See, Balance, Exchange

Building an Operational Toolkit for Active Distribution Networks under High DER Penetration

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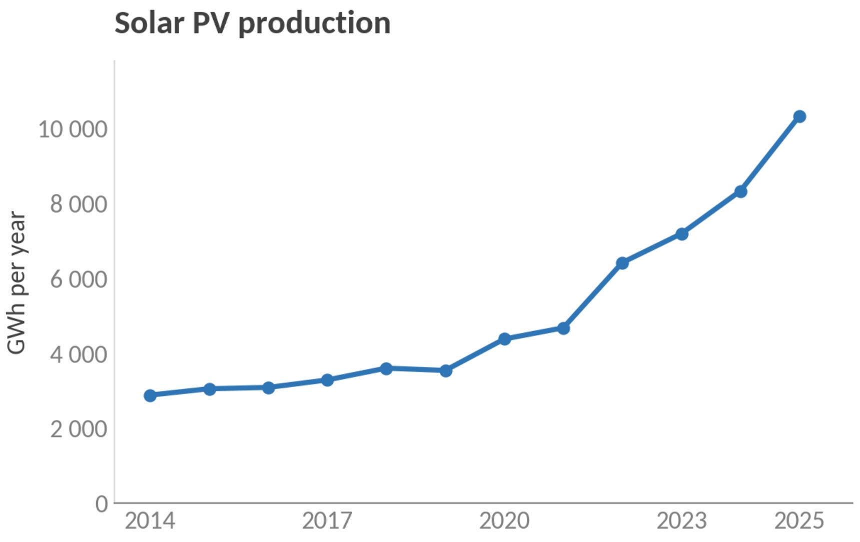
Maurizio Vassallo

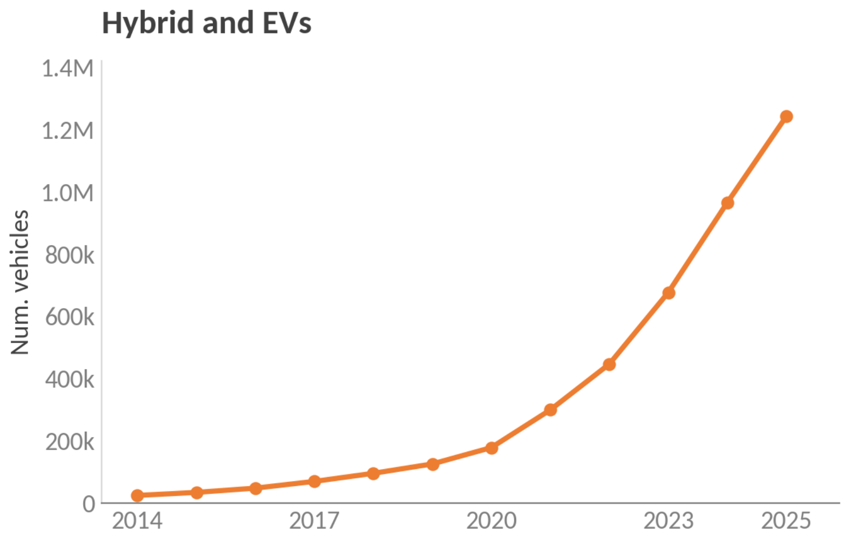
LIÈGE université

Electrification arrives at the low-voltage level

- Meeting decarbonization targets means electrifying heat, transport and industry
- Distributed energy resources (DERs) are installed at the low-voltage (LV) level: photovoltaic panels (PVs), heat pumps (HPs), electric vehicles (EVs)
- DERs are installed in months



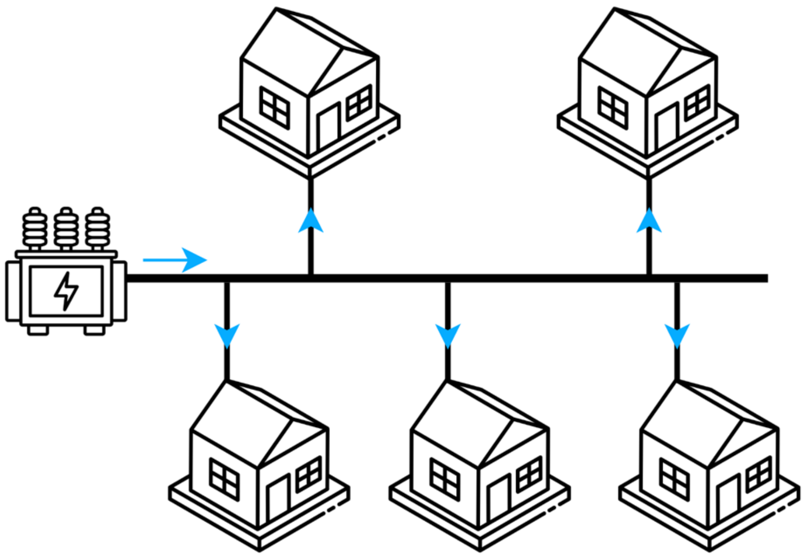
ELIA: Mix électrique (2025)

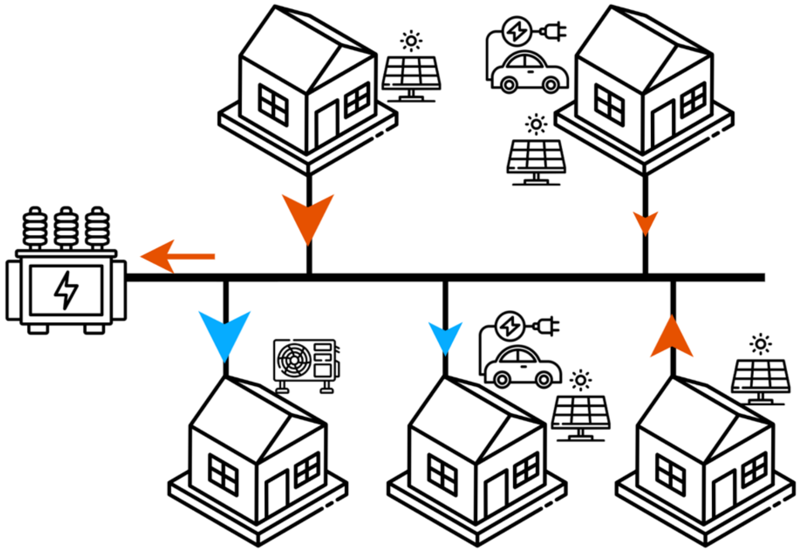


Statbel: Electric boom in Belgium (2025)

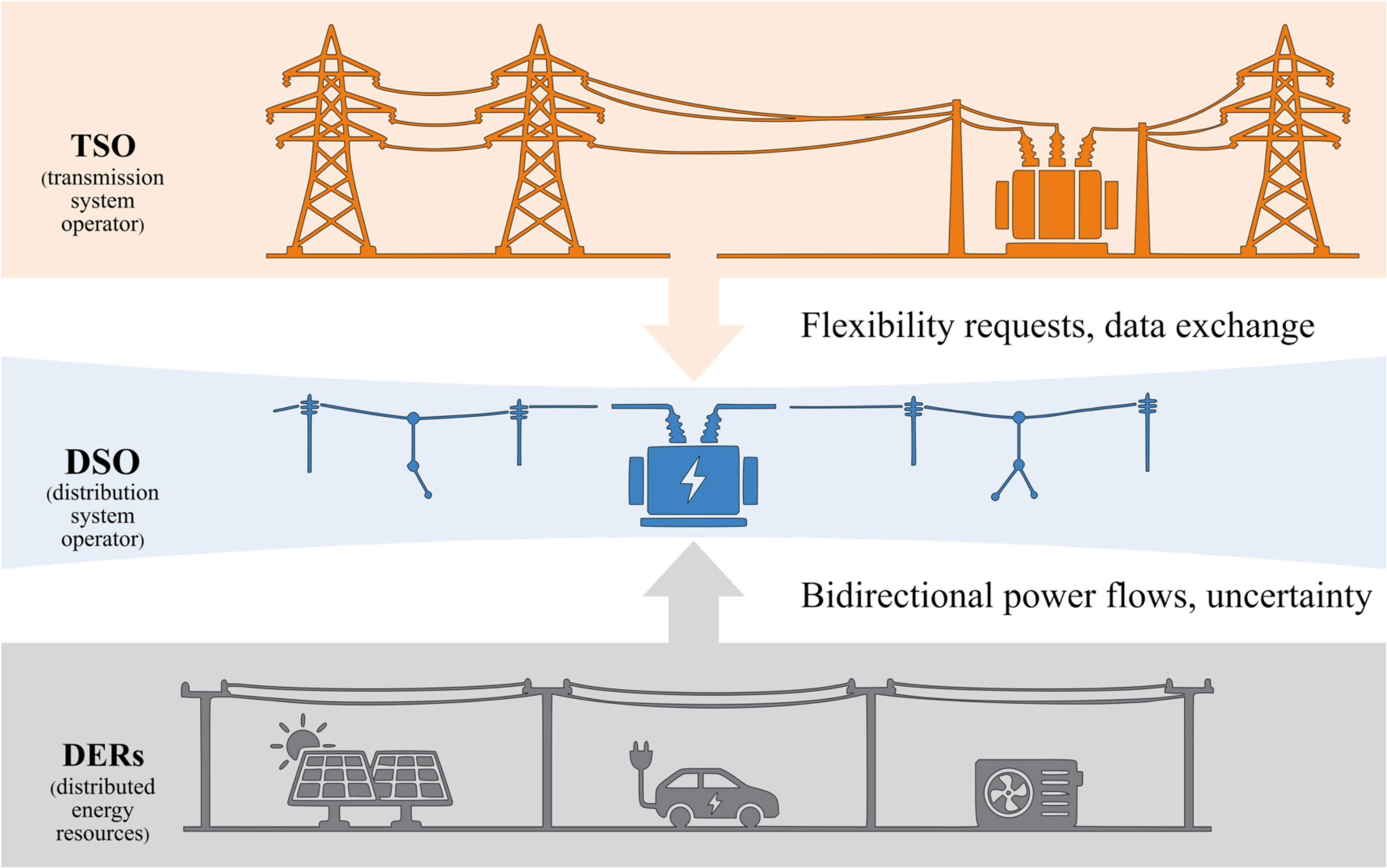
LV networks were not designed for this

- LV networks were built decades ago as passive infrastructure: one-directional flows, no measurement, sized on worst-case assumptions
- DERs break all three assumptions
- Reinforcement takes years





The DSO is squeezed from two directions



Consequence: DSOs must evolve from passive network owners into active system managers

This thesis in one sentence

A distribution operator has to identify its network, balance it, and distribute its capacity
This thesis builds one method for each capability

SEE	BALANCE	EXCHANGE
Network identification	Phase reconfiguration	Capacity trading

Deployment reality: what prevents coordination from reaching deployment?

