

Reliability modeling for multi-state power systems with decentralized renewable energy integration

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Abstract

The overall reliability of a microgrid hinges on the performance of its individual components. This paper proposes a probabilistic model based on Markov chains to evaluate the reliability of a microgrid embedded within a multi-state power system ($\mathcal{MSS}$). The microgrid comprises a wind turbine, photovoltaic panels, and a battery-based energy storage system, interconnected with the main grid. We develop a methodology integrating reliability analysis into power systems incorporating decentralized renewable generation via microgrids. This assessment serves both operational and economic objectives by quantifying the impact of distributed renewable units on distribution networks and mitigating outage costs from main grid failures. Reliability is modeled through Markov state-space diagrams that capture stochastic transitions via component-specific failure rates (λ) and repair rates (μ). Applied to an adapted IEEE 34-node test feeder, our results extend beyond conventional reliability indices. An importance analysis quantifies each component's contribution to system availability and expected energy shortfall, complemented by sensitivity analysis revealing their impact on overall performance. These findings equip network planners and operators with a decision-support tool for investment prioritization and grid resilience enhancement. Notably, estimating production capacity from stochastic renewable sources proves meaningful only when supported by highly reliable storage and power conversion systems.

1 Introduction

The planning of power distribution networks aims to meet evolving demand within operational constraints. Power system reliability means the ability of a power system network to generate adequate amount of electricity for an intended period of time under encountered operating conditions to meet the electricity demand. Distributed energy resources, such as generators using renewable energy sources (wind and solar) and storage, can enhance reliability by supplying power during outages [1-3]. However, the variability of these renewable sources limits their restoration effectiveness, a challenge addressed by storage and demand-side management [4].

Assessing this variability during outages is critical. Traditional binary reliability models (operational/failed) are insufficient. $\mathcal{MSS}$ models offer a nuanced representa-

tion with $\mathcal{M} + 1$ states, ranging from complete failure (0) to perfect operation ($\mathcal{M}$), including intermediate degraded performance states.

The behavior of an $\mathcal{MSS}$, governed by random failures and performance variations, is analyzed separately from power flow to quantify availability and interruptions. We formalize a generic $\mathcal{MSS}$ and construct a Markov model for its components, integrating state transition and occurrence probabilities—an established approach [5-6]. The model, based on exponentially distributed parameters, is presented in Section 2. The methodology is described in Section 3, and the discussion and conclusion are presented in Section 4.

2 Proposed model of microgrid

2.1. Model Assumptions

The system transitions through five performance levels: Perfect state: 100% power; Degraded state: 75% with reduced load; Semi-operational: 50% and 2% with load shedding; Complete failure: 0% energy delivery to feeder.

Binary-state Model

Let y (Measured performance variable, e.g. power in kW) be the substitution characteristic for the function of the component, and y_0 (Nominal performance value) the ideal or target value of y . Δ_0 is the half-width of the acceptable interval. In this two-state Markov process with a single transition from 1 to 0, the system works (is operational) if and only if it is in state 1.

$$x_i = \begin{cases} 1 & \text{if } y_0 - \Delta_0 \leq y \leq y_0 + \Delta_0 \\ 0 & \text{if the component has failed} \end{cases}$$

In a multi-state framework, different levels of performance can be defined according to intervals of $|y - y_0|$, corresponding to states such as excellent, degraded, or failed.

In a $\mathcal{MSS}$ with n different components, each component i is assumed to have $(m_i + 1)$ distinct performance levels, also called states. If $x_i(t)$ denotes the state of the i th component at time t , then $x_i(t) \in \{0, 1, \dots, m_i\}$ for $i = 1, 2, \dots, n \in \mathcal{N}$. When $x_i(t) = 0$, the i th component is in a state of total failure at time t . On the other hand, when $x_i(t) = m_i$, the i th component is in a state of perfect functioning at time t . For a static model that

considers the working efficiency of the system and its components at a fixed time t , the notation x_i is used instead of $x_i(t)$ to denote the performance state (working efficiency) of the i th component at that time. All components are assumed to be mutually independent, which means that the performance of each component is not affected by the performance of any other component.

