

ON THE APPLICATION OF BACKGROUND ORIENTED SCHLIEREN TO A TRANSONIC LOW-REYNOLDS TURBINE CASCADE

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ABSTRACT

Qualitative and quantitative visualizations of transonic turbomachinery flows provide essential information on compressibility and Mach number effects on boundary layer development, shock-boundary layer interactions (SBLI) and trailing edge flows. Background Oriented Schlieren (BOS) is a relatively new optical technique that allows capturing unsteady density gradient fields through turbomachinery cascades and thus, quantitative experimental data to validate transonic and supersonic blade designs. However, only very few experimental works in the open literature have successfully applied Background Oriented Schlieren (BOS) to transonic turbine or compressor flows. The current study presents the application of BOS to a transonic low pressure turbine cascade, the VKI SPLEEN C1 cascade for a range of Reynolds numbers (70,000 to 140,000) and transonic Mach numbers (0.90 to 1.00). The linear turbine cascade is tested at the von Karman Institute in the S-1/C high-speed wind tunnel. The test section is instrumented with different BOS optical setups to visualize the time-resolved density gradients through the turbine passage and at the trailing edge plane with dedicated field of views. The BOS images are processed using the classical cross-correlation algorithm, and

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the optical flow approach, recently introduced in BOS applications to gain spatial resolution and increased sensitivity. Steady-state density gradients of the cascade flow characterize the airfoil boundary layers, wakes and shock waves. The results from the two data reduction methods are assessed and compared against available RANS CFD predictions. Time-resolved measurements reveals the low-frequency motion of weak shock waves generated in the blade passage using POD and spectra analysis.

1 INTRODUCTION

The continuous decarbonization process arising in the aerospace sector requires more sustainable and efficient technologies. A promising solution is the ultra-high by-pass geared turbofan (GTF) architecture. This technology can effectively reduce the CO_2 , the NO_x emissions, as well as noise production, positioning it as a leading candidate to achieve the targets of the Flightpath 2050. The key feature of this architecture involves the use of a gearbox, that decouples the rotational speeds of the fan and of the turbine. This allows the installation of low-pressure turbines, characterized by a reduced number of stages, allowing significant savings in terms of engine weight, dimension, and fuel consumption [1, 2].

These LPTs operate at transonic exit conditions in terms of Mach number, and at low-Reynolds numbers, especially in the flight cruising phase. These specific flow conditions are challenging for achieving significantly high values of efficiencies: the compressibility effects interacting with the transitional boundary layer dictate the flow topology, hence the efficiency of operation [3, 4]. Visualization and quantitative characterization of density-varying flow features are essential for a better understanding of the flow physics in these specific environments and the validation of numerical tools. Low-Reynolds transonic cascades present a rather challenging environment for conventional and optical techniques. The low density within the cascade passages demands an extremely sensitive measurement setup for the visualization of weak unsteady shock waves [5].

Schlieren techniques are composed of relatively basic setups for visualization and quantification of density variations. Numerous turbomachinery flow studies used this optical technique to investigate the interaction among the shock-wave and the boundary layer [6, 5, 7], to check the flow periodicity [8], and to examine the trailing edge flows [9, 10, 11]. Yet, Schlieren setups require adequate optical access to the region of interest and, depending on the application, expensive lenses and mirrors. Moreover, the conventional setups do not provide directional information and shadowgraphy cannot be performed simultaneously. In this regard, Background Oriented Schlieren (BOS) provides a unique opportunity for visualization and quantitative characterization of refractive index variations without the need for a calibration [12]. Although there is a sensitivity and resolution degradation associated with BOS in comparison to the conventional Schlieren setups due to optical [13] and image-processing-related concerns [14], the latter can be mitigated using optical flow algorithms [15].

To the knowledge of the authors, the applications of BOS in turbomachinery facilities are rather scarce. Alhay et al. [16] used BOS in combination with particle image velocimetry (PIV) to retrieve the velocity field of the wake at an inlet Reynolds number of 1.6×10^6 , the density gradient at the cascade outlet and consequently estimate the blade loss. They found that the BOS results featured a good agreement with the pressure probe measurements, as the aerodynamic measurements fell within the 95% confidence interval of the BOS results. More recently, Gong et al. [17] performed BOS measurements on a high-pressure turbine cascade, obtaining a successful quantitative density distribution, with a significant agreement with the CFD results computed for the same cascade. Moreover, Sudhof et al. [18] used BOS on a supersonic turbine cascade to monitor the effectiveness of specific slotted tailboards installed in their test section to eliminate the shock wave reflections. They analyzed not only the average shock wave propagation from the blade trailing edges, but also acquired the images at high frequency (in the order of 150 kHz). This allowed analyzing the image brightness fluctuations in the frequency domain. Furthermore, Wernet [19] was able to develop a software able to post-process the BOS data directly live as the images were acquired, making the analysis phase faster and more practical, moreover he implemented a 4K HD monitor for reproducing the background instead of a printed paper, conferring a higher resolution for the background reproduction.

