

AEROELASTIC STABILITY CALCULATIONS OF A LAMINAR WING USING VISCOUS-INVISCID INTERACTION

Adrien Crovato¹, Cédric Liauzun¹, Paul Dechamps², Grigorios Dimitriadis², Vincent E. Terrapon²

¹DAAA, ONERA, Institut Polytechnique de Paris, 92320, Châtillon, France

adrien.crovato@onera.fr

cedric.liauzun@onera.fr

²Université de Liège, Place du 20 Aout 7, 4000 Liège, Belgium

paul.dechamps@uliege.be

gdimitriadis@uliege.be

vincent.terrapon@uliege.be

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Abstract: Laminar wing technology has drawn interest since the 1930s because of the potential reduction in drag and fuel consumption it can provide. Despite numerous studies carried out on this topic, multiple challenges remain to be solved before this technology can be integrated into the aircraft design process. In particular, the prediction of transition from laminar to turbulent flow and its effect on the aeroelastic behaviour of the aircraft are still active areas of research. Within this context, the present work proposes a novel approach for calculating the aeroelastic stability of laminar wings. A steady flow solution is first calculated by means of a viscous-inviscid interaction methodology, which is then used to correct an unsteady compressible source and doublet panel method. Finally, the flutter analysis is performed by means of a non-iterative p - k method. The methodology is illustrated for the case of the CAST 10-2 aerofoil, for which steady flow and flutter solutions are compared to experimental data. Although the present approach is unable to capture the complex unsteady physics inherent to transitional transonic flows, the results suggest that the methodology can still capture variations in the location of the transition, as well as their effects on the aerodynamic loads and aeroelastic stability of the aerofoil. Future research directions include improving the viscous-inviscid interaction method and investigating the entire flutter boundary.

1 INTRODUCTION

Laminar wing technology represents an interesting strategy to reduce drag, and thus fuel consumption. Since the stability of laminar flows is very sensitive to wing surface disturbances and to flight conditions, the boundary layer physics and the effect of transition from laminar to turbulent flow on wing aerodynamics have been widely studied by the scientific community [1–5]. Besides, transitional flows can have adverse effects on the aeroelastic stability of aircraft [6]. As a result, the study of laminar wings has also become an active research topic in the aeroelastic community. In particular, the German Aerospace Center (DLR) recently conducted an extensive experimental and numerical test campaign dedicated to studying the effect of laminar to turbulent transition on the flutter of the CAST 10-2 [6–18], an aerofoil initially designed to test advanced technologies. One of the main findings was that boundary layer transition could

displace and amplify the transonic flutter dip, *i.e.* the reduction of the flutter speed due to compressible flow effects.

In order to integrate laminar wing technology into the aircraft design process, it is essential to develop fast and robust numerical methods able to predict laminar to turbulent transition and its effect on aeroelastic stability. To that end, the present work aims at investigating the coupling of viscous-inviscid interaction methodologies, an unsteady panel method and a frequency matching technique to compute flutter. The main novelty of this work resides in the application of the approach to study laminar flutter. This paper is organized as follows. Section 2 presents the numerical methods and tools developed within the context of this work, and section 3 discusses the main aerodynamic and aeroelastic results. Finally, section 4 concludes this paper and summarises directions for future work.

2 METHODOLOGY

The numerical approach developed within the context of the present work relies on the combination of three techniques. A steady flow solution is first computed by means of a viscous-inviscid interaction methodology, whereby an inviscid full potential flow and a boundary layer equations solvers are coupled together. The inviscid flow variables obtained at the aerofoil surface are provided to the viscous solver, which in turns calculates a blowing velocity which is then fed back to the inviscid solver. Once the viscous-inviscid interaction process has converged, the derivative of the surface pressure coefficient with respect to the angle of attack, is computed using finite differences. Resorting to viscous-inviscid interaction is computationally less costly than solving the Reynolds-Averaged Navier Stokes (RANS) equations and offers a natural way of predicting the transition. Flutter is then calculated by coupling a structural pitch and plunge model, an unsteady compressible source and doublet panel method and a frequency matching technique. The mode shapes are first extracted from the structural model and then used as a boundary condition in the panel method in order to calculate the generalized aerodynamic force matrices at several reference reduced frequencies. These matrices are corrected using the steady pressure derivatives in order to model transonic and transitional flow nonlinearities. Finally the structural and aerodynamic matrices are given to a non-iterative p - k method in order to calculate the aeroelastic frequencies and damping ratios. The interest of using a panel method combined to the p - k method lies in the fact that this approach is fast and similar to industrial standards, such that it can be integrated easily in the aircraft design process. The overall process is depicted in figure 1.

2.1 Steady aerodynamic modelling

The viscous-inviscid interaction (VII) method relies on the coupling of an inviscid and a viscous solver. Two methods are used in the present work. In the first approach, the inviscid flow solution is first obtained by solving the steady full potential equation discretised by means of the finite element method,

$$\nabla \cdot (\rho \nabla \phi) = 0, \quad (1)$$

where ϕ is the velocity potential. The fluid density ρ is given by the isentropic flow relationship,

$$\rho = \rho_\infty \left[1 + \frac{\gamma - 1}{2} M_\infty^2 (1 - |\nabla \phi|^2) \right]^{\frac{1}{\gamma - 1}}, \quad (2)$$

where ρ_∞ is the freestream density, γ is the heat capacity ratio and M_∞ is the freestream Mach number. Although the full potential equation assumes that the flow is isentropic, it can still

