

# An Envelope-Based Operational Scheme for Distribution Networks: From Close-to-Real-Time Envelope Adjustment to Market-Based Reallocation

**Abstract**—In this paper, we propose a static security management scheme for distribution networks that aims to improve the utilization of the flexibility available in the network and increase users’ access to power. Prior to operation, all users have a bilateral agreement with the distribution system operator (DSO) defining their conditions of access to the network. This agreement defines a power envelope that provides the maximum level of power access, which we call a contractual envelope (CE). It comprises a firm part that is guaranteed and a flexible part that may be reduced close to real time. The reduced envelope is called an operating envelope (OE), that is, a power range within which the user can operate without compromising the static security of the network, under some assumptions. The scheme differentiates users whose power can be reliably predicted, the so-called predictable users, from those who cannot, referred to as unpredictable users. On the one hand, the power consumption/injection of a predictable user can be computed close to real time, such that the unused part of its CE can be leveraged to provide flexibility. Their OE remains secure as long as the predictable user consumes/injects power at its predicted value. On the other hand, unpredictable users seek flexibility because their power usage is unknown. Their OEs are at least as large as the firm part of the CEs established in the contractual agreements. The OEs are computed using the flexibility provided by predictable users to maximize the possible network usage of unpredictable users while satisfying contractual and regulatory constraints. In contrast to the predictable users, the unpredictable users’ OEs are computed to be secure for all power consumption/injection values within them. The OEs’ computation problem is formulated as a numerical optimization problem. Because the resulting OEs may nevertheless not match users’ needs, they are exchanged among users on a secondary market cleared close to real time to maximize social welfare while preserving static security for every power consumption/injection within the OEs.

## I. INTRODUCTION

The growing penetration of distributed energy resources (DERs), such as photovoltaic (PV) panels and batteries, has substantially increased the complexity of distribution network operation. This complexity increasingly compels distribution system operators (DSOs) to take close-to-real-time preventive actions to maintain network assets within their acceptable operating ranges.

At the same time, the electrification of transport, heating, and industrial processes is driving the electricity demand upward, inducing an associated surge in requests for new or increased grid connections. However, network reinforcement, which includes the installation or upgrade of substations [1], remains capital-intensive and slow, constrained by lengthy permitting and construction lead times [2]. Network hosting capacity therefore cannot grow at the required pace. Net-

work access is thus becoming a scarce resource, and users increasingly face connection delays [3]. In response, DSOs have introduced flexible connection contracts. Under these contracts, the consumption or injection of a user may be limited whenever network constraints would otherwise be violated.

This operational approach, referred to as active network management (ANM) [4, 5], allows the DSO to send curtailment orders to DERs and loads whenever a static security violation is anticipated. Authors in [6], for instance, show how ANM can help maintaining the distribution network within operational limits by performing, among others, Volt-VAR control and active power curtailment. While these measures ensure secure operation, they offer grid users little predictability. A user cannot know in advance how much power it will be allowed to exchange with the grid. An uncertain access to the grid may therefore undermine the profitability of the users’ business model. ANM also relies on real-time monitoring, which DSOs may be unable to perform given the limited observability of their networks.

Operating envelopes (OEs) offer a practical way to allocate network capacity while preserving operational security. An OE is a power range assigned by the DSO to a user within which the latter may freely withdraw or inject power without compromising the network’s static security [7]. Because the range is known prior to operation, an OE restores the predictability that curtailment orders lack. Using OEs also preserves confidentiality. The DSO grants each user a power range without accessing its internal costs or dispatch decisions, which the user optimizes privately within that range [8]. OEs have been applied in different contexts. Among others, they have been leveraged for guaranteeing fair network access within local energy markets [9], and for demand response [8, 10, 11]. OEs have also been proposed to enhance coordination between transmission and distribution system operators, as detailed in [12–14].

These works, however, treat the envelope as a single quantity, allocated by the DSO and used by the grid user as granted. Because DER behavior is uncertain, the DSO must size envelopes so that security holds for every operating point a user might adopt within its OE, which leads to conservative allocations, i.e., tight OEs. Market-based reallocation has been proposed to relieve this conservatism. The Secondary Congestion Option Platform Exchange lets a curtailed producer draw on local flexibility to relieve injection congestion [15], and SecuLex formalizes the same principle at the level of OEs, turning envelope bounds into assets traded on a continuously

cleared market [16]. Such mechanisms leave the allocation stage largely untouched. In [16], the envelope granted to a user remains a single quantity, sized to hold for every operating point of every user and allocated according to an egalitarian criterion.

In this paper, the envelope granted to a user departs from this definition. Prior to operation, a *contractual envelope* (CE) is agreed upon between the DSO and the user, combining two complementary parts: (i) a *firm envelope*, a time-invariant power range that guarantees the user a baseline access, and (ii) a *flexible extension*, which the DSO may reduce prior to each period according to production, consumption, and network conditions. The range resulting from this reduction is the *operating envelope* (OE), within which the user is required to operate over the period. Unlike the OEs of the literature, the OE proposed here might not be secure for every power consumption/injection it contains. Its security guarantee is conditional. Users whose power can be reliably predicted are represented by a predicted power consumption/injection value within their OE, and security is guaranteed at that value only. Users whose power cannot be predicted are covered over their whole OE.

The operational scheme proceeds in two sequential stages. Close-to-real-time, the DSO adjusts the flexible extensions to grant users as much operating range as possible while preserving this conditional security guarantee and following regulatory priority rules. The mismatch between resulting envelopes and users' needs motivates a second stage. This second stage consists in a secondary market on which users trade envelope bounds against financial compensation. The market is secondary in that users exchange among themselves the range already granted to them, rather than the DSO procuring it from them as in a local primary flexibility market, where it purchases flexibility and bears the associated cost [17].

The contribution of this paper is therefore threefold. First, the envelope granted to a user is derived from a contractual object, the CE, decomposed into a firm envelope, matching the firm capacity of a flexible connection agreement, and a flexible extension, reduced under a regulatory priority ordering to yield the OE of the period. Second, the security of the envelopes is made conditional on the predictability of grid users. Predictable users are represented in the adjustment problem by a predicted power consumption/injection, and only unpredictable users are covered over their whole envelope. Third, this flexible extension adjustment is combined with a market-based reallocation of the kind proposed in [16], organized here as a single pool auction per period and settled pay-as-clear.

The remainder of this paper is organized as follows. In Section II, an overview of the operational scheme through a running example is provided. Section III formalizes the close-to-real-time envelope adjustment problem, while Section IV formalizes the market-based envelope reallocation mechanism. Section V discusses regulatory aspects and Section VI concludes this work.

## II. AN INTUITIVE OVERVIEW OF THE ENVELOPE-BASED OPERATIONAL SCHEME

This section presents the envelope-based network operational scheme. In Section II-A, contractual and operating envelopes are formally defined and applied on an illustrative example, while Section II-B describes the close-to-real-time adjustment leading to the operating envelopes. Once these have been computed, a market opens to allow users to exchange their bounds, as illustrated in Section II-C.

### A. An envelope-based distribution network operation

#### Network and time model

Let us consider a balanced three-phase distribution network serving a set  $\mathcal{U} = \{u_1, \dots, u_N\}$  of  $N$  grid users. The time horizon is discretised into  $T$  periods, indexed by  $t \in \{0, \dots, T-1\} = \mathcal{T}$ . For each user  $u \in \mathcal{U}$  and period  $t$ , we denote by  $P_{u,t} \in \mathbb{R}$  the effective net power exchanged by user  $u$  with the network. This power is assumed to be constant over every period  $t$ . By convention, positive values correspond to power withdrawal (consumption) and negative values to power injection (production). Only active power is considered.

#### Contractual and operating envelope definitions

A *contractual envelope* (CE) is the power range agreed upon between the DSO and a grid user, defining the maximum level of power access granted to that user. It is a contractual object consisting of two components, a firm part and a flexible part, hereafter referred to as *firm envelope* and *maximum flexible extension*, respectively.

