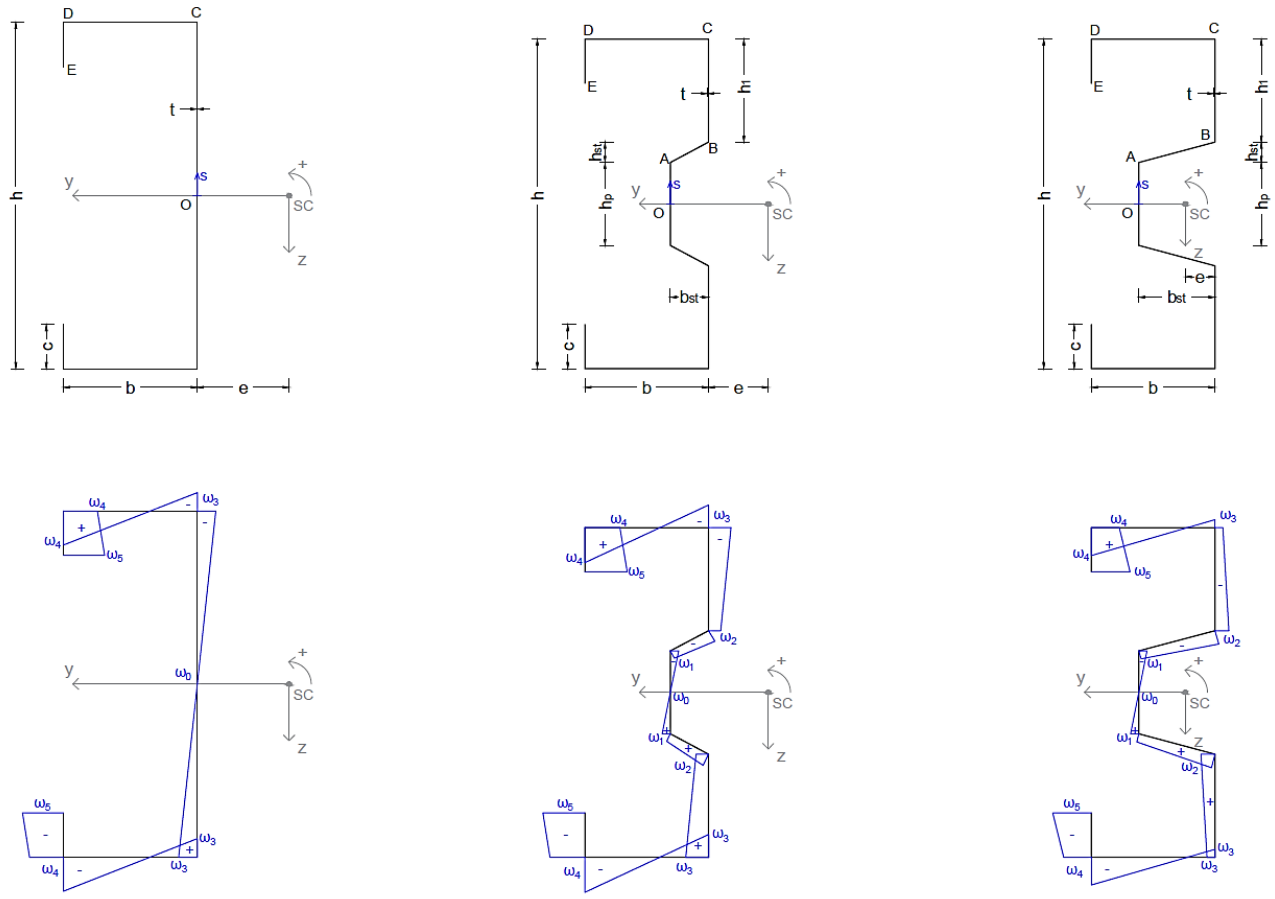


Figure 2: Sectorial coordinates of lipped-channel (left), type 1 sigma (middle) and type 2 sigma (right) sections