

MODELLING THE COMPLETE BEHAVIOUR OF DAPPED-END CONNECTIONS
ALONG RE-ENTRANT CORNER CRACKS

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Abstract

Reinforced concrete dapped-end connections, which are common in existing Gerber-beam bridges, typically feature an inclined corner crack at service loads due to high stress concentrations in the re-entrant corner. These cracks exacerbate corrosion issues and increase the stresses in the dapped-end reinforcement, which in the event of further deterioration and increased loading may lead to yielding and failure of the connection. This paper proposes a kinematics-based model that predicts the opening of the corner crack based on first principles: compatibility, equilibrium, and constitutive relationships. The model stems from an earlier model for strength of dapped-end connections failing along re-entrant corner cracks and extends it to predict the crack widths under increasing load up to failure. The modelling assumptions are discussed, and the crack width predictions are extensively validated using 42 large-scale tests. The results show good agreement between measured and predicted crack widths, as well as the peak flexural resistance of dapped-end connections. The average experimental-to-predicted ratio for the flexural strength is 0.98 and the coefficient of variations is 8.8%.

Keywords – reinforced concrete dapped-end connections; kinematics-based model; re-entrant corner crack; crack width prediction; flexural strength

1. Introduction

Reinforced concrete dapped-end connections are common in Gerber-beam bridges and other existing structures. Desnerck et al. (2018) reported that a total of 428 structures with dapped-end connections were identified in the UK Highways England Road network [1]. These connections were preferred in the past due to their statical determinacy and suitability for precast construction. However, dapped-end connections have exhibited major durability issues [2,3], and have proven challenging for assessment, monitoring, and retrofit [4]. These issues arise in part due to the formation of inclined cracks, propagating from the re-entrant corner of the connection. Therefore, there is a need for models that accurately predict the opening of the re-entrant corner cracks from their initiation up to failure.

To understand and model dapped-end connections, a number of experimental [4,5-11] and analytical [12-22] studies have been conducted in the past. Analytical studies have proposed strut-and-tie models [13-16], stress-field models [4,17], empirical and semi-empirical equations [18], nonlinear finite element methods [19-20], and kinematics-based methods [21-22]. Strut-and-tie models allow to visualize the flow of forces in dapped-end connections, where the compression in the concrete is modelled with struts and the tension in the reinforcement with ties. To refine such models, researchers have also developed stress field approaches where, instead of discrete struts, the concrete is modelled with compression stress fields. However, as these modelling approaches mainly consider the flow of stresses, they are not sufficient for evaluation of deformations throughout the response of the connection [17]. A notable exception is a recent strut-and-tie model proposed by Mata-Falcón and Yu et al. [23], which uses empirical factors to adapt the forces in the ties from ultimate to service load conditions.

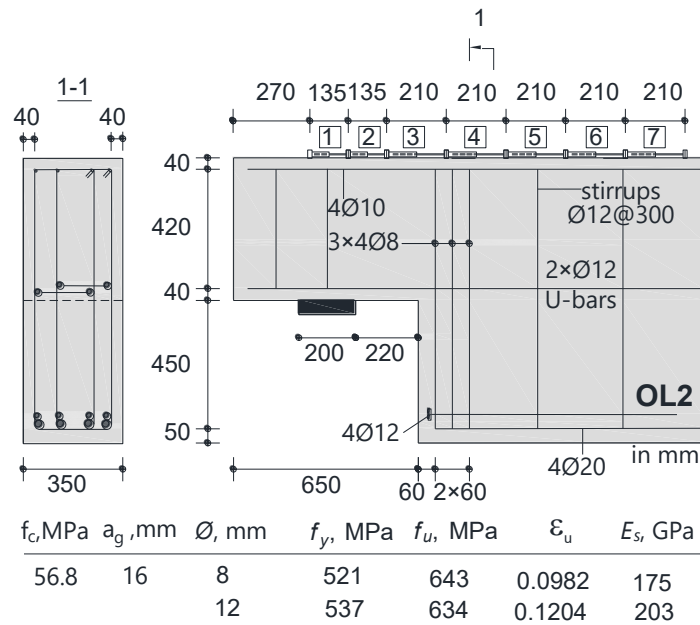
Recently, a kinematics-based model was proposed by the authors [21-22] for the strength of dapped-end connections failing along a re-entrant corner crack. Such failures typically occur with yielding of the main dapped-end reinforcement, and for this reason can be referred to as flexural failures. The model is based on a kinematic description of the deformations of the connection (compatibility of deformations), and also includes constitutive relationships and equilibrium conditions. It explicitly accounts for the crack kinematics such as the crack displacements at the level of the dapped-end reinforcement. In a recent study, separate closed-formed expressions were developed for evaluating crack widths [12] as a function of the load on the dapped end normalized by the peak load (normalized load).

In the current study, the main assumptions of the flexural strength model [21-22] are used to formulate an integrated mechanical approach for the complete flexural behaviour of dapped-end connections. This approach predicts the opening of the re-entrant corner crack as a function of the applied load from initiation of loading up to failure. Detailed experimental results from previous studies of the authors and others, including measurements of crack widths and strains, are used to validate the proposed approach.

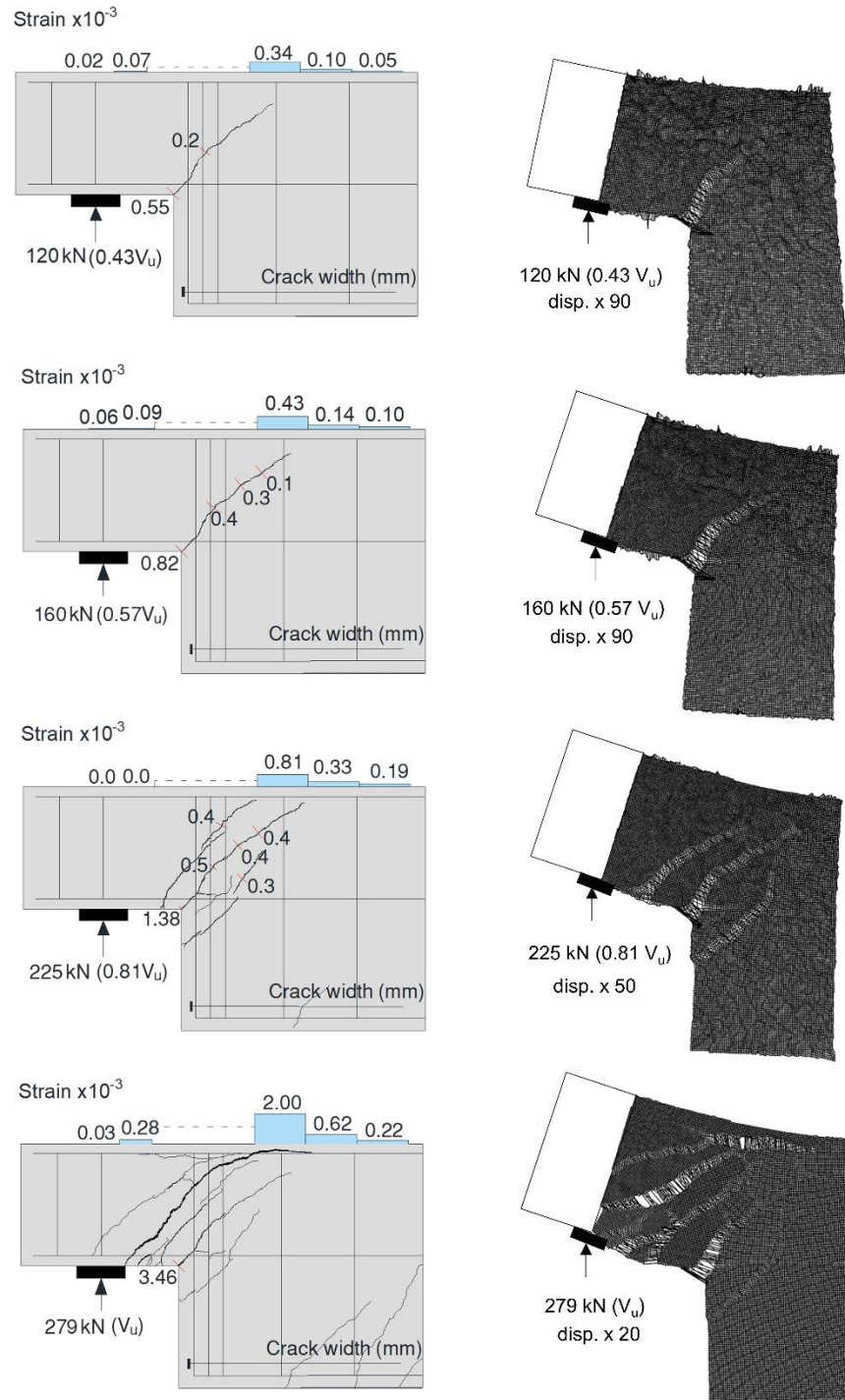
2. Observed flexural behaviour of dapped-end connections along re-entrant corner cracks

Figure 1 summarizes the properties and observed behaviour of a dapped-end connection tested by Rajapakse et al. [7] (specimen OL2). The total depth of the specimen was 1000 mm, the depth of the dapped end was 500 mm, and the width was 350 mm. The main reinforcement near the re-entrant corner consisted of 2 x $\phi 12$ mm U-bars (ϕ = bar diameter), as well as 12 $\phi 8$ mm vertical bars in the shape of stirrups. All details of the specimen are provided in Fig. 1a, including the measured material properties.

