

ARTICLE

Experimental and analytical investigations on the effect of steel fibers on the crack behavior and strength of dapped-end connections

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Abstract

Reinforced concrete (RC) dapped-end connections feature a sharp re-entrant corner subjected to high stress concentrations, leading to wide cracks under service loads and potential durability issues such as reinforcement corrosion. Effective crack control is therefore essential in the design of new dapped-end connections. Steel fiber reinforced concrete (SFRC) has shown potential to improve crack control while also enhancing ultimate strength. This study proposes a kinematics-based model to describe the response of SFRC dapped ends along the re-entrant corner crack. Appropriate constitutive relationships are used to link the deformations along the corner crack and the compression zone to the resisting forces: reinforcement and SFRC in compression and tension. The model is validated against eight large-scale tests with varying fiber amounts including detailed deformation measurements from which the resisting mechanisms and crack-opening behavior are derived. The experimental results show that a fiber amount of 70 kg/m³ reduces the crack width by about 40%, enabling compliance with code requirements. The validation demonstrates that the proposed model represents a reliable tool for designing SFRC dapped-end connections at both service and ultimate limit states.

KEYWORDS

crack behavior, dapped-end connections, kinematics-based model, service behavior, steel fiber reinforced concrete

1 | INTRODUCTION

Dapped-end connections are often encountered in reinforced or prestressed concrete supports, manufactured as precast elements. In the past, they were often used in

precast concrete bridges as they accommodate simple supports for bridge girders in a compact manner. However, these connections feature a sharp re-entrant corner with high stress concentrations which cause large cracks under service loads—see Figure 1a. The cracks widen further due to the corroding reinforcement caused by the ingress of water and de-icing salts, and fatigue deterioration because of repeated traffic loads. These deterioration

Karin Yu and Sameera Hippola contributed equally to this work and should be regarded as co-first authors.

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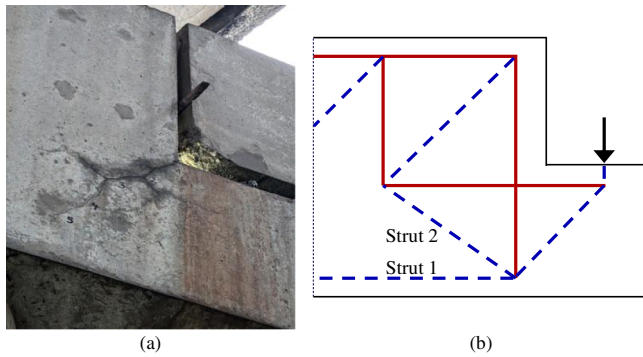


FIGURE 1 Reinforced concrete dapped-end connections: (a) re-entrant corner crack, and (b) typical strut-and-tie model for ultimate load design.

mechanisms negatively impact both the serviceability and ultimate behavior of dapped ends which could lead to catastrophic bridge collapses.^{1,2} Wide corner cracks are also of concern in precast structures of buildings, where dapped ends are currently often used. Therefore, ensuring adequate crack control at the re-entrant corner under service conditions is critical for the durability of newly designed dapped-end connections.

Several experimental and analytical studies have been carried out on reinforced concrete (RC) dapped-end connections concerning both service and ultimate behaviors. The experimental studies investigated the effects of the reinforcement layout,^{3–5} the reinforcement amount,^{6,7} the concrete strength,⁸ and the shear-span-to-depth ratio⁹ on the corner crack width, strength, and failure modes. Analytical studies include strut-and-tie models,^{10,11} kinematics-based models,^{12,13} and numerical analyses using finite element methods^{6,14,15} to predict the ultimate strength of dapped-end connections and corner crack widths. A typical strut-and-tie model used for the ultimate limit state design of connections with orthogonal reinforcement layout is shown in Figure 1b. These studies on RC dapped-end connections show that the re-entrant corner crack forms at very low load levels and remains difficult to control under service conditions, even with large reinforcement amounts.¹⁶ The addition of diagonal reinforcement can improve crack control. However, experimental studies^{7,11} including diagonal reinforcement showed that the reduction in the corner crack width was not sufficient to stay below code limits under service loads.

To improve crack control, researchers have explored the use of steel fiber-reinforced concrete (SFRC) in dapped-end connections.^{4,17,18} These experimental investigations demonstrate that steel fibers enhance strength, limit crack widths through fiber bridging, and improve the ductility of concrete in compression. Several

modeling approaches exist for predicting the ultimate strength of SFRC dapped ends—such as empirical formulations,¹⁹ strut-and-tie models,^{17,20} kinematics-based models,²¹ and finite element models.²² While these methods are useful for design at the ultimate limit state, there is still a lack of reliable models capable of predicting crack widths in SFRC dapped-end connections for serviceability assessments, which are essential for ensuring adequate durability.

To address this need, the present paper proposes a kinematics-based model to predict the opening of the corner crack as a function of the shear force in the SFRC dapped-end connections. The model is based on an existing kinematics-based formulation developed for RC dapped-end connections.¹³ Validation is performed using a comprehensive large-scale experimental program⁴ of eight tests featuring detailed digital image correlation (DIC) and strain measurements. The model is validated in terms of corner crack opening, resistance mechanisms throughout loading, and the ultimate strength of SFRC dapped-end connections.

2 | KINEMATICS-BASED MODELING OF SFRC DAPPED-END CONNECTIONS

This chapter introduces a kinematics-based model for the behavior of SFRC dapped-end connections along re-entrant corner cracks. The model is based on an existing kinematics-based approach developed by Rajapakse¹³ for the behavior of RC dapped-end connections. An overview of the existing model with slight modifications is first presented, followed by its extension to the behavior of SFRC dapped-end connections.

2.1 | Overview of the existing model for RC dapped ends

The modeling assumptions and expressions of the existing model, including compatibility of deformations, constitutive relationships, and equilibrium, are summarized in Figure 2 and Table 1. The kinematics of the model is illustrated in Figure 2a, where the re-entrant corner crack is explicitly modeled, separating the connection into two rigid blocks. The crack angle θ is taken as 45° , as the predictions are not highly sensitive to this assumption.^{12,13} The opening of the crack is associated with the relative rotation φ of the two rigid blocks about the tip of the crack. This relative rotation causes the formation of a compression zone above the tip of the crack, with a depth of x , and the engagement of reinforcement across the

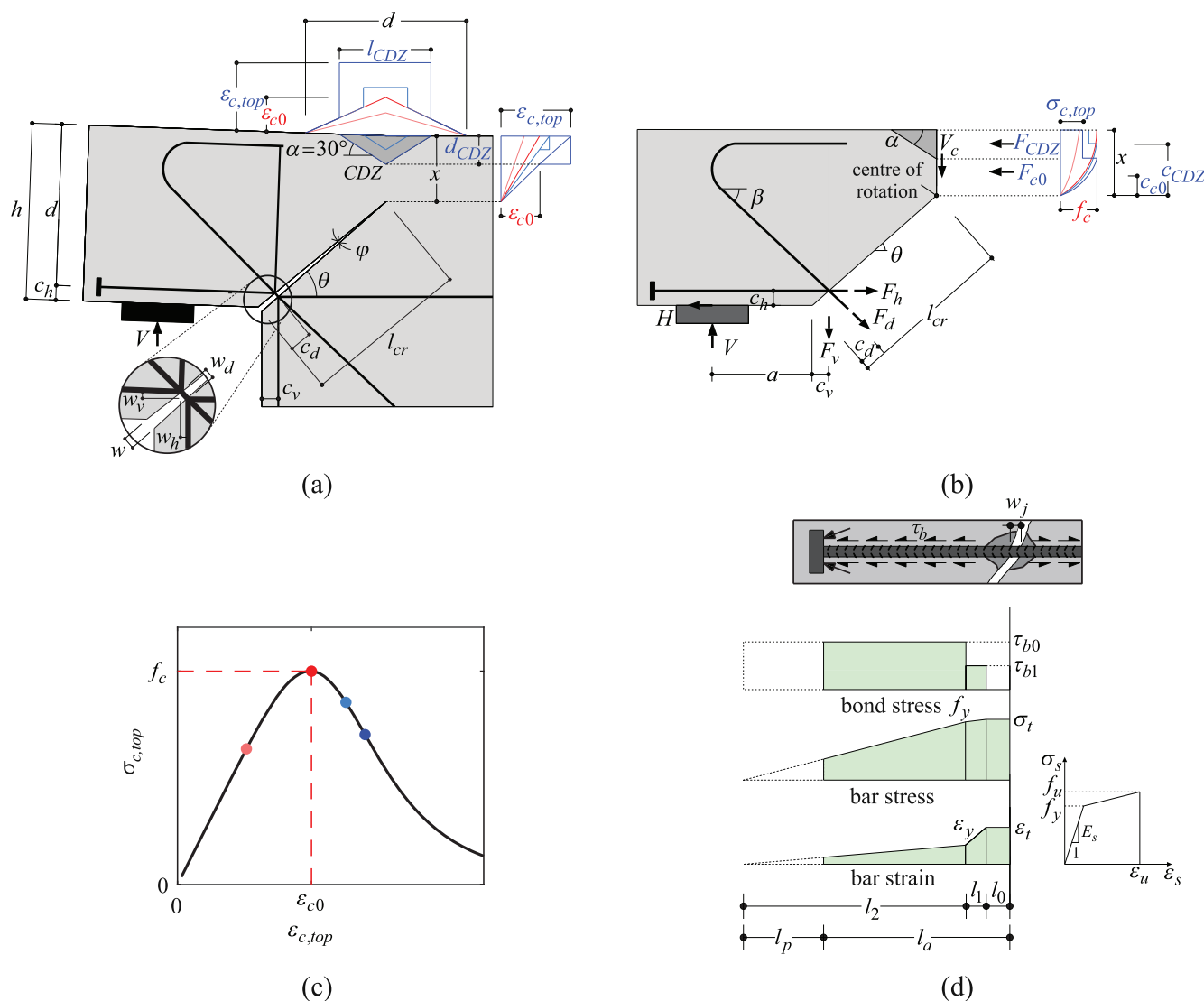


FIGURE 2 Kinematics-based model for the behavior of reinforced concrete dapped-end connections: (a) kinematics, (b) external and internal forces, (c) Stress-strain relationship for concrete,^{23,24} and (d) reinforcement strain and stress distributions in the vicinity of the corner crack.

crack. Constitutive relationships relate these deformations to the resisting forces shown in Figure 2b. The behavior of concrete in compression is shown in Figure 2c and the pull-out behavior of the reinforcement in Figure 2d. The complete response of the connection is obtained by increasing the strain at the top fiber $\epsilon_{c,top}$ in several load steps. The procedure for a single load step is discussed in the following.

