



Assessment of the Smoothed Particle Hydrodynamics formulations available in LS-DYNA for multiple-stage bird ingestion applications[☆]

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ABSTRACT

Smoothed Particle Hydrodynamics (SPH) is the standard tool for simulating high-velocity bird ingestion in aeroengines, yet LS-DYNA offers fifteen SPH formulations whose relative suitability for multi-stage ingestion has never been systematically assessed. This work presents a comprehensive evaluation of all fifteen formulations, combining (i) a physically representative single-impact benchmark, (ii) a sensitivity analysis of the governing numerical parameters, (iii) a double-impact configuration representative of sequential soft-body loading, and (iv) a validation against experimental data. The single-impact benchmark eliminates five formulations — symmetric (2, 3), adaptive SPH (9, 10), and moving least squares SPH (12) — due to unrealistic deformation, non-physical energy behavior, or core instability, narrowing the field to six candidates: default (0, 1), fluid particle (5, 6), and enhanced fluid (15, 16). Under double-impact loading, only the enhanced fluid formulations, which incorporate density filtering, retain stable shock propagation, coherent deformation, and accurate momentum partitioning; the remaining candidates suffer from either tensile-driven particle blow-up or unphysical clustering. Experimental validation against gelatin bird-surrogate impact tests confirms the predictive accuracy of the enhanced fluid formulations, with the renormalized variant (16) achieving the lowest displacement error and remaining numerically stable over extended computational domains where the standard renormalized formulation becomes computationally intractable. Formulation 16 is identified as the most robust and accurate SPH option for industrial bird-ingestion simulations in LS-DYNA, with formulation 15 offering a lower-cost alternative when computational efficiency outweighs the need for maximal accuracy.

1. Introduction

Bird strike, defined as the collision between a bird and an aircraft, has remained one of the most persistent hazards in aviation since the earliest days of flight, with the first recorded incident documented by the Wright brothers in 1905 [1] and the first fatal accident occurring only a few years later in 1912 [2]. The frequency of incidents has continued to increase due to rising air traffic density, larger propulsion systems, and wildlife resurgence [3]. While numerous bird strike events result in only limited damage, ingestion events defined as cases in which a bird enters an aircraft engine can be catastrophic, as they may lead to severe loss of

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thrust or even engine failure. This risk has driven stringent certification procedures that require engines and critical structures to demonstrate tolerance to specified bird masses and impact conditions through full-scale experimental tests. Because such certification tests are expensive, time-consuming, and limited in the range of conditions they can practically cover, numerical simulation has become indispensable for predicting structural response under large deformation and high strain rate conditions.

Classical Lagrangian finite element formulations have been extensively applied to fan blade impact simulations in which the bird is discretized using a Lagrangian mesh [4–7], but this approach suffers from severe mesh distortions and loss of accuracy in the large-deformation regime [8,9]. Eulerian [10,11] and Arbitrary Lagrangian–Eulerian (ALE) formulations [12], in which the bird material is represented within a fixed or partially moving mesh, mitigate distortion but introduce numerical dissipation, dispersion, or mesh quality issues, and require enlarged computational domains to accommodate material flow [13]. Meshless methods, particularly SPH, avoid these limitations by representing the projectile as an ensemble of interacting particles capable of sustaining extremely large deformation [11,14–16].

Originally developed for astrophysical applications, SPH has since evolved into a mature computational framework widely used in fluid dynamics, solid mechanics, impact engineering, geomechanics, and fluid–structure interaction (FSI). Recent reviews by Gotoh et al. [17], Khayyer et al. [18], and Zhang et al. [19] highlight the growing adoption of SPH and related particle methods for free-surface flows, wave-structure interaction, structural dynamics, and strongly coupled FSI problems involving large deformations, moving interfaces, fragmentation, or complex contact conditions. Within this broader context, bird strike and bird ingestion simulations constitute a particularly challenging class of transient FSI problems characterized by extreme deformation, rapid momentum transfer, and strong pressure wave propagation.

Benchmark studies [20–23] have demonstrated that SPH provides a favorable balance of accuracy, robustness, and computational cost for soft-body impact. This advantage is reinforced by the soft, fluid-like behavior of bird surrogates under high-velocity loading, for which a hydrodynamic constitutive law is particularly appropriate, as it captures their characteristic pressure-dominated response. These considerations become even more critical in modern propulsion architectures where downstream components are exposed, increasing the likelihood of multi-stage ingestion events in which a single projectile sequentially interacts with multiple parts of the structure.

Despite its versatility, SPH is known to suffer from several numerical deficiencies that can significantly affect predictive capability. Among the most important is tensile instability, which manifests through particle clustering, artificial stiffness, or particle blow-up under tensile stresses [24]. Because these pathologies can severely compromise solution quality, numerous stabilization techniques have been proposed by the SPH community. These include artificial stress approaches, tensile instability control methods [25], particle shifting techniques (PST) [26], transport velocity formulations, and more recently non-hourglass formulations specifically designed to suppress tensile instability while preserving consistency and conservation properties [27]. Although such developments are not currently available in the software used in this work, they demonstrate that particle clustering and particle blow-up are longstanding SPH challenges for which a broad range of remedies has been actively investigated.

Several numerical platforms currently provide SPH capabilities for impact and FSI simulations. Open-source solvers such as DualSPHysics [28], SPHinXsys [29], and LAMMPS [30] have become widely adopted research tools due to their advanced stabilization techniques, adaptive resolution strategies, and dedicated FSI capabilities. Commercial explicit dynamics packages, including LS-DYNA [31], Abaqus/Explicit [32], and AUTODYN [33], also offer SPH implementations and are extensively used in industrial impact analyses. Among these, LS-DYNA has become one of the most widely employed tools for bird strike certification studies [12,34], owing to its mature contact algorithms, extensive material library, and industrial validation history.

Despite the widespread reliance on SPH for bird strike applications, prior research has mainly compared different numerical methodologies, for instance, SPH against ALE, with limited attention to how the specific SPH formulation influences predictive capability, even though formulation choice is known to affect accuracy and stability [9,35]. LS-DYNA incorporates fifteen distinct SPH formulations based on different particle approximations, consistency corrections, and momentum conservation strategies. Nevertheless, to the best of the authors' knowledge, no comprehensive study has evaluated their relative performance for high-velocity bird strike and ingestion scenarios. The technical differences between these formulations are presented in Table 1 and detailed in Section 3.

The present work aims to fill this gap by systematically comparing all SPH formulations available in LS-DYNA, clarifying their theoretical differences, assessing their behavior in a representative single impact, identifying governing parameters through sensitivity analysis, analyzing their robustness in a multi-stage configuration, and validating the most promising candidates against experimental data. Particular emphasis is placed on numerical stability, tensile-instability-related failure mechanisms, physical fidelity, and robustness under sequential impacts representative of modern bird ingestion scenarios. Although the comparisons are performed within the LS-DYNA framework, the investigated influence of SPH formulation choices addresses a broader numerical modeling question and may provide useful guidance for similar bird strike simulations performed with other SPH implementations.

The paper is organized as follows. Section 2 introduces the fundamental background required to describe soft-body impacts. Section 3 presents the theoretical foundations of the fifteen SPH formulations implemented in LS-DYNA, while Section 4 summarizes the common numerical framework adopted throughout the study. Section 5 follows a progressive assessment methodology in which the formulations are evaluated through increasingly demanding benchmarks, consisting of the single-impact benchmark (Section 5.1), the sensitivity analysis (Section 5.2), the double-impact configuration (Section 5.3), and the experimental validation (Section 5.4). This progression progressively narrows the candidate formulations before validating the most promising ones against experimental data. Finally, the main findings are summarized in Section 6.

Table 1

Summary of the SPH formulations available in LS-DYNA and evaluated in this work. The table compares the underlying momentum discretization, kernel evaluation strategy, normalization Eq. (38), and density filtering Eq. (39) adopted by each formulation. “Evaluated” identifies formulations that are implemented and suitable for evaluation within the present assessment, whereas “Retained” denotes formulations that successfully reproduce the fundamental physics of soft-body impact.

ID	Formulation	Momentum	Kernel evaluation	Normalization	Density filtering	Evaluated	Retained
0	Default	(30)	Gather	×	×	✓	✓
1	Default renorm.	(30)	Gather	✓	×	✓	✓
2	Symmetric	(28)	Symmetric	×	×	✓	×
3	Symmetric renorm.	(28)	Symmetric	✓	×	✓	×
4	Tensor	N/A	N/A	N/A	N/A	×	–
5	Fluid particle	(29)	Gather	×	×	✓	✓
6	Fluid particle renorm.	(29)	Gather	✓	×	✓	✓
7	Total Lagrangian	N/A	N/A	N/A	N/A	×	–
8	Total Lagrangian renorm.	N/A	N/A	N/A	N/A	×	–
9	ASPH	(30)	Anisotropic	×	×	✓	×
10	ASPH renorm.	(30)	Anisotropic	✓	×	✓	×
12	MLSPH	N/A	N/A	N/A	N/A	✓	×
13	Incompressible	N/A	N/A	N/A	N/A	×	–
15	Enhanced fluid	(29)	Gather	×	✓	✓	✓
16	Enhanced fluid renorm.	(29)	Gather	✓	✓	✓	✓

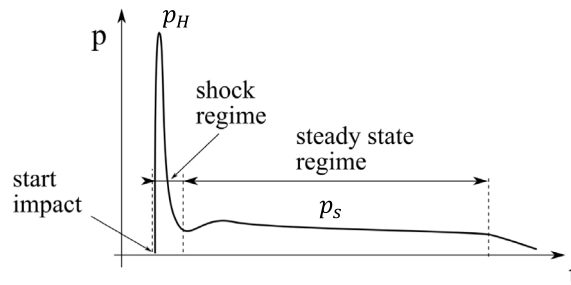


Fig. 1. Pressure profile for a soft-body impact showing the shock regime characterized by the Hugoniot pressure p_H followed by the steady flow regime characterized by the stagnation pressure p_s .

Source: Reproduced from Ref. [1].

2. Soft-body impact

The impact of a bird on engine blades is referred to as a soft-body impact, given the internal stresses generated during the collision that greatly exceed the mechanical strength of the projectile. Consequently, the bird undergoes significant deformation and flows upon contact, exhibiting a fluid-like behavior [36].

A central step in any bird strike analysis is the validation of a representative bird model through the predicted impact pressure. To support this step, a simplified but physically meaningful configuration is considered. A homogeneous cylinder impacts normally on a perfectly rigid flat plate at high velocity.

In this configuration, the projectile is treated as an incompressible inviscid fluid with no internal strength, which permits the use of hydrodynamic theory, i.e. impact response is governed solely by conservation of mass and momentum, with stresses arising from fluid motion rather than material strength. The response of the target is neglected by prescribing perfect rigidity, and both viscous effects and friction at the contact interface are disregarded.

The theory developed in this section follows the classical works of Wilbeck [36] and Barber et al. [37], as well as the more recent analysis of Allaeys [1].

Within this framework, the temporal evolution of the pressure is divided into two stages: the shock regime and the steady flow regime, presented in Sections 2.1 and 2.2, which together form the typical pressure response illustrated in Fig. 1.

2.1. Shock regime

The shock regime corresponds to the initial phase of the impact, during which the front of the projectile first interacts with the rigid target. This regime is brief and represents the transition from initial contact to the onset of steady flow.

The shock regime comprises four key stages illustrated in Fig. 2.

- (a) Before impact, the projectile approaches the plate with projectile velocity v_p .

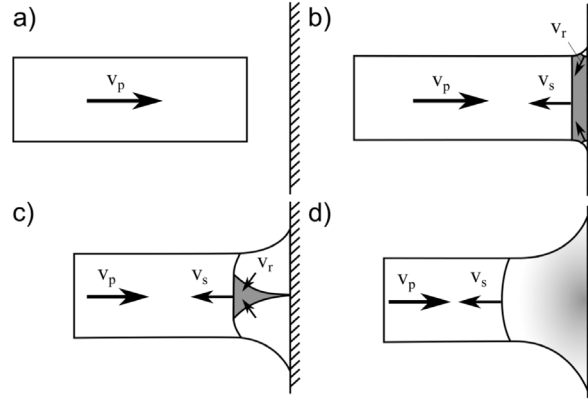


Fig. 2. Four characteristic stages of the shock regime during soft-body impact. The vectors v_p , v_s , and v_r denote the projectile velocity, the shock wave velocity, and the release wave velocity.

Source: Reproduced from Ref. [1].

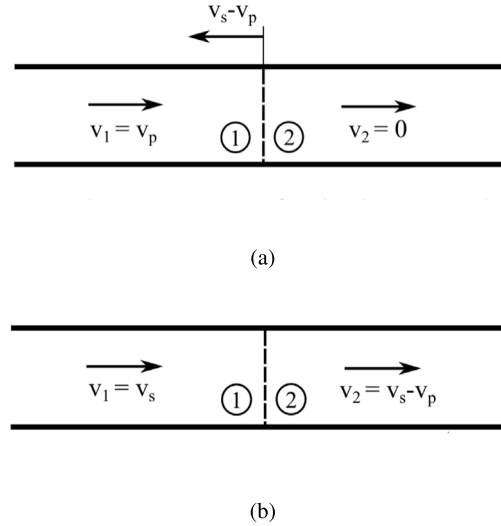


Fig. 3. One dimensional shock wave description for (a) a fixed shock configuration and (b) a standing shock configuration. The vectors v_1 and v_2 denote the velocities before and after impact.

Source: Reproduced from Ref. [1].

- (b) Upon contact, the leading particles are brought to rest relative to the target. This abrupt deceleration generates a compressive disturbance that propagates backward at velocity v_s as a shock wave. The edge particles, which are unconfined and subjected to high-pressure gradients, are accelerated outward and form a radial release wave that travels toward the cylinder axis at velocity v_r . Both waves travel significantly faster than the projectile velocity v_p . The higher velocity within the shocked region relative to the unshocked material permits the release wave to overtake the shock wave.
- (c) The arrival of the initial release wave at the cylinder axis marks the end of the shock pressure duration, since the target surface is no longer subjected to shock loading.
- (d) The release wave eventually catches the shock wave, eliminating the fully shocked region. From this point onward, a pressure wave propagates through the bird, and the impact process enters the steady flow regime during which the internal pressure decreases significantly.

Because the flow across the shock wave may be approximated as one dimensional, a 1D configuration is used to determine the shock velocity. As illustrated in Fig. 3, the shock may be described in either a stationary frame (a) or a frame moving with the shock (b). The standing shock configuration is particularly convenient, since it allows the conservation laws to be written in steady form:

$$\rho_1 v_s = \rho_2 (v_s - v_p), \quad (1)$$

$$p_1 + \rho_1 v_s^2 = p_2 + \rho_2 (v_s - v_p)^2, \quad (2)$$

forming the Rankine–Hugoniot jump conditions [36].

Assuming adiabatic and non-viscous flow, combination of (1) and (2) yields the pressure rise

$$p_H \triangleq p_2 - p_1 = \rho_1 v_s v_p, \quad (3)$$

commonly referred to as the Hugoniot pressure.

