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Analyzing changes in optimal variables in linear programming with uncertain parameters

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An introductory example



Investors

"They want to invest in a smart energy house."

Smart energy house



Solar PV



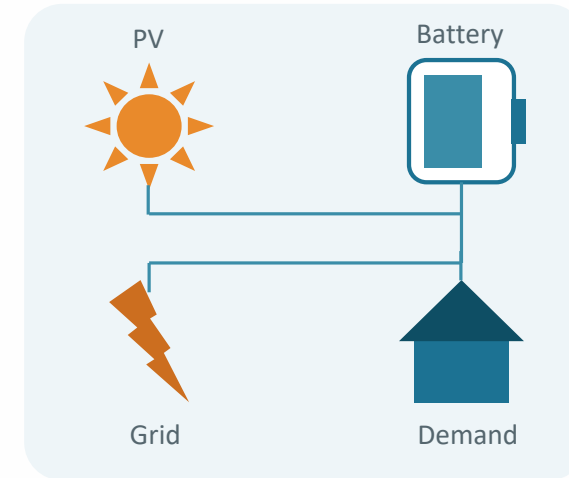
Battery storage



Household demand



Power grid connection



Objective

Minimize total system cost over the horizon, while satisfying demand at every stage.



Optimization model, and how it can be perturbed

The base LP

$$\begin{aligned} P \equiv \min \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

A scalar parameter $\lambda \in \Lambda$ perturbs one of three parts:

$P_c(\lambda)$

cost vector $(c + \lambda d)$

grid electricity prices

$P_b(\lambda)$

right-hand side $(b + \lambda d)$

demand

$P_A(\lambda)$

constraint matrix $(A + \lambda D)$

battery efficiency

The data c, b, A encode uncertain assumptions → how does the optimum react when they change?

Sensitivity analysis perturb **one** coefficient
of c, b or A at a time



Here: perturbation along a direction
one scalar λ moves the **vector** d (or D), all
assumptions vary together



Pertubating cost vector

Questions ?

"If grid electricity prices rise 20%, how does the total cost change ?"



Investors

$P_c(\lambda)$

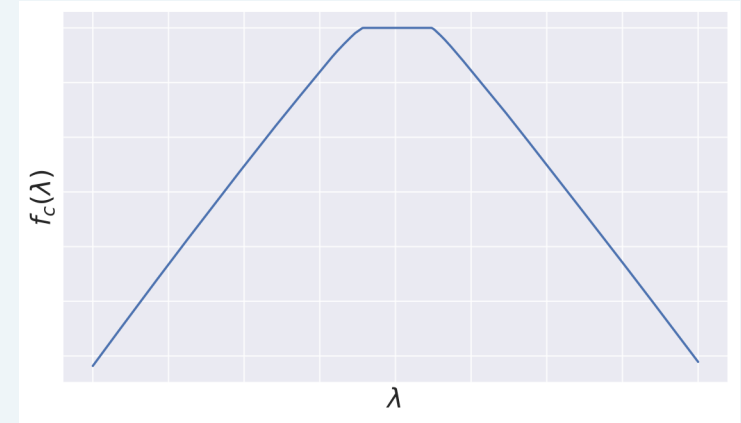
Economic conditions

Prices, CAPEX, OPEX, penalties

$$f_c(\lambda) = \min (c + \lambda d)^T x$$

$$\text{s.t. } Ax \leq b$$

$$x \geq 0$$



Continuous, concave, piecewise linear function



Pertubating right-hand side vector

Questions ?

*"If peak **demand** grows by 15%, how does the total cost change ?"*



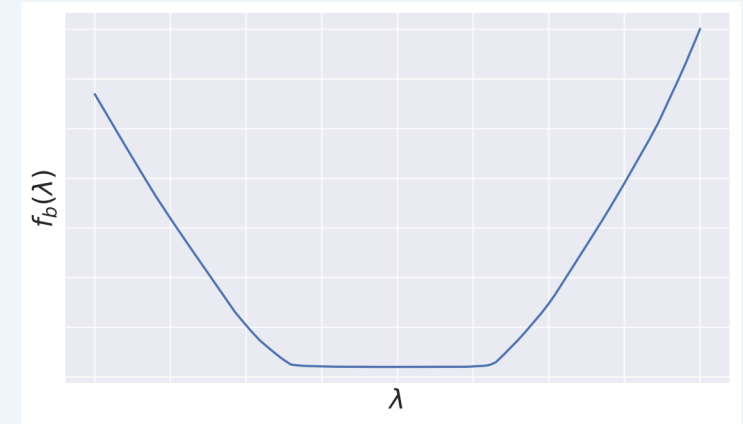
Investors

$P_b(\lambda)$

Demand & resources

Demand, grid import capacity

$$\begin{aligned} f_b(\lambda) &= \min c^T x \\ \text{s.t. } Ax &\leq (b + \lambda d) \\ x &\geq 0 \end{aligned}$$



Continuous, convex, piecewise linear function



Pertubating constraint matrix

Questions ?

*"If **battery efficiency** improves to 95%, how does the total cost change ?"*



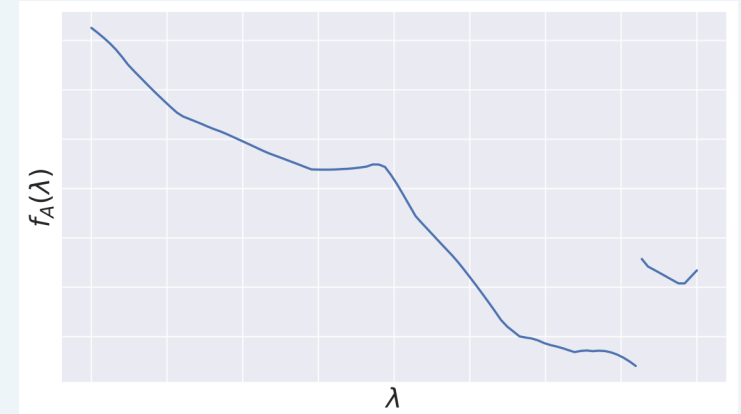
Investors

$P_A(\lambda)$

Technical coefficients

Efficiencies, yields, conversion

$$\begin{aligned} f_A(\lambda) &= \min c^T x \\ \text{s.t. } (A + \lambda D) x &\leq b \\ x &\geq 0 \end{aligned}$$



Piecewise rational function



The investor's what-if questions

Cost Vector

$$P_c(\lambda)$$

Prices, CAPEX, OPEX, penalties

*"If **grid electricity prices** rise 20%, ~~how does the total cost change?~~*



Investors

How much PV and battery should we install?"