As evident from Fig. 1b, initially the specimen developed a single inclined crack, which propagated from the re-entrant corner. The crack crossed the horizontal and vertical dapped-end reinforcement, and extended towards the top of the section. When the applied load V reached 43% of the failure load V_u , the measured width of the crack w was 0.55 mm. When the load was nearly doubled to 81% of V_u , the width of the crack increased to 1.38 mm. At this stage, strain gauges on the reinforcement showed that the horizontal reinforcement had yielded in the crack, while the stirrups were still elastic. As the crack was widening, compressive strains developed along the top edge of the specimen. These strains were measured with the chain of displacement transducers shown in Fig. 1a. The strains had a maximum value above the tip of the crack and diminished away from the tip in the dapped end. At peak load $V_u=279$ kN, all dapped-end reinforcement had yielded and the crack width measured 3.46 mm. The compressive strain above the crack tip reached -2×10^{-3} . This value was measured over a gauge length of 210 mm, and therefore the maximum local strain was likely significantly larger.



(a) Details of OL2 [7]



(b) Behaviour of specimen OL2 [7] (deformed shapes are scaled to a constant deflection of the dapped end)

Fig. 1 Specimen OL2 [7]

The large crack widths and compressive strains were associated with a large rotation of the dapped end relative to the rest of the specimen. This is evident from the measured deformed shapes of the specimen at failure shown in Fig. 1b. The deformed shapes were obtained with digital image correlation, and illustrate a typical flexural failure of dapped-end connections. Such failures are characterized by yielding of the reinforcement in the critical re-entrant corner crack, as well as a nearly pure rotation of the dapped end about the tip of the crack.

3. Overview of kinematics-based model for flexural strength of dapped-end connections

Such experimental observations were used by Rajapakse et al. [21] to formulate a mechanical model for the flexural strength of dapped-end connections. This model will serve as a basis for the development of a complete modelling approach for flexural behaviour. To facilitate the derivation of the complete approach, the original strength model is recapped in this section. Figure 2 summarizes the assumptions of the original model and its equations, including the conditions for compatibility of deformations, constitutive relationships, and equilibrium.

The kinematics of the model is illustrated in Fig. 2a, and can be compared to the measured deformed shapes in Fig. 1b. The model features a straight re-entrant corner crack inclined at angle θ , which divides the connection into two rigid blocks. Angle θ is estimated at 45° , or can be measured on site if an existing cracked dapped-end connection is being analysed. As the load is increased, the rigid blocks rotate relative to each other about the tip of the crack, leading to the localization of plastic deformations in the compression zone above the crack. It is assumed that this results in the development of a triangular compression damage zone (CDZ), similar in shape to the damage zones observed in beams failing in flexure [24]. With the defined geometry of the CDZ and the strain profile shown in Fig. 2a, the rotation ϕ about the tip of the crack at failure is obtained from Eq. 1. Failure is assumed to occur with crushing of the concrete in the CDZ when the strain of the top fibre reaches 3.5×10^{-3} .

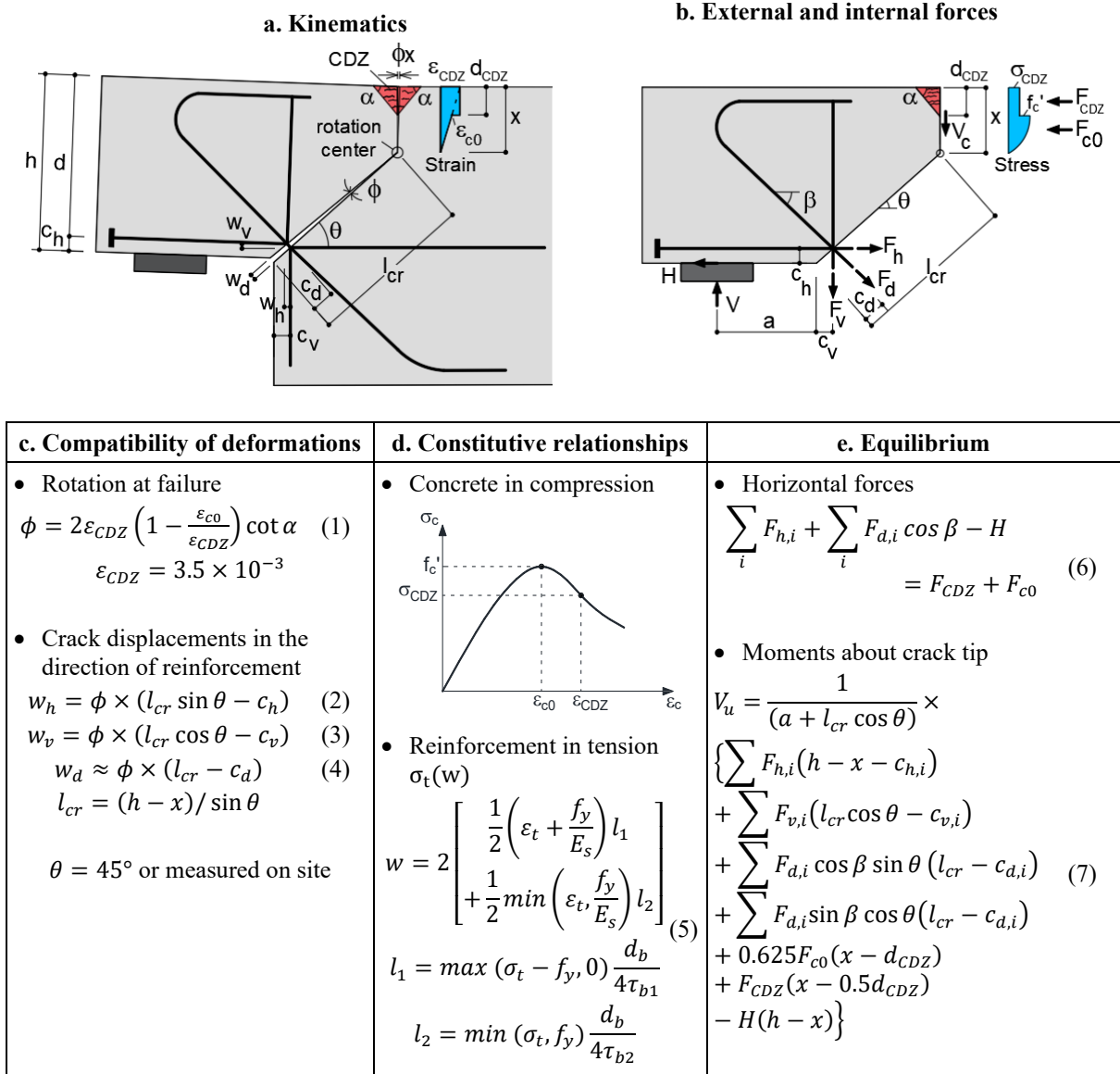


Fig. 2 Kinematics-based model for flexural strength of dapped-end connections [21]

The rotation ϕ is also associated with the opening of the corner crack, and thus with tension in the dapped-end reinforcement across the crack. For a given depth of the compression zone x and rotation ϕ , the crack width components in the direction of the reinforcement are computed based on small-displacement kinematics according to Eqs. 2-4. Widths w_h , w_v , and w_d

correspond to the horizontal, vertical, and diagonal reinforcement, respectively. Because ϕ is readily obtained from Eq. 1, the main unknown of the kinematics-based model is the depth x .

