

eurodyn 2026
27th September - 1st October
Hannover

# PEPPER: Discovery of Nonlinear Dynamics for Jumping Activities under an Auditory Stimulus


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LIÈGE université

fnrs LA LIBERTÉ DE CHERCHER

29th September 2026 – EURODYN in Hannover, Germany


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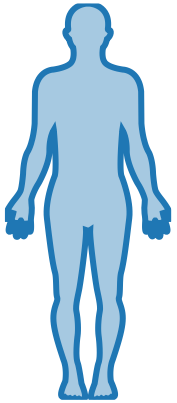
[4] F.R.S.-FNRS, National Fund for Scientific Research, Belgium

# Motivation





# Motivation

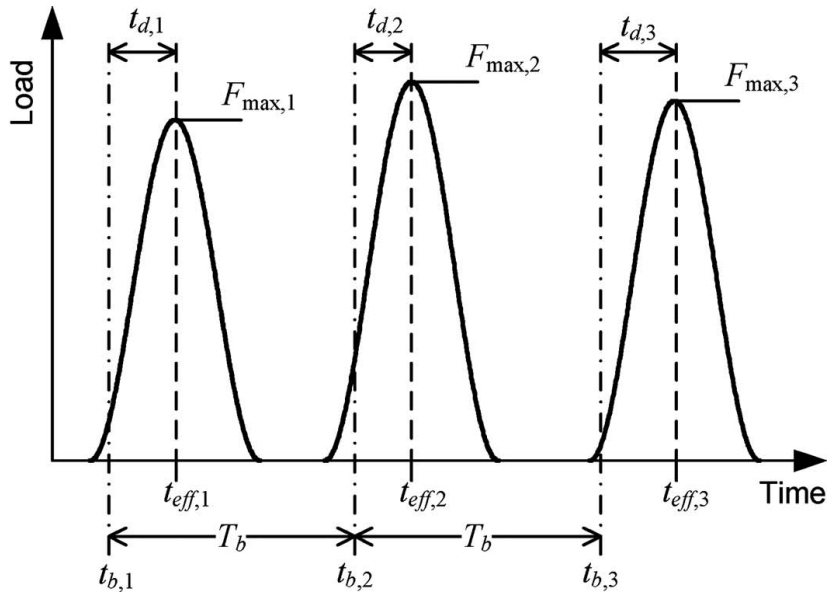






$$\ddot{Q}_k + 2\Gamma_k\Omega_k\dot{Q}_k + \Omega_k^2 Q_k = R_k\left[\{\ddot{z}_j\}_{j=1}^{J}\right]$$

## Experimental models




Sim et al., 2008

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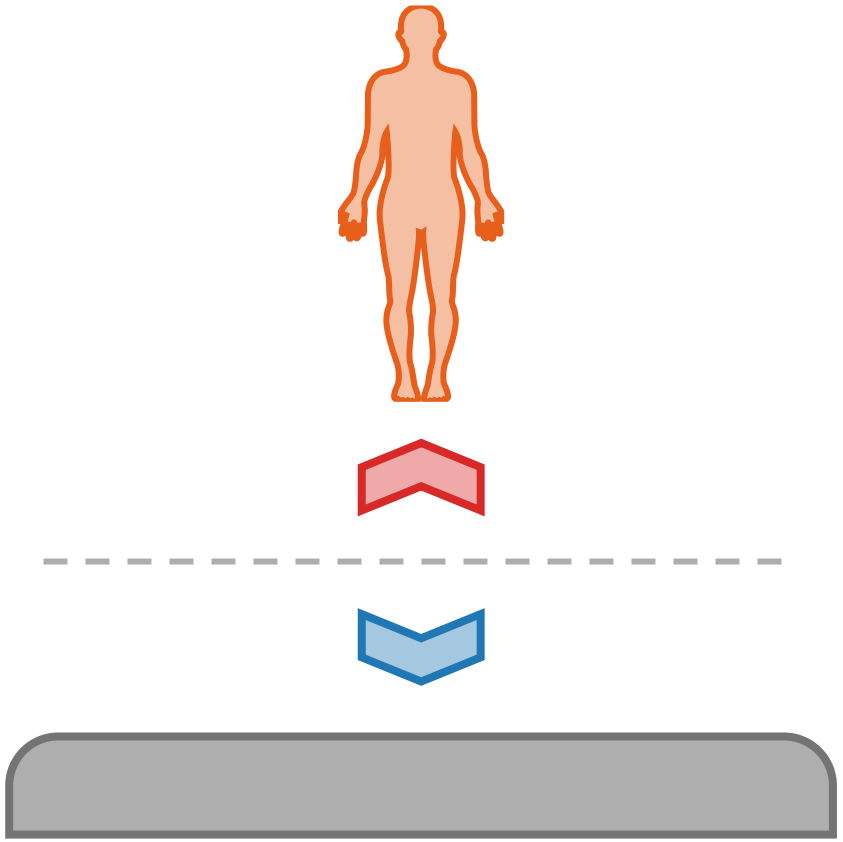

$$\ddot{z}_i - f(\dot{z}_i, z_i; \kappa_i, \varepsilon_i, \gamma_i, \eta_i, \overline{\omega}_i, \langle z_i \rangle) = r_i\left[\left\{\ddot{Q}_n\right\}_{n=1}^{N}\right]$$

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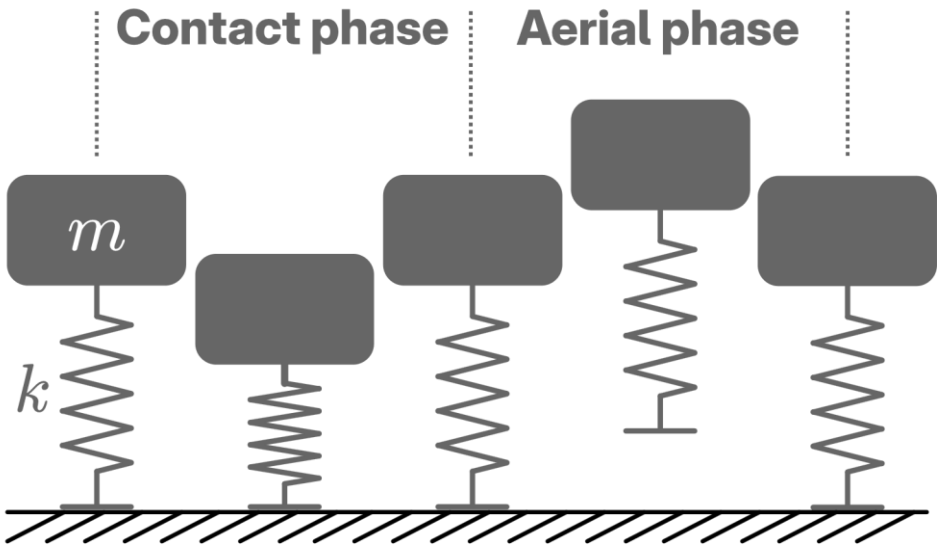
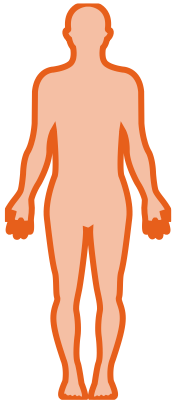
**Phenomenological models**




Blickhan, 1989

# Motivation



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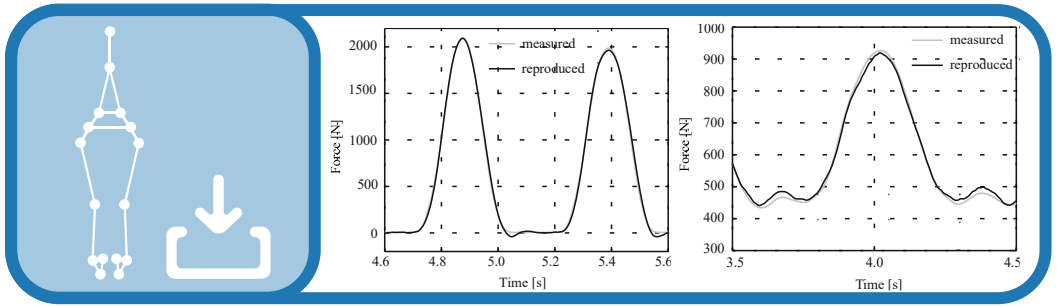


Racic et al., 2010

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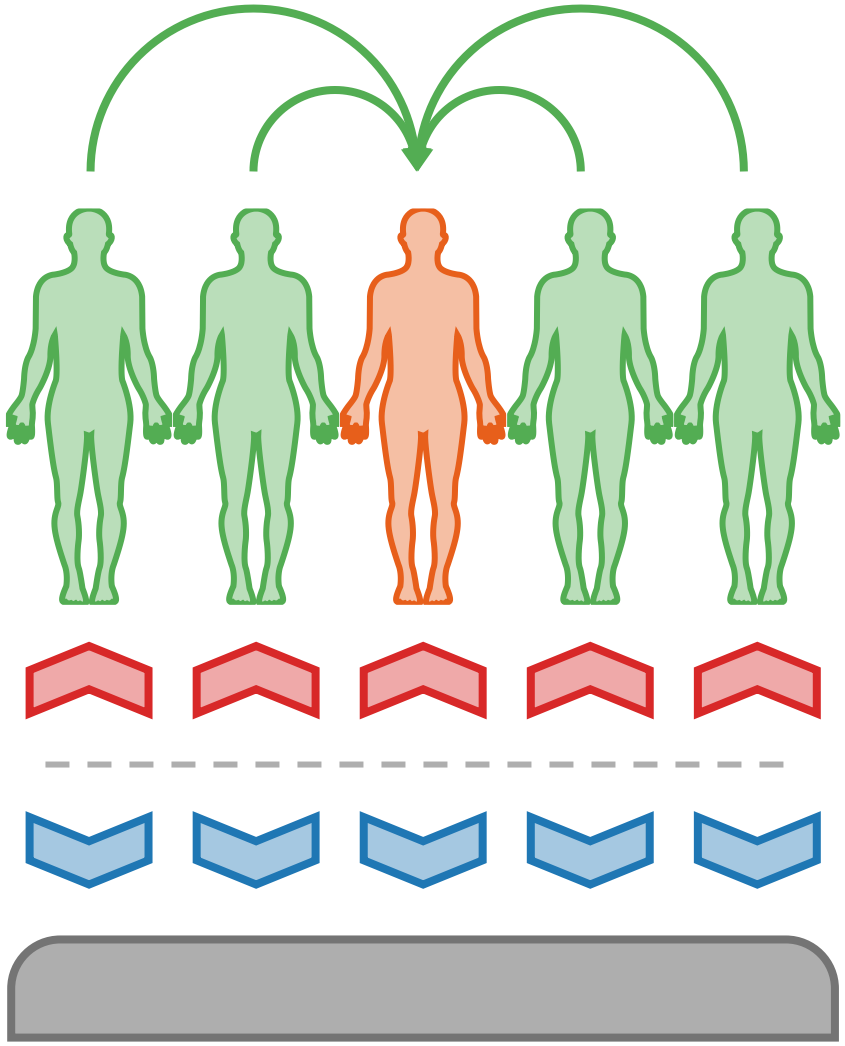

$$\ddot{z}_i - f(\dot{z}_i, z_i; \kappa_i, \varepsilon_i, \gamma_i, \eta_i, \overline{\omega}_i, \langle z_i \rangle) = r_i\left[\{\ddot{Q}_n\}_{n=1}^{N}\right] + s_i\left[\{\ddot{z}_j\}_{j=1}^{J}\right]$$

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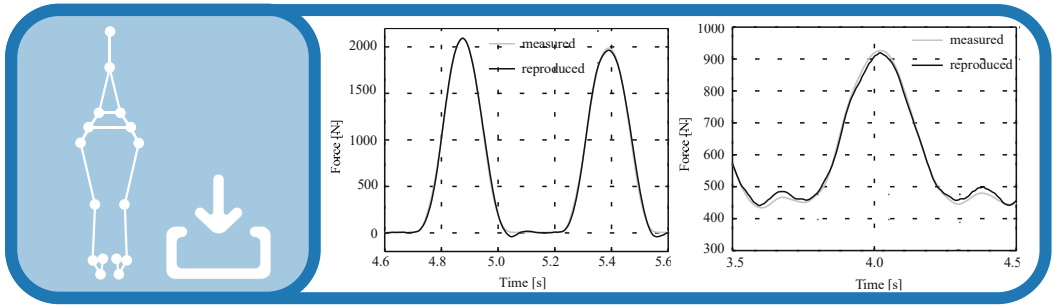