Right-hand side

$$P_b(\lambda)$$

Demand, grid import capacity

*"If peak **demand** grows by 15%, ~~how does the total cost change?~~*



Investors

What does the total electricity imported from the grid look like?"

Constraint matrix

$$P_A(\lambda)$$

Efficiencies, yields, conversion

*"If **battery efficiency** improves to 95%, ~~how does the total cost change?~~*



Investors

Can we install a smaller battery?"



A shift of perspective

Classical sensitivity analysis

How does the optimal **value** $f(\lambda)$ change?

Identifies which parameters move the objective, and by how much.

Our focus

How do the **decisions**, the variables x , change?

Decision-makers need to know which decision changes, not only the cost.

We analyze the variables, not only the objective value.



What we actually study

The problem in focus

$$\begin{aligned} g(\lambda) = \min \quad & \varphi^T x \\ \text{s. t. } & c^T x \leq f(\lambda) \\ & Ax \leq b \\ & x \geq 0 \end{aligned}$$

- $c^T x \leq f(\lambda)$: Additional constraint to keep the minimal cost.
- φ extracts the decision of interest: a single variable, a mean of variables, a difference of two decisions...

The main question

What does the function $g(\lambda)$ look like ?

- Is it a piecewise function ?
- Is it concave/convex ?
- Is it smooth ?



Outline

- 1 Cost vector
- 2 RHS
- 3 Constraint matrix
- 4 Dual
- 5 Conclusion



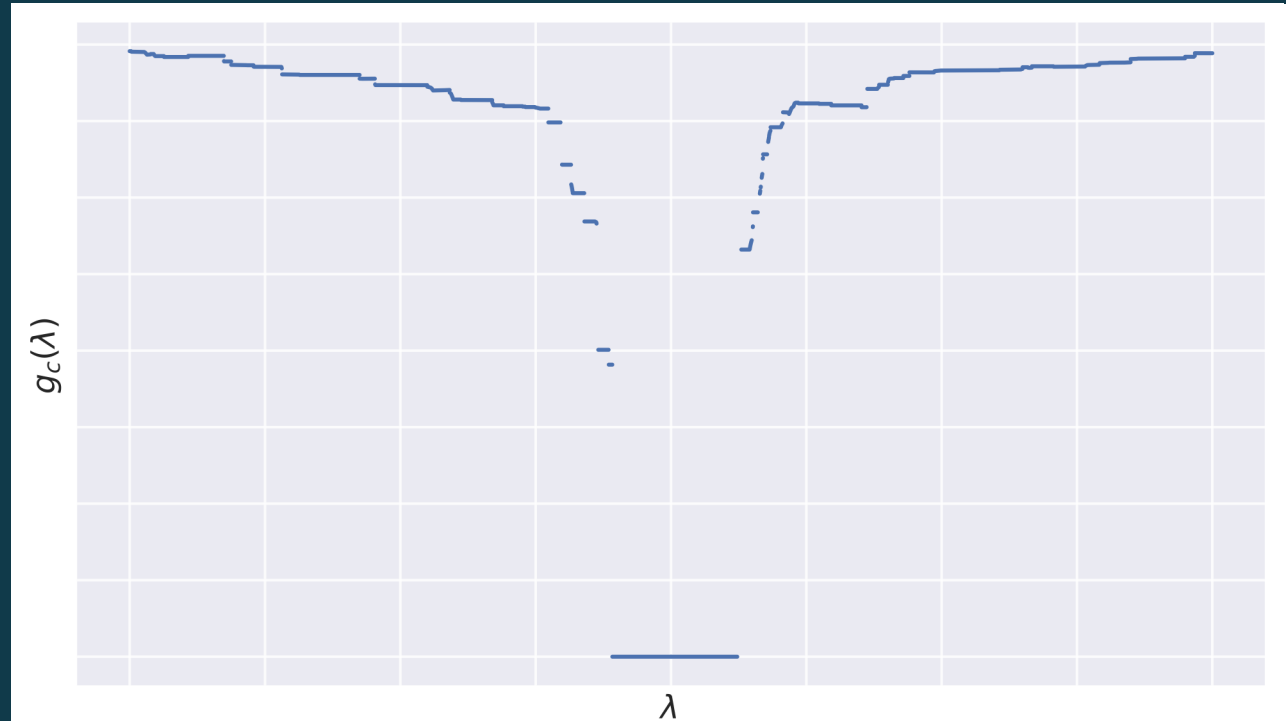
Focus: perturbing the cost vector c

The optimal value function

$$f_c(\lambda) = \min_{\substack{(c + \lambda d)^T x \\ \text{s.t. } Ax \leq b \\ x \geq 0}}$$

The problem in focus

$$g_c(\lambda) = \min_{\substack{\varphi^T x \\ \text{s.t. } (c + \lambda d)^T x \leq f_c(\lambda) \\ Ax \leq b \\ x \geq 0}}$$





What happens as λ varies?