Compressible phenomena are of paramount importance for turbomachinery flows as they influence the behavior of the boundary layer and the loss mechanisms within the cascade passage. Despite this importance, there is still a lack of employment of BOS to high-speed flows, especially in turbomachinery applications. The SPLEEN (Secondary and Leakage Flow Effects in High-Speed Low Pressure Turbines) test case dealt with the investigation of transonic low-pressure turbines, characterized by low Reynolds numbers, to redress the shortfall of experimental data in this

kind of turbomachinery. To complement the experimental investigations performed by Simonassi and Lopes [20, 21], the current study aims to perform BOS measurements for the characterization of compressible turbine flows, and to offer guidelines for applying this technique to the specific flow conditions under investigation. The images were processed with cross-correlation and optical-flow methodologies. The results obtained from the two different algorithms were also compared against CFD data presented in Ref. [22]. Furthermore, high-frequency data were treated in the frequency domain and compared with time-resolved pressure and quasi-wall shear stress measurements [23, 24].

2 METHODOLOGY

2.1 The turbine test rig

The experiments were carried out in the closed-loop high-speed, low-Reynolds number wind tunnel S-1/C located at the von Karman Institute. This facility allows the independent variation of the Reynolds and Mach numbers, hence the engine representative conditions can be effectively reproduced. The facility is driven by a 615 kW axial compressor, controlling the mass flux and therefore the Mach number, while the Reynolds number is fine-tuned by a vacuum pump that enables reaching pressure levels below 5000 Pa. Before admitting the air into the test section, a series of screens and honeycombs are used to ensure homogeneous and purely axial flow conditions. A passive turbulence grid is placed 400 mm upstream of the cascade central blade to elevate the free stream turbulence intensity to 2.4%.

The turbine cascade geometry is representative of the hub section of a high-speed low-pressure rotor of next-generation small-medium range geared turbofan engines. Hence, the SPLEEN C1 cascade was designed for transonic outlet Mach numbers, low flow acceleration, and high deflection. The linear cascade counts 23 blades with a span of 165 mm. More details about the turbine cascade geometry can be found in Ref. [20].

The BOS measurements were performed with and without the inlet turbulence grid to investigate the effect of the freestream turbulence on the cascade shock system and shock-wave boundary layer interactions. Additionally, the removal of the turbulence grid allowed operating the turbine cascade at supersonic exit Mach numbers. The flow conditions addressed in this investigation are reported in Table 1.

2.2 The background oriented schlieren setup

The BOS measurements were implemented with three optical configurations that use different digital CMOS detectors (Hamamatsu ORCA-FLASH 4.0 V3 and Imager SX 4M cameras for low-frequency measurements, and a Dantec Dynamics SpeedSense 2012 for time-resolved measurements). While both detectors utilized for steady measurements have similar spatial resolution specifications (2048 pix $\times$ 2048 pix for Hamamatsu ORCA and 2360 pix $\times$ 1776 pix for Imager Sx) the acquisition frequencies are limited to a maximum of 100 Hz. The Dantec SpeedSense 2012 enables image acquisitions up to 22000 fps with a spatial resolution of 1280 pix $\times$ 800 pix. The three optical setups use a Tamron SP 180 mm lens that allows magnification larger than one. A computer-generated speckle pattern is designed based on the guidelines provided by Cakir *et al.*[25]. A background pattern with a particle concentration of 0.027 ppp is laser printed on 220 g/m² paper at a resolution of 1200 dpi. The size of the background dots was optimized for each combination of camera and FOV. The test section features a single optical access, as visible in Fig. 1, therefore the background location was constrained to be on the cascade end-wall. The backgrounds were front-illuminated by two racks of eight Cree XP-E2 Q5 Green LEDs mounted on the outer surface of the optical window, as shown in Fig. 1.