A Markov model is a multi-state model with birth rates $\lambda_n \geq 0$, $n = 1, 2, \dots$ and death rates $\mu_n \geq 0$, $n = 1, 2, \dots$ describing repairable and non-repairable components. Each system state (combination of different generators) $x_i(t)$ has its own performance level. A state with zero capacity is a failed state, whereas a state with capacity equal to the rated production capacity is an available state. Adapted models are required to represent transient operating conditions. An operational state is a perfect state, that is, all components operate at rated power and satisfy the load demand. When modeling a system, it is always possible to break it down into its constituent parts. In reliability modelling, the objective is to establish a relationship between the failures of individual components or subsystems and the failure of the overall system, given a specified system performance objective.

3 Methodology

The system's state space is defined in terms of operational and failed configurations. At time t , the random variable X_t denotes the system state, where each component $i \in \{1, \dots, n\}$ occupies a binary state x_t^i (0 for failure, 1 for operation). The collection of all component states forms the finite state space $X_t = \{x_t^1, \dots, x_t^n\}$ which characterizes the behavior of the MSS. The system configuration may be expressed as $X_t = [P_{WT}^{(0)}, P_{PV}^{(0)}, P_{grid}^{(0)}, P_{BAT}^{(0)} | \dots | P_{WT}^{(T-1)}, P_{PV}^{(T-1)}, P_{grid}^{(T-1)}, P_{BAT}^{(T-1)}]$ together with the load demand vector $[P_{LOAD}^{(0)}, \dots, P_{LOAD}^{(T-1)}]^T$. Each component i is associated with an availability probability p and an unavailability probability $1 - p$, linked to the failure and repair rates λ and μ (in hours^{-1}). The system evolution is modeled as a continuous-time Markov process. Let $\mathbf{P}(t) = (P_1(t), P_2(t), P_3(t))$ denote the state probability (initial conditions) vector at time t . We consider the continuous-time Markov process governed by the Kolmogorov forward equations 1.

$$\frac{d\mathbf{P}(t)}{dt} = \mathbf{P}(t) \mathbf{Q} \quad (1)$$

where $\mathbf{Q}$ is the transition rate matrix. The diagonal entry of each row in the transition matrix equals $1 - \sum_{j \neq i} \lambda_{ij}$, where $\sum_{j \neq i} \lambda_{ij}$ represents the sum of the outgoing transition rates from state i .

3.1. Load Model

Load profiles are generally cyclic, with stochastic durations for peak loads (t_L) and off-peak loads (t_0), influencing the reliability of the distribution network. Assuming T_c and t_p represent the cycle time and the average peak

duration respectively, and that T_c is typically 24 hours, the duration of each state is $\Delta t = 1$ hour. The analyzed period may be, for example, one year ($T = 8760$ hours), and the transition rate between two time instants is $\lambda_{\Delta t} = 1$ occurrence per hour. The transition intensities $\lambda_{\text{loadPeak}}$ and λ_{loadLow} of the model can be obtained as follows 1.

$$\begin{cases} \lambda_{\text{load}}^{\text{Peak}} = \frac{1}{T_c - t_p} \\ \mu_{\text{load}}^{\text{Low}} = \frac{1}{t_p} \end{cases} \quad (2)$$

The load model is characterized by loads represented by a set of load levels or load states. The intensities $a_{i,i+1}$, for $i = 1, \dots, K$, representing the transitions from state i to state $i + 1$, are defined in terms of failure and repair rates.

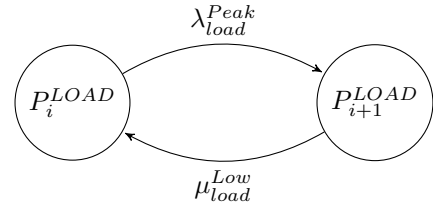


Figure 1: Power demand state-space diagram

3.2. Model Parameters: Low-Voltage Electrical Transformer

As a key component of an electrical grid, a transformer enables energy transfer between different voltage levels via inductively coupled windings. The state diagram in Figure 2 depicts the dynamic transitions and the temporal evolution of the transformer's operational state.