While v_p and ρ_1 may be specified in advance, the shock velocity v_s is generally unknown. For many solids and fluids, including water and air, the relation

$$v_s = c_0 + s v_p, \quad (4)$$

holds, where c_0 is the adiabatic sound speed and s a material constant [36]. This linear Hugoniot relation closes the system.

For a fluid with negligible shear modulus, the sound speed is given by

$$c_0 = \sqrt{\left. \frac{K}{\rho} \right|_{p=p_1}}, \quad (5)$$

where K is the bulk modulus. The release wave velocity corresponds to the sound speed evaluated in the shocked state,

$$v_r = \sqrt{\left. \frac{K}{\rho} \right|_{p=p_H}}. \quad (6)$$

For a linear equation of state in LS-DYNA, K is constant, and the increase in density under compression implies

$$\left. \frac{K}{\rho} \right|_{p=p_H} < \left. \frac{K}{\rho} \right|_{p=p_1}, \quad (7)$$

so that $v_r < c_0$.

Consequently, $v_r > c_0$ can only be achieved with a nonlinear equation of state, where the bulk modulus increases with compression, consistent with the physics of release wave propagation described above.

The shock duration is defined as the time required for the release wave, generated at the lateral boundary of the projectile, to reach the central axis. At this moment, the target no longer experiences shock-induced pressure. This characteristic time is

$$t_{\text{shock}} = \frac{r}{v_r}, \quad (8)$$

where r is the initial cylinder radius.

After multiple reflections within the material, the initial shock wave is dissipated and vanishes through conversion into heat.

2.2. Steady flow regime

For a cylindrical projectile with a length-to-diameter ratio greater than one, a steady flow regime develops after the shock regime [36].

Neglecting body forces and shearing forces allows the use of Bernoulli's equation to determine the pressure at the center of impact. Owing to symmetry, the cylinder axis forms a stagnation streamline that terminates at a point of zero velocity, as shown in Fig. 4. Along this streamline, Bernoulli's equation reads for an incompressible fluid

$$\int_{p_p}^{p_p+p_s} \frac{dp}{\rho} = \frac{v_p^2}{2}, \quad (9)$$

which yields the stagnation pressure

$$p_s = \frac{1}{2} \rho v_p^2, \quad (10)$$

Since the density of most materials increases under pressure, a more general inequality follows,

$$p_s^{\text{real}} \geq \frac{1}{2} \rho v_p^2. \quad (11)$$

The constant force F_s acting on the target cannot be obtained by directly multiplying (10) by the cross-sectional area, because the contact area evolves during impact. Instead, the force is determined by applying momentum conservation. The impulse imparted to the plate must equal the momentum lost by the projectile [36]:

$$\int_0^{t_{\text{imp}}^{\text{theo}}} F_s dt = \int_{v_p}^{v_{\text{reb}}} m_{\text{cyl}} dv \quad (12)$$

where $t_{\text{imp}}^{\text{theo}}$ is the impact duration and v_{reb} is the rebound velocity, which is negligible for soft-body impacts. The impact duration is approximated by the time required for the projectile to traverse its own length,

$$t_{\text{imp}}^{\text{theo}} = \frac{L_{\text{cyl}}}{v_p}, \quad (13)$$

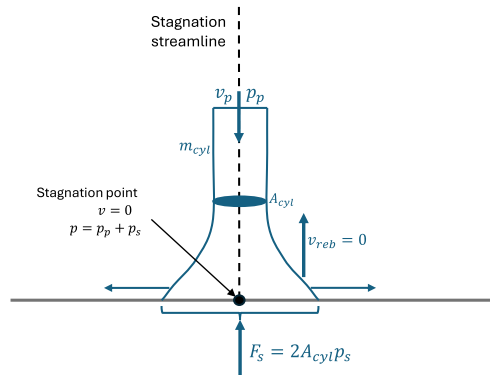


Fig. 4. Stagnation streamline along the axis of a soft body cylinder of mass m_{cyl} and initial cross-sectional area A_{cyl} impacting a rigid plate at velocity v_p . The rebound velocity v_r is negligible due to the soft body nature of the impact. The length-to-diameter ratio induces a stagnation flow that produces the stagnation pressure p_s . The impactor exerts a quasi-constant force F_s on the target.

which, inserted into (12), yields

$$F_s = \rho A_{cyl} v_p^2 = 2A_{cyl} p_s, \quad (14)$$

using (10) for the second equality.

A useful constant metric for the analysis in Section 5 is the *effective* stagnation pressure, defined as

$$p_{es} = \frac{F_s}{A_{cyl}}, \quad (15)$$

which is directly related to the stagnation pressure through $p_{es} = 2p_s$.

3. SPH formulations in LS-DYNA

This section aims to provide a clear and structured understanding of the SPH method with emphasis on both its theoretical foundations and its implementation in LS-DYNA. As the discussion unfolds, key differences between the available formulations will progressively be revealed.

3.1. Fundamentals

The SPH method is a meshless Lagrangian method that reconstructs smooth field variables from discrete particles through interpolation. This approach builds upon the exact integral representation of a scalar field $f(\mathbf{x})$ over a domain $\mathbb{V}$:

$$f(\mathbf{x}) = \int_{\mathbb{V}} f(\mathbf{x}') \delta(\mathbf{x} - \mathbf{x}') d\mathbf{x}', \quad (16)$$

where $\mathbf{x}$ is the current position vector and $\delta(\mathbf{x} - \mathbf{x}')$ is the Dirac delta function. While identity (16) is exact for continuous fields, it is unsuitable for numerical evaluation due to the singularity of δ . To address this, a continuous kernel function $W(\mathbf{x} - \mathbf{x}', h)$ with compact support defined by the smoothing length h is introduced.

The resulting kernel approximation takes the form

$$f(\mathbf{x}) \approx \int_{\mathbb{V}} f(\mathbf{x}') W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}', \quad (17)$$

which for symmetric, normalized, and positive-definite kernels, achieves second-order accuracy provided that f is sufficiently smooth [38].

Various kernel functions W may be defined, provided they satisfy a set of essential properties detailed in [39]. Throughout this work, the cubic B-spline kernel introduced by Monaghan and Lattanzio [40] is consistently employed. This kernel is widely adopted in SPH due to its similarity to a Gaussian function while maintaining compact support over a domain of diameter $4h$ [39].

The continuous kernel integral is transformed into a discrete summation by introducing a finite set of particles distributed across the domain $\mathbb{V}$ with each carrying physical attributes including mass m , Lagrangian position $\mathbf{x}$, velocity $\mathbf{v}$, and internal energy e [41]. Each particle is associated with a finite volume ΔV , related to its mass and density via:

$$m_j = \Delta V_j \rho_j, \quad (18)$$

for particle j .

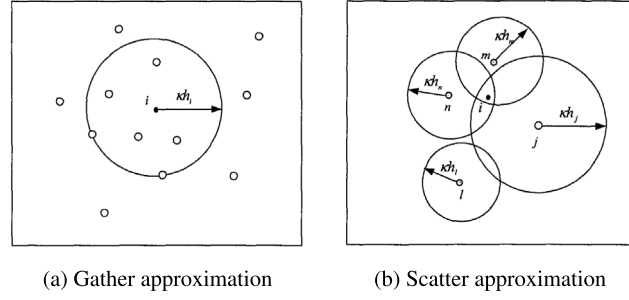


Fig. 5. Scatter and gather approximations in SPH. In the gather approximation (a), contributions are collected from particles within the support of i . In the scatter approximation (b), particle i is influenced by particles whose support domain includes it.

Source: Reproduced from [39].

For a reference particle i located at $\mathbf{x}_i$, the particle approximation generally reads as

$$f(\mathbf{x}_i) \approx \sum_{j=1}^N \frac{m_j}{\rho_j} f(\mathbf{x}_j) W(\mathbf{x}_i - \mathbf{x}_j, h), \quad (19)$$

with N the number of neighbors, i.e. the number of particles located within the compact support of W .

3.2. Smoothing length

The smoothing length is a central quantity in SPH, as it defines the support of the kernel function and thereby controls the interpolation quality, resolution, and computational cost.

The initialization of the smoothing length in LS-DYNA is governed by the user-defined parameter CSLH. For each SPH part, an initial smoothing length h_0 is computed as the maximum of the minimum inter-particle distances, scaled by CSLH. The default value, CSLH = 1.2, ensures that at least two layers of neighboring particles are included in the support domain, in line with the software guidelines [31] and industrial experience for bird strike scenarios [42].

The time evolution of the smoothing length is governed by:

$$\frac{dh}{dt} = h \nabla \cdot \mathbf{v}, \quad (20)$$

which mirrors the continuity equation and is designed to maintain an approximately constant number of neighbors [41]. To prevent uncontrolled growth or collapse of the smoothing length, it is constrained between user-defined bounds:

$$h_{\min} = \gamma_{\min} h_0, \quad h_{\max} = \gamma_{\max} h_0, \quad (21)$$

with default values $\gamma_{\min} = 0.2$ and $\gamma_{\max} = 2.0$, calibrated for bird strike scenarios [42].

A fundamental issue in SPH lies in the spatial variability of smoothing lengths: when particles i and j have different smoothing lengths h_i and h_j , it is unclear which should be used to evaluate $W(\mathbf{x}_i - \mathbf{x}_j, h)$ in (19). Using h_i yields the *gather* approximation, where i collects contributions from particles within its own support. Using h_j yields the *scatter* approximation, where i receives contributions from particles whose support includes i [43]. These interpretations are illustrated in Fig. 5.

A known consequence of spatially variable smoothing lengths is the potential violation of Newton's third law. Specifically, particle i may lie within the support of j , while j does not lie within that of i . In such cases, force symmetry is broken. To restore symmetry, a common solution is to use a symmetric smoothing length defined as the arithmetic mean of h_i and h_j [44]:

$$h_{ij} = \frac{h_i + h_j}{2}. \quad (22)$$

In LS-DYNA, the symmetric smoothing length defined in (22) is employed exclusively in formulations 2 and 3. For all other formulations, LS-DYNA adopts the gather approximation as it is the only approach for which a stable and consistent scheme can be rigorously established, provided the smoothing length remains bounded [45,46]. This theoretical result originates from the doctoral works of Lacombe [47] and Ben Moussa [48].

In problems with strongly directional deformation, such as bird ingestion, isotropic smoothing through a scalar smoothing length may be insufficient. Adaptive SPH (ASPH), rigorously introduced by Shapiro and Owen [49,50], addresses this by introducing an anisotropic kernel defined through a smoothing length tensor. This tensor allows for ellipsoidal support domains aligned with local deformation. In LS-DYNA, this approach is implemented in formulation 9 (ASPH) and its renormalized counterpart, formulation 10.

3.3. Artificial viscosity

Artificial viscosity is a critical numerical mechanism essential for stabilizing the numerical treatment of shocks. In physical systems, a shock wave corresponds to a narrow region over which kinetic energy is irreversibly converted into heat [39]. To replicate this dissipative behavior numerically, artificial viscosity is introduced into the momentum and energy equations to diffuse sharp gradients and suppress spurious oscillations [39].

The most widely adopted formulation is the Monaghan-type artificial viscosity [51], which is the default in LS-DYNA. It is defined as

$$\Pi_{ij} = \begin{cases} \frac{-\alpha_\pi \bar{c}_{ij} \phi_{ij} + \beta_\pi \phi_{ij}^2}{\bar{\rho}_{ij}}, & \text{if } \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} < 0, \\ 0, & \text{otherwise,} \end{cases} \quad (23)$$

with

$$\phi_{ij} = \frac{h_{ij} \mathbf{v}_{ij} \cdot \mathbf{x}_{ij}}{|\mathbf{x}_{ij}|^2 + \varphi^2}, \quad \bar{c}_{ij} = \frac{1}{2}(c_i + c_j), \quad \bar{\rho}_{ij} = \frac{1}{2}(\rho_i + \rho_j), \quad (24)$$

and

$$h_{ij} = \frac{1}{2}(h_i + h_j), \quad \mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j, \quad \mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j. \quad (25)$$

Key aspects include:

- (i) The parameters α_π and β_π control the linear and quadratic contributions, respectively. The linear term introduces bulk viscosity; the quadratic term mitigates interpenetration in high Mach number flows [39]. Both parameters can be controlled in LS-DYNA and are typically set to 1 by default.
- (ii) The regularization term $\varphi = 0.1h_{ij}$ avoids division by zero in the denominator of ϕ_{ij} .
- (iii) All quantities are symmetrized via arithmetic averaging, ensuring preservation of linear momentum.

3.4. Conservation equations

Apart from the total Lagrangian types implemented in LS-DYNA as formulation 7 (total Lagrangian) and formulation 8 (normalized total Lagrangian), which are inherently unsuitable in the context of large deformations due to their material description (the reader is referred to [52–55] for more details), the standard SPH forms of the continuity and energy equations commonly employed by all other SPH formulations read:

$$\frac{d\rho_i}{dt} = \rho_i \sum_j \frac{m_j}{\rho_j} \mathbf{v}_{ji} \cdot \nabla W_{ij}, \quad (26)$$

$$\frac{de}{dt} = - \sum_j m_j \mathbf{v}_{ji} \cdot \left(\frac{\boldsymbol{\sigma}_i}{\rho_i^2} + \frac{1}{2} \Pi_{ij} \mathbf{I} \right) \cdot \nabla W_{ij}, \quad (27)$$

where $\mathbf{I}$ is the identity tensor, $W_{ij} = W(\mathbf{x}_{ij}, h)$ and $\cdot$ denotes the dot product between vectors.

Three forms of the momentum equation are implemented in LS-DYNA [31]:

$$\frac{d\mathbf{v}_i}{dt} = - \sum_j m_j \left(\frac{\boldsymbol{\sigma}_i}{\rho_i^2} + \frac{\boldsymbol{\sigma}_j}{\rho_j^2} + \Pi_{ij} \mathbf{I} \right) \cdot \nabla W_{ij}, \quad (28)$$

$$\stackrel{\text{or}}{=} - \sum_j m_j \left(\frac{\boldsymbol{\sigma}_i}{\rho_i \rho_j} \cdot \nabla W_{ij} - \frac{\boldsymbol{\sigma}_j}{\rho_i \rho_j} \cdot \nabla W_{ji} + \Pi_{ij} \nabla W_{ij} \right), \quad (29)$$

$$\stackrel{\text{or}}{=} - \sum_j m_j \left(\frac{\boldsymbol{\sigma}_i}{\rho_i^2} \cdot \nabla W_{ij} - \frac{\boldsymbol{\sigma}_j}{\rho_j^2} \cdot \nabla W_{ji} + \Pi_{ij} \nabla W_{ij} \right). \quad (30)$$

(28) is only used for symmetric formulations, that is, formulations 2 (symmetric) and 3 (symmetric normalized). (29) is used in the fluid particle and enhanced fluid families, namely formulations 5 and 6 (fluid particle and fluid particle renormalized) as well as formulations 15 and 16 (enhanced fluid and enhanced fluid renormalized). The use of the mixed density product $\rho_i \rho_j$ improves numerical robustness in the presence of strong density gradients or pronounced stiffness contrasts [31]. (30) is used in all other cases, which corresponds to the standard implementation in LS-DYNA.