The turbine cascade operates at low-density levels and thus requires maximizing the optical system sensitivity to detect weak density gradients. The overall sensitivity of a BOS setup is controlled by the trade-off between the distances Z_A and Z_B [26] (Fig. 1). The optical system sensitivity was estimated through the following equation:

$$S = \frac{G \cdot Z_D \cdot M \cdot \Delta Z_{grad}}{l_{px}} \quad (1)$$

where G is the Gladstone-Dale constant, l_{px} is the pixel size of the camera, M is the setup magnification factor and ΔZ_{grad} is the thickness of the density object. For this optical setup, the distance Z_D is fixed (as the background pattern is attached to the cascade hub endwall), and the distance of the camera from the test section can be optimized.

As the camera lens has a fixed focal length, the distance Z_A was related to the magnification factor of the setup for the desired FOV. Regarding the system resolution, two sources can be considered: the optic resolution, directly related to the optical setup and the one connected to the image processing technique. The optical setup resolution can be calculated according to the following law:

$$r_t \approx \frac{f_{\#}}{2 \cdot M} \left(1 - \frac{Z_D}{Z_A} \right) + \frac{Z_D}{f_{\#}} \cdot \frac{M}{M+1} = K_1 \cdot f_{\#} + \frac{K_2}{f_{\#}} \quad (2)$$

Fig. 2 shows the considered FOVs for time-average tests, specifically for Fig. 2(a), the selected FOV had dimensions of 50 mm x 50 mm, and for Fig. 2(b)), the FOV had dimensions of 55 mm x 55 mm. For time-resolved measurements (Fig. 2(c)), the considered dimensions were 60 mm x 60 mm. Especially for the time-average acquisitions, three different FOVs were identified, namely the entire blade region, the passage and the blade wake. The final setup design was performed through the Matlab plug-in "BOS setup generator" [27], which embeds the aforementioned definitions of resolution and sensitivity and allows to compare and predict the performance of different BOS setups. Table 2 collects all the details of the equipment used in this experimental campaign. The magnification factor of the high-speed camera was increased as the pixel size of the sensor was four times larger than the other two cameras. At the same time, the minimum working distance of the used lens did not allow a further approach of the camera to the test section.

2.3 Data Processing

The process of Background Oriented Schlieren relies on the acquisition of a pair of images with and without the presence of density variations arising in the flow field. Due to the variation of the density gradient, the background random dots are visualized by the camera to move, allowing the density gradient reconstruction. The experimental activity consisted in three different macro-steps. A reference image, when no density gradients are present in the test section is captured, subsequently a series of images are acquired during the test, with the presence of density gradients. Displacement detection algorithms, namely cross-correlation and optical flow methodologies, are used for post-processing the images. This allows the extraction of all the flow relevant features.

2.3.1 Image Pre-processing

The density-induced displacements on the background pattern is performed by comparing a wind-on image with a reference one. Therefore, it is inevitable to end up with the secondary effects causing translation and rotation of the background pattern. Hence, a registration phase was necessary before applying any further algorithm to estimate the pixel displacement. The registration process was performed by means of the Speeded-Up Robust Features (SURF) algorithm [28].

The SURF algorithm is composed of three steps, such as feature extraction, description, and matching: a large number of features from the images were detected and they were used to register the particular image with respect to the reference one. The image correction was quantified in terms of average value and standard deviation by comparing the new registered image and the reference one. In Fig. 3, it is possible to visualize the comparison between the row image and the registered one, the trend of the total correction for all the acquired images $\epsilon_{tot} = \sqrt{\epsilon_x^2 + \epsilon_y^2}$, and the spectrum of the horizontal and vertical components. In Table 3, the image alignment accuracy is displayed in terms of average and standard deviation values for all the setups. The resulting registered image had the same macro features of the reference one, and therefore the further displacement detection algorithms, could be used for gathering the relevant flow features. After the registration phase, a Gauss filter was implemented on the images to reduce the spurious effects on light illumination and to enhance the image quality.