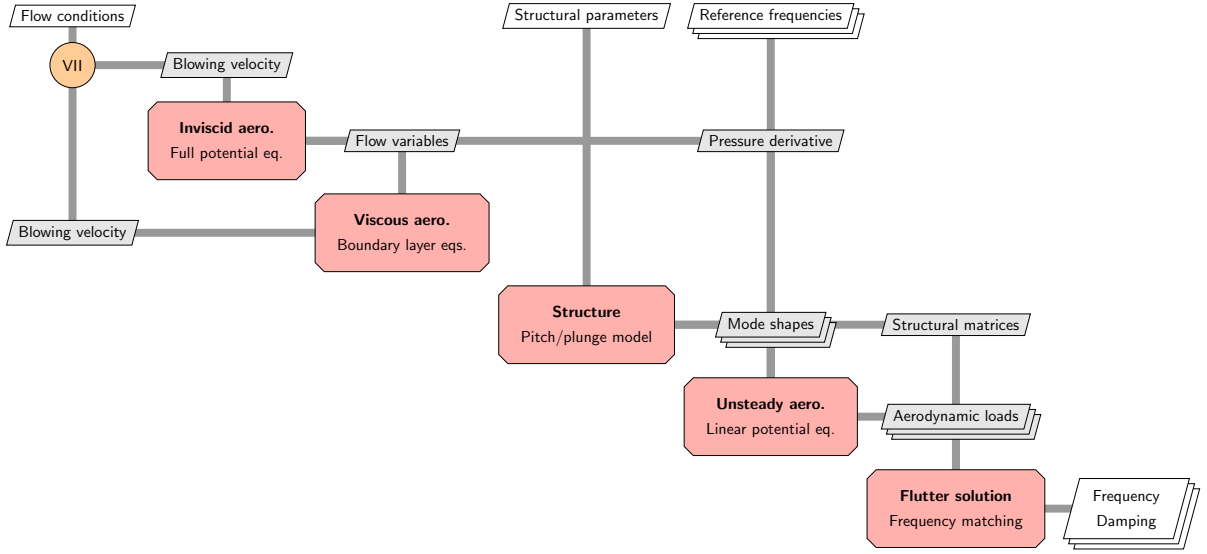


Figure 1: Overview of the steady aerodynamic and dynamic aeroelastic analyses developed within the context of the present work.

model transonic flows with weak shock waves. The viscous solution is then obtained by solving a pseudo-unsteady form of the integral boundary layer equations [19],

$$\begin{aligned}
 \frac{H}{u_e} \frac{\partial \theta}{\partial t} + \frac{\theta}{u_e} \frac{\partial H}{\partial t} + \frac{\theta H}{u_e^2} \frac{\partial u_e}{\partial t} &= \frac{\partial \theta}{\partial \xi} + (2 + H - M_e^2) \frac{\theta}{u_e} \frac{\partial u_e}{\partial \xi} - \frac{c_f}{2}, \\
 \frac{1 + H(1 - H^*)}{u_e} \frac{\partial \theta}{\partial t} + \frac{\theta(1 - H^*)}{u_e} \frac{\partial H}{\partial t} + \frac{\theta(2 - H^*H)}{u_e^2} \frac{\partial u_e}{\partial t} & \\
 = \theta \frac{dH^*}{d\xi} + (2H^{**} + H^*(1 - H)) \frac{\theta}{u_e} \frac{\partial u_e}{\partial \xi} - 2c_d + H^* \frac{c_f}{2}, & \quad (3)
 \end{aligned}$$

where ξ is the curvilinear coordinate along the aerofoil surface, θ is the momentum thickness of the boundary layer, H is the shape factor, c_f and c_d are the local friction and dissipation coefficients defined at the wall, H^* and H^{**} are the kinetic energy and density shape parameters, and where u_e and M_e are the inviscid velocity and Mach number calculated at the wall. When the flow is laminar, the e^N method [20, 21] is used to predict the transition. Equations (3) are solved together with a third equation representing the unsteady evolution of the amplification ratio N ,

$$\frac{\partial N}{\partial t} + \frac{\partial N}{\partial \xi} = \left(\frac{dN}{d\text{Re}_\theta} \frac{d\text{Re}_\theta}{d\xi} \right)_{\text{attached}} + A_{\text{separated}}, \quad (4)$$

where $A_{\text{separated}}$ modifies the envelope amplification rate in separated flows. When the flow is turbulent, equations (3) are solved together with the shear-lag equation [19],

$$\begin{aligned}
 \frac{\delta}{U_s u_e C_\tau} \frac{\partial C_\tau}{\partial t} + \frac{2\delta}{U_s u_e^2} \frac{\partial u_e}{\partial t} + \frac{\delta}{C_\tau} \frac{dC_\tau}{d\xi} & \\
 = 5.6 \left(C_{\tau_{EQ}}^{1/2} - C_\tau^{1/2} \right) + 2\delta \left\{ \frac{4}{3\delta^*} \left[\frac{c_f}{2} - \left(\frac{H_k - 1}{6.7H_k} \right)^2 \right] - \frac{1}{u_e} \frac{du_e}{d\xi} \right\}, & \quad (5)
 \end{aligned}$$

where δ is the boundary layer thickness, δ^* is the displacement thickness, C_τ is the shear-stress coefficient, $C_{\tau_{EQ}}$ is the equilibrium shear-stress coefficient, H_k is the kinematic shape parameter

and U_s is the equivalent normalized wall slip velocity, computed using an empirical correlation. The viscous equations are discretised using finite differences, and each point along the aerofoil is solved sequentially by space marching. At each station, the equations are marched in time until steady state is reached. A blowing velocity V_e is finally computed by differentiating the boundary layer displacement thickness and the inviscid density ρ_e as

$$V_e = \frac{1}{\rho_e} \frac{d}{dx} (\rho_e u_e \delta^*), \quad (6)$$

and fed back to the inviscid solver as a transpiration boundary condition. The inviscid and viscous solutions are coupled by means of a quasi-simultaneous algorithm, which can handle attached and mildly separated flows in a robust fashion. The full potential finite element solver is implemented in the computer code DART [22, 23]¹ and both the boundary layer solver and the coupling procedure are implemented in the computer code BLASTER [24, 25]².