The *firm envelope* imposed on user  $u$  is a time-invariant power range  $[P_u^{f,\min}, P_u^{f,\max}] \subset \mathbb{R}$ , guaranteed to the user at every period.<sup>1</sup>

The *maximum flexible extension* of user  $u$  is a pair of non-negative quantities  $(\bar{\delta}_u^{\min}, \bar{\delta}_u^{\max}) \in \mathbb{R}^+ \times \mathbb{R}^+$ , stating the largest downward and upward extensions of the firm envelope the DSO may ever grant. Like the firm envelope, it is time-invariant and fixed in the connection contract. The CE of user  $u$  therefore reads

$$\bar{\Delta}_u = [P_u^{f,\min} - \bar{\delta}_u^{\min}, P_u^{f,\max} + \bar{\delta}_u^{\max}]. \quad (1)$$

The extension actually granted for a given period is the *flexible extension*  $(\delta_{u,t}^{\min}, \delta_{u,t}^{\max}) \in \mathbb{R}^+ \times \mathbb{R}^+$ . It is adjusted by the DSO at each period in response to network conditions, within the maximum values stated in the contract,

$$0 \leq \delta_{u,t}^{\min} \leq \bar{\delta}_u^{\min}, \quad 0 \leq \delta_{u,t}^{\max} \leq \bar{\delta}_u^{\max}. \quad (2)$$

The *operating envelope* (OE) of user  $u$  at period  $t$  is the range resulting from this granted extension,

$$\Delta_{u,t} = [P_u^{f,\min} - \delta_{u,t}^{\min}, P_u^{f,\max} + \delta_{u,t}^{\max}], \quad (3)$$

so that  $[P_u^{f,\min}, P_u^{f,\max}] \subseteq \Delta_{u,t} \subseteq \bar{\Delta}_u$ . The user is required to operate within its OE during period  $t$ , i.e.,  $P_{u,t} \in \Delta_{u,t}$ .

<sup>1</sup>The firm envelope is guaranteed by default, but it is not inalienable. A user may temporarily release part of it on a secondary market in exchange for financial compensation, as further detailed in Section IV.

Whereas the CE only states the widest range the user may ever be granted, the OE is computed by the DSO so that the user may operate anywhere within it without compromising the static security of the network, under the conditions stated in Section II-B.

#### Illustrative example

To ground the concepts introduced above, we introduce a running example that will be used throughout the paper. We consider three grid users,  $u_1$ ,  $u_2$  and  $u_3$ , connected to a distribution network:

- $u_1$ : a wind farm of 10 MW installed capacity, which can only inject power,
- $u_2$ : an aggregated residential load of approximately 500 houses, with a peak withdrawal of 3 MW,
- $u_3$ : a battery rated at 6 MW, which can both withdraw and inject power.

The transformer connecting the distribution network to the rest of the grid has a nominal capacity of 10 MW, so the net power flowing through it must remain within the secure range  $[-10, 10]$  MW at all periods.

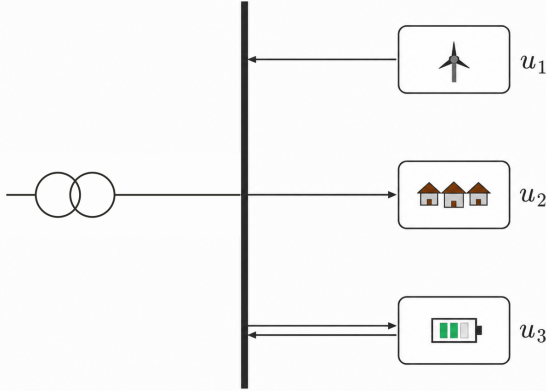


Fig. 1: Distribution network of the illustrative example: a wind farm ( $u_1$ ), an aggregated residential load ( $u_2$ ) and a battery ( $u_3$ ) share a common bus, connected to the upstream grid through the substation transformer. Arrows show each user's admissible direction of power exchange.

|       | $[P_u^{f,\min}, P_u^{f,\max}]$ | $\bar{\delta}_u^{\min}$ | $\bar{\delta}_u^{\max}$ |
|-------|--------------------------------|-------------------------|-------------------------|
| $u_1$ | $[-4, 0]$                      | 6                       | 0                       |
| $u_2$ | $[0, 3]$                       | 0                       | 0                       |
| $u_3$ | $[-2, 2]$                      | 4                       | 4                       |

TABLE I: Firm envelopes and maximum flexible extensions defining the CE of each user of the illustrative example (in MW).

The example is now instantiated at a given period  $t$ . The DSO adjusts the flexible extension of every grid user. The envelope adjustment process is further detailed in Section II-B. The resulting flexible extensions must keep the transformer within its secure operating range, i.e.,  $[-10, 10]$  MW. Table II

gives the flexible extension values  $\delta_{u,t}^{\min}, \delta_{u,t}^{\max}$  at period  $t$ , and the resulting OE  $\Delta_{u,t}$ .

At period  $t$ ,  $P_{u_1,t} = -8$  MW,  $P_{u_2,t} = 2$  MW, and  $P_{u_3,t} = -4$  MW. Neglecting losses, the net power seen by the transformer is  $(-8) + 2 + (-4) = (-10)$  MW, which lies within the range  $[-10, 10]$  MW. The transformer thus operates securely.

|       | $\delta_{u,t}^{\min}$ | $\delta_{u,t}^{\max}$ | $\Delta_{u,t}$ | $P_{u,t}$ |
|-------|-----------------------|-----------------------|----------------|-----------|
| $u_1$ | 6                     | 0                     | $[-10, 0]$     | -8        |
| $u_2$ | 0                     | 0                     | $[0, 3]$       | 2         |
| $u_3$ | 2                     | 3                     | $[-4, 5]$      | -4        |

TABLE II: Flexible extensions  $\delta_{u,t}^{\min}, \delta_{u,t}^{\max}$ , resulting OEs  $\Delta_{u,t}$ , and power consumption/injection  $P_{u,t}$  of each user at period  $t$  (MW).

#### B. Close-to-real-time envelope adjustment

At each period  $t$ , the DSO adjusts the flexible extensions for the upcoming period  $t+1$ , such that network assets are operated securely. Users are not all represented in the same way when the DSO computes the adjustment, and two categories are distinguished accordingly. *Predictable users* are those whose power can be reliably predicted given knowledge of their OE, whereas *unpredictable users* are those whose power cannot. For a predictable user, the DSO therefore associates with any possible OE a single power consumption/injection value. For an unpredictable user, no such point can be singled out.

The adjustment then proceeds in two steps. Firstly, the DSO establishes how each user is to be represented in the adjustment problem. A predictable user is represented by its power consumption/injection as a function of its OE. An unpredictable user is covered over the whole of that envelope, any point of which it might reach. Secondly, the DSO tightens the flexible extensions until the resulting OEs are consistent with security requirements. This tightening is performed while keeping the OEs granted to users as wide as possible, and is subject to the security constraints and to regulatory requirements such as a priority ordering over users. The adjustment is formalized as an optimization problem in Section III.

#### Illustrative example

We revisit the running example to illustrate how the OEs  $\Delta_{u_1,t+1}$ ,  $\Delta_{u_2,t+1}$ , and  $\Delta_{u_3,t+1}$  are computed at period  $t$  for the upcoming period  $t+1$ , such that the power flowing through the transformer stays within  $[-10, 10]$  MW.

In the first step, the DSO predicts the power consumption or injection of each predictable user as a function of its OE. The wind farm  $u_1$  is predicted to inject 10 MW at period  $t+1$  whenever its OE accommodates it, while the load  $u_2$  is predicted to withdraw 1 MW. The battery  $u_3$  is unpredictable, so no forecast can be used for it. Its power is instead taken over the whole envelope under consideration. With the extensions at their contractual maxima, this envelope is the contractual envelope  $\bar{\Delta}_{u_3} = [-6, 6]$  MW, and security

is verified against the bounds of that range. The net power flowing through the transformer can therefore take any value in  $[(-10)+1+(-6), (-10)+1+6] = [-15, -3]$  MW, whose lower end falls outside the transformer secure operating range  $[-10, 10]$  MW.