Having expressed the important deformations with x , the corresponding compressive stresses in the CDZ and tensile stresses in the reinforcement are computed using constitutive relationships – see Fig. 2d. The stresses in the compression zone are evaluated from the strains by using an appropriate constitutive relationship for concrete under uniaxial compression [25]. The tensile stresses σ_t in the reinforcement are computed from the crack widths w by using the tension chord model [26] (Eq. 5). In this model, the reinforcement is analysed as being anchored by bond in the concrete blocks on each side of the crack. Depending on which reinforcement is analysed, w is either w_h , w_v , or w_d from Eq. 2-4. The bond stresses τ_{b1} and τ_{b2} in Eq. 5 are calculated as recommended in [21,26]: f_{ct} and $2f_{ct}$, respectively, where f_{ct} is the tensile strength of the concrete [26]. If the bars are smooth, a constant reduced bond stress $\tau_{b1} = \tau_{b2} = 0.8f_{ct}$ is used [27].

Once the stresses/forces in the concrete and the reinforcement are obtained, the horizontal equilibrium of the free body in Fig. 2b is considered as per Eq. 6. This allows the calculation of the unknown depth of the compression zone x . Finally, the flexural strength of the connection V_u is obtained from Eq. 7, which expresses the moment equilibrium about the tip of the re-entrant corner crack. For complete information about this model, the reader is referred to the publication by Rajapakse et al. [21].

4. Model for complete flexural behaviour and crack width predictions

The kinematics-based model for crack width predictions will be formulated by extending the model for ultimate strength described in Section 3. In the extended model the strain in the compression zone will be increased from zero up to the failure of the connection. This will

allow to capture the complete behaviour in terms of applied load V versus the opening of the corner crack $w = \phi \times l_{cr}$.

The proposed model maintains the same equilibrium conditions shown Fig. 2e. The constitutive relationships for concrete in compression and reinforcement in tension also remain the same (Fig. 2d). However, in order to have realistic crack width predictions, a more refined modelling of the bond behaviour is required (stresses τ_{b1} and τ_{b2}), taking into account potential local concrete splitting and other effects.

The kinematics of the model in Fig. 2a is also maintained, albeit having one key difference. While the kinematic model for ultimate strength featured a compression damage zone (CDZ) associated with localized plastic deformations, this zone will not be present at low load levels when the concrete is nearly elastic. Therefore, a complete formulation is needed of the compression zone from initial loading up to failure, including the initiation of the CDZ.

4.1 Strains and rotation in the compression zone

The initiation of the CDZ can be linked to the strain in the top concrete fibre above the tip of the corner crack – see $\varepsilon_{c,top}$ in Fig. 3. When the stress in this location exceeds the compressive strength of the concrete f_c' , the material enters the post-peak regime, and localization of plastic strains occurs [21]. Therefore, it will be assumed that the CDZ is initiated when $\varepsilon_{c,top}$ reaches the strain corresponding to the peak stress of the concrete under uniaxial compression ε_{c0} . When $\varepsilon_{c,top} < \varepsilon_{c0}$, the strains across the depth of the compression zone x increases linearly from zero at the tip of the crack to $\varepsilon_{c,top}$ in the top fibre. When $\varepsilon_{c,top} \geq \varepsilon_{c0}$, the strain profile across x features a constant strain region within the CDZ similar to that in the model for flexural strength. The depth of the CDZ increases with increasing top strain according to:

$$d_{CDZ} = \max \left[0, \left(1 - \frac{\varepsilon_{c0}}{\varepsilon_{c,top}} \right) x \right] \quad (8)$$

Below the CDZ, the strains decrease linearly from ε_{c0} to zero at the tip of the crack. This strain profile will be used later to calculate the stresses and forces in the compression zone (Section 4.2).

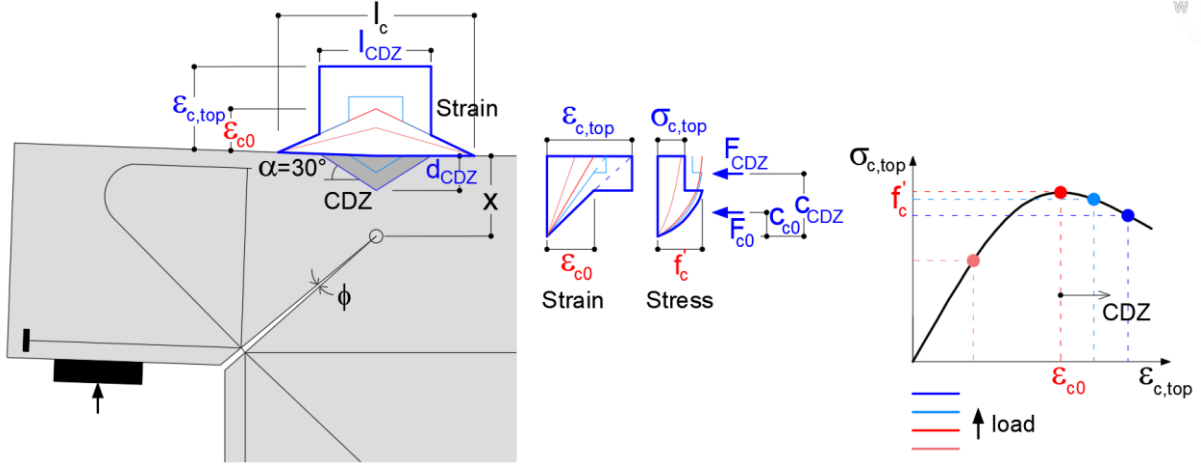


Fig. 3 Modelling of compression zone at various levels of top strain [25]

As the strain profile in the compression zone varies depending on the value of $\varepsilon_{c,top}$, it is necessary to modify Eq. 1 for the rotation about the tip of the corner crack ϕ . In general, the rotation is obtained by dividing the shortening of the top concrete fibre by the depth of the compression zone x . The shortening itself is evaluated by integrating the compressive strains along the top fibre, which contribute to the opening of the corner crack. Therefore, in addition to the strain profile across depth x , it is also necessary to approximate how the strains vary along the top concrete fibre before and after the formation of the CDZ.

To this end, it is instructive to revisit the strain diagrams of specimen OL2 shown in Fig. 1b. As pointed out earlier, the highest compressive strain along the top fibre occurred directly above the tip of the corner crack. To either side of this section, the compressive strains decreased gradually to nearly zero away from the crack. For specimen OL2, the measured

length affected by these strains under service loads was $l_c \approx 630$ mm, or approximately 1.4 times the effective depth of the dapped end d .

Based on these and other experimental observations, the strains along the top concrete fibre are approximated as shown in Fig. 3. Before the formation of the CDZ, the strains decrease linearly to zero over a length of $l_c/2$ on either side of the crack tip. Once $\varepsilon_{c,top}$ exceeds ε_{c0} and the CDZ forms, a constant strain of $\varepsilon_{c,top}$ is assumed along the length of the CDZ, l_{CDZ} . Outside the CDZ, the strains decrease to zero following the strain profile at $\varepsilon_{c,top} = \varepsilon_{c0}$. Based on these assumptions, the shortening of the top concrete fibre is expressed with $\varepsilon_{c,top}$ as follows:

If $\varepsilon_{c,top} < \varepsilon_{c0}$

$$\delta_{c,top} = 0.5\varepsilon_{c,top}l_c \quad (9a)$$

If $\varepsilon_{c,top} \geq \varepsilon_{c0}$

$$\delta_{c,top} = \varepsilon_{c,top}l_{CDZ} + 0.5\varepsilon_{c0} \times \max[0, (l_c - l_{CDZ})/l_c] \times \max[0, (l_c - l_{CDZ})] \quad (9b)$$

where the length of the CDZ and l_c are

$$l_{CDZ} = 2d_{CDZ}\cot\alpha \quad (9c)$$

$$l_c = d \quad (9d)$$

and $\alpha \approx 30^\circ$ [28].

Note that l_c is assumed equal to d (Eq. 9d), shorter than observed in test OL2. This is in part to account for the fact that the measured compressive strains are not associated only with the opening of the re-entrant corner crack, but also contribute to curvature away from the crack. The sensitivity of the predictions to this and other modelling parameters will be examined later in the study.

Having derived the shortening of the top concrete fibre $\delta_{c,top}$, it remains to express the rotation in the crack:

$$\phi = \frac{\delta_{c,top}}{x} \quad (10)$$

Therefore, the complete strain profile in the compression zone, as well as rotation ϕ , can be calculated for any given pair of top strain $\epsilon_{c,top}$ and depth of the zone x .

4.2 Stresses and forces in the compression zone

In addition to the strain profiles, Fig. 3 also shows the stress profiles in the compression zone pertaining to before (red curves) and after (blue curves) the formation of the CDZ. The stress profiles are obtained directly from the strain profiles by using a stress-strain relationship for concrete in uniaxial compression. In this study, the Popovics-Thorenfeld relationship is used [25] as extended by Collins and Porasz [29].