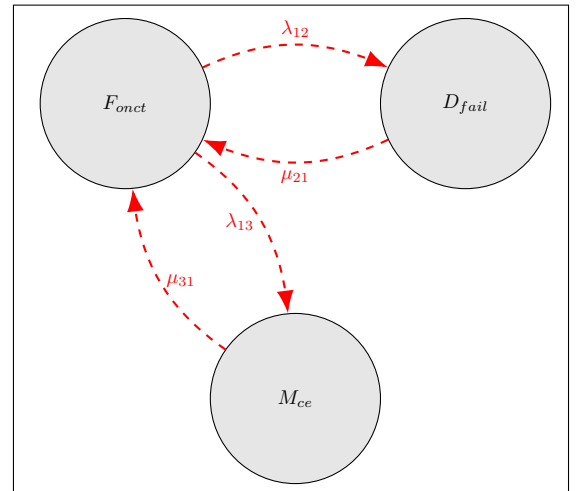


Figure 2: Assumptions on the mechanical states of a transformer: transformer operating $t_{F_{onct}}$, transformer failure $t_{D_{fail}}$, and transformer maintenance (repair) $t_{M_{ce}}$.

The random nature of the transformer's functional behavior is highlighted by its alternation among three states: (i) operational state $t_{Oper.}$, (ii) failure state $t_{D_{fail}}$, and (iii) maintenance state $t_{Maint.}$. The repair process is treated as a renewal process. Two stochastic variables are considered: the time to failure t_F with rate λ_{tF} , and the repair time t_R with rate μ_{tR} . These variables are assumed independent,

implying that transformer failure is not directly influenced by its load. The transformer's behavior is modeled using a Markov chain to describe the transitions between failure and repair states. Since the insulation temperature varies according to load cycles, the stochastic variables t_F , t_R , and the load-related variables t_L and t_{Lo} are assumed to follow exponential distributions. These parameters enable the modeling of transformer operation under varying load conditions and facilitate the derivation of operating and failure probabilities through differential equations.

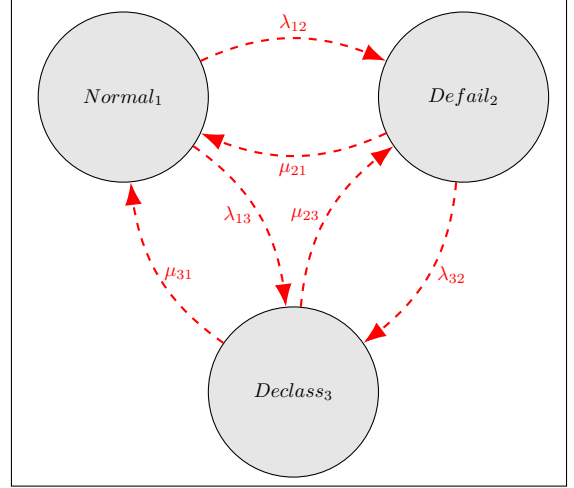
$$\begin{cases} f_{tF}(t) = \lambda e^{-\lambda t} \\ f_{tR}(t) = \mu e^{-\mu t} = \frac{1}{T_R} e^{-t/T_R} \\ f_{tL}(t) = \lambda_L e^{-\lambda_L t} = \frac{1}{T_L} e^{-t/T_L} \\ f_{tLo}(t) = \lambda_{Lo} e^{-\lambda_{Lo} t} = \frac{1}{T_{Lo}} e^{-t/T_{Lo}} \end{cases}$$

where λ is the failure rate in [1/year], μ is the repair rate in [1/year], the average outage duration T_R in [year], the average peak-load duration t_L in [year], and the average off-peak load duration t_{Lo} in [year]. The transition matrix corresponding to the three operating assumptions is defined as follows: the transformer is operational (t_{Fonct}), the transformer is in a failure state (t_{Dfail}), or the transformer is undergoing maintenance/repair (t_{Mce}) is given by,

$$\mathbf{Q} = \begin{pmatrix} -(\lambda_{12} + \lambda_{13}) & \lambda_{12} & \lambda_{13} \\ \mu_{21} & -\mu_{21} & 0 \\ \mu_{31} & 0 & -\mu_{31} \end{pmatrix}. \quad (4)$$