Racic et al., 2010



Chen et al., 2019

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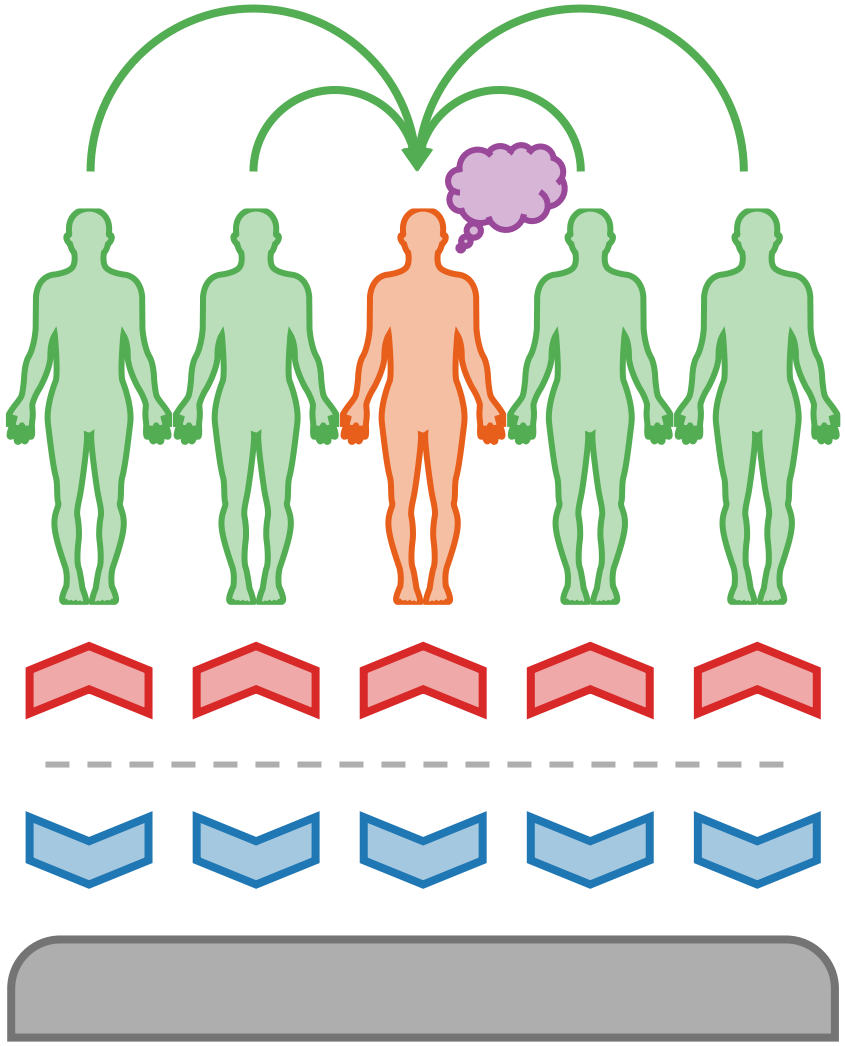

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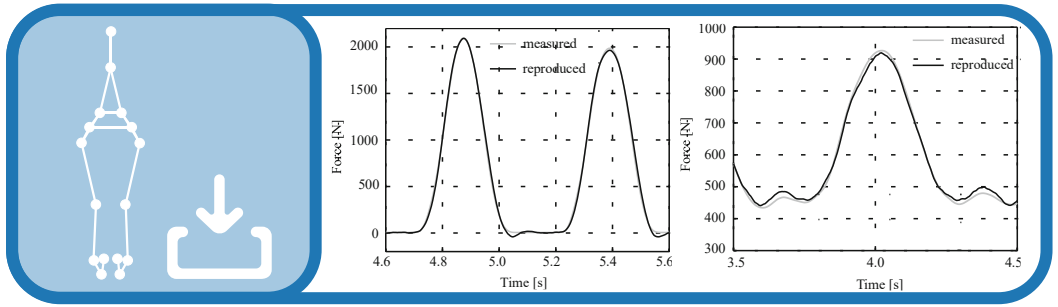
Cannon, 2021

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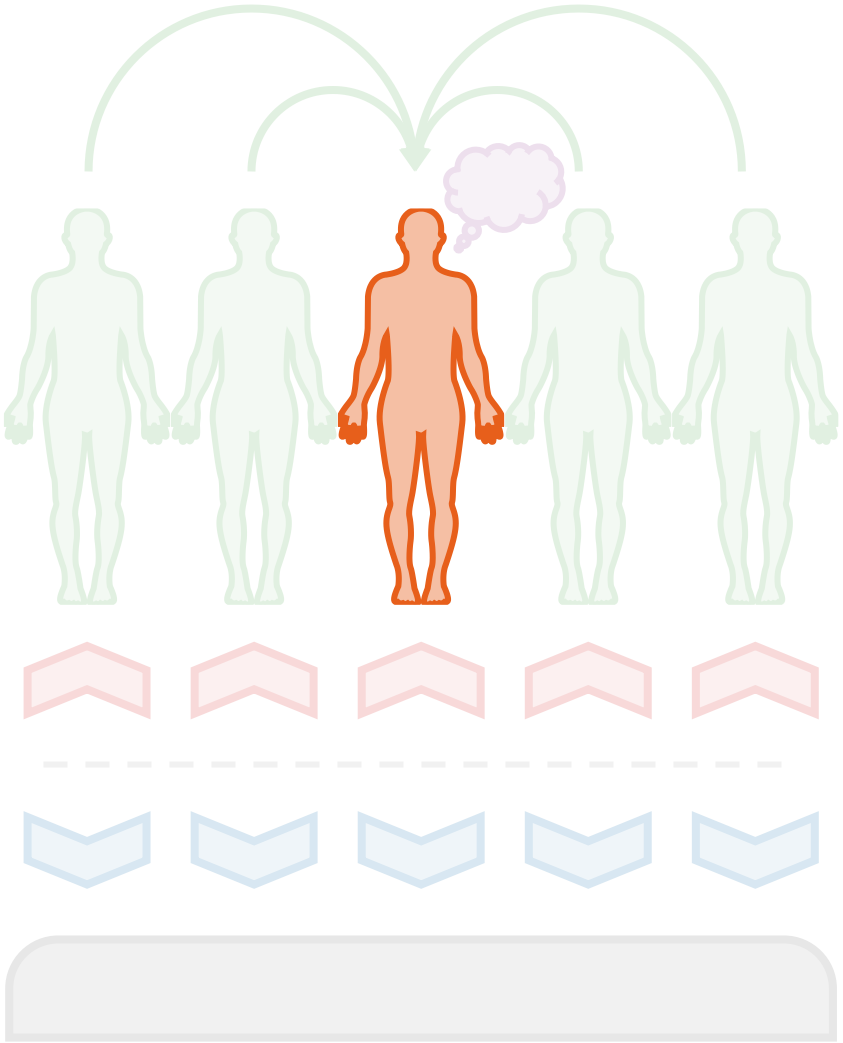

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Cannon, 2021




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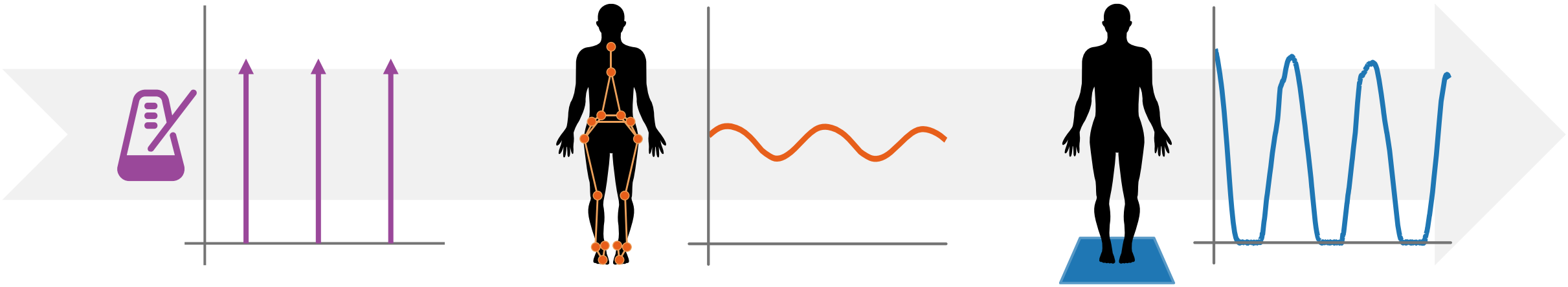
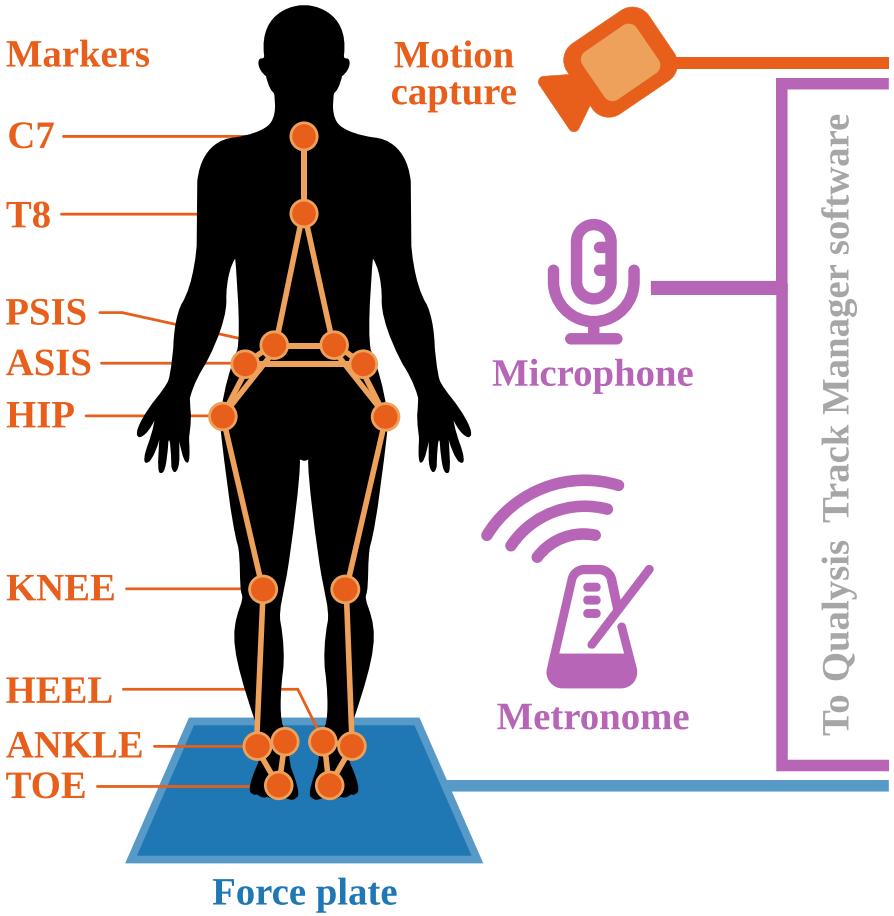
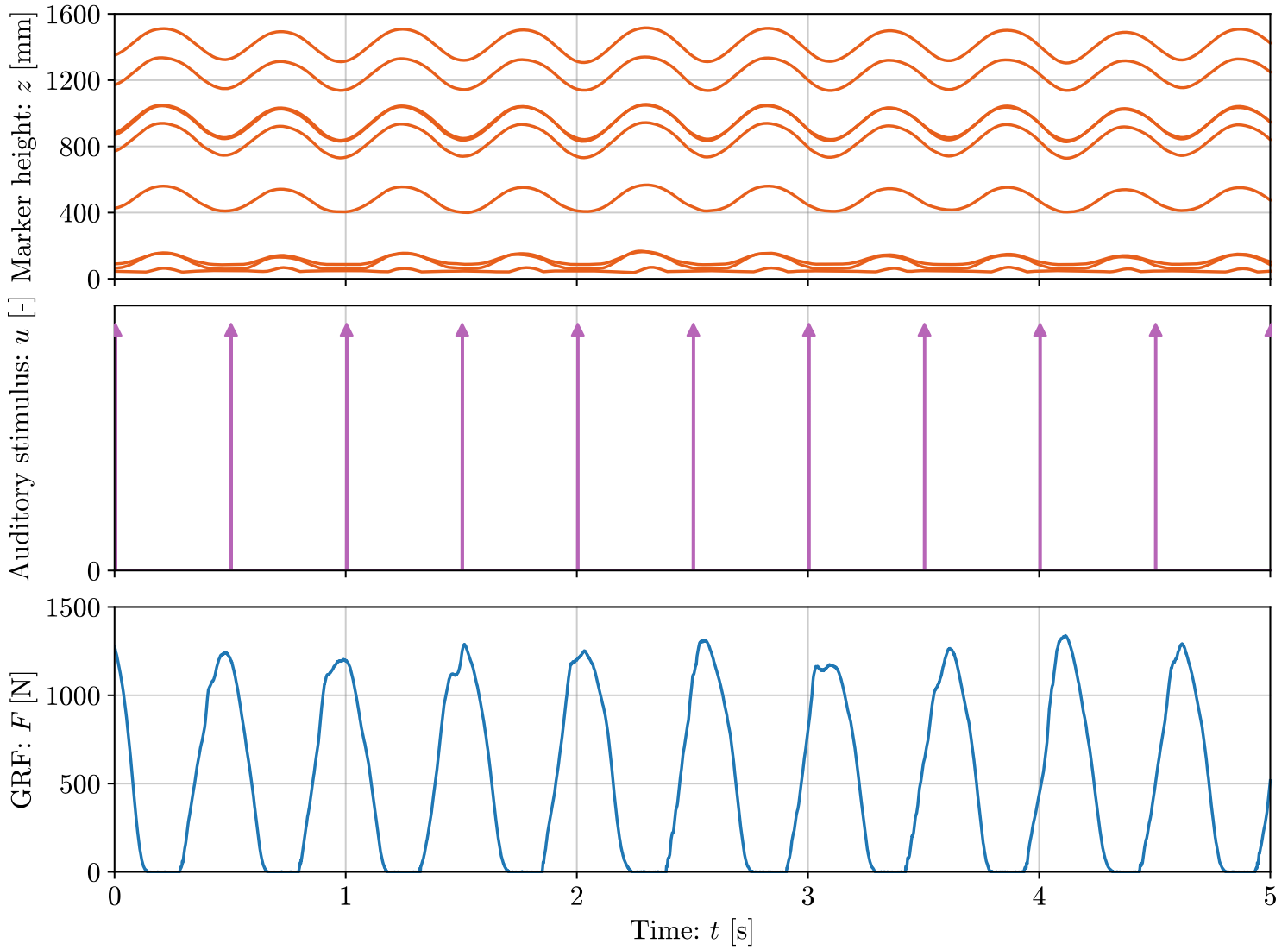
# Nonlinear Model Design