The second approach used in the present work is based on the so-called *defect formulation* proposed by Le Balleur et al. [26, 27]. It also relies on the coupling of the full potential equation to a form of the integral boundary layer equations. The terms required by the closure models are calculated by numerical integration of a set of parametric velocity profiles discretised in the direction normal to the streamlines calculated by the inviscid solver. These profiles are available for a wide variety of flows, ranging from fully attached and laminar to massively separated. In laminar flow regions, transition is taken into account naturally by combining several criteria, among which Michel's transition criterion [28]. The inviscid and viscous models are coupled together using a transpiration boundary condition and are solved either in direct or inverse mode by means of a semi-inverse method [29, 30]. The whole procedure, including the grid generation and automatic adaptation is implemented in the VIS07 code, which uses a finite difference discretisation. Because the velocity profiles are discretised along the height of the boundary layer, VIS07 requires a two-dimensional grid, contrary to the approach implemented in BLASTER. Note that VIS07 has already been successfully used to compute transonic transitional flows over aerofoils [31].

2.2 Dynamic aeroelastic modelling

The unsteady flow solution is calculated by means of a compressible source and doublet panel method, initially proposed by Morino [32–35]. The wing surface is discretised using quadrilateral panels onto which source and doublet singularities are superimposed. The source strengths σ are first calculated to enforce the non-penetration boundary condition at a reference reduced frequency k ,

$$\sigma(k) = -\mathbf{u}_c(k) \cdot \mathcal{B} \odot \mathbf{n}, \quad (7)$$

where $\mathbf{u}_c$ is the velocity on the surface induced by the motion of the wing, $\mathbf{n}$ is the unit normal to the surface, $\odot$ denotes the Hadamard product, and where the compressibility is taken into account by the factor $\mathcal{B}$, defined as

$$\mathcal{B} = \begin{bmatrix} 1/\beta \\ 0 \\ 0 \end{bmatrix}, \quad (8)$$

¹<https://gitlab.uliege.be/am-dept/dartflo>

²<https://gitlab.uliege.be/am-dept/blaster>

with $\beta = \sqrt{1 - M_\infty^2}$. For a combination of rigid pitch and plunge motions, $\mathbf{u}_c$ writes

$$\mathbf{u}_c(k) = \begin{bmatrix} 0 \\ 0 \\ u_{\infty, x} \tilde{\alpha} \end{bmatrix} + i \frac{k u_\infty}{l_{\text{ref}}} \begin{bmatrix} -\tilde{\alpha} (z - z_{\text{ref}}) \\ 0 \\ \tilde{\alpha} (x - x_{\text{ref}}) + \tilde{h} \end{bmatrix}, \quad (9)$$

where $\tilde{\alpha}$ and $\tilde{h}$ are the pitch and plunge amplitudes, x_{ref} and z_{ref} are the x and z coordinates of the reference center, u_∞ is the freestream velocity, l_{ref} is a reference length, and i is the imaginary unit. The doublet strengths $\boldsymbol{\mu}$ are then obtained by solving the linearized potential flow equation written in integral form,

$$\mathbf{A}(k) \boldsymbol{\mu}(k) = \mathbf{B}(k) \boldsymbol{\sigma}(k), \quad (10)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the unsteady influence coefficient matrices. The unsteady pressure coefficients at each frequency are then calculated from the unsteady Bernoulli equation and integrated on the wing surface to obtain the pressure loads. Finally, the generalised aerodynamic force matrix at reduced frequency k is calculated by multiplying the z -component of the surface displacement $\delta_{z_i} = \tilde{\alpha}_i (x - x_{\text{ref}}) + \tilde{h}_i$ of motion i by the z -component of the pressure loads due to motion j ,

$$Q_{ij}(k) = \delta_{z_i}(k) \cdot \mathbf{c}_{p_j}(k) \odot \mathbf{n}_z \odot \mathbf{S}_p, \quad (11)$$

where $\mathbf{c}_{p_j}$, $\mathbf{n}_z$, $\mathbf{S}_p$ are the vectors containing the unsteady pressure coefficients due to mode j , the z -component of the unit vectors normal to the wing and the panel surface areas, respectively. In order to account for nonlinearities due to compressible and viscous effects, the doublet strength are corrected by a diagonal correction matrix $\mathbf{D}^{\text{corr}}$ [36, 37]. The following equation must be solved to obtain the correction factors,

$$\nabla \mathbf{N}_x \mathbf{D}^{\text{corr}} \mathbf{A}^{-1}(0) \mathbf{B}(0) \mathbf{n}_z = \frac{\sqrt{1 - M_\infty^2}}{2} \mathbf{c}_{p_\alpha} - \mathbf{n}_x \odot \mathbf{n}_z, \quad (12)$$

where $\nabla \mathbf{N}_x$ is a matrix containing the x -component of the surface gradient operators, $\mathbf{c}_{p_\alpha}$ is the vector containing the derivatives of the steady pressure coefficient with respect to the angle of attack obtained from a reference steady solution computed using viscous-inviscid interaction, and where $\mathbf{n}_x$ contains the x -component of the unit vectors normal to the wing surface on each panel.