3.5. Shortcomings

Despite its versatility, the SPH method has some inherent deficiencies that impact its accuracy and stability in practice. Two shortcomings relevant to this work are presented: *consistency* and *tensile instability*.

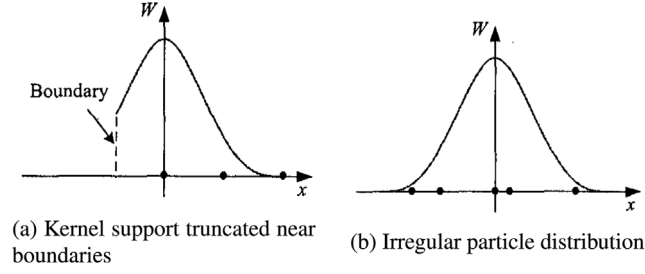


Fig. 6. Sources of particle inconsistency in SPH.
Source: Reproduced from [39].

3.5.1. Consistency

Numerical consistency refers to the ability of a discretization scheme to reproduce exact solutions as the discretization parameters tend to zero [56]. In SPH, consistency is affected by both the properties of the smoothing kernel and the discrete particle distribution, typically referred to as kernel and particle consistency [57,58].

Kernel consistency. To reproduce polynomials of degree n , the kernel W must satisfy successive moment conditions [38]:

$$\int_{\mathbb{V}} (\mathbf{x} - \mathbf{x}')^k W(\mathbf{x} - \mathbf{x}', h) d\mathbf{x}' = \begin{cases} 1 & \text{for } k = 0, \\ 0 & \text{for } 1 \leq k \leq n. \end{cases} \quad (31)$$

The zeroth- and first-order conditions are generally satisfied for a valid kernel in the domain interior. However, near boundaries, truncation of the kernel support breaks these conditions, leading to a loss of even zeroth-order consistency.

Particle consistency. The discrete summation of moment integrals over particles introduces further errors. Ideally, the zeroth- and first-order moment conditions should satisfy:

$$\sum_{j=1}^N W(\mathbf{x} - \mathbf{x}_j, h) \Delta V_j = 1, \quad (32)$$

$$\sum_{j=1}^N (\mathbf{x} - \mathbf{x}_j) W(\mathbf{x} - \mathbf{x}_j, h) \Delta V_j = 0. \quad (33)$$

However, these conditions are rarely fulfilled due to boundary truncation and irregular particle distribution. Fig. 6 illustrates these two effects. At the boundaries, the lack of full kernel support significantly degrades integration accuracy. Within the domain interior, irregular particle spacing distorts the summation weights, reinforcing LS-DYNA's recommendation to generate uniformly distributed particles [31].

3.5.2. Tensile instability

Another fundamental issue in SPH is *tensile instability*, where particles under tensile stress exhibit unphysical clumping, leading to artificial stiffness and potential computational failure [38].

A one-dimensional analysis by Swegle et al. [24] showed that instability occurs when the following condition is met:

$$W''\sigma < 0, \quad (34)$$

where σ is the stress (positive in compression, negative in tension) and W'' is the second derivative of the kernel. For standard cubic B-spline kernels, $W'' > 0$, and the inequality is only satisfied in tension, hence the term *tensile instability*.

This instability has been attributed to the lack of integration points in the SPH discretization, as the summation is performed solely over field particles [59]. Belytschko et al. [52,53,60] alternatively relate the instability to the incompatibility between a Lagrangian particle motion and an Eulerian kernel approximation.

Fig. 7 illustrates the onset of tensile instability: a regular initial configuration degenerates into a clustered, non-physical pattern.

3.6. Improvements

A variety of techniques have been developed throughout the years to alleviate the deficiencies presented in Section 3.5. Some improvements implemented in LS-DYNA are presented: *normalization* and *density filtering*.

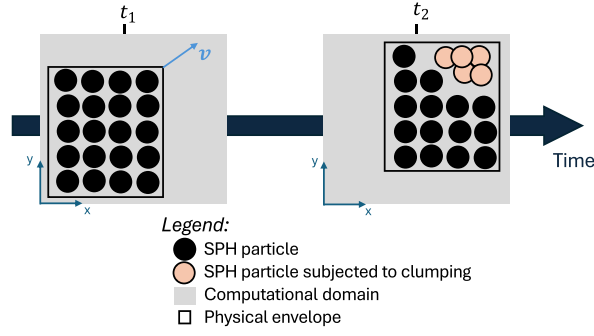


Fig. 7. Illustration of tensile instability in SPH.

3.6.1. Normalization

The term *normalization* is frequently encountered in the SPH literature, often referring to various strategies for restoring consistency in kernel-based approximations. The specific formulation adopted in LS-DYNA follows the approach developed by Randles and Libersky [61], which builds upon earlier work by Johnson and Beissel [62].

This normalization procedure is applied systematically to all normalized SPH formulations in LS-DYNA to improve the accuracy of spatial derivatives, particularly near boundaries.

Starting from the standard SPH approximation for the divergence of the stress tensor at particle i with $j = 1, \dots, N$ its neighboring particles:

$$(\nabla \cdot \sigma)_i = - \sum_j \frac{m_j}{\rho_j} (\sigma_j - \sigma_i) \cdot \nabla W_{ij}. \quad (35)$$

Given that this expression is generally not exact even for simple fields, the normalization procedure aims to ensure exactness for a benchmark case: a linear stress state.

Assume a stress field of the form $\sigma = \mathbf{Q}_1 + \mathbf{Q}_2 \cdot \mathbf{x}$, where $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are second- and third-order tensors, respectively, both symmetric in their first two indices. To exactly reproduce such a field, a second-order correction tensor $\mathbf{N}$ is introduced. The divergence approximation is then modified as:

$$(\nabla \cdot \sigma)_i = \left(- \sum_j m_j (\sigma_j - \sigma_i) \otimes \frac{\nabla W_{ij}}{\rho_j} \right) : \mathbf{N}, \quad (36)$$

where $\otimes$ denotes the dyadic product and $:$ the double contraction symbol.

Substituting the linear stress field into (36) yields

$$\mathbf{Q}_2 : \mathbf{I} = \mathbf{Q}_2 : \left[\left(- \sum_j m_j (\mathbf{x}_j - \mathbf{x}_i) \otimes \frac{\nabla W_{ij}}{\rho_j} \right) \cdot \mathbf{N} \right], \quad (37)$$

which is satisfied for any $\mathbf{Q}_2$ if and only if

$$\mathbf{N} = \left(- \sum_j m_j (\mathbf{x}_j - \mathbf{x}_i) \otimes \frac{\nabla W_{ij}}{\rho_j} \right)^{-1}. \quad (38)$$

This normalization strategy restores zeroth-order consistency in the approximation of the divergence of the stress tensor near boundaries, while preserving first-order consistency in the interior under regular particle distributions. This was further emphasized by Chen et al. [63], whose corrective SPH method recovers the one-dimensional form of Randles and Libersky's [61] normalization for this operator.

3.6.2. Density filtering

The density filtering procedure described is implemented in LS-DYNA exclusively for formulations 15 and 16, corresponding to the enhanced fluid and its normalized variant.

This approach was introduced to mitigate the pronounced pressure oscillations commonly observed in fluid simulations using standard SPH formulations [64]. These oscillations originate from noise in the particle density field, which directly propagates into the pressure field.

The filtering process consists in applying a Shepard filter, which is constructed using the kernel functions. In the literature, this operation is sometimes referred to as *density normalization* [65] or *density reinitialization* [66]. The resulting filtered density is given by:

$$\rho_i = \frac{\sum_j^N m_j W_{ij}}{\sum_j^N m_j \frac{W_{ij}}{\rho_j}}, \quad (39)$$

where the summation is performed over all neighboring particles j of particle i .

This form is based on the work of Colagrossi [67,68], who studied several reinitialization schemes, including the Shepard-type.

Interestingly, (39) corresponds precisely to a boundary deficiency alleviation strategy originally proposed by Randles and Libersky [61]. In their formulation, this expression was specifically derived to correct the density of boundary particles by using only the contributions from interior neighbors in the summations.

This density filtering technique restores zeroth-order consistency in the density approximation near boundaries while preserving first-order accuracy in the interior, provided the particle distribution remains regular. Chen et al. [63] emphasized this behavior, showing that their corrective SPH formulation recovers relation (39) for the density field.

An alternative SPH formulation available in LS-DYNA is the *Moving Least Squares Smoothed Particle Hydrodynamics* (MLSPH) method, implemented as formulation 12. This approach aims to improve the accuracy of standard SPH by employing a least-squares-based interpolation framework. The implementation follows the quasi-linear reproducing kernel formulation introduced by Yreux [64,69]. As MLSPH is not central to the present study, it is not discussed further.

Based on this discussion, the reader now has the necessary context to clearly interpret the formulations summarized in Table 1, as presented in the introduction.

4. Practical considerations

The previous section described the SPH formulations investigated in this work and highlighted the principal differences between them. Besides the particle approximation itself, their implementation relies on a common numerical framework governing neighbor identification, contact treatment, and time integration. These aspects directly influence the computational efficiency, numerical stability, and reproducibility of the simulations. The following sections therefore summarize the numerical framework common to all SPH formulations investigated in this work.

4.1. Neighbor identification

The SPH approximation requires each particle to identify neighboring particles located within the compact support of its kernel function at every time step. Since this operation is repeated throughout the simulation, neighbor identification constitutes one of the most computationally demanding components of an SPH analysis [39].

Neighbor searching is consistently performed using a linked-list or bucket-sort algorithm [39,55] throughout this work. The computational domain is subdivided into regular cells, and particles are assigned according to their current position. The neighborhood of each particle is then constructed by searching only its own cell and the surrounding cells rather than the complete particle set. This substantially reduces the computational cost while preserving the exact particle neighborhood [31].

4.2. Contact treatment

The interaction between the SPH projectile and the finite-element target is enforced through a penalty-based contact formulation. Limited interpenetration between the contacting bodies generates restoring forces, while the interface stiffness is determined automatically from the material properties of the interacting bodies [31]. The same contact formulation and parameters are employed throughout all simulations.

4.3. Time integration

The governing conservation equations are integrated using the explicit central-difference scheme [31]. Numerical stability is governed by the Courant–Friedrichs–Lewy (CFL) condition, which limits the admissible time increment according to

$$\Delta t = C_{\text{CFL}} \min_i \left(\frac{h_i}{c_i + \|\mathbf{v}_i\|} \right), \quad (40)$$

where h_i denotes the smoothing length, c_i the local speed of sound, $\mathbf{v}_i$ the particle velocity, and C_{CFL} is a safety coefficient [39]. The time increment is updated adaptively throughout the simulation according to the local particle conditions. The same integration scheme and adaptive time-stepping strategy are employed for all SPH formulations considered in this work.

5. Results and discussion

The SPH formulations are evaluated through a sequence of increasingly demanding benchmark cases to identify the most suitable approach for bird strike and multi-stage bird ingestion simulations. The assessment progresses from a single-impact benchmark (Section 5.1), to a sensitivity analysis (Section 5.2), a double-impact configuration (Section 5.3), and finally experimental validation (Section 5.4), allowing unsuitable formulations to be progressively eliminated while assessing the robustness of the remaining candidates.

In accordance with Table 1, the tensor (4), total Lagrangian (7 & 8), and incompressible (13) formulations are excluded from the present assessment, as they are either non-executable(4) or fundamentally incompatible with bird strike simulations owing to their material description (7 & 8) or implicit incompressibility assumption (13).

Table 2

Key parameters used in the single-impact configuration. All values in SI units.

Parameter	Value	Unit
Bird mass	1	kg
Bird geometry	Cylinder	–
Length-to-diameter ratio	2	–
Bird length	0.17	m
Impact velocity	116	m/s
Simulation time	2×10^{-3}	s
Inter-particle distance	4.4×10^{-3}	m
Bulk modulus	9.5117×10^8	Pa
Target diameter	0.4	m
Target height	0.03	m
Artificial viscosity	(1, 1)	–
TSSFAC	0.67	–

5.1. Single impact

The selection of suitable SPH formulations begins by assessing whether they reproduce the fundamental hydrodynamic response expected during a simple soft-body impact. To this end, a single normal impact of a cylindrical projectile onto a rigid flat target is considered. This configuration serves as a minimal yet rigorous benchmark: any formulation failing to capture the fundamental hydrodynamic response described in Section 2 is excluded from subsequent analyses. Accurate representation of a single soft-body impact is a necessary precondition for reliable multi-impact or ingestion simulations.

5.1.1. Model description

The bird surrogate is a right circular cylinder of mass 1 kg, length $L_{\text{cyl}} = 174.8$ mm, and length-to-diameter ratio 2. It is discretized using 12 000 SPH particles with initial spacing 4.4 mm and launched at $v_0 = 116$ m/s. The simulation extends to 2 ms, beyond the theoretical impact duration $\frac{L_{\text{cyl}}}{v_0} = 1.5$ ms, ensuring the capture of the full stagnation phase.

The bird is represented using a hydrodynamic constitutive model, initially coupled with a linearized equation of state with bulk modulus $K = 951.17$ MPa,

$$p = -K \epsilon_v = K \ln \left(\frac{\rho}{\rho_0} \right), \quad (41)$$

which is adopted to simplify the analysis.

The target is a rigid flat cylinder ($\varnothing = 0.4$ m, height 0.03 m), meshed with 5904 rigid elements. Standard artificial viscosity coefficients $(\alpha_\pi, \beta_\pi) = (1, 1)$ [39] are applied, and the time step is scaled using a Time Step Scaling Factor (TSSFAC) of 0.67.

The SPH kernel parameters are initially set to default intervals $\gamma_{\min} = 0.2$, $\gamma_{\max} = 2$, and smoothing length scale (CSLH) of 1.2, which control the neighbor domain and kernel support width.

Table 2 summarizes the main model parameters.

5.1.2. Comparison metrics

The physics of the reference scenario is well established, enabling quantitative criteria to assess the SPH formulations. Three comparison metrics are employed:

- (i) *Energy conservation.* During a simulated soft-body impact, kinetic energy is dissipated into internal energy due to artificial viscosity. A physically correct response exhibits a transfer between kinetic and internal energies while maintaining a constant total energy. Numerical accuracy is measured through the *energy ratio* [70]:

$$e_r(t) = \frac{E_{\text{total}}(t)}{E_{\text{total}}^0 + W_{\text{ext}}(t)}, \quad (42)$$

where $E(t)$ denotes the total energy of the system at time t , E^0 the initial total energy, and $W_{\text{ext}}(t)$ the work performed by external forces. In the present study, no external work is applied, such that $W_{\text{ext}} = 0$. Deviations of $e_r(t)$ from unity therefore indicate numerical dissipation or instability.