When the stresses in the compression zone are integrated, they produce the forces in the zone corresponding to the compression damage zone F_{CDZ} and the region below the CDZ F_{c0} :

$$F_{CDZ} = b d_{CDZ} \sigma_{c,top} \quad (11a)$$

$$F_{c0} = b \int_0^{x-d_{CDZ}} \sigma_c dx \quad (11b)$$

where b is the width of section and x is the depth of compression zone. Stresses $\sigma_{c,top}$ and σ_c denote respectively the stress along the top fibre and concrete stress for a given strain.

The distance from force F_{CDZ} to the tip of the corner crack is

$$c_{CDZ} = x - 0.5d_{CDZ} \quad (11c)$$

and that from force F_{c0} to the tip is

$$c_{c0} = \frac{\int_0^{x-d_{CDZ}} \sigma_c x_0 dx}{\int_0^{x-d_{CDZ}} \sigma_c dx} \quad (11d)$$

where x_0 is a coordinate along the depth of compression zone (zero at the crack tip). These integrals are evaluated numerically by discretizing the depth of compression zone into a sufficient number of layers (fibres) as commonly done in flexural calculations.

4.3 Tension in the reinforcement and bond with the concrete

As mentioned earlier, in the kinematics-based modelling framework, the relationship between the tensile strains/stresses in the reinforcement in the crack and the corresponding crack width is established according to Eq. 5 [26]. This relationship is significantly affected by the values of the bond τ_{b1} and τ_{b2} between the reinforcement and the concrete. In the model for flexural strength (Section 3), τ_{b1} and τ_{b2} were estimated in a simple manner [26] that reflects the pullout behaviour of bars in uncracked concrete under uniaxial conditions. However, as the conditions in dapped-end connections are more complex, a refined modelling of the bond is presented in the following.

There are at least two phenomena that can result in reduction of bond in dapped-end connections. First, in the vicinity of the inclined re-entrant corner crack, there are two (or three) layers of reinforcement crossing the crack in different directions (horizontal, vertical and/or diagonal). This can result in local debonding of the reinforcement in the re-entrant corner, which has been observed even in uniaxial tests [30]. Second, in the presence of big bars and/or small concrete cover, splitting cracks can occur along the bars, exacerbated by dowel action of the bars. This results in a loss of lateral confinement, and thus reduction of bond strength.

To account for the first effect, all bars are considered unbonded over a length l_0 of 3 bar diameters on each side of the re-entrant corner crack [30]. If the bars have different diameters,

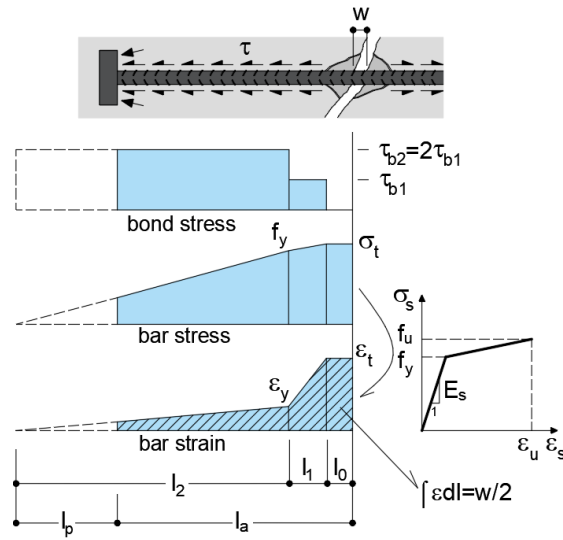
the largest diameter is used in the calculations. To account for the second effect, an expression for bond strength in the presence of local splitting is adopted from Tepfers [31]:

$$\tau_{b2} = \min \left(2, \frac{c + d_b/2}{1.664d_b} \right) f_{ct} \quad (12a)$$

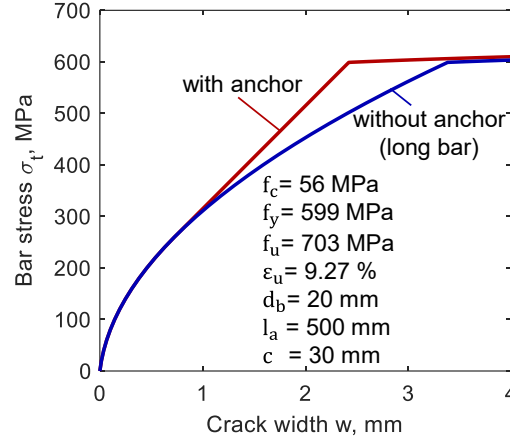
$$\tau_{b1} = \tau_{b2}/2 \quad (12b)$$

where c is the minimum concrete cover for each layer of reinforcement (horizontal, vertical and diagonal), d_b is the bar diameter, and $f_{ct} \approx 0.33\sqrt{f_c}$ (MPa).

These modifications are illustrated in Fig. 4a, which shows the variation of bond stresses, bar stresses, and bar strains on one side of the crack. The activated length of the reinforcement is divided into three parts: l_0 near the crack face where the reinforcement is unbonded (zero bond), l_1 where the reinforcement yields and the bond is τ_{b1} , and l_2 farther in the concrete where the reinforcement is elastic and the bond is τ_{b2} .



(a) Strain and stress distribution in the vicinity of a crack



(b) Sample behaviour of reinforcement with and without anchorage

Fig. 4 Modelling the behaviour of reinforcement including the effect of end anchorage

For a given stress in the bar in the crack σ_t , the stress diagram along the bar is obtained from equilibrium using the bond values. The strains along the bar are in turn obtained from the stresses using the bilinear stress-strain diagram shown in the figure. When the strains are integrated on both sides of the crack, they produce the crack width w :

$$w = 2 \left[l_0 \varepsilon_t + \frac{1}{2} \left(\varepsilon_t + \frac{f_y}{E_s} \right) l_1 + \frac{1}{2} \min \left(\varepsilon_t, \frac{f_y}{E_s} \right) l_2 - \frac{1}{2} \left(\min \left(\varepsilon_t, \frac{f_y}{E_s} \right) \frac{l_p}{l_2} \right) l_p \right] \quad (13a)$$

$$l_0 = 3d_b \quad (13b)$$

$$l_1 = \max (\sigma_t - f_y, 0) \frac{d_b}{4\tau_{b1}} \quad (13c)$$

$$l_2 = \min (\sigma_t, f_y) \frac{d_b}{4\tau_{b2}} \quad (13d)$$

$$l_p = \max (l_0 + l_1 + l_2 - l_a, 0) \quad (13e)$$

This refined equation is used to replace Eq. 5 of the original model for flexural strength. It is applied separately to the horizontal, vertical and diagonal reinforcement, and therefore w represents the crack displacement in the direction of the reinforcement w_h , w_v and w_d , respectively.

The last term of Eq. 13a is added in this study to account for the possible presence of a stiff anchorage at the end of the bar, such as for example an anchor head or a loop. In such cases, if the length of the bar l_a is smaller than $(l_0 + l_1 + l_2)$, the strain diagram along the bar is curtailed at the location of the anchor (shaded area in Fig. 4a). This results in a smaller crack width for a given reinforcement stress in the crack as illustrated in Fig. 4b. This plot compares the results from Eqs. 13 applied to a 20 mm bar with and without an anchor. The two $\sigma_t - w$ curves deviate when the strain penetration along the bar reaches the anchor. Following this point, the anchored bar exhibits a stiffer behaviour. It is important to note that both scenarios examined in Fig. 4b involve either sufficient anchorage length or a stiff anchor which prevents anchorage failure. The case of insufficient anchorage length without a stiff anchor, and its associated failure mode, is not considered in the present study, and requires further investigation.

Finally, the proposed model also accounts for bar ruptures in the re-entrant corner crack. If the strain in any horizontal, vertical or diagonal bar/layer reaches the rupture strain of the reinforcement ε_u , the stress in the bar is brought to zero.

4.4 Solution procedure of the kinematics-based model for complete flexural behaviour

The modifications proposed above can be seamlessly integrated into the kinematics-based modelling framework. With these changes, the support shear force for a given strain in the top fibre $\varepsilon_{c,top}$ can be computed from the moment equilibrium taken about the tip of the corner crack:

$$\begin{aligned}
V = & \frac{1}{(a + l_{cr} \cos \theta)} \\
& \times \left\{ \sum F_{h,i} (h - x - c_{h,i}) \right. \\
& + \sum F_{v,i} (l_{cr} \cos \theta - c_{v,i}) + \sum F_{d,i} \cos \beta \sin \theta (l_{cr} - c_{d,i}) \\
& \left. + \sum F_{d,i} \sin \beta \cos \theta (l_{cr} - c_{d,i}) + F_{c0} c_{c0} + F_{CDZ} c_{CDZ} - H(h - x) \right\}
\end{aligned} \tag{14}$$

The notations in this expression are illustrated in Fig. 2 and Fig. 3.