The general problem

$$g_c(\lambda) = \min \varphi^T x$$

$$\text{s.t. } (c + \lambda d)^T x \leq f_c(\lambda)$$

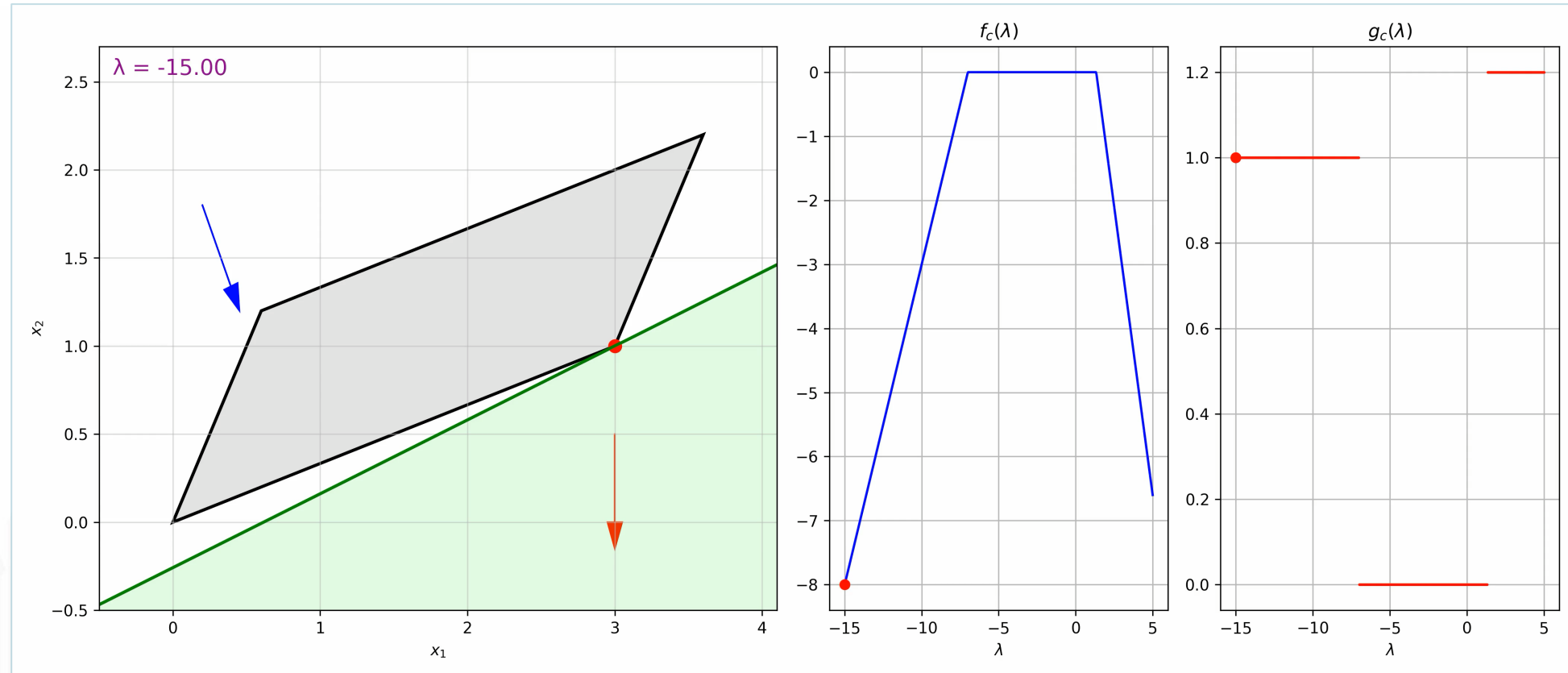
$$Ax \leq b$$

The example problem

$$g_c(\lambda) = \min x_2$$

$$\text{s.t. } (c + \lambda d)^T x \leq f_c(\lambda)$$

$$Ax \leq b$$





Continuous alternative : an ε -tolerance (1/2)

Accepting near-optimal solutions

Introducing $\varepsilon > 0$, the problem becomes :

$$\begin{aligned} g_c(\lambda) = \min \quad & \varphi^T x \\ \text{s.t.} \quad & Ax \leq b \\ & (c + \lambda d)^T x \leq f_c(\lambda) + \varepsilon \\ & x \geq 0 \end{aligned}$$

Why this is natural

Data are noisy (measurements, forecasts).

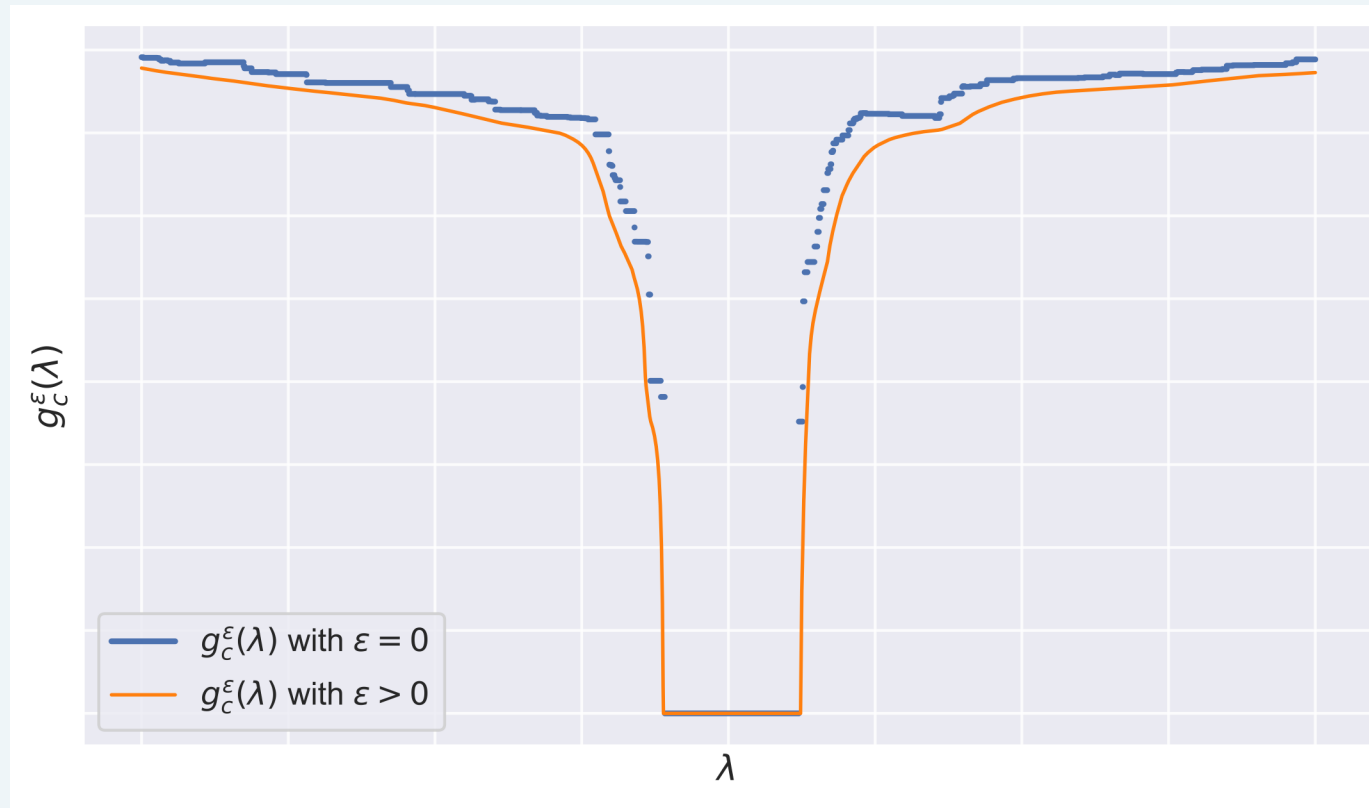
Solvers themselves never give the exact optimum.