In the second step, the DSO tightens the flexible extensions until network security is ensured. The DSO minimizes the total tightening of the flexible extensions required to restore network security. This objective determines the overall amount of flexibility that must be withdrawn, but it does not identify the users that should bear the corresponding reductions, since several allocations may yield the same minimum value. This ambiguity is resolved through a regulatory priority ordering that specifies how the flexible extension adjustment is distributed among users. In this paper, we adopt a last-in-first-out (LIFO) rule, under which users are ranked according to their connection date. The most recently connected users have their envelopes modulated first, while earlier users are modulated only once the flexibility of all more recent users has been exhausted. In the illustrative example, the battery  $u_3$  is assumed to be the most recently connected user and is therefore modulated before the wind farm  $u_1$ . The latter is tightened only once the battery's downward flexible extension has been fully exhausted.

Accordingly, the lower flexible bound of the battery  $\delta_{u_3,t+1}^{\min}$  is first reduced from its maximum value of 4 to 0 MW, removing 4 MW of injection and yielding the OE  $[-2, 6]$  MW. This alone is insufficient, since the possible exchange power range at the transformer is  $[(-10)+1+(-2), (-10)+1+6] = [-11, -3]$  MW. This range still has a lower end which falls outside the transformer secure operating range. The wind farm is therefore modulated next. Its lower flexible bound  $\delta_{u_1,t+1}^{\min}$  is reduced from 6 to 5 MW, yielding an OE of user  $u_1$  of  $[-9, 0]$  MW. As the forecast production of 10 MW exceeds the lower bound of this OE,  $u_1$  is curtailed to 9 MW at period  $t+1$ . The OE of  $u_2$  remains at its firm value  $[0, 3]$  MW.

Once the flexible extensions are modulated, the net power through the transformer is guaranteed to remain within  $[(-9)+1+(-2), (-9)+1+6] = [-10, -2]$  MW, which is compatible with the secure operating interval of the transformer  $[-10, 10]$  MW. The resulting envelopes are summarized in Table III. Note that the upward flexible bounds are left at their maximum values. Since only the injection side is reaching the secure operating range bounds, tightening them would bring no security benefit while needlessly shrinking the users' operating range.

|       | $\delta_{u,t+1}^{\min}$ | $\delta_{u,t+1}^{\max}$ | $\Delta_{u,t+1}$ | $P_{u,t+1}$ |
|-------|-------------------------|-------------------------|------------------|-------------|
| $u_1$ | 5                       | 0                       | $[-9, 0]$        | -9          |
| $u_2$ | 0                       | 0                       | $[0, 3]$         | 1           |
| $u_3$ | 0                       | 4                       | $[-2, 6]$        | 6           |

TABLE III: Flexible extensions  $\delta_{u,t+1}^{\min}$ ,  $\delta_{u,t+1}^{\max}$ , resulting operating envelopes  $\Delta_{u,t+1}$ , and power consumption/injection  $P_{u,t+1}$  of each user at period  $t+1$  (MW).

### C. Market-based envelope reallocation

The flexible extension adjustment of Section II-B does not guarantee that the resulting envelopes fulfill all the users' needs. Some users may not exploit the range granted to them, while others may wish to operate beyond their bounds. To address this, we propose a secondary market on which users trade envelope bounds against financial compensation. The market is organized as a pool, cleared once per period. After the DSO has notified the envelopes computed for the upcoming period  $t+1$ , users submit their orders to the market operator. At gate closure, the operator clears the whole order book in a single auction, maximizing social welfare while preserving network security, and notifies the resulting envelopes before the period begins. This mechanism is illustrated hereafter and formalized in Section IV.

#### Illustrative example

We revisit the running example to illustrate the market mechanism. After the envelope adjustment, the downward flexible extension of the battery  $u_3$  has been reduced to zero. Its injection capability is thus limited to the negative part of the firm envelope, which remains guaranteed. The wind farm  $u_1$  has retained part of its downward extension and is therefore limited to 9 rather than 10 MW. At period  $t+1$ , the battery does not expect to use its injection capacity. It is therefore willing to raise its lower bound by up to 2 MW, provided it receives at least 20 €/MW. The wind farm, in contrast, wants to lower its own bound by 1 MW so as to inject its full forecast production, and is willing to pay up to 50 €/MW for it. The resulting order book is given in Table IV.

| User  | Type | Side | Quantity (MW) | Price (€/MW) |
|-------|------|------|---------------|--------------|
| $u_1$ | buy  | min  | 1             | 50           |
| $u_3$ | sell | min  | 2             | 20           |

TABLE IV: Order book submitted to the market at period  $t+1$ .

The two orders are matched, since the price offered by the wind farm exceeds the price required by the battery. The buy order limits the cleared quantity to 1 MW, so the order of  $u_3$  is only partially accepted and sets the clearing price at 20 €/MW. The lower bound of  $u_1$  is then lowered to -10 MW and that of  $u_3$  raised to -1 MW, against a payment of 20 € from  $u_1$  to  $u_3$ .

The megawatt gained by  $u_1$  is exactly the one released by  $u_3$ , so the sum of the lower bounds is unchanged and the largest injection the transformer may see remains -10 MW, which sits within its secure operating range. The range released by  $u_3$  belongs to its firm envelope, its downward extension having already been reduced to zero. The firm envelope is thus guaranteed but not inalienable. A user may give it up voluntarily, for a single period and against compensation. Conversely, the range acquired by  $u_1$  brings its envelope back to the widest one its connection contract allows. Table V summarizes the envelopes before and after clearing and the operational scheme is illustrated in Figure 2.

|       | $[P'_{u,t}, \min, P'_{u,t}, \max]$ | $\delta'_{u,t+1}, \min$ | $\delta'_{u,t+1}, \max$ | $\Delta'_{u,t+1}$ | $P'_{u,t+1}$ |
|-------|------------------------------------|-------------------------|-------------------------|-------------------|--------------|
| $u_1$ | $[-4, 0]$                          | 6                       | 0                       | $[-10, 0]$        | -10          |
| $u_2$ | $[0, 3]$                           | 0                       | 0                       | $[0, 3]$          | 1            |
| $u_3$ | $[-1, 2]$                          | 0                       | 4                       | $[-1, 6]$         | 6            |

TABLE V: Updated firm envelopes, flexible extensions, operating envelopes, and operating points at  $t + 1$  after market clearing (MW).

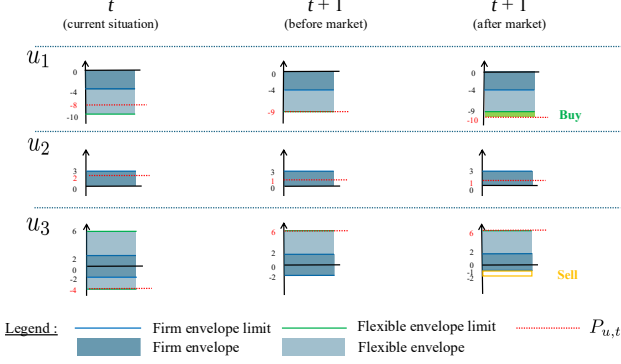


Fig. 2: Evolution of the operating envelopes of the three users across the operational scheme: the initial allocation at period  $t$ , the LIFO-based flexible extension adjustment for period  $t + 1$ , and the result after market clearing.

### III. COMPUTATION OF FLEXIBLE ENVELOPE EXTENSIONS

This adjustment leads to OEs carrying the conditional security guarantee of Section II-A, i.e., every asset remains within its secure operating range provided predictable users operate at their predicted power consumption/injection, and this for every power consumption/injection the unpredictable users may adopt within their OEs. Consistent with the running example, we restrict the formulation to active power. Moreover, the security constraints considered are the thermal capacities of the network edges, each edge corresponding to a physical asset: the substation transformer, or a line or cable. In Section III-A, the model of the radial network and the resulting security conditions are presented. Section III-B formally states the flexible extension adjustment problem, and Section III-C derives a tractable counterpart and applies it to the running example.