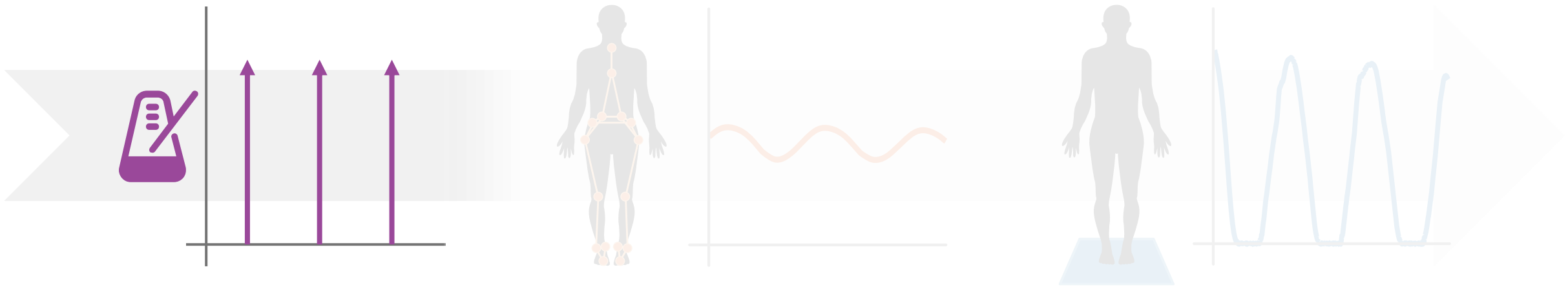
# Experimental Setup

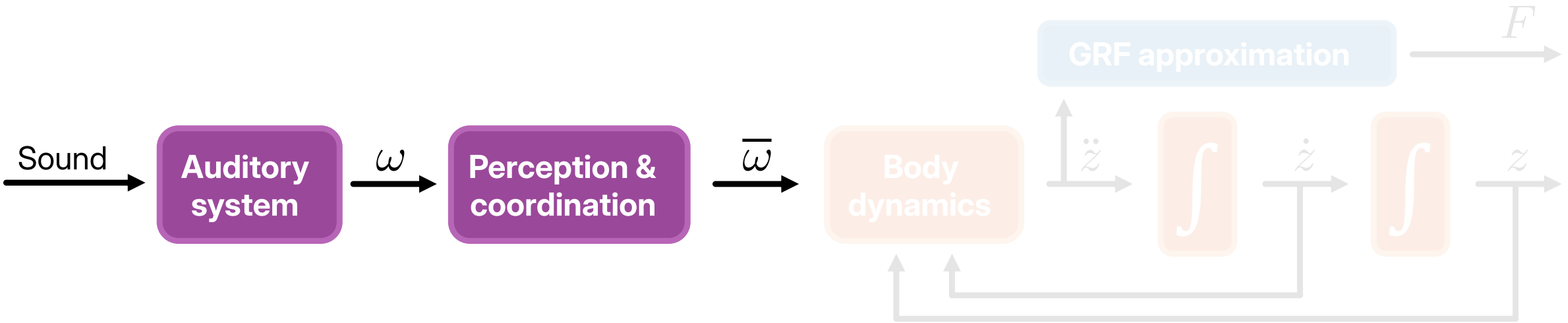


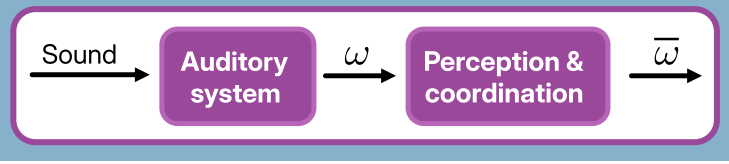

GRF: Ground Reaction Force

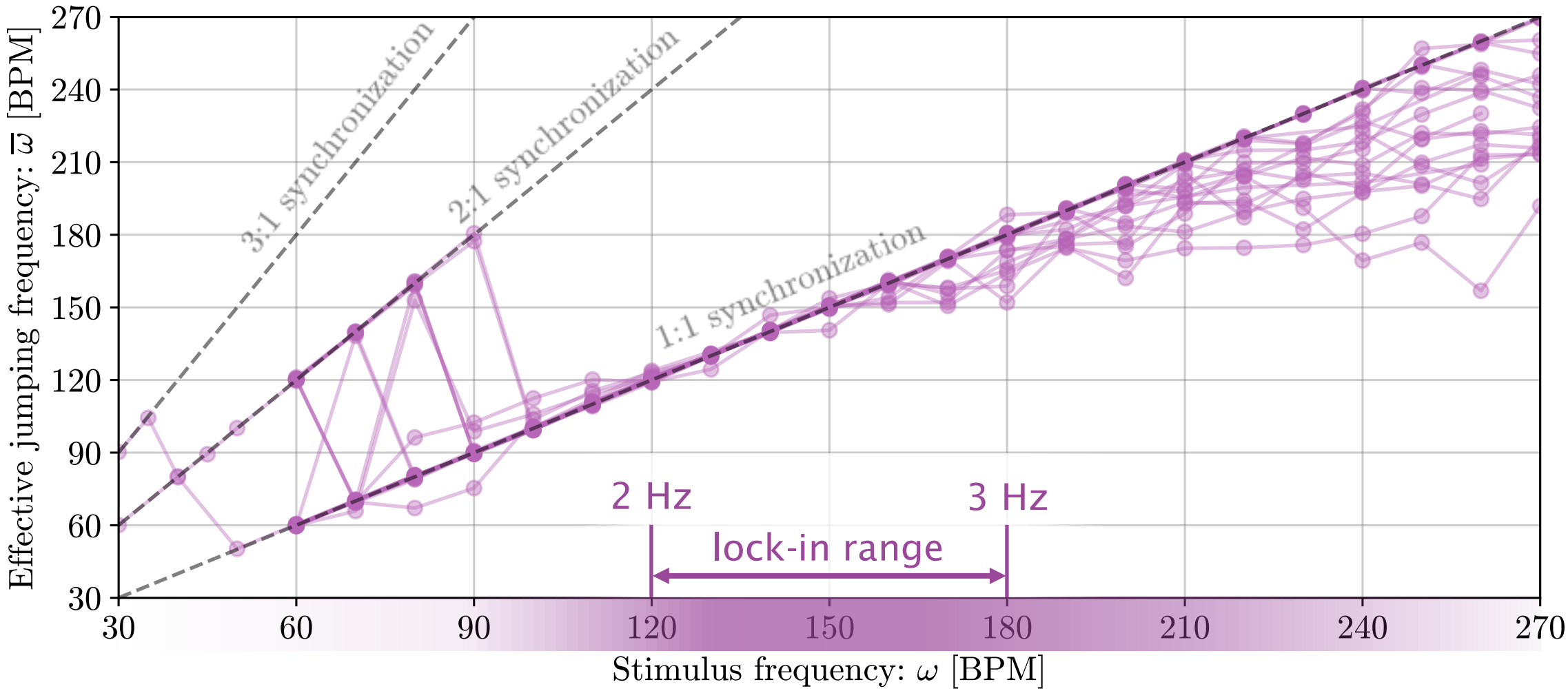
# Nonlinear Model Design





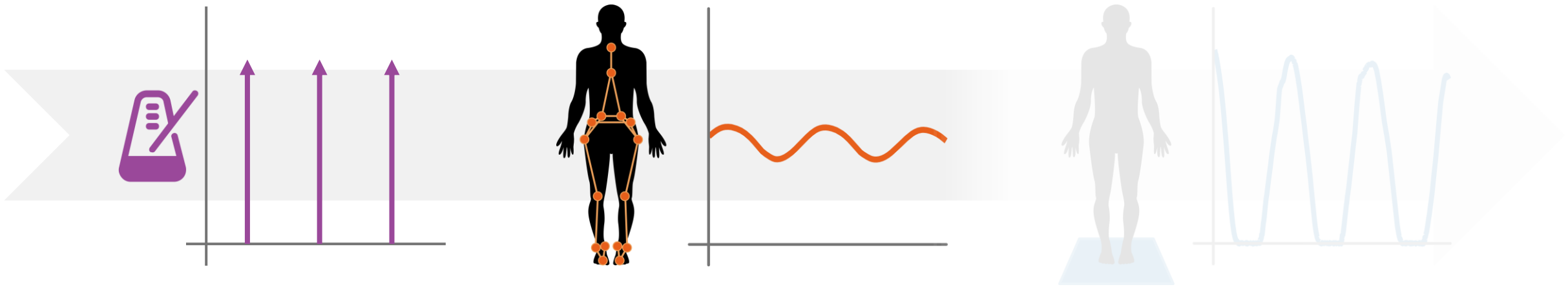
# Synchronization

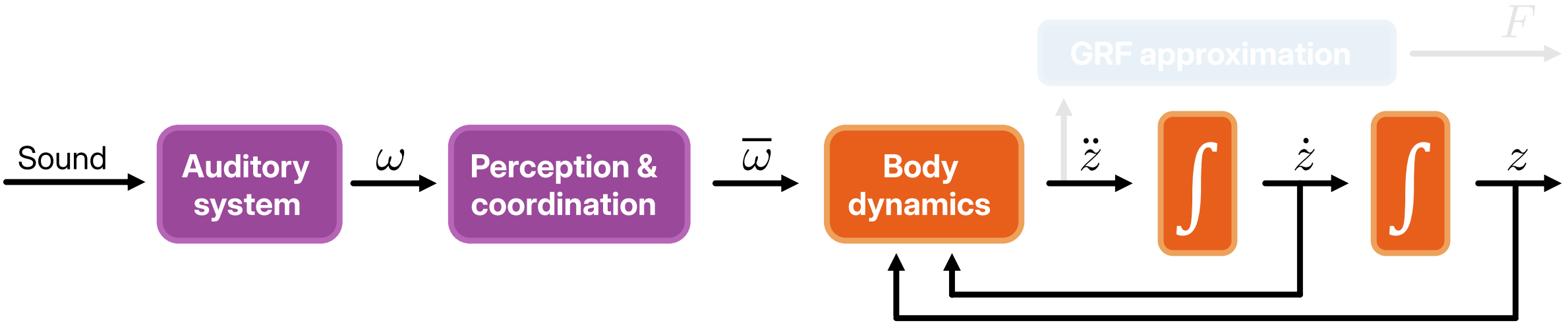


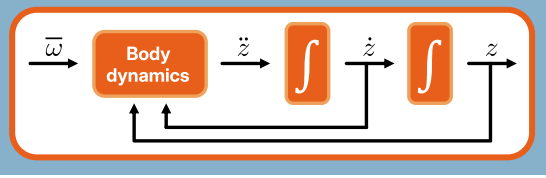

21 participants = 13 males + 8 females ($21 \pm 3$ years old)

