

A shot in the dark: searching for dark substructures in the RX J0437+00 galaxy cluster

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Accepted 2026 June 10. Received 2026 June 6; in original form 2026 January 23

ABSTRACT

Obtaining a census of dark matter structures at low mass ($\leq 10^9 M_\odot$) can provide strong constraints on the nature of dark matter, though identifying such structures remains difficult. In this work, we study the galaxy cluster RX J0437.1+0043, taking advantage of its powerful ‘exotic’ Hyperbolic-Umbilic (HU) lensing configuration to search for substructure candidates. Using a combination of high-resolution imaging, integral field unit spectroscopy, and gravitational lensing modelling, we report on a tentative detection of a dark matter subhalo ($m_{\text{halo}} = 2.27 \pm 0.83 \times 10^9 M_\odot$) near the vicinity of one of the largest HU images. We stress that this result is still preliminary and that deeper data and more advanced modelling techniques are needed to ultimately confirm this detection. Nevertheless, this work outlines the first steps towards understanding subhalo properties in dense cluster environments, developing HU cluster lenses as a potential new tool for investigating dark matter.

Key words: gravitational lensing: strong – techniques: imaging spectroscopy – galaxies: clusters: individual: RX J0437.1+0043 – dark matter.

1 INTRODUCTION

Dark matter (DM) has been a major topic of modern astronomical research for several decades. Although it accounts for nearly 90 per cent of the Universe’s mass budget, little is known about its physical properties; this is largely due to the fact that, to date, direct detection of a DM particle has never been achieved (e.g. L. E. Strigari 2013; M. Misiasek & N. Rossi 2024). Thus, we currently lack observational evidence of (proposed) DM characteristics such as its particle mass, its interaction cross-section, and its decay rate. Since insights into the exact nature of DM have important implications in both astrophysics and cosmology, it is imperative that we continue to investigate this elusive substance.

By interacting through gravity, DM is known to cluster and grow over cosmological time. As a result, we can indirectly detect evidence of its presence through other ways, particularly by measuring the *total* mass of collapsed (or collapsing) structures. Several methods exist for ‘weighing’ a DM halo, including measuring galaxy rotation curves in H I gas (e.g. A. Banerjee & C. J. Jog 2011; E. M. Di Teodoro et al. 2023), probing Doppler shifts in Cosmic

Microwave Background photons via the Sunyaev-Zel’dovich (SZ) Effect (e.g. D. P. Marrone et al. 2012; Planck Collaboration et al. 2013; Í. Zubeldia & A. Challinor 2019; A. Huchet & J.-B. Melin 2024), or tracing the temperature profile of X-ray gas in galaxy clusters (e.g. M. Sereno et al. 2017; C. Tchernin et al. 2018; J. F. V. Allingham et al. 2024; B. Beauchesne et al. 2025). However, of all the possible techniques, one method stands out as an especially efficient mass probe: gravitational lensing. Gravitational lensing directly provides a total (dark+baryon) mass estimate for an object, without needing to invoke simplifying assumptions about the object’s morphology, kinematic properties, or dynamical state. Therefore, when used in conjunction with other observables that intrinsically trace baryons (such as galaxy brightness or emission-line strength), a lensing-mass estimate can cleanly disentangle the mass contributions of luminous matter from DM (e.g. M. Limousin et al. 2008; J. Richard et al. 2010; M. Jauzac et al. 2012; C. Grillo et al. 2015; D. J. Lagattuta et al. 2017; C. Cerny et al. 2018; G. Mahler et al. 2018; J. M. Diego et al. 2020; J. F. V. Allingham et al. 2023; A. Niemiec et al. 2023; D. Perera et al. 2024; G. Rihtaršič et al. 2025).

At the same time, lensing is equally suited to measuring mass at both large and small scales. This allows for a simultaneous estimate of the larger-scale ‘smooth’ mass component of the un-

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derlying halo, along with the distribution of smaller ‘clumpy’ subhaloes that exist within the primary mass. The subhalo mass function is a fundamental parameter of the cosmological model and, as such, can provide additional clues to the nature of DM. According to cosmological simulations, the standard concordance Lambda cold dark matter (Λ -CDM) model predicts that of thousands of low-mass ($< 10^9 M_\odot$) DM haloes should survive to the present day within the gravitationally bound confines of a galaxy cluster, as a consequence of the hierarchical build-up of the Universe’s large-scale structure (e.g. S. D. M. White & C. S. Frenk 1991). Conversely, alternative models such as a relativistic ‘warm’ DM particle (WDM) or a nominally higher DM cross-section (self-interacting DM; SIDM) can predict the existence of far fewer subhaloes (e.g. M. R. Lovell et al. 2012; J. Zavala et al. 2019; E. O. Nadler et al. 2025); though see e.g. A. Ragagnin et al. (2024) for conditions where SIDM and Λ -CDM subhalo populations become less divergent. With an accurate census of low-mass substructures then, we can begin to discriminate between competing cosmological models. As many of these objects are expected to be extremely faint or even entirely dark (A. Benitez-Llambay & C. Frenk 2020), lensing once again serves as a powerful tool for their detection.

Lensing-based efforts to detect DM subhaloes and estimate their mass function have become more common in recent years, both observationally (e.g. S. Vegetti et al. 2012; Y. D. Hezaveh et al. 2016; J. W. Nightingale et al. 2024) and computationally (e.g. G. Despali & S. Vegetti 2017; R. Li et al. 2017; N. C. Amorisco et al. 2022). In all cases, the technique is largely the same: (i) identify the multiple images of a lensed background galaxy; (ii) look for distortions in either the position or brightness of individual images relative to its counterparts (as ‘copies’ of the same source, multiple images should inherently appear identical, after accounting for lensing magnification); and finally, (iii) attempt to correct these deformations by adding a perturbing mass (i.e. a subhalo) to the lens model, in the vicinity of the distortion. However, at present, nearly all efforts in this domain have focused on galaxy-scale mass distributions, i.e. galaxy–galaxy lenses (GGLs).

In this work, we focus on gravitational lensing in galaxy clusters, as clusters have two advantages over their galaxy-scale counterparts. First, they are physically larger and more massive than individual galaxies, which (again invoking the hierarchical nature of the Universe) implies that they contain more subhaloes than GGLs (J. E. Doppel et al. 2026) and also span a wider mass range. This suggests that there is a higher probability of detecting subhaloes in clusters, and that the statistics of these detections will be more diverse. Second, background objects being lensed by clusters lie, on average, further away from bright galaxies compared to GGLs. This means that they are less contaminated by cluster galaxy light, making it easier to identify the subtle deformations, possibly highlighting the presence of nearby small-scale subhaloes.

Objects lensed by clusters can appear in a variety of configurations, ranging from common ‘stable’ orientations (such as double images, quadruples, and fully connected Einstein-rings) to more ‘exotic’ forms, such as swallowtail and umbilic shapes (e.g. A. O. Petters, H. Levine & J. Wambsganss 2001; G. Orban de Xivry & P. Marshall 2009; A. K. Meena & J. S. Bagla 2020; see Fig. 1). Previous work has shown that these exotic systems are more sensitive to the presence of subhaloes compared to stable patterns (A. K. Meena & J. S. Bagla 2023), with one specific exotic configuration, the Hyperbolic-Umbilic (HU), providing the most optimal ‘net’ for catching sub-

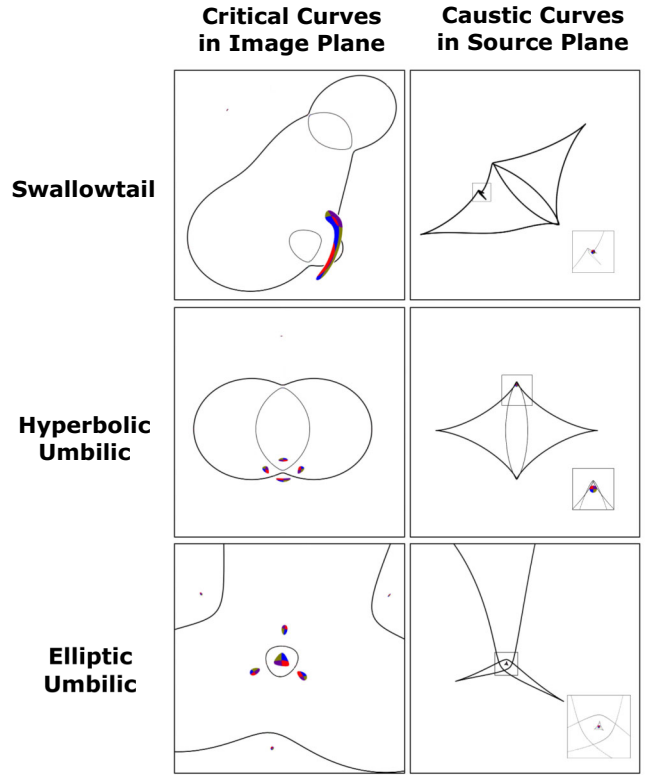


Figure 1. Examples of rare, ‘exotic’ lens configurations. The left column displays the critical curves in the (observed) image plane, with the characteristic image formations and image parity shown by the multicoloured regions. The right column shows the corresponding caustics in the (demagnified, intrinsic) source plane. [Adapted from figs 1–3 in A. K. Meena & J. S. Bagla (2020).] Lenses in exotic configurations are more sensitive to the effects of nearby substructures than more common ‘stable’ lensed images, making them ideal probes to look for low-mass subhaloes. The HU lensing configuration seen in the middle panels is the subject of this work.

structures, in terms of its physical size and number of lensed images.

Therefore, we here investigate the possible presence of substructures in the galaxy cluster RX J0437.1+0043 (RX J0437; $z = 0.285$). First studied in D. J. Lagattuta et al. (2023), the cluster has a largely elliptical shape with 12 spectroscopically confirmed multiple-image systems. Remarkably, despite being historically rare, *three* of the RX J0437 systems appear in HU arrangements (System 1, $z_s = 1.97$; System 2, $z_s = 2.97$; and System 10, $z_s = 6.02$), each lying within the cluster core ($R < 100$ kpc from the centre; see Fig. 2). High-resolution imaging from the *Hubble Space Telescope* (HST) shows that Systems 1 and 2 are both clumpy, highly structured galaxies, while integral-field spectroscopy shows that all three systems have bright, extended emission-line features, making them ideal test cases to search for low-mass perturbers.