3.3. Markov model for the photovoltaic generator

The PV system contains many power electronic components[7]. The risk is significant because solar PV power depends on intermittent irradiation, which affects the temperature of the components: **State 1**: corresponds to good sunshine conditions. This means that the PV system is operating at full capacity. **State 2**: the system operates with partial degradation failures but remains fully functional. **State 3**: the system has major faults and operates only partially, with output power reduced relative to its nominal capacity. This situation occurs, for example, during cloudy periods, in the absence of sunshine, or at night.



(3) Figure 3: Three-state Markov diagram of the photovoltaic system: "normal", "failed", and "derated" with failure rate λ and repair rate μ .

The failure model of each component is analyzed and parameterized by the failure rate;

$$\begin{cases} \lambda_I^{PV}(P^{PV}, U^{PV}, T^{PV}) = \lambda_C^{PV} + \sum_i (\lambda_{Di}^{PV} + \lambda_{Gi}^{PV}) \\ \mu_I^{PV}(P^{PV}, U^{PV}, T^{PV}) = \frac{1}{\lambda_I^{PV}} \left[\lambda_C^{PV} \mu_C^{PV} + \sum_i (\lambda_{Di} \mu_{Di}^{PV} + \lambda_{Gi} \mu_{Gi}^{PV}) \right] \\ A^{PV} = (P^{PV}, U^{PV}, T^{PV}) \frac{\frac{1}{\mu_I^{PV}}}{\lambda_I^{PV} + \frac{1}{\mu_I^{PV}}} \\ P_i = \frac{N_i}{Total_Samples} \end{cases} \quad (5)$$

where λ_I^{PV} , μ_I^{PV} , and A^{PV} denote the failure rate, repair time, and availability of the photovoltaic inverter, respectively. The corresponding model parameters are $\lambda_I(P, U, T)$, $\mu_I(P, U, T)$, and $A_I(P, U, T)$, which denote the failure rate, the repair rate, and the availability of the photovoltaic inverter, respectively. P_i and N_i are the probability and number of occurrences of each state i , and N_{total} is the total number of samples. The subscripts G , D , and C refer to the failure rate and repair time of the IGBT, diode, and capacitor components, respectively, while the subscript i denotes the i -th component of the system. Here, P is the power, T is the temperature, and U is the voltage. The transition rates between the states are given by λ_{12} , λ_{13} , λ_{32} , μ_{21} , μ_{31} , and μ_{23} as shown in Figure 3.

$$\mathbf{Q} = \begin{pmatrix} -(\lambda_{12} + \lambda_{13}) & \lambda_{12} & \lambda_{13} \\ \mu_{21} & -(\mu_{21} + \lambda_{32}) & \lambda_{32} \\ \mu_{31} & \mu_{23} & -(\mu_{31} + \mu_{23}) \end{pmatrix}. \quad (6)$$

3.4. Markov model for wind generator

The combination of operational states includes the turbine and its subcomponents, along with the wind speed

behavior 4. Components WT_1 , GE , and EP have the status UP , indicating normal operation, while in WT_2 , WT indicates a $DOWN$ status; in WT_3 , GE is $DOWN$; and in WT_4 , EP is $DOWN$. The efficient use of wind energy remains a major challenge, due to the uncertainty in wind speed along with the occurrence of wind turbine failures. The number of states depends on the precision required by the model, and the exponential distribution assumes a constant transition rate between states. The failure rates λ_T^{WT} , λ_{GE}^{WT} , and λ_{EP}^{WT} correspond to the wind turbine, the permanent magnet generator, and the electrical panel, respectively;