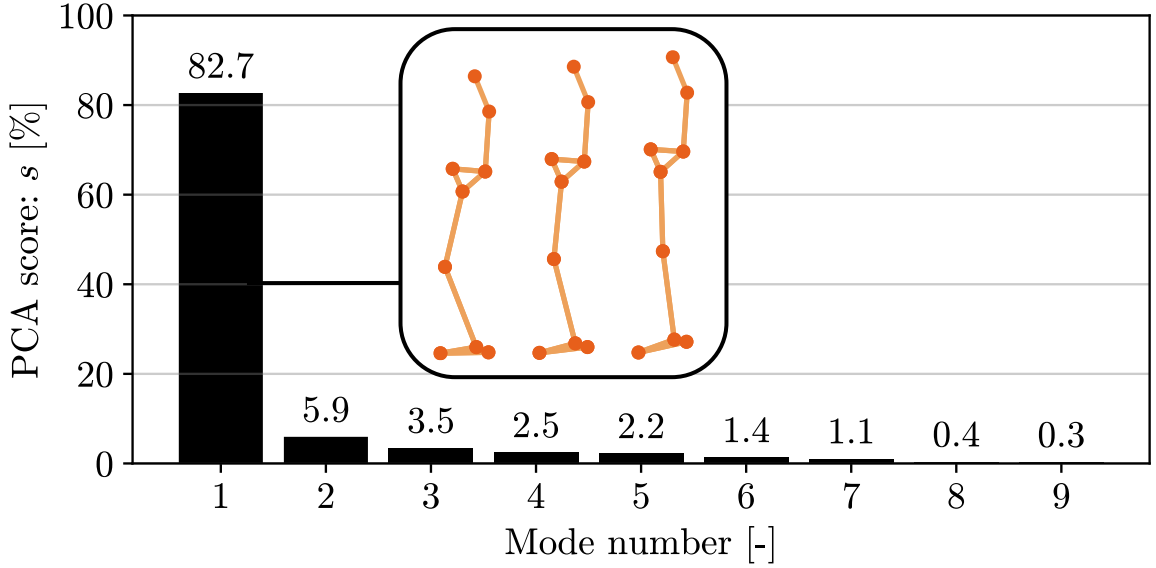
# Nonlinear Model Design

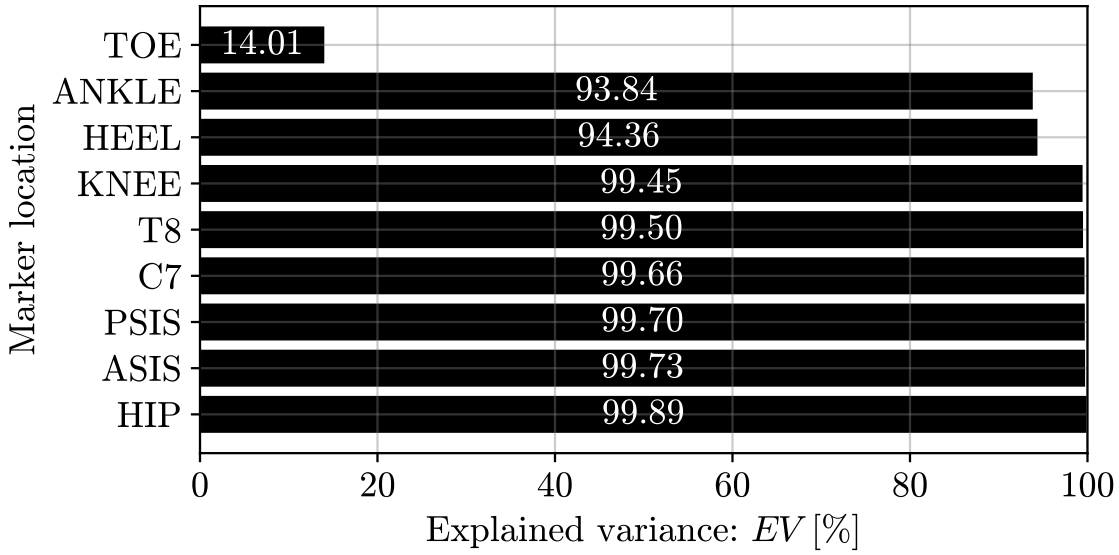


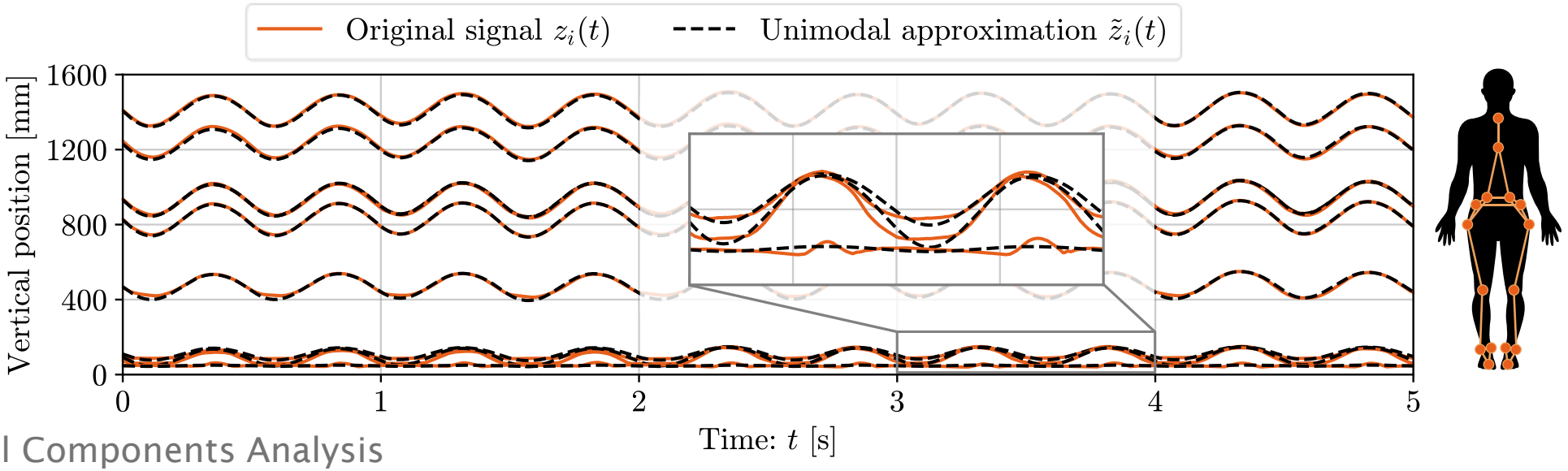


# Unimodal Modeling



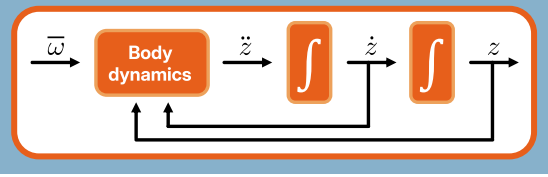



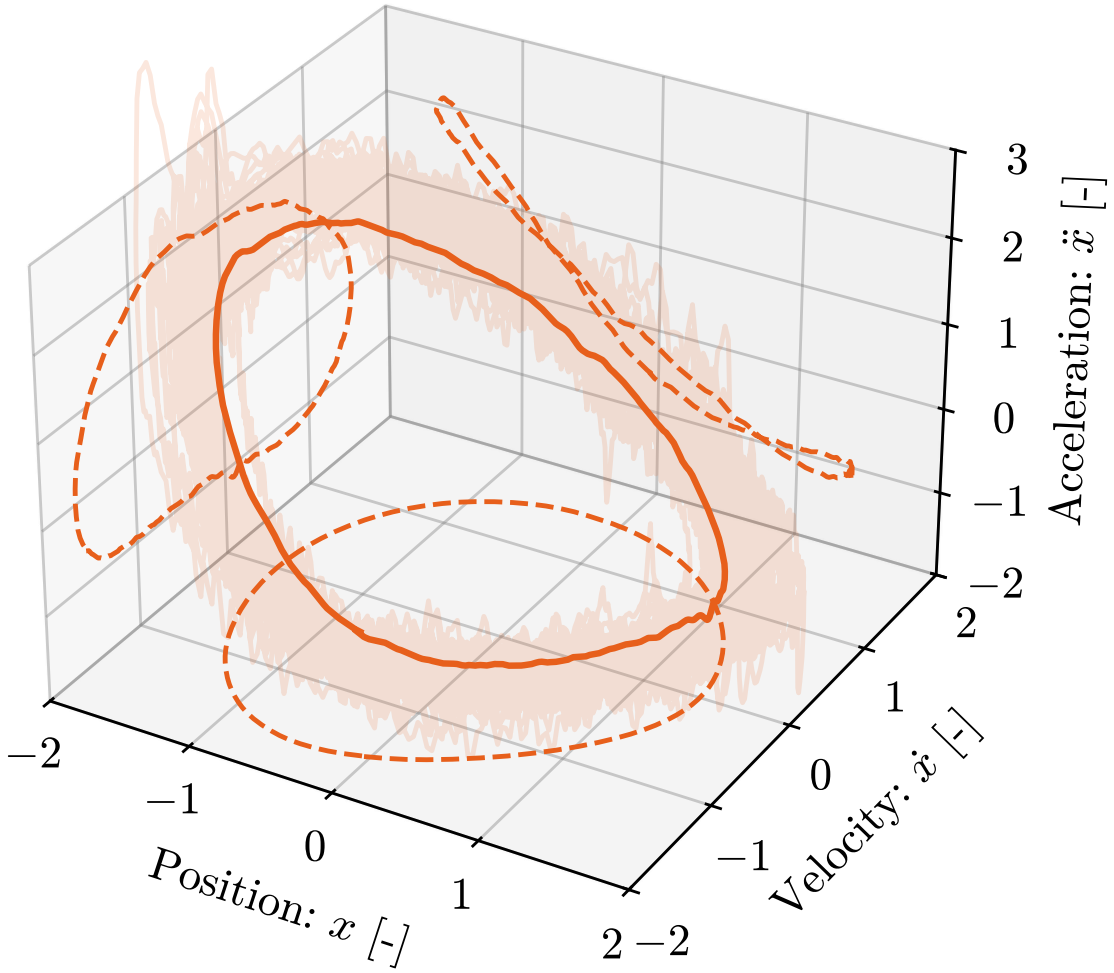




PCA: Principal Components Analysis

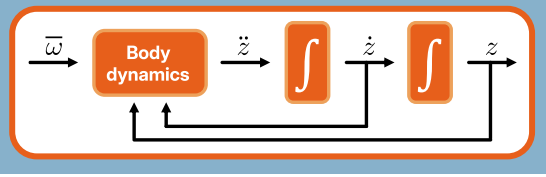
# Nonlinear Modeling



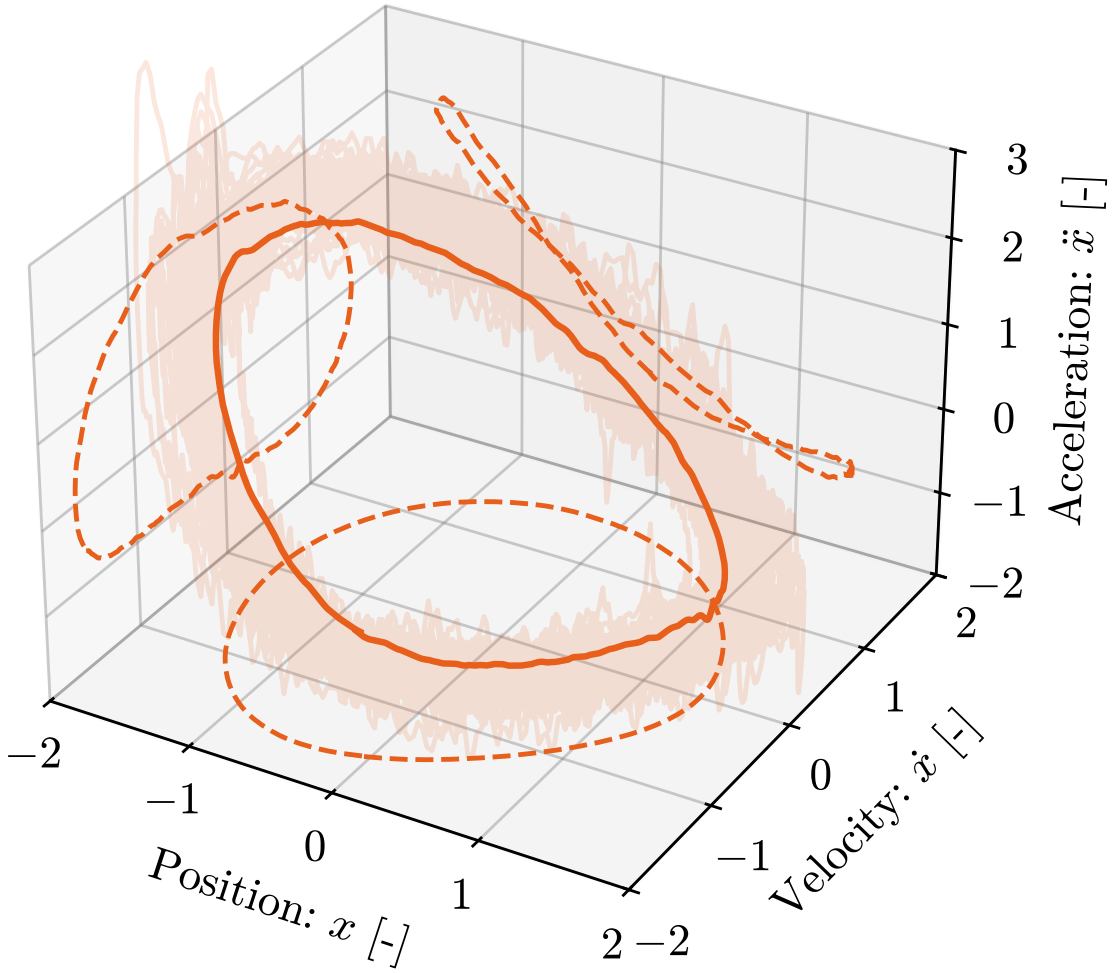

$$\ddot{z}(t) = f(\dot{z}, z)$$



# Nonlinear Modeling



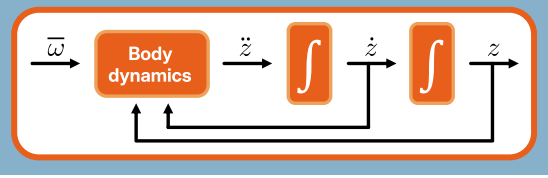

$$\ddot{z}(t) = f(\dot{z}, z) \;\leftrightarrow\; \ddot{x}(\tau) = \varphi(\dot{x}, x)$$




Dimensionless form:

$$\tau = \bar{\omega} t \qquad x = (z - \langle z \rangle)\frac{\bar{\omega}^2}{g}$$