Aeroelastic stability is assessed by calculating the aeroelastic eigenvalues of the system, which can be found by solving the flutter equation

$$\left[\frac{u_\infty^2}{l_{\text{ref}}} p_i^2 \mathbf{M} + \mathbf{K} - \frac{1}{2} \rho_\infty u_\infty^2 \mathbf{Q}(k) \right] \mathbf{q}_i = 0, \quad (13)$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, and where p_i and $\mathbf{q}_i$ are the eigenfrequencies and eigenmodes. The eigenfrequency p is expressed as

$$p = (g + i)k, \quad (14)$$

where g denotes the damping. In order to compare numerical and experimental results, the freestream density and velocity are expressed in terms of stagnation pressure p_0 and temperature T_0 using the isentropic flow relations and the perfect gas law,

$$\begin{aligned} u_\infty &= M_\infty \sqrt{\gamma R T_0 \left(1 + \frac{\gamma - 1}{2} M_\infty^2 \right)^{-1}}, \\ \rho_\infty &= \frac{p_0}{R T_0} \left(1 + \frac{\gamma - 1}{2} M_\infty^2 \right)^{\frac{-1}{\gamma - 1}}, \end{aligned} \quad (15)$$

where R is the perfect gas constant. In the present work, equation (13) is solved using the non-iterative version of the $p - k$ method with mode tracking proposed by van Zyl [38, 39]. The flutter equation is first solved once for each reference reduced frequency using the corresponding generalised aerodynamic force matrix calculated using the panel method. The difference between the imaginary part of each eigenfrequency solution and each reference reduced frequency $\delta_i = \Im(p_i) - k_i$ is then computed. Because the imaginary part of the eigenfrequency is the reduced frequency, a change in the sign of the differences δ_1 and δ_2 between two consecutive eigenfrequencies p_1 and p_2 indicates that a valid solution exists between these two eigenfrequencies. This solution can then be found by linear interpolation. Once the imaginary part of the solution has been interpolated, the real part of the eigenfrequency can also be interpolated to obtain the damping. The panel method is implemented in the computer code SDPM [40]³ and the non-iterative $p - k$ method is implemented in PYPK [37]⁴.

3 RESULTS

The methodology developed within the context of the present work is used to calculate the flow over the CAST 10-2/DOA-2 aerofoil and to predict its flutter behaviour at transonic speed. The CAST 10-2 is a supercritical aerofoil designed to feature natural laminar flow and extensively studied by the National Aeronautics and Space Administration (NASA) and DLR since the 1980s [41]⁵. Recently, a rectangular wing of 30 cm chord and 1 m span using this aerofoil was mounted on a pitch and plunge mechanism and tested in the Transonic Wind Tunnel Göttingen [9, 13, 15]. The experimental measurements were made at a Reynolds number $Re = 2 \times 10^6$ and at several Mach numbers in the range $M_\infty = [0.70; 0.82]$. The aerofoil with leaf and torsional spring models is depicted in figure 2.



Figure 2: CAST 10-2 aerofoil with leaf and torsional springs.

3.1 Steady aerodynamics

BLASTER and VIS07 are used to calculate the steady flow over the CAST 10-2 aerofoil at a freestream Mach number $M_\infty = 0.75$, a Reynolds number $Re = 2 \times 10^6$ and angles of attack α between -1.5° and $+1.5^\circ$. The inviscid unstructured grid for BLASTER is generated using Gmsh [42]⁶ by choosing a cell size of 1% of the chord on the aerofoil surface and a growth ratio of 1.2 in the field, yielding a mesh with 4750 triangles. The viscous structured mesh contains 350 points along the chord of the aerofoil. Since the two grids are different at their interface, radial basis functions are used to interpolate the data from one mesh to another. Two different approaches were used to obtain the results. Firstly, by fixing the location of the transition from laminar to turbulent flow at the leading edge, hence producing a fully turbulent boundary layer. Secondly, by letting the transition develop freely. On the other hand, VIS07 creates and adapts its own structured meshes during the calculation. The final inviscid and viscous grids contain

³<https://gitlab.uliege.be/am-dept/sdpm>

⁴<https://gitlab.uliege.be/am-dept/pypk>

⁵The coordinates of the CAST 10-2 used in this work have been taken from Stanewsky et al. [41].

⁶<https://gmsh.info>

441 $\times$ 108 points in the field and 441 $\times$ 61 points in the boundary layer, respectively. Only transitional flows were simulated using VIS07.