$$\lambda_{WT} = \frac{\sum_i n_i}{T - \sum_k D_k} \quad (7)$$

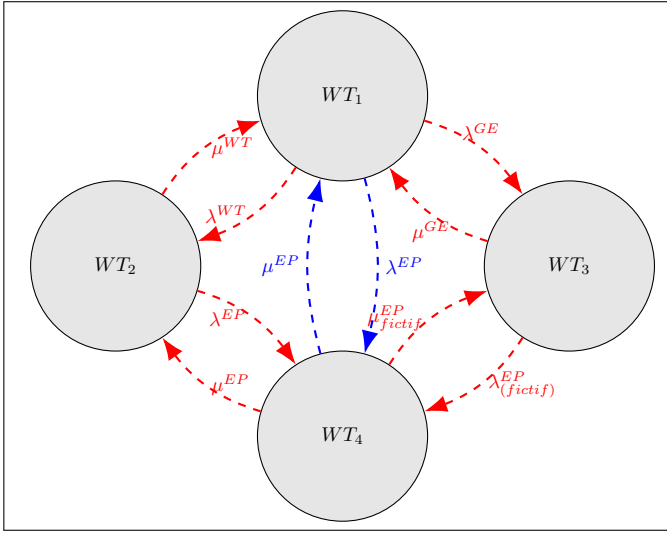


Figure 4: Markov model representing a wind turbine as a function of wind speed and its random failures.

where n_i is the number of failures of WT subsystem i over the total recording period (e.g., in SCADA data), and D_k is the downtime associated with each failure event k , for a total number of outage events k . The failure rate of each individual component considering the effect of wind speed can also be calculated by weighting the discrete probability distribution of wind energy:

$$\begin{cases} \mu_{WT} = \frac{\prod_{j=1}^n \mu_j \cdot \sum_{j=1}^n \lambda_j}{\sum_{j=1}^n (\lambda_j \cdot \prod_{k=1, k \neq j}^n \mu_k)} \\ \underline{a}_{WT}^{i+1(T)} = \frac{\lambda_{WT}}{\lambda_{WT} + \mu_{WT}} \end{cases} \quad (8)$$

where μ_j is the repair rate of wind subsystems, and $\underline{a}_{WT}^{i+1(T)}$ is the unavailability. The following random variables are considered in the Markov model for the wind system: λ_{WT} and μ_{WT} for the wind turbine; λ_{GE} and μ_{GE} for the permanent magnet generator; and λ_{EP} and μ_{EP} for the power electronics. We consider a continuous-time Markov chain with four states WT_1 , WT_2 , WT_3 , and WT_4 as shown in Figure 4. The infinitesimal generator matrix $\mathbf{Q}$ is given by:

$$\mathbf{Q} = \begin{pmatrix} -(\lambda^{WT} + \lambda^{GE} + \lambda^{EP}) & \lambda^{WT} & \lambda^{GE} & \lambda^{EP} \\ \mu^{WT} & -(\mu^{WT} + \lambda^{EP}) & 0 & \lambda^{EP} \\ \mu^{GE} & 0 & -(\mu^{GE} + \lambda^{EP}) & \lambda^{EP} \\ \mu^{EP} & \mu^{EP} & \mu_f^{EP} & -(2\mu^{EP} + \mu_f^{EP}) \end{pmatrix} \quad (9)$$

3.5. Stochastic model for battery

The Markov model ensures availability despite renewable variability and grid failures (BESS mitigates power quality issues in grid-connected/islanded modes) with rates λ_{BAT}, μ_{BAT} ; λ_{INV}, μ_{INV} ; $\lambda_{BAT,C}, \mu_{BAT,C}$. The BESS model diagram (Fig. 5) shows the following states: BAT_1 : All UP (perfect operation); BAT_2 : $BAT+BATC$ DOWN, INV UP, BAT_3 : $BAT+BATC$ UP, INV DOWN, BAT_4 : All DOWN (complete outage). With λ_{BAT}, μ_{BAT} (battery), λ_{INV}, μ_{INV} (inverter), and $\lambda_{BAT,C}, \mu_{BAT,C}$ (battery controller/charger).