# Nonlinear Modeling

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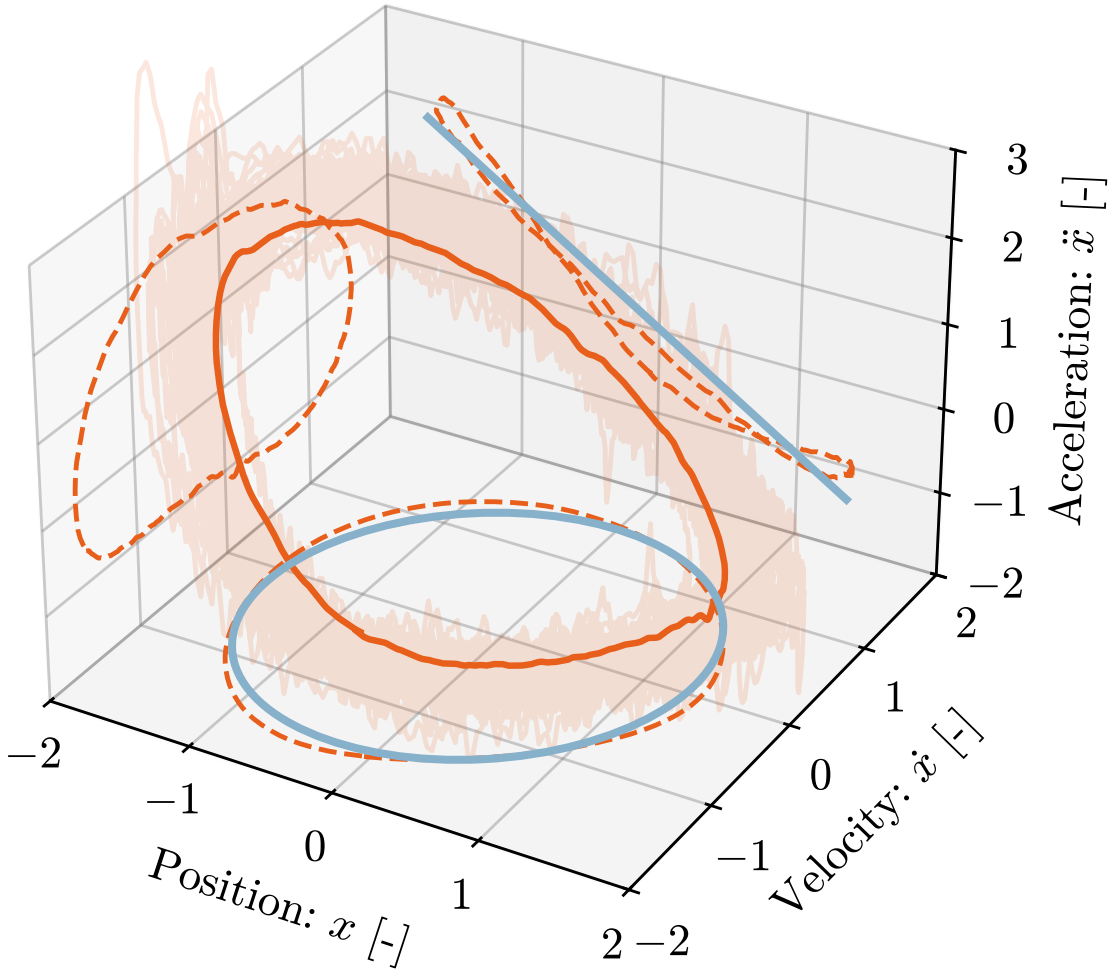


Dimensionless form:

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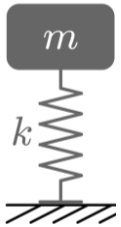
At first order:

Harmonic oscillator $\kappa x$



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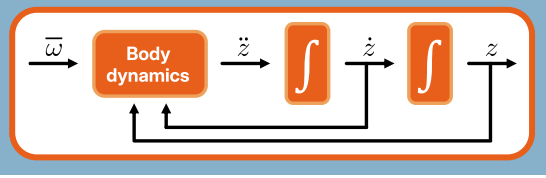


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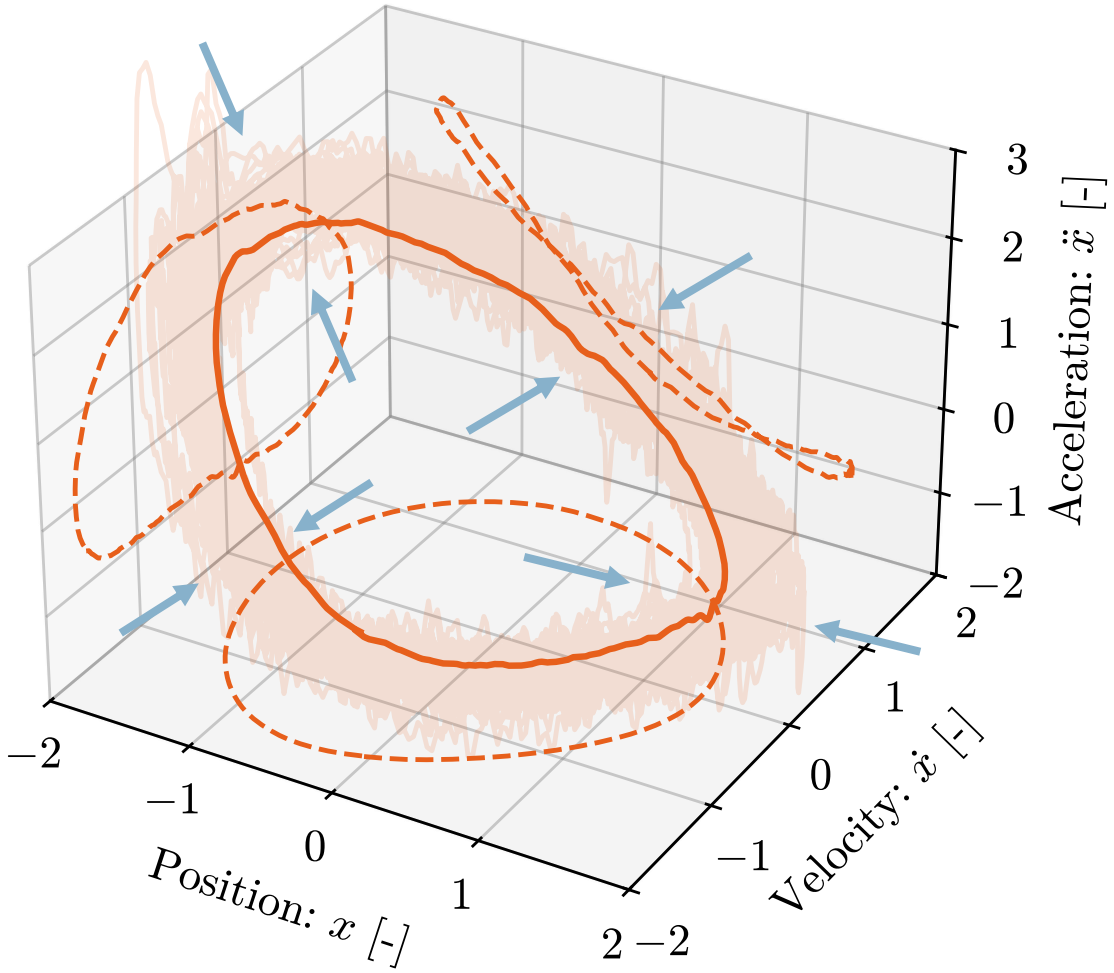


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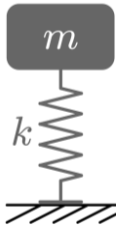
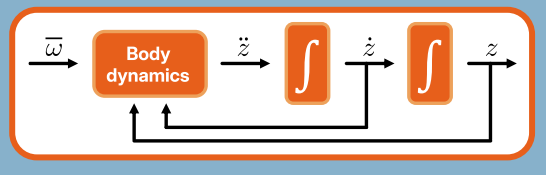



Small corrections ($0 < \varepsilon \ll 1$):

Stable limit cycle $\varepsilon(\gamma x^2 - 1)\dot{x}$ (VdP-like)

# Nonlinear Modeling

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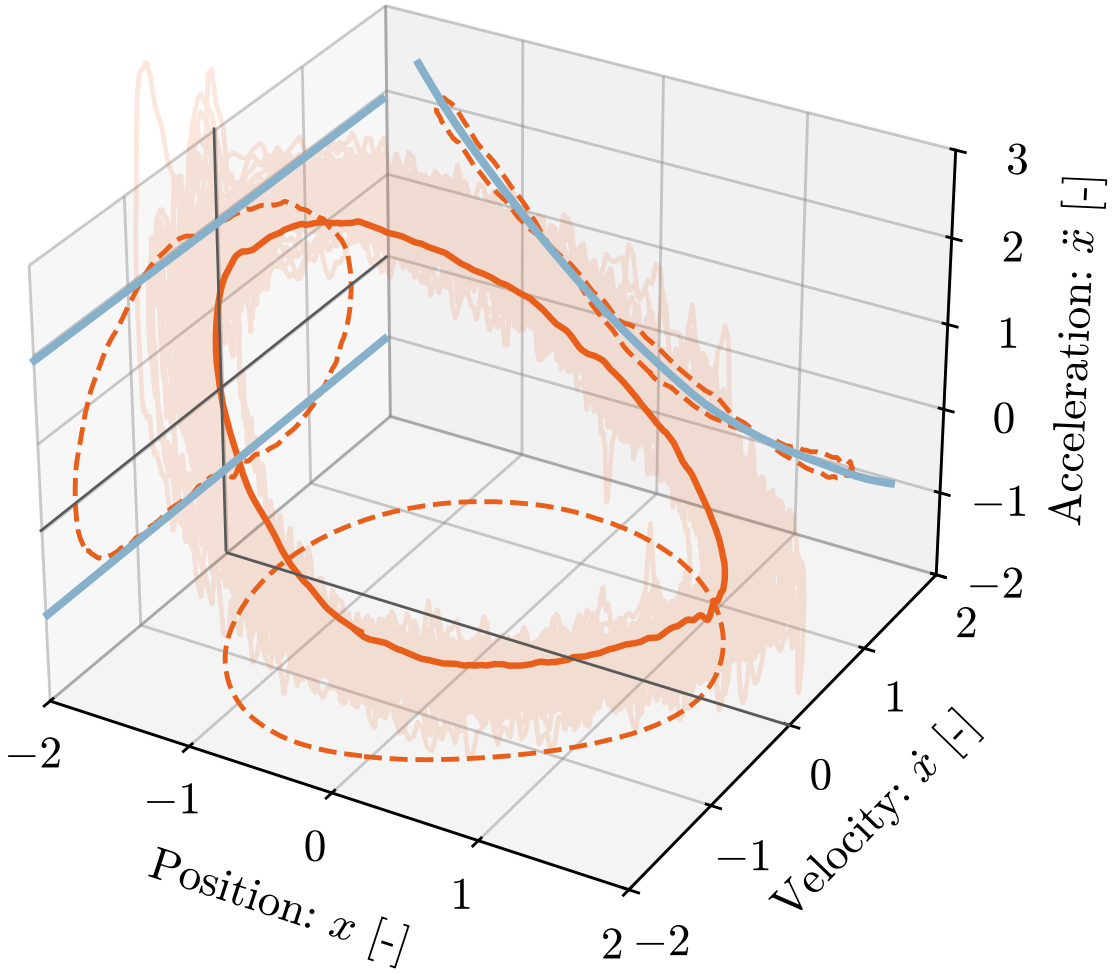


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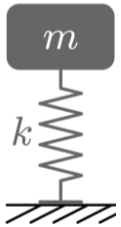


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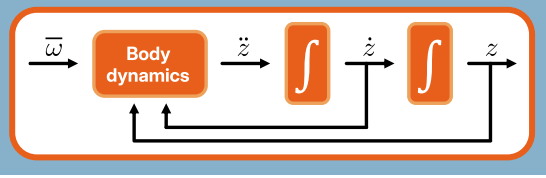
# Nonlinear Modeling

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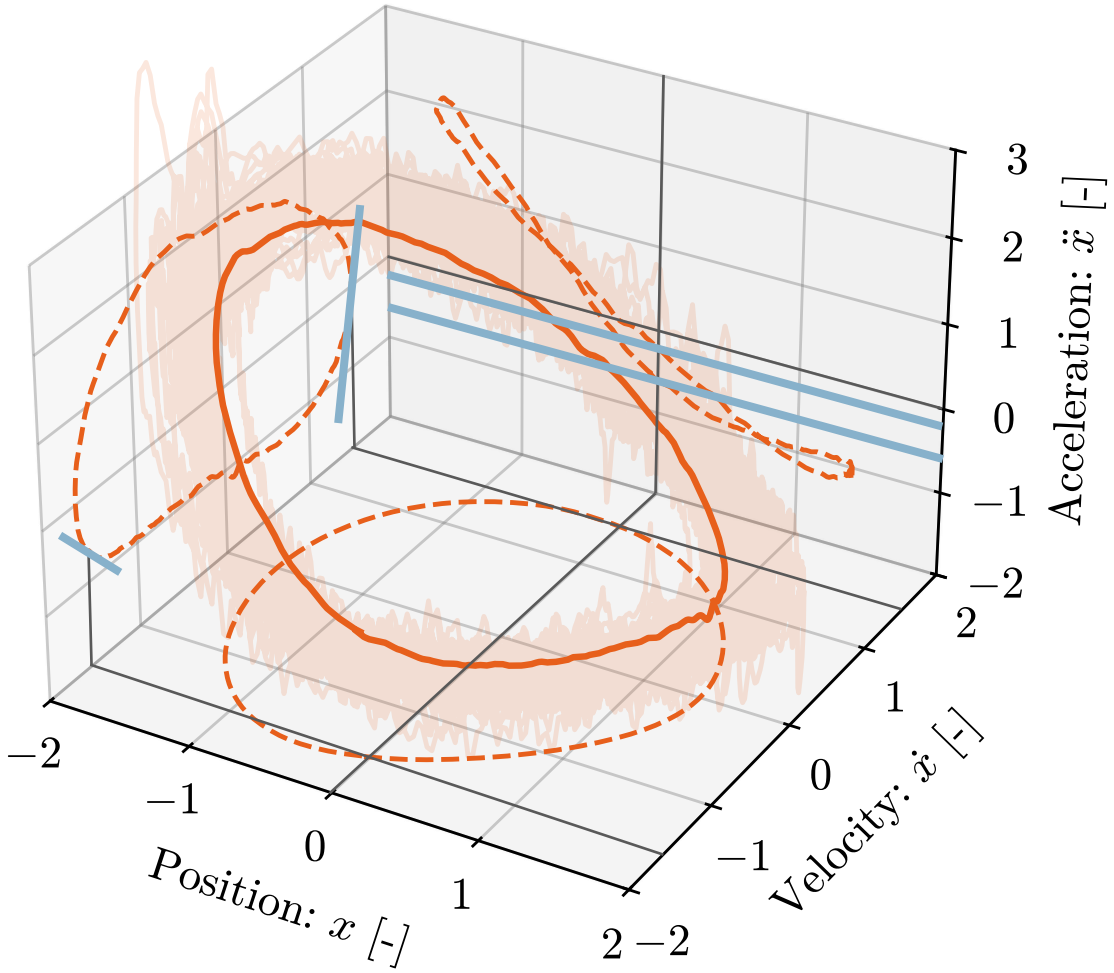