Figure 3 shows the lift and moment coefficients, and the location of the transition on the suction side of the CAST 10-2 aerofoil for angles of attack α between -1.5° and $+1.5^\circ$, calculated using BLASTER and VIS07, and compared to experimental data gathered by Braune et al. [15] and Hebler et al. [9]. Both BLASTER and VIS07 are able to capture the change in the location of the transition as well as the associated loss of lift and decrease in moment coefficient, which is of paramount importance for flutter prediction. Nevertheless, the results calculated using VIS07 are shifted compared to the experimental data. A similar offset in the lift coefficient and transition location between experimental and numerical results have already been observed by Hebler et al. [9]. In this challenging case, BLASTER requires only thirty seconds per angle of attack on a laptop computer, which is significantly faster than traditional RANS-based solvers. However, the software currently lacks robustness for treating transonic transitional flows. This can lead to spurious oscillations in the results, *e.g.* at $\alpha = -0.2^\circ$, or even failure to converge, *e.g.* at $\alpha = +1.4^\circ$. On the other hand, VIS07 requires about two hours per calculation. This can be attributed to the higher grid density and the more complex procedures implemented within the code, such as the numerical integration of the velocity profiles along the boundary layer height. Note that VIS07 is a legacy code and its performance could certainly be improved.

Figure 4 shows the pressure coefficient and its derivative with respect to the angle of attack along the chord of the CAST 10-2 aerofoil at an angle of attack of $\alpha = 0^\circ$ computed using BLASTER and compared to experimental measurements obtained by Braune et al. [15] and Hebler et al. [9]. The results obtained using VIS07 are also depicted but at an angle of attack $\alpha = 0.4^\circ$, chosen to match the lift coefficient and the transition location observed experimentally. When the transition is triggered at the leading edge, there is a good agreement between BLASTER and the experimental data, the pressure coefficient only differing slightly on the suction side between 20% and 40% of the chord. When the transition is left free, its location predicted by BLASTER on the pressure side lies slightly upstream of that measured experimentally, while that predicted by VIS07 matches the experimental point at zero degree. On the suction side, the transition computed by the two codes is in good agreement with experimental data and lies at 46% and 45% of the chord, respectively. Downstream of the transition, the flow decelerates sharply then re-accelerates smoothly, remaining supersonic throughout the process. The supersonic region is terminated by a shock near 65% of the chord. The first deceleration, which can be attributed to the transition, and the shock obtained using VIS07 are both sharper than those calculated using BLASTER. Besides, the pressure coefficient computed using VIS07 oscillates near 60% of the chord, upstream of the shock wave. On the other hand, Braune et al. [15] observed a first shock between 40% and 45% of the chord, directly followed by a second supersonic re-acceleration and terminated by a shock close to 65% of the chord. While they noted a strong interaction between the shocks and the boundary layer, they do not indicate where the transition occurs. In their experiments, Hebler et al. [9] observed intermittent flow between 40% and 55% of the chord. Defining the location of the transition as the point with 50% intermittency, they reported that transition occurred at 46% of the chord on average. However, the pressure distribution they measured does not feature the first shock between 40% and 45% of the chord. Note that their experiment was conducted at an angle of attack of 0.05° rather than 0° . Both authors also observed that the flow was highly unsteady and gave time-averaged pressure distributions. This can explain the discrepancies between the numerical results and the experimental data. The derivatives of the pressure with respect to the angle of attack obtained using BLASTER and VIS07 are calculated by means of central finite differences by varying the angle of attack by