For a given strain at the top fiber $\varepsilon_{c,top}$ and a compression zone depth x , the shortening of the top concrete fiber $\delta_{c,top}$ can be calculated from Equations (2) to (4). These expressions are derived assuming the formation of a compression damage zone (CDZ) with localized plastic strains once the top fiber reaches the compressive strength of the concrete f_c . Figure 2a illustrates the possible compressive strain profiles on the top face and across

the compression zone with increasing $\varepsilon_{c,top}$ and load. Once $\delta_{c,top}$ is known, the rotation φ is calculated using Equation (1). With known φ , the crack displacements in the direction of the reinforcement— w_h , w_v and w_d —are computed using Equations (5–7).

Having expressed all the important deformations for given $\varepsilon_{c,top}$ and x , the stresses in the compression zone and tensile stresses in the reinforcement are determined using the constitutive relationships shown in Table 1. The compression zone is discretized into several layers and a model for concrete under compression^{23,24} is applied to each layer to obtain the stresses in the compression zone σ_c . Subsequently, these stresses are integrated in order to calculate the forces F_{CDZ} and F_{c0} as well as the respective lever arms c_{CDZ} and c_{c0} from the crack tip as given in Equations (10–12).

TABLE 1 Expressions of the kinematics-based model for reinforced concrete dapped ends: compatibility of deformations, constitutive relationships, and equilibrium, according to Rajapakse.¹³

Compatibility of deformations	Constitutive relationships	Equilibrium
- Rotation and top fiber strain $\varphi = \frac{\delta_{c,top}}{x} \quad (1)$ if $\varepsilon_{c,top} < \varepsilon_{c0}$ $\delta_{c,top} = 0.5 \varepsilon_{c,top} d$ if $\varepsilon_{c,top} \geq \varepsilon_{c0}$ $\delta_{c,top} = \varepsilon_{c,top} l_{CDZ}$ (2) $+ 0.5 \varepsilon_{c0} \cdot \max\left(0, \frac{l_c - l_{CDZ}}{l_c}\right)$ $\cdot \max(0, (l_c - l_{CDZ}))$ $l_{CDZ} = 2d_{CDZ} \cot(\alpha) \quad (3)$ $d_{CDZ} = \max\left(0, \left(1 - \frac{1 - \varepsilon_{c0}}{\varepsilon_{c,top}}\right)x\right) \quad (4)$ - Crack displacements in the direction of reinforcement $w_h = \varphi(l_{cr} \sin(\theta) - c_h) \quad (5)$ $w_v = \varphi(l_{cr} \cos(\theta) - c_v) \quad (6)$ $w_d = \varphi(l_{cr} - c_d) \sin(\theta + \beta) \quad (7)$ $l_{cr} = (h - x) / \sin(\theta) \quad (8)$ $w = l_{cr} \varphi \quad (9)$	- Concrete in compression $F_{CDZ} = bd_{CDZ} \sigma_{c,top} \quad (10)$ $F_{c0} = b \int_0^{x-d_{CDZ}} \sigma_c dx \quad (11)$ $c_{c0} = \frac{\int_0^{x-d_{CDZ}} \sigma_c x_0 dx}{\int_0^{x-d_{CDZ}} \sigma_c dx} \quad (12)$ - Reinforcement strain and crack width $w_j = 2 \left(l_0 \varepsilon_t + \frac{1}{2} \left(\varepsilon_t + \frac{f_y}{E_s} \right) l_1 + \frac{1}{2} \min \left(\varepsilon_t, \frac{f_y}{E_s} \right) l_2 - \frac{1}{2} \left(\frac{\min \left(\varepsilon_t, \frac{f_y}{E_s} \right) l_p}{l_2} \right) l_p \right) \quad (13)$ $l_0 = 3\phi \quad (14)$ $l_1 = \frac{\max(\sigma_t - f_y, 0) \phi}{4\tau_{b1}} \quad (15)$ $l_2 = \frac{\min(\sigma_t, f_y) \phi}{4\tau_{b0}} \quad (16)$ $l_p = \max(l_0 + l_1 + l_2 - l_a, 0) \quad (17)$ $\tau_{b0} = \min \left(2, \frac{c + \frac{\phi}{2}}{1.664 \phi} \right) f_{ct} \quad (18)$ $\tau_{b1} = \frac{\tau_{b0}}{2} \quad (19)$	- Horizontal equilibrium $F_{CDZ} + F_{c0} = \sum_i F_{h,i} + \sum_i F_{d,i} \cos(\beta) - H \quad (20)$ - Moment equilibrium around crack tip $V_u = \frac{1}{a + l_{cr} \cos(\theta)} \left(\sum_i F_{h,i} (h - x - c_{h,i}) + \sum_i F_{v,i} (l_{cr} \cos(\theta) - c_{v,i}) + \sum_i F_{d,i} \cos(\beta) \sin(\theta) (l_{cr} - c_{d,i}) + \sum_i F_{d,i} \sin(\beta) \cos(\theta) (l_{cr} - c_{d,i}) + F_{c0} c_{c0} + F_{CDZ} (x - 0.5 d_{CDZ}) - H(h - x) \right) \quad (21)$

The tensile stresses in the reinforcement at the crack σ_t are computed from the crack displacements w_j by using the Tension Chord Model (TCM).²⁵ This model assumes a rigid-plastic bond law with bond stresses τ_{b0} and τ_{b1} for elastic and plastic regions of the reinforcement, respectively. Each reinforcement is analyzed as being anchored by bond in the rigid concrete blocks on each side of the crack. Depending on the orientation of the reinforcement, w_j in Equation (13) corresponds to either w_h , w_v , or w_d from Equations (5–7). Equation (13)

accounts for pull-out of the reinforcement in four regions of the reinforcement. The first term considers local debonding (i.e., zero bond) near the crack within a length of $l_0 = 3\phi$ caused by a cone-shaped local splitting. The second and third terms account for the pull-out from the plastic and elastic regions of the reinforcing bar. The last term considers the reduction of pull-out due to the presence of a stiff anchorage at the end of the bar (an anchor head or a loop). The bond stress τ_{b0} is calculated using Equation (18) where c is the minimum

concrete cover for the considered reinforcement layer, ϕ is the bar diameter, and $f_{ct} \approx 0.33\sqrt{f_c}$ (MPa) according to Bentz.²⁶ The first term of Equation (18) accounts for the pull-out behavior of bars in uncracked concrete under uniaxial conditions, while the second term accounts for the splitting behavior²⁷ in the presence of bars with large diameter and/or small concrete cover.

The described modeling of the reinforcement represents the only modification of the original model proposed in Rajapakse.¹³ The original approach included an empirical reduction factor for the bond as a function of the amount of reinforcement. In this study, the empirical factor is replaced by a more refined modeling of the bond shear stresses. More precisely, the debonded length l_0 and the bond expression in Equation (18)²⁷ accounting for local splitting are introduced.

Once the stresses and forces in the compression zone and reinforcement are known, the horizontal equilibrium of the free body shown in Figure 2b is calculated according to Equation (20). This allows for iterative calculations of the depth of the compression zone x such that the horizontal equilibrium is satisfied. Finally, the corresponding shear force V is computed using the moment equilibrium around the crack tip as per Equation (21). This resembles a conventional flexural strength calculation. As mentioned before, the complete response is obtained by repeating the above calculation procedure for different top fiber strains $\epsilon_{c,top}$ varying from 0.01% to 0.35%. By following this procedure, the variation of the corner crack width w with the shear force V and the strength of the connection can be obtained. For more details of this model, the reader is referred to Rajapakse.¹³

2.2 | Model for behavior of SFRC dapped-end connections along re-entrant corner crack

As mentioned before, the proposed model for SFRC dapped-end connections is based on the existing model for RC connections presented before. The model is extended to account for the behavior of SFRC in compression and tension in this section. The extensions include the adaptation of suitable constitutive models for the enhanced ductility of the compression zone and the forces carried by the fibers across the re-entrant corner crack.

2.2.1 | Behavior of SFRC under compression

To model the behavior of SFRC in compression, the constitutive relationship proposed by Lee et al.²⁸ is adopted.

This model can be used to calculate the stress σ_c for a given strain ϵ_c in different fibers of the compression zone, and will replace the model for plain concrete shown in (Figure 2c):

$$\sigma_c = f_c \cdot \frac{A\left(\frac{\epsilon_c}{\epsilon_{c0}}\right)}{A - 1 + \left(\frac{\epsilon_c}{\epsilon_{c0}}\right)^B} \quad (22)$$

$$A = \begin{cases} \frac{1}{1 - \frac{f_c}{\epsilon_{c0}E_c}}, & \text{if } \epsilon_c \leq \epsilon_{c0} \\ 1 + 0.723RI^{-0.957}, & \text{otherwise} \end{cases} \quad (23)$$

$$B = \begin{cases} A, & \text{if } \epsilon_c \leq \epsilon_{c0} \\ \max\left[\left(\frac{f_c}{50}\right)^{0.064} (1 + 0.882RI^{-0.882}), A\right], & \text{otherwise} \end{cases} \quad (24)$$

where the index $RI = \rho_f l_f / \phi_f$ is defined based on the fiber volumetric ratio ρ_f , length l_f , and diameter ϕ_f of the steel fibers. The strain at peak stress ϵ_{c0} and the elastic modulus E_c are assumed as $(0.0003RI + 0.0018)f_c^{0.12}$ and $(-367RI + 5520)f_c^{0.41}$, which were also empirically derived by Lee et al.²⁸

A comparison of the stress-strain relationships for different fiber volumetric ratios is shown in Figure 3a. The fibers have a strong impact on the post-peak regime of the concrete. The greater the amount of fibers, the more ductile (or less brittle) the concrete is. It will be shown later that this property of SFRC can have a significant positive effect on the ultimate load-bearing capacity of dapped-end connections.