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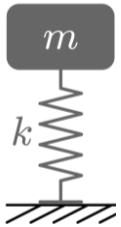


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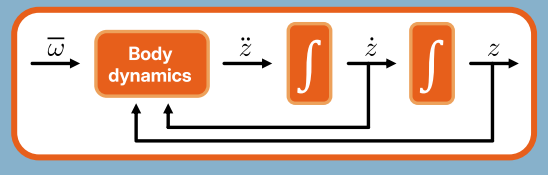
Quadratic stiffness $\varepsilon\lambda x^2$

Quadratic damping $\varepsilon\eta\dot{x}^2$

# Nonlinear Modeling

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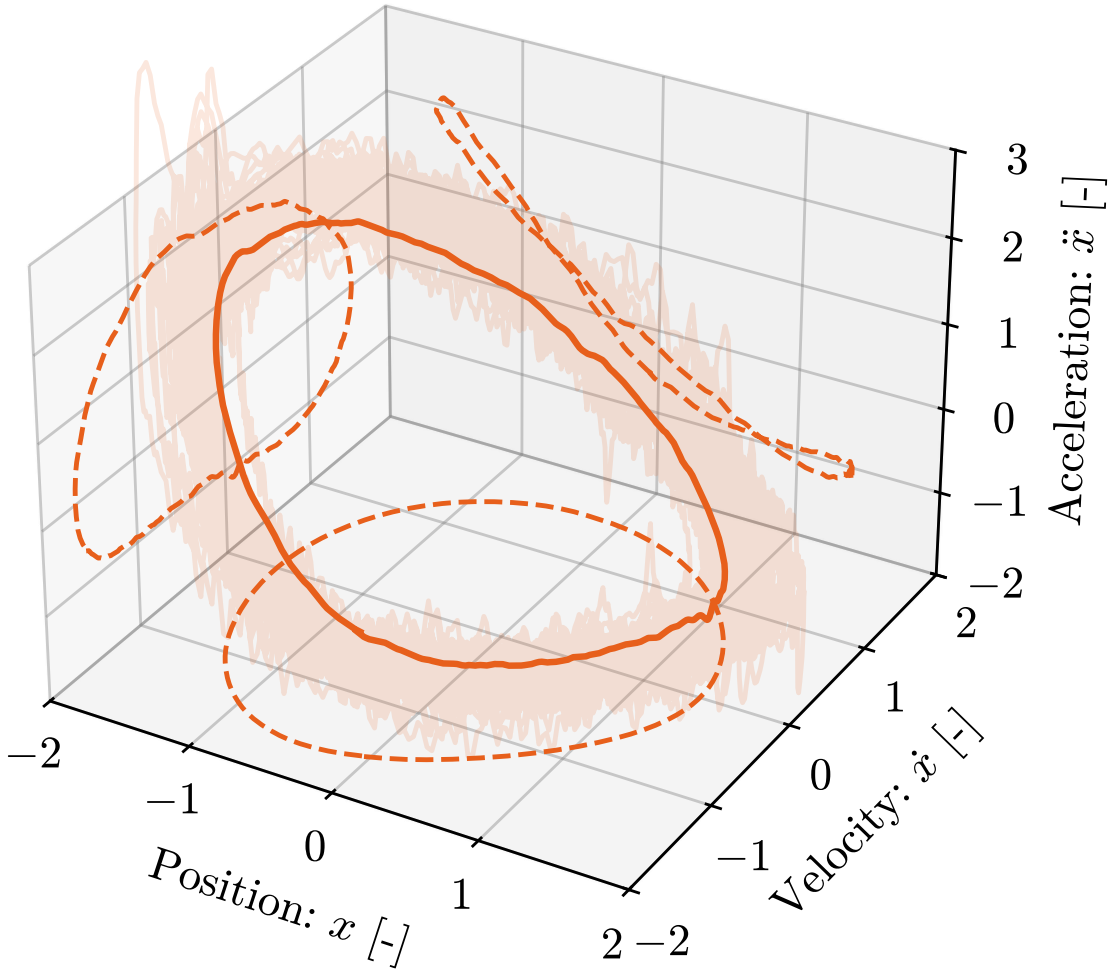


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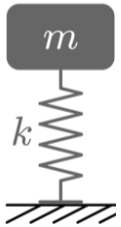



Small corrections ($0 < \varepsilon \ll 1$):

| | |
|---|---|
| Stable limit cycle | $\varepsilon(\gamma x^2 - 1)\dot{x}$ (VdP-like) |
| Quadratic stiffness | $\varepsilon\lambda x^2$ |
| Quadratic damping | $\varepsilon\eta\dot{x}^2$ |

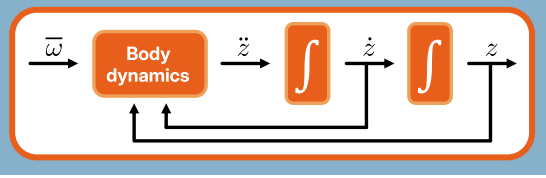
Secularity conditions:

$$\lambda = -\eta \qquad \kappa = 1 + \varepsilon^2\left(\frac{4\eta^2}{3\gamma} + \frac{1}{8}\right)$$

$$\ddot{x} + \kappa x + \varepsilon(\gamma x^2 - 1)\dot{x} + \varepsilon\eta(\dot{x}^2 - x^2) = 0$$

only 3 indep. parameters: $\varepsilon, \gamma, \eta$

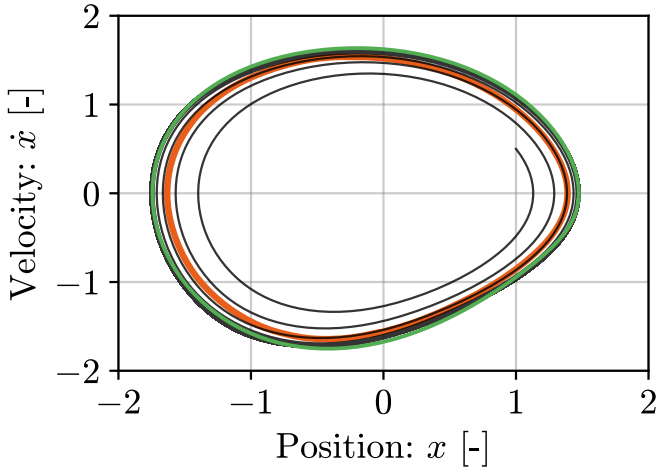
# Validation

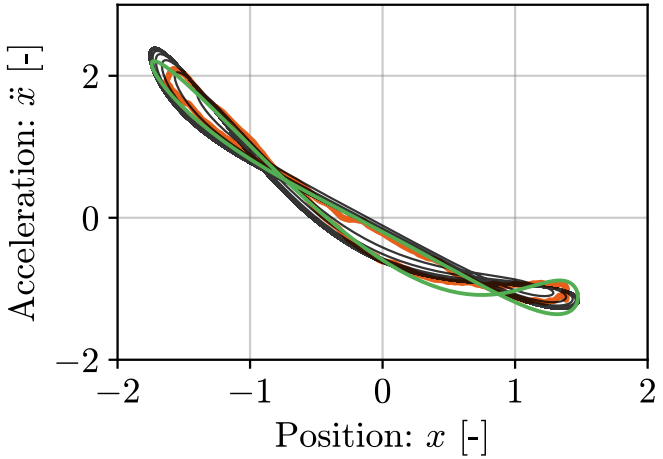


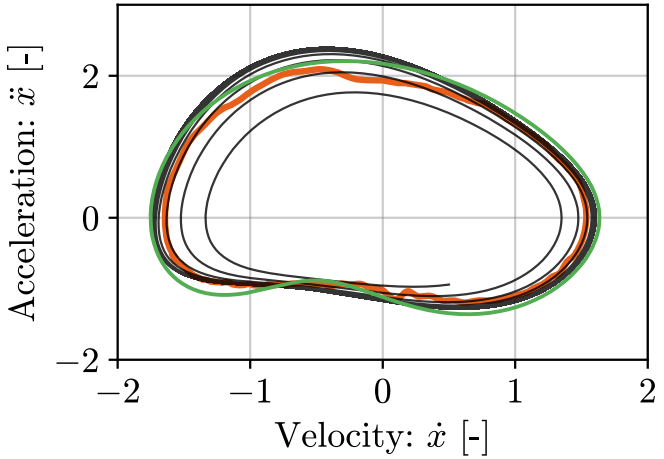

Experimental

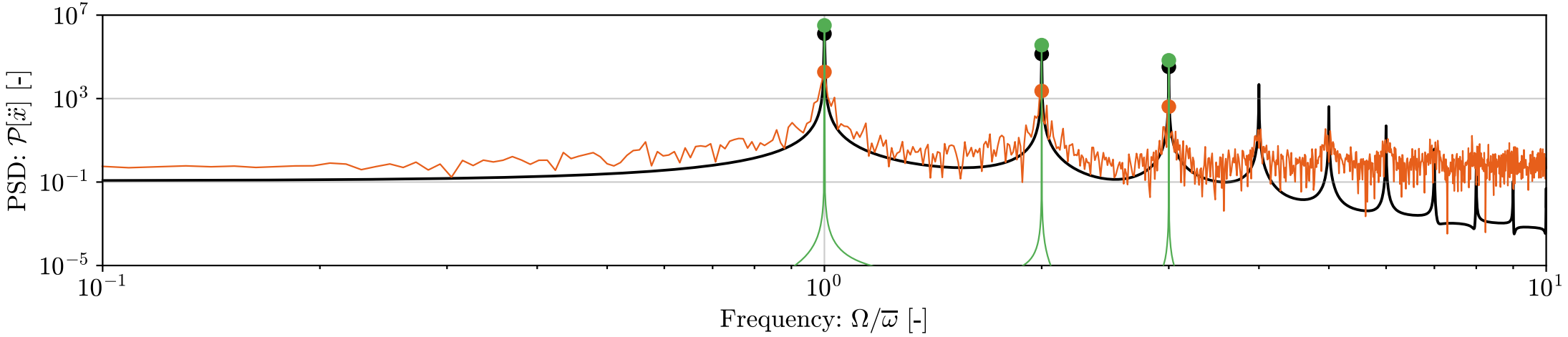
$$\ddot{x} + \kappa x + \varepsilon(\gamma x^2 - 1)\dot{x} + \varepsilon\eta(\dot{x}^2 - x^2) = 0$$

$$x(\tau) \approx \frac{2}{\gamma^{1/2}}\cos(\tau) - \varepsilon\left(\frac{4\eta}{3\gamma}\cos(2\tau) + \frac{\gamma}{4\gamma^{3/2}}\cos(3\tau)\right)$$