2.3.2 Displacement Detection

The processing of BOS images for detection of displacements induced by density gradients was performed via two different approaches. The first approach was the well-known cross-correlation based block matching method, commonly employed for particle image velocimetry applications [29]. The underlying theory of spatial cross-correlation

computes a measure of similarity of two intensity (illumination) maps as a function of displacement with respect to the original position of the selected blocks of pixels referred to as interrogation windows (IW) [30]. Hence, the corresponding displacement vectors represent an average refractive index variation within the IW [31]. However, as the minimum size of the IW is governed by the necessary signal-to-noise ratios of intensity maps for higher correlation coefficients to be established, resolution of the resulting vector field becomes limited. This limitation not only reduces the resolution, but also yields the gradient information being vulnerable to over-smoothing due to the averaging effects [32]. Even though multi-pass interrogation schemes exist to enable a larger dynamic range of pixel displacements to be covered [33], these approaches are developed for tracing fluid particle motions within a bulk flow which makes them ineffective against very localized flow features of density variations. The employed cross-correlation algorithm, was based on a Fast-Fourier Transform window deformation process where the original interrogation window size was composed by $16 \text{ pix} \times 16 \text{ pix}$. A multi-pass approach with 3 passes was used, as a consecutive window size reduction to $8 \text{ pix} \times 8 \text{ pix}$, and finally $6 \text{ pix} \times 6 \text{ pix}$. The selection of the final window size and of the subsequent size reductions was related to the expected pixel displacements, that for the current test conditions attained the order of 0.6 pixels: a too large interrogation window would not have been suitable for assessing the such small pixel displacements.

Nevertheless, the problem of inferior sensitivity and resolution can be addressed through the application of optical flow (OF) image processing technique which defines the motion of objects within the image plane by means of brightness patterns [34]. The greatest advantage of using OF for BOS is its capability to capture the displacement of each pixel enabling one vector-per-pixel resolution. This is proven to enable OF go beyond the resolution and accuracy characteristics of PIV algorithms for reconstructing density field information [35]. Optical flow algorithms can be classified into gradient-based or variational approaches. The gradient-based approaches formulate a governing set of equations for the apparent motion of illumination intensities based on the assumption of constant brightness. However, as the resultant equation system is under determined, it demands additional constraints. This is provided in the form of an energy functional introduced by Horn and Schunk [36], which contains a quadratic penalization of brightness constancy residuals and large displacement gradients with a regularization factor (λ). Accordingly, the Horn-Schunk optical flow is utilized for the processing of BOS images following a hierarchical scheme [37] composed of 6 pyramid levels with 10 iterations at each level. The information transfer between the resolutions of each pyramid level is performed via bi-cubic interpolations while the brightness constancy residuals and high intensity second order gradients of density are regularized with a regularization constant of $\lambda=1000$.

2.3.3 Shock Detection

In order to quantify the physical characteristics of the reconstructed flow structures and the differences achieved with various methods, the physical alignment of the shock waves was computed by identifying locations of higher magnitudes of second order density gradients. Accordingly, displacement fields with the dominant direction of density variation across the shock waves were captured for each measurement configuration (vertical or horizontal) over which regions of interest are selected for the reconstruction of shock wave shape and orientation. Then, second order gradients within the direction of high amplitude density gradient variations over the shock waves were computed. Thus, the location of the shock wave over each line was extracted based on the location of maximum amplitude of second order gradient of density perpendicular to the local shock surface.

3 RESULTS

3.1 Comparison Among Different Algorithms and CFD Results

The assessment of the results and of the different algorithms was performed by comparing the displacements obtained through the different methodologies. As is evident in Fig. 4, the two different algorithms were successfully able to capture all the relevant flow features. The shock wave can be detected not only in the same location for the two algorithms, but also in terms of intensity. To quantify the correspondence of the shock intensity detected by the two algorithms, it can be observed how the vertical displacement across the shock wave is fairly correspondent for the two cases. From a qualitative perspective, smoother variations of the displacement within the analyzed region characterized the optical flow results. This effect is highly related to the calculation approach, in fact optical flow is able to achieve one vector/pixel resolution providing higher sensitivity. This allowed to use higher regularization parameters which assure the additional smoothness without losing sensitivity.

For a better comparison of the two methods, an extraction of the displacements across the shock-wave, in the boundary layer (in Fig. 4, and in the wake region of the blade (in Fig. 7) was performed. Also in this case, a rather good agreement among the two different algorithms was registered. Another relevant aspect is the capability of the optical flow images to better reconstruct the shape of the shock at the impinging point on the blade suction side.

From a theoretical standpoint, the expected shock-wave boundary layer interaction appears to be similar to the one represented in Fig. 5. The shock waves impinging on the blade suction side always lead to a sudden pressure rise. When a laminar boundary layer interacts with a shock wave, the sharp pressure change may force the local boundary layer to separate causing a thickening of it. The separation bubble produces an additional displacement of the flow, leading first to subsequent expansion waves and then, to a final compression shock.