#### A. Radial network and security envelopes

The distribution network is modeled as a directed tree  $\mathcal{G} = (\mathcal{N}, \mathcal{E})$  rooted at the substation node  $n_0$ , where it connects to the upstream grid. Edges are directed away from  $n_0$ . Each user  $u \in \mathcal{U}$  is attached to a unique node  $n_u \in \mathcal{N}$ . For each node  $n$  and period  $t$ ,  $P_{n,t} \in \mathbb{R}$  denotes the net active power exchanged with the grid, following the sign convention of Section II, i.e., positive for withdrawal, and negative for injection. Nodes without a user have  $P_{n,t} = 0$ . The active power flowing from node  $m$  to node  $n$  along edge  $(m, n) \in \mathcal{E}$  is denoted  $P_{m,n,t} \in \mathbb{R}$ .

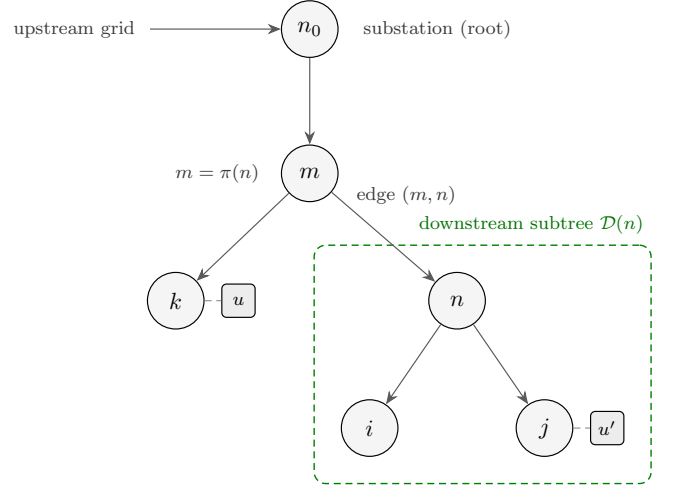


Fig. 3: Radial distribution network as a directed tree rooted at the substation node  $n_0$ . Edges point away from the root, giving each node  $n \neq n_0$  a unique parent  $\pi(n)$ . The dashed region is the downstream subtree  $\mathcal{D}(n)$ , and users (e.g.  $u, u'$ ) are attached to nodes.

The network is assumed lossless. Active power is then conserved at every node, so the power balance at node  $n \neq n_0$  reads

$$P_{\pi(n),n,t} = P_{n,t} + \sum_{k:(n,k) \in \mathcal{E}} P_{n,k,t}, \quad (4)$$

where  $\pi(n)$  is the parent of node  $n$  in the tree. Let us further define  $\mathcal{U}(n) = \{u \in \mathcal{U} : n_u \in \mathcal{D}(n)\}$  the set of users connected to the downstream subtree  $\mathcal{D}(n)$  of node  $n$ . The generic distribution network is illustrated in Figure 3.

Each edge  $(m, n) \in \mathcal{E}$  is secure whenever the flow does not exceed its thermal rating  $\bar{P}_{m,n} \in \mathbb{R}^+$ , assumed constant over time. Each edge is therefore associated to a security envelope, mathematically written as  $\Sigma_{m,n}$ , which defines the secure operating range of edge  $(m, n)$ :

$$P_{m,n,t} \in \Sigma_{m,n} \triangleq [-\bar{P}_{m,n}, \bar{P}_{m,n}], \quad \forall (m, n) \in \mathcal{E}, \quad (5)$$

and the network is secure at period  $t$  if (5) holds on every edge.

#### B. A generic flexible extension adjustment problem

The flexible extension adjustment problem, first illustrated in Section II-B, can be formalized as an optimization problem. At period  $t - 1$ , the DSO computes the extensions  $\{\delta_{u,t}^{\min}, \delta_{u,t}^{\max}\}_{u \in \mathcal{U}}$  for the next time period, so that all edges operate within their security envelopes.

##### Predictable and unpredictable users

Let  $\mathcal{U}_p \subseteq \mathcal{U}$  and  $\mathcal{U}_u \subseteq \mathcal{U}$  denote the sets of predictable and unpredictable users introduced in Section II-B, with  $\mathcal{U}_p \cup \mathcal{U}_u = \mathcal{U}$  and  $\mathcal{U}_p \cap \mathcal{U}_u = \emptyset$ . For each predictable user  $u \in \mathcal{U}_p$ , the DSO has access to a prediction function which maps any candidate OE to the power consumption/injection the user is predicted

to adopt within it. The candidate OE being parameterized by the flexible extensions, this function is defined as

$$P_{u,t} = \text{Pred}(u, \delta_{u,t}^{\min}, \delta_{u,t}^{\max}, t) \in \Delta_{u,t}, \quad (6)$$

where the only requirement is that the predicted power consumption lies within the envelope considered. The function is otherwise generic. The different possible instantiations of this function are discussed in Section V. No prediction function exists for an unpredictable user  $u \in \mathcal{U}_u$ , whose power is taken over the whole of its OE  $\Delta_{u,t}$ , itself a function of the extensions being optimized.

#### Regulatory requirements

Several combinations of extensions may satisfy security requirements. Security alone therefore does not determine the adjustment, and the choice among the admissible combinations is a regulatory matter. It decides which users bear the reduction when the network cannot accommodate every extension at its contractual maximum. A user  $u$  is said to be reduced on side  $s \in \{\min, \max\}$  whenever  $\delta_{u,t}^s < \bar{\delta}_u^s$ , and exhausted on that side whenever  $\delta_{u,t}^s = 0$ . A rule then specifies the order in which users are reduced, or the proportions in which the reduction is shared among them.

To express this requirement as a constraint of the flexible extension adjustment problem, we define the violation indicator

$$\text{Reg}(\delta_t) = \begin{cases} 1, & \text{if } \delta_t \text{ violates the regulatory rule,} \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

with  $\delta = (\delta_{u,t}^{\min}, \delta_{u,t}^{\max})_{u \in \mathcal{U}}$ . An adjustment complies with the regulatory framework if and only if  $\text{Reg}(\delta_t) = 0$ . This condition is considered within the adjustment problem (8).

#### Optimization problem

Let us denote by  $\rho_{u,t}^s = \bar{\delta}_u^s - \delta_{u,t}^s$  the reduction imposed on user  $u$  on side  $s$ . At period  $t - 1$ , the DSO solves the following optimization problem, whose solution gives the flexible extensions of period  $t$ , subject to the modifications

the market may subsequently bring:

$$\mathcal{P}_1 \equiv \underset{\delta_{u,t}^{\min}, \delta_{u,t}^{\max}}{\text{minimize}} \quad \sum_{u \in \mathcal{U}} (\rho_{u,t}^{\min} + \rho_{u,t}^{\max}) \quad (8a)$$

$$\text{s.t. (a)} \quad 0 \leq \delta_{u,t}^{\min} \leq \bar{\delta}_u^{\min}, \quad 0 \leq \delta_{u,t}^{\max} \leq \bar{\delta}_u^{\max}, \quad \forall u \in \mathcal{U}, \quad (8b)$$

$$(b) \text{ For each } (P_{u,t})_{u \in \mathcal{U}_u} \in \prod_{u \in \mathcal{U}_u} \Delta_{u,t},$$

and for every  $(m, n) \in \mathcal{E}$ , we have

$$P_{m,n,t} \in \Sigma_{m,n}$$

with the different  $P_{m,n,t}$

being the solution of the set of equations:

$$P_{m,n,t} = P_{n,t} + \sum_{k: (n,k) \in \mathcal{E}} P_{n,k,t}, \quad \forall (m, n) \in \mathcal{E}, \quad (8c)$$

$$P_{u,t} = \text{Pred}(u, \delta_{u,t}^{\min}, \delta_{u,t}^{\max}, t), \quad \forall u \in \mathcal{U}_p, \quad (8d)$$

$$(c) \text{ Reg}(\delta_t) = 0. \quad (8e)$$

The objective (8a) grants users as much flexible operating range as possible. Condition (a), given by (8b), keeps each extension within its contractual bounds, so that the OE remains within the CE. Condition (b) enforces network security for every power consumption/injection the unpredictable users may adopt within their OEs. Each predictable user produces or consumes at the value returned by the prediction function (8d). Condition (c), given by (8e), imposes the regulatory rule through the indicator (7).

