HIGH-ORDER SIMULATION OF THE SPLEEN CASCADE WAKE GENERATOR: IMPACT OF THE STABILISATION METHOD

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INTRODUCTION

Over the last decade the discontinuous Galerkin method (DGM, or DG) has emerged as one of the best candidates among numerical methods for scale-resolving simulations, such as Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES), combining the accuracy of high-order finite element methods (FEM), with the stability for convective problems provided by the finite volume method (FVM). In addition to high accuracy on unstructured meshes, enabling to tackle complex geometries, DGM provides excellent efficiency and scalability due to its inherent locality of the data.

However, the non-linear nature of the Navier-Stokes equations (NSE, or NS) can cause complex physical phenomena that are difficult to capture numerically. A first issue is turbulent flow features, which occur on a large range of scales, most of which are very small with respect to the geometry; tracking these features requires a very high accuracy. At transonic and supersonic flows shocks develop. The near-discontinuity of the flow field over a shock can not be resolved by any scheme, and pose significant challenges for high order solvers, *e.g.* by creating false negatives, which may lead to the blow up of the simulation. In this case, it may be required to stabilise the discretisation, thus trading accuracy for stability. Practical aeronautic applications are turbulent and trans- or supersonic, which results in direct interactions between turbulence and shocks. In such cases the choice of the stabilisation method is of paramount importance, in order to obtain the best balance between stability and accuracy. A first step is then to ensure the stabilisation strategy is only activated when strictly necessary.

One of the most common stabilisation strategies consists in adding *artificial viscosity* (AV) to the resolved equations such that the nonlinear terms are counterbalanced in regions containing under-resolved features. It was adapted to DG first by Persson and Peraire [20]. They introduce an additional uniform dissipation term on all equations, activated and tailored by a sensor computed on the basis of the regularity of the solution in the cell. Later, more advanced AV strategies have been proposed, *e.g.* Fernandez et al. [7] presents an approach enhancing physical dissipation mechanisms, activated on the basis of physical considerations.

The main issue about these strategies is that the resolved physics itself is modified, as it basically aims at thickening the shock features so that it can be resolved by the mesh. Fortunately, this has no impact on shock location or speed, but it will also dissipate turbulent flow features. These methods have shown to be capable of stabilising high speed simulations up to the hypersonic regime, see for example Ching et al. [6].

More recently, new strategies that do not require the addition of dissipation have been proposed, which stabilise the simulation by enforcing a discrete analogue of the second law of thermodynamics, taking their inspiration from finite volume (FVM) and finite difference methods (FDM). They are referred to as *entropy stable* DGM (ESDG).

The second law of thermodynamics is enforced discretely in the sense that the discretisation scheme cannot lead to an increase in mathematical entropy, defined as $U = -\rho s$, where ρ denotes the density and s the specific entropy. The scheme for discretizing convective terms inherently conserves entropy within cells, and is combined with a compatible flux dissipating the mathematical entropy at cell interfaces, to render the method *entropy stable*. They were introduced in DG by Gregor J. Gassner et al. [8], who noticed similarities between differentiation operators of DG spectral element methods (DGSEM), *i.e.* DG methods using the collocated interpolation and quadrature points along with a corresponding quadrature rule, and derivation operators appearing in FDM that intrinsically conserve entropy. The two main ingredients for such a strategy in DG are: 1. a so called *summation-by-part* (SBP) operator, imitating the integration by part operation at the discrete level, and 2. a *two-points entropy compatible volume flux* equipped with conservation and consistency properties. These different variants of ESDG have then gained particular interest in the literature, *e.g.* see [5, 21].

Although these precautions are taken, false negative values of key quantities, such as density or pressure, may still lead to the violation of the second law of thermodynamics. To solve this problem, a positivity preserving strategy was introduced in [9, 12, 22], consisting of a convex blending of the solution computed with high order ES with its low order ES counterpart. *Strong stability preservation* Runge Kutta of 4th order with 5 stages (SSPRK54) time-stepping scheme, proposed by Spiteri and Ruuth [25], is then used to maintain positivity.

Harsh conditions that may lead to the above-mentioned issues, and therefore require stabilisation, can be encountered in high-speed (HS) low pressure turbines (LPT) of novel geared turbofan (GTF) architectures, due to the low Reynolds and high Mach number.