2.2.2 | Modeling the tensile behavior of steel fibers across cracks

To accurately capture the pull-out behavior of the fibers, a constitutive relationship for the tensile behavior of steel fibers across the re-entrant corner crack needs to be considered. The tensile stress of the SFRC σ_f includes the stress due to the frictional bond behavior of steel fibers σ_{st} and the stress due to the mechanical anchorage of end-hooked fibers σ_{eh} . These stresses are calculated using the Simplified Diverse Embedment Model (SDEM) developed by Lee et al.²⁹ as a function of the crack width w_k . The mathematical expressions of the SDEM are as follows:

$$\sigma_f = \sigma_{st} + \sigma_{eh} \quad (25)$$

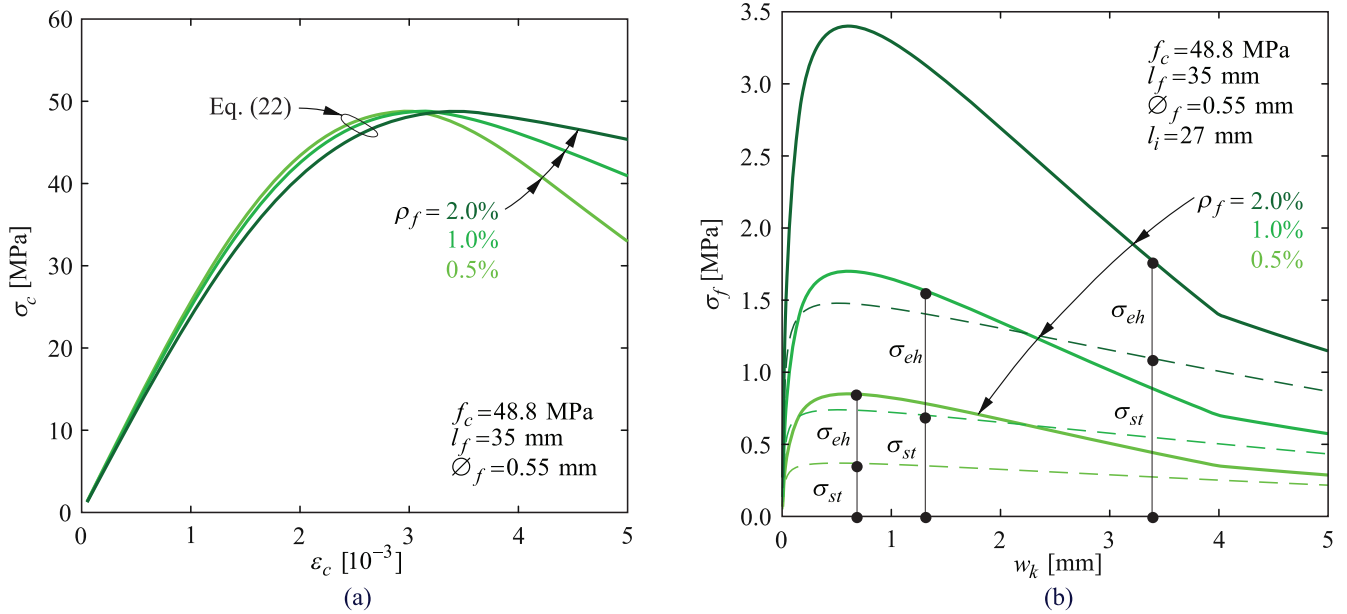


FIGURE 3 Behavior of Steel fiber reinforced concrete (SFRC) under compression²⁸ and tension²⁹ varying the fiber volumetric ratio ρ_f .

$$\sigma_{st} = \alpha_f V_f K_{st} \tau_{f,\max} \frac{l_f}{\phi_f} \left(1 - \frac{2w_k}{l_f}\right)^2 \quad (26)$$

$$K_{st} = \begin{cases} \frac{\beta_f w_k}{3s_f}, & \text{if } w_k < s_f \\ 1 - \left(\frac{s_f}{w_k}\right)^{0.5} + \frac{\beta_f}{3} \left(\frac{s_f}{w_k}\right)^{0.5}, & \text{if } w_k \geq s_f \end{cases} \quad (27)$$

where $\alpha_f = 0.5$ is the fiber orientation factor, $\tau_{f,\max} = 0.396\sqrt{f_c}$ (MPa) the frictional bond strength of the fiber, $\beta_f = 0.67$, and $s_f = 0.01$ mm, and

$$\sigma_{eh} = \alpha_f V_f K_{eh} \tau_{eh,\max} \frac{2(l_i - 2w_k)}{\phi_f} \quad (28)$$

$$K_{eh} = \begin{cases} \beta_{eh} \left(\frac{2w_k}{3s_{eh}} - \frac{1}{5} \left(\frac{w_k}{s_{eh}} \right)^2 \right), & \text{if } w_k < s_{eh} \\ 1 + \left(\frac{7\beta_{eh}}{15} - 1 \right) \left(\frac{s_{eh}}{w_k} \right)^{0.5} - \frac{2(w_k^{0.5} - s_{eh}^{0.5})^2}{l_f - l_i}, & \text{if } w_k < \frac{l_f - l_i}{2} \\ \left(\frac{l_i - 2w_k}{2l_i - l_f} \right)^2 K_{eh,i}, & \text{if } w_k < \frac{l_i}{2} \end{cases} \quad (29)$$

where $\tau_{eh,\max} = 0.429\sqrt{f_c}$ (MPa) is the maximum bond strength due to the mechanical anchorage of a end-hooked fiber, l_i is the distance between the mechanical anchorages, $\beta_{eh} = 0.76$, $s_{eh} = 0.1$ mm, and $K_{eh,i}$ is K_{eh} at $w_k = (l_f - l_i)/2$. These stress-crack width relationships

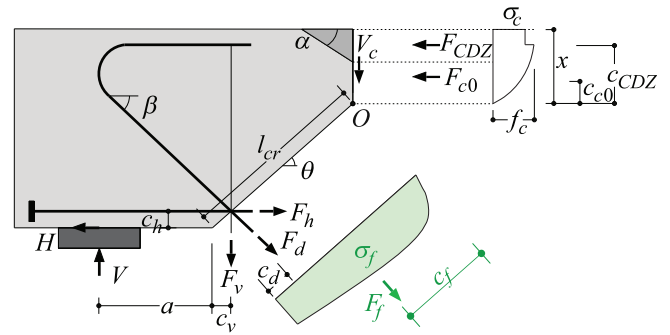


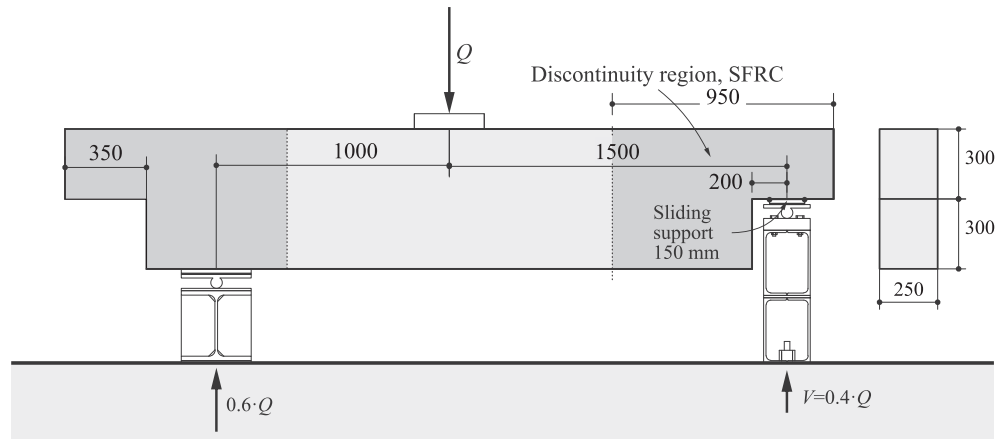
FIGURE 4 Free-body diagram of the rigid block including the effects of fibers.

are plotted in Figure 3b for different fiber volumetric ratios.