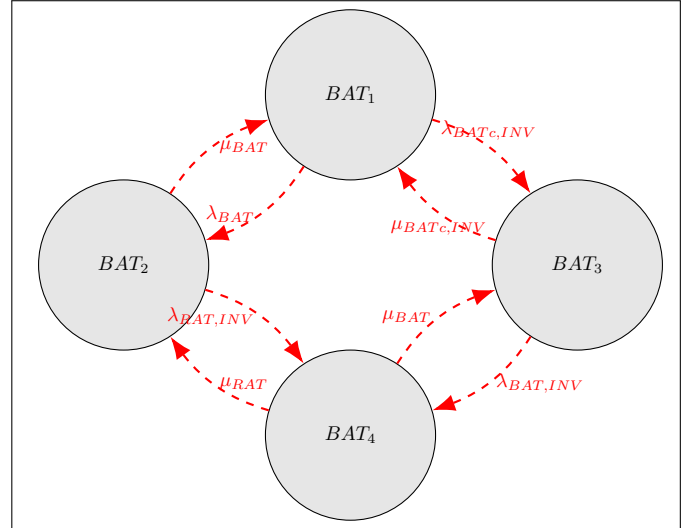


Figure 5: The state-space diagram of the battery

The parameter λ denotes the failure rate, defined as the probability per unit time of a system component (battery, controller, or inverter) transitioning from an operational to a failed state. Conversely, μ represents the repair rate, corresponding to the probability per unit time that a failed component is restored to full operational functionality.

$$\begin{cases} \lambda_{BAT}^{Unit} = \lambda_{BAT} + \lambda_{BAT,C} + \lambda_{INV} \\ \mu_{BAT}^{Unit} = \frac{\lambda_{BAT} + \lambda_{BAT,C} + \lambda_{INV}}{\left[\left(1 + \frac{\lambda_{BAT}}{\mu_{BAT}}\right) \left(1 + \frac{\lambda_{BAT,C}}{\mu_{BAT,C}}\right) \left(1 + \frac{\lambda_{INV}}{\mu_{INV}}\right) - 1 \right]} \end{cases} \quad (10)$$

where μ_{BAT} is the battery repair rate, $\mu_{BAT,C}$ is the battery charge controller repair rate and μ_{INV} is the inverter repair rate. A stochastic transition probability matrix is subsequently formulated to model the system state space figure 5. This matrix is mathematically described by the following 11 below;

$$\begin{bmatrix} 1-\lambda^{BAT}-\lambda_{INV}^{CC} & \lambda_{INV}^{CC} & \lambda^{BAT} & 0 \\ \mu^{BAT} & 1-\lambda^{BAT}-\mu_{CC,INV}^{BAT} & 0 & \lambda_{INV}^{CC} \\ \mu_{INV}^{BAT} & 0 & 1-\mu_{BAT,INV}^{BAT}-\mu_{INV}^{CC} & \lambda^{BAT} \\ 0 & \mu_{CC,INV}^{BAT} & \mu_{CC,INV}^{BAT} & 1-2\mu_{CC,INV}^{BAT} \end{bmatrix} \begin{matrix} p_0^{PV,WT,grid,BAT}(0) = 1, \\ p_4^{PV,WT,grid,BAT}(0) = 0, \\ p_3^{PV,WT,grid,BAT}(0) = p_2^{PV,WT,grid,BAT}(0) = p_1^{PV,WT,grid,BAT}(0) = 0. \end{matrix} \quad \begin{matrix} (12a) \\ (12b) \\ (14) \end{matrix}$$

where $\lambda = \frac{1}{TTF} = \frac{1}{\lambda_{BAT} + \lambda_{CC}}$.

3.6. CASE STUDY

The microgrid is assumed connected to the IEEE 34-bus test feeder, a standard reference system for distribution studies. Considering a constant power of 242 kW, the energy over the study period of 24 hours amounts to 5808 kWh. The state space is partitioned into operational states (up-states) and failure states (down-states). To validate the proposed model, a combined production-demand model is used. The main network is represented by its intrinsic state (20/0.4 kV, 630 kVA feeder supplied by 20 kV/0.4 kV, 630 kVA transformer).