This paper is organized as follows. We describe the data used in this analysis in Section 2, then lay out the techniques used to classify features in lensed galaxies and investigate differences between their multiple images in Section 3. Based on the results of this census, we create a series of strong-lensing mass models that we present in Section 4, allowing us to detect perturbations in the vicinity of lensed galaxies. We discuss modelling results in Section 5, presenting evidence for a possible low-mass DM

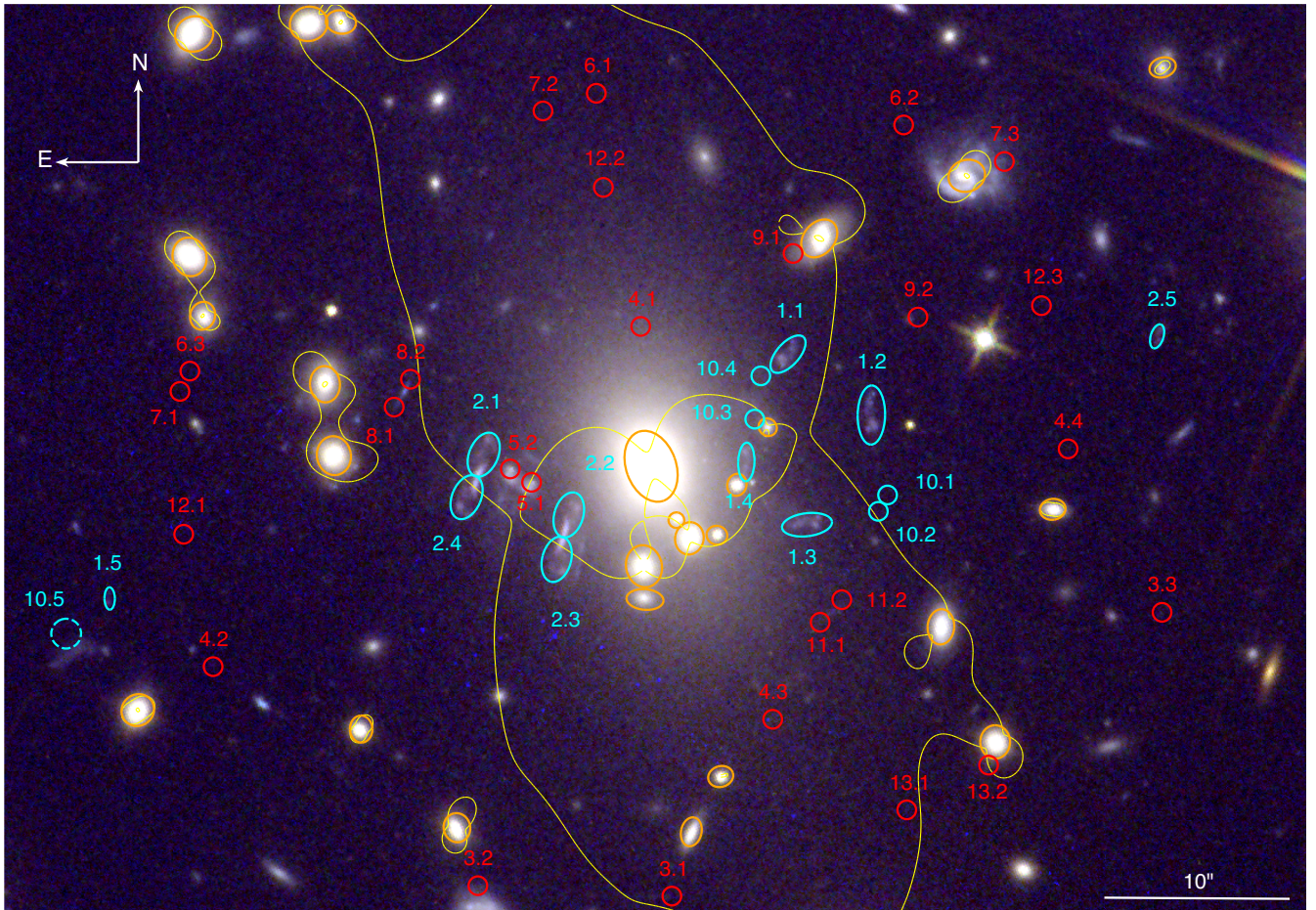


Figure 2. Spectroscopically confirmed multiply-imaged sources in the RXJ0437 field, overlaid on multiband *HST* imaging (*F140W*/*F110W*/*F814W*). Sources identified as HU systems appear in cyan, while all other systems are shown in red. The model-predicted image 10.5 (not seen in the MUSE data) appears as a dashed circle. The numbering convention of all systems follows that presented in D. J. Lagattuta et al. (2023). For reference, we also include a model critical curve (yellow line) at $z = 2.5$, a redshift roughly halfway between the two major HU systems (1 and 2) used in this work. Individual cluster members used to construct the lens model (including the BCG) are highlighted as orange ellipses.

substructure close to System 1. Finally, we conclude and discuss future work in Section 6.

Throughout this work, we assume a flat cosmological model with parameters set to $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Under these conditions, $1''$ covers 4.297 kpc at the cluster redshift ($z=0.285$). Finally, unless otherwise specified, all magnitudes are presented in the AB system (J. B. Oke 1974).

2 OBSERVATIONS

Our analysis is based on high-resolution space-based imaging with the *HST* and ground-based spectroscopy with the MUSE spectrograph at the Very Large Telescope (VLT). The reduced data used in this work are nearly identical to those presented in D. J. Lagattuta et al. (2023), as are the techniques we employ to reduce them. Full details of the data and specific reduction methods can be found in Section 2 of D. J. Lagattuta et al. (2023), but we provide a summary of the information here and highlight the minor differences in the final reduction to aid the reader.

2.1 Hubble Space Telescope imaging

HST imaging of RXJ0437 exists in three broad-band filters: *F814W*, *F110W*, and *F140W*. The *F814W* data were acquired on

21 November 2021 using the *Advanced Camera for Surveys* (ACS), while the *F110W* and *F140W* data were acquired on 25 July 2022 with the *Wide-Field Camera 3* (WFC3). All data were taken as part of a SNAPshot imaging campaign (GO-16670; PI: H. Ebeling) targeting massive galaxy clusters. A summary of the relevant observational parameters is shown in Table 1.

For this analysis, we use high-level data products in each band from the Mikulski Archive for Space Telescopes (MAST¹), which provides reduced and stacked images for multiband analysis. Reduction was performed using the publicly available DRIZZLEPAC software², which aligned individual exposures to a common World Coordinate System, eliminated systematic artefacts such as cosmic-ray strikes and hot pixels, corrected for the effects of geometric distortion, and (in the case of the ACS image) removed photometric smearing caused by charge-transfer inefficiency tails. Unlike in D. J. Lagattuta et al. (2023), however, we applied one final correction to the stacked images: we resampled each image from its native (non-rotated) pixel scale to a common registered pixel grid ($0.05 \text{ arcsec pixel}^{-1}$, oriented

¹<https://mast.stsci.edu/search/ui/>

²<https://www.stsci.edu/scientific-community/software/drizzlepac>

Table 1. Summary of the *HST* observations for RXJ0437. Column 1 gives the instrument and the band. Columns 2 and 3 list the ID of the proposal and respective PI. Columns 4 and 5 give the total exposure time in seconds and the observation date.

Band	PID	P.I.	Exposure time (s)	Observation Date
ACS/F814W	16 670	H. Ebeling	1200	2021-11-21
WFC3/F110W	“	“	706	2022-07-25
WFC3/F140W	“	“	1059	2022-07-25

North up, East left) using SWarp³ (E. Bertin 2010). This provided a more uniform pixel coverage between bands, making it easier to compare photometric estimates from band to band (Section 3.2).

2.2 VLT/MUSE spectroscopy

The VLT/MUSE data used in this work is gathered from two observing programs targeting RXJ0437. The first one is a single 1 h exposure taken on 15 February 2020 as part of the KALEIDOSCOPE Cluster survey (PID 0104.A-0801; PI A. Edge). These data were obtained in WFM-NOAO-N mode, centred on the brightest cluster galaxy (BCG) located at ($\alpha = 69.289677$, $\delta = 0.73114470$). We combine this frame with a larger $1.5 \text{ arcmin} \times 1.5 \text{ arcmin}$ mosaic of eight stacked MUSE pointings observed between 15 January and 13 February 2021 (PID 106.21AD; PI D. Lagattuta). The mosaic is again centred on the BCG, but the data are instead acquired using Adaptive Optics (AO) corrections in WFM-AO-N mode. The AO system significantly improves the resolution of the MUSE Data, with AO-corrected images achieving an effective point spread function (PSF) of 0.4 arcsec (near the Nyquist limit), compared to the average seeing-limited 0.85 arcsec PSF.

All nine data frames are reduced using the method outlined in Section 2.2 of D. J. Lagattuta et al. (2022). This procedure largely follows the standard MUSE data reduction pipeline (P. M. Weilbacher et al. 2020), but includes an additional autocalibration step to reduce flux variation between integral field unit (IFU) image slices, and applies the Zurich Astrophysical Purge (ZAP) software (K. T. Soto et al. 2016) to better remove sky line residuals. The combined data set spans a wavelength range from 4750 to 9350 Å, though the mosaic frames have a coverage gap between 5804 and 5967 Å due to the AO laser-blocking notch filter. The mean spectral resolution of the final combined data cube is $R = 3000$, and the exposure time ranges from 20352 s in the cluster centre to 2544 s in the outskirts. A summary of the different MUSE observation parameters is given in Table 2. Crucially, the MUSE data provide robust redshift information for all confirmed multiple-image systems in RXJ0437; a full discussion on the spectral features of the HU systems can be found in Section 7.1 and fig. 9 of D. J. Lagattuta et al. (2023). However, to summarize here, we identify ~ 10 distinct line features in the spectrum of System 1, ~ 5 in the spectrum of System 2, and 1 (Ly α) in the spectrum of System 10.