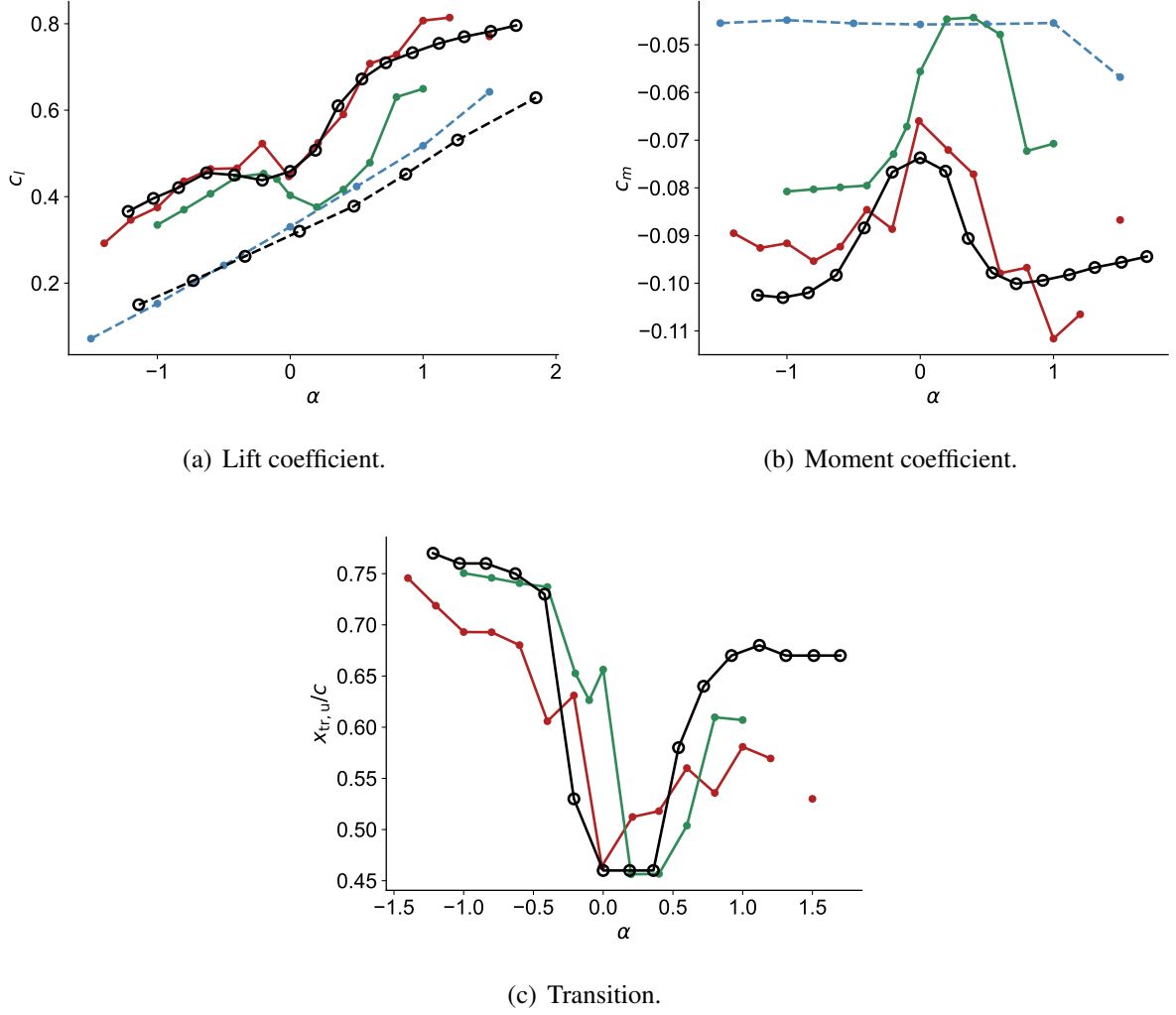


Figure 3: Lift and moment coefficients, and location of the transition on the suction side of the CAST 10-2 aerofoil at $M_\infty = 0.75$, $Re = 2 \times 10^6$ and $\alpha = [-1.5; +1.5]^\circ$ computed using BLASTER and VIS07, and compared to experimental data [9, 15]. Dashed blue: BLASTER (fully turbulent), solid red: BLASTER (free transition), solid green: VIS07 (free transition), dashed black: experimental data (fully turbulent), solid black: experimental data (free transition).

0.02° . Note that the two codes are very sensitive to variations in the angle of attack in this challenging case. These results must therefore be interpreted with caution. The two codes predict similar trends, except in front of the transition, where the magnitude of the pressure derivative calculated by VIS07 is higher, and in the trailing edge region, where VIS07 yields a nonzero lift derivative. The location and the strength of the pressure gradient vary significantly when the transition is triggered at the leading edge and when the flow transitions naturally. This is probably due to different interaction mechanisms between the transition, boundary layer and shocks. Although BLASTER does not capture the unsteady nature of the boundary layer, the code is able to predict differences between fully turbulent and transitional flows at least qualitatively, which will certainly affect the flutter behaviour of the aerofoil.

3.2 Dynamic aeroelasticity

The methodology developed in this work is used to calculate the flutter of the CAST 10-2 wing at a freestream Mach number $M_\infty = 0.75$, a Reynolds number $Re = 2 \times 10^6$ and an angle of