In the multi-state Markov model, each generating unit has two mechanical states: working (p) and failed ($1-p$). The transformer (connected to the grid and operating at power P_{grid}) is modeled according to three operational states: $t_{Functioning}$, $t_{Failure}$, and t_{Repair} , with a rated power of 5,000 kW, and $\lambda_{grid} = 0.0004 \text{ year}^{-1}$ and $\mu_{grid} = 0.013 \text{ year}^{-1}$. The intrinsic state of the PV panels corresponds to a total capacity of 45.3 kW. In the multi-state Markov model, the panels have two mechanical states: working (p , during the day) or failed ($1-p$, at night or end of day). Three PV panel scenarios are considered: $P_{PV} = 15 \text{ kW}$, $P_{PV} = 8 \text{ kW}$, and $P_{PV} = 22 \text{ kW}$, connected to buses 812, 814, and 850, respectively. The failure rate is $\lambda_{PV} = 0.0005 \text{ h}^{-1}$, and the repair rate is $\mu_{PV} = 0.013 \text{ h}^{-1}$.

The intrinsic state of the wind turbine corresponds to a total capacity of 7 kW. In the multi-state Markov model, it has two mechanical states: working (p) or failed ($1-p$). The total solar and wind production capacities are 45.3 kWp and 7 kW, respectively. The wind turbine may experience forced outages. To account for this effect, the sequential up-down-up cycle can be aggregated based on $\lambda_{WT} = 0.0005 \text{ h}^{-1}$ and $\mu_{WT} = 0.013 \text{ h}^{-1}$. The state probabilities can be solved and used to calculate the reliability function.

The intrinsic state of the energy storage system corresponds to a total capacity of 132.2 kWh. In the multi-state Markov model, it has two natural states: charging ($1-p$, end of discharge) and discharging (p). The phenomena of battery failure and repair are assumed to be negligible. The state probability $p_0(t)$ can be readily employed to compute the reliability function, as it characterizes reliability through the relationship. The system is defined by the initial distribution $\mathbf{p}_i(t) = (1, 0, 0, 0)$ and by the transition probability matrix $\mathbf{Q}$ of the embedded Markov chain. The initial conditions are specified as follows:

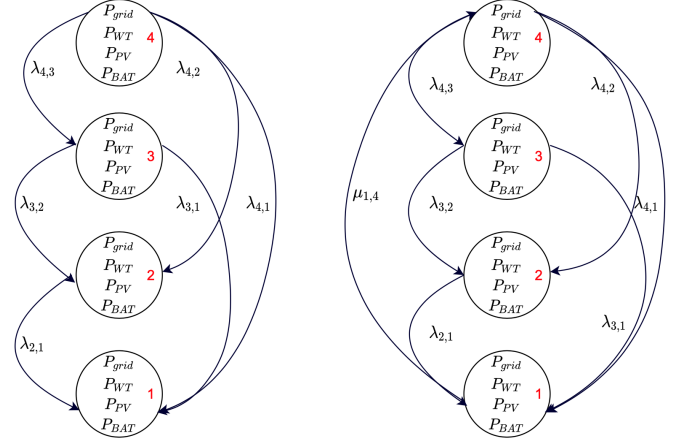


Figure 6: Transient behavior. Non-repairable system (left) with small imperfection (right)