- (ii) *Impact pressure.* The pressure transmitted to the rigid plate reflects the formulation's ability to reproduce shock physics. Comparisons are carried out against Hugoniot and stagnation pressures and the expected profile shape shown in Fig. 1.
- (iii) *Computational cost.* The runtime required to complete the simulation is recorded. Because large-scale ingestion simulations are computationally intensive, formulations with unstable behavior and/or excessive cost are disqualified.

5.1.3. Pressure measurements

Different measurement strategies lead to systematically different pressure values. One possible approach consists in sampling pressure locally at the contact interface. However, such local measurements depend on the choice of the averaging region and can

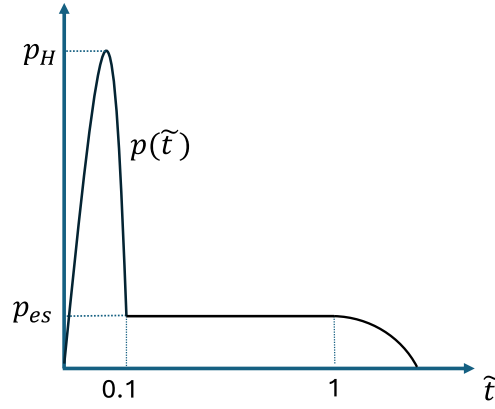


Fig. 8. Idealized measured pressure profile for a soft-body impact on a rigid target. p_H denotes the Hugoniot pressure while p_{es} the effective stagnation pressure. The highlighted interval $[0.1, 1]$ denotes the assumed stagnation phase used for computing the effective stagnation pressure.

therefore be sensitive to user-defined settings. In the present work, pressure is instead evaluated from the total reaction force (rcforc) generated during impact, normalized by the bird's initial cross-sectional area. This force-based definition is preferred because:

- (i) it provides a direct equivalent to the theoretical Hugoniot pressure, which is defined at the onset of impact relative to the initial area;
- (ii) it enables a consistent comparison between the measured stagnation pressure and its theoretical counterpart, the *effective stagnation pressure* defined in (15), which reflects the same force-based measurement approach;
- (iii) it reproduces the expected temporal pressure evolution characteristic of soft-body impacts.

Pressure readings from SPH particles are not used as a primary metric because:

- (i) particle locations become ill-defined as the material undergoes severe deformation, degrading measurement consistency;
- (ii) the primary quantity of interest is the load imposed on the rigid target, not the internal pressure field of the projectile, which is largely irrelevant in a hydrodynamic regime.

The Hugoniot pressure is taken as the maximum pressure immediately upon impact. Theoretical values are computed from (3). Preliminary analyses indicate that the Hugoniot relationship (4) can be approximated as linear, which implies a constant shock speed equal to the sound speed, i.e. $s = 0$.

Determining the effective stagnation pressure is less straightforward. Attempts to identify the stagnation onset using the theoretical shock duration based on (8) proved inconclusive, due to the growing physical contact area which is not accounted for in the present measurement methodology. A pragmatic approach is therefore adopted: the stagnation pressure is obtained from a constant regression over the normalized time interval $[0.1, 1]$, which represents a post-shock plateau region.

A schematic illustration of the idealized pressure profile, as inferred from the theoretical framework and adopted measurement strategy, is provided in Fig. 8 to clarify the expected pressure evolution and the definition of the effective stagnation phase.

For the sake of clarity, the term “effective” will be omitted in the remainder of this document. Henceforth, the stagnation pressure will refer to the effective stagnation pressure as defined above.

Three physical time scales relevant to this work are defined:

- (i) the shock duration $t_{\text{shock}}^{\text{theo}}$, corresponding to the radial release wave transit to the projectile's centerline;
- (ii) the shock propagation time $t_{\text{prop}}^{\text{theo}}$, equal to the longitudinal shock-front travel time to the back of the projectile.
- (iii) the impact duration $t_{\text{imp}}^{\text{theo}}$, representing the time for material to flow through its own length;

Finally, all time-dependent quantities are expressed in dimensionless form $\tilde{t} = t/t_{\text{imp}}^{\text{theo}}$ to preserve scale invariance and enable direct comparisons.

5.1.4. Initial screening

A preliminary screening is performed to eliminate SPH formulations that clearly fail to reproduce fundamental soft-body impact physics. Two minimal criteria are applied:

- (i) numerical instabilities must not develop within the bulk of the projectile, i.e. instability localization must remain restricted to boundary regions;
- (ii) the projectile must flow upon impact, consistent with soft-body impact theory.

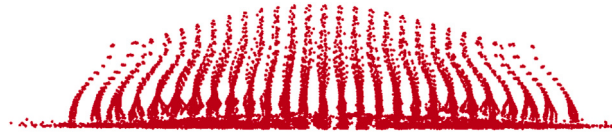


Fig. 9. Side view at $\tilde{t} = 1.32$ illustrating the abnormal particle accumulation exhibited by formulation 9 (adaptive SPH). Instead of flowing along the surface, particles stack upon impact, indicating unrealistic interaction dynamics.



Fig. 10. Trajectory history of a representative SPH particle located in the projectile core using formulation 12 (moving least-squares based). Over $\tilde{t} \in [0, 1.12]$, the position exhibits pronounced unphysical oscillations, indicating numerical instability within the bulk.

Formulation 9 (adaptive SPH with anisotropic smoothing tensor) is excluded at this stage. Despite its intended suitability for large-deformation problems, the simulated material response is dominated by abnormal particle accumulation, as shown in Fig. 9. Particles exhibit poor sliding behavior, both internally and against the rigid target, forming stacked clusters rather than a continuous flow. While this behavior may originate from a software implementation issue rather than from the anisotropic tensor itself, the formulation from LS-DYNA fails to reproduce basic hydrodynamic characteristics and is therefore rejected.

Formulation 12 (moving least-squares based) also fails the screening. Contrary to expectations (given that the method was introduced to improve tensile stability, accuracy, and performance in large deformation scenarios [31]), instabilities arise directly within the projectile core. Fig. 10 illustrates the unphysical particle oscillations observed. Nevertheless, this is consistent with guidance from DynaS+ technical support, which discourages the use of this formulation for high-velocity impact scenarios. The implementation by Yreux [64] focused on validation cases where the impacted structure, not the projectile, was SPH-discretized, suggesting a mismatch with the current use case.

5.1.5. Wave propagation analysis

The retained formulations are evaluated during the shock phase, which governs the early impact response. A longitudinal cross-section along the cylinder centerline is examined, and the pressure field serves as the primary diagnostic, following the theoretical description in Section 2.1. The phase entails the formation of a shock wave at the contact interface and a radial release wave at the projectile boundaries, followed by the release wave overtaking the shock front.

All retained formulations correctly reproduce the essential shock dynamics. A representative result is shown in Fig. 11 for formulation 0. Immediately upon impact, the leading particle layer is arrested, producing a high-pressure zone. A release wave, originating at the unconfined edges, propagates radially toward the cylinder axis. Interaction between the release wave and the shock front generates the characteristic droplet-shaped pressure region, which then travels upstream along the projectile.

The time required for this shock front to traverse the entire cylinder length defines the theoretical propagation time $t_{\text{prop}}^{\text{theo}}$ and enables comparison between theoretical and simulated shock velocities.

With the linear Hugoniot in Eq. (4) and $s = 0$, the theoretical shock speed equals the sound speed,

$$v_s = c_0 = \sqrt{\frac{K}{\rho}} = 998.46 \text{ m/s},$$

which yields the theoretical propagation time

$$t_{\text{prop}}^{\text{theo}} = \frac{L_{\text{cyl}}}{v_s} = 0.175 \text{ ms} \implies \tilde{t}_{\text{prop}}^{\text{theo}} = 0.12.$$

Table 3 summarizes the simulated propagation times and relative errors. Formulation 3 achieves the lowest error (12.6%), followed by formulation 10 (14.3%). Formulations 0 and 5, as well as 1 and 6, produce identical results, indicating that the momentum equation form (cf. Eqs. (29), (30)) does not significantly affect the initial shock dynamics. No systematic trend is observed between normalized (1, 3, 6, 16) and non-normalized (0, 2, 5, 15) formulations, suggesting that boundary normalization does not strongly influence propagation speed at this early stage.

Remark. The theoretical propagation time $t_{\text{prop}}^{\text{theo}} = L_{\text{cyl}}/v_s$ neglects the projectile displacement $v_p t_{\text{prop}}^{\text{theo}}$, which partly explains the relatively large errors observed in Table 3. Nevertheless, this first-order estimate remains sufficient for the present analysis.

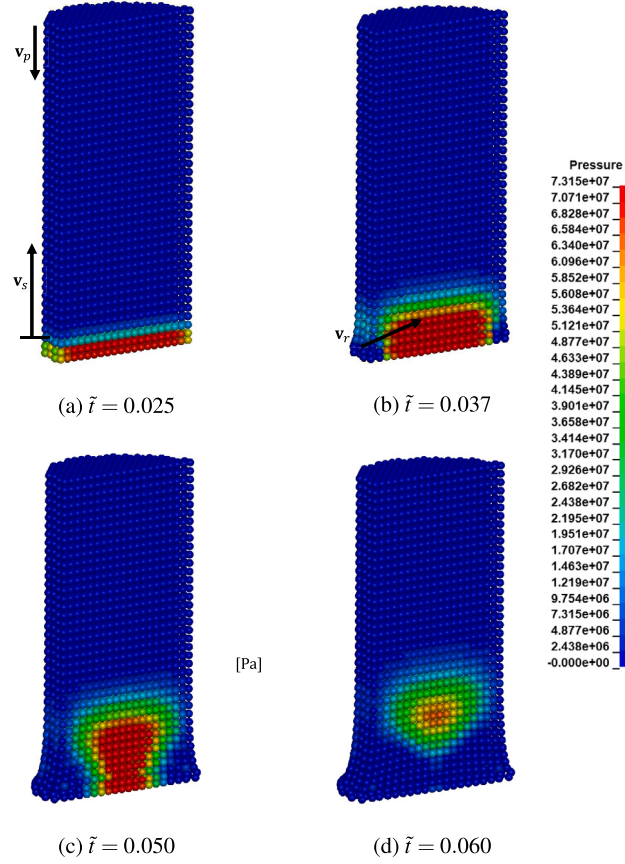


Fig. 11. Shock formation and propagation within the projectile along a longitudinal section at four selected dimensionless times. Arrows indicate projectile velocity v_p , shock velocity v_s , and release wave velocity v_r ; arrow lengths scale with magnitude. Results correspond to formulation 0.

Table 3

Comparison of simulated and theoretical normalized shock propagation times. The theoretical value is $\tilde{t}_{\text{prop}}^{\text{theo}} = 0.12$. ϵ indicates the relative error.

Formulation	Simulated time [–]	ϵ [%]
Default (0)	0.098	15.4
Default renorm. (1)	0.089	23.4
Symmetric (2)	0.098	15.4
Symmetric renorm. (3)	0.101	12.6
Fluid particle (5)	0.098	15.4
Fluid particle renorm. (6)	0.089	23.4
ASPH renorm. (10)	0.099	14.3
Enhanced fluid (15)	0.090	22.2
Enhanced fluid renorm. (16)	0.091	21.1

Radial release wave behavior is examined in Fig. 12. The release front moves inward before switching to upstream propagation. The nearly constant extent of the compressed zone (gray) indicates that the release velocity v_r remains slightly below v_s , as predicted in Section 2.1. Only nonlinear equations of state allow $v_r > c_0$, which is not the case here.

Theoretical release speed follows from the linearized equation of state in Eq. (41):

$$v_r = \sqrt{\frac{K}{\rho_H}}, \quad (43)$$

where, using (41), the Hugoniot density satisfies

$$\rho_H = \rho_0 e^{p_H/K}. \quad (44)$$

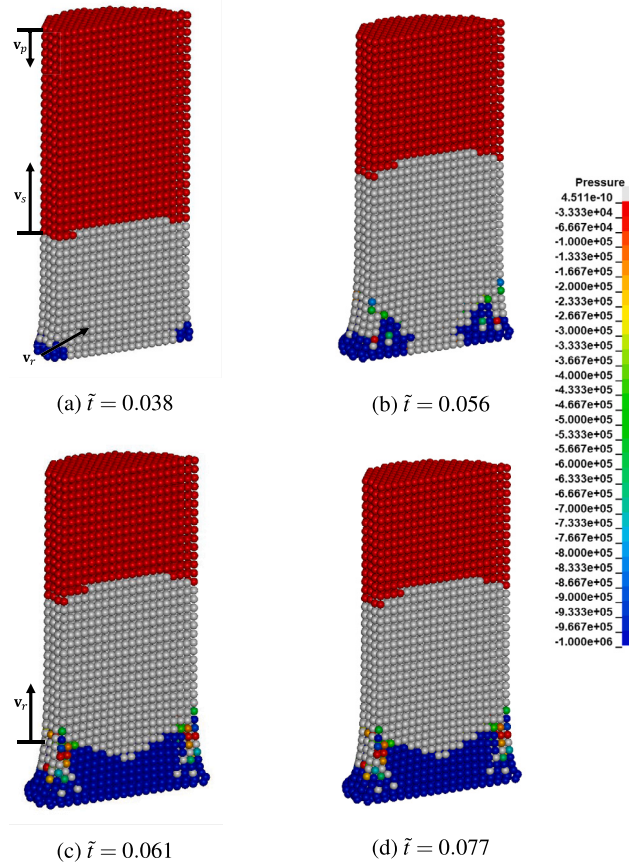


Fig. 12. Radial release wave propagation within the projectile at four selected dimensionless times, shown along a longitudinal section. Gray regions denote compression, red regions undeformed states, and blue regions tension. Results correspond to formulation 0. Velocity vectors $\mathbf{v}_p$, $\mathbf{v}_s$, and $\mathbf{v}_r$ are overlaid; vector lengths scale with magnitude.

Table 4

Comparison of simulated and theoretical normalized shock durations. The theoretical value is $\tilde{t}_{\text{shock}}^{\text{theo}} = 0.031$. ϵ indicates the relative error.

Formulation	Simulated time [-]	ϵ [%]
Default (0)	0.026	16.2
Default renorm. (1)	0.028	7.3
Symmetric (2)	0.026	16.2
Symmetric renorm. (3)	0.025	17.1
Fluid particle (5)	0.026	16.2
Fluid particle renorm. (6)	0.028	7.3
ASPH renorm. (10)	0.027	10.5
Enhanced fluid (15)	0.025	20.9
Enhanced fluid renorm. (16)	0.027	12.1

This yields $v_r = 942.12 \text{ m/s}$, strictly below v_s . The corresponding shock duration is

$$\tilde{t}_{\text{shock}}^{\text{theo}} = \frac{r_{\text{cyl}}}{v_r} = 0.046 \text{ ms} \implies \tilde{t}_{\text{shock}}^{\text{theo}} = 0.031.$$

Simulation results are reported in Table 4. Formulations 1 and 6 exhibit the lowest relative error (7.3%), followed by formulation 10 (10.5%) and formulation 16 (12.1%). Formulations 0, 2, and 5 form an intermediate group (16.2%), indicating that momentum equation form has negligible influence at this stage. Formulations 3 (17.1%) and 15 (20.9%) show the largest deviations. Across all pairs, renormalized formulations (1, 3, 6, 16) outperform their non-renormalized counterparts (0, 2, 5, 15), indicating that renormalization improves boundary handling and release wave propagation.