3 SUBSTRUCTURE ANALYSIS

The DM substructures we are interested in are considerably lower-mass and more compact compared to the larger, cluster-scale haloes in which they are embedded. As a result, any vis-

ible deflections they cause will be small, extending only to ~ 1 arcsec from their position (e.g. Q. He et al. 2022). In practice, this means that while these substructures will not significantly alter the global lens model, their presence can create localized perturbations, subtly altering the appearance of nearby objects in the (observed) lens plane. Given a suitable benchmark, these perturbations can be detected, particularly when looking at the (ostensibly identical) multiple images of a lensed background galaxy. Although any multiply-imaged galaxy in a cluster can, in principle, be used for such a comparison, the unique nature of HU systems makes them ideally suited for this task.

Specifically, HU images experience a high level of magnification (up to $\sim 100\times$) that is nearly isotropic, providing a larger, more uniform view of the source galaxy compared to, e.g. lensed giant arcs, which only appear strongly stretched in one direction. This is because the HU images lie close to both radial and tangential critical curves (lines of formally infinite magnification for point sources, see Fig. 1). As a result, we are better able to identify unique and distinct features in the galaxy’s morphology, making it easier to look for unexpected discrepancies between images.

In this section, we describe the techniques used to study the properties of the HU images in RXJ0437 and identify any perceptible differences between them. Before discussing the efforts in detail however, we first highlight the main discovery. Specifically, we identify a small, but significant, flux-ratio anomaly in the northernmost region of HU Image 1.2 (labelled clump 7 in Fig. 3). Interestingly, this is very close to the location where D. J. Lagattuta et al. (2023) simulated the effects of DM substructures on HU images (see fig. 13 in that work), demonstrating that moderate mass substructures could create noticeable, localized discrepancies when comparing fluxes between images. We stress that the results of that work are entirely different from what we present here, as this is an empirical detection rather than a simulation. That said, however, it is in some ways not surprising that we would find an anomaly in Image 1.2, since it is both the largest counterimage of System 1 and the furthest from the BCG, making it the most sensitive to subtle flux variations.

While we do identify other potential anomalies, the image 1.2 anomaly is the most significant, and ultimately, reveals the presence of a candidate DM substructure mass (Section 4). Nevertheless, in the following sections, we still describe the investigations of all HU image components, in order to provide context about the overall search procedure.

3.1 HU galaxy images subcomponents identification

We begin by visually inspecting each image in a given lens system, looking for distinct morphological features that can be compared across all images. We note here that, while each HU lens has five unique multiple images, our analysis focuses only on the four central images of a given system (see Fig. 2),

³<https://www.astromatic.net/software/swarp/>

Table 2. Summary of VLT/MUSE observations for RX J0437. Columns 1 and 2 indicate the ID of the ESO programme and the respective PI. For each pointing, we give the observation date in Column 3, the instrument set-up in Column 4, the total exposure time in Column 5, the right ascension (R. A.) and declination (Decl.) of the centre of the field of view in Columns 6 and 7, and the mean FWHM of the seeing during the observations in Column 7.

ESO programme	P.I.	Observation date	Instrument set-up	Exposure time (s)	RA (J2000)	Decl. (J2000)	Seeing (arcsec)
0104.A-0801	A. Edge	2020-02-15	WFM-NOAO-N	2910	69.289583	0.731167	0.49
106.21AD	D. Lagattuta	2021-01-15	WFM-AO-N	2544	69.293333	0.735667	1.05
“	“	“	“	“	69.286958	0.735611	1.49
“	“	2021-01-17	“	“	69.286917	0.735528	0.41
“	“	2021-01-19	“	“	69.293208	0.726389	0.70
“	“	“	“	“	69.286792	0.726472	0.79
“	“	“	“	“	69.293250	0.735750	0.70
“	“	2021-02-13	“	“	69.286917	0.735694	0.65
“	“	2021-02-17	“	“	69.293292	0.726306	0.83

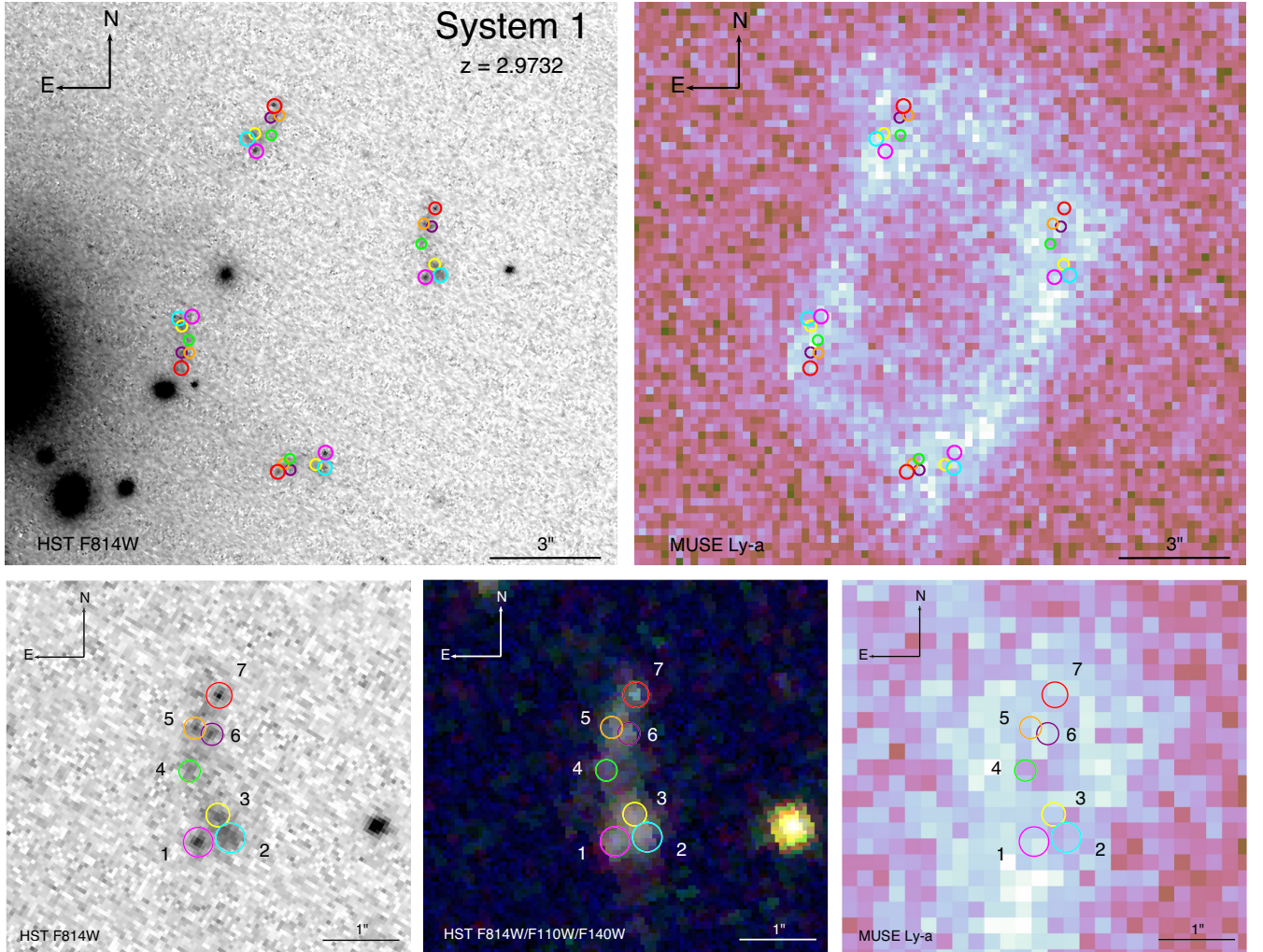


Figure 3. Multiple images of HU System 1 ($z = 2.97$), seen in several data sets. In the top left panel, we see a greyscale image of the complete system, observed in *HST*/F814W imaging, the highest resolution image available. A zoom-in of one image (Image 1.2) is seen in the bottom left cutout. From this data, we have identified seven distinct morphological features (‘clumps’, labelled 1–7), which we identify with coloured apertures. A three-band colour image of the same system is shown in the bottom middle panel. This adds colour information, but individual clumps appear less distinct, due to the lower resolution of the *HST*/F110W and F140W imaging. The top right panel shows how the system appears in VLT/MUSE imaging, in this case, a complete ring structure composed of $\text{Ly}\alpha$ emission. A zoom in (of the same individual image) is shown in the bottom right panel. Contrasting with the *HST* imaging, the MUSE view of the system shows a broad, smooth feature, rather than distinct clumps. While this is partially due to the inherent structure of the $\text{Ly}\alpha$ gas, atmospheric smearing also plays a role, with typical natural seeing values ranging between 0.5 and 1.0 arcsec during the observations (see Table 2).

as these are the most highly-magnified, allowing us to identify unique structural features. To make full use of all available information, we measure the object's appearance using both *HST* (broad-band imaging) and MUSE (narrow-band imaging, centred around known emission-line features) data. This is because each of the RXJ0437HU galaxies is a young, highly star-forming system, still actively assembling. As a result, the stellar and gas disks of the galaxy are not necessarily aligned, so the two components can provide independent sight-lines through the foreground cluster environment. Of the three *HST* bands at our disposal (ACS/F814W, WFC3/F110W and WFC3/F140W), the F814W imaging has the highest resolution (with an average PSF full width at half-maximum (FWHM) of 0.07 arcsec, compared to the 0.14 and 0.16 arcsec values obtained, respectively, in F110W and F140W), providing the clearest picture for detecting distinct, clumpy subcomponents. We therefore use the F814W data specifically for our initial feature detection, but supplement it with colour information from the other two bands.