The boundary layer on the blade suction side was investigated at different curvilinear coordinates via extracting the displacement information over the wall normal direction. The specific values of the boundary layer thickness were determined based on a qualitative standpoint: the beginning of the boundary layer was identified with the sudden increase of the background particle displacements close to the blade suction side. Fig. 6 collects the boundary layer thickness obtained by means of the two displacement detection algorithms and by means of the numerical simulations. BOS and CFD do not show an acceptable level of correspondence, especially in the prediction of the boundary layer, where the trends retrieved by the BOS and the CFD do not match.

Analyses of the wake were performed by comparing the density gradient obtained by cross-correlation, optical flow and CFD at different distances downstream the blade trailing edge, specifically $x = 0.03C_{ax}$, $x = 0.11C_{ax}$ and $x = 0.23C_{ax}$. Also in this case, the results among the two different algorithms are rather similar. However, the absolute values determined experimentally underestimate the numerical ones, especially in the closest position to the blade trailing edge. In the other two positions, the CFD and experimental results are more in agreement as shown in Fig. 7.

The optical flow procedure was proved to be more robust compared to the cross-correlation one and it allowed to obtain the reliable pixel displacement in all the relevant regions of the considered FOV.

3.1.1 Uncertainty analysis

To quantify the uncertainty of the results obtained by the BOS measurements, the image pre-processing and the displacement detection algorithms were considered. For the image pre-processing, the uncertainty was determined by means of the standard deviation on the absolute correction displacement $\epsilon_{tot,std}$ by means of the following formula: $U_{Pre-Proc} = 3\epsilon_{tot,std}/\sqrt{N}$, where N is the number of acquired images. Regarding the contribution of image processing stage to the uncertainty of the displacement fields, both approaches (cross-correlation and optical flow) have their respective methods of uncertainty quantification based on the underlying mathematical background. Accordingly, the uncertainty of the displacement detection was estimated for the cross-correlation method: a correlation statistics technique was used for this purpose [38]. This methodology focuses on quantifying the differences between two interrogation windows that have been mapped back in each other by the detected displacement field. A statistical analysis is performed on how each pixel in the interrogation window contributes to the cross-correlation peak, and the uncertainty value of the displacement field is retrieved. Similarly, the initial stage of uncertainty estimation from optical flow is rather similar in which the wind-on image is warped by the estimated displacement field and mapped on to the reference. However, as the algorithm itself makes use of the intensity variations across the image plane, the mismatch of background feature intensities is computed as a subsequent step. Afterwards, the gray scale intensity differences are normalized with the dynamic range of the photo sensor with which the image acquisition is performed. Finally, in order to compute the influence of the local displacement magnitude on the uncertainty, a weighted averaging is performed utilizing the displacement vector field at each time instant and the uncertainty contribution is represented as a time averaged information. The corresponding uncertainty propagation through the data analysis is determined for the three different setups and the results are summarized in Table 4.

3.2 Shock-wave Structure and Orientation

The BOS experiment allowed to visualize the shock wave impinging on the blade suction side. In fully convergent turbine cascades, when sonic conditions within the blade passage are reached, shocks and expansion waves inside the cascade emanate from the blade trailing edge, and impinge on the adjacent blade suction side [39]. For all the Reynolds number tested, only when the Mach number was superior than 0.95, a shock wave could be visualized. The typical flow structures of a shock wave within a turbomachinery row were clearly detected: just upstream the shock wave, the

flow in the passage keeps accelerating due to the passage geometry. When the sonic condition is reached, the passage chokes. Due to the further contraction of the passage (boundary layer growth on SS and PS), the flow suddenly compresses through a normal shock [40].

The shock wave impinging on the blade suction side induces an adverse pressure gradient on the boundary layer which travels upstream in the subsonic part of the boundary layer. This does not necessarily mean that the boundary layer separates at the shock impingement. The separation arises only if the adverse pressure gradient originating from the shock is too intense for the incoming laminar boundary layer [5]. From the BOS images in Fig. 8, it is possible to visualize that the lowest Reynolds number tested (70k), the shock intensity is the lowest possible: this fact reflects the reduced density environment arising for this Reynolds number. As the Reynolds number increases to 120k, the shock becomes more intense, as the detected displacement grew by almost 57% with respect to the previous case. When the Mach number increases up to unity, the shock wave intensity reaches the highest value and the impingement point is advanced towards the blade trailing edge, as the blade is reaching the limit loading condition.