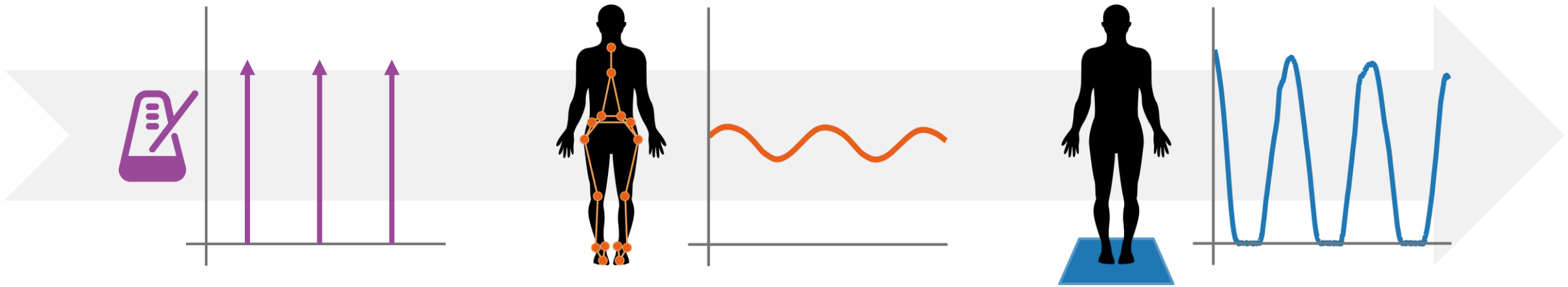


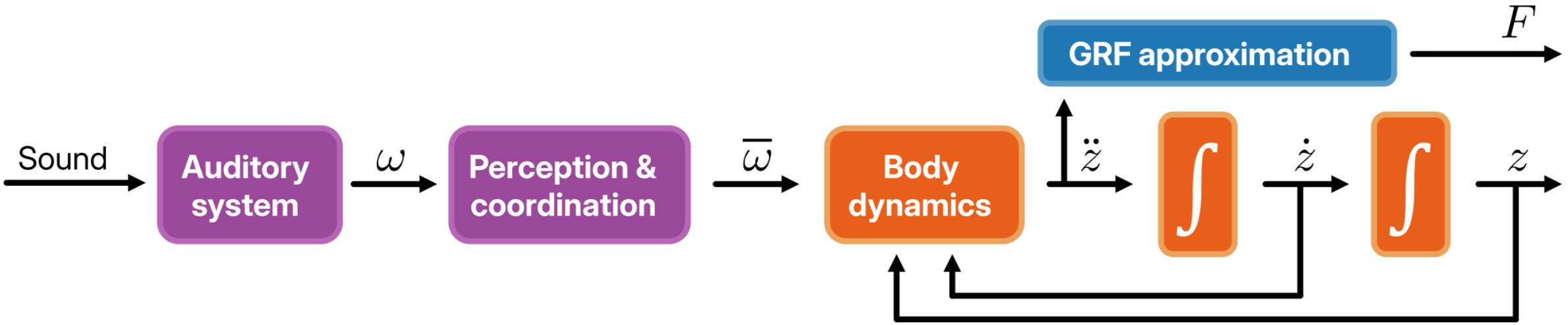



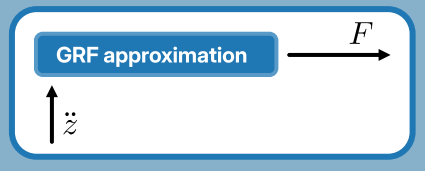
# Nonlinear Model Design

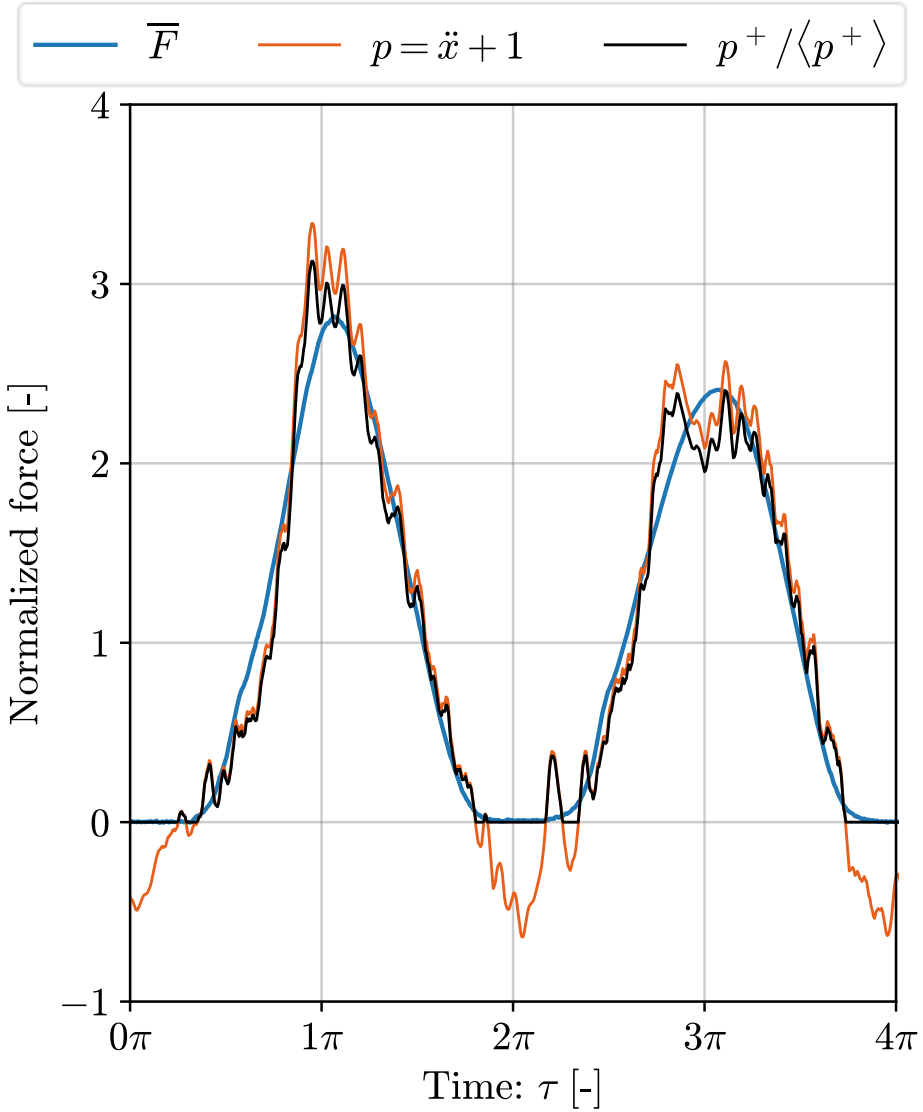


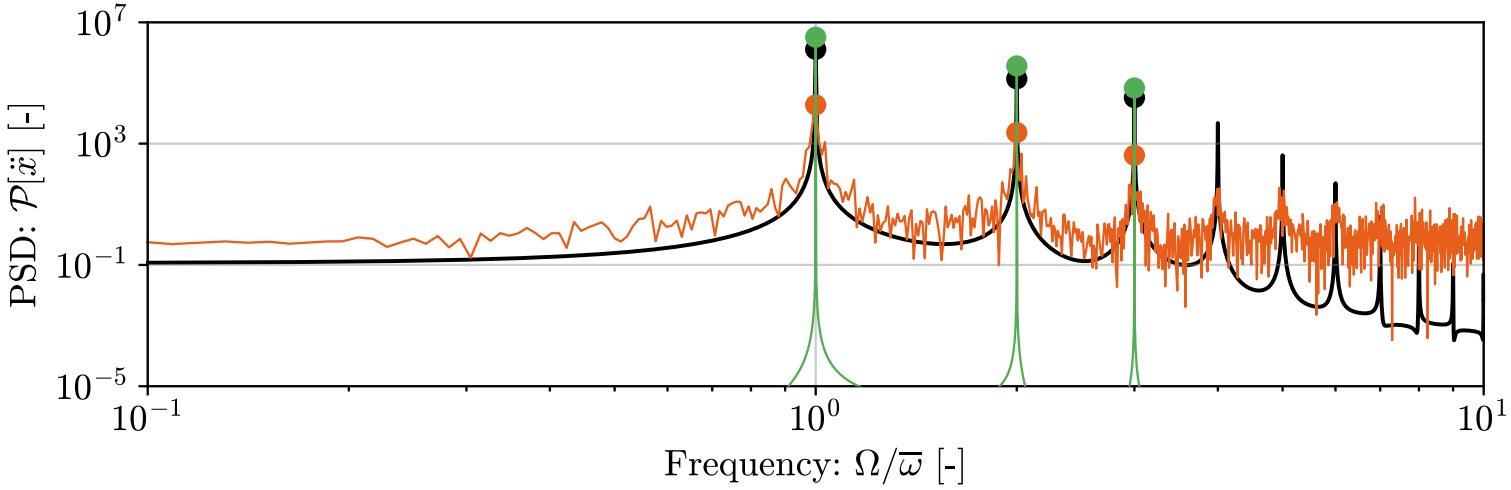


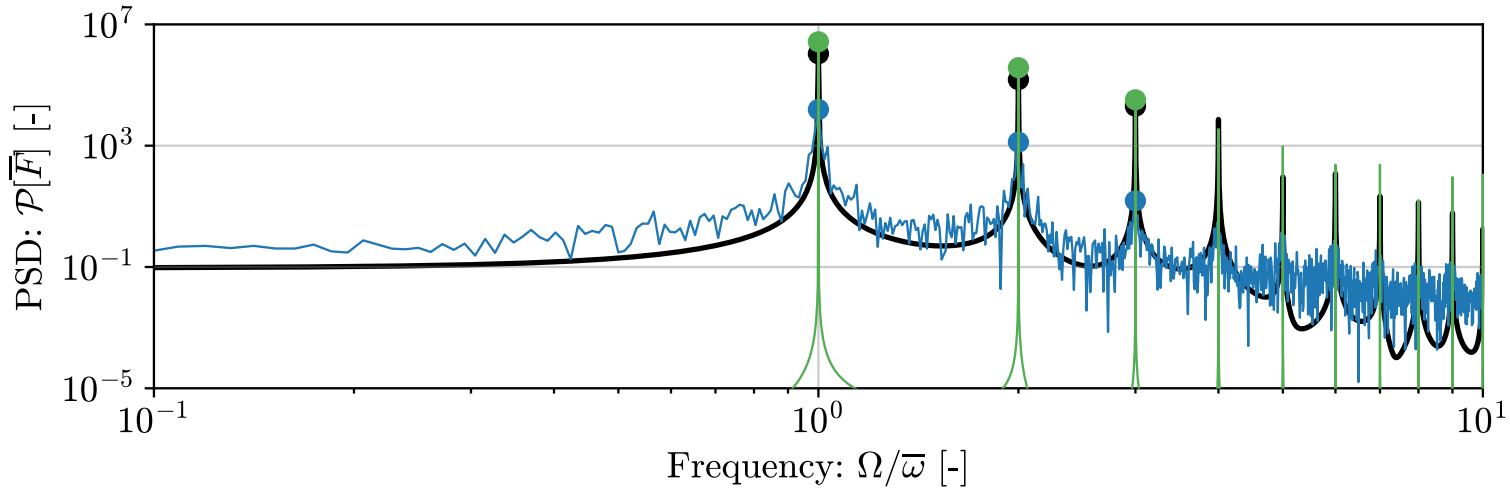

GRF: Ground Reaction Force

# GRF approximation



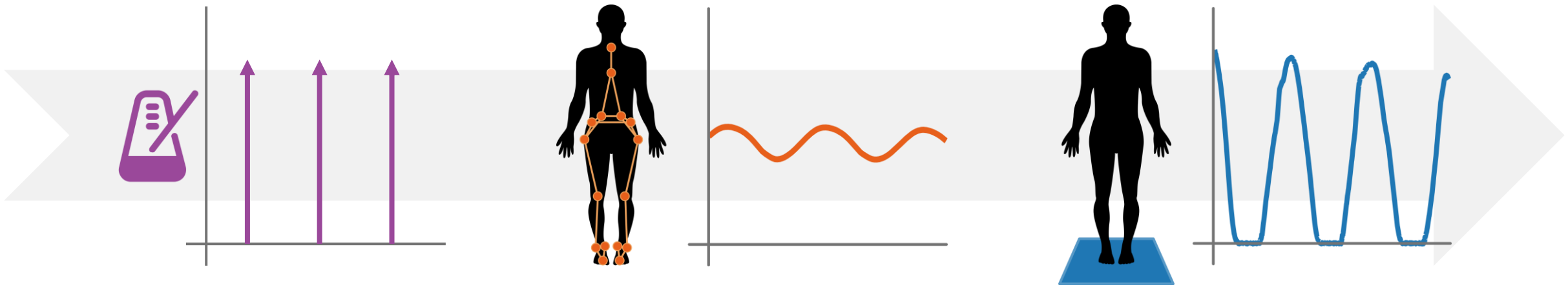



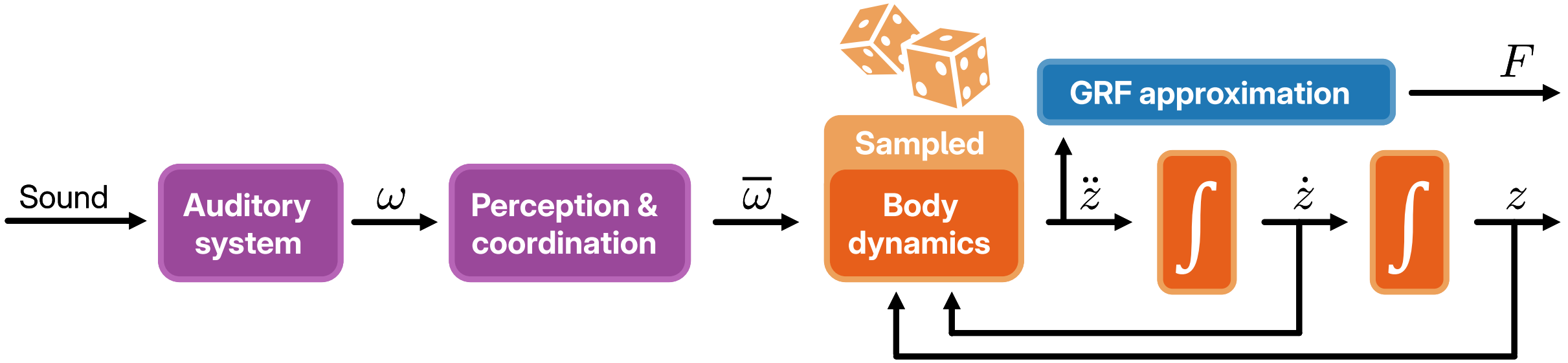




GRF: Ground Reaction Force

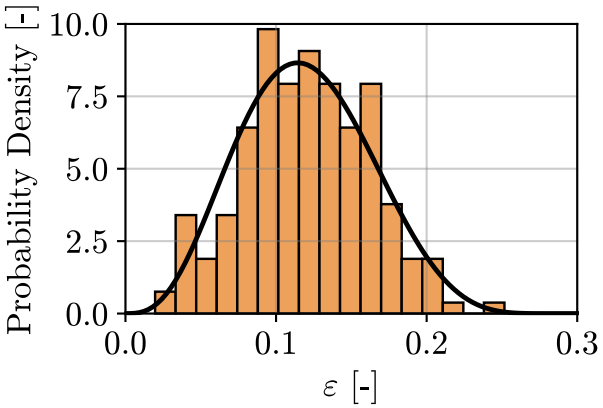
# Nonlinear Model Design

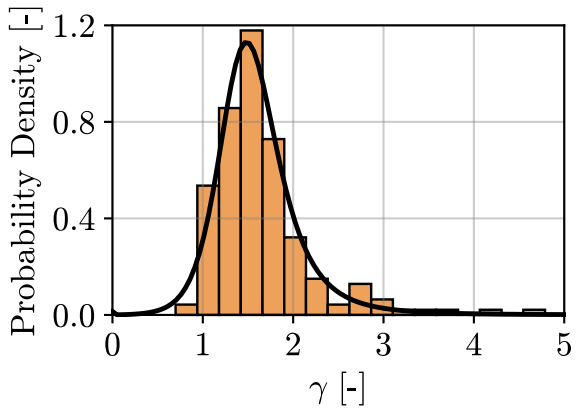


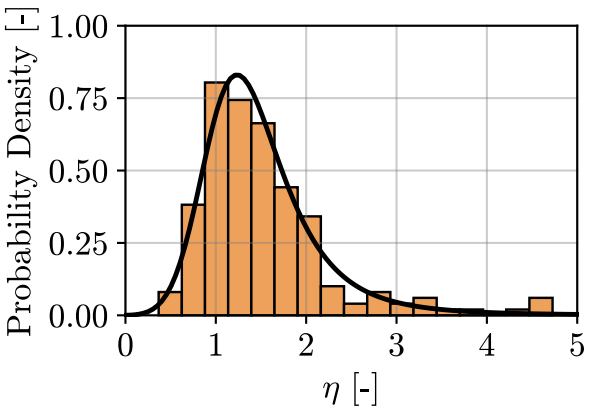


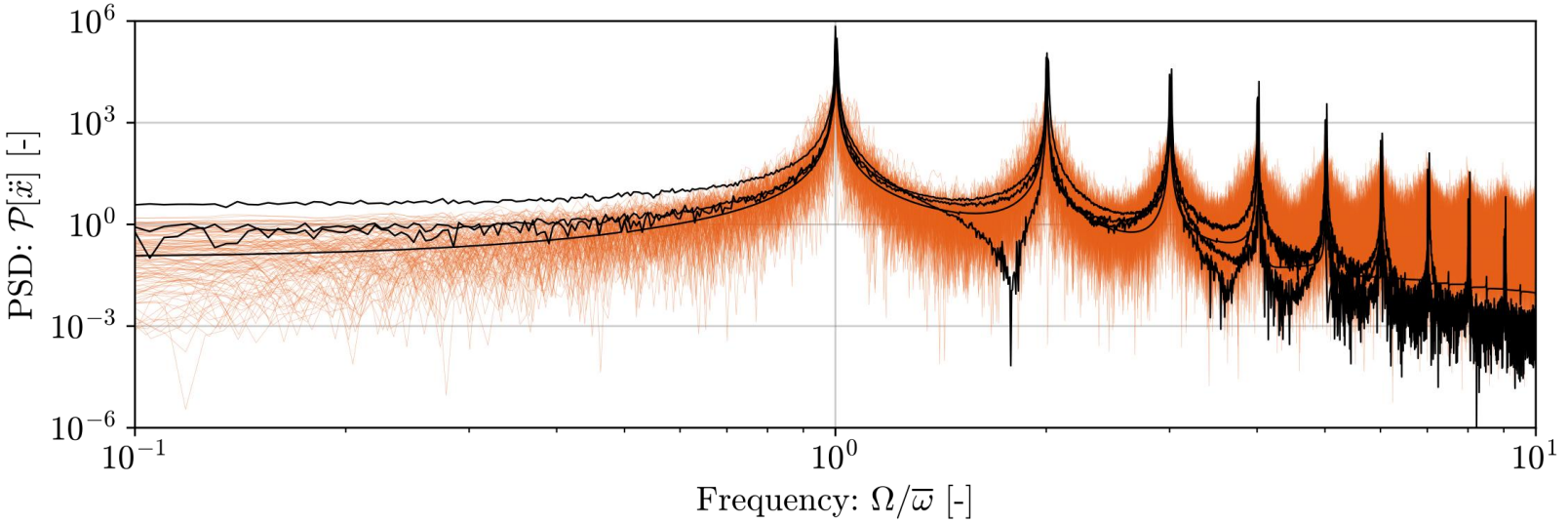

GRF: Ground Reaction Force

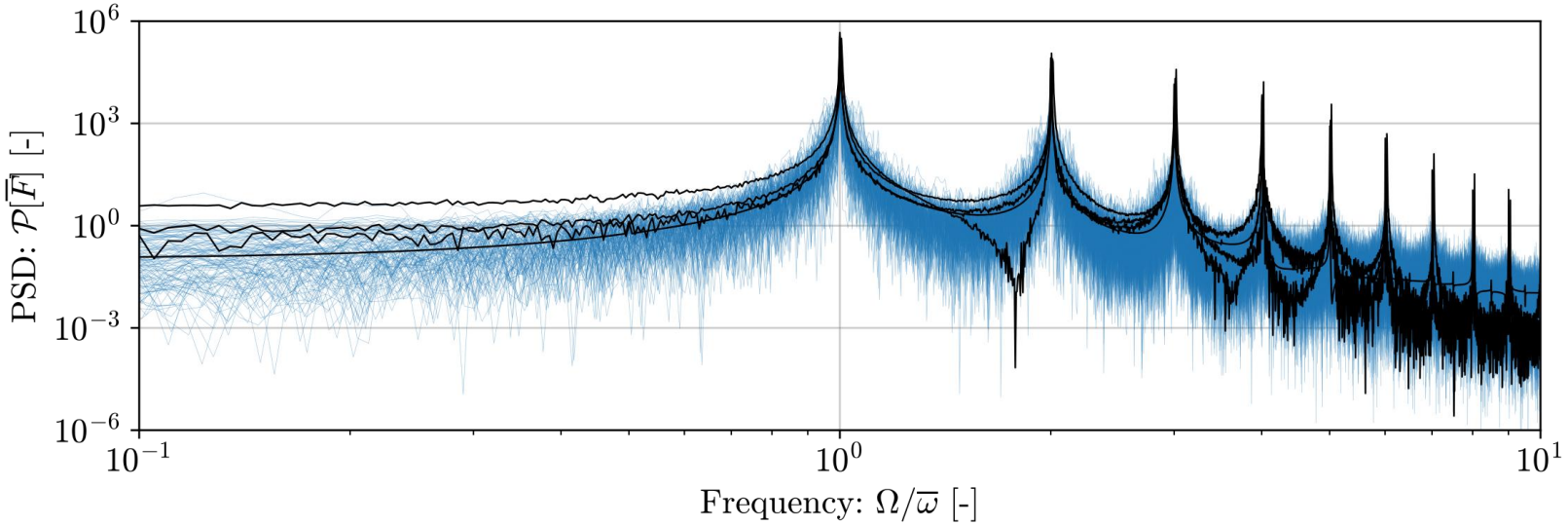
# Sampling New Realizations



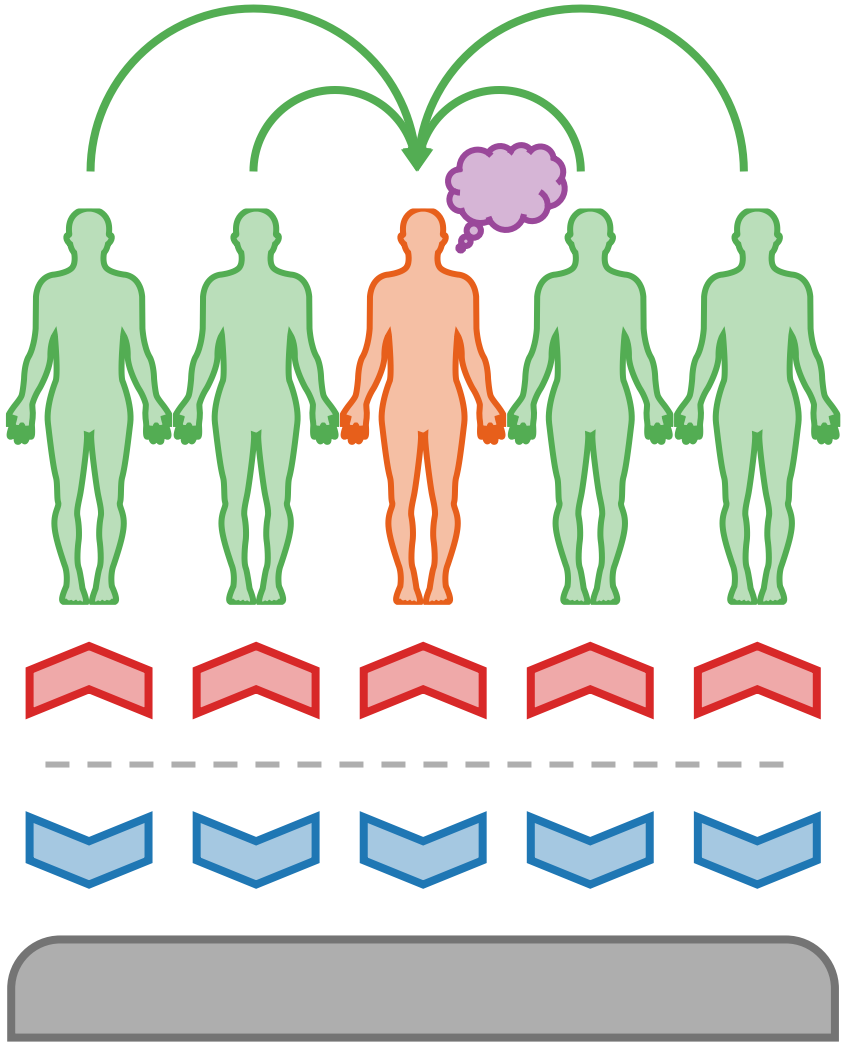