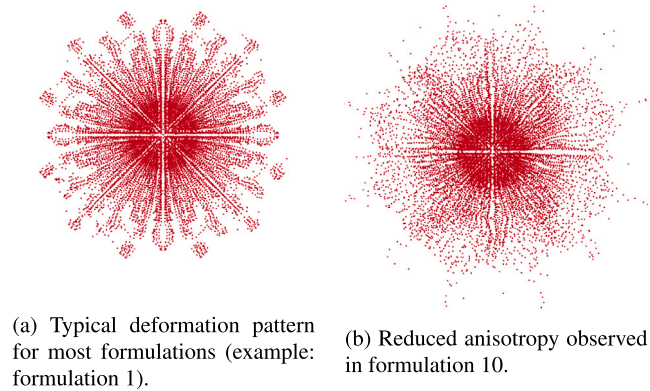


Fig. 13. Front-view (facing the plate) deformation at $\tilde{t} = 0.77$. The characteristic double-cross shape appears in all formulations except formulation 10.

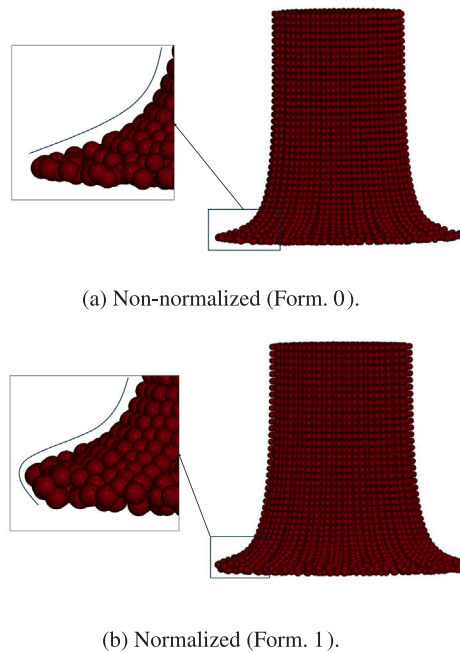


Fig. 14. Boundary deformation at $\tilde{t} = 0.17$. Only the normalized formulation recovers the expected curvature reversal [42].

5.1.6. Qualitative analysis

The deformation of the bird upon impact presents a recurring double-cross pattern across all formulations except formulation 10, where the effect is attenuated (Fig. 13). This feature does not originate from the formulation itself but stems from anisotropy introduced during SPH discretization, which is a limitation within LS-DYNA.

A physically meaningful feature during radial expansion is the inflection point along the lateral boundary. This curvature reversal originates from the velocity gradient induced by plate contact: particles sliding along the plate decelerate relative to those above, altering the radial profile [42]. Normalized formulations improve boundary treatment, enabling this inflection to form. As shown in Fig. 14, formulation 1 captures the curvature reversal, whereas its non-normalized counterpart (formulation 0) produces a monotonically curved boundary.

Another key distinction involves the behavior of particles near the boundary of the radially-outward flow. Non-normalized variants, such as formulation 0, exhibit spurious ejections caused by kernel support loss. Normalized formulations maintain coherence and eliminate these artificial expulsion events (Fig. 15).

At later times, branch-like features emerge radially from the impact zone. In non-normalized formulations, these branches exhibit abnormally high stiffness and coincide with persistent tensile regions (Fig. 16(a)), characteristic of tensile instability [24,41].

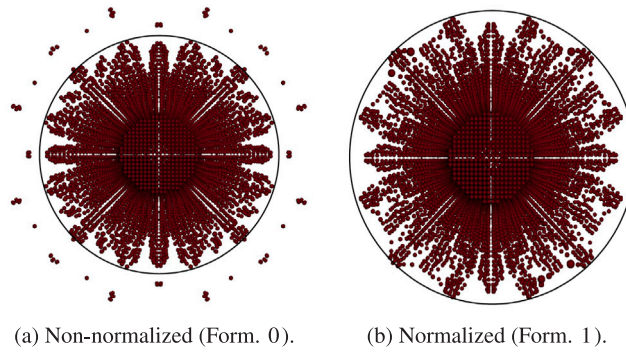


Fig. 15. Front-view at $\tilde{t} = 0.53$. Normalization suppresses non-physical particle ejection. The black circle denotes the radial expansion of the bulk of the bird which is the same for both cases.

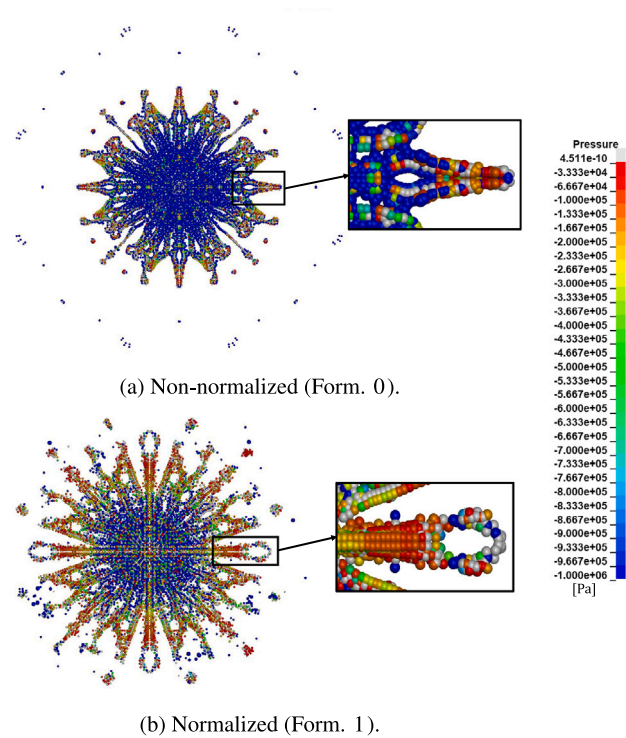


Fig. 16. Front-view (facing the plate) pressure field at $\tilde{t} = 1.32$. Tensile instability generates stiff radial branches in non-normalized SPH; normalization reduces their severity.

Clustering and artificial rigidity follow, consistent with known SPH pathologies. Normalized formulations still form branches (due to anisotropy introduced by LS-DYNA's SPH discretization), but they remain diffuse and mechanically softer (Fig. 16(b)), demonstrating partial mitigation.

Normalized formulations remain susceptible to tensile instability through particle blow-up, where tensile-state particles are expelled from the domain. This phenomenon is more severe than clustering because it introduces non-physical energy growth. Formulations 1, 3, and 6 exhibit this behavior to different extents, with formulation 6 being the most unstable (Fig. 17). In contrast, formulations 10 and 16 (normalized as well) show no signs of such blow-up in this configuration, though instability may still develop under different conditions.

Remark. This mechanism differs from the bulk instability of formulation 12, which originated in the interior and was not driven by tension stresses.

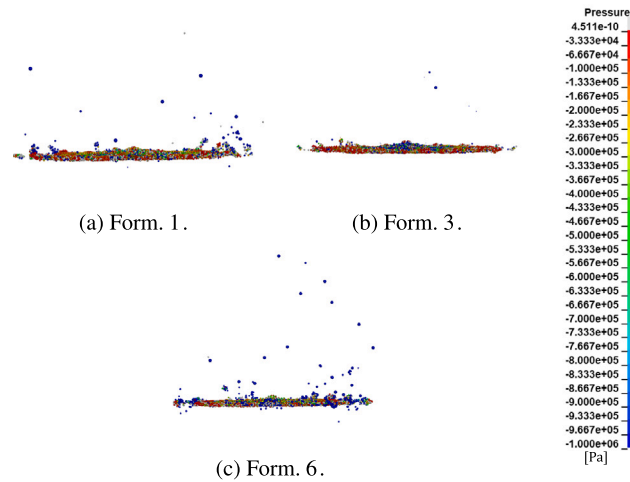


Fig. 17. Top-view (facing the bird's side) pressure at $\tilde{t} = 1.32$. Tensile-driven particle blow-up affects formulations 1, 3, and 6.

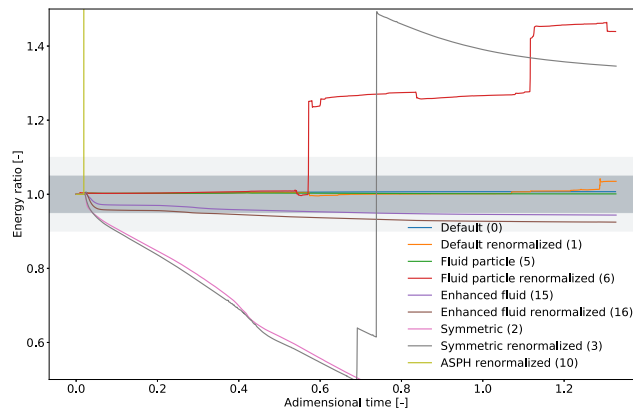


Fig. 18. Evolution of the energy ratio as a function of normalized time for all SPH formulations.

5.1.7. Quantitative analysis

The first comparison metric is the energy ratio introduced in Section 5.1.2. Two thresholds are used for evaluation: an *ideal* range corresponding to less than 5% variation, and an *acceptable* range permitting up to 10%. Fig. 18 shows that formulations 2 (pink), 3 (gray), and 10 (golden) exhibit clearly unphysical behavior. Formulation 10 diverges catastrophically, reaching an energy ratio of approximately 4×10^8 at the end of the simulation. Formulations 2 and 3 present a sustained decay of total energy, with formulation 3 additionally displaying sudden increases due to unstable particle blow-ups.

A decomposition of energy components shows that internal energy is the dominant driver of these discrepancies. In formulation 10, internal energy increases without bound, indicating a failure to correctly dissipate shock-induced deformation. In formulations 2 and 3, kinetic energy decreases as expected, but internal energy does not increase correspondingly, resulting in a net energy loss. This behavior is consistent with the known shortcomings of symmetric formulations in LS-DYNA, which are explicitly discouraged in all use cases [31]. Since these formulations fail to satisfy the fundamental requirement of energy conservation, they cannot be regarded as physically reliable. They are therefore discarded from the subsequent analysis.

Formulation 6 (red) initially behaves correctly but becomes unstable later in the simulation. A milder version of this effect occurs in formulation 1 (orange), supporting the earlier observation of its superior robustness relative to formulation 6.

In contrast, formulations 0 (blue) and 5 (green) exhibit the most satisfactory energy behavior. Their energy ratios remain close to unity and no discontinuities are observed, consistent with the absence of instability-induced particle loss.

Formulations 15 (violet) and 16 (brown) initially show a modest energy decrease followed by stabilization within acceptable bounds. Their longer-term response will therefore be decisive in assessing their applicability to extended ingestion scenarios. Notably, formulation 16 is the only renormalized strategy that remains free of discontinuities, highlighting its superior numerical robustness.

The second comparison metric is the impact pressure. Fig. 19 shows the reaction-force-based pressure evolution for formulations 0, 1, 5, 6, 15, and 16. All formulations reproduce the expected stages outlined in Section 5.1.2: a sharp rise at the Hugoniot pressure, a decay following shock release, a stagnation plateau, and a smooth return to zero. Two trends are noteworthy. First,

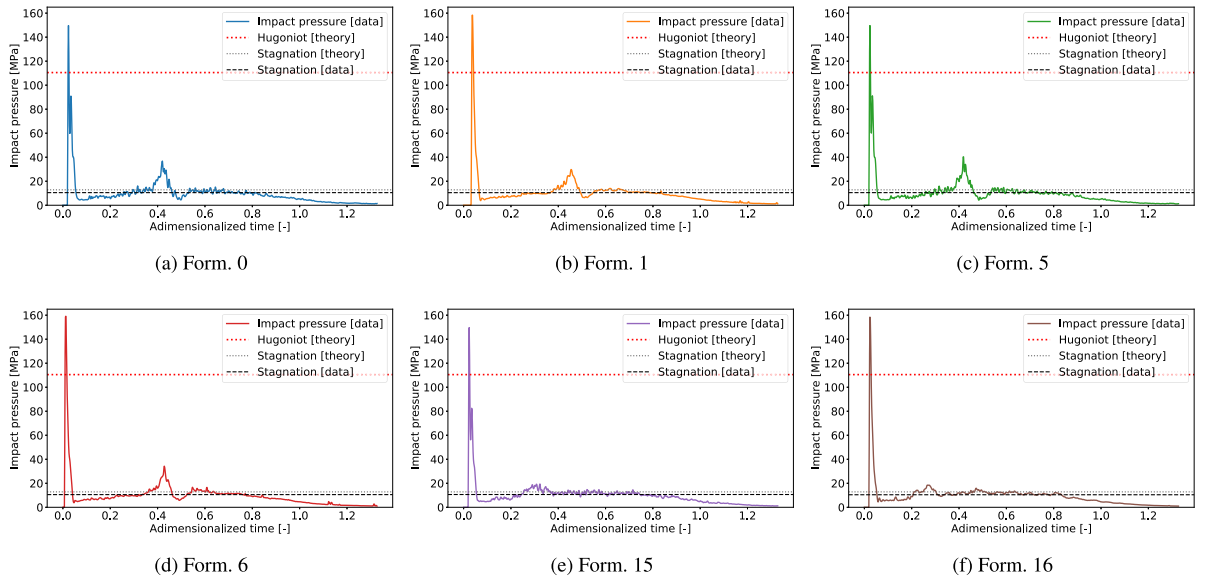


Fig. 19. Impact pressure evolution for all retained formulations.

Table 5

Simulated and theoretical Hugoniot p_H and stagnation pressures p_{es} obtained via the reaction-force method. Theoretical values: $p_{es}^{\text{theo}} = 12.84$ MPa and $p_H^{\text{theo}} = 110.51$ MPa. $\epsilon_{H/es}$ indicate the corresponding relative errors.

Formulation	p_H [MPa]	ϵ_H [%]	p_{es} [MPa]	ϵ_{es} [%]
Default (0)	149.63	35.4	10.50	18.2
Default renormalized (1)	158.98	43.9	10.48	18.4
Fluid particle (5)	149.63	35.4	10.51	18.2
Fluid particle renormalized (6)	159.12	44.0	10.57	17.7
Enhanced fluid (15)	149.63	35.4	10.64	17.1
Enhanced fluid renormalized (16)	158.39	43.3	10.44	18.7

normalized variants (1, 6, 16) exhibit smoother profiles suggesting that the normalized kernels attenuate noise in the data for the used measurement methodology. Second, all except formulations 15 and 16 display a secondary peak around $\tilde{t} = 0.4$, caused by shock wave reflection inside the projectile. Its suppression in formulations 15 and 16 is attributed to density filtering.