Computations are undertaken jointly by the university of Liège and Cenaero to reproduce tests performed by the von Karman Institute for Fluid Dynamics (VKI) in collaboration with Safran Aircraft Engines during the SPLEEN project, aimed at studying the main and cavity flows in high-speed LPT, typical for novel Geared Turbofans (GTF). In order to distinguish the impact of incoming turbulence from the turbulence generated by separation or wakes of the blade itself, three configurations were considered: (1) clean inlet, (2) turbulence injection, (3) wake reproduced by a set of rotating bars generator. Three different Mach numbers are considered to check for compressibility and in particular the impact of choking. Furthermore, 2 Reynolds numbers are considered each time.

Several publications and databases detail the experimental campaign in SPLEEN [11, 13, 14, 15, 16, 17, 18, 19, 23, 24] whereas first DNS have been presented in [2, 3, 4] for clean test cases.

During the studies involving the wake generators [26], stability issues emerged due to the formation of a very strong shock on the wake generator, especially during the transient, but also once the flow is established. Therefore, rather invasive stabilisation techniques needed to be used to perform the calculations.

In order to reduce the impact of the stabilisation on further simulations, this contribution aims to compare stabilisation strategies, including ESDGSEM, positivity preserving, and AV, with regard to the topology of the flow around the wake generators. The goal is to identify trade-offs between accuracy, robustness, physical fidelity, and computational efficiency, providing guidance for the selection of stabilisation strategies in industrial compressible turbulent flow simulations.

The methods presented in this paper are implemented in *ArgoDG* [10], a discontinuous Galerkin (DG) solver of Cenaero. *ArgoDG* is able to capture turbulence and strong shocks using, among others, AV methods, *hp*-adaptation, ESDGSEM, and positivity preservation. The scope of this paper is limited to quad meshes. The SBP operators, the implementation of two-points volume fluxes, and the assembly of the volume convective fluxes with the *flux differencing* strategy are built following the process detailed in [1, 5, 21].

RESULTS

The results provided in this section were obtained with a fourth order simulation of the SPLEEN cascade wake generator, with and without AV [27].

Figure 1 shows snapshots of the Schlieren in the domain. It can be seen that, without AV, the amplitude of the discontinuity is more rigid, which increases the oscillations around the shock.

Figure 2 shows snapshots of the entropy in the domain with and without AV. Without AV, the entropy generated by the shock is not dissipated and is carried away by the flow.

FUTURE WORK

With regard to methods for preserving positivity, the use of a second-order low order ES counterpart can improve accuracy while maintaining stability near discontinuities. Extending positivity preserving to triangular meshes also allow for broader comparison and simplifies mesh creation.

Following this analysis, which aims to identify the most effective and least intrusive methods, a study can be conducted on the interaction between wake and HSLPT using several conditions.

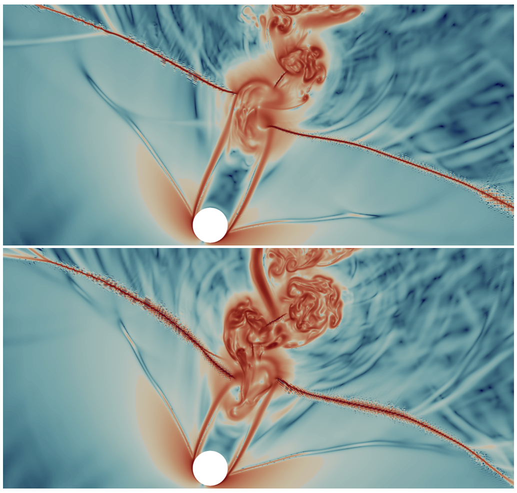

Figure 1: Snapshots of the schlieren in the wake generator domain, before AV removal at $t = t_0$ (top) and after AV removal at $t = t_0 + 3.6 \times 10^{-5}$ s (bottom); with a timestep of $\mathrm{d}t = 10^{-8}$ s and a convective time unit of $t_c = 10^{-3}$ s.

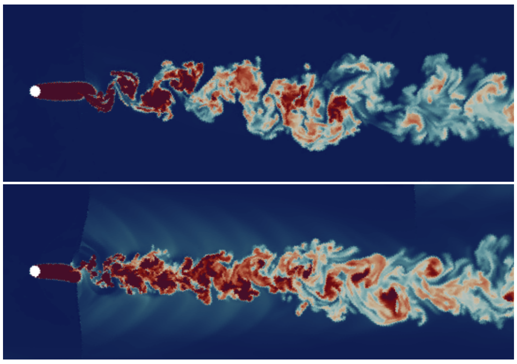

Figure 2: Snapshots of the entropy field in the wake of the cylinder with (top) and without AV (bottom).

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