We identify seven distinct features in the *HST* imaging of System 1, which appear to be individual pockets of active star formation. These knots are also seen in the MUSE data – specifically in the wavelength range covering C III] emission and UV stellar continuum – but they appear to fuse into two broader clumps (Fig. 3) due to the lower resolution of MUSE. Similarly, we detect three features in System 2, including a bright central component representing the main galaxy core, along with two additional stellar knots in the outskirts of the system. The MUSE view is again at lower resolution and, in this case, we can only confidently identify the bright central core, though the separation between individual multiply-imaged components is still distinct (Fig. 4). Finally, while we attempt to identify features of System 10, we see no evidence of galaxy continuum in any *HST* bands, though given the shallowness of the exposures (and a redshift $z \sim 6$) this is perhaps unsurprising. Instead, System 10 appears only as a single, nearly point-like feature in the MUSE data, with the two Northern-most images appearing as distinct objects, and the two Southern components appearing as a merging pair. Unlike the other HU galaxies, the images in this system appear only as a Ly α (Ly α) emission line, with no discernible continuum emission elsewhere in the data cube (Fig. 5). Given the lower resolution of the MUSE-only detection, coupled with the lack of colour information and difficulty in identifying unique features in each component – the smoothness of the Ly α flux in the merging pair prevents a distinct separation of images – we decided to focus solely on Systems 1 and 2 in the remainder of this analysis. However, we note that it would be interesting to revisit System 10 in the future using deeper *HST* or *JWST* observations.

3.2 HU galaxy images subcomponents properties

After identifying individual clumps, we characterize some of their key physical properties, making separate measurements in each HU image, to look for cross-image discrepancies. We specifically focus on two features for each clump: its centroid position and total (multiband) flux, as differences between images in these quantities (known as astrometric and flux-ratio anomalies, respectively) can indicate the presence of a nearby low-mass perturber.

We first attempt to make these measurements using SOURCEEXTRACTOR (E. Bertin & S. Arnouts 1996), in order to automatically detect each clump's centroid and standardize the aperture

used to measure its flux. To maintain consistency between measurements in different bands, we run SOURCEEXTRACTOR in dual image mode, using the F814W band as the detection image, and a second filter (including F814W itself) as the photometric image. However, after several attempts (including modifications to our choice of SOURCEEXTRACTOR detection/deblending parameters), the software was unable to detect and isolate all identified clumps, leaving some gaps and blending some crowded objects into a single detection. Therefore, we manually select apertures on the basis of the visual inspection of the previous step. We centre each aperture at the brightest pixel of a given clump, using a radius that matches its observed (magnified) size in the F814W band. An example of our selection can be seen in Fig. 3.

To search for astrometric anomalies, we compare our identified centroid positions with the D. J. Lagattuta et al. (2023) lens model predicted positions. In doing so, we find that no clump is more than 0.45 arcsec from its expected location, which falls within the global model uncertainty provided in D. J. Lagattuta et al. (2023), namely 0.50 arcsec. This is to be expected, as many of these knots are used as constraints in the model (particularly System 1) leading to minimized predictions during model optimization. Nevertheless, we note that the largest model residuals are located near Images 1.2, 1.3, and 2.2, providing a starting point to look for perturbers.

In addition to the astrometric search, we also compare the conjugate fluxes of each clump. Unlike astrometric positions, the clump magnitude is not used as a constraint in the lens model, thus serving as a more independent probe of anomalies. To measure the flux of each clump, we create a self-consistent estimator by summing a scaled version of the flux contained within each aperture. Specifically, we scale the empirically-measured flux in each pixel by its model-defined magnification value. This provides a measurement of intrinsic (i.e. non-magnified) flux, which should remain constant from image to image in the absence of low-mass perturbers.

To improve the contrast between clump images, and to minimize any contamination due to cluster light, we first subtract the BCG flux from each filter before measuring aperture fluxes. We model the BCG light using MGEFIT, a publicly available PYTHON package based on the multi-Gaussian expansion (MGE) galaxy fitting technique presented in M. Cappellari (2002). A list of our fitting parameters can be seen in Table 3.

As previously mentioned, we calculate uncorrected fluxes within the manually constructed clump identification apertures. While this is entirely sensible for the flux measurement, we note that there is some discrepancy in the magnification value, since the model-predicted final position of each clump can be slightly offset from its observed position due to the uncertainty inherent in the D. J. Lagattuta et al. (2023) lens model. Therefore, to understand how these offsets might affect our magnification-corrected fluxes, we compare the difference between the magnification at the observed (μ_{observe}) and model-predicted (μ_{model}). In most cases, this difference is small (≤ 0.5), though there are a few exceptions.

Most prominently, Clump 1 of System 2 shows a high variation in magnification, both between multiple-image instances and when moving around its local neighbourhood. This is because (as a merging image) the clump straddles the cluster critical curve at its redshift, leading to larger-than-normal variations in magnification in this region. (We particularly note that there is a large magnification difference in Image 2.2 relative to the other

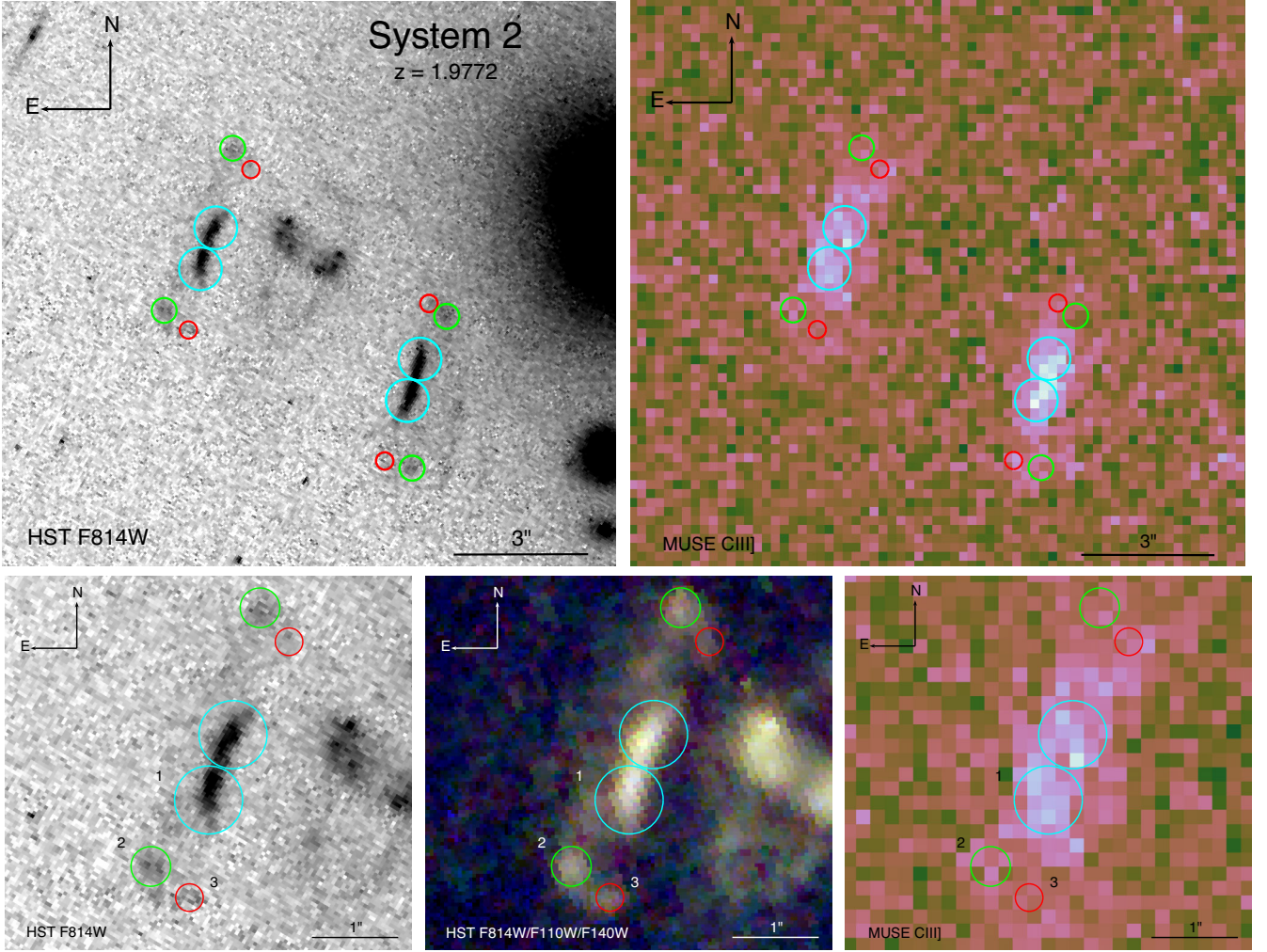


Figure 4. Multiple images of HU System 2 ($z = 1.98$), seen in several data sets. The layout and composition of these images follow that of Fig. 3, though the zoomed-in image this time is the combined merging images of Images 2.1 and 2.4. In System 2, we identify three distinct clumps in the high-resolution imaging, which, due to their larger sizes, are still distinct in the colour panel. The bright central component is readily identified in MUSE data (here CIII emission), while the two edge components are less bright.

counterimages in that family, which is likely enhancing the observed flux ratio anomaly here.) To incorporate these variations in our flux correction, we calculate the standard deviation of magnifications in a new set of apertures, centred at the observed location of each clump, but with an expanded radius reaching the model-predicted location (as highlighted in Fig. 6.) In this way, we conservatively account for the full range of positional variation in the base lens model. We take these new values as the model magnification error, which we incorporate into the uncertainty of the final flux correction.

We inspect clump fluxes in all three *HST* bands, as well as the MUSE flux of the brightest emission line for each system (in this case, Ly α for System 1 and CIII] for System 2). However, we only report here fluxes measured in F814W and F110W. This is because the flux uncertainties in the other data sets are much higher, due to heavy component blending caused by a larger PSFs and, in the case of MUSE, a considerably broader pixel scale. As a result, values measured for a given clump in these filters are always formally consistent with one another, removing any discriminatory power. The results of our flux correction procedure are shown in Fig. 7, and numerical values are provided in

Table A1.