A quantitative comparison of Hugoniot and stagnation pressures is reported in Table 5. All formulations reproduce stagnation pressure with similar accuracy. Formulation 15 is the most accurate, while formulation 16 yields the highest deviation.

All non-normalized versions (0, 5, 15) overestimate the theoretical Hugoniot pressure by 35.4%. Normalized variants (1, 6, 16) perform slightly worse, with errors between 43.3 and 44.0%. Density filtering has negligible influence on transferred pressure using the reaction-force-based measurement.

Remark. Although the numerical Hugoniot pressures consistently exceed the analytical prediction, the two values are obtained using different procedures. The analytical reference is based on the simplified linear Hugoniot relation introduced in Section 2.1, whereas the numerical pressure is obtained from the target reaction force. As reported by Hedayati et al. [71], the evaluated Hugoniot pressure depends on the adopted methodology, including whether it is determined from local pressure measurements or the target reaction force, as well as the averaging procedure and projectile geometry. Likewise, the analytical prediction depends on the assumptions inherent to the selected Hugoniot relation. Consequently, discrepancies between analytical, numerical, and experimental Hugoniot pressures are common in the bird-strike literature. Nevertheless, the predicted pressures remain within the range reported in previous studies (e.g., Hedayati et al. [71]). The objective of this benchmark is not to maximize the accuracy of the Hugoniot pressure predictions, but to compare the SPH formulations within a consistent framework.

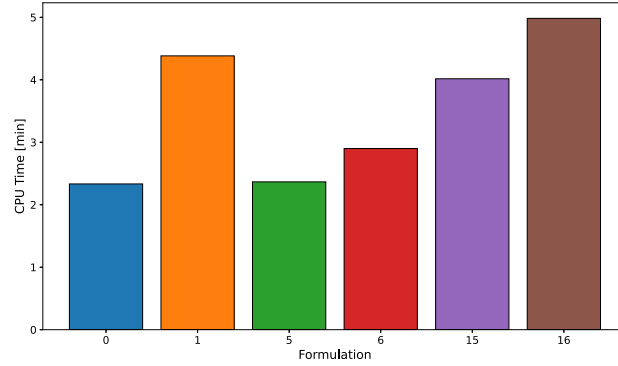
Particle-based pressure measurements are employed to further investigate the Hugoniot pressure (Table 6). Density filtering induces a marked underestimation in formulations 15 and 16 (approximately -18%). All other formulations overestimate the value, with non-normalized variants (0 and 5) performing best (error $\sim 13\%$), and normalized counterparts (1 and 6) exhibiting larger deviations.

The final comparison criterion is computational cost (Fig. 20). Formulations 0 and 5 are the most efficient due to the absence of corrective operations. Formulation 6, formulation 1, and formulation 15 incur moderate overhead from renormalization or density filtering. Formulation 16 is the most expensive because it combines both.

Table 6

Hugoniot pressures p_H obtained directly from SPH particles. Theoretical value: $p_H^{\text{theo}} = 110.51$ MPa. ϵ_H denotes the corresponding relative error.

Formulation	p_H [MPa]	ϵ_H [%]
Default (0)	124.58	12.7
Default renormalized (1)	130.01	17.7
Fluid particle (5)	124.68	12.8
Fluid particle renormalized (6)	129.52	17.2
Enhanced fluid (15)	90.66	−18.0
Enhanced fluid renormalized (16)	90.66	−18.0

**Fig. 20.** Total CPU time for all retained SPH formulations using 8 CPUs and 20 GB RAM.**Table 7**

Parameters and values examined in the sensitivity analysis; default values are highlighted in orange.

Parameter	Values tested
Inter-particle distance	8.89, 4.44, 2.22 mm
Impact velocity	100, 116, 150, 200, 300 m/s
Initial smoothing length constant CSLH	1.05, 1.20, 1.30
Smoothing length scheme	Varying, Constant
Time step scaling factor TSSFAC	0.2, 0.4, 0.678, 0.9
Linear artificial viscosity α_π	0.6, 0.8, 0.9, 1.0, 1.1, 1.2, 1.4
Quadratic artificial viscosity β_π	0.6, 0.8, 0.9, 1.0, 1.1, 1.2, 1.4

It will be shown later that the increasing instability in formulations 1 and 6 reduces their effective performance, while formulation 16 remains stable and permits a larger time-step scale factor TSSFAC, ultimately yielding a lower total simulation time.

5.2. Sensitivity analysis

Having identified the formulations capable of reproducing the fundamental hydrodynamic response, the next step is to assess the robustness of the single-impact results and identify the parameters most critical to the simulation of soft-body impacts. Seven parameters were systematically varied: inter-particle distance, impact velocity, initial smoothing length constant CSLH, smoothing length scheme (constant or adaptive), time step scaling factor TSSFAC, and the linear and quadratic artificial viscosity coefficients α_π and β_π . These parameters were evaluated for their effect on physical accuracy, numerical stability, and computational cost, using formulation 0 as a reference for stable cases and formulations 1 and 6 for instability-prone behaviors. The tested values are summarized in Table 7.

The inter-particle distance primarily governs SPH resolution. Reducing the spacing improves local shock resolution and energy dissipation, whereas coarser spacing stabilizes formulations 1 and 6 by broadening the support domain.

Impact velocity directly scales Hugoniot and stagnation pressures and modulates numerical stability; lower velocities reduce tensile-driven instabilities in formulations 1 and 6.

The initial smoothing length CSLH significantly affects the number of neighbors, dissipation, and stability: larger CSLH increases kernel overlap, smooths gradients, and stabilizes otherwise unstable formulations. Allowing the smoothing length to vary somewhat preserves neighbor counts during large deformations, whereas constant smoothing lengths lead to a more pronounced depletion.

The time step scale factor TSSFAC influences both stability and computational cost: smaller values stabilize formulations 1 and 6 by better resolving local gradients but increase CPU time, whereas larger values accelerate the simulation at the risk of instability.

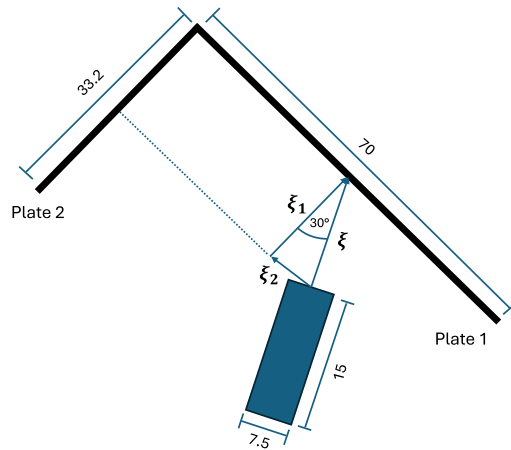


Fig. 21. Top-view schematic of the double-impact configuration (not to scale). The incident linear momentum ξ and its decompositions along the normal directions of each plate (ξ_1 and ξ_2) are shown.

Finally, artificial viscosity plays a key role in stability: increasing the linear term α_π enhances energy dissipation and suppresses tensile instabilities, while variations of the quadratic term β_π have negligible effects under the studied conditions.

Overall, the sensitivity analysis identifies the most influential parameters (inter-particle distance, CSLH, TSSFAC, α_π , and impact velocity) and provides guidance for stabilizing previously unstable formulations. In contrast, β_π is of secondary importance.

5.3. Double impact

The retained formulations are subsequently evaluated in a more representative bird ingestion scenario involving two consecutive impacts. This configuration constitutes a first controlled step toward realistic multi-stage loading where projectile fragments interact sequentially with engine components. The objective is to evaluate the retained SPH formulations, 0 (default), 1 (default renormalized), 5 (fluid particle), 6 (fluid particle renormalized), 15 (enhanced fluid), and 16 (enhanced fluid renormalized), under this more demanding configuration, and to establish a rigorously justified ranking based on their physical fidelity, numerical robustness, and computational efficiency.

The analysis is structured as follows. The physical and numerical characteristics of the test case are defined in Section 5.3.1. The metrics used to compare the retained formulations are presented in Section 5.3.2. These metrics are applied in Section 5.3.3, and the key findings summarized.

5.3.1. Case specification

The configuration consists of two rigid plates arranged orthogonally, forming a corner geometry. A right circular cylindrical projectile representing the bird strikes the first plate with an incidence angle of 30° relative to its normal. A top-view schematic of the setup is provided in Fig. 21.

The projectile is a gelatin-based surrogate with a mass of 634 g, length 15 cm, and diameter 7.5 cm. It impacts the first plate centrally at 120 m/s. No experimental validation is attempted at this stage: the deformation and flow are analyzed qualitatively, acknowledging the limitations of the hydrodynamic constitutive law.

Sequential momentum transfer is induced by introducing a second plate perpendicular to the first, with dimensions 130 cm $\times$ 33.2 cm, ensuring interception of the post-impact projectile flow. The inclusion of a second target is motivated by two factors:

- (i) In SPH gather approximations (see Fig. 5(a)) with spatially varying smoothing lengths, momentum conservation is non-trivial. The formulation proposed by Lacombe [47] is reported to recover consistency [45].
- (ii) In bird ingestion, load transmission across successive components is governed predominantly by progressive momentum partitioning.

To isolate the influence of the double-impact configuration, all other modeling parameters are kept consistent with the single-impact study. Only the following modifications are introduced:

- *Equation of state*: the non-linear relation is employed to better capture strong compressibility effects during high-velocity impact.
- *Termination time*: the simulation is extended to 8.5 ms to fully capture both impacts and subsequent flow evolution.
- *Resolution*: the inter-particle spacing is set to 2 mm to appropriately capture the bird's fragmentation.

An overview of the full three-dimensional model and associated SPH domain is presented in Fig. 22. The governing parameters are summarized in Table 8 for completeness and reproducibility.

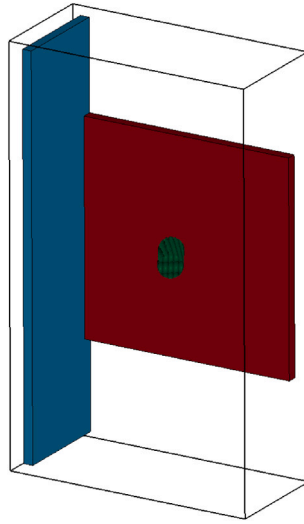


Fig. 22. Three-dimensional LS-DYNA model of the double-impact setup. The black box indicates the SPH computational domain.

Table 8

Primary model parameters defining the double-impact configuration. All quantities are expressed in SI units.

Parameter	Value	Unit
Bird mass	0.634	kg
Bird geometry	Cylinder	–
Length-to-diameter ratio	2	–
Bird length	0.150	m
Impact velocity	120	m/s
Impact angle	30	deg
Simulation time	8.5×10^{-3}	s
Inter-particle distance	2×10^{-3}	m
Plate 1 dimensions	0.70×0.70	m $\times$ m
Plate 2 dimensions	1.30×0.332	m $\times$ m

5.3.2. Ranking metrics

Evaluation under the double-impact configuration requires metrics that reflect the dual constraints of ingestion simulations: physical realism and numerical stability. The retained formulations are therefore assessed with respect to four complementary criteria:

- (i) *Physical deformation*: the projectile flow is examined qualitatively. Formulations that preserve continuous material coherence, avoid spurious fragmentation, and reproduce realistic spreading receive favorable scores.
- (ii) *Numerical stability*: assessed using the energy ratio e_r , defined in Section 5.1.2. Formulations demonstrating sustained closeness to unity and avoiding abrupt fluctuations are considered robust.
- (iii) *Computational efficiency*: measured through total CPU time. Lower runtimes are favorable when achieved without compromising stability or realism.
- (iv) *Momentum transfer*: the momentum transmitted to each plate is compared against theoretical expectations based on mass, velocity, and incidence. Evaluation considers both normal components and total transmitted momentum, representing the formulation's ability to conserve and partition momentum through sequential impacts.

Given the intrinsic difference between these metrics, no direct quantitative aggregation is attempted. Instead, a qualitative scoring system is introduced, assigning discrete values from 1 to 5, where 1 indicates very good performance and 5 very poor performance. The scoring scale is summarized in Table 9.

5.3.3. Selection process

All retained SPH formulations are evaluated using the double-impact configuration defined in Section 5.3.1. One exception concerns the time step scale factor (TSSFAC) for formulations 1 and 6, which require a reduced value of 0.4 to mitigate numerical instabilities, as identified in Section 5.2. Evaluation is performed according to the four criteria introduced in Section 5.3.2.

Table 9

Qualitative scoring scale for multi-criteria formulation ranking.

Score	Interpretation
1	Very good
2	Good
3	Neutral
4	Poor
5	Very poor

Table 10Simulated versus theoretical momentum transfer for plate 1 (ξ_1), plate 2 (ξ_2), and total (ξ). $\epsilon_{./1/2}$ indicate the corresponding relative errors.

Formulation	ξ_1 [kg m/s]	ϵ_1 [%]	ξ_2 [kg m/s]	ϵ_2 [%]	ξ [kg m/s]	ϵ [%]
Default (0)	76.7	16.4	38.4	0.8	85.8	12.7
Default renormalized (1)	76.4	15.9	39.3	3.2	85.9	12.9
Fluid particle (5)	76.8	16.5	37.9	0.3	85.6	12.5
Fluid particle renormalized (6)	76.4	16.0	39.1	2.8	85.9	12.9
Enhanced fluid (15)	76.4	15.9	38.5	1.2	85.5	12.4
Enhanced fluid renormalized (16)	75.6	14.7	38.7	1.6	84.9	11.6

Physical deformation. The first criterion assesses the physical plausibility of the bird surrogate's deformation, illustrated through snapshots at five representative times in Fig. 23. All formulations exhibit flow-like behavior, consistent with the hydrodynamic constitutive model, but differences are evident:

- Formulations 0 and 5, as well as 1 and 6, yield identical deformation patterns, confirming that the choice of momentum equation (default or fluid particle) has negligible impact on global deformation in typical bird strike scenarios.
- Normalized formulations (1, 6, 16) maintain superior boundary coherence; leading particles remain attached to the bulk upon impact, avoiding detachment observed in non-normalized variants.
- Tensile instabilities persist in formulations 1 and 6 despite reduced TSSFAC, manifesting as particle blow-up.
- Non-normalized formulations exhibit particle clumping under tensile stress, mitigated in formulation 15 through density filtering. Normalized formulations produce the most coherent and physically consistent deformation fields.

Numerical stability. The second criterion evaluates stability through the energy ratio e_r over time (Fig. 24). Formulations 0 and 5 remain stable. Enhanced fluid formulations 15 and 16 show initial decay but stabilize over longer simulation times, demonstrating suitability for extended multi-impact simulations. Formulations 1 and 6 exhibit tensile-driven particle blow-up despite the reduced TSSFAC, with formulation 6 diverging significantly.

The mean number of neighbors (Fig. 25) illustrates the correlation between neighbor count and stability. Following the first impact, particle dispersion reduces neighbors across all formulations. Non-normalized variants maintain artificially higher counts due to particle clumping, whereas normalized formulations exhibit more realistic distributions.

