

Full paper

An interaction formula method for buckling design of steel angles bolted through one leg at their ends

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Abstract

Steel angles used as bracings in lattice towers or other types of structures are typically connected through one of their legs at their ends. These semi-rigid connections are eccentric to the central axis of the member inducing bending moments under axial loading. Nevertheless, the relevant European design standards treat angle bracings as simply supported, centrally loaded members accounting for end joint effects empirically through an effective slenderness. Previous studies have shown that this method cannot provide accurate resistance predictions. The present study proposes an alternative design approach based on the interaction formula method, explicitly accounting for the end restraints and the bending moments due to eccentricities. A numerical parametric study considering angles with bolted end joints is also presented. The accuracy of both the proposed method and the existing design rules is assessed against the numerical results. The comparisons demonstrate that the proposed method achieves high accuracy with significantly lower variability, thus providing more reliable resistance predictions, compared to the normative design rules.

Keywords

steel angles, lattice towers, bolted joints, stability, design methods, FEM, Eurocode 3

1 Introduction

Steel angles have wide application in several types of structures, commonly used as bracing members. Lattice towers, in particular, are often built exclusively from angles due to their ease of connection. Angle bracings are typically bolted through one of their legs to the adjoining structural members. This connection is eccentric to central axis of the member; therefore, although bracings are primarily used to transmit axial forces, bending moments also arise due to the eccentricities induced by the end joints.

Two European design Standards, namely EN 1993-3-1 [1] and its new version FprEN 1993-3 [2], as well as EN 50341-1 [3] provide rules for the stability verification of angle bracings in lattice towers. The design rules of both standards are based on the effective slenderness method. This method assumes that bracings are simply supported, axially loaded members and proposes a buckling verification to pure compression. The effects of rotational restraints and eccentricities are accounted for through an effective slenderness. The differences between the standards lie in the proposed buckling curve, the consideration or other-

wise of the torsional-flexural buckling mode, and the expressions for calculating the effective slenderness.

Previous studies by the authors [4,5] as well as other researchers [6,7] have demonstrated that the existing design rules cannot accurately predict the buckling resistance of angles with single-leg bolted end joints. The present study proposes an alternative design approach for such members based on the interaction formula method. By explicitly accounting for the end restraints and the interaction between axial force and bending moments, this approach is expected to yield improved resistance predictions. In the following, the proposed method is first outlined. Then, a numerical study is presented which generates the required input values for the proposed interaction formula, as well as reference member resistances for the subsequent assessment of the accuracy of the analytical design methods. Finally, conclusions regarding the applicability and accuracy of the proposed method are drawn from the obtained results and further research is proposed. The study is limited to members made of S460 steel with Class 2 and Class 3 cross-sections. Members made of various steel grades, including Class 4 profiles, are currently being studied by the authors, and relevant

results will be presented in a subsequent work.

2 The interaction formula method

EN 1993-1-1 [8] provides interaction formulae for stability verification of members subjected to compression and bending. Nevertheless, these formulae have not been validated for angle members and are therefore inapplicable for their design.

During the RFCS project ANGELHY [9], interaction formulae exclusively for angle members were developed in compliance with Eurocode 3 provisions and validated for pin-ended members in [10]. In the present study, a modified interaction formula, building upon the ANGELHY framework, is proposed. The proposed formula, given in Eq. (1), reflects the fact that angles which are bolted through one leg at their ends fail in a torsional-flexural buckling mode where 2nd order effects appear predominantly about their minor axis (see §3.1 and [4]).

$$\left[\frac{N_{Ed}}{N_{b,Rd}} + k_{vu} \frac{M_{u,Ed}}{M_{bu,Rd}} \right]^\xi + k_{vv} \frac{M_{v,Ed}}{M_{v,Rd}} \leq 1 \quad (1)$$

where N_{Ed} , $M_{u,Ed}$ and $M_{v,Ed}$ are the axial force and the maximum bending moments about the principal axes along the member. $N_{b,Rd}$ is the member buckling resistance under compression, $M_{bu,Rd}$ is the member lateral torsional buckling under major axis bending and $M_{v,Rd}$ is the cross-sectional bending resistance about minor axis. k_{ij} are interaction factors and the exponent ξ depends on the susceptibility of the cross-section to local buckling.