For all the different flow conditions, the shock impinging position was investigated. It was found that the turbulence level did not have any effect on the shock position impingement and intensity, while the Reynolds number seemed to have a modest effects on the location of the shock wave impingement and a much higher effect on the shock intensity. The Mach number, instead, affected both the shock impingement position and shock wave intensity. Lopes et al. predicted on the same cascade under examination, for $M_2 = 0.95$, a boundary layer separation at $\frac{x}{L} \approx 0.68$, confirming in this way the results obtained from the BOS experiments [21]. Table 5 contains all the details about the detected shock impingement points. The FOV capturing the rear part of the SPLEEN blade (on the bottom of Fig. 8) reveals how the shock is emitted from the trailing edge. For $M_2 = 0.95$, only from the pressure side part of the trailing edge a shock wave is barely emitted. This behaviour was observed also by Michalek et al. [41]. For the highest Mach number ($M_2 = 1$), the shock waves are emitted from both sides of the trailing edge. The pressure side portion of the shock wave is clearly preceded by an expansion fan, which impinges on the nearby blade suction side at the same place as the shock wave. In all the considered cases, the shock waves are emitted from a location that is detached from the blade trailing edge. The boundary layers upstream the trailing edge detach immediately downstream, forming the shear layers. The flow attached to the shear layers, instead, accelerates passing through the Prandtl-Meyer expansion fans. The shear layers merge downstream the blade, forming in this way the wake. A base region, characterized by low momentum and low pressure flow, is embedded among the two shear layers. At the point of convergence between the two shear layers, the flow is forced to compress due to the forced changing of flow direction, hence a system of strong shock waves is formed [42]. The shock waves for the different Reynolds and Mach numbers tested was plotted. As a general trend, the shock wave is emanated from a closer region to the trailing edge, as the Reynolds number increases. This fact could be connected to the state of the boundary layer approaching the blade trailing edge, as observed by Rossiter when studying trailing edge flows [11].

3.3 Shock-wave intensity

The shock wave intensity was detected based on the displacement registered across the shock-wave. A line perpendicular to the shock wave was considered and the displacement along this line was extracted to understand and compare the shock-wave intensity for the different flow conditions. To make the comparison more reliable and evident among the different test conditions, the displacement mainly representative of the shock was normalized with respect to the displacement relative to the flow conditions that characterized the strongest shock intensity. The results can be found in Fig. 9. The major factor influencing the shock-wave intensity is the Mach number: the shock is almost 40% stronger with $M_2 = 1$ than the cases at $M_2 = 0.95$. The Reynolds number effect is much more limited on the shock-wave intensity: only an appreciable change can be detected from the lowest Reynolds case ($Re_2 = 70k$) to the highest Reynolds number values ($Re_2 = 120k$ and $Re_2 = 140k$).

3.4 Shock-wave unsteadiness

One of the most relevant outcomes from this experimental analysis was the detection of the shock wave unsteadiness. As visible in Fig. 10, some specific positions were considered for the analysis of the unsteadiness. For the assessment, some specific regions in the blade passage were identified. For the lowest Mach number case ($M_2 = 0.90$), the shock-wave could not be visually detected. Hence, two selected area in the region of the shock were considered and the average displacement vector was identified for each image acquired. This allowed to reconstruct time-series of dis-

placements and to perform a spectral analysis, highlighting the characteristic frequencies. For the cases of $M_2 = 0.95$ and $M_2 = 1.00$, the shock displacement was reconstructed in time domain for the specific pixel corresponding to the 0%, 50% and 100% of the shock pitch-wise coordinate, called x_s , as it can be seen in Fig. 10. The frequency domain analysis was performed only on the shock-wave fluctuations and the power spectral density was obtained to understand the specific frequencies characterizing the shock movement. As visible in Fig. 10, for the $M_2 = 0.90$ case, after the first peak at $19Hz$, a sharp drop followed and subsequently an almost steady trend was observed. For the higher Mach number cases ($M_2 = 0.95$ and $M_2 = 1.00$) some specific frequency contents were clearly detected. The results of the main frequency contents are summarized and reported in Table 5.

The fact that some specific frequency bands could be identified only for the highest Mach number cases ($M_2 = 0.95$ and $M_2 = 1.00$) is an indication that the shock-wave at $M_2 = 0.90$ does not arise. This is also confirmed by the time-average BOS results that struggled to capture the shock for the lowest Mach number cases. Similar results were detected by Boerner [5] that investigated the shock-wave boundary layer interactions by means of hot-films and Schlieren visualizations. He found that for all the investigated Reynolds number (of the same order of magnitudes of the ones of the current experiment), the most excited frequencies were the lowest ones, between 160 and $215Hz$, and the middle ones, namely $717Hz$ and $1814Hz$.