To solve all equations of the proposed model, a step-by-step iterative procedure is illustrated with the flowchart in Fig. 5.

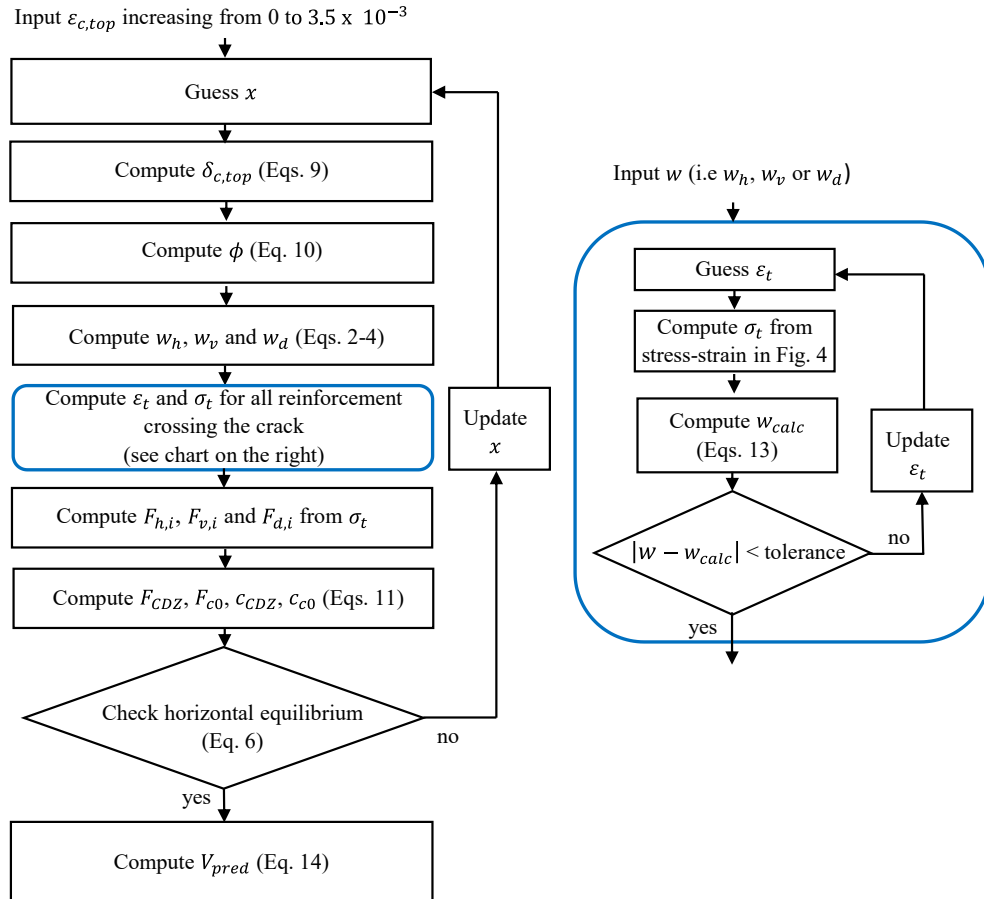


Fig. 5 Solution procedure

The computations are performed for $\varepsilon_{c,top}$ varying from 0 to 3.5×10^{-3} . Larger compressive strains are not analysed as this is associated with excessive damage and significant uncertainties. The main unknown of the method is the depth of the compression zone x , which is obtained iteratively on the basis of horizontal equilibrium. For each value of x , the deformations are obtained from compatibility of deformations, and the stresses/forces from the constitutive relationships discussed above.

4.5 Failure modes

According to the proposed kinematics-based model, failure along the re-entrant corner crack can be triggered as follows:

1. Yielding of all reinforcing bars at the re-entrant corner, followed by rupture of the reinforcement.
2. Yielding of all reinforcing bars at the re-entrant corner, followed by crushing of the concrete.
3. Crushing of the concrete accompanied with yielding of some reinforcing bars at the re-entrant corner.

The first two failure modes are typically observed in lightly reinforced dapped ends, with the first mode governing in connections that use low-ductility reinforcement. The third failure mode is generally observed in heavily reinforced specimens, where concrete crushing occurs prior to the yielding of some reinforcing bars.

5. Validation of model for complete behaviour and crack width predictions

5.1 Specimen OL2

The kinematics-based model formulated above is used to predict the complete flexural response of specimen OL2 (Section 2). The response is expressed in terms of applied force V versus

crack width at the re-entrant corner $w = \phi \times l_{cr}$. Figure 6a compares the predicted and measured responses of the specimen up to failure. In addition, the plot is used to study the sensitivity of the predictions to the assumed length of the compression zone l_c (Fig. 3). Three values of l_c are used for comparisons: $0.5d$, $1d$ and $1.5d$, where d is the effective depth of the dapped end.

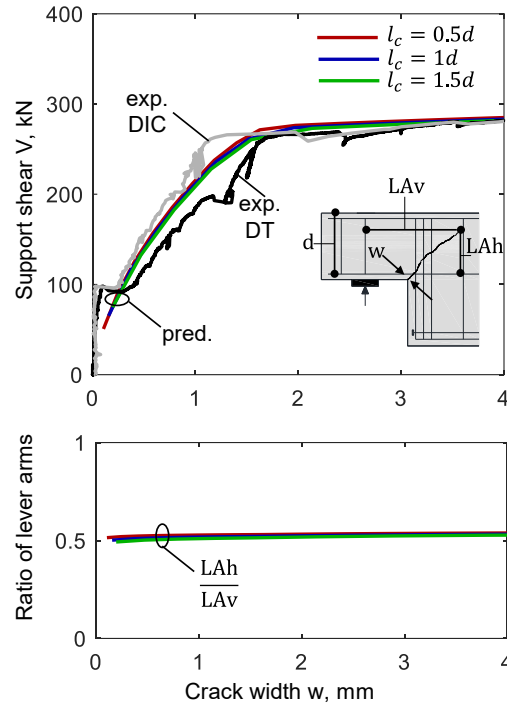
It can be seen from Fig. 6a that the difference between the three prediction curves is negligible. It can also be seen that the predictions agree very well with the measured response (The crack widths were measured with DIC on one face and displacement transducers DT on the opposite face). Following the crack formation at $V \approx 100$ kN, the measured response softened gradually in an exponential manner until all reinforcement yielded at the re-entrant corner. The yielding initiated a long plastic plateau characterized by uncontrolled opening of the corner crack. The proposed kinematics-based model captured well both the pre- and post- yielding regime, including the flexural strength of specimen OL2. The strength experimental-to-predicted ratio is $V_{exp}/V_{pred} = 283 \text{ kN}/310 \text{ kN} = 0.91$.

The low sensitivity of the predictions to length l_c can be explained by analysing the resistance components in Eq. 14. This equation is analysed for a fixed crack displacement w and different values of l_c . Each component of the equation contributing to the load V is obtained as a product of two quantities: a resisting force ($F_{h,i}$, $F_{v,i}$, $F_{d,i}$, F_{c0} or F_{CDZ}), and the ratio of the lever arm of the force to the lever arm of the load V . As the strains and stresses in the reinforcement are directly linked to the crack width, reinforcement forces $F_{h,i}$, $F_{v,i}$, and $F_{d,i}$ are unaffected by l_c for a given w . In turn, by considering the horizontal equilibrium, the forces in the concrete F_{c0} and F_{CDZ} are also unaffected by l_c . Therefore, it can be concluded from Eq. 14 and Fig. 6a that the ratios of the lever arms remain nearly unchanged as well. This is confirmed in the bottom plot of Fig. 6a, where the ratio of the lever arms of forces F_h and $V - LAh/LAV$ – is plotted for

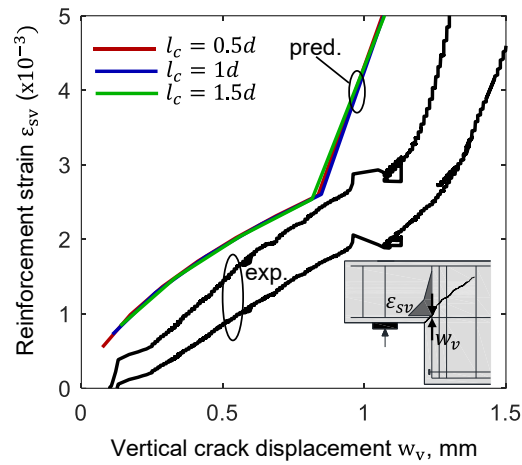
the three l_c values, showing nearly identical results. This demonstrates that the simplification $l_c = d$ made earlier is appropriate.