- Compression buckling resistance $N_{b,Rd}$

$$N_{b,Rd} = \chi \frac{A \cdot f_y}{\gamma_{M1}} \quad (2)$$

where

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1.0 \quad (3)$$

$$\phi = 0.5[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2] \quad (4)$$

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} \quad (5)$$

A is the cross-sectional area, f_y is the yield stress, γ_{M1} is the partial safety factor, N_{cr} is the minimum elastic critical buckling load, and α is the imperfection factor according to §8.3.1.4 and Tables 8.2-8.3 of EN 1993-1-1 [8].

- Lateral torsional buckling resistance $M_{bu,Rd}$

$$M_{bu,Rd} = \chi_{LT} \frac{W_u \cdot f_y}{\gamma_{M1}} \quad (6)$$

where

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq \min\left(1; \frac{1}{\bar{\lambda}_{LT}^2}\right) \quad (7)$$

$$\phi_{LT} = 0.5[1 + \alpha_{LT}(\bar{\lambda}_{LT} - 0.4) + \bar{\lambda}_{LT}^2] \quad (8)$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_u f_y}{M_{cr}}} \quad (9)$$

W_u is the section modulus about major axis, f_y is the yield stress, γ_{M1} is the partial safety factor, M_{cr} is the elastic critical buckling moment, and α_{LT} is the imperfection factor

corresponding to buckling curve a of EN 1993-1-1 [8] independent of steel grade [10].

- Minor axis bending resistance $M_{v,Rd}$

$$M_{v,Rd} = \frac{W_v \cdot f_y}{\gamma_{M0}} \quad (10)$$

where W_v is the section modulus about minor axis, f_y is the yield stress, and γ_{M0} is the partial safety factor.

- Interaction factors k_{ij}

$$k_{vu} = C_u \quad (11)$$

$$k_{vv} = \frac{C_v}{1 - \frac{N_{Ed}}{N_{cr}}} \quad (12)$$

$$C_i = 0.6 + 0.4 \cdot \psi_i \geq 0.4 \quad (13)$$

where $-1 \leq \psi_i \leq 1$ is the ratio between end moments.

- Exponent ξ

$$\xi = \begin{cases} 2, & \text{if } c/t \leq 16\varepsilon \\ 1 + 0.5 \left(\frac{26.3\varepsilon - c/t}{26.3\varepsilon - 16\varepsilon} \right), & \text{if } 16\varepsilon < c/t < 26.3\varepsilon \\ 1, & \text{if } c/t \geq 26.3\varepsilon \end{cases} \quad (14)$$

where c is the width and t the thickness of the outstand angle leg as shown in Figure 1, and $\varepsilon = \sqrt{235/f_y [\text{MPa}]}$ is a material factor.

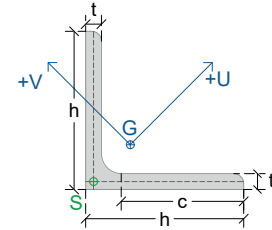


Figure 1 Geometry of equal leg angle

Application of the proposed interaction formula requires four quantities, i.e. the critical buckling load N_{cr} , the critical bending moment M_{cr} , and the developing moments due to the eccentricities $M_{u,Ed}$ and $M_{v,Ed}$ for which no analytical expressions are currently available in the literature for members with elastic and oblique supports. In this study, these quantities are therefore determined numerically. Although this approach is not practical for routine design, it enables investigation of the applicability and accuracy of the interaction formula method for angles with single-leg bolted end joints, with the development of analytical expressions identified as future work.

3 Numerical study

The numerical study involved two types of analyses. First, a parametric study conducted, from which the ultimate resistance of several members was determined through geometrically and materially nonlinear analyses with imperfections (GMNIA). The results of GMNIAs were used as reference for the assessment of the analytical design methods. Additionally, a series of elastic geometrically nonlinear analyses (GNA) were performed to calculate the required parameters to be used as input in the interaction formula design method.