SEE

Customer topological path identification (TPI)

The problem

Topological Path Identification problem (TPI)
Identifying the path of each customer inside a power distribution network

DSOs lack information about the customer paths	+	Heterogeneous information spread across different files	+	Limited and/or inaccurate data about network elements

=

The DSO cannot reliably identify the customer topological paths

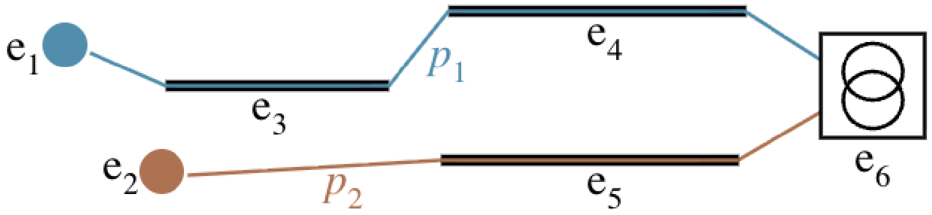
Benefits

Operationally
Locate faults and restore supply faster

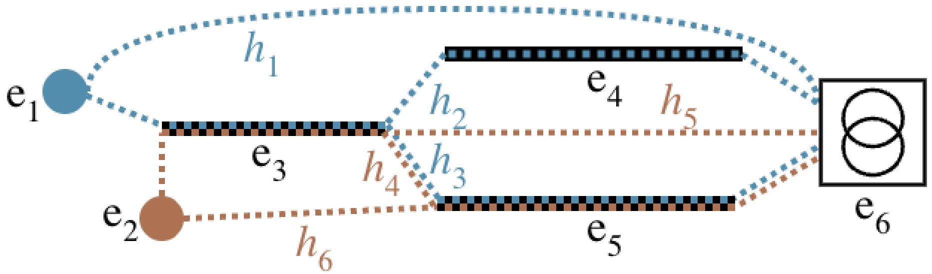
Analytically
Perform network analysis and anticipate potential power quality issues

Planning
Make informed decisions such as network investments

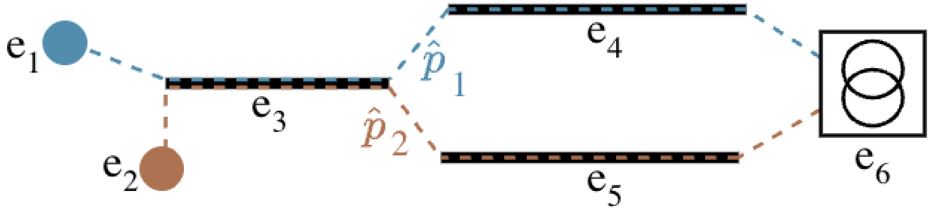
Paths



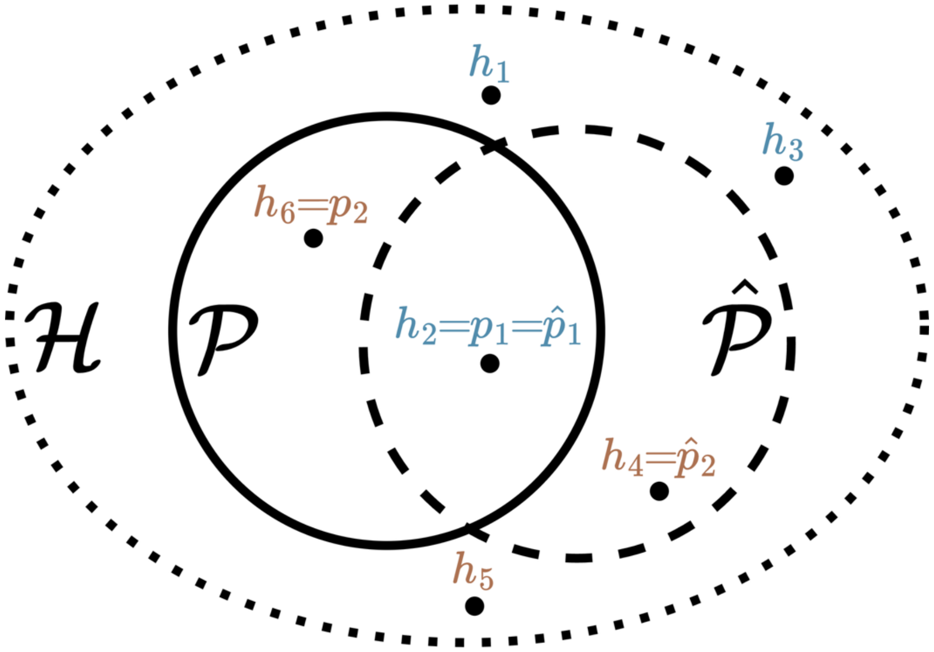
Real paths (P)



Hypothetical paths (H)



Estimated paths $(\hat{P})$



The methodology

Raw information → Transform into explicit rules → Generate candidates (A*) → Optimize (ILP) → Diagnose

Inputs: static GIS records and the customer-to-feeder-junction assignments held by the DSO

Output: feasible paths for the customers, plus a list of issues

Generate candidates, then select the best set

1. Generate: A* search

Enumerate all physically plausible routing options connecting each customer to the distribution grid

Max connection distance: 50 m

Max total path length: 500 m

Max paths for customer: 35

2. Select: ILP optimization

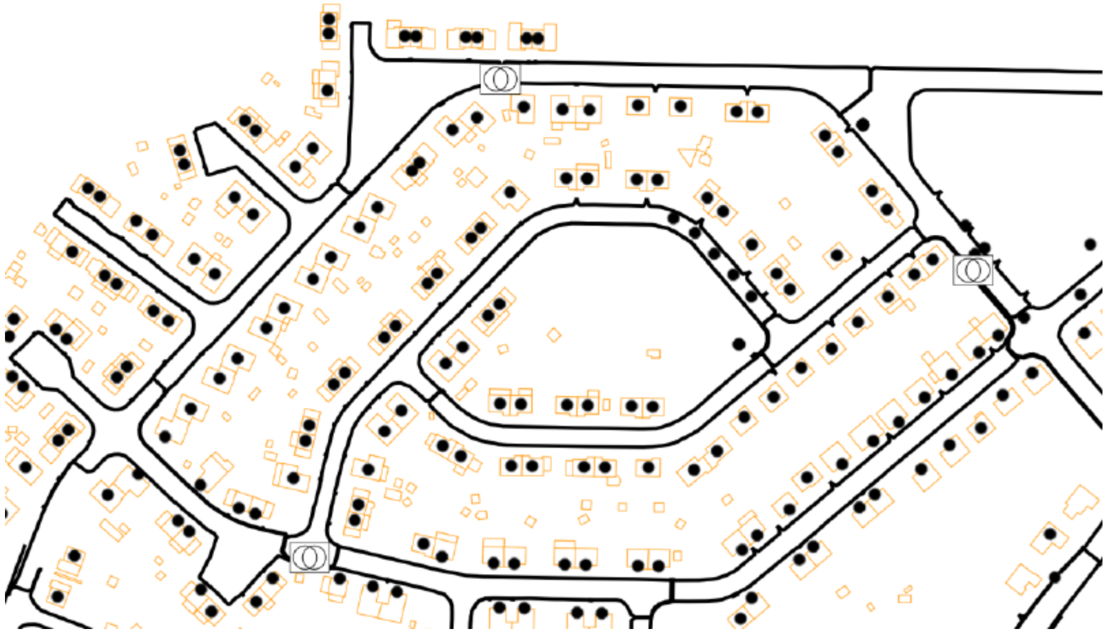
Maximize identified customer connections while penalizing unnecessary use of grid elements

Path Validity: Path elements must share a feeder

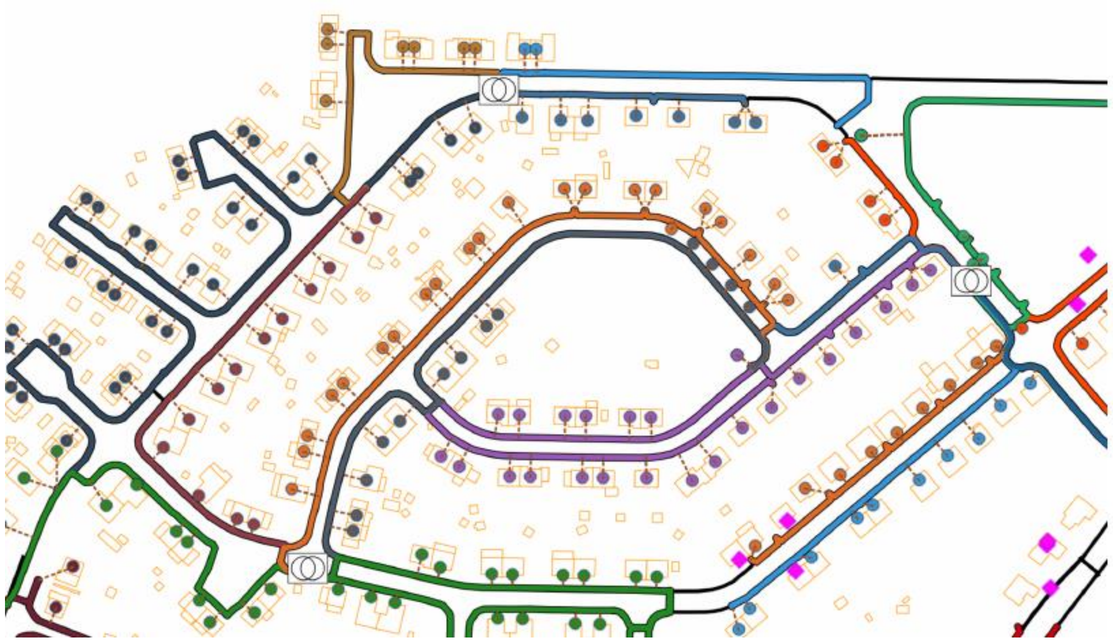
Unique customer-path: each customer is associated with at most one selected path