Every solver stops within the limits of a numerical tolerance.

An ε is like one cent on a €1,000,000 project: practically nothing.



Continuous alternative : an ε -tolerance (2/2)



What happens as λ varies ? Add a tolerance

The general problem

$$g_c(\lambda) = \min \varphi^T x$$

$$\text{s.t. } (c + \lambda d)^T x \leq f_c(\lambda) + \varepsilon$$

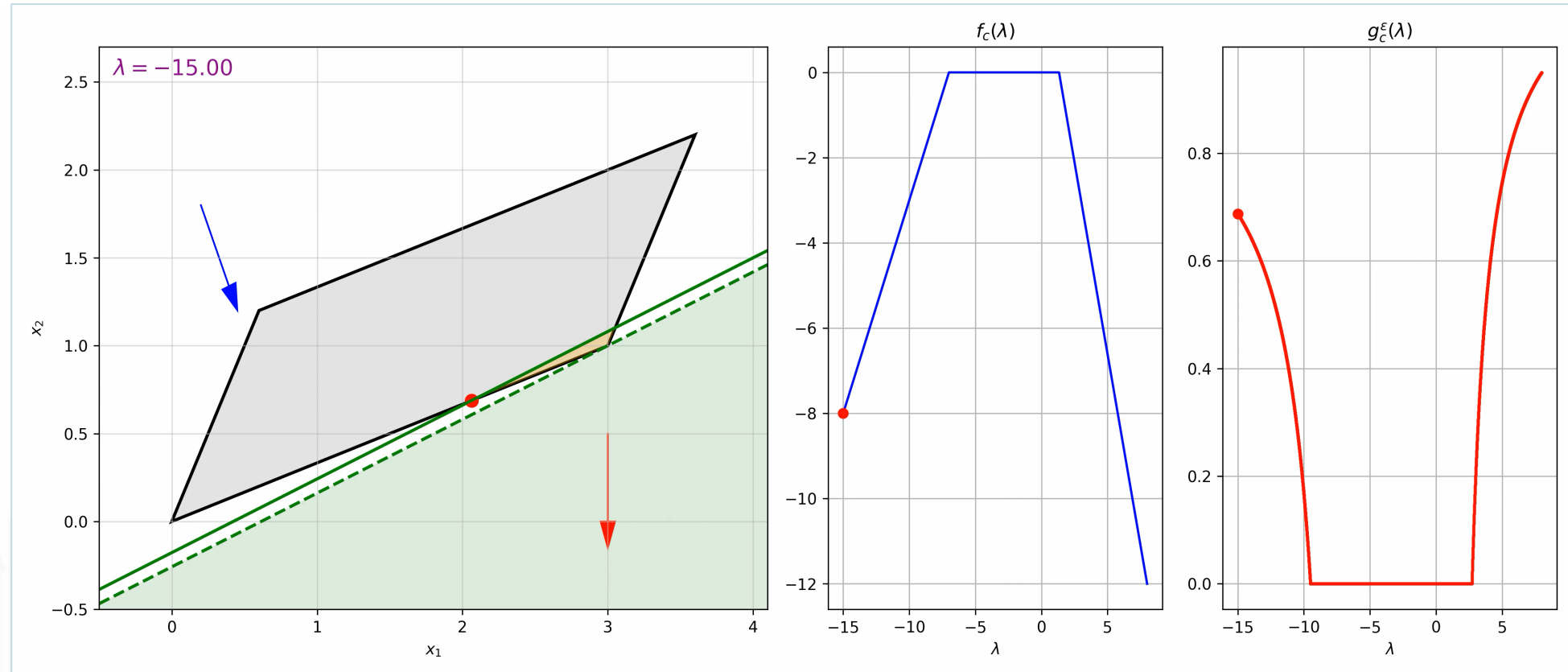
$$Ax \leq b$$

The example problem

$$g_c(\lambda) = \min x_2$$

$$\text{s.t. } (c + \lambda d)^T x \leq f_c(\lambda) + \varepsilon$$

$$Ax \leq b$$





Continuity and structure of $g_c^\varepsilon(\lambda)$

Theorem 1

If $\varepsilon > 0$, the value function $g_c^\varepsilon(\lambda)$ is continuous at every $\lambda \in \Lambda$.