Following the same procedure, it is also possible to evaluate the assumption for a triangular strain profile within l_c (Fig. 3). This is because both l_c and the strain profile affect the shortening of the top compressive fibre $\delta_{c,top}$ in the same way – see Eq. 9a. If l_c is doubled, $\delta_{c,top}$ doubled as well. If the fullness of the strain diagram is doubled (i.e., from triangular to a block diagram), $\delta_{c,top}$ also doubles. Therefore, it can be concluded that the shape of the strain diagram has an equally limited impact on the predicted V - w response as the length l_c .

In addition to the V - w response in Fig. 6a, Fig. 6b shows the relationship between the crack width in the direction of the vertical reinforcement w_v and the strain in the reinforcement ε_{sv} of specimen OL2. Both the measured and predicted ε_{sv} - w_v curves are presented, where the prediction is generated with Eqs. 13. The measured strains ε_t were reduced by 0.4×10^{-3} to account approximately for shrinkage strains imposed by the concrete on the reinforcement (shrinkage was not measured directly in the test). It can be seen that Eqs. 13, used with the bond strength expression given by Eq. 12, capture reasonably well the measured response.



(a) Measured and predicted V - w response of specimen OL2 for different values of l_c



(b) Measured and predicted $\varepsilon_{sv} - w_v$ response of OL2 for different values of l_c

Fig. 6 Effect of length l_c on the kinematics-based model predictions

5.2 Tests by Rajapakse et al. [5],[7]

Specimen OL2 was part of a test series of 16 dapped-end connections – see Table 1. Eight of the specimens had orthogonal reinforcement (OL set) and eight had both orthogonal and

diagonal reinforcement (DL set). Specimen OL1 had the smallest amount of reinforcement and exhibited a pure a flexural failure along a dominant re-entrant corner crack. As the reinforcement amount was increased in the other specimens, the failure mode shifted gradually to a combination of flexure and shear. The shear cracks developed in the dapped end and extended towards the top of the main vertical reinforcement (stirrups). Nevertheless, strain gauge measurements show that in all 16 specimens the main dapped-end reinforcement yielded (or nearly yielded) prior to failure [5],[7], and therefore all specimens reached their flexural capacity along the re-entrant corner crack. The observed and predicted failure modes of the individual specimens are reported in Table 1.

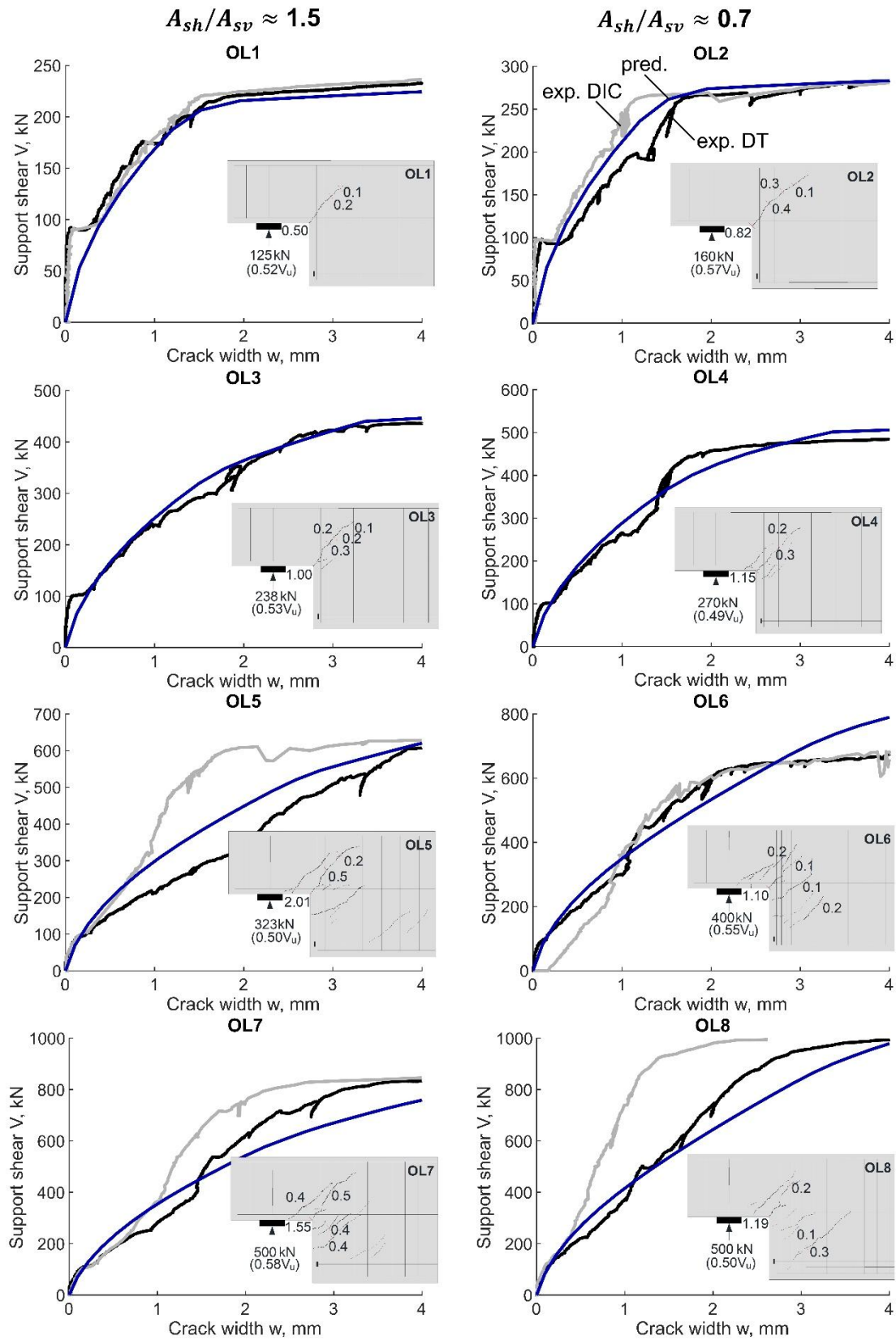


Fig. 7 Kinematics-based model predictions of corner crack widths of test specimens by Rajapakse et al. (2022) [7]