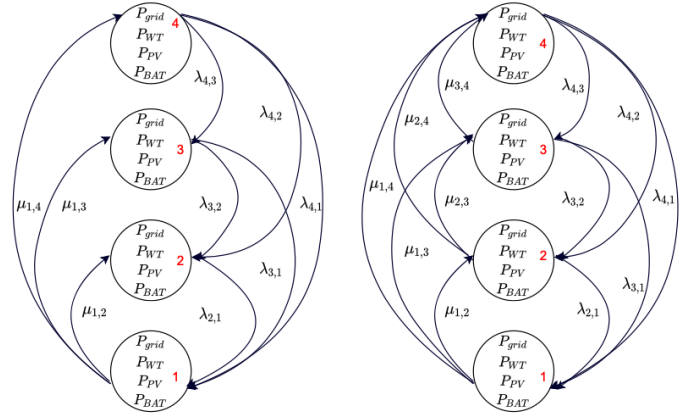
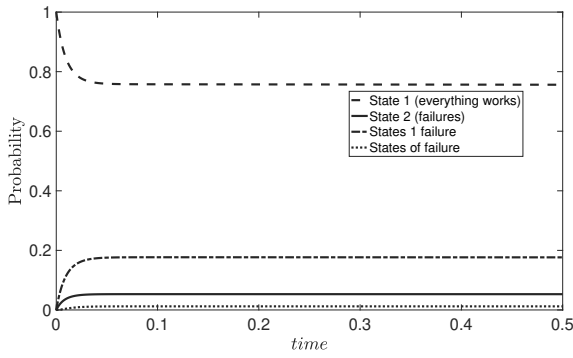
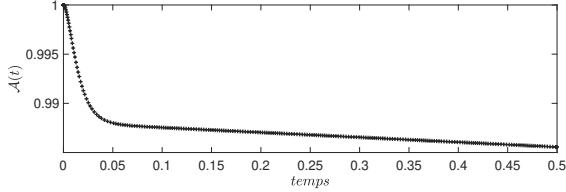
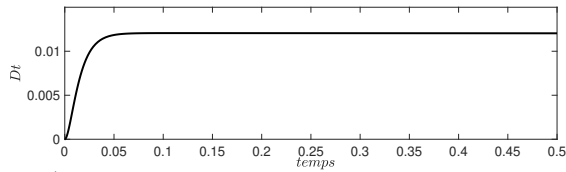
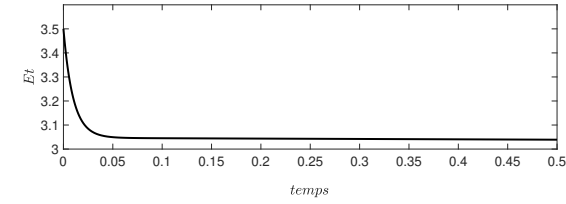


Figure 7: Transient behavior of the system with repair

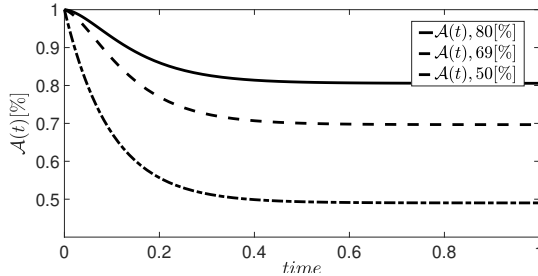
Key metrics quantify individual component contributions to overall system performance. The probabilities of the system state groups are indicated 9 (a). The performance deficit Dt is the probability that the system is in a state of failure, and Et is the expected output and availability $\mathcal{A}(t)$. Here, the system's performance gradually declines due to failure 9(b) and average availability in 9 (c).



(a) probabilities of the groups of states of the system



(b) Expected power deficiency



(c) Availability of the electrical system

Figure 9: Reliability measures

4 Discussion and Conclusion

For system reliability assessment, it is essential to develop appropriate models for wind power generation, pho-

tovoltaic power generation, and load demand. This paper presents a model based on a multi-state system framework that integrates the uncertainties associated with wind speed, solar irradiation, and the failures of wind turbines and photovoltaic panels.

To ensure the reliable operation of wind and solar energy systems, it is crucial to determine when and how much power (in kW) is generated by the microgrid. Accordingly, a reliability assessment technique has been developed by considering individual state diagrams. The proposed method makes it possible to determine whether the microgrid can supply the required energy to meet the load demand and to evaluate the impact of mechanical failures in wind turbines and solar panels. Availability of power from a DG system or a traditional main network power system depends on the reliability of that system. Due to these factors, the generation capacity may vary over time. A Markov process is employed to characterize the dynamic state transitions and to model the random behavior of the power system, which evolves in a discrete manner.

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