# Conclusion

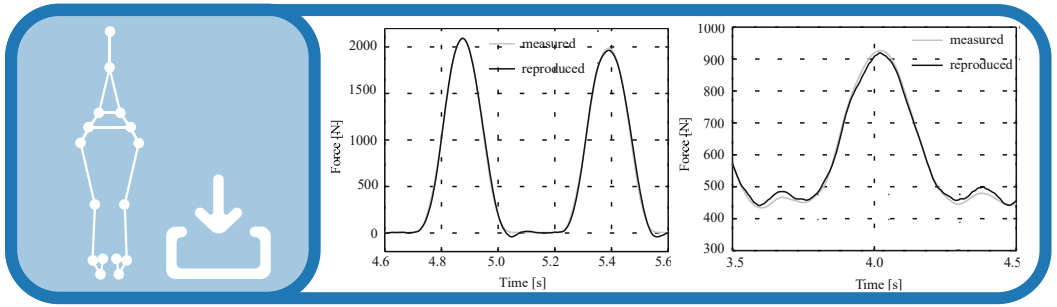


$$\ddot{z}_i - f(\dot{z}_i, z_i; \kappa_i, \varepsilon_i, \gamma_i, \eta_i, \overline{\omega}_i, \langle z_i \rangle) = r_i\left[\{\ddot{Q}_n\}_{n=1}^{N}\right] + s_i\left[\{\ddot{z}_j\}_{j=1}^{J}\right] + W_i(t)$$

$$\ddot{Q}_k + 2\Gamma_k\Omega_k\dot{Q}_k + \Omega_k^2 Q_k = R_k\left[\{\ddot{z}_j\}_{j=1}^{J}\right]$$



Cannon, 2021




Racic et al., 2010



Chen et al., 2019



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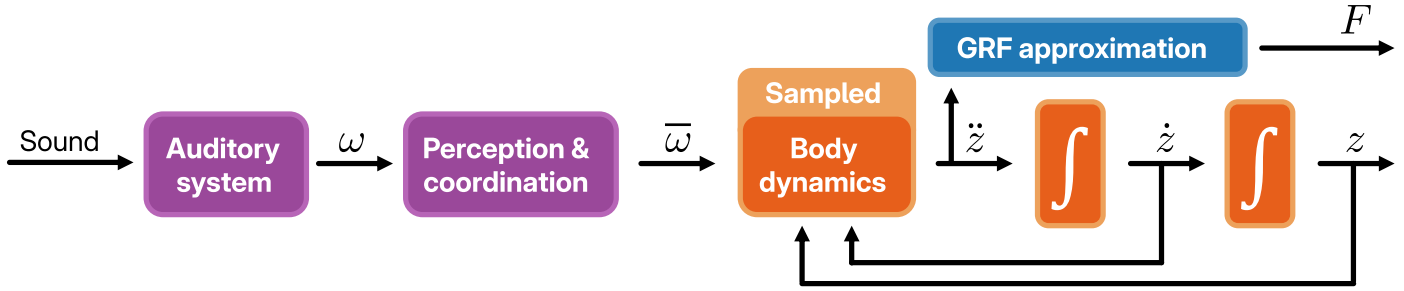
# Thank you for your attention!




$$\ddot{x} + \kappa x + \varepsilon(\gamma x^2 - 1)\dot{x} + \varepsilon\eta(\dot{x}^2 - x^2) = 0$$

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