Theorem 2

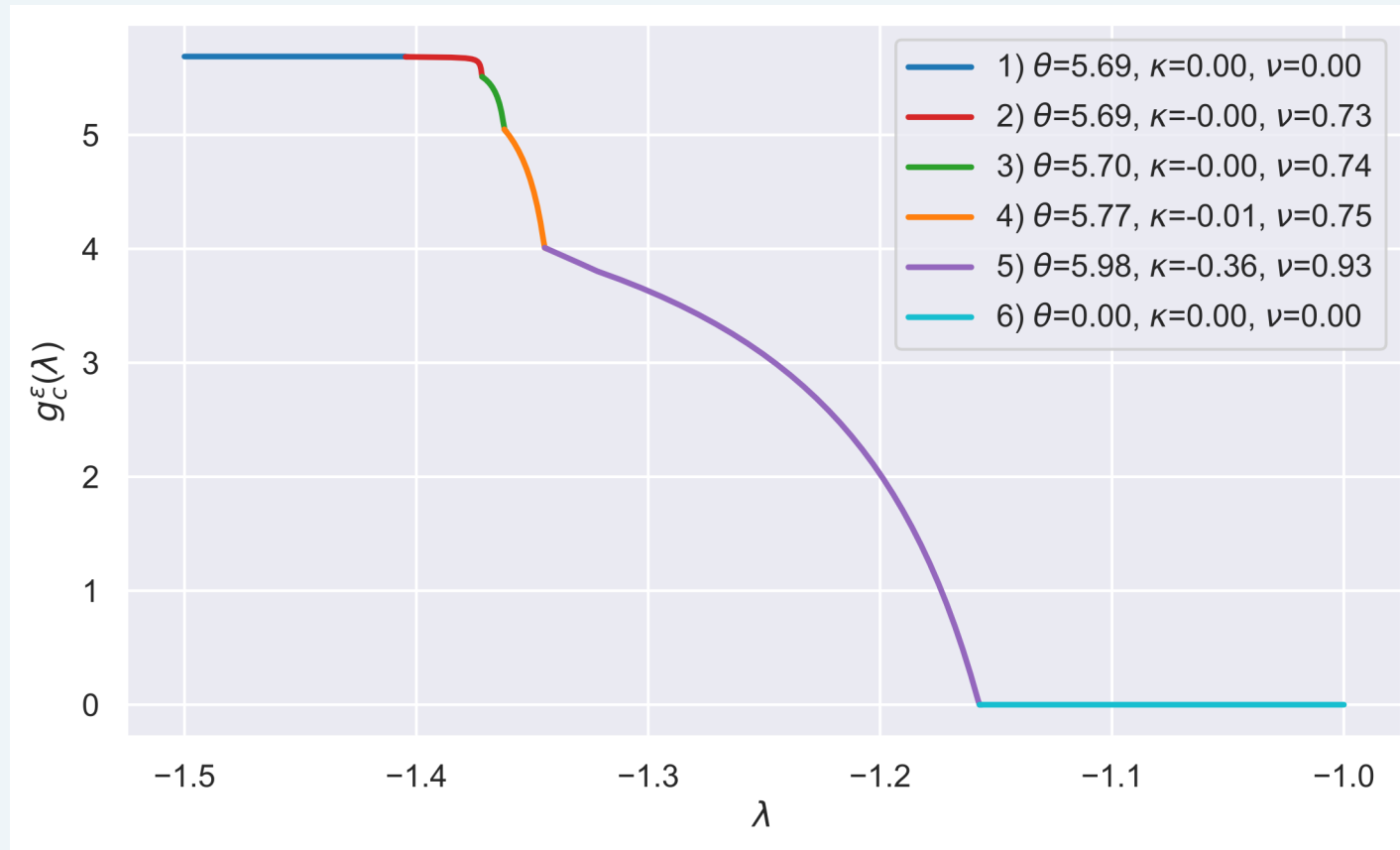
On each interval I_j , where the same basis B remains optimal, the function $g_c^{\varepsilon(j)}(\lambda)$ is homographic, described by only 3 scalars $(\theta_B, \kappa_B, \nu_B)$:

$$g_c^{\varepsilon(j)}(\lambda) = \theta_B + \frac{\lambda \kappa_B}{(1 + \lambda \nu_B)}$$

Proofs: Set-valued mappings & Berge's maximum theorem (Thm 1); Sherman–Morrison formula (Thm 2).



Structure of $g_c^\varepsilon(\lambda)$





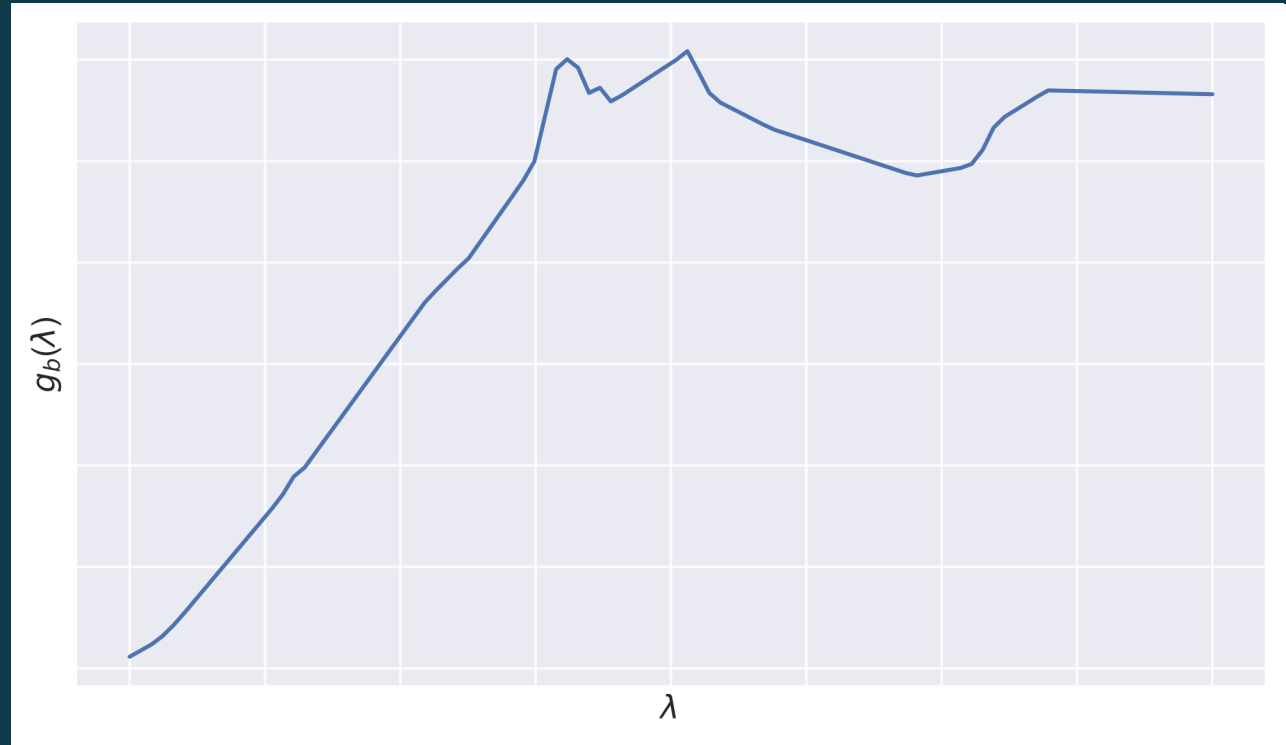
Perturbing the right-hand side b

The optimal value function

$$\begin{aligned} f_b(\lambda) = \min \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b + \lambda d \\ & x \geq 0 \end{aligned}$$