Figure 4 shows the free body from Figure 2b, which now includes the tensile stresses across the re-entrant corner crack due to fibers. The stress profile σ_f is obtained by discretizing the crack into several segments and analyzing each of them using Equations (25–29) based on the respective crack widths $w_k = l_{cr,k}\phi$, where $l_{cr,k}$ is the distance measured along the crack from point O. The σ_f profile is integrated to calculate the force in the fibers F_f and its lever arm c_f from the crack tip. The equilibrium equations given in Table 1 are modified as follows to include the effect of force F_f (in green):

$$F_{CDZ} + F_{c0} = \sum_i F_{h,i} + \sum_i F_{d,i} \cos(\beta) - H + F_f \sin(\theta) \quad (30)$$

FIGURE 5 The experimental set-up of the three-point bending test (dimensions in mm, adapted from Mata-Falcón et al.¹⁰).



$$\begin{aligned}
 V_u = & \frac{1}{a + l_{cr} \cos(\theta)} \left(\sum_i F_{h,i} (h - x - c_{h,i}) \right. \\
 & + \sum_i F_{v,i} (l_{cr} \cos(\theta) - c_{v,i}) + \sum_i F_{d,i} \cos(\beta) \sin(\theta) (l_{cr} - c_{d,i}) \\
 & + \sum_i F_{d,i} \sin(\beta) \cos(\theta) (l_{cr} - c_{d,i}) + F_{c0} c_{c0} \\
 & \left. + F_{CDZ} (x - 0.5 d_{CDZ}) - H(h - x) + F_f c_f \right) \quad (31)
 \end{aligned}$$

3 | EXPERIMENTAL VALIDATION

3.1 | Test campaign

The performance of the kinematics-based model is examined for a series of eight dapped-end tests, part of a larger experimental campaign.⁴ Six of the dapped-end tests were conducted on steel-fiber reinforced beam specimens, whereas the remaining two did not contain any steel fibers and served as reference experiments using conventional concrete. All specimens had the same configuration of reinforcing bars and varied only in their fiber content. The experimental campaign is described in the following sections (more details can be found in Mata-Falcón⁴).

3.1.1 | Specimens and materials

The span of the beams was 2500 mm with a rectangular 250 × 600 mm cross-section which reduced to 250 × 300 mm at the dapped end (see Figure 5). Each side of the beam specimen was tested independently, resulting in two tests for each configuration. For the beams including steel fibers, only the discontinuity region around the support area was cast with SFRC (up to 600 mm from the

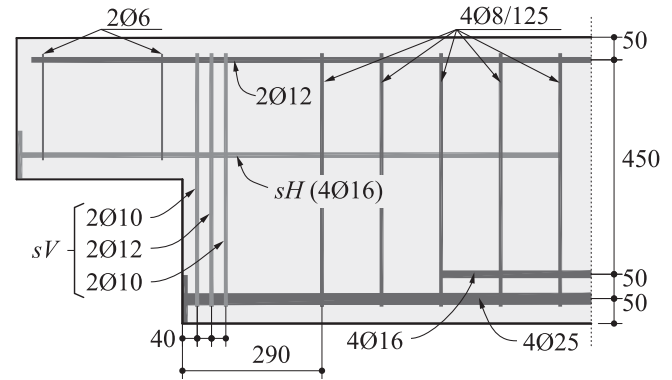


FIGURE 6 Notations of the reinforcement and geometry (dimensions in mm, adapted from Mata-Falcón et al.¹⁰).

TABLE 2 Concrete strength f_c and fiber dosages of test specimens (volume V_f and fiber volumetric ratio ρ_f).

Specimen	DEB-1.6	DEB-3.7	DEB-3.8	DEB-3.9
f_c [MPa]	31.1	45.5	48.8	48.4
V_f [kg/m ³]	0	40	70	100
ρ_f [%]	0.00	0.51	0.88	1.26

re-entrant corner, as shown in Figure 5). In the remaining area, not relevant for the local behavior at the dapped end, normal-strength concrete without fibers was used. Figure 6 shows the dapped-end geometry and the reinforcement configuration used for all specimens. The main dapped-end reinforcement—stirrups at the re-entrant corner and horizontal reinforcing bars—are labeled as sV and sH , respectively. The vertical mechanical concrete cover was 50 mm, whereas the horizontal one was 40 mm.

Table 2 contains the concrete strength and steel fiber content of each beam specimen. DEB-1.6 served as a reference experiment of a beam specimen without steel fibers. The amount of steel fibers increased from 40 kg/m³

Specimen		Reinforcement bar diameter [mm]				
		ø8	ø10	ø12	ø16	ø25
DEB-1.6	f_y [MPa]	532.3	544.2	546.1	549.6	569.9
	f_u [MPa]	672.1	654.3	658.5	672.8	695.9
DEB-3.7, 3.8, 3.9	f_y [MPa]	509.3	542.4	551.7	529.7	540.9
	f_u [MPa]	659.9	645.4	651.8	637.6	649.8

TABLE 3 Mechanical properties of the reinforcement of the test specimens.

(DEB-3.7, fiber volumetric ratio $\rho_f = 0.51\%$) to 70 kg/m^3 (DEB-3.8, $\rho_f = 0.88\%$) and 100 kg/m^3 (DEB-3.9, $\rho_f = 1.26\%$).

The normal-strength concrete used in specimen DEB-1.6 had a characteristic strength of 30 MPa and a maximum aggregate size of 20 mm, and was delivered from a local ready-mix concrete plant, whereas for the SFRC a self-consolidating concrete with a maximum aggregate size of 12 mm was produced in the lab. The SFRC mix is as follows: 340.0 kg/m^3 CEM I 42.5R, 210.0 kg/m^3 water, 1810.2 kg/m^3 aggregates, 5.4 kg/m^3 superplasticizer, and the respective fiber dosages of each specimen. The steel fibers were DRAMIX RC-65/35-BN with a slenderness of $l_f/\phi_f = 35/0.55 = 64$, a tensile strength of 1100 MPa, and single-hooked ends. The concrete compressive strength was determined by testing two $150 \times 300 \text{ mm}$ cylinders for each specimen at the same time as the corresponding tests were carried out, see Table 2. The scatter due to the difference in concrete age was lower than the differences induced by the different fiber amounts. Conventional deformed reinforcing bars were used for the reinforcement and their mechanical properties were determined by two tensile tests per batch. The averaged mechanical properties for each bar diameter can be found in (Table 3).

3.1.2 | Test setup, instrumentation, and post-processing

Three-point non-symmetric bending tests as illustrated in Figure 5 were conducted on simply supported beam specimens. The load was applied by means of a 2500 kN hydraulic actuator at 1500 mm distance from the tested dapped end under displacement-controlled conditions of 0.25 mm/min . From equilibrium, the load V at the tested support was 40% of the applied load. Once the first dapped end failed, the experimental setup was inverted to test the second dapped end.

The supports were designed to allow for both rotations and horizontal displacements which was achieved by using a system of Teflon (PTFE) and polished 5 mm thick stainless-steel plates. On top of these plates, a $300 \times$

$150 \times 20 \text{ mm}$ neoprene was placed to ensure a uniform load distribution.

For the instrumentation, two additional measurement techniques were chosen besides the load cell. Strain gauges glued to the reinforcing bars were used to measure steel bar deformations, whereas photographic images were taken at a 1 Hz frequency to track concrete surface cracks and deformations using DIC post-processing tools. Compared with displacement transducers, DIC offers the advantage of measuring cracks precisely without needing to know in advance their location or pattern. The reinforcing bars were instrumented with several strain gauges at strategic locations. In this study, only the deformations of the main vertical bars in the vicinity of the diagonal crack were analyzed for DEB-3.8 (T2) to estimate the vertical load carried by the reinforcement. Although the strain gauges were strategically placed close to the expected cracking location, they rarely fall together in reality. Hence, the measured experimental strains $\varepsilon_{s,\text{exp}}$ need to be adapted to estimate the reinforcement strains at the crack ε_t based on the distance between the locations of the crack and the strain gauge Δ .¹¹ Assuming a two-step bond distribution τ_b according to the TCM,^{25,30} the maximum strain of the reinforcement at the crack ε_t can be determined as follows:

$$\varepsilon_t = \varepsilon_{s,\text{exp}} + \frac{\Delta}{\phi} \frac{4\tau_{b0}}{E_s} \text{ if } \varepsilon_{s,\text{exp}} \leq \varepsilon_{s,y} \text{ and } \varepsilon_t \leq \varepsilon_{s,y} \quad (32)$$

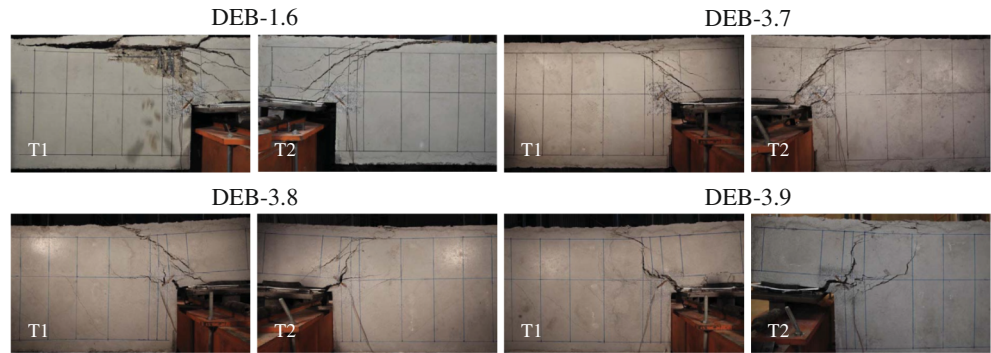
$$\varepsilon_t = \varepsilon_{s,y} + \left(\Delta - \frac{(\varepsilon_{s,y} - \varepsilon_{s,\text{exp}})\phi E_s}{4\tau_{b0}} \right) \frac{4\tau_{b0}}{\phi E_s} \text{ if } \varepsilon_{s,\text{exp}} \leq \varepsilon_{s,y} \text{ and } \varepsilon_t > \varepsilon_{s,y} \quad (33)$$

$$\varepsilon_t = \varepsilon_{s,\text{exp}} + \frac{\Delta}{\phi} \frac{4\tau_{b1}}{E_s} \text{ if } \varepsilon_{s,\text{exp}} > \varepsilon_{s,y} \text{ and } \varepsilon_t > \varepsilon_{s,y} \quad (34)$$

where E_s is the Young's modulus equal to 200 GPa, ϕ the diameter of the steel reinforcement, $\tau_{b0} = 2f_{ct}$, and $\tau_{b1} = f_{ct}$ with the mean tensile strength f_{ct} according to Bentz.²⁶

In a post-processing step, the DIC commercial software VIC-2D from Correlated Solutions Inc. was used to

FIGURE 7 Post-peak pictures of the tested dapped-end beams.



analyze the images.³¹ For all specimens, the full field deformation and displacement fields were obtained using subset size = 29 px, step size = 8 px, and strain filter size = 7. For the tests DEB-3.7 (T1) and (T2), DEB-3.8 (T2), and DEB-3.9 (T1), the measurements exhibited excessive noise, so that a 1-D median time filter with size 10 was applied to reduce this noise. Based on this information, the crack widths close to the re-entrant corner were manually extracted by computing the difference between the displacement fields of both crack lips.