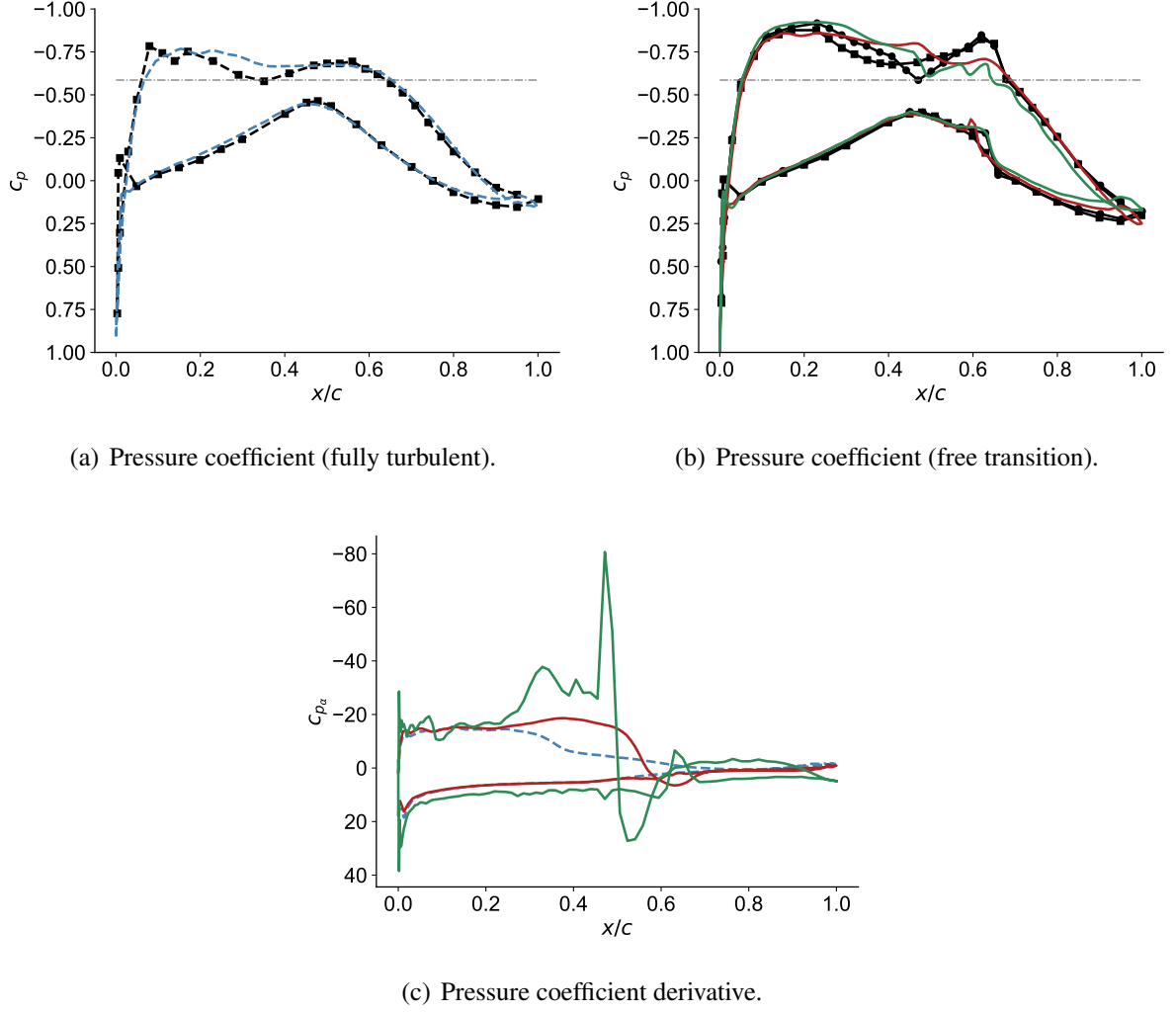


Figure 4: Steady pressure coefficient and derivative with respect to angle of attack along the chord of the CAST 10-2 aerofoil at $M_\infty = 0.75$, $Re = 2 \times 10^6$ and $\alpha = 0^\circ$ obtained using BLASTER and VIS07, and compared to experimental measurements [9, 15]. Dashed blue: BLASTER (fully turbulent), solid red: BLASTER (free transition), solid green: VIS07 ($\alpha = 0.4^\circ$, free transition), dashed black squares: experimental data (fully turbulent) [9], solid black circles: experimental data (free transition) [15], solid black squares: experimental data (free transition) [9], horizontal dash-dotted grey line: critical pressure coefficient.

attack $\alpha = 0^\circ$ tested by Hebler et al. [13] and Braune et al. [15] in the Transonic Wind Tunnel Göttingen. The pitch and plunge mechanism was attached at the wing quarter chord and its structural parameters are given in table 1.

Table 1: Structural parameters of the CAST 10-2 wing and its pitch/plunge mechanism mounted in the wind tunnel. Reproduced from [13].

Parameter	Symbol	Value	Unit
Mass	m	26.252	kg
Moment of inertia	I_α	0.0776	kg/m ²
Static imbalance	S_α	0.2659	kg m
Leaf spring stiffness	K_h	8.902×10^5	N/m
Torsional spring stiffness	K_α	7.282×10^3	N m/rad

The mass and stiffness matrices are given by

$$\mathbf{M} = \begin{bmatrix} m, & S_\alpha \\ S_\alpha, & I_\alpha \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} K_h, & 0 \\ 0, & K_\alpha \end{bmatrix}, \quad (16)$$

and the reference length is taken to be half the chord length. The modes shape φ_i are obtained by solving the flutter equation at zero velocity

$$(-\omega_i^2 \mathbf{M} + \mathbf{K}) \varphi_i = 0, \quad (17)$$

where ω_i are the natural circular frequencies, yielding

$$\omega = [182.4, 314.7] \text{ rad/s}, \quad \varphi = \begin{bmatrix} 0.469, & -0.015 \\ 0.883, & 1.0 \end{bmatrix}, \quad (18)$$

where the modes are organised per column. The natural frequencies are close to those measured experimentally [13] and the mode shapes are identical to those obtained by Hebler et al. [12]. The generalised aerodynamic force matrices corresponding to these modes are calculated using SDPM at seven reduced frequencies $k = [0.001, 0.1, 0.2, 0.3, 0.5, 0.7, 1.0]$. The grid is made of 40 quadrilateral panels along the chord of the wing and 1 along the span on both the suction and the pressure sides. The pressure derivatives calculated using BLASTER and VIS07, shown in figure 4(c), are used to correct SDPM. Finally, PYPK is used to calculate flutter by solving equation (13) for stagnation pressures varying between 40 and 130 kPa. Figure 5 shows the evolution of the frequencies and damping ratios of the plunge and pitch modes calculated numerically without any correction, and corrected using the steady pressure derivatives with fixed and free transitions. Using the correction method lowers the critical pressure, and this reduction becomes even more pronounced as the transition is left free. When VIS07 data is used to correct the panel method, the frequency of the pitch mode increases with the stagnation pressure, while it decreases when BLASTER data is used. This is most probably due to the differences noted in the pressure derivatives. However, there is no change in the flutter mechanism. Fehrs [10] obtained a similar evolution of the frequencies and damping ratios, as well as comparable reduction in the flutter pressure, using unsteady RANS calculations combined to a k method. Hebler et al. [13] also reported limit cycle oscillations and flutter, but with both modes becoming unstable. On the other hand, Braune et al. [15] observed limit cycle oscillations and single degree of freedom flutter whereby the pitch mode became unstable.