Proper Orthogonal Decomposition (POD) analysis was performed on a focused region containing the shock wave. The aim of the POD analysis was the detection of the main modes of vibrations of the shock wave. The POD was performed for the highest Mach number case ($M_2 = 1.00$), and the results can be found in Fig. 11. The POD analysis focused on the fluctuation displacement across the shock wave. Mode 1 is the most energetic mode and it represents the average displacement. The center of the shock wave is characterized by a positive displacement region held between negative displacement regions. Mode 2 exhibits closely spaced displacement peaks indicating the unsteady complex structure of the density field around the shock wave. Conversely, the spatial function of Mode 3 resembles that of Mode 1 but with the opposite sign. This mode accounts for the phase-oriented relocation of the shock wave. This analysis was verified by comparing the reconstructed displacement field. By considering only Mode 1 and 2, the shock wave center remains stationary while its width and intensity vary over time. However, including Mode 3 results in the shock wave moving in the streamwise direction. Further investigation is needed to uncover the sources of the corresponding shock wave instability.

4 CONCLUSION

BOS measurements were performed in a high-speed ($0.90 < M_2 < 1.00$), low-Reynolds ($70k < Re_2 < 140k$) LPT cascade. The simple setup required by BOS made possible density gradient measurements in a single optical access turbine cascade. The flow tested conditions required the adoption of highly sensitive setups for the effective capturing of all the relevant flow features. The flow field was investigated from time-average and time-resolved perspectives. Time-average measurements were performed with cameras aiming at maximizing the resolution, while time-resolved acquisitions were carried out by means of a lower resolution high speed camera. Two displacement detection algorithms were utilized, specifically the cross-correlation algorithm and the optical-flow. The results derived from the two methodologies well agreed among each other and with the numerical results obtained for the SPLEEN C1 cascade. Optical flow results presented not only smoother trends in the displacement, compared to the cross correlation derived ones, but also more reliable displacement values in all the relevant regions of the FOV. Shock-wave patterns, boundary layer thickness, shock-wave boundary layer interaction, trailing edge flows and shock unsteadiness were effectively detected by means of this optical technique. Time-resolved measurements allowed to detect the main frequencies at which the shock wave was observed to move. The unsteady data were treated by means of a shock reconstruction process and a POD analysis, providing the dominant frequency and structure of the density field attributed to the shock wave instability. The results agreed as well with previous studies of shock-wave boundary layer interactions investigated for LPT cascades at similar conditions of Mach and Reynolds numbers.

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NOMENCLATURE

Greek letters

$\epsilon_{tot,m}$ Average alignment accuracy [px]

$\epsilon_{tot,std}$ Standard deviation of alignment accuracy [px]

Dimensionless groups

Re Reynolds number, $\rho v c_a x / \mu$

M Mach number, v/a

Abbreviations

G Gladstone-Dale constant [m^{-1}]

Z_D Distance background - gradient plane [m]

M Magnification factor [-]

ΔZ_{grad} Density gradient region thickness [m]

l_{px} Camera pixel size [m]

r_t Optical system resolution [mm]

r_{IP} Image processing resolution [mm]

$f_{\#}$ f-number [-]

Tu Turbulence Intensity [%]

FOV Field of view

f_{ps} Frames per second

IW Interrogation window

f_{samp} Sampling frequency [Hz]

S Optical system sensitivity $\left[\frac{\frac{px}{kg}}{m^4} \right]$

CFD Computational fluid dynamics

GTF Geared turbofan

LPT Low pressure turbine

FR Fast-response sensor

HF Hot-film sensor

HF Hot-film sensor

U_{CC} Uncertainty for Cross-Correlation method [px]

U_{OF} Uncertainty for Optical- Flow method [px]

$U_{\Delta\rho,CC}$ Uncertainty on Density Gradient $\left[\frac{kg}{m^4} \right]$

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TABLES

Table 1. Test conditions in terms of $Re_{\text{out, is}}$, $M_{\text{out, is}}$ and Tu

TI	$Re_{\text{out, is}}$	$M_{\text{out, is}}$
0.90%	70 k	0.90, 0.95
0.90%	120 k	0.90, 0.95, 1.0
0.90%	140 k	0.90, 0.95
2.5%	70 k	0.90, 0.95
2.5%	120 k	0.90, 0.95
2.5%	140 k	0.90, 0.95