#### C. A tractable flexible extension adjustment problem

Problem  $\mathcal{P}_1$  cannot be passed to a solver as it stands. Its condition (b) must hold for every power consumption/injection the unpredictable users may adopt within their OEs, which amounts to infinitely many constraints. Two properties of the network model of Section III-A reduce them to a finite number.

Firstly, summing (4) over the nodes of  $\mathcal{D}(n)$  makes the flows internal to that subtree cancel,  $(m, n)$  being the only edge connecting it to the rest of the network. Each flow is therefore the sum of the user powers it carries,

$$P_{m,n,t} = \sum_{u \in \mathcal{U}(n)} P_{u,t}, \quad \forall (m, n) \in \mathcal{E}, \quad (9)$$

and is affine in them.

Secondly, each unpredictable user may take any power consumption/injection within its own OE, independently of the others. Since (9) is a plain sum, the flow  $P_{m,n,t}$  is smallest when every unpredictable user of  $\mathcal{U}(n)$  sits at the lower bound of its OE, and largest when they all sit at the upper bound. Checking condition (b) at these two power consumption/injection profiles is thus enough to guarantee it over all of them, which yields the following problem, where conditions (b) and (c) are understood to hold for every edge  $(m, n) \in \mathcal{E}$ .

$$\mathcal{P}_2 \equiv \underset{\delta_t}{\text{minimize}} \sum_{u \in \mathcal{U}} (\rho_{u,t}^{\min} + \rho_{u,t}^{\max}) \quad (10a)$$

$$\text{s.t. (a) } 0 \leq \delta_{u,t}^s \leq \bar{\delta}_u^s, \quad \forall u \in \mathcal{U}, \forall s, \quad (10b)$$

$$\begin{aligned} (b) \quad P_{m,n,t}^{\min} &= \sum_{u \in \mathcal{U}_p(n)} P_{u,t} \\ &+ \sum_{u \in \mathcal{U}_u(n)} (P_u^{f,\min} - \delta_{u,t}^{\min}), \end{aligned} \quad (10c)$$

$$\begin{aligned} P_{m,n,t}^{\max} &= \sum_{u \in \mathcal{U}_p(n)} P_{u,t} \\ &+ \sum_{u \in \mathcal{U}_u(n)} (P_u^{f,\max} + \delta_{u,t}^{\max}), \end{aligned} \quad (10d)$$

$$(c) \quad P_{m,n,t}^{\min}, P_{m,n,t}^{\max} \in \Sigma_{m,n}, \quad (10e)$$

$$(d) \quad P_{u,t} = \text{Pred}(u, \delta_{u,t}^{\min}, \delta_{u,t}^{\max}, t), \quad \forall u \in \mathcal{U}_p, \quad (10f)$$

$$(e) \quad \text{Reg}(\delta_t) = 0, \quad (10g)$$

where  $\mathcal{U}_p(n) = \mathcal{U}_p \cap \mathcal{U}(n)$  and  $\mathcal{U}_u(n) = \mathcal{U}_u \cap \mathcal{U}(n)$  denote the predictable and unpredictable users downstream of node  $n$ .

Conditions (b) and (c) replace condition (b) of  $\mathcal{P}_1$ . Condition (b) defines, through (10c) and (10d), the two extreme flows of edge  $(m, n)$ , obtained when every unpredictable user downstream operates at the lower, respectively the upper, bound of its OE, while predictable users produce or consume at their predicted value. Condition (c) keeps both flows within the security envelope  $\Sigma_{m,n}$ , which by the argument above enforces (5) for every power consumption/injection the unpredictable users may adopt.

The objective (10a) and conditions (a) to (c) are affine in the decision variables, so the tractability of  $\mathcal{P}_2$  rests entirely on the two remaining conditions, which are left generic. Should the prediction function of condition (d) and the regulatory rule of condition (e) both admit a formulation that is linear in the extensions,  $\mathcal{P}_2$  is a linear program, and can be solved as such. *Back to the illustrative example*

The running example of Section II is a particular case of  $\mathcal{P}_2$ , obtained by fixing the three objects it leaves generic. The network first: all users share a single bus, so the tree reduces to the single edge  $(n_0, n_1)$  representing the substation transformer, of security envelope  $\Sigma_{n_0, n_1} = [-10, 10]$  MW, and conditions (b) and (c) are written for that edge only, with  $\mathcal{U}(n_1) = \mathcal{U}$ . The prediction function next:  $u_1$  and  $u_2$  are predictable, with forecasts  $\hat{P}_{u_1,t} = -10$  MW and  $\hat{P}_{u_2,t} = 1$  MW, and condition (d) is instantiated by (19), whereas  $u_3$  is unpredictable and is therefore covered over the whole of its OE.

Regarding the regulatory rule, as announced in Section II-B, we adopt a last-in-first-out (LIFO) rule, under which the flexibility of a more recently connected user is reduced before that of any earlier one. Users are indexed by connection order, with  $r(u) \in \{1, \dots, N\}$  the rank of user  $u$ . Practically, this means that  $r = 1$  is the rank of the earliest user and  $r = N$  that

of the most recently connected one. The LIFO rule requires that an earlier user be reduced only once every later user is exhausted, so that the regulatory condition (10g) instantiates as

$$\delta_{u_i,t}^s < \bar{\delta}_{u_i}^s \implies \delta_{u_j,t}^s = 0, \quad \forall u_i, u_j \in \mathcal{U} \text{ with } r(u_j) > r(u_i), \quad (11)$$

for both  $s \in \{\min, \max\}$ . Condition (11) cannot be passed to a solver as it stands. However, rank-dependent weights  $w_u = \beta^{N-r(u)}$ , with  $\beta > 1$ , can, under some conditions, reproduce the LIFO rule, as established in Lemma 1. Assuming  $u_1$ ,  $u_2$  and  $u_3$  to have been connected in this order, and substituting the values of Table I, yields

$$\begin{aligned} \mathcal{P}_3 \equiv \underset{\delta_{u,t}^{\min}, \delta_{u,t}^{\max}}{\text{minimize}} \quad & \beta^2 (\rho_{u_1,t}^{\min} + \rho_{u_1,t}^{\max}) + \beta (\rho_{u_2,t}^{\min} + \rho_{u_2,t}^{\max}) \\ & + (\rho_{u_3,t}^{\min} + \rho_{u_3,t}^{\max}) \end{aligned} \quad (12a)$$

$$\text{s.t. (a) } 0 \leq \delta_{u_1,t}^{\min} \leq 6, \quad \delta_{u_1,t}^{\max} = 0, \quad (12b)$$

$$\delta_{u_2,t}^{\min} = \delta_{u_2,t}^{\max} = 0, \quad (12c)$$

$$0 \leq \delta_{u_3,t}^{\min} \leq 4, \quad 0 \leq \delta_{u_3,t}^{\max} \leq 4, \quad (12d)$$

$$(b) \quad P_{n_0, n_1, t}^{\min} = P_{u_1, t} + P_{u_2, t} - 2 - \delta_{u_3, t}^{\min}, \quad (12e)$$

$$P_{n_0, n_1, t}^{\max} = P_{u_1, t} + P_{u_2, t} + 2 + \delta_{u_3, t}^{\max}, \quad (12f)$$

$$(c) \quad -10 \leq P_{n_0, n_1, t}^{\min}, \quad P_{n_0, n_1, t}^{\max} \leq 10, \quad (12g)$$

$$\begin{aligned} (d) \quad P_{u,t} &= \text{Pred}(u, \delta_{u,t}^{\min}, \delta_{u,t}^{\max}, t), \quad \forall u \in \{u_1, u_2\}, \\ \text{i.e., } P_{u_1,t} &= -4 - \delta_{u_1,t}^{\min}, \quad P_{u_2,t} = 1. \end{aligned} \quad (12h)$$