3.1 Parametric study

The parametric study included three parameters: cross-section, member slenderness, and number of bolts in the end joints. Members with three equal-leg angle profiles made of S460 steel were considered (L45×5, L80×8, and L150×13), with end joints having either one or two bolts and six slenderness ratios ranging from 50 to 180. The gusset plates in the end joints were three times thicker and twice as wide as the connected angle leg, made of S460 steel. The profiles were bolted to the gusset plates with the largest possible bolts, and the gusset plates were fixed at their outer faces.

A finite element model (see Figure 2) was developed in ABAQUS [11] software and validated against experimental results obtained by the authors [4,5]. The solid C3D8I element was employed, while mesh size was selected following sensitivity studies. The interactions between different parts of the model were described using the hard contact formulation in the normal direction and the penalty formulation in the tangential direction with a friction coefficient of 0.35 corresponding to galvanised surfaces [12]. The angles and the gusset plates were modelled with a linear elastic-perfectly plastic material (see Figure 2c) with nominal material properties. For the bolts and the regions of the leg around the bolt holes an elastic steel material was employed in order to suppress any joint-related failure modes. The boundary conditions and loading were introduced at single support points, which were coupled with the nodes of the outer surfaces of the gusset plates, as shown in Figure 2a. All degrees of freedom at the support points were fixed except for the longitudinal translation at the point where the loading was applied as an imposed displacement.

Initial geometric imperfections and residual stresses were also introduced in the model. The geometric imperfections had the shape of the first buckling mode, as obtained from a linear buckling analysis (LBA), and they were introduced in the more unfavourable direction with an amplitude equal to $L/1000$ (see Figure 2b) in accordance with [12]. For the residual stresses, a 3-point linear model with an amplitude equal to 70 MPa (see Figure 2d), as prescribed by [13], was adopted.

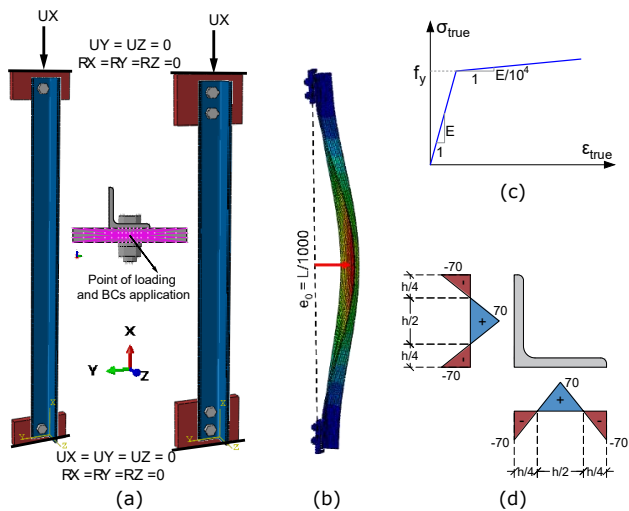


Figure 2 FEM model: (a) Geometry, boundary conditions and loading, (b) geometric imperfections, (c) material model, and (d) residual stresses model

The ultimate resistance of the members, taken as the maximum resisted axial load, was calculated with a series of GMNIAs. The results are summarised in Figure 3, where the vertical axis represents the member resistance normalised with the cross-section resistance, while the horizontal axis the reference length normalised with the radius of gyration about the minor axis. The reference length is defined as the distance between the (external) bolts of the end joints, while the normalised reference length equals the slenderness ratio of the corresponding pin-ended member. As can be seen from Figure 3, member resistance can reach the 20-50% of the cross-section resistance for members with one-bolt end joints and the 30-70% for members with two-bolt end joints.