The problem in focus

$$\begin{aligned} g_b(\lambda) = \min \quad & \varphi^T x \\ \text{s.t.} \quad & c^T x \leq f_b(\lambda) \\ & Ax \leq b + \lambda d \\ & x \geq 0 \end{aligned}$$





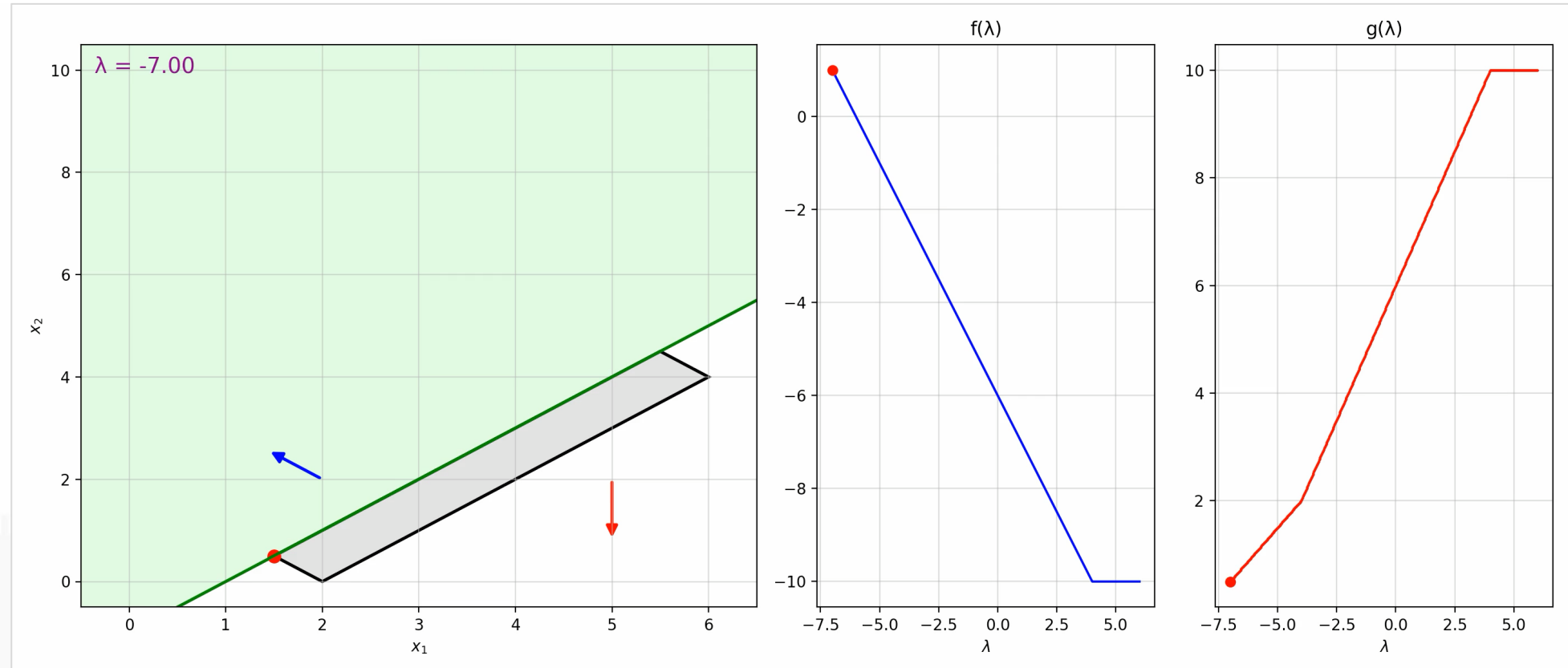
Perturbing b : a translation

The general problem

$$\begin{aligned} g_b(\lambda) = \min \varphi^T x \\ \text{s.t. } c^T x \leq f_c(\lambda) \\ Ax \leq b + \lambda d \end{aligned}$$

The example problem

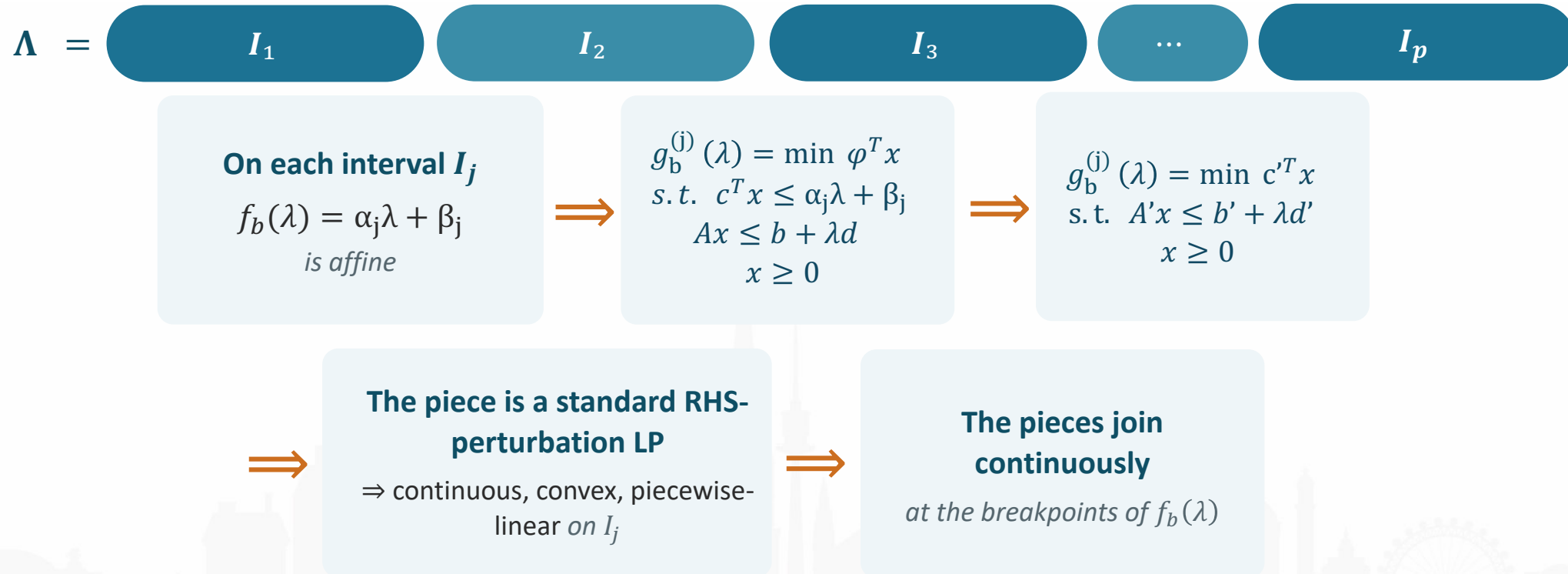
$$\begin{aligned} g_b(\lambda) = \min x_2 \\ \text{s.t. } c^T x \leq f_b(\lambda) \\ Ax \leq b + \lambda d \end{aligned}$$