**Lemma 1** (LIFO through weighted reductions). *Suppose the transformer is the only asset which is associated to a security envelope, and that on each side  $s$  the corresponding security envelope bound depends on the extensions only through the aggregate  $\sum_{u \in \mathcal{U}} \delta_{u,t}^s$ . Then, for any  $\beta > 1$ , every solution of  $\mathcal{P}_3$  satisfies (10g) and is optimal for the objective (10a) of  $\mathcal{P}_2$ .*

*Proof.* The security bound on a side  $s$  depends on the extensions only through  $\sum_u \delta_{u,t}^s$ , hence only through  $\sum_u \rho_{u,t}^s$ , so any redistribution of the reductions leaving this aggregate unchanged is feasible.

Suppose a solution of  $\mathcal{P}_3$  violates (10g), i.e. there exist  $u_i, u_j$  with  $r(u_j) > r(u_i)$ ,  $\rho_{u_i,t}^s > 0$  and  $\rho_{u_j,t}^s \bar{\delta}_{u_j}^s$ . Let  $\epsilon = \min\{\rho_{u_i,t}^s, \bar{\delta}_{u_j}^s - \rho_{u_j,t}^s\} > 0$  and transfer  $\epsilon$  of reduction from  $u_i$  to  $u_j$ . The aggregate is unchanged, so the new point is feasible, and both reductions remain within their bounds. The objective (12a) varies by  $\epsilon(w_{u_j} - w_{u_i})0$ , since  $r(u_j) > r(u_i)$  gives  $w_{u_j} < w_{u_i}$ , contradicting optimality. No such pair therefore exists.

Finally, the weights are positive, so (12a) increases with every  $\rho_{u,t}^s$ . Its minimiser makes each aggregate  $\sum_u \rho_{u,t}^s$  as small as the security bound permits, hence each  $\sum_u \delta_{u,t}^s$  as large as possible, which is (10a).  $\square$

The example satisfies the hypotheses of Lemma 1. Indeed, the single bus leaves the transformer as the only asset, and the binding security condition (12g), once (12e) and (12h) are substituted into it, reads  $\delta_{u_1,t}^{\min} + \delta_{u_3,t}^{\min} \leq 5$ , which depends on the downward extensions only through their sum. Problem  $\mathcal{P}_3$  may therefore be solved in place of  $\mathcal{P}_1$ . As  $u_3$  is the most recently connected user, it carries the smallest weight, so the optimum first drives  $\delta_{u_3,t}^{\min}$  from its bound of 4 down to 0, then sets  $\delta_{u_1,t}^{\min} = 5$ . This recovers the operating envelopes  $\Delta_{u_1,t} = [-9, 0]$ ,  $\Delta_{u_2,t} = [0, 3]$  and  $\Delta_{u_3,t} = [-2, 6]$  of Table III.

#### IV. FORMALIZATION OF THE MARKET-BASED REALLOCATION MECHANISM

The flexible extension adjustment problem of Section III guarantees that the OEs allocated by the DSO are compatible with the security envelopes of the network, but it does not ensure that these envelopes match the users' needs. Some users may not require the full range allocated to them, while others would value an extension of their OE. To address this, we formalize a market-based reallocation mechanism, first introduced in Section II-C. Organized after the DSO has solved the flexible extension adjustment problem, this market reallocates OEs' bounds between users while preserving security.

After the market clearing, the OE of user  $u$ , originally denoted  $\Delta_{u,t} = [\underline{P}_{u,t}, \bar{P}_{u,t}]$ , evolves into

$$\Delta'_{u,t} = [\underline{P}'_{u,t}, \bar{P}'_{u,t}]. \quad (13)$$

Note that what are exchanged on the market are downward and upward parts of OEs, irrespective of the decomposition into a firm and a flexible part. A user may therefore release range that belongs to its firm envelope, and not only the flexible extension granted for the period. The firm envelope is therefore guaranteed but not inalienable. It may be given up voluntarily, for a single period and against compensation.

##### A. Order format

An order submitted by a user on this market is defined as the tuple

$$o = (id, u, \text{type}, s, \bar{q}, \pi), \quad (14)$$

where  $id \in \mathbb{N}$  is a unique identifier,  $u \in \mathcal{U}$  is the user submitting the order,  $\text{type} \in \{\text{buy}, \text{sell}\}$  indicates whether the user wants to acquire or release operating range,  $s \in \{\text{min}, \text{max}\}$  specifies whether the lower or the upper bound is concerned,  $\bar{q} \in \mathbb{R}_+$  is the maximum quantity offered, and  $\pi \in \mathbb{R}_+$  is the limit price.

A buy order on side min means that the user wants to decrease its lower bound, thereby obtaining more room to inject. A sell order on side min means that the user accepts to increase its lower bound, thereby releasing part of this room. Conversely, a buy order on side max increases the upper bound, whereas a sell order on side max decreases it.

Let  $\mathcal{O}$  denote the set of submitted orders. Clearing an order consists in accepting all or part of the offered quantity. Therefore, we associate with each order  $o$  a *cleared quantity*

$q_o \in [0, \bar{q}_o]$ , the decision variable of the market-clearing problem. To lighten the notation, we introduce the sign

$$\sigma_o = \begin{cases} +1, & \text{type} = \text{buy}, \\ -1, & \text{type} = \text{sell}, \end{cases} \quad (15)$$

so that a cleared order widens the envelope when  $\sigma_o = +1$  and tightens it when  $\sigma_o = -1$ . The post-market bounds of user  $u$  then read

$$\underline{P}'_{u,t} = \underline{P}_{u,t} - \sum_{\substack{o \in \mathcal{O}: \\ u_o = u, s_o = \text{min}}} \sigma_o q_o, \quad (16)$$

$$\bar{P}'_{u,t} = \bar{P}_{u,t} + \sum_{\substack{o \in \mathcal{O}: \\ u_o = u, s_o = \text{max}}} \sigma_o q_o. \quad (17)$$

##### B. Market-clearing problem

Network security after the market clearing is enforced exactly as in Section III-B, with the post-market OEs in place of the adjusted ones, so that the post-market OEs carry the same conditional security guarantee. Predictable users are represented by their predicted power consumption/injection, obtained by evaluating the prediction function on the post-market OE, i.e., (19) with the post-market bounds  $\underline{P}'_{u,t}$  and  $\bar{P}'_{u,t}$  in place of the adjusted ones. No prediction is relied upon for unpredictable users, whose power is taken over the whole of the post-market OE  $\Delta'_{u,t}$ .