Computational cost. The total CPU time required for each formulation is presented in Fig. 26. In contrast to the single-impact case, no monotonic relationship emerges between algorithmic complexity and computational cost. Although formulation 15 employs an additional density-filtering step relative to formulations 0 and 5, it executes approximately 20% faster. Despite its elevated per-step workload, formulation 16 completes the simulation 44% faster than formulation 1. This improvement stems from enhanced numerical stability, which supports a higher TSSFAC. By contrast, formulations 1 and 6 exhibit instability-driven particle blow-up. Even with reduced TSSFAC, these instabilities impose stringent CFL constraints, substantially increasing total runtime.

Momentum transfer. The final criterion evaluates momentum transfer to the two plates. With an incidence angle of 30° (Fig. 21), theoretical values are:

$$\xi_1^{\text{theo}} = 65.9 \text{ kg m/s}, \quad (45)$$

$$\xi_2^{\text{theo}} = 38.0 \text{ kg m/s}, \quad (46)$$

$$\xi^{\text{theo}} = 76.1 \text{ kg m/s}. \quad (47)$$

Numerical values and relative errors are summarized in Table 10. All formulations conserve total momentum within 11.6%–12.9% error. The largest deviations occur in the first plate, while the second plate receives momentum close to theoretical predictions. Formulation 16 exhibits the lowest overall error.

5.3.4. Ranking and performance overview

The final ranking synthesizes the comparative analysis across deformation realism, numerical stability, computational cost, and momentum transfer. Table 11 summarizes the individual scores and their cumulative ranking, with lower values indicating better overall performance. The scores provide a qualitative synthesis of the comparative assessment presented in Section 5.3.3. Although

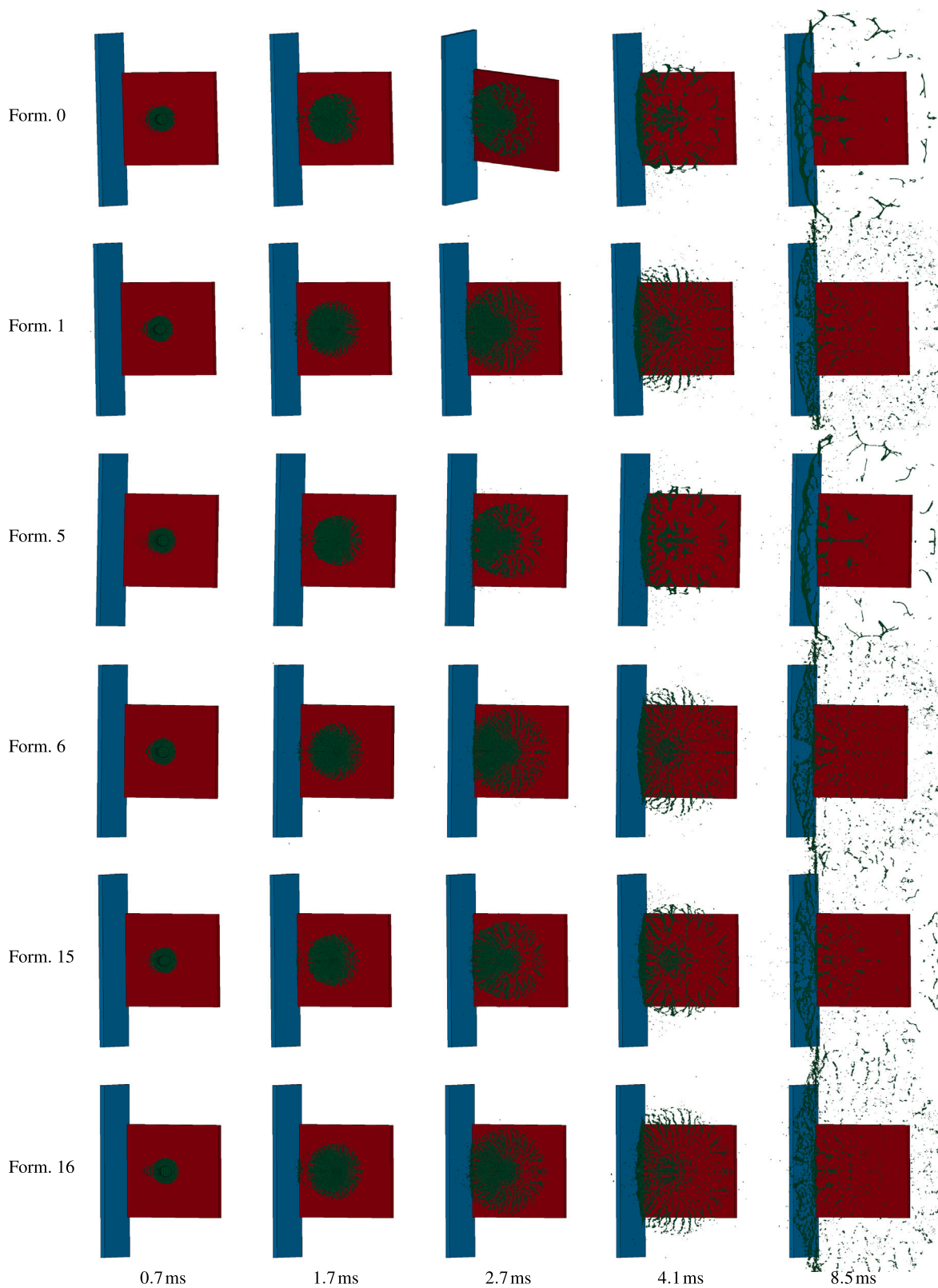


Fig. 23. Time evolution of bird deformation for all retained SPH formulations under the double-impact configuration. Rows correspond to formulations; columns to simulation times.

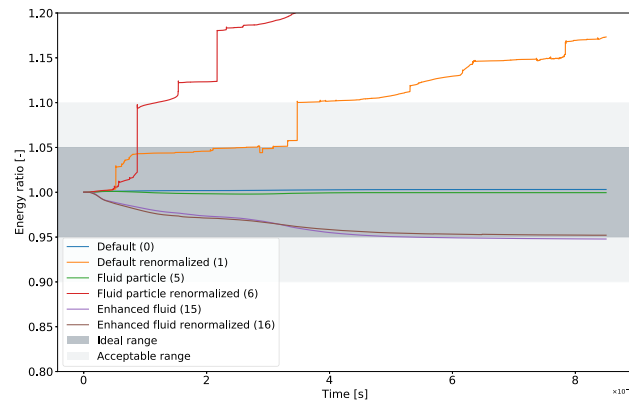


Fig. 24. Energy ratio e_r as a function of time for all retained SPH formulations under double-impact conditions.

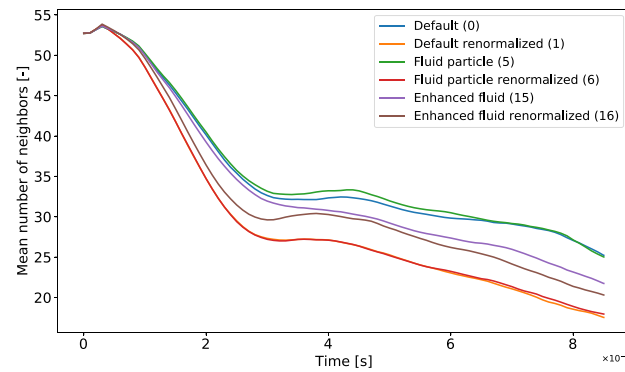


Fig. 25. Time evolution of the mean number of neighbors per particle for all retained formulations. Normalized formulations yield lower, physically representative neighbor counts.

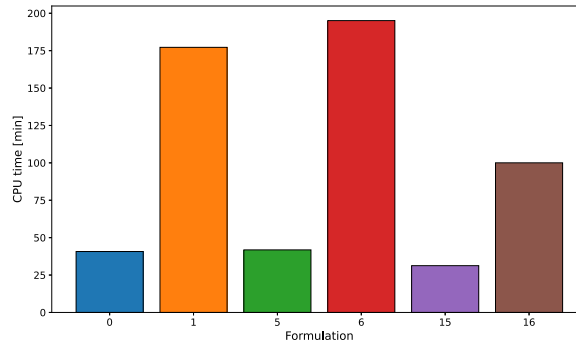


Fig. 26. Total CPU time for all retained SPH formulations under double-impact conditions (MPP, 8 MPI ranks, 20 GB RAM).

their assignment involves expert judgment, each score is consistently based on the objective observations discussed in the preceding sections, including the severity of the identified physical and numerical behaviors.

Interpretation of scores.

- Physical deformation: Normalized formulations (1, 6, 16) produce the most realistic deformation with smooth boundaries and minimal particle clumping, scoring 1. Non-normalized formulations 0 and 5 exhibit spurious clustering and boundary ejection (score 4), while formulation 15 shows moderate improvement in terms of clustering (score 3).
- Computational cost: Formulation 15 benefits from density filtering that stabilizes particle motion, completing 20% faster than 0 and 5 (score 1). Formulations 0 and 5 are moderately efficient (score 2). Normalized formulations 1, 6, and 16 have longer runtimes due to stricter time step requirements and are conservatively scored 4.

Table 11

Ranking of retained SPH formulations based on four performance criteria. Lower scores indicate superior performance.

Formulation	Deformation	CPU	Stability	Momentum	Total
Default (0)	4	2	1	2	9
Default renormalized (1)	1	4	4	2	11
Fluid particle (5)	4	2	1	2	9
Fluid particle renormalized (6)	1	4	5	2	12
Enhanced fluid (15)	3	1	2	2	8
Enhanced fluid renormalized (16)	1	4	2	1	8

- Numerical stability: Formulations 0 and 5 remain stable (score 1). Enhanced formulations 15 and 16 exhibit minor energy drift but stabilize (score 2). Formulations 1 and 6 are prone to particle blow-up, with formulation 6 being the most unstable (scores 4 and 5, respectively).
- Momentum transfer: All formulations achieve similar momentum transfer, with formulation 16 showing slightly superior accuracy (score 1). All others are scored 2.

Overall assessment. The enhanced fluid formulations (15 and 16) demonstrate the most balanced performance under double-impact conditions. Formulation 16 (*enhanced fluid renormalized*) excels in physical realism, deformation fidelity, numerical robustness, and momentum conservation, making it the most accurate choice for demanding simulations. Formulation 15 (*enhanced fluid*) offers a compelling compromise, minimizing computational cost while maintaining acceptable physical consistency and avoiding severe tensile instabilities.

By contrast, non-normalized formulations 0 and 5, though stable and efficient, exhibit unphysical deformation due to tensile locking and boundary particle projection. Normalized formulations 1 and 6 improve deformation fidelity but remain unstable, with formulation 6 particularly sensitive to tensile-driven particle blow-up.

5.4. Experimental validation

The preceding numerical benchmarks identified formulations 15 and 16 as the most promising candidates. Although originally developed in LS-DYNA for weakly compressible flows, the underlying density filtering of Randles and Libersky [61] is not inherently restricted to such applications. Their applicability to bird strike is therefore assessed by comparison with single-impact experiments, using the default renormalized formulation 1 as the reference.

5.4.1. Test description

The test, conducted by the University of Dayton Research Institute for Safran Aero Boosters, involves a gelatin-based bird surrogate impacting a static blade. The projectile has a mass of 44.54 g and a geometry reconstructed from high-speed footage. The impact velocity is 470.3 m/s at approximately three-quarters of the blade length. The computational domain and orthonormal reference frame are illustrated in Fig. 27.

Three SPH formulations are evaluated:

- Default renormalized (1), serving as the industrial baseline;
- Enhanced fluid (15), offering the greatest computational efficiency;
- Enhanced fluid renormalized (16), providing the highest physical fidelity.

5.4.2. Visual correlation

Figs. 28 and 29 compare simulation snapshots from formulations 1, 15, and 16 with experimental footage at $t = 0.045, 0.20$, and 0.30 ms.

Top and side views (Figs. 28 and 29, respectively) show both enhanced formulations capturing the splitting of the projectile into a coherent mass and trailing cluster, connected by a fine stream consistent with experimental observations.

5.4.3. Quantitative correlation

Displacements of the leading and trailing-edge tips along x and y axes are plotted in Fig. 30. All formulations closely match experimental amplitude and phase.

Root Mean Square Error (RMSE) analysis over $[0, 2.5]$ ms (Table 12), beyond which post-processing accuracy is not guaranteed, confirms that formulation 16 achieves the lowest average error, 2.85 mm, outperforming formulation 15 (2.89 mm) and formulation 1 (3.08 mm).

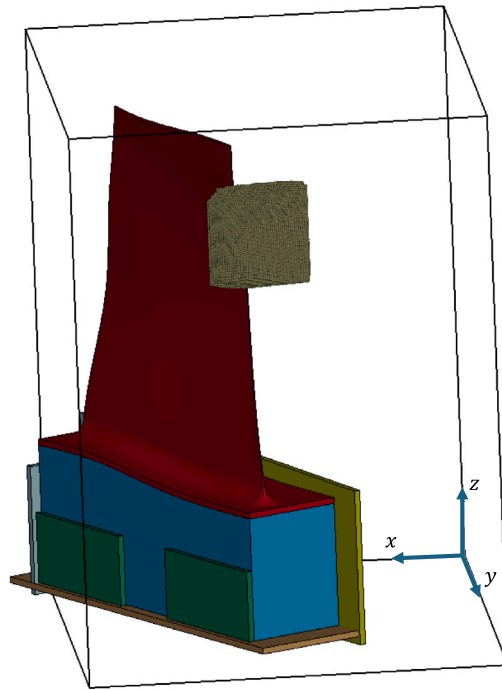


Fig. 27. Simulation setup for experimental validation. The black box represents the SPH domain. The gelatin projectile impacts a fixed blade at 470.3 m/s.

Table 12

RMSE of leading-edge (LE) and trailing-edge (TE) displacements relative to experiment over [0, 2.5] ms.

Formulation	LE x [mm]	LE y [mm]	TE x [mm]	TE y [mm]
Default renormalized (1)	5.06	1.57	3.83	1.89
Enhanced fluid (15)	4.05	1.35	4.22	1.94
Enhanced fluid renormalized (16)	3.75	1.45	4.26	1.95

Computational considerations. When computations are restricted to the SPH domain (black box in Fig. 27), the runtimes remain comparable: 15 achieves 10 h 18 min, 1 reaches 10 h 31 min, and 16 requires 10 h 51 min (TSSFAC = 0.9). In contrast, extending the SPH calculations to the full domain reveals a pronounced divergence. Formulation 16 remains stable, completing in 15 h 49 min, whereas formulation 1 becomes impractical due to CFL-constrained time steps, leading to an estimated runtime exceeding 270 h.

Further investigations were conducted using formulations 0, 5, 6, and 15 to isolate the mechanism responsible for the observed instability. The comparison between formulations 0 and 1, as well as between 5 and 6, shows that the introduction of renormalization systematically leads to a loss of stability under extended computational domains, independently of the momentum equation employed. In contrast, comparing formulations 6 and 16, which differ only by the addition of density filtering, demonstrates that this operation restores stability and prevents the CFL time step collapse observed in the renormalized formulation without filtering. These results identify renormalization as the source of the instability and density filtering as the stabilizing mechanism.