Unique element-feeder: each element belongs to only one feeder

Real Belgian LV network



Before

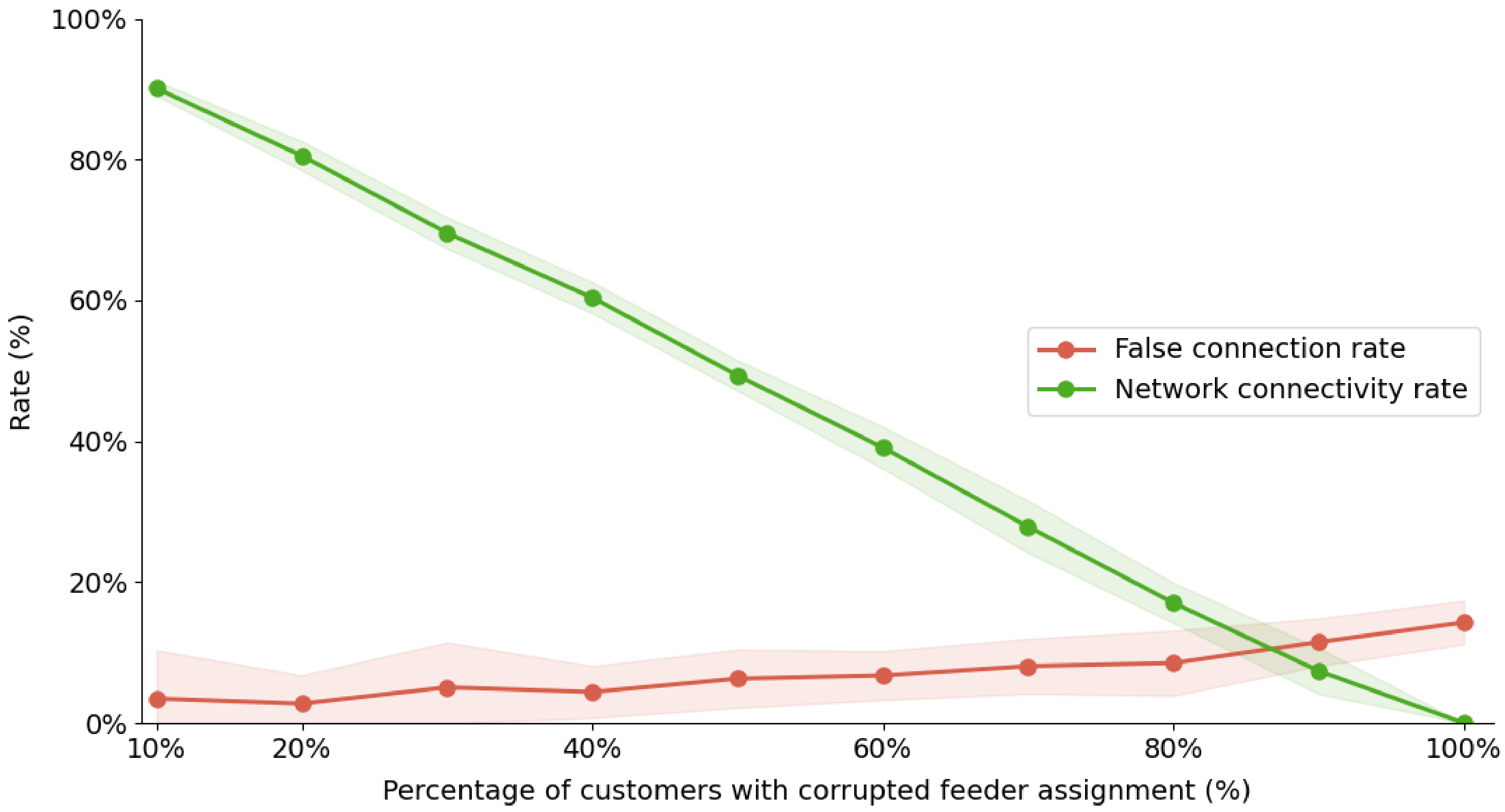


After

Network elements	Customers	Candidate paths	A* generation	ILP solving	Customers identified
1 089	526	17 335	264 s	5.8 s	88%

What if the records are wrong?

- **Robustness analysis**: randomly corrupt 10-100% of the customer-to-feeder assignments on a sub-network



Takeaways

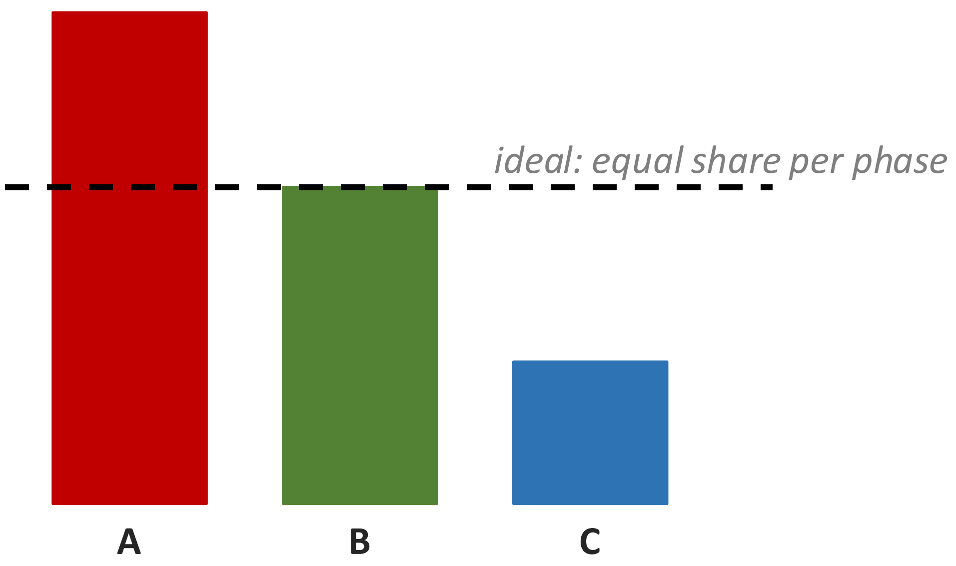
- A feasible topological path is recovered for 88% of 526 customers on a real Belgian LV network
- Static GIS records are enough: no smart-meter time series, no new hardware
- The ILP reaches optimality in seconds once the A* stage has pruned the candidate set
- Under corrupted records the framework refuses bad connections

BALANCE

Customer phase reconfiguration

Problem: unbalanced grids

- Power distribution networks are built for balanced 3-phase loads, but the rapid growth of DERs disrupts this balance



Phase unbalance leads to:

- Difficulty maintaining voltage levels
- Higher energy losses
- Accelerated equipment aging
- Increased operational costs for DSOs

Our solution

No power flow calculation
The objective is the load unbalance itself, which keeps the problem tractable

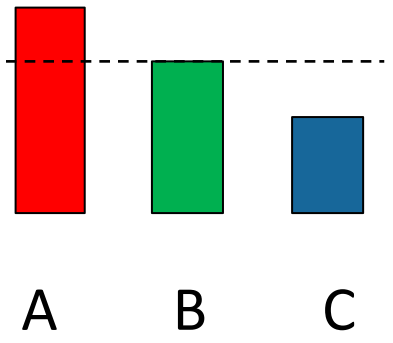
Diverse DERs included
PVs, EVs and HPs, each with its own temporal profile

Year-long optimization
A whole year at 15-minute resolution (about 35k timesteps)

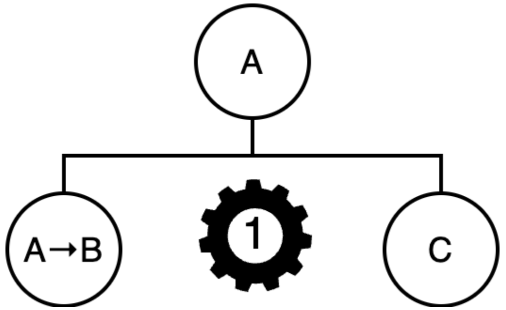
Optimization Problem

$$\min_{\mathbf{B}} \; \lambda^{P} \cdot P^{unb} + \lambda^{\Psi} \cdot \Psi^{chg} + \lambda^{D} \cdot D^{wgt} \qquad \text{(a)}$$

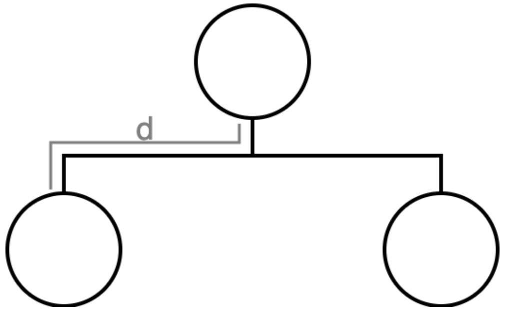
Load phase unbalance



Reconfiguration count



Distance impact



Optimization Problem

$$\min_{\mathbf{B}} \; \lambda^{P} \cdot P^{unb} + \lambda^{\Psi} \cdot \Psi^{chg} + \lambda^{D} \cdot D^{wgt} \qquad \text{(a)}$$

$s.t.$

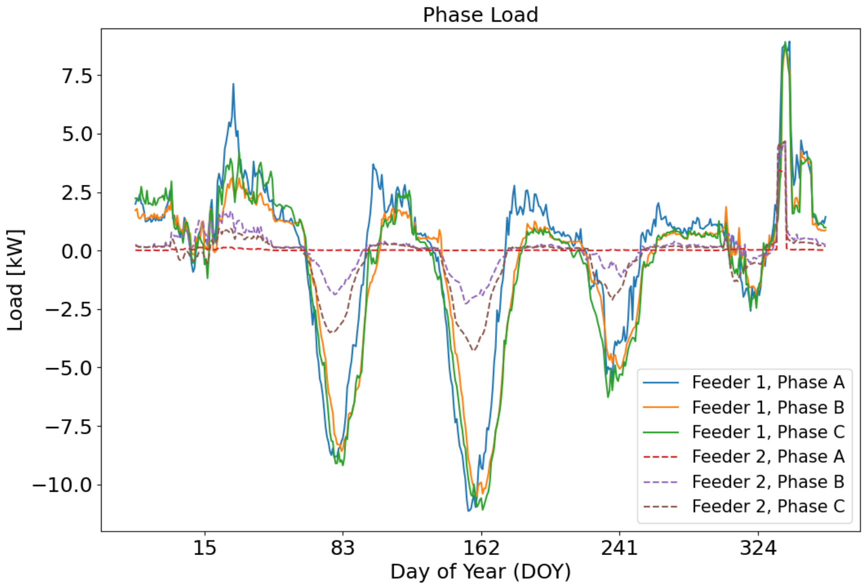
$$\sum_{\psi}^{\Psi} \mathbf{B}_{c,\psi} = 1 \qquad \forall c \in \mathcal{C} \qquad \text{(b)}$$

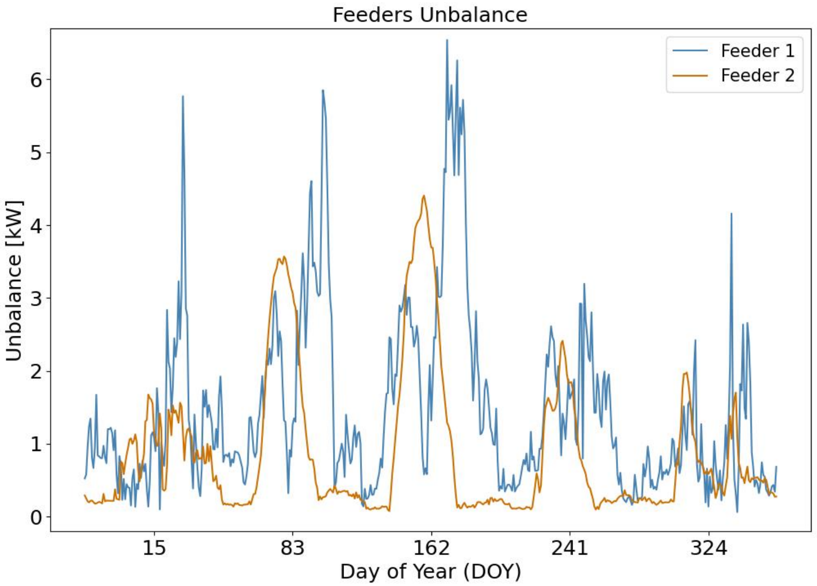
$$\mathbf{B}_{c,\psi} \leq \mathbf{B}_{c,\psi}^{feas} \qquad \forall c \in \mathcal{C}, \forall \psi \in \Psi \qquad \text{(c)}$$

$$\mathbf{B}_{c,\psi} \in \{0, 1\} \qquad \forall c \in \mathcal{C}, \forall \psi \in \Psi \qquad \text{(d)}$$