Again, the largest discrepancies are found in image 1.2 (specifically clump 7), with slightly smaller discrepancies seen in Images 1.3 and 2.2. This is similar to the results of the astrometric discrepancy search. For context, the ‘typical’ (i.e. non-anomalous) variation seen in the other HU clumps is entirely consistent with variations in the other multiply-imaged systems that appear in the *HST* data (Systems 5, 8, and 11). While these systems do not have significant morphological features within counterimages, they nonetheless set a baseline for expected flux variation within an HU image. We therefore consider the identified anomalous clumps as our best candidates for searching for possible dark substructures.

4 GRAVITATIONAL LENSING MASS MODELLING

To investigate the possible presence of dark substructures in the RXJ0437 field, we modify the parameters of the strong lensing mass model presented in D. J. Lagattuta et al. (2023). Specifically, we add small perturbing mass components to the original D.

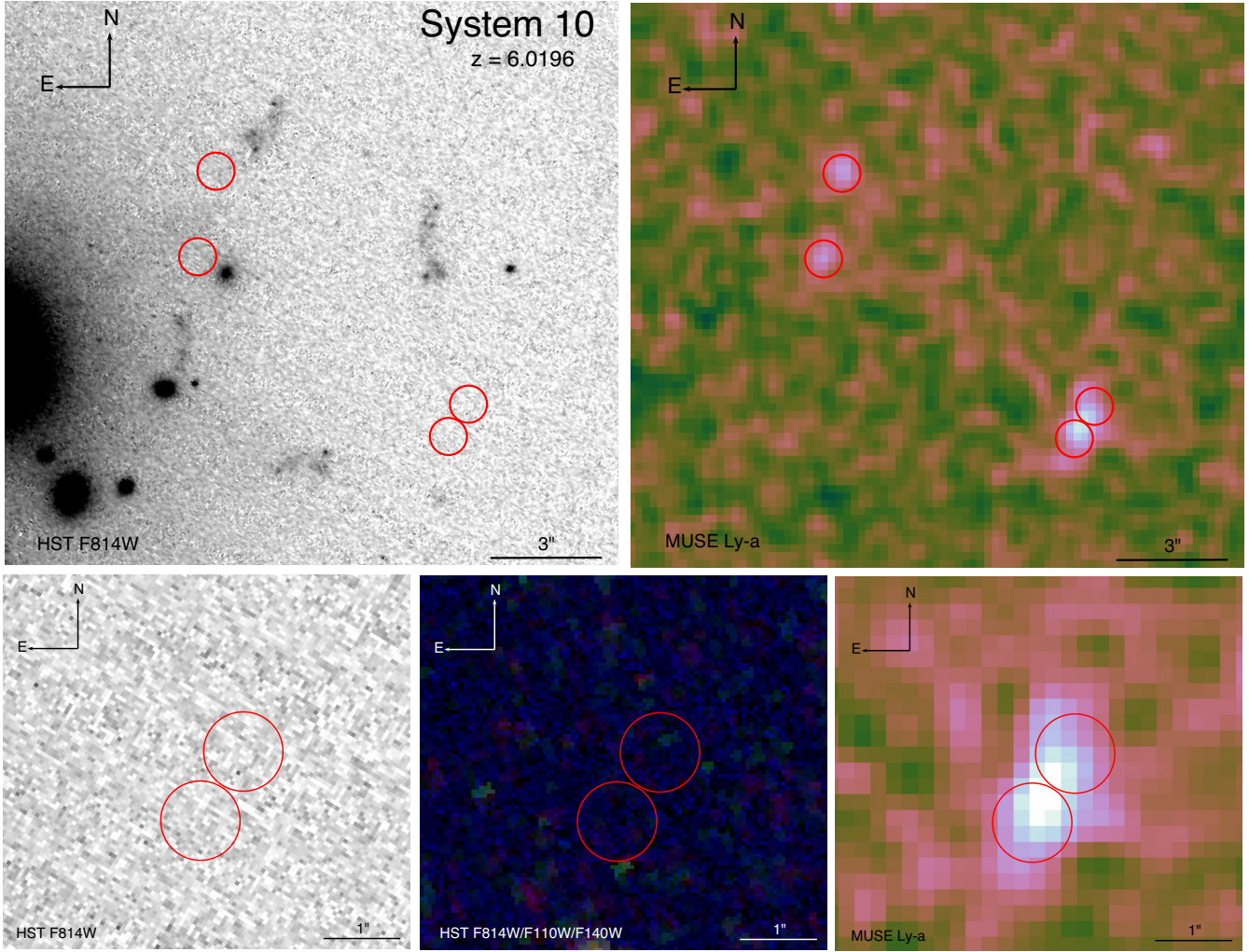


Figure 5. Multiple images of HU System 10 ($z = 6.02$), seen in several data sets. The layout and composition of these images follows that of Fig. 3, though the zoomed-in image this time is the combined merging images of Images 10.1 and 10.2. Unlike Systems 1 and 2, System 10 is not seen in any broad-band images, having been solely identified in deep MUSE observations as a faint Ly α emitter. Future, deeper imaging may yet reveal continuum emission and more complex morphological features for this system.

Table 3. Model parameters used during the Multi Gauss Expansion (MGE) subtraction of the BCG. Here, PA is defined in degrees East of North, and ellipticity is given by $\varepsilon = (a^2 - b^2)/(a^2 + b^2)$, where a and b are the semimajor and semiminor axes of the ellipse, respectively.

Gauss components	PSF size (arcsec)	Fit mask radius (arcsec)	Fit mask PA (degree)	Fit mask ellipticity	Fit mask RA (degree)	Fit mask Dec (degree)
15	1.8	21.75	279	0.29	69.2895902	0.7313656

J. Lagattuta et al. (2023) model, and then optimize the added parameters to see if they significantly improve the overall fit. Although there are several lens modelling codes that can do this, we choose to use the parametric modelling software LENSTOOL (E. Jullo et al. 2007).

4.1 Parametrization of substructures

We model each DM perturber candidate as a single, compact subhalo, in-line with the appearance of substructures in numerical simulations (e.g. G. Despali & S. Vegetti 2017; A. Benítez-Llambay & C. Frenk 2020). Following the convention of D. J. Lagattuta et al. (2023), we parametrize these subhaloes using a Pseudo-

Isothermal Elliptical Mass Distribution (PIEMD; Á. Elíasdóttir et al. 2007). This choice is not arbitrary, as due to the self-similar nature of DM, it is standard to use a common profile for both large and small structures in lensing studies (e.g. M. Jauzac et al. 2015; D. J. Lagattuta et al. 2019; G. B. Caminha et al. 2023; L. J. Furtak et al. 2024; J. F. V. Allingham et al. 2025). Additionally, a PIEMD halo is physically similar to the more universal NFW profile (J. F. Navarro, C. S. Frenk & S. D. M. White 1997), but is computationally faster to evaluate. At the same time, a PIEMD model provides a central flattening of the halo (a ‘core’ component), allowing for more control over the mass distribution.

The PIEMD halo has seven tunable parameters: centroid position (α, δ), ellipticity (ε), position angle (θ), two characteristic

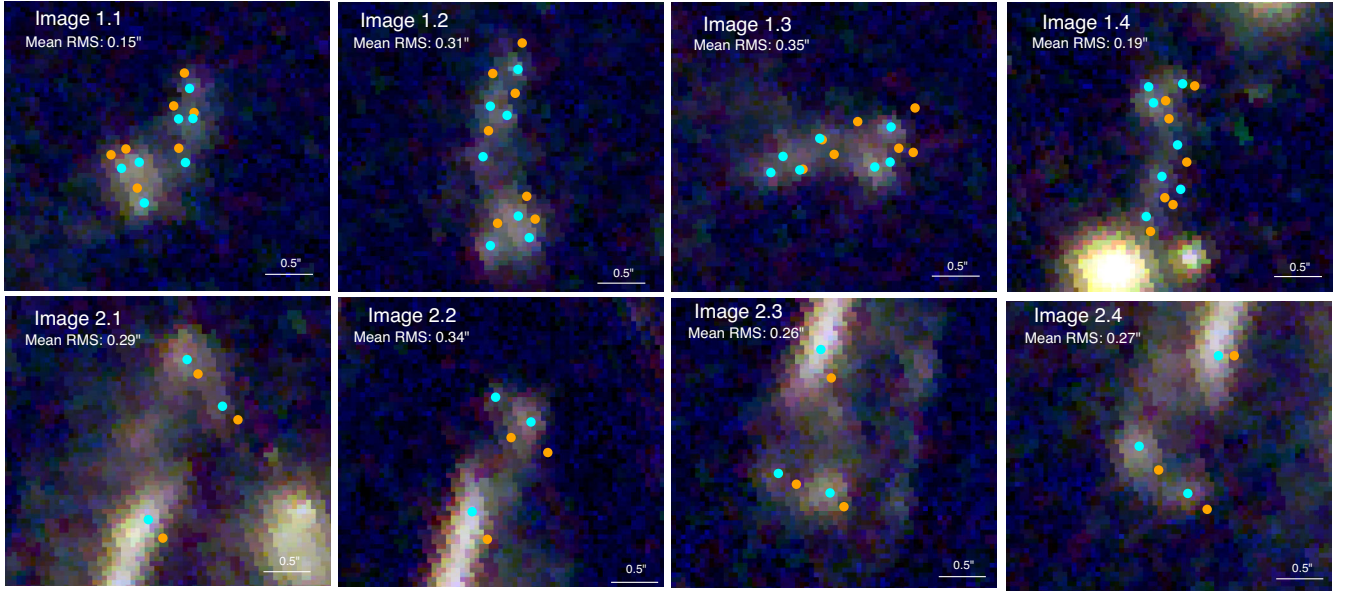


Figure 6. A comparison of the observed stellar clumps in the HU systems (cyan) to the model-predicted positions (orange). The largest discrepancies (~ 0.35 arcsec) can be found around Images 1.2, 1.3, and 2.2.

radii (r_{core} and r_{cut}) that control the inner and outer-most mass distributions (where the halo deviates from a purely isothermal profile), and finally a velocity dispersion (σ) which, when combined with the characteristic radii, encodes the total halo mass. For larger mass components, i.e. cluster-scale and galaxy-scale haloes, several of these components can be set by empirical information. For example, the overall distribution of cluster member galaxies can guide the ellipticity of a cluster halo, while the positions and morphologies of individual cluster members can be observed directly in broadband imaging.