The critical stagnation pressures obtained at flutter are given in table 2 and compared to those measured experimentally by Hebler et al. [13] and by Braune et al [15]. The critical pressure predicted using SDPM without any correction is much higher than that observed experimentally. When the generalised aerodynamic forces are corrected using BLASTER and VIS07, the critical pressure decreases dramatically. This is consistent with the transonic dip phenomenon. Tripping the transition at the leading edge in BLASTER yields a higher flutter pressure than when the transition is left free. This trend is similar to that observed experimentally even though there are some discrepancies in the actual critical pressure value. Such differences between numerical and experimental results have already been noted in previous works by the authors [36, 37]. They can be linked to the inaccuracy of the pressure derivative used to correct the panel method, especially in this challenging case where the physics is unsteady and features a breathing laminar bubble, not well captured by VII. It is also worth noting that Hebler et al. [12] used a doublet lattice method corrected by wind tunnel data in order to calculate the flutter of the CAST 10-2 aerofoil. They also observed that the transonic dip was more pronounced with a transitional flow than with a fully turbulent flow. However, the flutter was calculated at an angle of attack $\alpha = 1^\circ$. Consequently, a direct comparison with their results is not relevant.

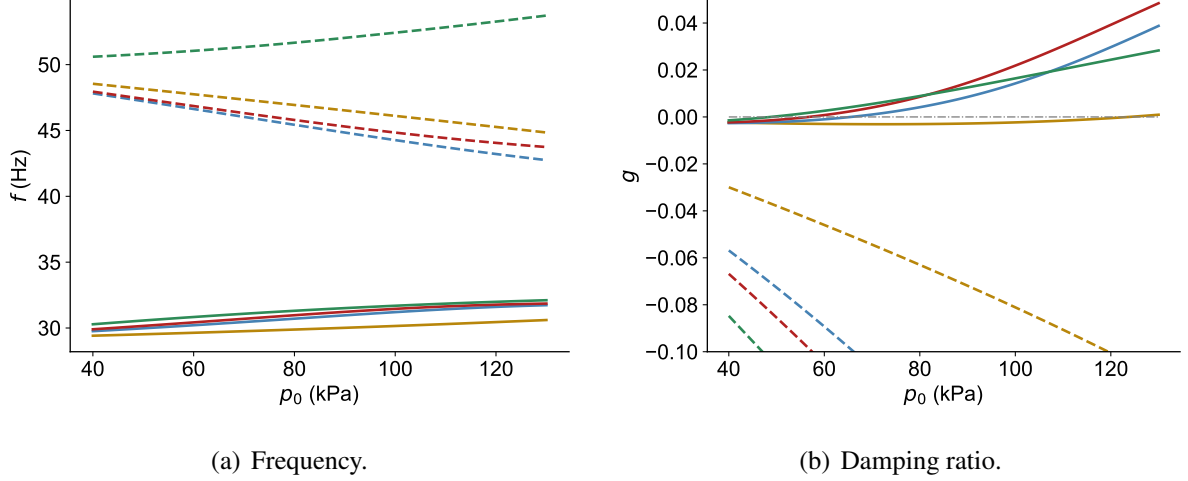


Figure 5: Frequency and damping ratio of the CAST 10-2 wing as a function of the stagnation pressure at $M_\infty = 0.75$, $Re = 2 \times 10^6$ and $\alpha = 0^\circ$ obtained using SDPM, alone and corrected by BLASTER and by VIS07. Blue: SDPM-BLASTER (fully turbulent), red: SDPM-BLASTER (free transition), green: SDPM-VIS07 (free transition), yellow: SDPM. The solid lines represents the plunge mode while the dashed lines represent the pitch mode. The horizontal dash-dotted grey line represents zero-damping.

Table 2: Flutter stagnation pressure at $M_\infty = 0.75$, $Re = 2 \times 10^6$ and $\alpha = 0^\circ$ obtained using SDPM, alone and corrected by BLASTER and by VIS07.

Method	Transition	p_0 (kPa)
SDPM	-	124
SDPM-BLASTER	Fixed	65
SDPM-BLASTER	Free	56
SDPM-VIS07	Free	48
Hebeler's experiments [13]	Fixed	70
Hebeler's experiments [13]	Free	50
Braune's experiments [15]	Free	45

4 CONCLUSION

The present work investigated the combination of the steady viscous-inviscid interaction codes BLASTER and VIS07 to the unsteady panel method SDPM and to the non-iterative p - k methodology implemented in PyPK in order to calculate flutter on the CAST 10-2 aerofoil. The results were then compared to experimental measurements. Due to their steady nature, the viscous-inviscid interaction methods used in this work could not faithfully reproduce the complex unsteady physics inherent to the transonic and transitional flow over the aerofoil. However, the location of the transition, as well as the nonlinear trend in the lift and moment coefficients calculated using BLASTER and VIS07 at several angles of attack showed good agreement with experimental data. Furthermore, BLASTER qualitatively captured consistent shifts in the aerodynamic behaviour between flows whereby transition is triggered at the leading edge and those with free transition. Using the steady pressure derivatives calculated by means of viscous-inviscid interaction to correct the unsteady panel method allowed improving flutter prediction. More specifically, the critical stagnation pressure was found to decrease for the case with free transition compared to the case where it is fixed at the leading edge. This is similar to what had been observed experimentally, although there are some discrepancies in the value of the stagnation pressure at flutter.

Although the results obtained using the methodology developed within the scope of this study are in agreement with experimental data, several points must be improved. Firstly, the results obtained using BLASTER at certain angles of attack were not accurate and, in some cases, the code did not converge at all. The robustness of BLASTER must therefore be improved. Secondly, the present work investigated only one Mach number, and the study should be extended to several Mach numbers. Finally, the flutter mechanism predicted by SDPM and PYPK is not consistent with that observed experimentally, and should consequently be investigated further. Ultimately, the methodology will also be demonstrated on more complex configurations, such as the wing of the European Clean Aviation project AWATAR ⁷.

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⁷<https://cordis.europa.eu/project/id/101140588>

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