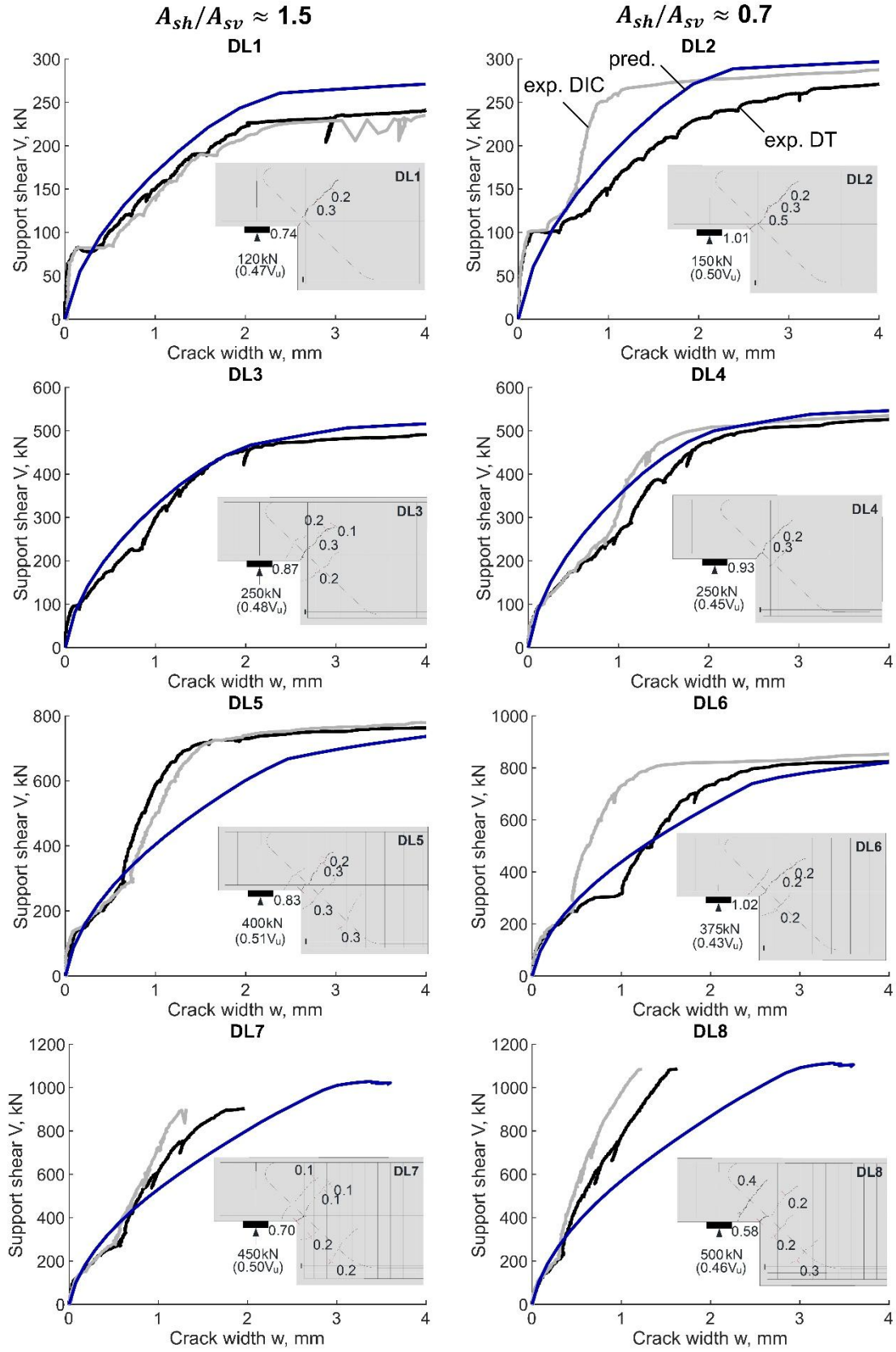


Fig. 8 Kinematics-based model predictions of corner crack widths of test specimens by Rajapakse et al. (2024) [5]

Table 1 : Tests of dapped-end connections

No.	Ref.	Beam name	a (mm)	h (mm)	b (mm)	c_h (mm)	$c_{2,h}$ (mm)	Hor. bars	f_{yh} (MPa)	c_v (mm)	Vert. bars	f_{yv} (MPa)	c_d (mm)	Diag. bars	f_{yd} (MPa)	f_c' (MPa)	$V_{u,exp}$ T1 (kN)	$V_{u,exp}$ T2 (kN)	Exp. failure mode	w_{exp} T1 (mm)	w_{exp} T2 (mm)	w_{pred} (mm)	$V_{u,pred}$ (kN)	$\frac{V_{u,exp}T1}{V_{u,pred}}$	Pred. failure mode
1	5,7	OL1	320	500	350	40	40	4d _b 12	537	32	6d _b 8	521	-	-	-	52.1	245	-	HV	0.45	-	0.58	249	0.98	HVC
2		OL2	320	500	350	40	40	4d _b 12	537	32	12d _b 8	521	-	-	-	52.1	283	-	HVC	0.66	-	0.52	310	0.91	HVC
3		OL3	320	500	350	40	40	4d _b 16	599	30	8d _b 10	509	-	-	-	51.3	472	-	HVC	1.05	-	0.90	463	1.02	HVC
4		OL4	320	500	350	40	40	4d _b 16	599	30	14d _b 10	509	-	-	-	51.3	555	-	HVC	1.20	-	0.94	531	1.05	HVC
5		OL5	320	500	350	40	40	4d _b 20	598	28	8d _b 12	537	-	-	-	48.1	628	-	HVC	1.81	-	1.09	659	0.95	VC
6		OL6	320	500	350	40	40	4d _b 20	598	28	16d _b 12	537	-	-	-	48.1	728	-	HVC	1.07	-	1.07	833	0.87	VC
7		OL7	320	500	350	40	40	4d _b 25	540	28	12d _b 12	537	-	-	-	46.2	868	-	HVC	1.46	-	1.39	770	1.13	VC
8		OL8	320	500	350	40	40	4d _b 25	540	28	24d _b 12	537	-	-	-	46.2	995	-	HVC	1.30	-	1.33	996	1.00	VC
9		DL1	320	500	350	40	40	2d _b 12	537	30	2d _b 10	509	165	1d _b 20	598	56.8	257	-	HVD	0.77	-	0.63	294	0.88	HVDC
10		DL2	320	500	350	40	40	2d _b 12	537	30	4d _b 10	509	165	1d _b 20	598	56.8	299	-	HVD	0.98	-	0.69	323	0.92	HVDC
11		DL3	320	500	350	40	40	2d _b 16	599	32	6d _b 8	521	56	3d _b 16	599	56.4	524	-	HVDC	0.91	-	0.68	532	0.98	HVDC
12		DL4	320	500	350	40	40	2d _b 16	599	32	10d _b 8	521	56	3d _b 16	599	56.4	559	-	HVDC	1.09	-	0.68	566	0.98	HVDC
13		DL5	320	500	350	40	40	2d _b 20	598	28	4d _b 12	537	60	4d _b 16	599	49.6	782	-	HVDC	0.70	-	0.95	754	1.04	HVDC
14		DL6	320	500	350	40	40	2d _b 20	598	28	8d _b 12	537	60	4d _b 16	599	49.6	876	-	HVDC	1.15	-	1.00	839	1.04	HVDC
15		DL7	320	500	350	40	40	2d _b 25	540	28	6d _b 12	537	65	3d _b 25	540	52.0	903	-	HVDC	0.72	-	0.76	1019	0.88	VDC
16		DL8	320	500	350	40	40	2d _b 25	540	28	12d _b 12	537	65	3d _b 25	540	52.0	1085	-	HVDC	0.62	-	0.92	1102	0.98	VDC
17	4	DEB-1.1	200	300	250	45	45	5d _b 10	567	35	2d _b 10+2d _b 8	587*	-	-	-	41.1	194	-	HV	0.83	-	0.84	186	1.04	HVC
18		DEB-1.2	200	300	250	45	45	3d _b 10	567	35	2d _b 10+2d _b 8	587*	-	-	-	39.3	146	133	HV	0.48	0.61	0.64	147	0.99	HVC
19		DEB-1.3	200	300	250	45	45	5d _b 10	567	36	2d _b 8	619	-	-	-	39.9	121	133	VC ^{T2}	0.42	0.34	0.61	140	0.87	VC
20		DEB-1.4	200	300	250	45	45	5d _b 10	567	36	2d _b 8	619	-	-	-	40.4	183	170	HV	0.51	0.83	0.88	175	1.05	VC
21		DEB-1.5	200	300	250	45	45	3d _b 10	567	36	2d _b 8	619	-	-	-	40.8	125	-	HV	0.59	-	0.57	137	0.91	HVC
22		DEB-1.6	200	300	250	42	42	4d _b 16	550	32	2d _b 10	544	-	-	-	31.1	309	251	HVC	0.61	0.99	1.15	252	1.23	VC
23		DEB-1.7	200	300	250	44	44	4d _b 12	546	34	2d _b 10	544	-	-	-	30	194	189	HC	0.58	0.74	0.72	220	0.89	VC
24		DEB-1.8	200	300	250	44	44	5d _b 12	546	34	2d _b 10	544	-	-	-	32.2	195	199	HVC	0.62	0.57	0.79	200	0.98	VC
25		DEB-1.9	200	300	250	44	44	3d _b 12	546	34	2d _b 10	544	-	-	-	31.9	142	146	HVC	0.43	0.45	0.56	175	0.81	HVC
26		DEB-2.1	200	300	250	45	45	3d _b 10	567	36	3d _b 8	619	55	2d _b 10	567	40.2	195	200	HVD	0.56	0.73	0.55	183	1.07	HVDC
27		DEB-2.2	200	300	250	44	44	4d _b 12	546	34	4d _b 10	544	56	2d _b 12+1d _b 10	545*	33.3	322	330	HVDC	0.54	0.52	0.65	297	1.09	VDC
28		DEB-2.3	200	300	250	44	44	3d _b 12	546	32	2d _b 12	546	56	2d _b 12	546	33.3	241	-	HVDC	0.48	-	0.57	238	1.01	VDC
29		DEB-2.4	200	300	250	45	45	4d _b 10	548	33	2d _b 12	552	55	2d _b 12+1d _b 16	548*	36.9	312	309	HVDC	0.43	0.58	0.62	303	1.03	VDC
30		DEB-2.5	200	300	250	46	46	2d _b 8	554	36	2d _b 8	554	54	2d _b 16+1d _b 12	546*	37.1	265	295	HVDC	0.41	0.31	0.73	301	0.88	HVDC
31		DEB-2.6	200	300	250	42	42	4d _b 16	544	34	2d _b 8	554	58	2d _b 16+1d _b 12	546*	38.3	328	-	VDC ^{T2}	0.55	-	0.55	346	0.95	VDC

a = distance from the support to the re-entrant corner, h = depth of dapped end, b = width of dapped end, $c_{1,h}$, $c_{2,h}$ = cover to Hor. bars, f_{yh} = yield strength of Hor. bars, c_v = cover to Vert. bars, f_{yv} = yield strength of Vert. bars, c_d = cover to Diag. bars, f_{yd} = yield strength of Diag. bars, f_c' = concrete compressive strength, $V_{u,exp}$ = measured strength, w_{exp} = measured crack width at $0.5V_{u,exp}T1$, w_{pred} = predicted crack width at $0.5V_{u,exp}T1$, $V_{u,pred}$ = predicted strength, H: horizontal bars yield, V: vertical bars yield, D: diagonal bars yield, C: concrete crushing