In DEB-3.8 (T2), a more refined analysis of the crack behavior was conducted with the help of the Automatic Crack Detection and Measurement tool (ACDM).^{32,33} ACDM can automatically detect and measure all existing cracks based on the full field deformation and displacement fields delivered by the DIC analysis. For this refined analysis, slightly different DIC parameters were selected (subset size = 33 px, step size = 8 px, and strain filter size = 11) and a median time filter with size three was applied to reduce the noise. For the crack detection method in ACDM using edge detection, the low and high thresholds were selected at 1 and 10 mm/m. At the end, crack kinematic smoothing with a filter span of two, the interpolation method “nearest” and the smoothing method “moving” was applied.

3.2 | Test results

3.2.1 | Ultimate behavior

The failure of dapped-end beams with reinforcement amounts and concrete cover sizes, similar to the ones analyzed in this study, is usually governed by yielding of the main reinforcement.¹⁰ However, a brittle failure of the upper concrete cover limits, in many cases, the ultimate loads and reduces the ductility.¹⁰ This spalling failure initiates at the top of the vertical dapped-end reinforcement (see e.g., failure of specimen DEB-1.6 (T1) in Figure 7). The addition of steel fibers was shown to prevent the spalling failure of the concrete cover by

incorporating a certain residual tensile strength into the concrete cover (see experimental post-peak pictures in Figure 7). This resulted in an enhanced ductility and strength of the SFRC elements. Hence, all steel-fiber reinforced beam specimens failed with yielding and rupture of the reinforcement, whereas both tests of DEB-1.6 failed in a brittle manner, where concrete spalling occurred right after yielding of the reinforcement. The results of the ultimate loads are shown in Table 4. The addition of fibers increased the strength of the dapped ends by between 18% and 30%. The ultimate load increased with increasing quantity of steel fibers until a ρ_f of approximately 0.88%. The additional steel fibers in DEB-3.9 ($\rho_f = 1.26\%$) compared with DEB-3.8 ($\rho_f = 0.88\%$) did not lead to a significant increase in load-bearing capacity. That is, the strength saturated beyond $\rho_f \cong 0.88\%$. This trend is explained with the help of the kinematics-based model in Section 4.

The sequence of yielding of the reinforcement is also listed in Table 4, where $sV1$ indicates the closest and $sV3$ the farthest stirrup from the re-entrant corner. The first layer of stirrups yields first, confirming that bars with smaller diameters engage faster.

3.2.2 | Crack behavior

Generally, the use of steel fibers does not modify the cracking load since the steel fibers cannot be activated without cracking. In all cases, the first crack started at the re-entrant corner of the dapped end, which is favored by the notch effect due to the geometrical discontinuity where a concentration of stresses occurs.

Figure 8 shows the load-deformation behavior of all tests, particularly the corner crack width depending on the load. Additionally, Table 4 shows the average corner crack width of the two twin tests conducted in each beam specimen at selected loads. Note that there is a scatter between the corner crack widths of twin tests, which was also observed in experiments without steel fibers,¹¹ due to uncertainties from preshrinkage, a high variance of

TABLE 4 Main experimental results (sequence of yielding, crack width results show the mean of twin tests for each beam specimen and their comparison with the specimen without fibers at different loads).

Specimen	Concrete type	Sequence of yielding		Ultimate load V_u [kN]		Corner crack width w [mm]					
		T1	T2	T1	T2	$V = 110$ kN		$V = 150$ kN		$V = 190$ kN	
DEB-1.6	Conventional	$sV1-sH-sV2-sV3$	$sV1-sH-sV2$	309.2	250.9	0.54	REF	0.77	REF	1.01	REF
DEB-3.7	SFRC, $\rho_f = 0.51\%$	$sV1-sV2-sH-sV3$	$sV1-sV2-sH-sV3$	332.7	328.3	0.46	−16%	0.69	−10%	0.94	−7%
DEB-3.8	SFRC, $\rho_f = 0.88\%$	$sV1-sV3-sV2-sH$	$sV1-sV2-sH-sV3$	363.3	361.6	0.36	−33%	0.49	−37%	0.54	−46%
DEB-3.9	SFRC, $\rho_f = 1.26\%$	$sV1-sH-sV2-sV3$	$sV1-sH-sV2$	354.5	370.0	0.27	−50%	0.51	−41%	0.67	−33%

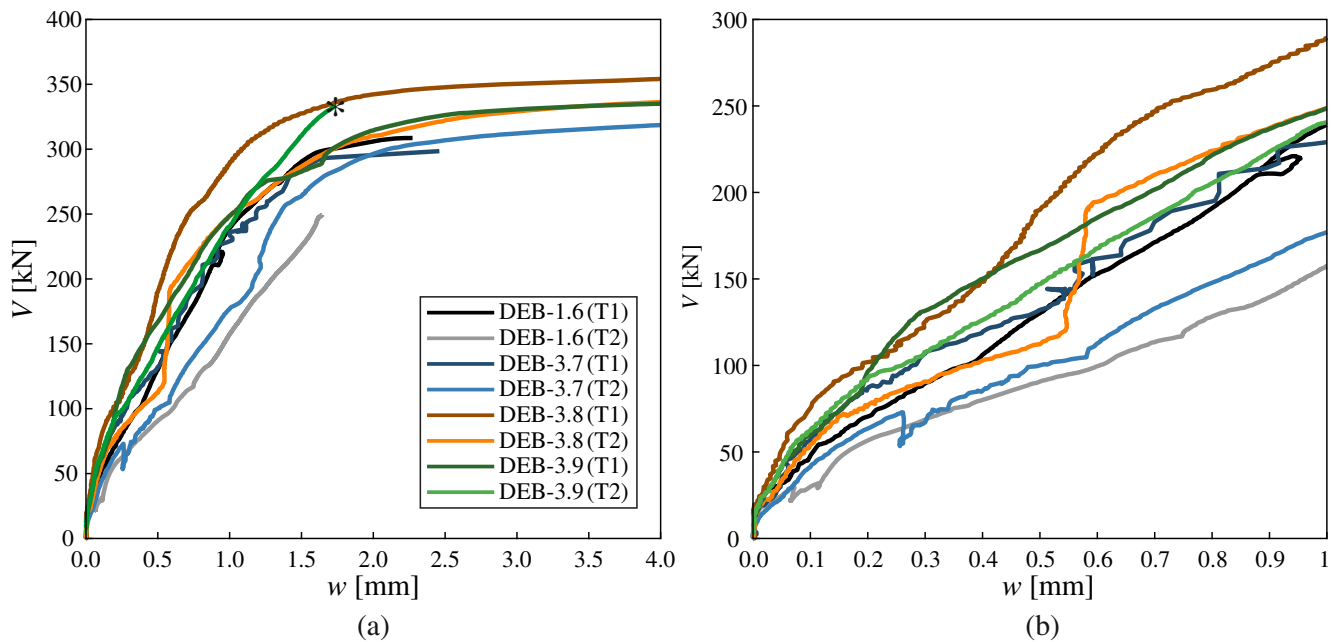


FIGURE 8 Corner crack width plots depending on the load for all tests: (a) for crack width range $w \in [0, 4]$ and (b) for $w \in [0, 1]$ (* indicates that the DIC measurements stopped before the ultimate load was reached in DEB-3.9 (T2)).

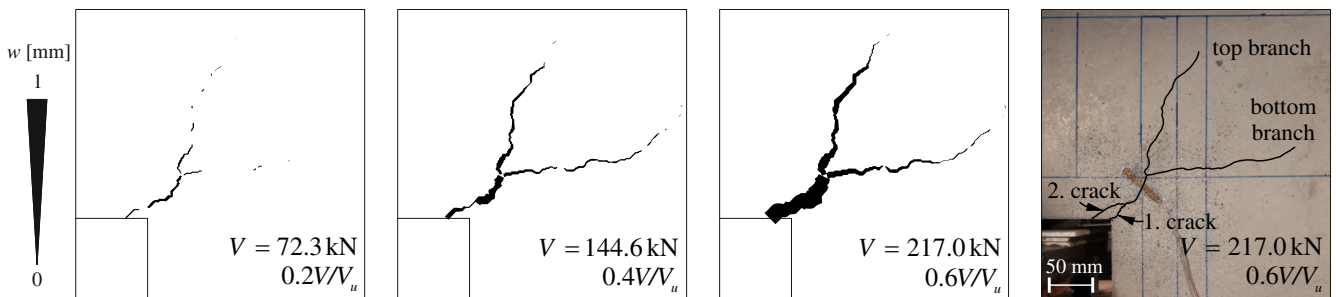


FIGURE 9 Crack behavior at different load levels for DEB-3.8 (T2). The threshold of crack visualization was set to 0.05 mm.

crack patterns and highly concentrated local stresses. However, a related experimental campaign³⁴ showed that the twin tests had similar global deformations. For both tests DEB-3.8 (T2) and DEB-3.9 (T1), first a crack at the re-entrant corner formed but then at approximately

$V = 125$ kN and $V = 90$ kN, respectively, a second crack next to it formed and fused with the first one. The second crack became dominant, see Figure 9 for the case of DEB-3.8 (T2). Therefore, for those two cases, the maximum width of the two cracks, was chosen as a