Case study: Belgian network

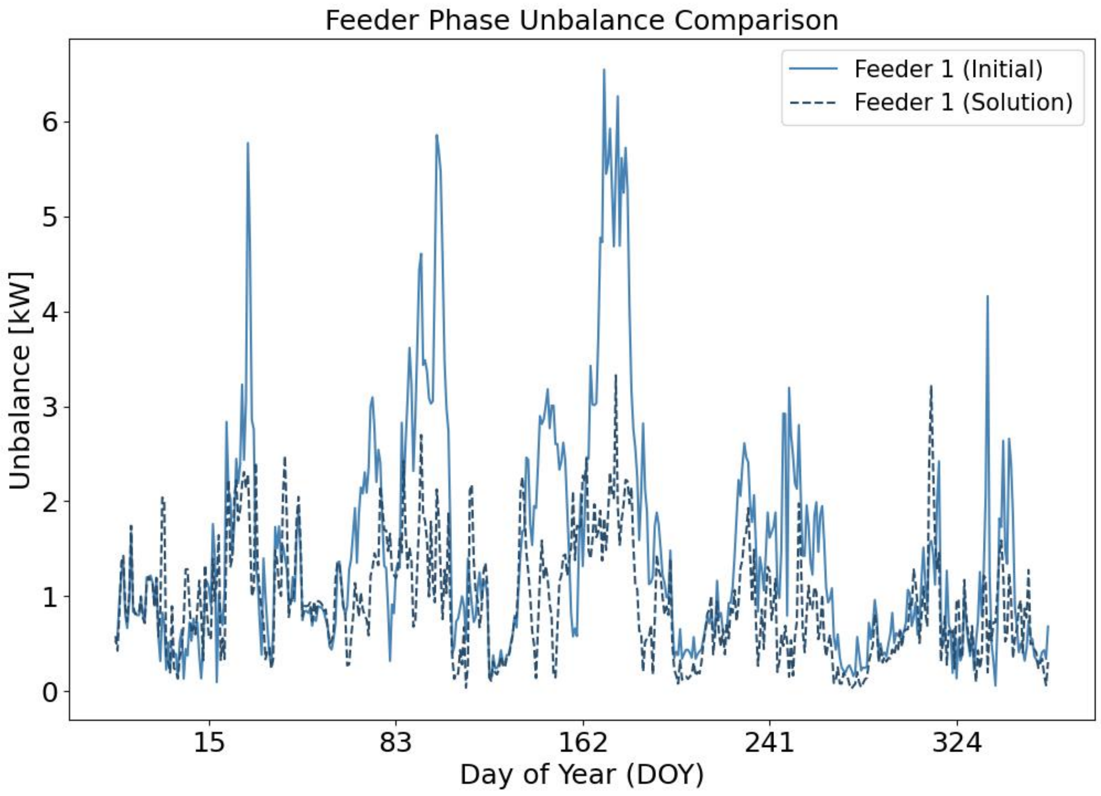
- 23 customers: 14 single-phase, 9 multi-phase, on two feeders
- High DER scenario: 90% PV, 60% EV, 50% HP penetration
- Analysis period: whole year, 15-minute resolution

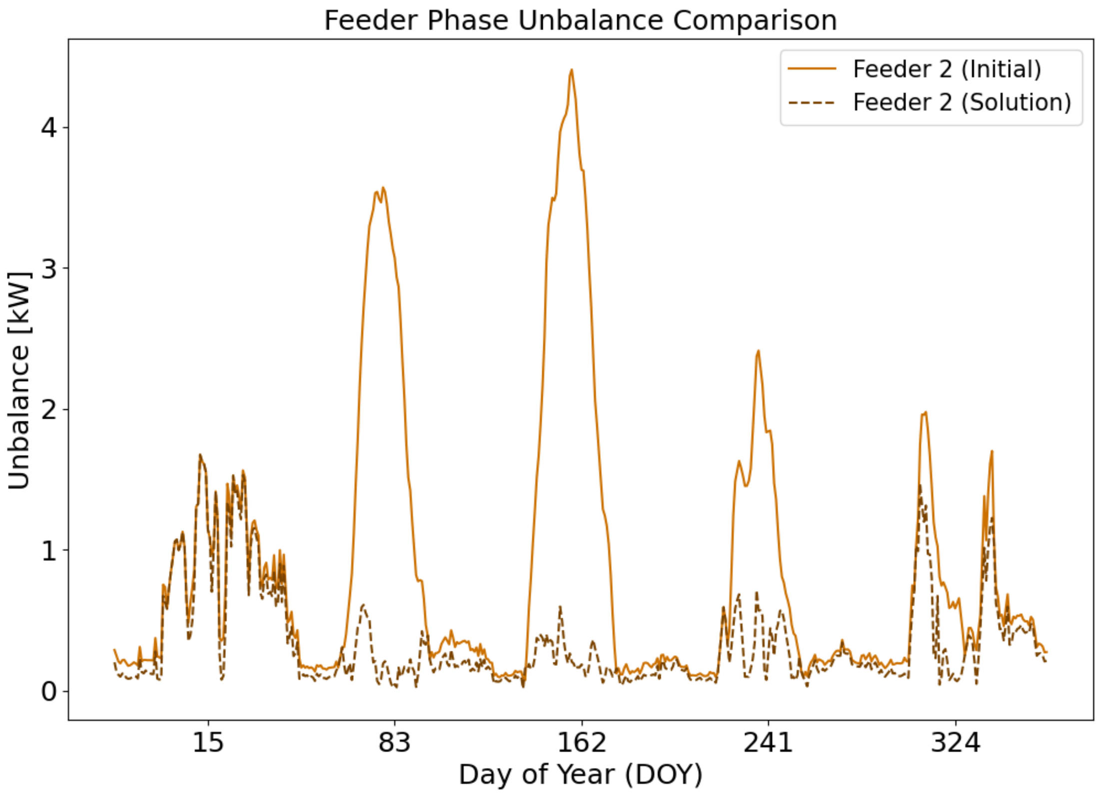




Results: load unbalance

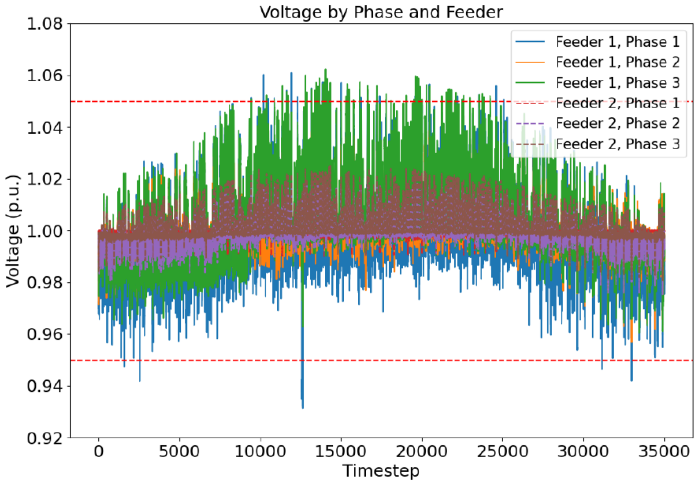
-36 % unbalance on feeder 1 and -62 % on feeder 2, obtained with 8 reconfigurations in total





Results: voltages and losses

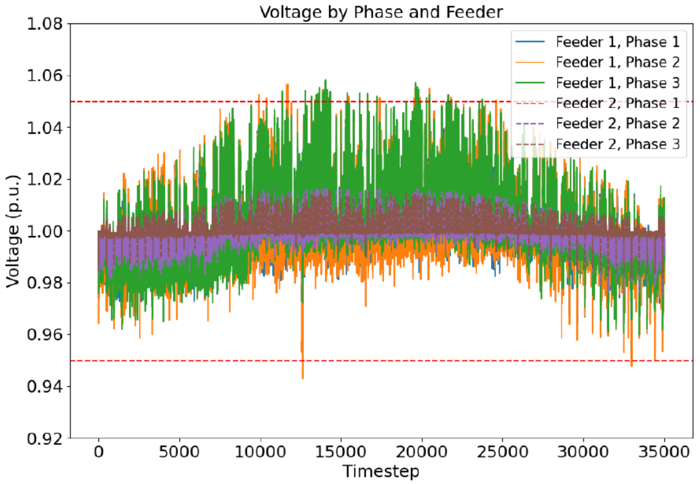
Feeder	Phase	Voltage max [p.u.]		Voltage min [p.u.]		Line Losses [kWh]	
		before	after	before	after	before	after
1	A	1.0609	1.0482	0.9313	0.9505	2784	2122
	B	1.0424	1.0565	0.9568	0.9430	1835	2592
	C	1.0622	1.0582	0.9604	0.9597	2999	2651
2	A	1.0014	1.0085	0.9806	0.9861	14	72
	B	1.0181	1.0163	0.9752	0.9754	313	279
	C	1.0250	1.0154	0.9870	0.9818	258	124



before

Overloaded phases improve

Feeder 1, phase A: maximum and minimum voltage back within limits. Voltage issues: 535 → 227



after

Total line losses

8206 → 7842 kWh per year across the network. -4.4% in total losses

Trade-off

Feeder 1, phase B: voltage worsens

Takeaways

- Load unbalance reduced by 36% and 62% on the two feeders, with 8 customer moves in total
- Line losses fall 4.4% and voltage issues 58%
- Balancing shifts load rather than removing it as some phases end up carrying more than before

EXCHANGE

Secure limit exchange market

Balancing is not enough

- Re-phasing uses the existing network better, but it cannot adapt to what happens on a given day
- Under high DER penetration the question becomes how much power each customer may inject or withdraw at a given moment
- Today limits are either implicit or imposed in real-time through curtailment
- Neither reflects who needs the capacity, nor rewards those who can do without it

Existing approaches, and what they miss

Approach	What it does	What it misses
Active network management (ANM)	Curtails or reschedules DERs in real-time	Poor scalability and observability
Dynamic operating envelopes (DOEs)	Ensure network security	Fixed and non-transferable
Local energy markets (LEM)	Trade energy (kWh)	Not the network capacity (kW) itself