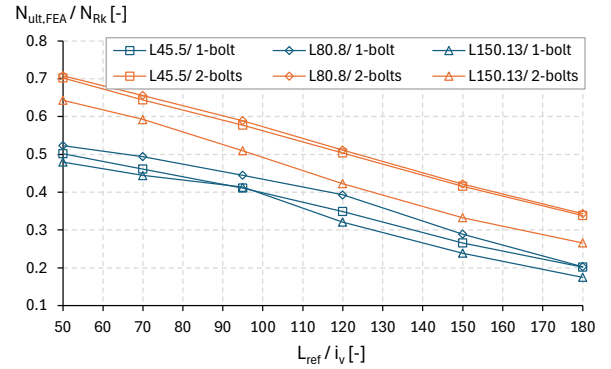


Figure 3 Results of the numerical parametric study: normalised member resistance, GMNIA

Figure 4 illustrates the equilibrium path and the failure mode of an indicative member. At collapse, displacements along both principal axes as well as twist have been developed, indicating a torsional-flexural buckling mode. Nevertheless, displacements along the major axis are considerably larger, which shows that second-order effects about the minor axis were predominant.

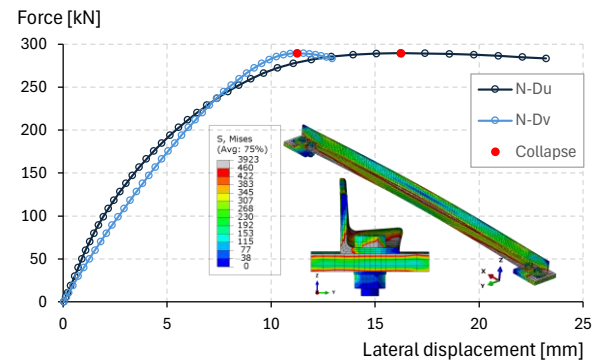


Figure 4 Force-Lateral displacement curves at half-length and failure mode of an indicative specimen from GMNIA (indicative specimen: L80x8, 2-bolt end joints, slenderness ratio = 120)

3.2 Critical buckling load N_{cr}

The elastic critical buckling load under compression was calculated with a series of GNAs, taken as the maximum axial force resisted by the member. The same model as the one employed for the GMNIAs was adopted, with the difference that elastic material was used for all component parts and that no imperfection was introduced. It should be noted that linear buckling analysis (LBA) is not able to provide realistic critical buckling loads due to the nonlinear nature of the contacts in the end joints.

The equilibrium path as well as the elastic buckling mode of an indicative member as obtained from the GNA are presented in Figure 5. The buckling mode involves displacements along both principal axes and twist corresponding to torsional flexural buckling. The displacement along the major axis are almost two times larger than the ones along minor axis indicating that the second-order effects about the minor axis govern.

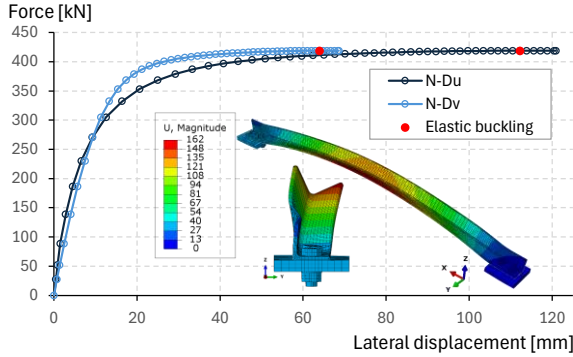


Figure 5 Force-Lateral displacement curves at half-length and elastic buckling mode from GNA (indicative specimen: L80x8, 2-bolt end joints, slenderness ratio = 120)

Figure 6 presents the critical buckling loads of the studied members normalised with the critical minor axis flexural buckling loads of the corresponding pin-ended members, which were calculated analytically. The beneficial influence of the bolted support conditions is evident. In the studied sample, the benefit from the supports increases with the member slenderness because the configuration of the end joints is identical for members with the same cross-section, so the rotational stiffness of the joints becomes relatively higher compared to the stiffness of the member. For the shortest specimen with L150x13 cross-section and one-bolt end joints the ratio of the critical loads is less than one. This is because twist was the predominant deformation component of its buckling mode. Consequently, the critical load corresponding to minor axis flexural buckling was not the most representative to be used as reference load for comparison.