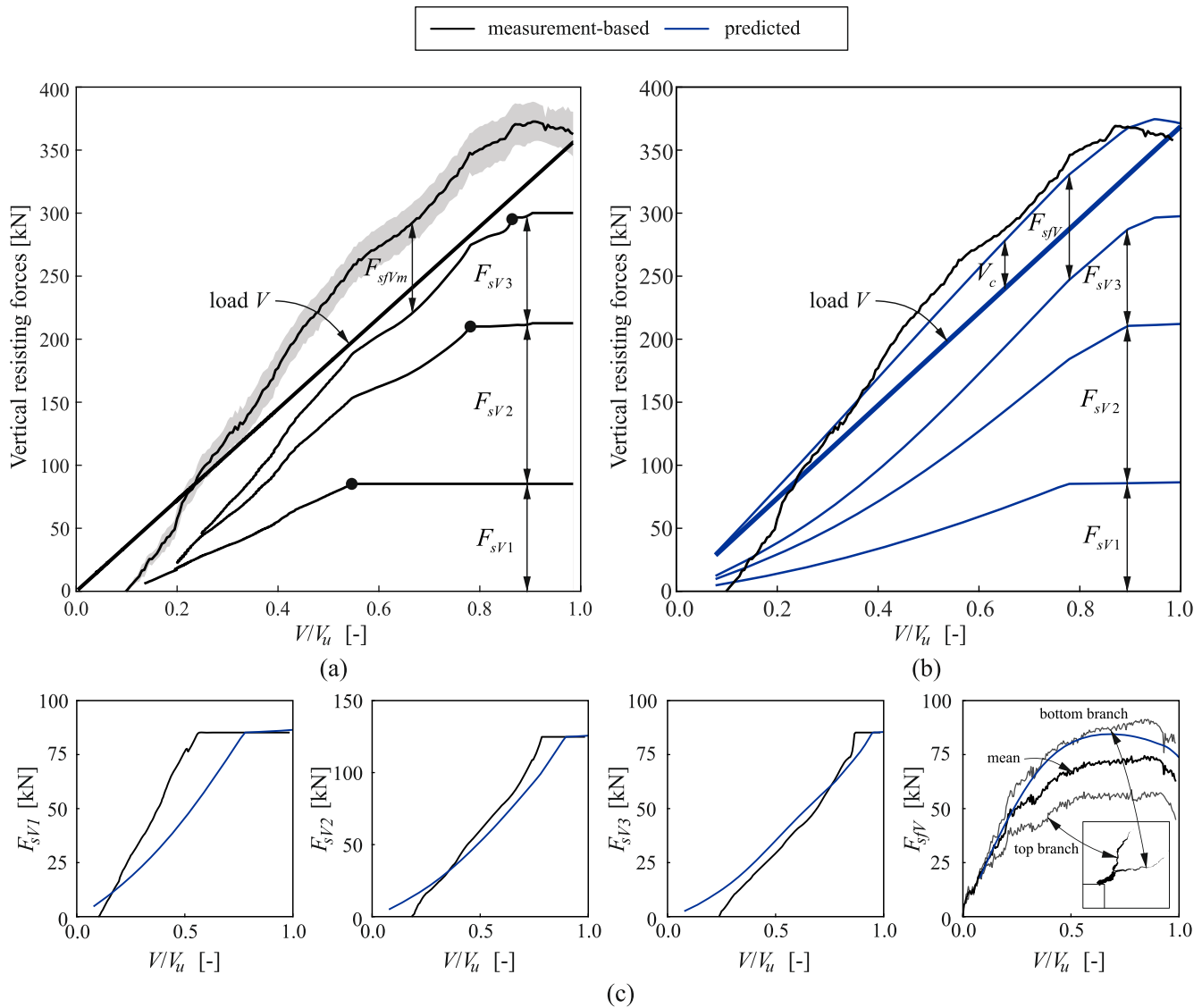


FIGURE 10 Resistance mechanisms for DEB 3.8 (T2): (a) estimation based on measured crack kinematics (dots indicate onset of yielding), (b) prediction of the kinematics-based model, and (c) comparison of the individual resistance components.

representative result. The formation of the second crack explains the kink that can be observed for DEB-3.8 (T2) in Figure 8. The addition of fibers was very effective in reducing the corner crack width. At a load of $V = 110$ kN, the corner crack widths of both DEB-3.8 ($\rho_f = 0.88\%$) and DEB-3.9 ($\rho_f = 1.26\%$) were approximately 33% and 50% lower, respectively, than that of DEB-1.6 without steel fibers. At the higher load of $V = 190$ kN, this reduction increased to 46% for DEB-3.8 but decreased to 33% for DEB-3.9. The lower fiber content was less effective for crack control: For specimen DEB-3.7 ($\rho_f = 0.51\%$), the corner crack width reduction with respect to the specimen without fibers was only 10%.

3.3 | Estimation of resistance mechanisms from measured deformations

To grasp a better understanding of the relative contribution of the different resistance mechanisms, the vertical force contributions of the steel fibers and vertical reinforcements are estimated in this subsection based on the measured crack kinematics and reinforcement strains, respectively. This analysis is conducted for test DEB-3.8 (T2). While the previous subsection described the overall crack behavior based on the result of the crack width at the re-entrant corner (result of test DEB-3.8 (T2) shown with a solid orange line in Figure 8), this part examines the complete kinematics of the main crack. To this end,

the DIC results were processed with ACDM^{32,33} as described in Section 3.1.2. Figure 9 plots the crack kinematics at various load levels: 20%, 40%, and 60% of ultimate load. First, a main crack (1. crack) formed at the re-entrant corner, which then split into two crack branches shortly after crossing the reinforcement. At a load of $V = 125$ kN ($V/V_u \approx 0.35$), a second crack (2. crack) about 2 cm next to the re-entrant corner formed, which morphed into the main crack and became dominant. Due to the subset size used and the way ACDM conducts the crack detection, ACDM measured these two near cracks in the vicinity of the re-entrant corner as a single crack.

The SDEM by Lee et al.,²⁹ already described in Section 2.2.2, was taken to approximate the stresses transferred through the crack thanks to the activation of the steel fibers. The magnitude and orientation of the fiber stresses at each crack point were estimated on the basis of the measured crack widths and crack angles. The total vertical force resisted by the fibers (F_{sfV}) was obtained by integrating the vertical components of the fiber stresses along the crack. A separate estimation was conducted considering either the top or bottom branches of the crack. The right plot in Figure 10c shows the forces carried by the steel fibers depending on the load level for the two different crack branches. The force carried when considering the bottom branch was higher due to its lower inclination. The top branch started to show a plateau around $0.5V/V_u$ until $0.9V/V_u$. This behavior is observed later in the bottom branch at around $0.6V/V_u$. For adding the stirrup contribution to the fibers, the mean value between the forces carried by the fibers in the top and bottom crack branches will be considered.

The forces in the stirrups (F_{svi}) were determined based on the measured reinforcement strains by using a simple bilinear stress-strain relationship with Young's modulus $E_s = 200$ GPa and ultimate steel strain $\varepsilon_{s,u} = 10\%$. In those cases where the strain gauge was not located exactly at the crack, the reinforcement strain at the crack was estimated as described in Section 3.1.2. Furthermore, the measurement-based curves were corrected to account for the effects of shrinkage. The first strain gauge measurements were taken at the beginning of loading when shrinkage had already developed and precompressed the reinforcement. An estimated value of -0.4×10^{-3} was assumed for the unrestrained shrinkage of the concrete, as shrinkage measurements were not available. Moreover, a linear-elastic analysis of the concrete and reinforcing bars showed that the prestrain of the reinforcement at the re-entrant corner was approximately equal to the unrestrained shrinkage of concrete. Therefore, this estimated value was used to shift the reinforcement strain measurements. Figure 10a shows the

combination of the vertical forces carried by the different resistance mechanisms and the comparison to the acting load V from the experimental measurements. A dot in the figure indicates the onset of yielding in the respective stirrup. It can be seen that the sum of the forces in the stirrups and steel fibers exceeds the applied load significantly for a wide range of loads. This suggests a possible formation of Strut 2 shown in Figure 1b. Within the framework of the kinematics-based model, this suggests an upward force V_c in the free body diagram in Figure 2a.

3.4 | Model predictions

These measurement-based results are compared to the predictions of the kinematics-based model. Figure 10b presents the load-resisting mechanisms of specimen DEB-3.8 (T2) as predicted by the model in blue. The plot shows that the sum of the forces in the vertical bars and the fibers surpasses the applied load and agrees well with the curve derived from the experiment (in black). Therefore, the model also predicts a formation of Strut 2 (negative component V_c). The validation of the predictions of the individual load-bearing mechanisms is conducted in Figure 10c. The measurement-based curves are reasonably well captured by the model, particularly for the second and third layers of stirrups. As for the force in the fibers, the model prediction, assuming a single crack at 45° , nearly overlaps with the measurement-based curve obtained from the flatter crack in the test.

The model predictions for the complete load-crack width response of all specimens are shown in Figure 11. The pairs of nominally identical tests T1 and T2 show that there can be a significant natural scatter in the corner crack widths (compare black and gray experimental curves), as described in Section 3.2.2. The model predictions lie either between the two experimental curves or closer to the tests with larger crack widths. The model also captures the peak resistance well: the average ratio between the experimental and the predicted ultimate load is 0.96. According to the predicted load-bearing components, the vertical reinforcement reaches a plateau for all specimens except the reference dapped end (DEB-1.6). As it can be expected, the contribution from the steel fibers increases with increasing amount of fibers. In all specimens, $V_c < 0$ is predicted for service loads, which corresponds to the development of Strut 2 shown in Figure 1b. However, this strut action vanishes near failure for the specimens with fibers ($V_c = 0$). This is consistent with the test results in Figure 10a for test DEB-3.8 (T2). The main experimental and prediction results of the ultimate load are summarized in Table 5.