The market operator clears the orders by maximizing social welfare:

$$\mathcal{P}_4 \equiv \underset{\{q_o\}_{o \in \mathcal{O}}}{\text{maximize}} \quad \sum_{o \in \mathcal{O}} \sigma_o \pi_o q_o \quad (18a)$$

$$\text{s.t. (a) } 0 \leq q_o \leq \bar{q}_o, \quad \forall o \in \mathcal{O}, \quad (18b)$$

$$(b) \quad \sum_{o: s_o = s} \sigma_o q_o = 0, \quad \forall s \in \{\text{min}, \text{max}\}, \quad (18c)$$

$$(c) \quad \Delta'_{u,t} \subseteq \bar{\Delta}_u, \quad \underline{P}'_{u,t} \leq \bar{P}'_{u,t}, \quad \forall u \in \mathcal{U}, \quad (18d)$$

$$(d) \text{ For each } (P'_{u,t})_{u \in \mathcal{U}_u} \in \prod_{u \in \mathcal{U}_u} \Delta'_{u,t},$$

and for every  $(m, n) \in \mathcal{E}$ , we have

$$P'_{m,n,t} \in \Sigma_{m,n}$$

with the different  $P'_{m,n,t}$

being the solution of the set of equations:

$$P'_{m,n,t} = P'_{n,t} + \sum_{k: (n,k) \in \mathcal{E}} P'_{n,k,t}, \quad (18e)$$

$$\forall (m, n) \in \mathcal{E},$$

$$P'_{u,t} = \text{Pred}(u, \delta_{u,t}^{\min}, \delta_{u,t}^{\max}, t), \quad (18f)$$

$$\forall u \in \mathcal{U}_p.$$

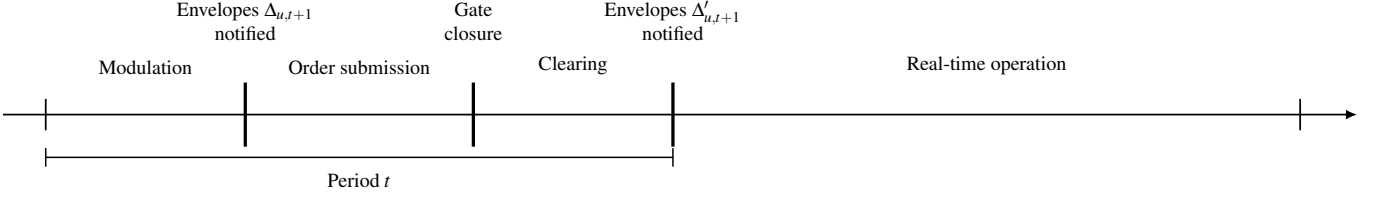


Fig. 4: Timeline of the envelope-based operational scheme. During period  $t$ , the DSO adjusts the flexible extensions and notifies the resulting OEs  $\Delta_{u,t+1}$ , after which users submit their orders until gate closure. The market operator then clears the order book and notifies the post-market OEs  $\Delta'_{u,t+1}$  before period  $t+1$  begins, over which each user operates within its post-market OE.

The objective (18a) maximizes social welfare. Condition (a), given by (18b), limits each cleared quantity to the quantity offered. Condition (b), given by (18c), states that, on each side, the signed cleared quantities sum to zero, so that the market reallocates already available operating range. Condition (c), given by (18d), keeps each post-market OE within the CE of the user and non-empty. Condition (d) enforces network security for every power consumption/injection the unpredictable users may adopt within their post-market OEs. It is condition (b) of  $\mathcal{P}_1$ , imposed on the post-market OEs: the flows solve the nodal balance equations (18e), in which each predictable user contributes the power consumption/injection returned by the prediction function (18f) evaluated on its post-market OE, and are required to lie within the security envelopes of the edges. The same vertex argument as in Section III-C applies, so that  $\mathcal{P}_4$  admits a tractable counterpart obtained by writing condition (d) at the bounds of the post-market OEs only.

### C. Operational scheme timeline

The market is organized as a double-sided pool, cleared once per period by a market operator. All orders are collected over a submission window and cleared simultaneously in a single auction. The market for period  $t+1$  is operated during period  $t$ , along the following timeline:

- 1) the DSO solves the flexible extension adjustment problem (8) and notifies each user of its OE  $\Delta_{u,t+1}$ ;
- 2) the submission window opens, during which users send their orders to the market operator;
- 3) at gate closure, the order book  $\mathcal{O}$  is fixed and the market operator clears it by solving (18);
- 4) the resulting post-market OEs  $\Delta'_{u,t+1}$  are notified to the users before period  $t+1$  begins.

This timeline is depicted in Figure 4. Each user then operates within its post-market OE over the whole of period  $t+1$ , and the procedure is repeated for the following period. The envelopes traded are thus valid for a single period, after which the firm envelope is restored and the flexible extensions are recomputed by the DSO. The market is settled pay-as-clear, with a single clearing price per side. Every cleared order on a side is therefore priced identically, and the payments of the buyers exactly cover the remuneration of the sellers.

*Back to the illustrative example*

The order book of Table IV contains a buy order from  $u_1$ , willing to pay up to 50 €/MW to lower its bound by at most 1 MW, and a sell order from  $u_3$ , willing to release up to 2 MW of its lower bound for at least 20 €/MW. The buyer's price exceeds the seller's, so the trade increases the social welfare (18a) and the largest feasible quantity is cleared. The buy order limits it to 1 MW, so that the sell order of  $u_3$  is only partially cleared, for a social welfare of  $(50 - 20) \times 1 = 30$  €. Being the marginal order, the offer of  $u_3$  sets the clearing price at 20 €/MW, so that  $u_1$  pays 20 € to  $u_3$  for the megawatt of downward range acquired and the whole welfare accrues to the buyer. After clearing, the lower bound of  $u_1$  is extended to -10 MW and that of  $u_3$  is tightened to -1 MW. The megawatt gained by  $u_1$  is exactly the one released by  $u_3$ , so the largest potential injection through the transformer is unchanged and network security is preserved. The resulting envelopes are those of Table V, i.e., the wind farm can now inject its full forecast production of 10 MW, while the battery, which did not intend to use its injection range, is compensated for releasing it.

## V. DISCUSSIONS

Different aspects of the operational scheme deserve to be further detailed. The regulatory framework under which the scheme would be deployed and the representation of predictable users are two of them. They enter the formalization of Sections III and IV through the function  $\text{Reg}(\delta_t)$  and the prediction function  $\text{Pred}(\cdot)$ , respectively. Both functions are left generic, so that their practical instantiation remains to be settled. The network model, for its part, has been chosen for its tractability rather than for its realism.

Moreover, a commercial actor may operate assets at several locations of the grid, in which case it is not represented as a single user of the scheme but as a collection of users, one per connection point, each holding its own OE.

To discuss these aspects, this section is organized as follows. Section V-A confronts the scheme with the regulatory framework, at the European level and in its regional transposition. The Walloon region, in Belgium, is taken as a candidate for deployment, and the extent to which its regulation accommodates the scheme is examined. Section V-B discusses the prediction function and the binary classification of users it rests on. Section V-C examines how commercial actors operating assets at several network locations are represented and how they may

participate in the market. Finally, Section V-D reviews the limitations of the simplified network model.

#### A. Regulatory framework

The practical implementation of the scheme presupposes a regulation that (i) recognizes the envelope as a contractual object, (ii) entitles the DSO to adjust the flexible part of the envelope without owing compensation to the user, and (iii) allows users to exchange envelope bounds among themselves. These conditions are only partially met today.

The envelope proposed in this paper can be considered as an operational implementation of the flexible connection agreements introduced in European law. Directive (EU) 2019/944, as amended by Directive (EU) 2024/1711 [18, 19], requires a framework under which system operators offer flexible connection agreements in areas where network capacity is limited. Article 6a provides that such agreements may specify maximum firm injection and withdrawal capacities, together with additional flexible capacities. This distinction maps directly onto the two components of the contractual envelope of Section II, namely the firm envelope  $[P_u^{f,\min}, P_u^{f,\max}]$  and the maximum flexible extensions  $(\delta_u^{\min}, \delta_u^{\max})$ .

In Wallonia, taken here as a candidate region for deployment, the decree of 19 December 2025 transposes Article 6a and extends to withdrawal a flexibility regime that previously concerned injection only [20–23]. Connection contracts concluded in congested areas must state the firm capacity together with the additional flexible capacity, and the user must install a certified power control system. Walloon law thus already provides a firm-and-flexible contractual object closely matching the envelope of Section II, and places compliance on the user, as the proposed scheme requires. Two further features are favorable. No compensation is due for an adjustment remaining within the agreed flexible capacity, and the Government is empowered, on a proposal of the CWaPE, to lay down the criteria governing the order of priority for power consumption/injection limitations. Whether LIFO, as proposed in Section III, is the appropriate criterion is another matter. Ranking users by connection date differentiates otherwise comparable users, and must be reconciled with the prohibition of discrimination laid down in Article 31(2) of Directive (EU) 2019/944.