For our candidate subhalo perturbers, since we do not have any empirical knowledge of their appearance, we make some simplifying assumptions during the mass modelling. First, we assume each halo has spherical symmetry, removing the need to fit ε and θ . This follows the convention of previous subhalo detection studies (e.g. S. Vegetti et al. 2012; Q. He et al. 2022; J. W. Nightingale et al. 2024), who find that differences between elliptical and spherical perturbers on the lensing signal are, as of yet, still too weak to be reliably discerned. For similar reasons, we fix the core radius component of each clump to a small (but non-vanishing) 0.15 kpc. This increases the compactness of the perturbers, increasing their lensing efficiency while minimizing their mass. In this way, we can treat any obtained values as a lower limit on the derived subhalo mass estimate. This, when combined with an upper limit derived from a (non) detection of a luminous component, constrains the possible phase space of allowable DM mass. After making these assumptions, we are left with only four variable parameters (α , δ , σ , and r_{cut}) for each halo, which we optimize independently for each component.

4.2 Construction and optimization of the mass model

4.2.1 Construction

After deciding on the form of our candidate perturbers, we include them in the model. Following the abundance of anomalies in the observational data and the original lens model (Sec-

tion 3.2), we treat the area around Images 1.2, 1.3, and 2.2 as the most obvious regions to make our initial guesses. To be conservative (in the absence of any other information), we place a single candidate DM halo in the vicinity of each image. Given the low mass, compact nature of our assumed subhalo components, any effects they have on the global lens model will remain confined to a small area around their positions. Since the three ‘most anomalous’ lens images are at a considerable distance from one another, this suggests that individual subhaloes should not interact with or affect one another in any significant way, leaving them as independent probes (Fig. 8).

To test this idea, we construct four separate ‘substructure-inclusive’ models of the RXJ0437 field: three in which we place a single subhalo candidate close to one of the affected multiple images (Models 1–3) and a fourth (Model 4) that includes one subhalo around each image simultaneously. Model 1 focuses on the area around HU Image 1.2, Model 2 around Image 1.3, and Model 3 around Image 2.2. We label the three elements of Model 4 as Models 4.1–4.3 (with e.g. Model 4.1 corresponding to the subhalo in Model 1). We stress, however, that these three components are fit simultaneously and are not considered as separate models. By constructing the models in this way, we can compare the final halo parameters between models more easily, giving us a measure of parameter stability but also an indication of component confidence: if a candidate substructure remains in a similar position and maintains a similar mass after both its own individual model optimization and the optimization of combined candidates, we can be more confident in its possible existence.

4.2.2 Optimization

In order to improve the computational efficiency of our models, rather than re-optimizing all elements, we instead only allow the subhalo components to vary, fixing the remaining (larger-scale) parameters to the final values presented in D. J. Lagattuta et al. (2023). This significantly increases the speed of a model run and also prevents the small substructure components from being overwhelmed by larger neighbours. Using LENSTOOL, we opti-

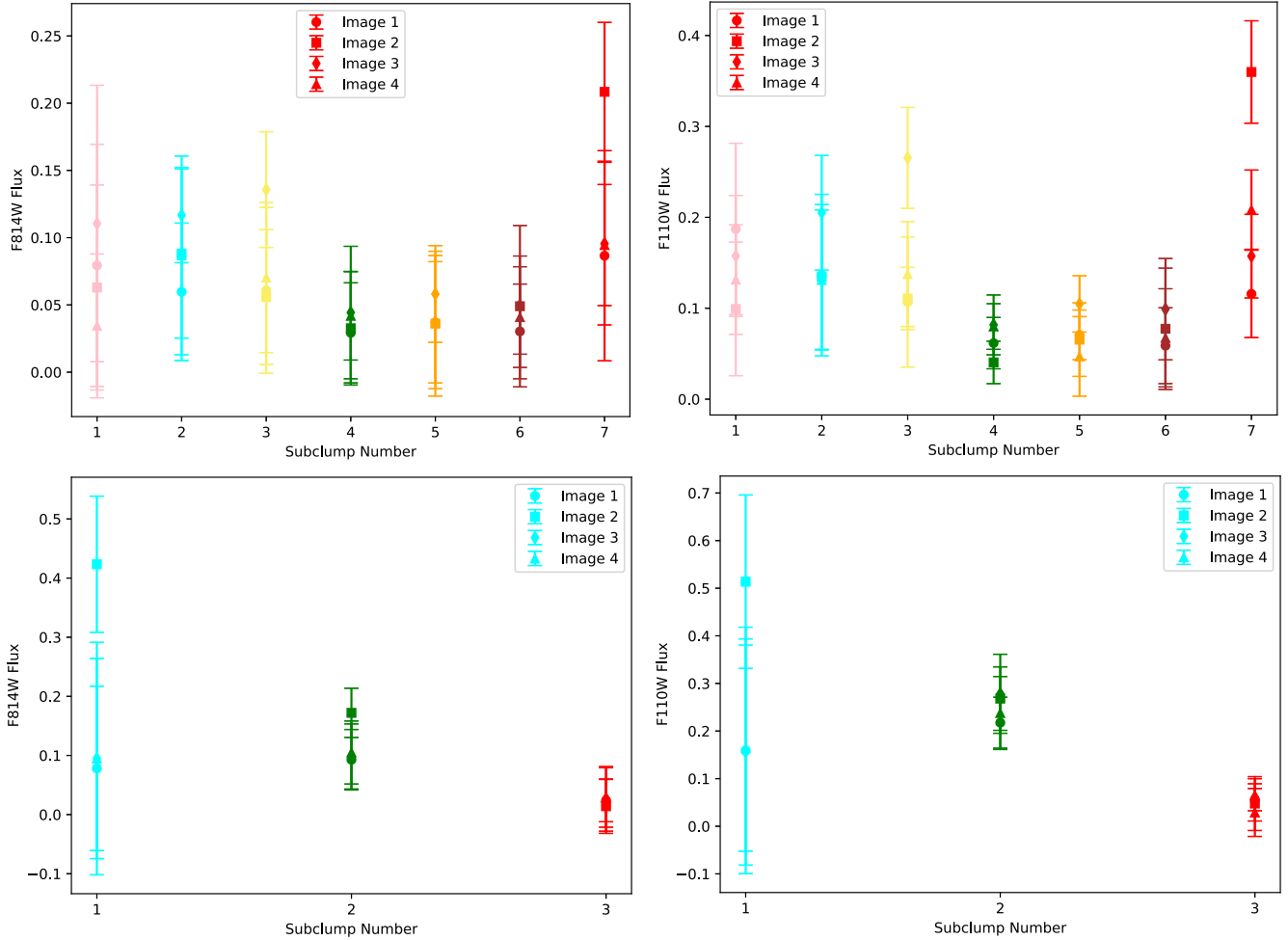


Figure 7. Top panels: Measured aperture fluxes in the *F814W* and *F110W* bands for each of the seven distinct clumps identified in System 1, after correcting for magnification effects. An example of apparently discrepant points can be seen in the *F110W* plot. Clump colours correspond to those seen in Fig. 3. Bottom panels: same, but for System 2. Here, clump colours correspond to those seen in Fig. 4. In all cases, flux errors are reported at the 3σ level.

mize each models (Models 1–4) separately, then compare their parameters after they have converged.

During optimization, we apply moderate constraint limits to each subhalo parameter. This allows us to effectively explore parameter space, while preventing the subhalo components from drifting into non-physical regimes. Specifically, we allow the centroid of each subhalo (α , δ) to vary within a $6 \text{ arcsec} \times 6 \text{ arcsec}$ box, centred around the mean coordinates of all observed sub-components of a given HU image. We similarly constrain the subhalo velocity dispersion, σ , to vary between 0 and 50 km/s, and the cut-off radius, r_{cut} , between 1 and 10 kpc (e.g. G. Mahler et al. 2023; J. E. Doppel et al. 2026). Combined, these constraints maintain the shape of the subhalo as small and compact, and keep it within the effective area of the target HU image while still allowing for the possibility of a non-detection.

The optimization process itself is handled by LENSTOOL, using its built-in fitting routine. During optimization, LENSTOOL attempts to minimize the distance between the observed and model-predicted positions (in the image plane) of the multiple-image constraints. Model predictions are determined by casting the multiple images of a given object to the source plane, calculating a common barycentre and then recasting the barycentre position back to the image plane. The barycentre itself is determined

using inverse magnification weighting, where less-magnified images contribute more strongly to the barycentre position. After each model converges to its best-fitting parameter set, the code robustly explores uncertainties in parameter space, generating 1000 Markov Chain Monte Carlo (MCMC) realizations of each model. The results of this exploration can be seen in Table 4, and a graphical representation of the final halo candidates can be seen in Fig. 8.