Continuity of $g_b(\lambda)$

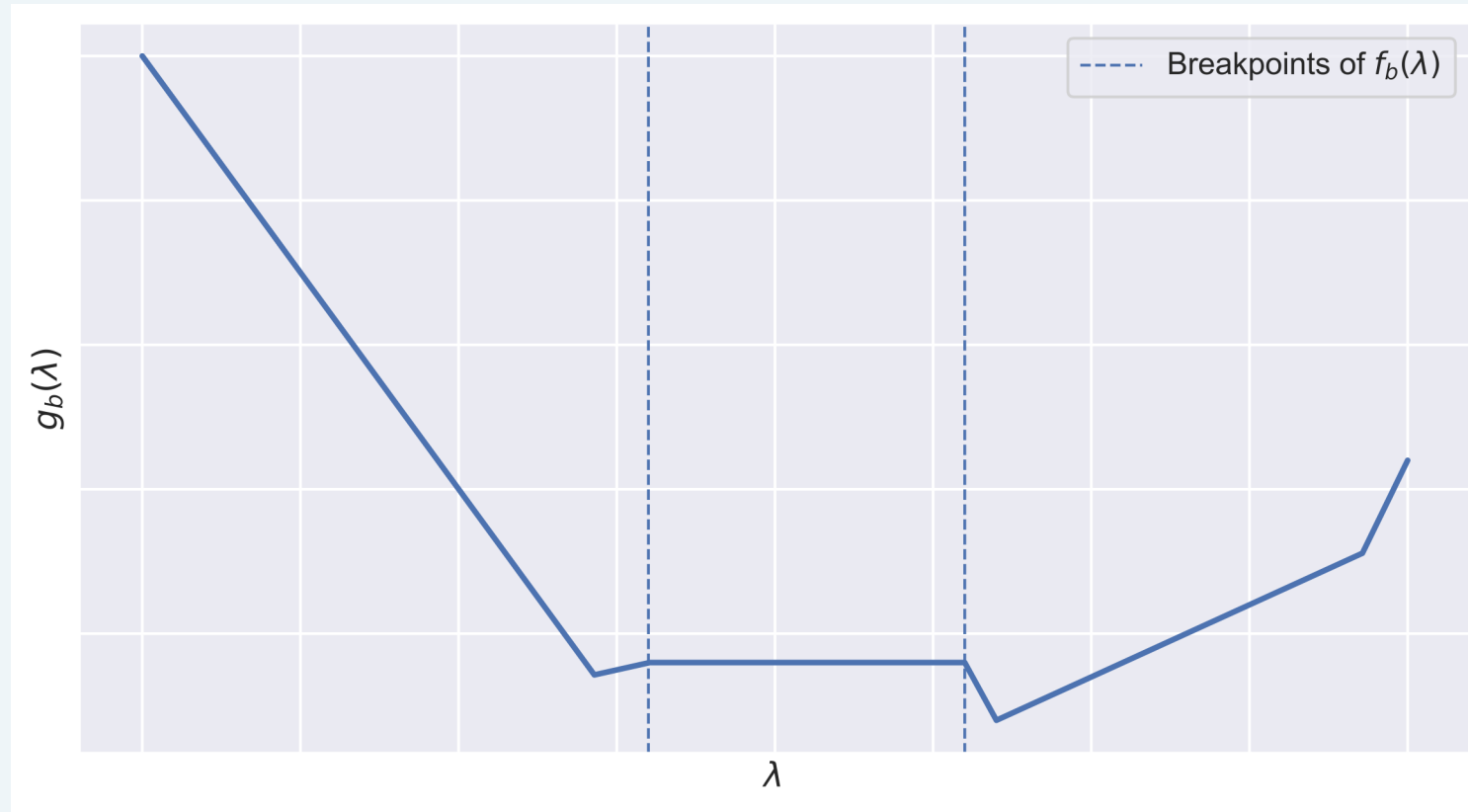
Globally, $g_b(\lambda)$ is built piece by piece over the intervals where $f_b(\lambda)$ is affine.



⇒ $g_b(\lambda)$ is continuous (even with $\varepsilon = 0$) and piecewise function composed of piecewise linear convex functions.