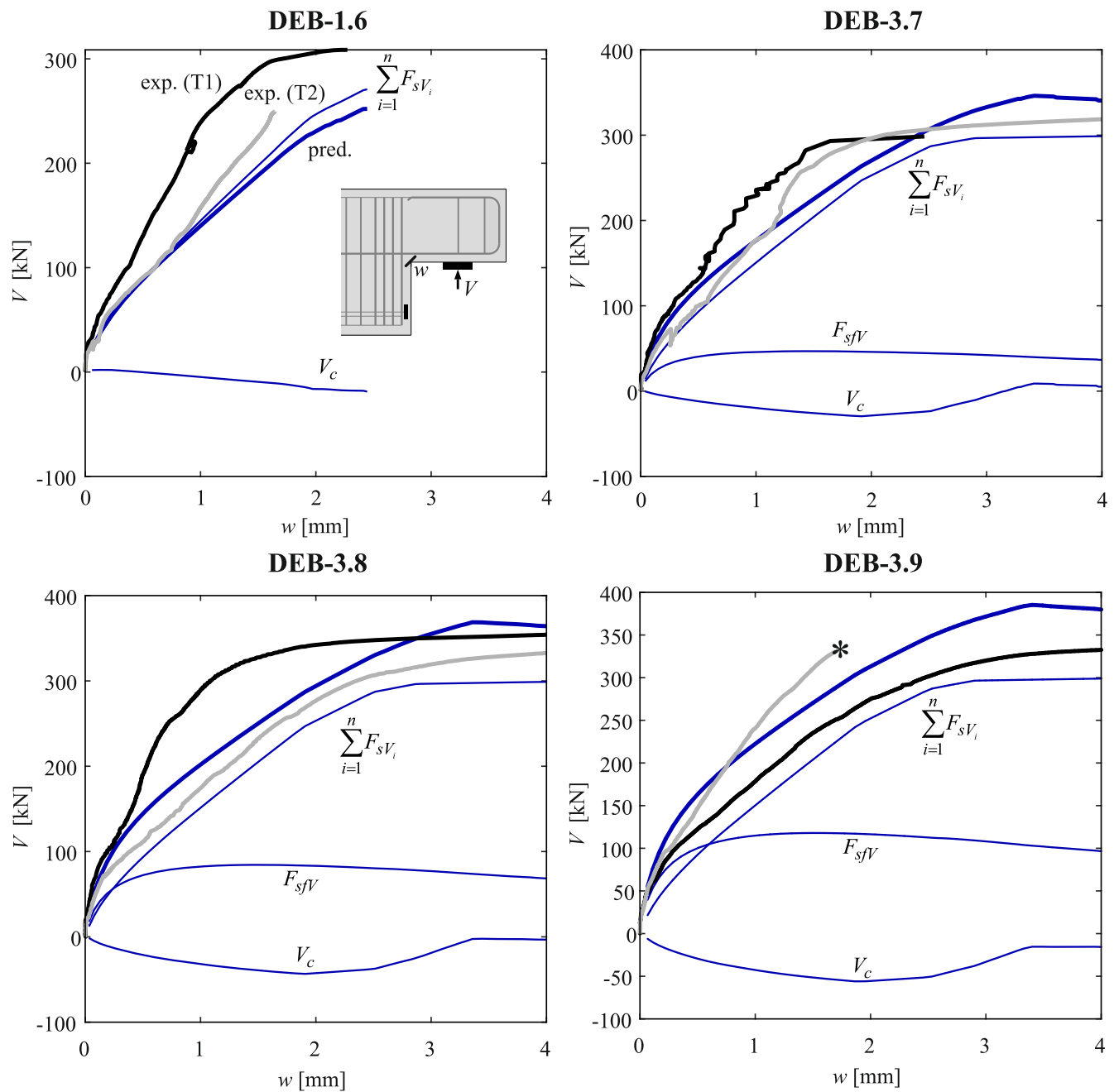


FIGURE 11 Comparison of predicted and measured corner crack widths for all tests (* indicates that the digital image correlation measurements stopped before the ultimate load was reached in DEB-3.9 (T2)).

TABLE 5 Predicted and measured ultimate strengths of dapped-end beams with varying fiber content.

Specimen	Concrete type	$V_{u,expT1}$ [kN]	$V_{u,expT2}$ [kN]	$V_{u,pred}$ [kN]	$V_{u,expT1}/V_{u,pred}$ [-]	$V_{u,expT2}/V_{u,pred}$ [-]
DEB-1.6	Conventional	309.2	250.9	252.1	1.23	1.00
DEB-3.7	SFRC, $\rho_f = 0.51\%$	332.7	328.3	346.2	0.96	0.95
DEB-3.8	SFRC, $\rho_f = 0.88\%$	363.3	361.6	368.8	0.99	0.98
DEB-3.9	SFRC, $\rho_f = 1.26\%$	354.5	370.0	385.2	0.92	0.96

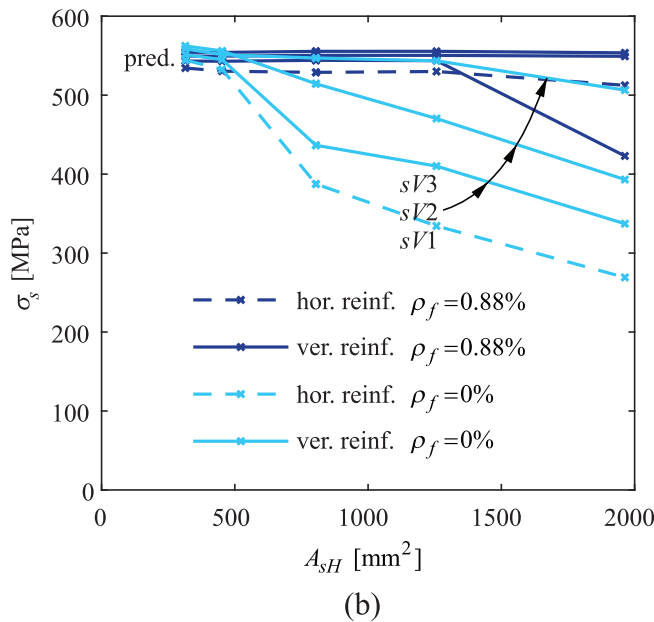
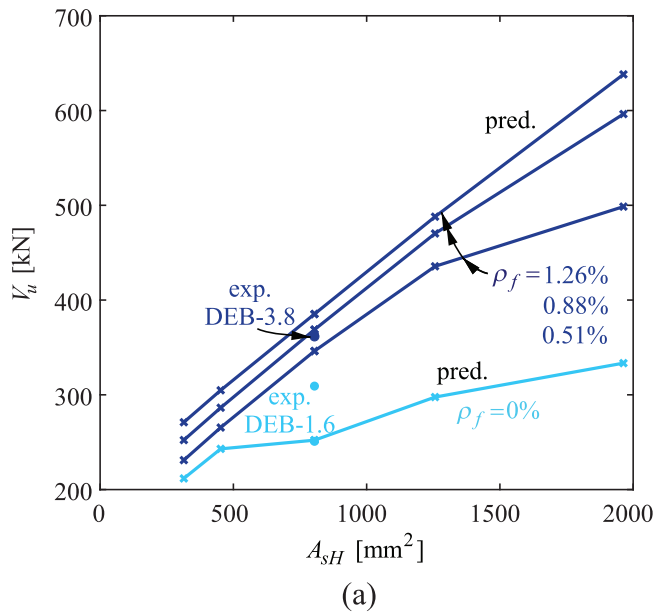


FIGURE 12 Influence of horizontal reinforcement on the ultimate behavior of dapped ends without diagonal reinforcement: (a) predicted strength, and (b) reinforcement stresses at failure.

4 | EFFECT OF DAPPED-END PROPERTIES ON STRENGTH AND CRACK CONTROL

To further understand the experimentally validated model and the behavior of SFRC dapped ends, a parametric study is conducted by systematically varying the properties of the test specimens. In particular, the amounts of the three main reinforcements—fibers, horizontal, and vertical reinforcing bars—are varied one at a time as shown in Figures 12–14. Furthermore, the effect of the