4.3 Model results

Looking at the final outcomes, we can compare the best-fitting parameters of the individual halo models (Models 1–3) to the combined model (Model 4). Doing this, we see that the subhalo positions around Images 1.3 and 2.2 move significantly between cases, with centroids that are inconsistent within uncertainty measurements. At the same time, the velocity dispersion parameters between models also diverge, and, in some cases, the values are consistent with zero. This suggests that either the models have not converged, or there is not enough information to truly constrain a subhalo here. Given the wide-ranging MCMC exploration in the previous step and the fact that the each model provides

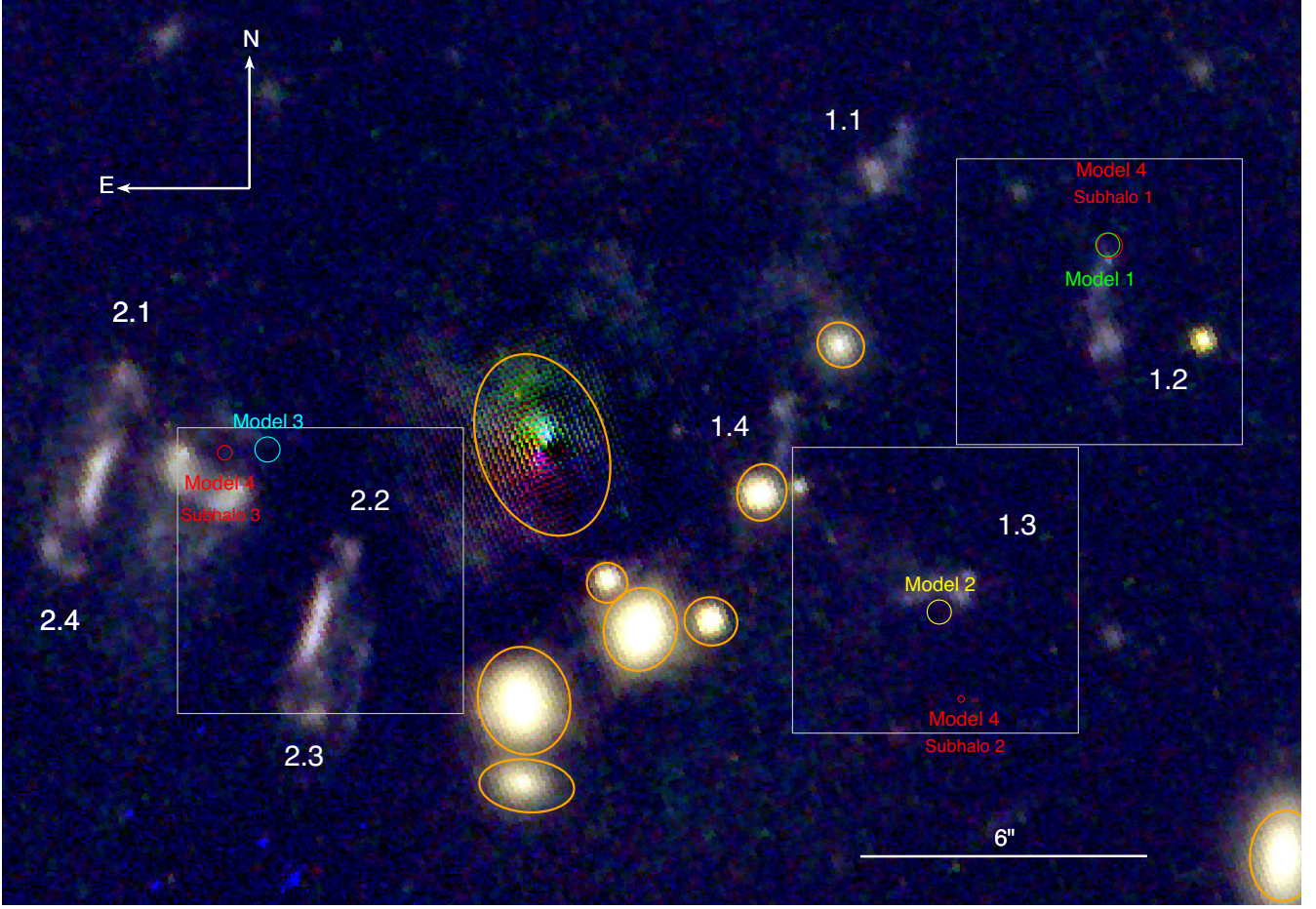


Figure 8. Graphical representation of the best-fitting model results, where the BCG has been subtracted to improve the contrast (following the procedure described in Section 3.2). Grey squares represent the 6×6 arcsec region of space over which a given component can move during model optimization. Each box is centred on an HU image that contains a possible flux ratio anomaly. Additionally, cluster members included in the overall mass model are highlighted with orange ellipses. Single-subhalo models are shown in green (Model 1), yellow (Model 2), and cyan (Model 3), respectively, while the multisubhalo model (Model 4) appears in red. The final position of each halo is represented by a circle, with sizes representing a scaled combination of velocity dispersion, which serves as a proxy for halo mass. We see that the final parameters for the subhalo around HU Image 1.2 (Models 1 and 4, Subhalo 1) are entirely consistent, while the other candidate subhaloes change more significantly between models.

Table 4. Best-fitting values of the candidate subhaloes for all four models. Column 1 lists the model considered. Columns 2 and 3 give the best-fitting values of Right Ascension and Declination (and their respective errors) in arcseconds, relative to the BCG ($\alpha = 69.2896877$, $\delta = 0.7311409$) at the centre of RXJ0437. Column 4 presents the dPIE cut-off radius, while Column 5 lists velocity dispersion. Model 4 is run with all three subhalo candidates inserted; however, for clarity, we list the best-fitting position of each individual subhalo.

Model	Δ_α (arcsec)	Δ_δ (arcsec)	r_{cut} (kpc)	σ (km s^{-1})
Model 1	$11.87^{+3.47}_{-0.47}$	$4.20^{+1.79}_{-1.86}$	$1.45^{+0.16}_{-0.68}$	38^{+1}_{-3}
Model 2	$8.33^{+2.70}_{-0.65}$	$-3.51^{+0.54}_{-1.80}$	$2.91^{+0.28}_{-0.95}$	38^{+1}_{-7}
Model 3	$-5.53^{+1.25}_{-0.75}$	$-0.10^{+1.30}_{-1.19}$	$2.97^{+0.21}_{-1.29}$	40^{+1}_{-6}
Model 4 (Subhalo 1)	$11.91^{+1.42}_{-2.50}$	$4.19^{+2.41}_{-1.93}$	$1.38^{+0.09}_{-0.25}$	40^{+1}_{-2}
Model 4 (Subhalo 2)	$8.79^{+2.26}_{-0.13}$	$-5.34^{+0.89}_{-1.07}$	$2.52^{+0.29}_{-1.09}$	10^{+2}_{-6}
Model 4 (Subhalo 3)	$-6.67^{+1.31}_{-1.57}$	$-0.17^{+1.32}_{-0.69}$	$1.02^{+0.13}_{-0.72}$	23^{+5}_{-1}

a similar final rms (0.28–0.29 arcsec), non-convergence seems unlikely, making it more probable that the signal is simply not strong enough to convincingly reveal the presence of a subhalo.

Conversely, the two models that include HU Image 1.2 show good agreement between them, with overlapping centroid positions and a velocity dispersion that is internally consistent and also inconsistent with zero. This provides stronger evidence for the existence of a subhalo, warranting further exploration. Specifically, we compare the fit statistics of Model 1 to the original one presented by D. J. Lagattuta et al. (2023), finding that the new model has both a modestly smaller rms (0.29 arcsec $\rightarrow$ 0.28 arcsec) and lower total χ^2 (101.92 $\rightarrow$ 98.18). Using these values to calculate the Bayesian Information Criterion (BIC) metric, we find that the substructure model also has a slightly lower total BIC ($-4.52 \rightarrow -5.83$). While this does not differ significantly, it nonetheless reduces the likelihood of overfitting. Collectively, then, this information strengthens the idea that the subhalo near Image 1.2 is our best DM candidate.

Finally, we can convert the model parameters to a physical mass value. For a PIEMD profile, this procedure is described in

Table 5. Best-fitting masses for all substructure models.

Model	Mass ($M_{\odot}$)
Model 1	$(2.27 \pm 0.83) \times 10^9$
Model 4 (Subhalo 1)	$(2.39 \pm 0.86) \times 10^9$
Model 2	$(4.59 \pm 1.26) \times 10^9$
Model 4 (Subhalo 2)	$(2.75 \pm 0.71) \times 10^8$
Model 3	$(5.19 \pm 1.86) \times 10^9$
Model 4 (Subhalo 3)	$(5.79 \pm 0.43) \times 10^8$

Appendix A7 of Á. Elíasdóttir et al. (2007):

$$\sigma^2(r_{\text{core}}) = \frac{2G}{\Sigma(r_{\text{core}})} \times \int_{r_{\text{core}}}^{\infty} \frac{M(r_{\text{cut}})\rho(r_{\text{cut}})}{r_{\text{core}}^2} \sqrt{r_{\text{cut}}^2 - r_{\text{core}}^2} dr_{\text{cut}}, \quad (1)$$

which when solved and rearranged becomes

$$M(r_{\text{cut}}) = \frac{\sigma^2(r_{\text{core}})}{G * \frac{r_{\text{cut}}}{r_{\text{core}}^2} * \frac{1}{2\pi} * \frac{4}{3}}. \quad (2)$$

Substituting in the values from Model 1, we measure a final subhalo candidate mass of $(2.27 \pm 0.83) \times 10^9 M_{\odot}$. A full list of masses for all subhaloes over all models can be found in Table 5.

5 DISCUSSION

5.1 Reliability of detections

The possible detection of an invisible mass perturber in a galaxy cluster is an intriguing possibility, but we stress that this result is preliminary and care must be taken to interpret it. In particular, we note that the *HST* imaging data we have used for this analysis is shallow, limiting our ability to detect and discriminate between perturbers. Relevant to this work is the fact that, in addition to our candidate substructure, there are a few other small, faint objects located within the bounding boxes of the model fit (Fig. 8) which we do not include as mass components in the model. The brightest of these (the yellow dot to the west of Image 1.2 is a spectroscopically confirmed star, which will have no impact on the mass model. However, we have no information about the other two objects (a faint grey dot roughly halfway between Images 1.1 and 1.2, and a smaller yellow dot close to Image 1.4.) Like the known star, their compact, point-like sizes and low brightnesses suggest they could also be Galactic in origin, but we cannot rule out the possibility that they are low-mass galaxies that could also have a hand in creating lensing flux anomalies. (We point out, though, that the grey spot – which might most affect the interpretation of the candidate substructure – is nearly 3 arcseconds away from Image 1.2, meaning that, unless it is exceptionally massive, its Einstein radius will not extend to the identified flux-ratio anomaly.) To truly break this degeneracy, we will need deeper imaging and or spectroscopic data.