SecuLEx makes the limits tradeable, while keeping the network secure

SecuLEx: allocate fairly, then trade

1. Allocate

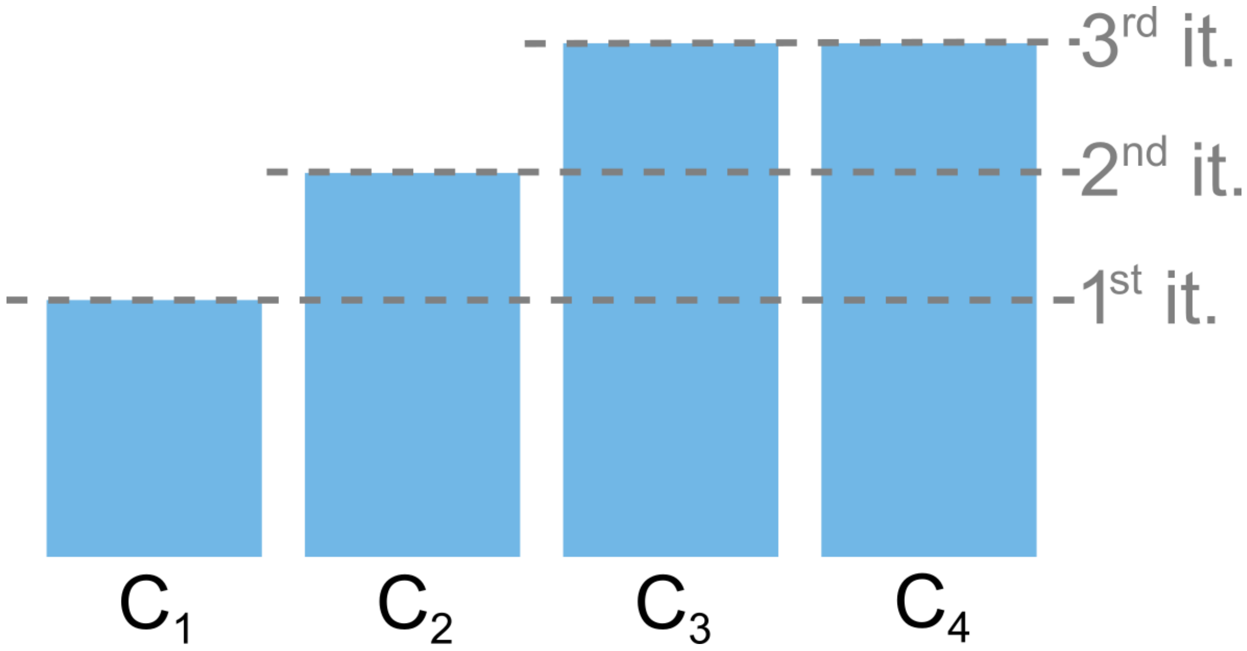
- The DSO assigns the initial DOEs to all customers
- Uses a lexicographic max-min optimization program
- Guarantees fairness and network security

2. Trade

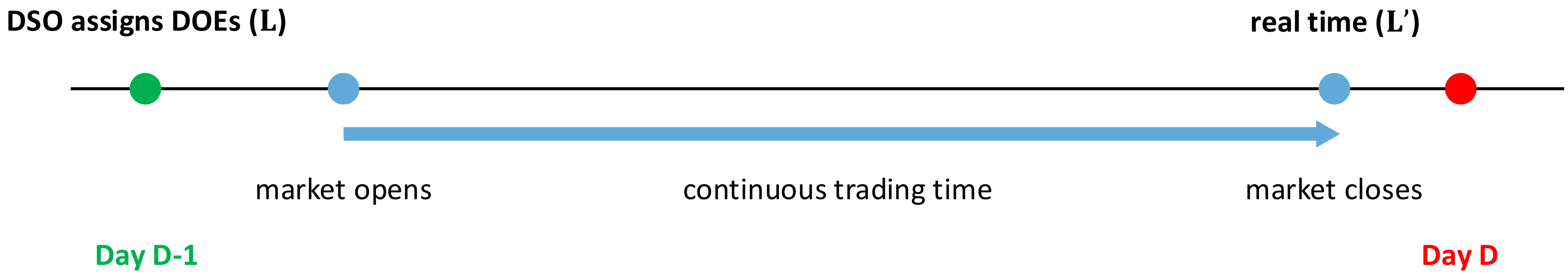
- A continuous market allows customers to buy and sell portions of their envelopes
- Matches limits to actual real-time needs (day-ahead up to real-time)
- Network security is verified before every cleared trade

DOEs and security

- Each customer receives DOEs:
  - $\mathbf{L_c} = [\underline{P}_c, \overline{P}_c] \ \forall c \in \mathcal{C}$
- Security is guaranteed when limits are respected
- Initial allocation by lexicographic max-min: maximize the smallest envelope first, fix it, repeat



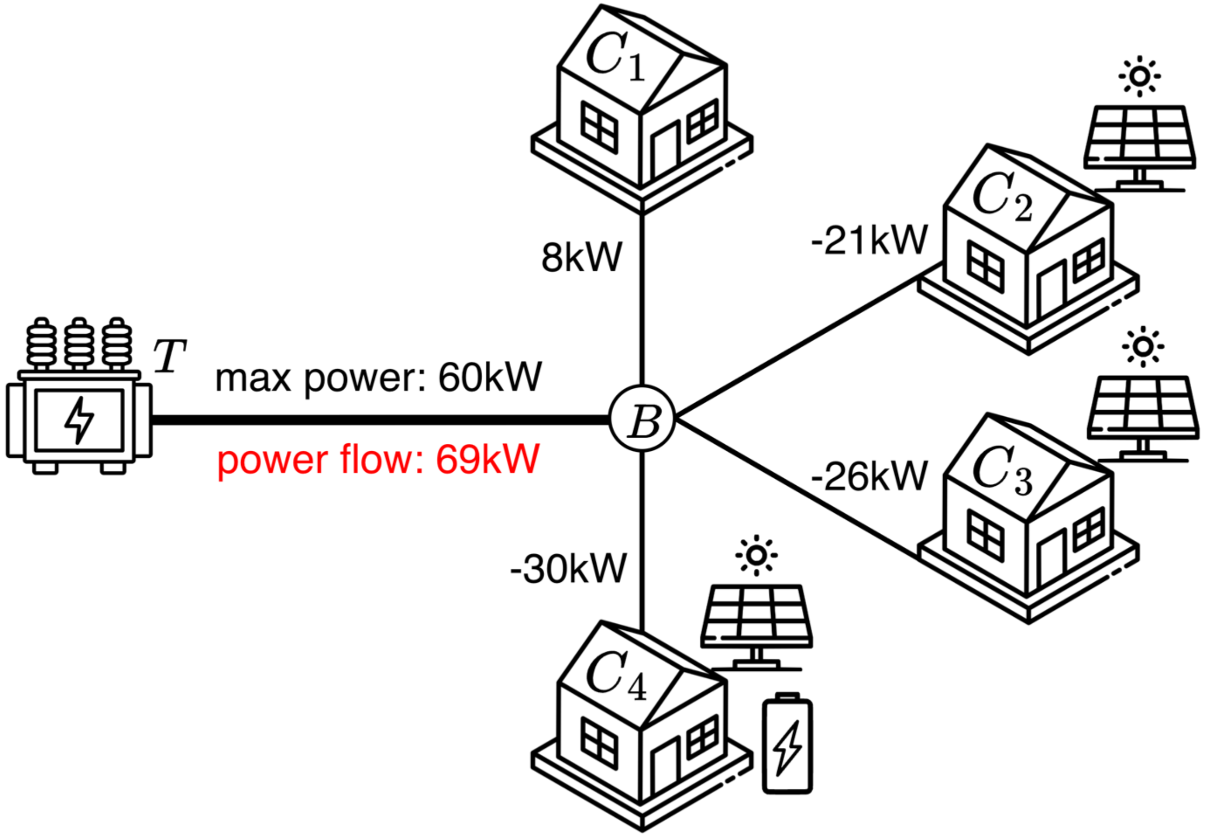
A continuous market for limits



Orders. Buy/sell a portion of the lower/upper limit for a given price and a specific time

Clearing. Maximize social welfare ($\sum_{b \in \mathcal{B}} \pi_b a_b - \sum_{s \in \mathcal{S}} \pi_s a_s$), subject to the security check on the updated limits $\mathbf{L}'$. A linear program under the DC approximation

Illustrative example



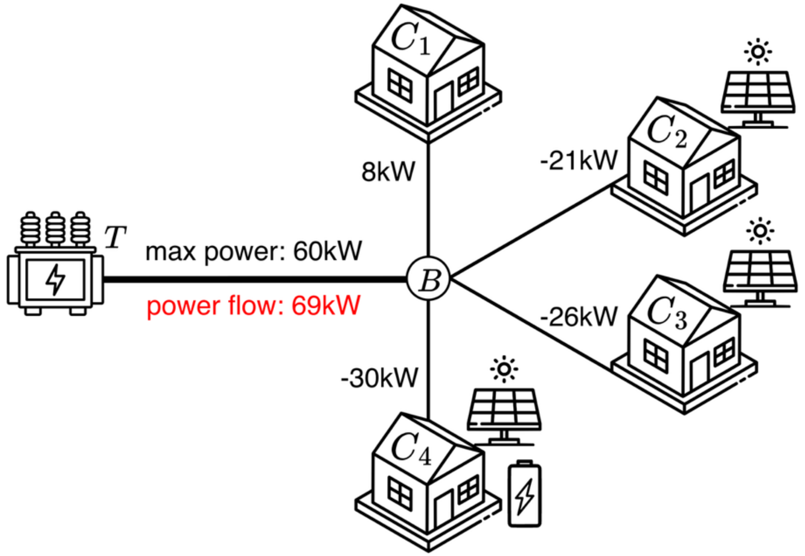
Expected midday positions

Cust	Type	Net (kW)	Buy (€/kWh)	Feed-in (€/kWh)
C_1	load	+8	0.15	—
C_2	PV	−21	0.16	0.06
C_3	PV	−26	0.15	0.04
C_4	PV + batt.	−30	0.15	0.02

Four ways to handle the congestion

- **No control:** 9kW overload. The network is insecure
- **Centralized ANM:** DSO curtails 9kW in real time (equally over C2-C4); opportunity loss 0.36€. No access to customer batteries
- **DOEs:** Fixed day-ahead DOEs. 7 to 17kW curtailed depending on C4's choice (0.30-0.50€)
- **SecuLEx:** Customers trade limits to use their flexibility. Only 1kW curtailed (0.04€)