Table 2. Optical setups magnification, camera resolution, working distances, pixel size, system sensitivity and acquisition frequency

Setup	Camera	M	FOV	Z_A	Camera resolution	f_{samp}	$S \left[\frac{\text{pix}}{\frac{\text{kg}}{\text{m}^3}} \right]$	r_t [mm]
I	Imager SX 4M	0.19	50 x 50 mm^2	1010 mm	2360 x 1776 px	15Hz	0.110	0.89
II	Hamamatsu	0.24	55 x 55 mm^2	841 mm	2048 x 2048 px	11Hz	0.114	0.59
III	SpeedSense 2012	0.37	60 x 60 mm^2	580 mm	1280 x 800 px	4000Hz	0.041	1.91

Table 3. Raw image alignment accuracy

Setup	$\epsilon_{\text{tot}, m}$ [px]	$\epsilon_{\text{tot}, m}$ [mm]	$\epsilon_{\text{tot}, \text{std}}$ [px]	$\epsilon_{\text{tot}, \text{std}}$ [mm]
I	0.168	0.006	0.147	0.005
II	0.170	0.013	0.103	0.008
III	0.190	0.005	0.168	0.005

Table 4. Uncertainty quantification for different setups

Uncertainty	Setup I	Setup II	Setup III
$U_{Pre-Proc}[px]$	0.044	0.038	0.022
$U_{CC}[px]$	0.093	0.167	0.04
$U_{OF}[px]$	0.044	0.054	N/A
$U_{tot,CC}[px]$	0.103	0.171	0.045
$U_{tot,OF}[px]$	0.062	0.058	N/A
$U_{\Delta\rho,CC} [\frac{kg}{m^3}]$	0.930	1.500	1.097
$U_{\Delta\rho,OF} [\frac{kg}{m^3}]$	0.600	0.511	N/A

Table 5. Shock-wave impingement position and shock-wave main frequencies

Re_2 [-]	M_2 [-]	Tu [%]	x/L [-]	$f_{1^{st}}$ [Hz]	$f_{2^{nd}}$ [Hz]	$f_{3^{rd}}$ [Hz]
70k	0.95	2.5	0.696	n/a	n/a	n/a
70k	0.95	0.9	0.692	15	94	179
120k	0.90	n/a	n/a	18	n/a	n/a
120k	0.95	2.5	0.685	n/a	n/a	n/a
120k	0.95	0.9	0.683	21	94	183
120k	1.00	0.9	0.743	28	93.5	246
140k	0.95	2.5	0.681	n/a	n/a	n/a
140k	0.95	0.9	0.677	21	89	187.5

LIST OF FIG CAPTIONS

Fig. 1. Photos of the BOS setup (1st row-left), background installation on the cascade end-walls (1st row-middle) and illuminated optical test section (1st row-right). Isometric (2nd row) and meridional (3rd row) views of the SPLEEN test section.

Fig. 2. (a) FOV used for setup 1, (b) FOV used for setup 2, (c) FOV used for setup 3

Fig. 3. Image correction.

Fig. 4. (Left) Qualitative comparison between CC and OF algorithms, (Center) Comparison of vertical displacement obtained from CC and OF, (Right) Comparison of horizontal displacement obtained from CC and OF algorithms.

Fig. 5. Attached and separated normal shock wave boundary layer interaction [43].

Fig. 6. Comparison of experimental and numerical boundary thickness (a) $Re_2 = 70k$ $M_2 = 0.90$, (b) $Re_2 = 70k$ $M_2 = 0.95$, (c) $Re_2 = 120k$ $M_2 = 0.90$, (d) $Re_2 = 120k$ $M_2 = 0.95$.

Fig. 7. Wake density gradients obtained by CC, OF and CFD.

Fig. 8. BOS visualization of shock impingement on the blade suction side and wake region for $Re_2 = 70k$ $M_2 = 0.95$ (1st column), $Re_2 = 120k$ $M_2 = 0.95$ (2nd column), $Re_2 = 140k$ $M_2 = 0.95$ (3rd column), $Re_2 = 120k$ $M_2 = 1.00$ (4th column) and reconstructed shock waves (5th column).

Fig. 9. Comparison of shock wave intensity for different flow conditions.

Fig. 10. Shock-wave detection points and SPLEEN blade instrumented (left) and Spectra of shock wave displacement, hot-film and Kulites for different flow conditions.

Fig. 11. Proper Orthogonal Decomposition (POD) analysis analysis on shock-wave dynamics.