At the same time, the total mass of the candidate subhalo ($\sim 2 \times 10^9 M_{\odot}$), while many orders of magnitude smaller than the cluster halo, is on the higher side of potential subhalo detections (e.g. S. Vegetti et al. 2012; Y. D. Hezaveh et al. 2016; Q. Minor et al. 2021; D. M. Powell et al. 2025), lessening its ability to discriminate between DM models, as significant deviations only become apparent at cluster masses $< 10^8 M_{\odot}$. Ultimately, a more detailed study and analysis will require a statistical approach, based on several halo detections. None the less, this work represents an important first step in detecting low mass deflectors in the densest

regions of galaxy clusters, increasing the likelihood of finding other examples, with more varied halo masses, in the future.

5.2 Is the subhalo candidate truly ‘dark’?

Along similar lines, it is important to note that, inherently, our analysis has assumed that the subhalo perturbers are embedded within the main cluster halo. However, this is not the only plausible scenario: it is also possible that these objects lie along the line of sight (LoS). Cosmological simulations have previously shown that, while a cluster dominates the mass budget of its LoS, the distribution of objects responsible for the observed lensing signal has a much broader distribution (A. Robertson et al. 2020).

This can be especially problematic for detecting and distinguishing DM perturbers, as these cannot be reliably redshifted, regardless of imaging quality. Indeed, if this is the case here, it can significantly change our interpretation of the result: a more distant LoS deflector would have a different mass and a lower observable flux density. Combined, this could make the deflector a less viable candidate for discriminating between DM models. Specifically, a more massive, distant object would be even less sensitive to differences between CDM and its alternatives, and may not be truly dark – rather, luminous but below the detectable flux limit.

Importantly, recent work has shown that the observable effects of LoS deflectors are sensitive to image depth, with deeper imaging more sensitive to these interlopers than shallow imaging (Q. He et al. 2022). Given the shallow nature of our current imaging data, we are likely less sensitive to LoS perturbers and can, at the moment, attribute any distortions to a subhalo in the cluster field itself. Nevertheless, the possibility of interlopers remains. To truly confirm or reject this possibility, we must turn to deeper and higher-resolution data.

6 CONCLUSIONS AND FUTURE WORK

In this work, we have studied mass distributions in the galaxy cluster RXJ0437, taking advantage of three high-magnification, ‘exotic’ HU gravitational lenses (designated Systems 1, 2, and 10 in D. J. Lagattuta et al. 2023) to search for low-mass ($m \lesssim 10^9 M_{\odot}$) subhaloes in the cluster core. We summarize our results as follows:

(i) Using a combination of high-resolution *HST* imaging and VLT/MUSE spectroscopy, we identify seven distinct luminous subcomponents (clumps) in System 1, three in System 2, and a single bright $\text{Ly}\alpha$ emission region in System 10.

(ii) After subtracting contamination due to the nearby BCG and correcting for lensing magnification, we compare the brightnesses between the multiply-imaged clumps of each system (which should ostensibly be identical). Doing so, we find that segments of Images 1.2, 1.3, and 2.2 are noticeably different from their counterparts.

(iii) We also compare the observed positions of each HU clump to its predicted location in the D. J. Lagattuta et al. (2023) lens model, again finding the largest discrepancies in Images 1.2, 1.3, and 2.2. Together with the luminosity discrepancies, we focus on these regions for our substructure search.

(iv) To investigate the presence of substructure, we embed several candidate DM subhaloes in the D. J. Lagattuta et al. (2023) lens model, using the parametric modelling software LENSTOOL. We specifically add these candidates in the vicinity of Images 1.2, 1.3, and 2.1.

(v) After optimizing the new lens models, we see that one candidate subhalo (the object in the region of Image 1.2) maintains a consistent position and total mass: $(2.27 \pm 0.83) \times 10^9 M_\odot$ which we consider as our best possible substructure candidate.

While potentially interesting, we stress that this detection is still preliminary: our best available imaging data are shallow (*HST* SNAPshot images) resulting in lower confidence measurements. To gain confidence in the result, we will need deeper and higher resolution data. This will soon be available in the form of *James Webb Space Telescope* (*JWST*) data (PID: 6207, PI D. Lagattuta), allowing us to explore this cluster further. In particular, the new *JWST* data will enhance our situation in two key ways, by (1) reducing the uncertainty on the ‘global’ lens model by identifying new multiply-imaged constraints, thereby reducing model uncertainty (Cerny et al., in preparation) and (2) better resolving the structure of the HU systems, making it easier to detect the subtle perturbations caused by small-scale concentrations of DM. However, with this work, we demonstrate the feasibility of using galaxy cluster lenses, and their HU systems, as laboratories to study the distribution of low-mass substructures.

Besides acquiring further data, however, we are also endeavouring to expand the sample of known HU lenses. This will allow us to investigate DM in a statistical way, using a sample of subhalo detections to place more robust constraints on the low-mass end of the subhalo mass function. To that end, we have already identified additional, promising HU candidates, both in archival *HST* data and in recent *JWST* imaging—particularly the Strong Lensing and Cluster Evolution (SLICE) sample: a collection of 124 galaxy clusters observed with *JWST* as part of a survey programme (PID: 5594; PI G. Mahler; C. Cerny et al. 2026). We will continue to pursue these efforts in follow-up work.

ACKNOWLEDGEMENTS

DJL, MJ, and JED acknowledge support by the United Kingdom Research and Innovation (UKRI) Future Leaders Fellowship ‘Using Cosmic Beasts to uncover the Nature of Dark Matter’ (grant number MR/X006069/1).

DATA AVAILABILITY

All imaging and spectroscopic data used in this work are publicly available in standard archives (see the information provided in Section 2 of this manuscript). The final parameters of all lens models are provided in the body of the text.

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APPENDIX A: FLUX AND MAGNIFICATION VALUES

In this appendix, we present the empirical and model-dependent measurements of individual HU clumps, which are used to create Fig. 7. In addition to reproducing the plot, these mea-

measurements also help to highlight and add context to the significance of the flux ratio anomalies identified in HU Images 1.2, 1.3, and 2.2 (Section 3.2). As described in that section, flux measurements and magnification values for each clump are measured in circular apertures, centred on the clump position, with a diameter matching the clump's extent in the F814W

band. Magnification uncertainties are estimated in a slightly larger aperture, which is coincident with the position of the clump but with a wider diameter, encompassing positional uncertainty in the original D. J. Lagattuta et al. (2023) lens model (Table A1).

Table A1. Relevant measurements of the HU clumps used to investigate the presence of substructures.

Image	Clump	F814W flux (electron s ⁻¹)	F814W error (electron s ⁻¹)	F110W flux (electron s ⁻¹)	F110W error (electron s ⁻¹)	Model Magnification	Magnification error
1.1	1	1.38	0.03	3.25	0.03	17.7	0.2
1.1	2	0.89	0.02	2.04	0.03	15.0	0.3
1.1	3	0.91	0.02	1.59	0.02	14.9	0.3
1.1	4	0.51	0.01	1.07	0.01	17.4	0.3
1.1	5	0.61	0.02	1.16	0.01	16.4	0.3
1.1	6	0.45	0.01	0.87	0.01	14.7	0.1
1.1	7	1.23	0.03	1.64	0.02	14.2	0.3
1.2	1	1.58	0.03	2.49	0.02	25.1	0.9
1.2	2	1.79	0.02	2.73	0.03	20.3	0.4
1.2	3	1.17	0.02	2.31	0.01	20.9	0.5
1.2	4	0.69	0.01	0.85	0.01	21.1	0.8
1.2	5	0.74	0.02	1.34	0.01	20.5	0.6
1.2	6	0.92	0.02	1.45	0.02	18.8	0.9
1.2	7	3.64	0.02	6.28	0.02	17.4	0.5
1.3	1	1.61	0.03	2.30	0.02	14.6	0.6
1.3	2	1.42	0.01	2.49	0.02	12.1	0.4
1.3	3	1.70	0.01	3.32	0.02	12.5	0.4
1.3	4	0.60	0.02	1.10	0.01	13.4	0.7
1.3	5	0.74	0.01	1.33	0.01	12.7	0.5
1.3	6	0.57	0.01	1.13	0.02	11.4	0.6
1.3	7	1.05	0.02	1.73	0.02	11.0	0.4
1.4	1	0.46	0.02	1.76	0.02	13.4	0.5
1.4	2	0.90	0.03	1.36	0.03	10.4	0.5
1.4	3	0.74	0.02	1.45	0.02	10.5	0.5
1.4	4	0.45	0.01	0.86	0.01	10.8	0.4
1.4	5	0.32	0.02	0.40	0.02	8.5	0.4
1.4	6	0.33	0.01	0.55	0.02	8.2	0.4
1.4	7	0.94	0.02	2.06	0.02	9.9	0.3
2.1	1	10.72	0.05	21.84	0.09	137.1	32.9
2.1	2	2.75	0.02	6.44	0.02	29.6	0.8
2.1	3	0.71	0.01	1.66	0.01	29.9	1.0
2.2	1	23.81	0.04	28.89	0.06	56.2	9.3
2.2	2	2.93	0.01	4.56	0.02	17.0	1.3
2.2	3	0.29	0.02	0.99	0.02	20.8	2.2
2.3	1	13.24	0.07	21.79	0.08	139.8	54.5
2.3	2	2.46	0.02	6.97	0.03	25.0	0.9
2.3	3	0.80	0.02	1.84	0.01	30.3	1.2
2.4	1	10.98	0.06	18.98	0.07	115.8	13.2
2.4	2	3.17	0.02	7.17	0.02	30.1	1.6
2.4	3	0.95	0.02	0.95	0.02	32.8	1.6

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