SecuLEx: the trades



Updated lower limits

- **C_2:** −20kW → −21kW
- **C_3:** −20kW → −25kW
- **C_4:** −20kW → −14kW

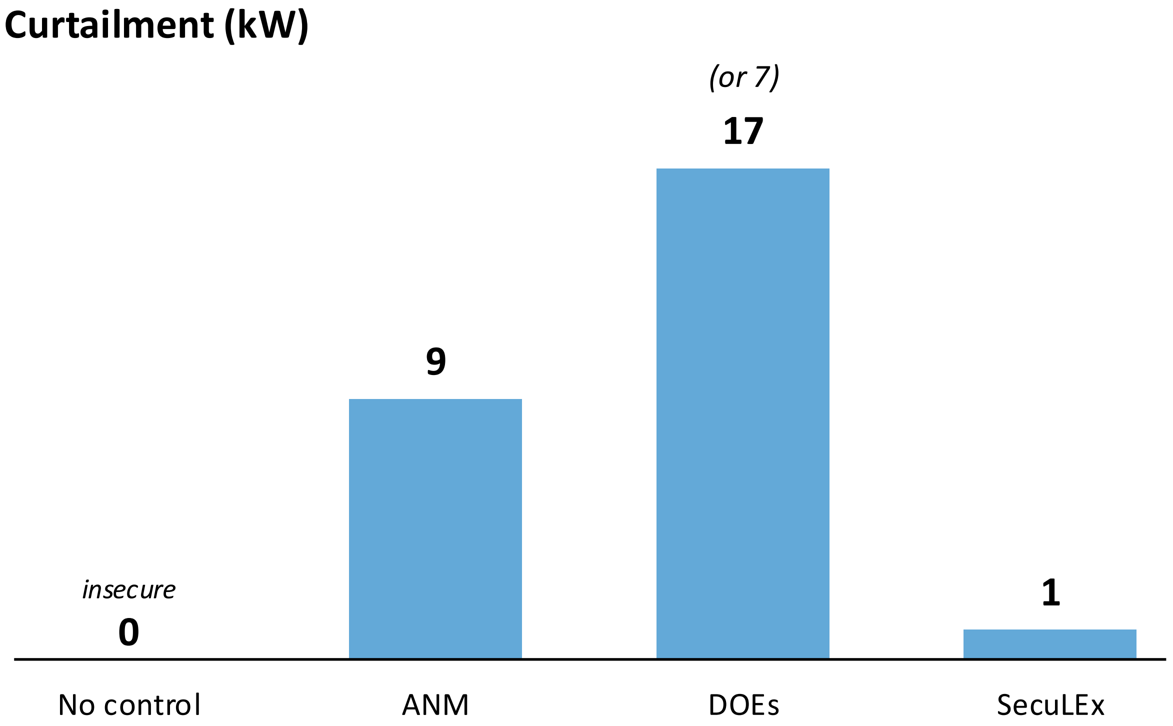
Order book

Cust.	**Type**	**Bound**	**Price** (€/kW)	**Qty** (kW)
C_2	buy	lower	0.03	1
C_3	buy	lower	0.02	6
C_4	sell	lower	0.02	6

Pay-as-bid:

- **C_2:** pays 0.03€
- **C_3:** pays 0.10€
- **C_4:** receives 0.12€
- **Social welfare:** 0.01€ (goes to the DSO)

Summary results



Metric	No ctrl	ANM	DOEs	SecuLEx
Curtailment (kW)	0	9	17 (or 7)	**1**
Renewable use (%)	100	87	76 (90)	**99**
Security violation	Yes	No	No	**No**
Incentivizes flex.	No	No	Partial	**Yes**
Social welfare (€)	–	–	–	**0.01**

Takeaways

- An egalitarian allocation gives every customer a secure starting point, and the market reallocates it towards real need
- Under radial network and DC approximation, security verification reduces to two boundary checks, so allocation and clearing are both LPs
- Lower curtailment and higher renewable use, with no real-time operation

COORDINATION

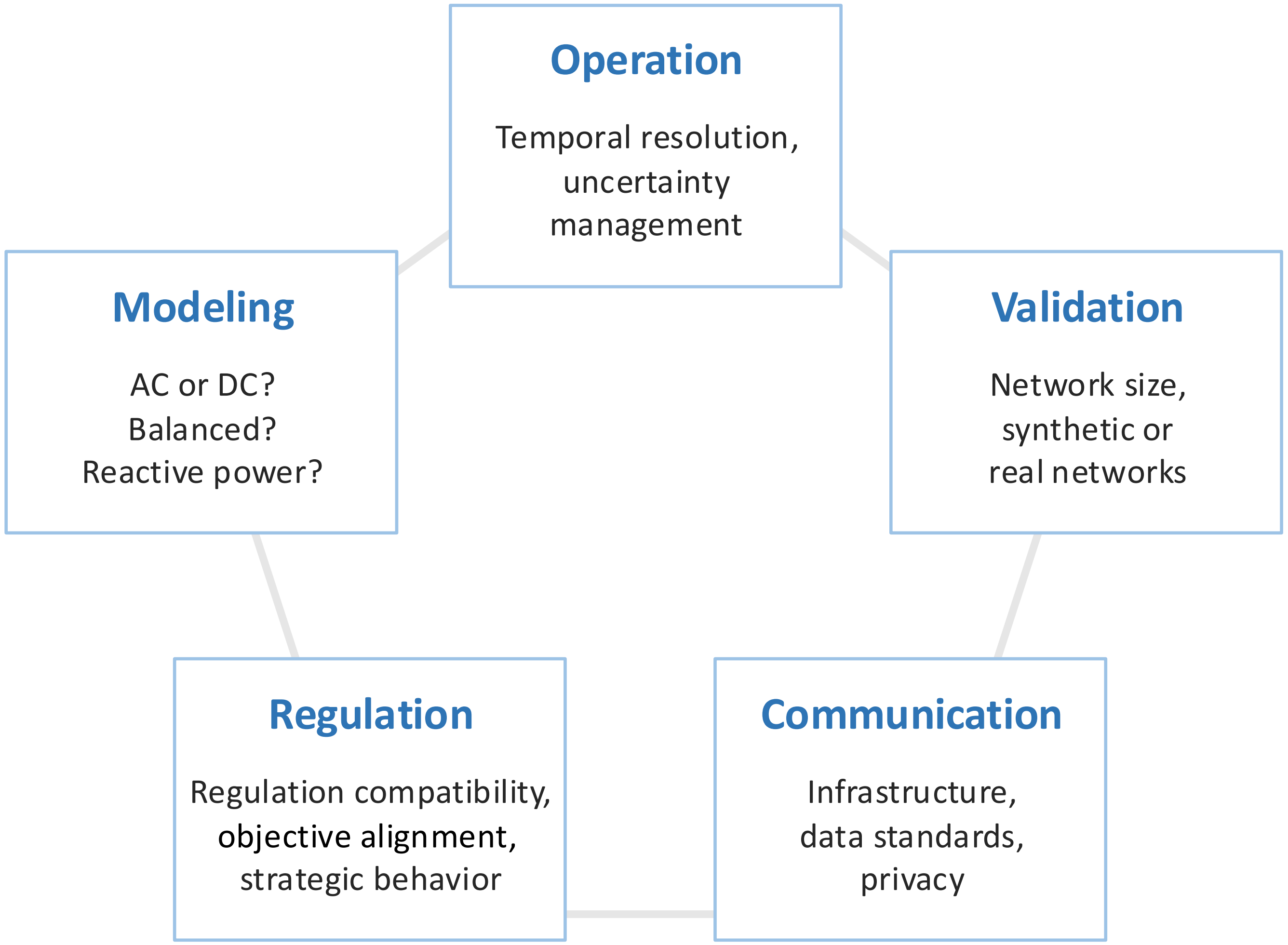
Why methods rarely reach deployment

From optimisation to coordination

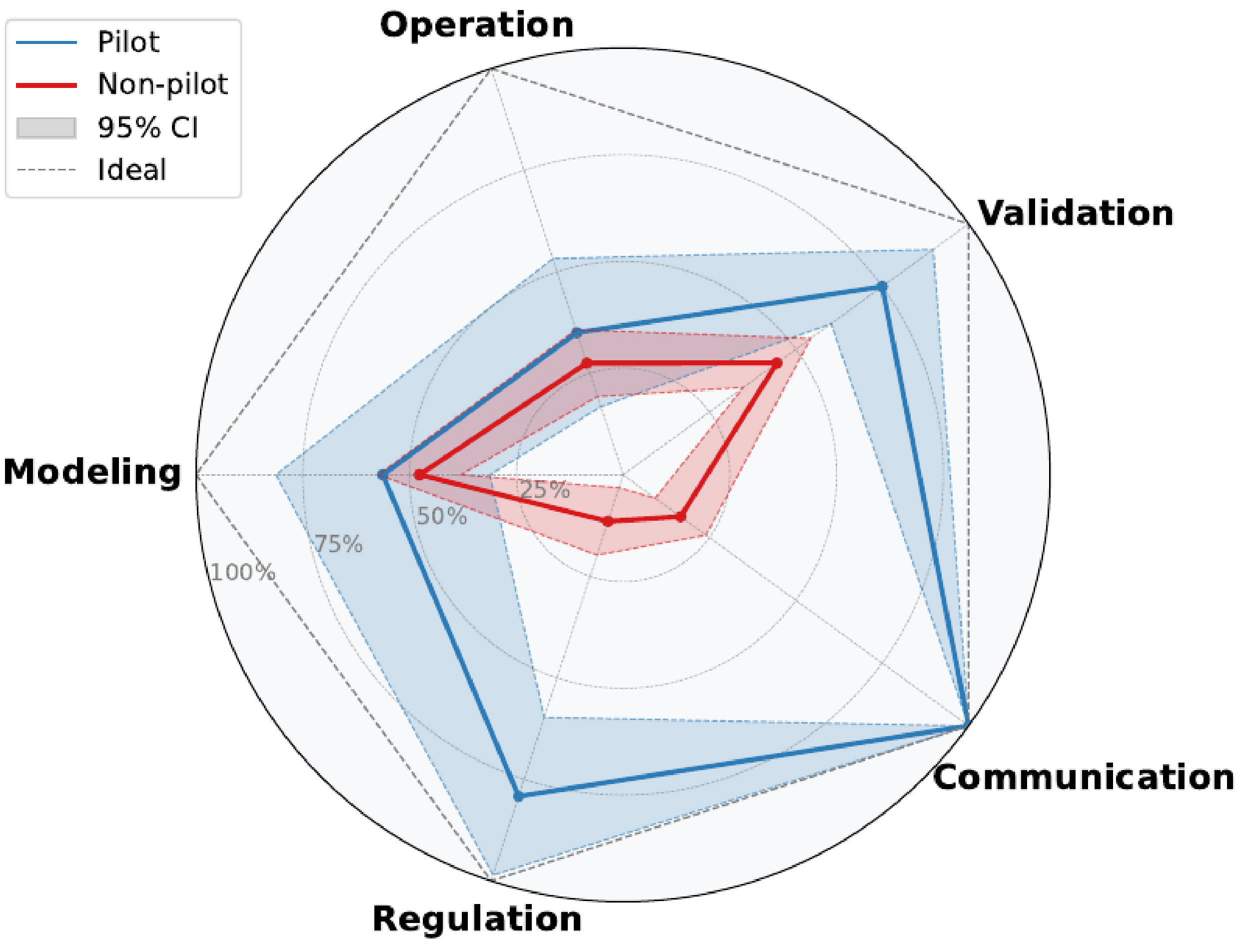
- The three capabilities address the pressure from below. The pressure from above is still open
- At the interface, the problem becomes one of coordination
- TSO-DSO coordination: large literature, several pilots, almost no deployments
- The usual explanation is that the models are not yet sophisticated enough
- If the gap were algorithmic, pilots would model the network more carefully than papers do