Structure of $g_b(\lambda)$





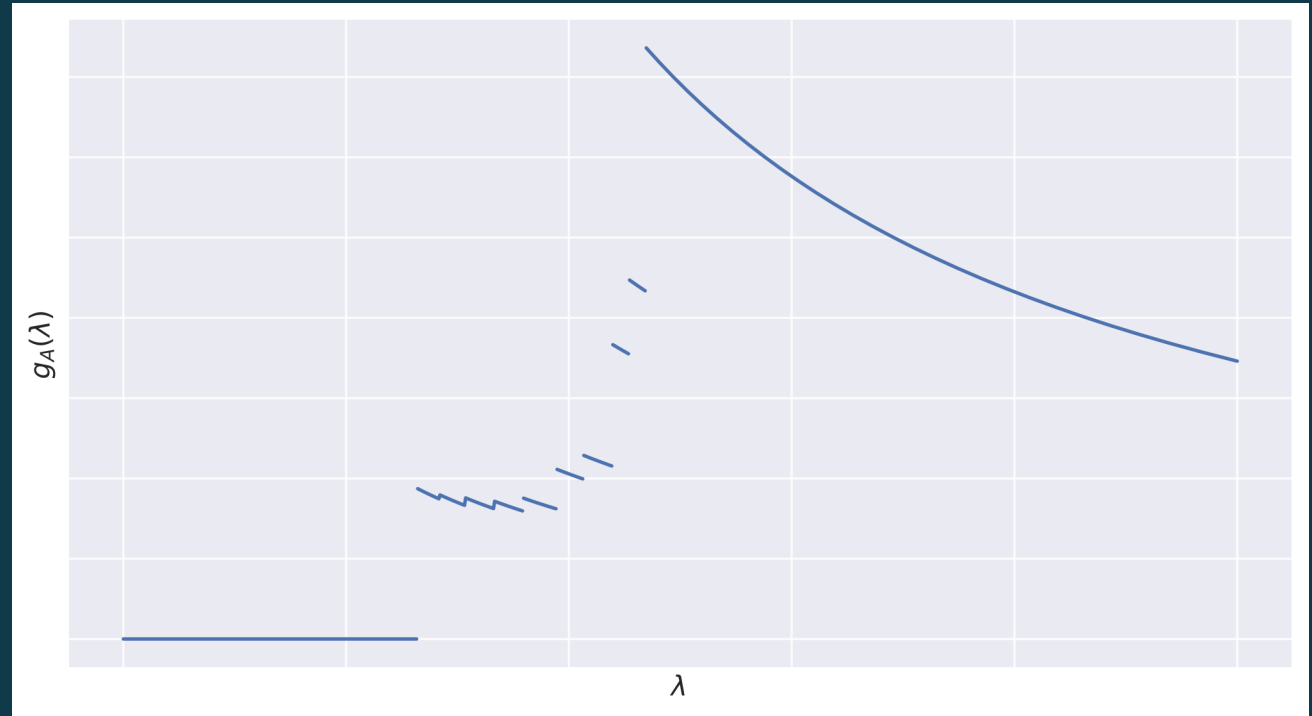
Perturbing the constraint matrix A

The optimal value function

$$\begin{aligned} f_A(\lambda) = \min \quad & c^T x \\ \text{s.t.} \quad & (A + \lambda D)x \leq b \\ & x \geq 0 \end{aligned}$$

The problem in focus

$$\begin{aligned} g_A(\lambda) = \min \quad & \varphi^T x \\ \text{s.t.} \quad & c^T x \leq f_A(\lambda) \\ & (A + \lambda D)x \leq b \\ & x \geq 0 \end{aligned}$$





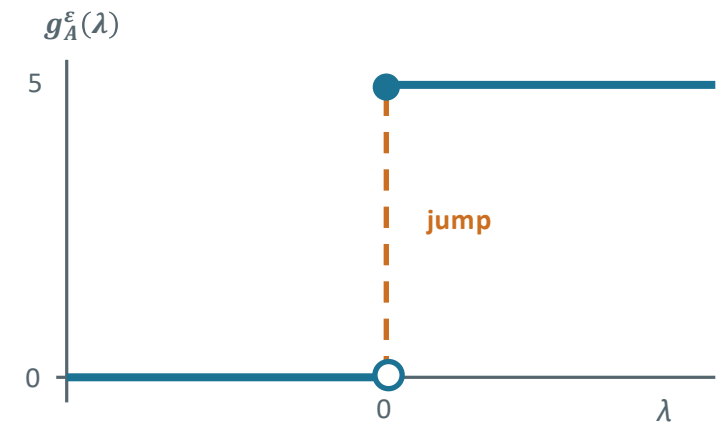
Perturbing A: ε does not restore continuity

Observation 1

Unlike $g_c^\varepsilon(\lambda)$ and $g_b^\varepsilon(\lambda)$, the function $g_A^\varepsilon(\lambda)$ can be discontinuous even with a tolerance $\varepsilon > 0$.

Using
 $\varphi = 1$

Example	$\lambda < 0$	$\lambda \geq 0$
$f_A(\lambda) = \max x$ $s.t. \ x \leq 5$ $\lambda x \geq 0$	$x \leq 5$ $x \leq 0$	$x \leq 5$ $x \geq 0$
	$f_A(\lambda) = 0$	$f_A(\lambda) = 5$
$g_A^\varepsilon(\lambda) = \max x$ $s.t. \ x \geq f_A(\lambda) - \varepsilon$ $x \leq 5$ $\lambda x \geq 0$	$x \geq 0 - \varepsilon$ $x \leq 5$ $x \leq 0$	$x \geq 5 - \varepsilon$ $x \leq 5$ $x \geq 0$
	$g_A^\varepsilon(\lambda) = 0$	$g_A^\varepsilon(\lambda) = 5$





The dual side: marginal prices

"If grid electricity prices rise 20%, how does the total cost change?"

How much PV and battery should we install?"

Investors

What's the shadow price of my grid-import limit ?

Marginal prices - the dual

- ⚡ Demand → **electricity price (€/kWh)**
- 🔌 Grid-import limit → **capacity value (€/kW)**
- 🔋 Battery capacity → **storage value (€/kWh)**

Where the ϵ -tolerance acts

- 🎯 **Primal** → relax the objective
- 🔑 **Dual** → relax the constraints



Primal-dual asymmetry + Conclusion

	Primal $g(\lambda)$	Dual $h(\lambda)$
Cost vector	Needs $\varepsilon > 0 \rightarrow$ continuous Piecewise homographic function	Continuous (even $\varepsilon = 0$) Piecewise function composed of piecewise linear convex functions
Right Hand Side	Continuous (even $\varepsilon = 0$) Piecewise function composed of piecewise linear convex functions	Needs $\varepsilon > 0 \rightarrow$ continuous Piecewise homographic function
Constraint Matrix	Discontinuous even with ε Erratic function	Discontinuous even with ε Erratic function



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Thank you !

Questions ?

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