Figures 7 and 8 show the measured and predicted $V-w$ responses of the specimens with orthogonal and diagonal reinforcement layouts, respectively. The observed crack patterns at $\sim 50\%$ of the failure load are also shown in the plots. In both figures, each row shows two specimens with identical horizontal (and diagonal) dapped-end reinforcement, but different vertical reinforcement. The specimens on the left had a horizontal-to-vertical reinforcement ratio A_{sh}/A_{sv} of approximately 1.5, and those on the right of ~ 0.7 .

It is evident from Figs. 7 and 8 that the model predicts reasonably well the corner crack widths for all specimens, except for the cases with the heaviest reinforcement OL7, OL8, DL7 and DL8. The main reason for this can be inferred from the crack diagrams in the figure. While the lightly reinforced specimens OL1 and OL2 developed a major crack in the re-entrant corner, the increase in reinforcement amounts resulted in additional (secondary) cracks. These cracks occurred both in the dapped end and in the full-depth section, and were approximately parallel to the corner crack. In the specimens with the heaviest reinforcement, the additional cracking occurred early and controlled the corner crack. This is clearly observed in the measured $V-w$ diagrams of specimens OL7 and OL8, where the response “stiffens” upon secondary cracking. Nevertheless, even for these specimens the proposed model captures well the initial response, which is of interest for serviceability limit state checks. In terms of flexural strength, the model provides adequate predictions across the entire range of reinforcement amounts. The effect of secondary cracking on the width of the corner crack, as well as the influence of restrained shrinkage in the concrete, have been recently modelled in more detail by Mihaylov [12].

5.3 Tests by Mata-Falcón (2015) [4]

An extensive experimental campaign featuring global and local deformation measurements was also reported by Mata-Falcón [4]. The total depth of the specimens was 600 mm, and the depth of the dapped end was 300 mm (Table 1). The aspect ratio of the dapped end a/d was

200 mm/250 mm=0.8 and the concrete compressive strength was in the range of 30 MPa to 41 MPa. Pairs of nominally identical dapped ends were tested to study the scatter in the response (tests T1 and T2). The crack widths at the re-entrant corner were measured continuously using digital image correlation.

Figure 9 shows comparisons of the measured re-entrant corner crack widths (black and ash lines for T1 and T2) with the model predictions (blue line). It can be seen that the crack widths under service loads, as well as the shape of the V - w response, are reasonably well captured by the model with some exceptions. As it can be expected, the model produces the same V - w response for each pair of nominally identical dapped ends. However, in certain cases a significant scatter is noticeable between the replicated tests.

Finally, as evident from Figs 7-9, the proposed model for complete flexural behaviour captures well the strength of all 42 specimens in Table 1 with a wide range of test variables. The average strength experimental-to-predicted ration is 0.98, and the coefficient of variation is 8.8%.

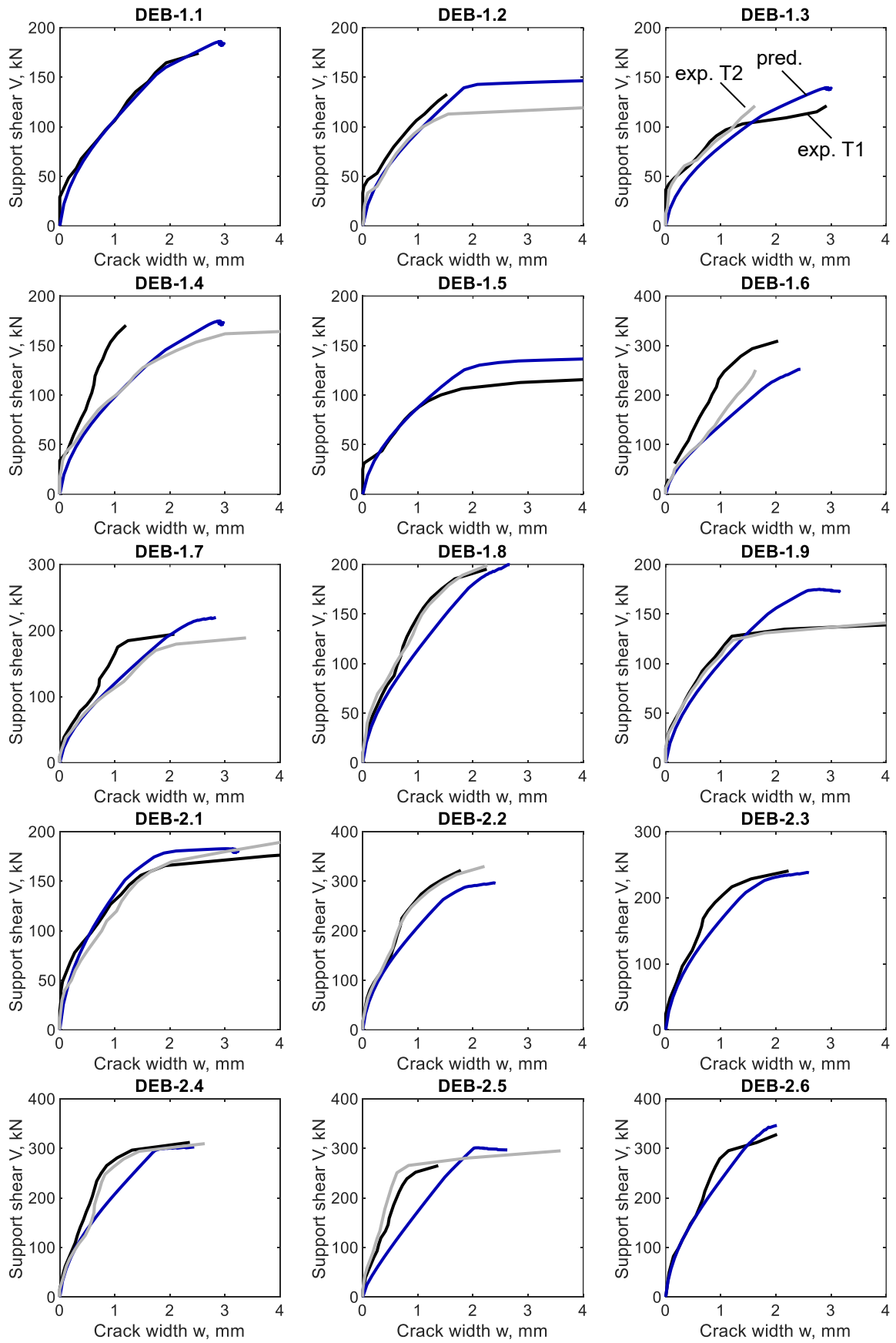


Fig. 9 Kinematics-based crack width prediction of test specimens by Mata-Falc3n (2015) [4]

Conclusions

This paper presented the formulation of a kinematics-based model for complete flexural behaviour of dapped-end connections. The re-entrant corner crack width and strength predictions of the model were validated with 42 specimens from two experimental studies, featuring a wide range of test variables. The main conclusions of the study are:

- The kinematics-based model captured reasonably well the complete load versus crack width response of the test specimens, including the peak resistance and the local engagement of the reinforcement in the cracks.
- Very large amounts of reinforcement can induce significant secondary cracks and restrained shrinkage, which are not explicitly accounted for in the model.
- For the 42 tests specimens modelled in this study, the proposed model produced strength experimental-to-predicted ratios with an average value of 0.98 and a coefficient of variation of 8.8%.
- The proposed model offers a suitable framework for future studies on crack-based assessment of existing connections. It also offers a suitable basis for modelling deterioration effects such as fatigue and corrosion.

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Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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