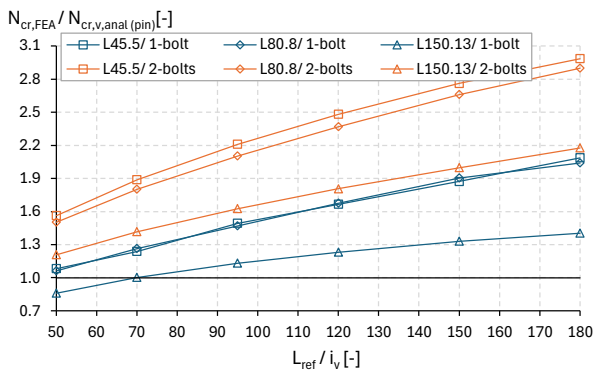


Figure 6 Normalised elastic critical buckling loads under compression as obtained from GNA

3.3 Critical bending moment M_{cr}

The elastic critical bending moment under major axis bending was calculated similarly to the elastic critical buckling load with a series of GNAs, adopting the same model, but modifying the applied loading. In order to impose a constant major axis moment along the member

without preventing warping, two opposite couples of (opposite) longitudinal displacements were applied at the legs of the end cross-sections of the members, as illustrated in Figure 7.

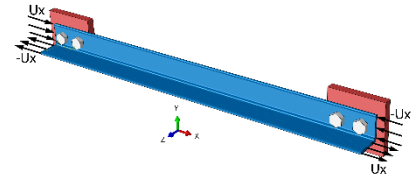


Figure 7 Applied loading for the calculation of the elastic critical bending moment

The critical bending moment was taken equal to maximum major axis bending moment developed at half-length of the member before instability appears. The equilibrium path and the corresponding buckling mode of an indicative member are shown in Figure 8.

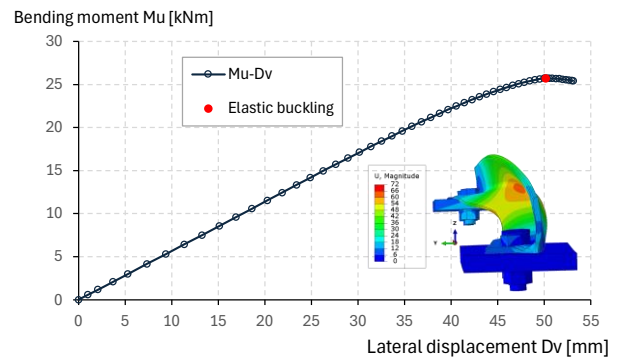


Figure 8 Major axis bending moment – Minor axis displacement curve at half-length and elastic buckling mode from GNA (indicative specimen: L80x8, 2-bolt end joints, slenderness ratio = 120)

Figure 9 presents the elastic critical buckling moments of the studied members normalised with the critical buckling moments of the corresponding pin-ended members, which were calculated analytically with the Eq. (F.11) of FprEN 1993-3 [2]. The beneficial effect of the supports is evident, although less pronounced than the effect on the critical load under compression.

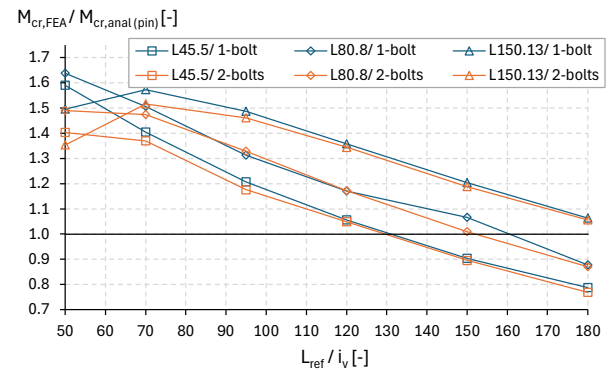


Figure 9 Normalised elastic critical buckling moments under major axis bending as obtained from GNA

3.4 Bending moments due to eccentricities

The bending moments developed in the angle members due to loading eccentricities induced by the end joints are constant along the member. The first-order moments, in particular, which should be used as input in the interaction formula, are a linear function of the applied axial force.

Their magnitude depends on the stiffness of the end joints, ranging from 0 to $N \cdot e$ (Axial force $\cdot$ Eccentricity) for fixed and pinned support conditions, respectively.