Five dimensions

A multidimensional analysis of the TSO-DSO coordination literature studies scored across five dimensions, with pilot projects compared against purely theoretical work



Pilots versus academy



Modeling and operation: no gap
Pilots use the same simplifying assumptions as academic studies

Validation, communication, regulation: large gap
Pilots handle all three; theoretical studies mostly ignore them

The asymmetry
Research effort concentrates where the gap is smallest, not where it is largest

From pilot to commercial deployment

Pilots score higher on validation, communication and regulation, and still do not become operational systems

- **Tractability:** even pilots fall back on simplified models
- **Scenario uncertainty:** errors undermine operation
- **LV visibility:** weak coordination
- **Communication:** far more engagement than anticipated
- **Market design:** illiquidity and fragmentation

Limitations of the framework

Publication bias
Failures are rarely published, so the scores describe a best case

Equal weights
Scores are not comparable across dimensions, only within one

Missing dimensions
Technical threats such as cybersecurity sit outside the five

Not independent dimensions
Privacy rules constrain communication; modelling limits validation at scale

Diagnostic tool, not a readiness score

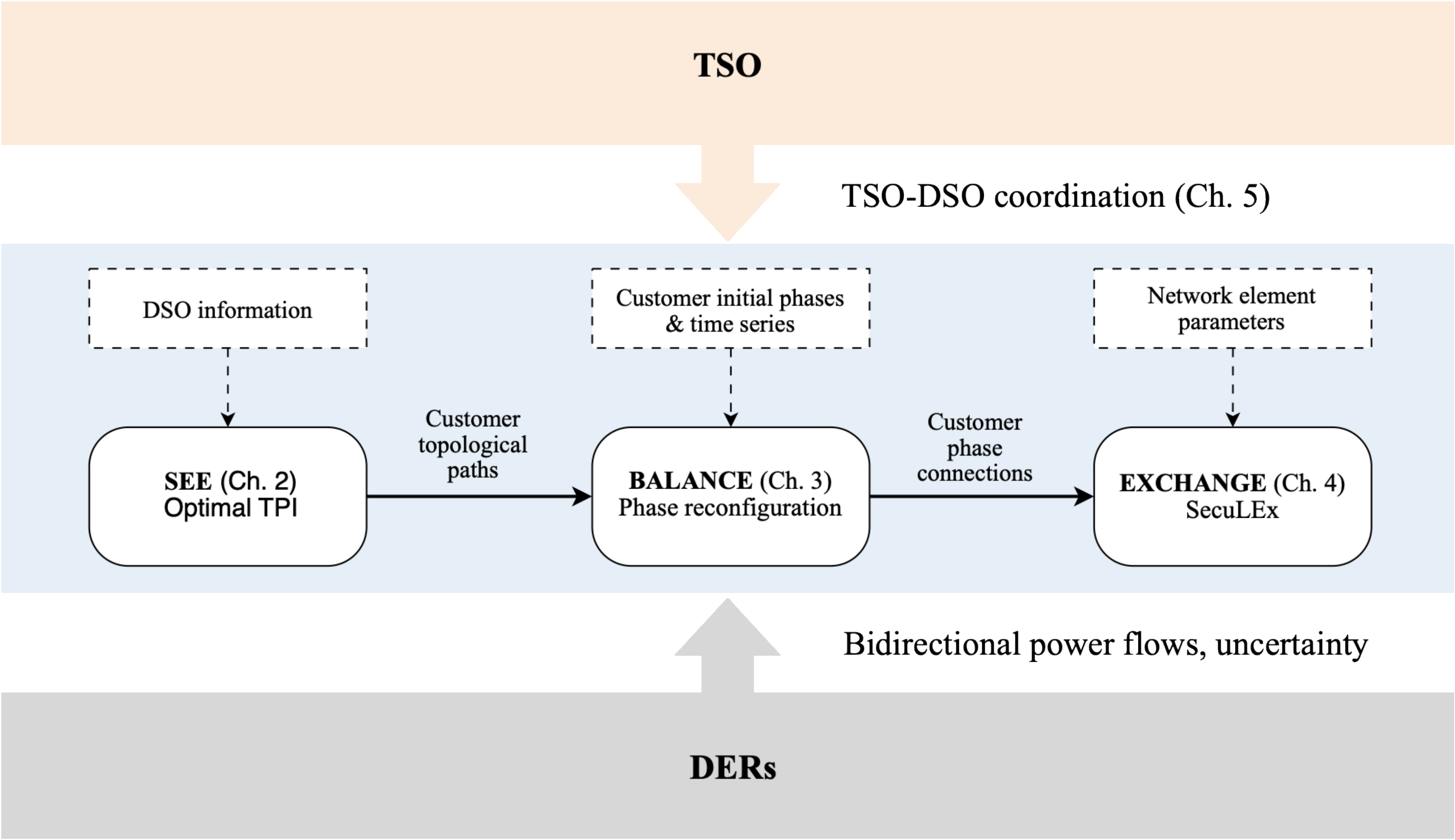
Takeaways

- Modeling and operation show no measurable gap between pilot projects and theoretical studies
- Validation, communication and regulation are addressed by every pilot but by only few theoretical studies
- The bottleneck may not be algorithmic complexity, but validation at scale, communication infrastructure and regulatory alignment
- Pilots are not ready to be deployed either: the gap to commercial deployment remains open

CONCLUSION

What the toolkit is, and what comes next

Putting the toolkit together



Gaps, and the directions they open

What the toolkit does not do yet	Next steps
Phase identification is not part of the ILP See	Extend the ILP to assign phases with the topological paths
Balance and exchange do not share a network model	AC multi-phase unbalanced formulations for SecuLEx
The three methods are not chained on one network	Integrated case study on the Belgian network
Re-phasing rests on a single deterministic year	Robust re-phasing across DER and weather scenarios

Beyond optimization: validation at scale, communication standards, regulatory alignment, and participant behaviour

Thank you

Back-up Slides

A* candidate generation

$$f(e) = g(e) + h(e)$$

$$g(e) = \sum_{k=1}^{|p|-1} \left(\mathrm{Dist}(p_k, p_{k+1}) + w_{p_k} \right) \qquad h(e) = \mathrm{Dist}(e, t)$$

- **Neighbour lookup:** Each point element runs a radius query (*scipy.spatial.KDTree.query_ball_point*) at 50 m; each line element runs the same query at both of its extremes
- **Diversifying the N paths:** Every time an element is used, its weight w is increased by a scaling factor, so the next search is pushed towards a different route. The accumulated weight enters g(e) alongside the distance
- **Pruning:** Candidates are dropped on maximum connection distance D and maximum total path length L; the search stops at N valid paths per customer
- On the Belgian network N = 15 gave 17 335 candidates in 264 s. **This stage, not the ILP, is the bottleneck**

Candidate paths as matrices

$$\mathbf{H} = \begin{array}{c} \\ h_1 \\ h_2 \\ \vdots \\ h_k \\ \vdots \\ h_{|\mathcal{H}^{\mathcal{I}'}|} \end{array} \begin{array}{c} \begin{array}{cccc|cccc|cccc} c_1 & c_2 & \cdots & c_{|C|} & r_1 & r_2 & \cdots & r_{|R|} & t_1 & t_2 & \cdots & t_{|T|} \end{array} \\ \left(\begin{array}{cccc|cccc|cccc} 0 & 1 & \cdots & 0 & 1 & 0 & \cdots & 1 & 0 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 1 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{array}\right) \\ \begin{array}{ccc} \mathbf{H}_C & \mathbf{H}_R & \mathbf{H}_T \end{array} \end{array} \tag{2.15}$$

$$\mathbf{T}_R = \begin{array}{c} \\ t_1 \\ t_2 \\ \vdots \\ t_k \\ \vdots \\ t_{|T|} \end{array} \begin{array}{c} \begin{array}{ccccccc} r_1 & r_2 & \cdots & r_m & \cdots & r_{|R|-1} & r_{|R|} \end{array} \\ \left(\begin{array}{ccccccc} 0 & 1 & \cdots & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 1 \end{array}\right) \end{array} \tag{2.18}$$

$$\hat{\mathbf{P}} = \begin{array}{c} \begin{array}{ccccc} h_1 & \cdots & h_k & \cdots & h_{|\mathcal{H}^{\mathcal{I}'}|} \end{array} \\ \left(\begin{array}{ccccc} 1 & \cdots & 1 & \cdots & 0 \end{array}\right) \end{array} \tag{2.19}$$

- **H:** One row per candidate path, one column per element. Split into H_C (customers), H_R (lines and junctions) and H_T
- **T_R and $\hat{P}$ are the decision variables:** which element belongs to which feeder terminal, and which candidate paths are selected

The ILP in full

$$\max_{\hat{\mathbf{P}},\mathbf{T}_R} \quad \omega \cdot \sum_{k}^{|\mathcal{H}^{I'}|} \hat{\mathbf{P}}(k) \; - \; \sum_{m}^{|\mathbb{T}|} \sum_{n}^{|\mathbb{R}|} \mathbf{T}_R(m,n) \qquad \text{(a)}$$

$$\text{s.t.} \quad \sum_{k}^{|\mathbb{R}|} \mathbf{H}_R(m,k) \cdot \hat{\mathbf{P}}(m) \cdot \mathbf{H}_T(m,n) \; \leq \; \big(\mathbf{T}_R \times \mathbf{H}_R^\top \cdot \mathbf{H}_T^\top\big)(n,m) \qquad \text{(b)}$$

$$\big(\hat{\mathbf{P}} \times \mathbf{H}_C\big)_{(k)} \; \leq \; 1 \qquad \text{(c)}$$

$$\sum_{k}^{|\mathbb{T}|} \mathbf{T}_R(k,m) \; \leq \; 1 \qquad \text{(d)}$$

$$\hat{\mathbf{P}}(k) \in \{0,1\}, \qquad \mathbf{T}_R(m,n) \in \{0,1\} \qquad \text{(e–f)}$$

(a) maximise connected customers, penalise assigning elements to feeders without necessity ($\omega = 10$)

(b) all elements of a path share one feeder

(c) one path per customer

(d) one feeder per element

Hyperparameters and runtime

Category	Metric	Value
Network profile	Total elements (E)	1 089
	Customers (C)	526
	Lines (R)	441
	Feeder junctions (T)	96
	MV/LV transformers (F)	26
Hyperparameters	Max candidate paths per customer (N)	35
	Max connection distance (D)	50 m
	Max path length (L)	500 m
	Objective weighting factor (ω)	10
Statistics	Candidate paths generated ($\|H^{l'}\|$)	17 335
	A* generation time	264 s
	ILP optimization time	5.8 s
Result	Customers connected	88%

Phase configurations

$$\Psi = \{ (A), (B), (C),$$
$$(A,B), (A,C), (B,A), (B,C), (C,A), (C,B),$$
$$(A,B,C), (A,C,B), (B,A,C), \ldots, (C,B,A) \}$$