Three gaps nevertheless remain. First, Walloon law empowers the operator to limit injection or withdrawal within the contracted flexible capacity, but does not require communicating the adjustment ahead of operation. The technical regulation would have to define the operating envelope as a range notified before each period, which is what restores the predictability that curtailment lacks. Second, the market of Section IV has no legal basis, as grid power access capacity remains attached to a connection point and cannot be transferred. Their tradability should be recognized and a market operator designated. Third, the flexibility requirements apply only to new connections above a power threshold in congested areas, whereas the proposed scheme assigns an envelope to every user of the network.

#### B. The notion of predictability

The formalization of Section III rests on two modeling choices: (i) a binary classification of users into predictable and unpredictable ones, and, for the former, (ii) a prediction function  $\text{Pred}(u, \delta_{u,t}^{\min}, \delta_{u,t}^{\max}, t)$ . Any function that returns, for each user and each pair of flexible extensions, a power consumption/injection value lying within the resulting OE is a priori valid. Both choices deserve discussion.

Different instantiations of the prediction function may be proposed. The one used in the illustrative example is made explicit hereafter. Each predictable user announces, ahead of each period, the power  $\hat{P}_{u,t}$  it intends to exchange with the network, hereafter called the forecast. The predicted power consumption/injection is the forecast whenever the OE accommodates it, and the bound of  $\Delta_{u,t}$  closest to the forecast otherwise:

$$\text{Pred}(u, \delta_{u,t}^{\min}, \delta_{u,t}^{\max}, t) = \begin{cases} \max\{\hat{P}_{u,t}, P_u^{f,\min} - \delta_{u,t}^{\min}\}, & \hat{P}_{u,t} < 0, \\ \min\{\hat{P}_{u,t}, P_u^{f,\max} + \delta_{u,t}^{\max}\}, & \hat{P}_{u,t} \geq 0. \end{cases} \quad (19)$$

This instantiation places the forecasting burden on the user, who knows its own installation best, and keeps the adjustment problem (8) tractable, the clipping being affine in the extensions once the saturated bound is known. Alternatively, the DSO may produce the forecast itself, from historical measurements or exogenous data such as weather predictions. The function may also encode a behavioral model of the user, for instance the anticipated response of a price-driven asset to a market signal. The instantiation also determines who bears the consequences of a prediction error. Under (19), the conditional security guarantee presupposes that the user honors its announcement, so the connection contract must make the announcement binding. Under a DSO-produced forecast, by contrast, a deviation within the envelope cannot be held against the user, and the operator must cover the residual uncertainty by other means.

The binary classification itself may prove too restrictive. It amounts to assuming that the DSO either holds perfect information on the future behavior of a user, who is then called predictable, or no information at all, in which case the user is unpredictable. Many real cases sit between these two extremes. An aggregated load may be predictable fifteen minutes ahead but not two hours ahead, a battery may be predictable only when its operation is governed by a contractual agreement, and renewable generation typically comes with a quantifiable forecast error. In all these cases, the information available is partial but not absent. The information may be encoded in different forms, such as a probability distribution over the future power consumption/injection of the user.

Two generalizations bridge the gap between the categories. First, an uncertainty set might be defined around the predicted power consumption/injection, and network security is guaranteed for every value within it. The two categories of this paper are then the extreme cases, the set degenerating to a single value for a predictable user and to the whole OE for an unpredictable one. Secondly, a small risk of security

envelope violation is accepted, and the OEs are determined based on probabilistic forecasts, as first proposed in [24]. The tractability of (8) and (18) depends, in all cases, on the instantiation chosen. The choice among these options is, moreover, partly regulatory, as a critical network constraint may require a strict guarantee, whereas economic operation may tolerate a probabilistic criterion combined with corrective measures.

### C. Multi-site portfolios and aggregated participation

The illustrative example runs on a single bus, but the formalization of Sections III and IV does not. Users may be attached to any node of the tree, and security is enforced on every edge. A commercial actor operating assets at several network locations is therefore represented as a set of users, one per connection point, each holding its own contractual envelope. The internal constraints and load shapes of these assets remain private. The actor dispatches its portfolio freely within the OEs granted, which is precisely the confidentiality property that motivates the envelope-based approach.

Commercial actors may already use the market of Section IV to reallocate range within its own portfolio, by submitting matching sell and buy orders for two of its sites. Whether the trade clears, however, depends on the network location of both. The clearing problem verifies the security of the post-market OEs on every edge, so range released at one site can be acquired at another only if the edges affected by the second site accommodate it.

Participation as a single market party, rather than as a collection of users, requires at least two extensions of the proposed secondary market. First, the order format of Section IV-A would have to accommodate orders spanning several sites. For instance, an order that clears only if the corresponding order at another site clears as well. Second, predictability could be assessed at the portfolio level. A portfolio may be predictable in aggregate while its individual sites are not, as aggregation smooths out individual variability. The aggregated residential load of the illustrative example already relies on this effect. The prediction function of Section III would then be defined on a set of users rather than on a single one. Both extensions are left for future work.

### D. Network model limitations

The network model of Section III-A assumes a balanced three-phase, lossless network with a radial topology, and enforces security through the thermal capacities of the edges only, the substation transformer included. Only active power is considered. While these assumptions preserve the tractability of the adjustment and market-clearing problems, they provide only a rough representation of real-world three-phase distribution networks, where phase unbalance is common.

A more accurate representation of the network might be explored. Voltage constraints should be considered, particularly in networks with significant photovoltaic generation, and cannot be captured by a per-edge power range. They couple

the users nonlinearly and would require security envelopes of a different nature. Reactive power and losses must be taken into account, the latter breaking the exact aggregation of user powers along the tree. The topology may also be meshed or looped, in which case flows no longer follow uniquely from the nodal balance (4).

These extensions affect the two stages of the scheme unevenly. The flexible extension adjustment problem generalizes naturally, at the price of a more complex, possibly non-convex, security description. The market-based reallocation, by contrast, relies on a property of the simplified model: since edge flows aggregate the downstream user powers one-for-one, a megawatt of OE released by one user exactly offsets a megawatt acquired by another user of the same subtree, which the balance constraint (18c) exploits. In a more detailed model, one megawatt injected at the end of a feeder does not have the same impact as one megawatt injected close to the transformer. Envelope trades then cease to be location-blind, and the clearing must either weight the exchanged quantities by nodal sensitivity factors or verify the network constraints explicitly on the post-market envelopes, as (18e) already does in the simplified setting.

## VI. CONCLUSIONS

This paper proposes a static security management scheme for distribution networks built on both contractual and operating envelopes, addressing the lack of grid power access users might face. Each contractual envelope combines a firm part, guaranteed over the long term, and a flexible part that the DSO adjusts according to network conditions. The scheme is organized in two successive stages. First, close-to-real-time, the DSO adjusts the flexible extensions to grant users an OE as wide as possible. We formalize this flexible extension adjustment as an optimization problem. We also propose a tractable algorithm for implementing it under the assumptions that the regulatory constraint consists solely in a LIFO rule. Resulting OEs ensure that every network asset remains within its secure operating range, provided each predictable user operates at its predicted power consumption/injection.

Because the resulting OEs may not satisfy the users' needs, a secondary market allows users to trade OE bounds against financial compensation, reallocating operating range to those who value it most while preserving security. A running example on a radial network illustrates both stages. The work could be extended along several lines. First, the formulation could be extended to complex power envelopes, so as to capture voltage-related constraints. Second, the scheme could be assessed on realistic, unbalanced three-phase networks. Third, the binary classification of users could be relaxed, by defining probabilistic forecasts and uncertainty sets around the predicted power consumption/injection. And, finally, the strategic behavior of users bidding into the OE's market can be studied.

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