As mentioned earlier, the employed models cannot be analysed with a linear analysis due to the inherent nonlinear behaviour of the contacts used in the end joints. For this reason, a series of GNAs was performed adopting the same model and loading as used for the calculation N_{cr} . In order to obtain the first-order bending moments about both principal axes, the developed longitudinal stresses were integrated over a cross-section located close to the ends of the member in its undeformed configuration (see Figure 10).

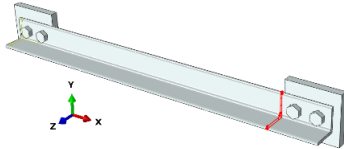


Figure 10 Reference cross-section (highlighted in red) for the calculation of bending moments

Expressions for estimating first-order bending moments, in the form of $M = aN + b$, were derived from the numerical (N , M) data extracted from the initial loading steps using linear regression. Figure 11 illustrates the adopted procedure for an indicative specimen.

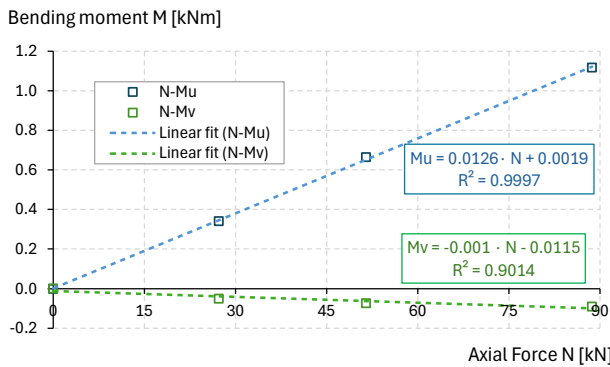


Figure 11 Derivation of linear regression expressions for estimating first-order bending moments from GNA results (indicative specimen: L80x8, 2-bolt end joints, slenderness ratio = 120)

4 Assessment of alternative design rules

This section assesses the accuracy of the interaction formula method, as implemented in the previous paragraphs, as well as the effective slenderness method, as prescribed in the relevant European standards, against the results of the numerical parametric study. Comparisons between the analytically determined resistances in accordance with a) EN 50341-1-Annex J [3] combined with the provisions of the corresponding Belgian Annex [14], b) EN 1993-3-1-Annex G, H [1], FprEN 1993-3-Annex C [2], and d) the proposed interaction formula method are presented in Figure 12 - 15.

The accuracy of the resistance predictions of the normative design rules following the effective slenderness method varies across the slenderness range. Specifically, these rules tend to overestimate the resistance of short members (especially when bolted with a single bolt), whereas they significantly underestimate the resistance of

slender members (especially when bolted with two bolts). In contrast, the resistance estimates of the proposed interaction formula method are consistently on the safe side and highly accurate for all studied members.

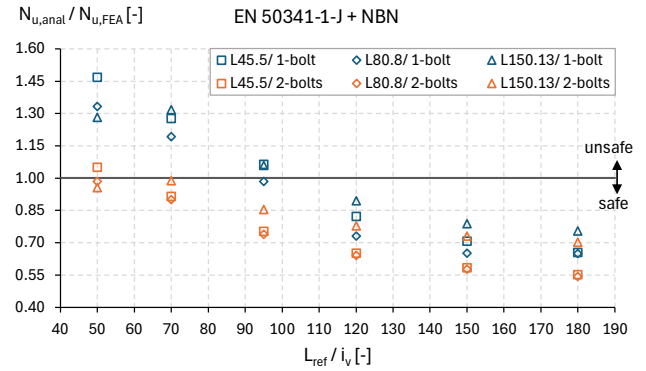


Figure 12 Comparison between numerical and analytical resistance in accordance with EN 50341-1-Annex J + Belgian Annex