Integrated assessment. The enhanced fluid formulations reproduce experimental deformation and displacement with high fidelity. RMSE confirms superior performance relative to the baseline formulation 1. Formulation 16 maintains full numerical stability under extended domains, making it the most reliable predictive option, whereas formulation 15 provides a computationally efficient alternative with minor trade-offs in accuracy.

6. Conclusions

This work presented a comprehensive evaluation of fifteen smoothed particle hydrodynamics (SPH) formulations implemented in LS-DYNA with the objective of identifying those most suitable for simulating multi-stage bird ingestion in aeroengine environments. The investigation employed multiple complementary analyses to rigorously evaluate physical fidelity, numerical stability, and computational efficiency: a single-impact benchmark, a sensitivity study, a double-impact configuration representative of multi-stage ingestion, and an experimental validation campaign.

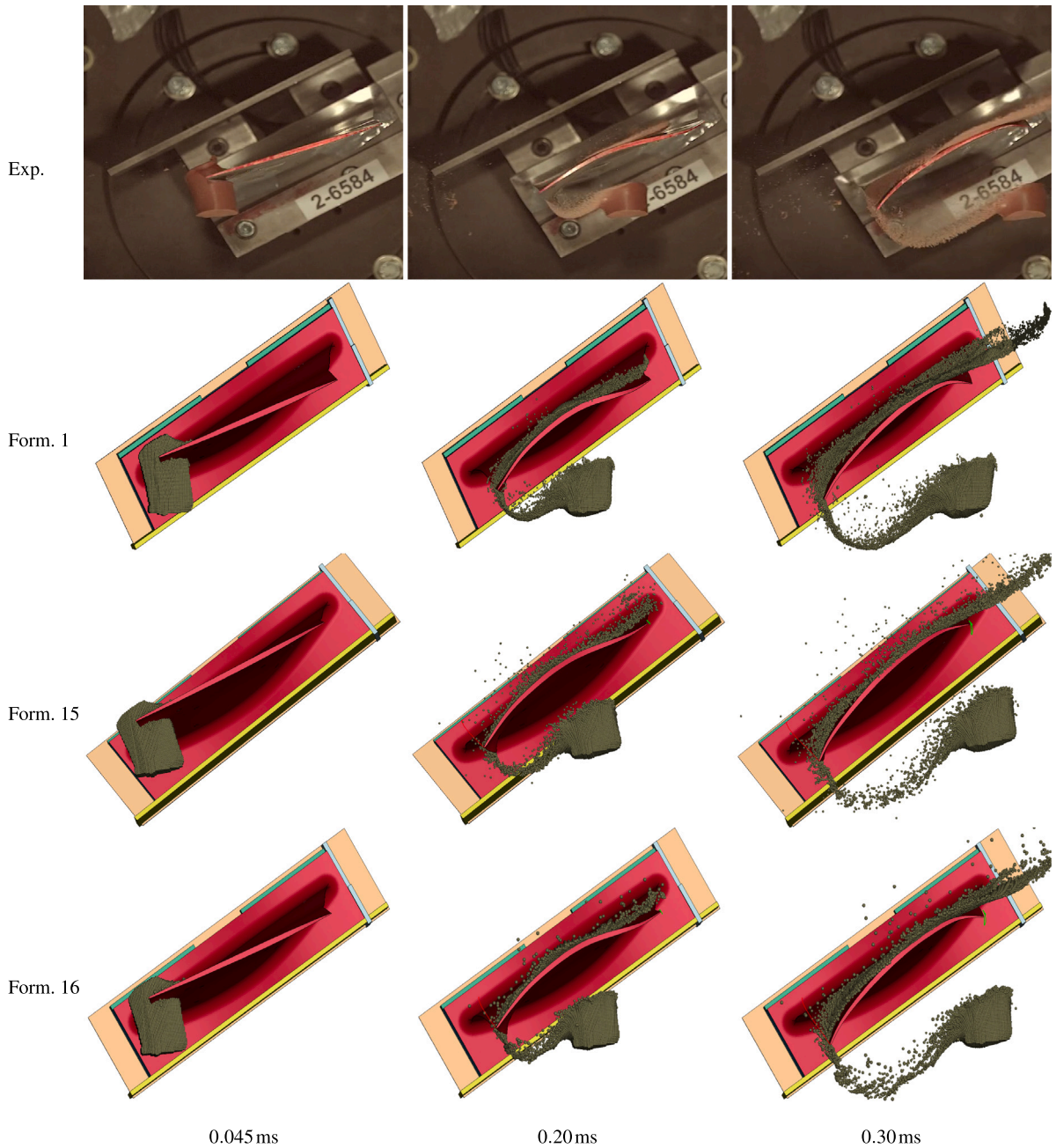


Fig. 28. Top view comparison between SPH simulations and experimental footage at three time instants. Red and green curves indicate the trajectories of the leading-edge and trailing-edge tips. The enhanced formulations reproduce the observed flow footprint and gross deformation features.

The single-impact study established a physically representative baseline scenario and enabled an initial screening of the fifteen formulations, from which formulations 9 (ASPH) and 12 (MLSPH) were eliminated due to unphysical particle behavior. Quantitative assessment of the remaining nine candidates then led to the further elimination of formulations 2 (symmetric), 3 (symmetric renormalized), and 10 (ASPH renormalized) due to unphysical energy behavior or numerical instability. Six formulations remained viable: 0 (default), 1 (default renormalized), 5 (fluid particle), 6 (fluid particle renormalized), 15 (enhanced fluid), and 16 (enhanced fluid renormalized). The sensitivity analysis identified the key parameters governing robustness in high-velocity soft-body impact and clarified the stabilization strategies needed for the renormalized variants. The double-impact configuration provided a more stringent stress test and revealed that formulations 15 and 16 outperform all others in terms of stability and physical realism, with formulation 16 being the only renormalized approach to remain stable throughout the entire event. Finally, the experimental

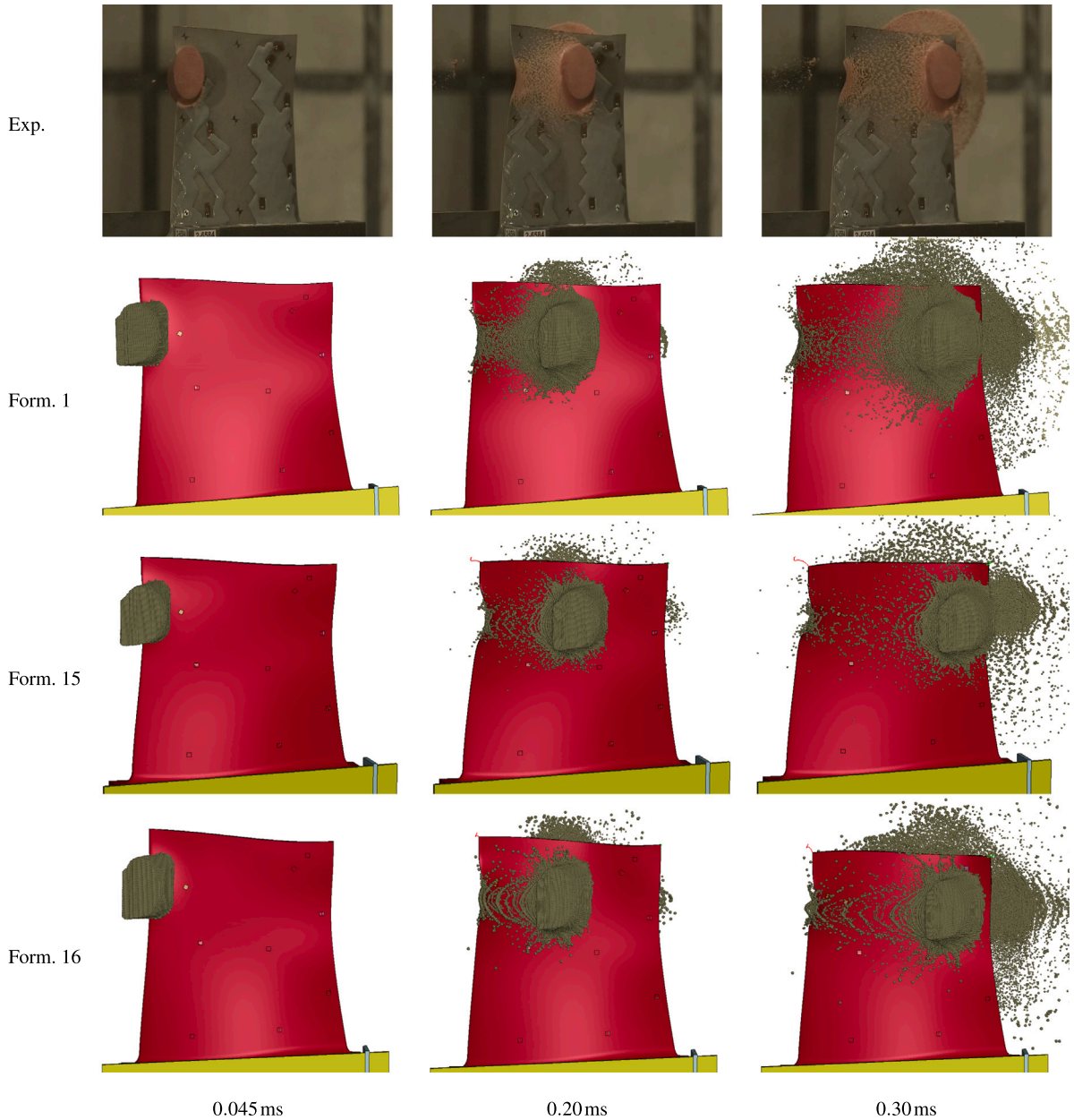


Fig. 29. Side view comparison between SPH simulations and experimental footage at three time instants. Red and green curves indicate the trajectories of the leading-edge and trailing-edge tips. The enhanced formulations reproduce the observed flow footprint and gross deformation features.

validation confirmed that formulations 15 and 16 achieve predictive accuracy consistent with physical tests, consolidating their suitability for realistic industrial applications.

From these findings, two formulations emerge as recommended choices for multi-stage ingestion scenarios. Formulation 15 offers the lowest computational cost while maintaining physically credible behavior. Formulation 16 provides the best overall compromise between realism, stability, and efficiency, benefiting from a stable renormalization procedure, improved behavior under large deformation, and increased allowable TSSFAC. In contrast, formulations 0 and 5, despite their computational speed, suffer from tensile locking and clustering artifacts, whereas formulations 1 and 6 remain vulnerable to tensile-driven particle instabilities that limit their applicability in extended ingestion sequences.

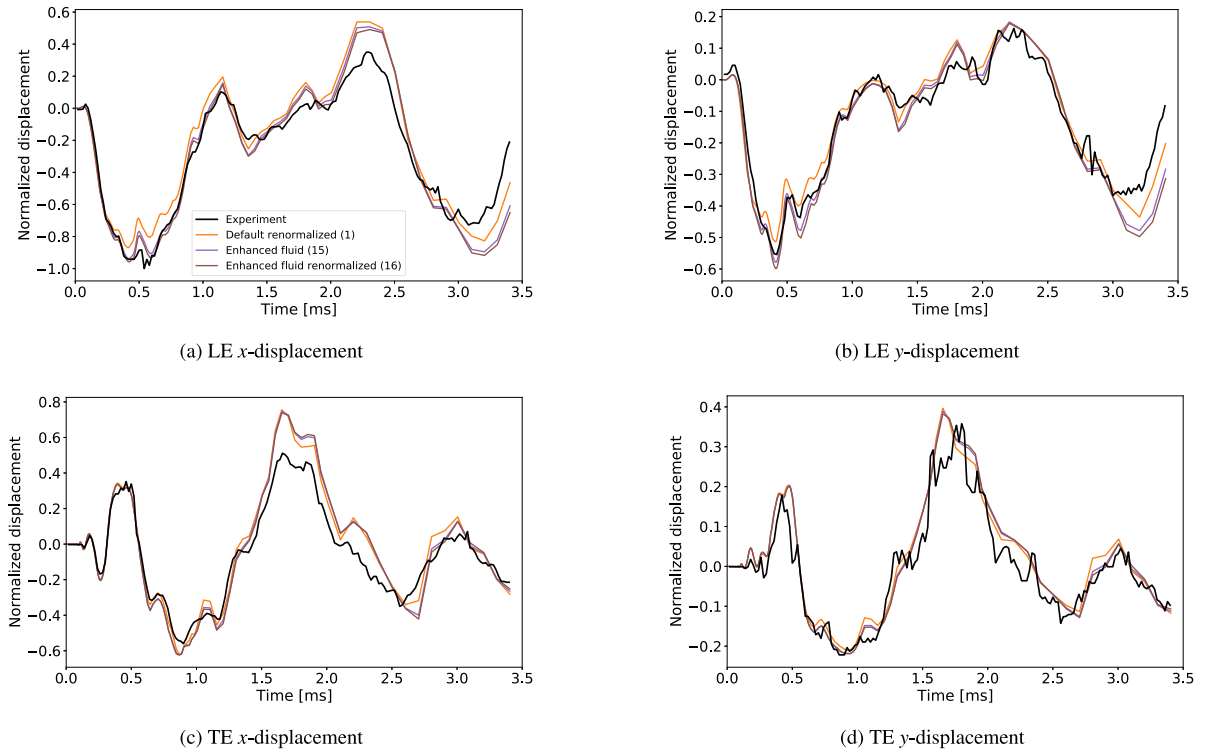


Fig. 30. Normalized tip displacements of leading edge (LE) and trailing edge (TE) in x and y , comparing SPH simulations and experiment.

The results open several promising avenues for future research. First, the two validated formulations should be exercised in a full-scale multi-stage ingestion simulation to confirm their robustness and predictive capability under operational conditions. Second, their performance should be evaluated for other soft-body projectiles such as ice, enabling broader applicability across certification-relevant scenarios. Third, the impact of alternative constitutive laws, including hyperelastic models [72], should be explored to assess the generality of the conclusions drawn here. In the longer term, the degradation of particle resolution observed during extended simulations highlights a fundamental limitation of SPH for large-deformation, long-duration events. Methods such as the particle finite element method (PFEM) [73], which incorporate remeshing mechanisms capable of preserving spatial resolution, represent promising candidates for overcoming these constraints and warrant systematic investigation.

CRediT authorship contribution statement

E. Smeyers: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **A. Radermecker:** Writing – review & editing, Validation, Methodology. **R. Barale:** Writing – review & editing, Validation. **J.-P. Ponthot:** Writing – review & editing, Validation, Funding acquisition.

Declaration of generative AI and AI assisted technologies

During the preparation of this work, the authors used generative artificial intelligence tools such as ChatGPT to assist with wording and language editing. After using these tools, the authors carefully reviewed and edited the content and take full responsibility for the content of the published article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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