- **|Ψ| = 15:** three single-phase configurations, six ordered two-phase configurations, and the six permutations of the three phases
- **Order matters:** (A, B, C) and (A, C, B) are different configurations

Configuration matrices

$$\mathbf{B}^{init}_{example} = \begin{array}{c} \\ c_1 \\ \vdots \\ c_i \\ \vdots \\ c_{|\mathcal{C}|} \end{array} \!\!\begin{array}{c} \begin{array}{cccccc} \psi_1 & \psi_2 & \cdots & \psi_{10} & \cdots & \psi_{|\Psi|} \end{array} \\ \begin{pmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 \end{pmatrix} \end{array}. \tag{3.2}$$

$$\overline{\mathbf{B}}^{init} = \mathbb{1} - \mathbf{B}^{init}, \tag{3.3}$$

$$\mathbf{B}^{feas}_{example} = \begin{array}{c} \\ c_1 \\ \vdots \\ c_{|\mathcal{C}|} \end{array} \!\!\begin{array}{c} \begin{array}{cccccccc} \psi_1 & \psi_2 & \psi_3 & \psi_4 & \cdots & \psi_{10} & \cdots & \psi_{|\Psi|} \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & \cdots & 1 \end{pmatrix} \end{array}. \tag{3.4}$$

- $\mathbf{B}^{init}$ is the configuration before optimization**;** $\mathbf{B}^{feas}$ masks the configurations a customer may move to (same phase count); **B** is the decision variable
- $\mathbf{B}^{init}$ is the complement

Choosing the scaling factors

Phase unbalance term:

$$P^{unb} = \sum_{t}^{\mathcal{T}} \sum_{f}^{\mathcal{F}} \sum_{\phi}^{\Phi} |\mathcal{A}_{\phi,f,t} - \mu^{\mathcal{A}}_{f,t}|$$

$$\mathcal{A}_{\phi,f,t} = \sum_{c}^{\mathcal{C}_f} P_{\phi,c,t}$$

$$\mu^{\mathcal{A}}_{f,t} = \frac{\sum_{\phi}^{\Phi} \mathcal{A}_{\phi,f,t}}{|\Phi|}$$

Number of reconfigurations term:

$$\Psi^{chg} = \sum_{c}^{\mathcal{C}} \sum_{\psi}^{\Psi} \mathbf{B}_{c,\psi} \cdot \overline{\mathbf{B}}^{init}_{c,\psi}$$

Customer distance term:

$$D^{wgt} = \sum_{c}^{\mathcal{C}} \frac{1}{c_{dist}} \cdot \sum_{\psi}^{\Psi} \mathbf{B}_{c,\psi} \cdot \overline{\mathbf{B}}^{init}_{c,\psi}$$

- **λ^P - cost of unbalance:** technical losses and the operational inefficiency caused by uneven load between phases
- **λ^Ψ - cost of a switch:** labour and potential downtime for physically re-phasing a connection
- **λ^D - position weight:** how much more a change is worth when the customer sits far down the feeder, where voltage drop and losses are largest
- Set iteratively: unbalance is given priority first, then the weights are adjusted to correct terms that dominate or vanish. **Final values $\lambda^P = 0.015$, $\lambda^\Psi = 6$, $\lambda^D = 1$**

Cost-benefit analysis

Metric	Before optimization	After optimization
Network losses (kWh/year)	8 206	7 842
Loss cost (€/year, at €0.20/kWh)	1 641	1 568
Savings (€/year)	-	73
Switching cost (€, 8 switches at €40)	-	320
Voltage issues	535	227
Net benefit (€, year 1)	-	-247
Break-even time (years)	-	4.4

Orders and the order book

$$o = (id,\ c,\ type,\ bound,\ power,\ \Delta,\ \pi,\ t)$$

Field	Domain	Meaning
id	$\mathbb{N}$	Unique identifier
c	$\mathcal{C}$	Customer submitting the order
type	{buy, sell}	Distinguishes demand from supply
bound	{lower, upper}	Which side of the DOE the order concerns
power	{active, reactive}	Nature of the requested change
Δ	$\mathbb{R}_+$	Requested change in limit (kW or kVAR)
π	$\mathbb{R}$	Offered price (€/kW or €/kVAR)
t	$\mathcal{T}$	Target time period (product time)

Lexicographic max-min allocation

$$
\begin{aligned}
\operatorname{lex}\max_{\mathbf{L}}\ \min\quad & \left|\overline{P}_c - \underline{P}_c\right| && \forall c \in \mathcal{C} \quad \text{(a)}\\
\text{s.t.}\quad & \underline{P}_c^{contr} \le \underline{P}_c \le \underline{P}_c^{guar} && \forall c \in \mathcal{C} \quad \text{(b)}\\
& \overline{P}_c^{guar} \le \overline{P}_c \le \overline{P}_c^{contr} && \forall c \in \mathcal{C} \quad \text{(c)}\\
& \underline{P}_c \le \overline{P}_c && \forall c \in \mathcal{C} \quad \text{(d)}\\
& \mathbf{L}_c = [\underline{P}_c, \overline{P}_c], \qquad VerifyLimits(\mathcal{G}, \mathbf{L}) \le 0 && \text{(e–f)}
\end{aligned}
$$

- Egalitarian rule: the smallest envelope is maximized first, then the next smallest, and so on

- P^{guar} are minimum guaranteed withdrawal and injection limits; P^{contr} are contractual extrema set by equipment capacity

Solving the allocation iteratively

$$
\begin{aligned}
\max_{w,\mathbf{L}} \quad & w && \text{(a)} \\
\text{s.t.} \quad & \underline{P}_c^{contr} \leq \underline{P}_c \leq \underline{P}_c^{guar}, \qquad \overline{P}_c^{guar} \leq \overline{P}_c \leq \overline{P}_c^{contr} && \forall c \notin \mathcal{C}' \quad \text{(b–c)} \\
& \underline{P}_c = \underline{P}_c^*, \qquad \overline{P}_c = \overline{P}_c^* && \forall c \in \mathcal{C}' \quad \text{(d–e)} \\
& \underline{P}_c \leq \overline{P}_c && \forall c \in \mathcal{C} \quad \text{(f)} \\
& w \leq \overline{P}_c - \underline{P}_c && \forall c \notin \mathcal{C}' \quad \text{(g)} \\
& VerifyLimits(\mathcal{G}, \mathbf{L}) \leq 0 && \text{(h–i)}
\end{aligned}
$$

- Solve for the largest common width w. Customers whose constraint (g) is tight at the optimum have their envelope frozen, join C**’**, and the LP is re-solved for the rest until every customer is fixed

- One LP per iteration, at most |C| iterations; variables and constraints scale linearly with the number of customers

Market clearing

$$\max_{a,\mathbf{L}'} \quad \sum_{b\in\mathcal{B}} \pi_b a_b \; - \sum_{s\in\mathcal{S}} \pi_s a_s \qquad \text{(a)}$$

$$\text{s.t.} \quad 0 \le a_o \le \Delta P_o \qquad \forall o \in \mathcal{O} \quad \text{(b)}$$

$$\underline{P}'_c = \underline{P}_c + \sum_{s\in\mathcal{S}\cap\mathcal{L}:c_s=c} a_s \; - \sum_{b\in\mathcal{B}\cap\mathcal{L}:c_b=c} a_b \qquad \forall c \in \mathcal{C} \quad \text{(c)}$$

$$\overline{P}'_c = \overline{P}_c - \sum_{s\in\mathcal{S}\cap\mathcal{U}:c_s=c} a_s \; + \sum_{b\in\mathcal{B}\cap\mathcal{U}:c_b=c} a_b \qquad \forall c \in \mathcal{C} \quad \text{(d)}$$

$$\underline{P}'_c \le \overline{P}'_c, \qquad \mathbf{L}' = [\underline{P}', \overline{P}'], \qquad VerifyLimits(\mathcal{G}, \mathbf{L}') \le 0 \qquad \text{(e–g)}$$

$$\text{Settlement:} \quad \Pi_c = \sum_{b\in\mathcal{B}:c_b=c} a_b \pi_b \; - \sum_{s\in\mathcal{S}:c_s=c} a_s \pi_s \qquad SW = \sum_{c\in\mathcal{C}} \Pi_c$$

- Maximise social welfare subject to partial acceptance, the updated limits, and the security check on L′. Under DC and radial assumptions this is an LP that scales linearly with the number of orders

Scoring table

Dimension	Sub-dimension	Characteristic	Score	Total	Non-Pilot	Pilot
Modeling	Transmission network	DC formulation	0	29 (76%)	23 (77%)	5 (63%)
		AC formulation	1	9 (24%)	7 (23%)	3 (37%)
	Distribution network	DC formulation	0	6 (16%)	5 (17%)	1 (13%)
		AC formulation	1	32 (84%)	25 (83%)	7 (87%)
	Phase balance	Balanced system	0	31 (82%)	26 (87%)	5 (63%)
		Unbalanced system	1	7 (18%)	4 (13%)	3 (37%)
	Reactive power	Not considered	0	11 (29%)	9 (30%)	2 (25%)
		Considered	1	27 (71%)	21 (70%)	6 (75%)
Operation	Temporal resolution	Single timestep	0	12 (32%)	10 (33%)	2 (25%)
		Hours	1	18 (47%)	15 (50%)	3 (37%)
		Minutes	2	5 (13%)	3 (10%)	2 (25%)
		Seconds	3	3 (8%)	2 (6%)	1 (13%)
	Uncertainty management	Deterministic	0	20 (53%)	17 (57%)	3 (37%)
		Scenarios	1	15 (40%)	10 (33%)	5 (63%)
		Robust optimization	2	3 (8%)	3 (10%)	0
Validation	Network size	Small	0	18 (47%)	16 (54%)	2 (25%)
		Medium	1	11 (29%)	7 (23%)	4 (50%)
		Large	2	9 (24%)	7 (23%)	2 (25%)
	Network type	Not mentioned	0	1 (3%)	1 (3%)	0
		Standard test-case	1	22 (57%)	22 (74%)	0
		Real network	2	15 (40%)	7 (23%)	8 (100%)
Communication	Infrastructure	Not considered	0	27 (69%)	27 (87%)	0
		Considered	1	12 (31%)	4 (13%)	8 (100%)
	Data standardization	Not considered	0	29 (74%)	29 (94%)	0
		Considered	1	10 (26%)	2 (6%)	8 (100%)
	Privacy	Not considered	0	22 (56%)	22 (71%)	0
		Considered	1	17 (44%)	9 (29%)	8 (100%)
Regulation	Regulation incompatibility	Not considered	0	28 (72%)	28 (90%)	0
		Considered	1	11 (28%)	3 (10%)	8 (100%)
	Objective alignment	Not considered	0	24 (63%)	23 (77%)	1 (13%)
		Considered	1	14 (37%)	7 (23%)	7 (87%)
	Strategic behavior	Not considered	0	30 (83%)	26 (93%)	4 (50%)
		Considered	1	6 (17%)	2 (7%)	4 (50%)