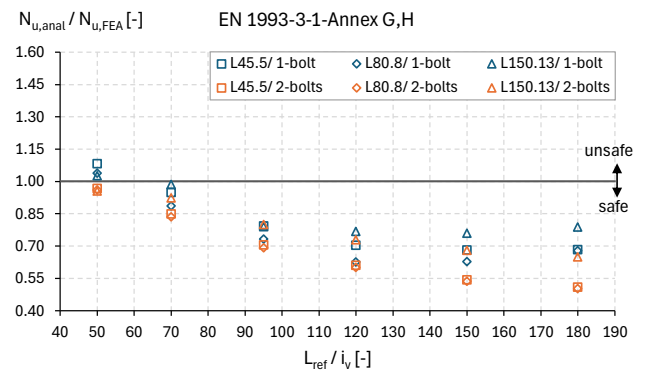


Figure 13 Comparison between numerical and analytical resistance in accordance with EN 1993-3-1-Annex G,H

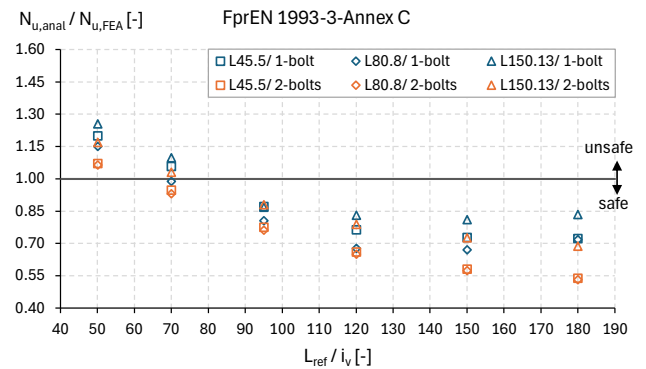


Figure 14 Comparison between numerical and analytical resistance in accordance with FprEN 1993-3-Annex C

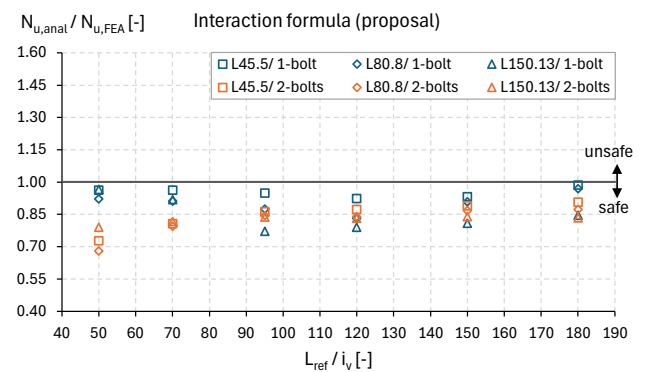


Figure 15 Comparison between numerical and analytical resistance in accordance with the interaction formula design method

Table 1 presents a statistical evaluation of the results. It confirms that the proposed design method outperforms the effective slenderness-based design rules prescribed by the normative standards, achieving high accuracy with significantly lower variability.

Table 1 Statistical evaluation of predictive accuracy of design rules

	$N_{u,analytical} / N_{u,FEA} [-]$			
	EN 50341-1-J+NBN	EN 1993-3-1-G,H	FprEN 1993-3-C	Interaction formula
Mean	0.88	0.77	0.84	0.87
CoV	28.2%	20.6%	23.2%	8.2%

5 Conclusions and outlook

The study proposes an interaction formula method, in compliance with the Eurocode 3 provisions, for the stability verification of steel angles which are bolted through one leg at their extremities.

The accuracy of the proposed method as well as the accuracy of the design rules prescribed in the relevant European standards is assessed against numerical results obtained from GMNIAs with an experimentally validated finite element model. The comparisons demonstrate that the proposed interaction formula method outperforms the effective slenderness-based design rules specified by the standards. The proposed method provides safe and fairly accurate resistance estimates within the examined slenderness range, which encompasses most members encountered in practice. In contrast, the normative design rules exhibit high variability in the accuracy of their predictions, yielding unsafe results for short members and overly conservative for slender members.

In conclusion, the proposed interaction formula method offers a more accurate alternative to existing design rules for the design of angles with single-leg bolted end connections. To enable practical application of the proposed method, the authors are currently developing analytical expressions for evaluating the stiffness of the end joints, the elastic critical buckling loads, and the bending moments arising from joint eccentricity.

Acknowledgements

Part of the study was conducted in the framework of the project "New Steel" funded by Elia Group (Transmission Operator Belgium).

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