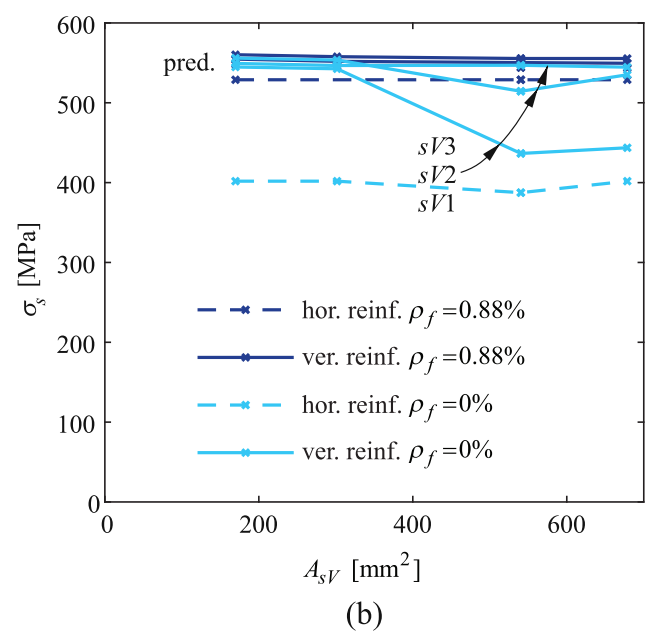
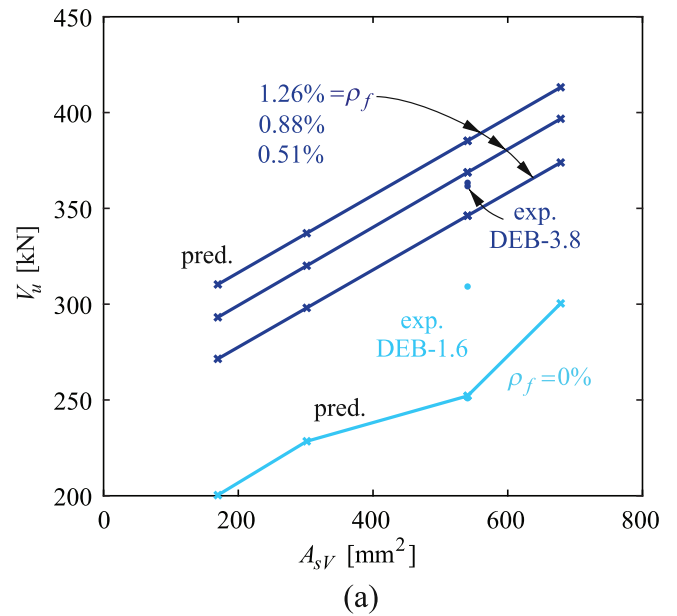
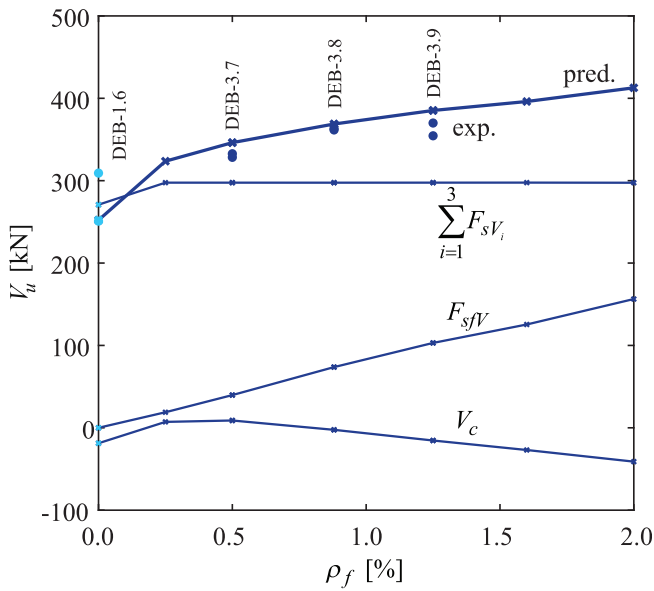


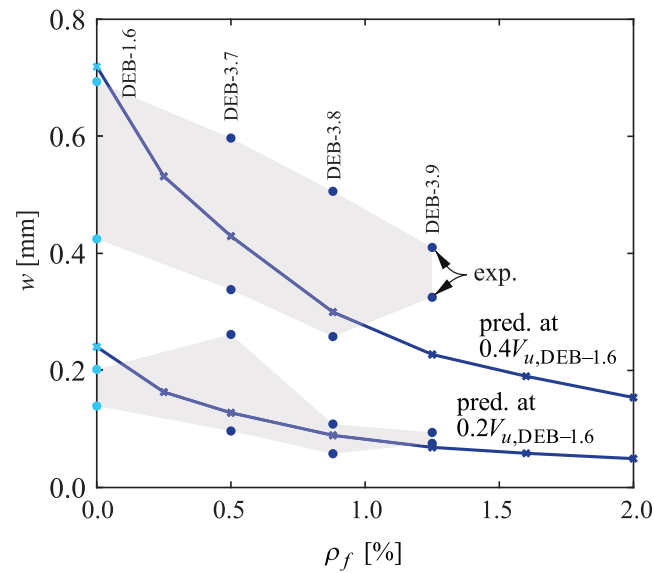
FIGURE 13 Influence of vertical reinforcement on the ultimate behavior of dapped ends without diagonal reinforcement: (a) predicted strength, and (b) reinforcement stresses at failure.

fiber volumetric ratio on a reinforcement layout with diagonal reinforcement is presented in Figure 15.

In Figure 12, the horizontal reinforcement of the test specimens (4016) was varied to 4010, 4012, 4020, and 4025. The plot shows that the increase of horizontal reinforcement results in a substantial increase of the ultimate strength for dapped ends with fibers (based on DEB-3.7, DEB-3.8, and DEB-3.9). In comparison, this effect is significantly less pronounced for dapped ends without fibers (based on DEB-1.6). This notable difference is explained in the bottom plot of the figure which shows the stresses



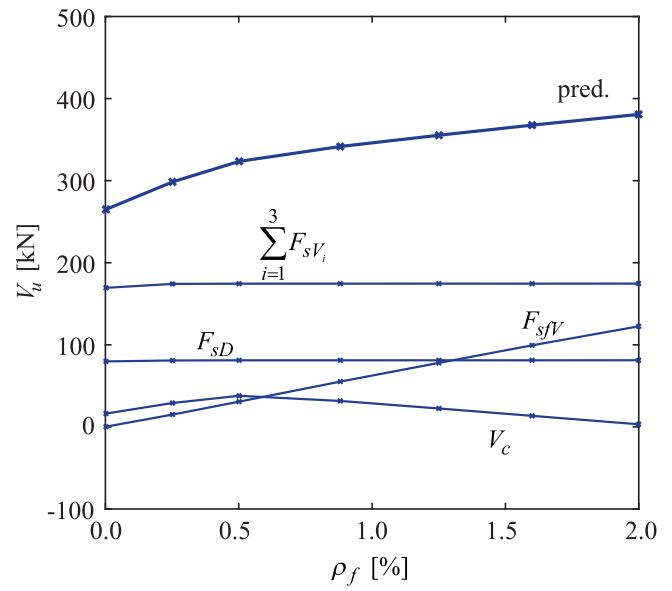
(a)



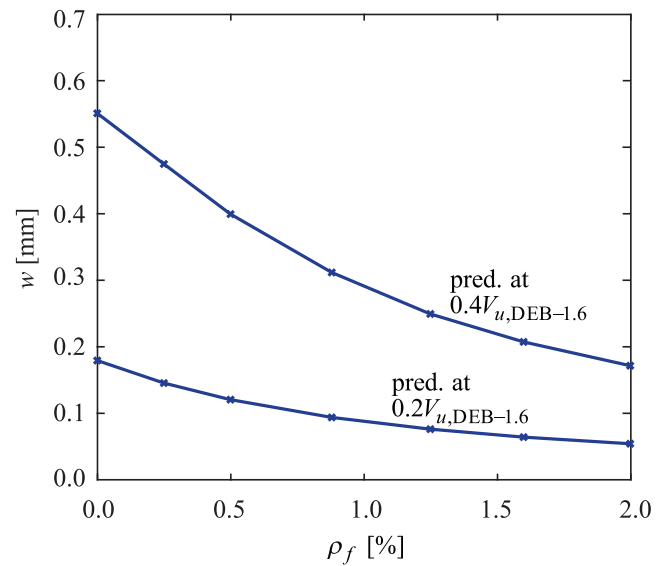
(b)

FIGURE 14 Influence of fiber volumetric ratio in dapped-end connections without diagonal reinforcement on (a) the ultimate behavior, and (b) crack control.

in the reinforcement at failure for $\rho_f = 0.88\%$ and $\rho_f = 0\%$. In configurations with fibers, all reinforcing bars are almost fully utilized at failure (both horizontal and vertical reinforcement). However, if no fibers are added, most of the reinforcement remains elastic and the reinforcement stress at failure decreases with increasing area of horizontal bars. This difference can be explained by the effect of the fibers in the compression zone. The fibers enhance the ductility of the compression damage zone and allow for larger plastic deformations to occur until the reinforcement is fully engaged and yields.



(a)



(b)

FIGURE 15 Influence of fiber volumetric ratio in dapped-end connections with diagonal reinforcement on (a) the ultimate behavior, and (b) crack control.

In contrast, this effect is not observed when the vertical reinforcement of the test specimens ($4\phi 10 + 2\phi 12$) was varied to $6\phi 6$, $6\phi 8$, and $6\phi 12$ in Figure 13. This is because the stirrups have smaller diameters and require less plastic rotation at the critical crack to yield. Therefore, the model predicts that nearly all vertical reinforcing bars yield in all cases with and without fibers. The strength difference between the two cases is then solely due to the contribution of the fibers in tension. This explains why the difference between the two strength curves remains approximately constant for the entire studied range of vertical reinforcement amount.

Finally, the amount of steel fibers is varied in Figures 14 and 15, considering fiber contents larger than those tested in the reference experimental campaign and for two different reinforcement layouts. Figure 14 shows results for the configuration tested in the experimental campaign, while Figure 15 provides results for a reinforcement layout with diagonal reinforcement. The diagonal layout was designed to achieve the strength of DEB-1.6 without fiber contribution and consists of 3ø16 horizontal, 2ø10 + 2ø10 vertical, and 1ø16 diagonal reinforcing bars. The predicted failure behavior (Figure 14a) captures the trend of the experimental points well. The fiber contribution F_{sfV} increases linearly; however, this trend is countered by the increasing negative contribution of the component V_c . This explains the observed saturation in strength gain due to the fibers in tests with fiber contents exceeding 70 kg/m^3 ($\rho_f = 0.88\%$).

The most positive effect of the fibers is illustrated in the bottom plots of Figures 14 and 15 for the same set of dapped ends. The plots show the corner crack widths at two levels of load: 20% and 40% of the average measured ultimate strength of tests DEB-1.6 (T1) and (T2) (56.0 kN and 112.0 kN) without fibers. Both the experimental and predicted results show a substantial reduction of the crack width for increasing amount of fibers. As a reference, the addition of a fiber amount of 70 kg/m^3 ($\rho_f = 0.88\%$) to the orthogonal layout reduces the corner crack width by approximately 58%, while the reduction is 45% in the case of the diagonal layout. The diagonal layout provides improved crack control compared to the orthogonal layout in the absence of fibers, although it remains insufficient to comply with code provisions. The addition of fibers therefore addresses the primary vulnerability of dapped-end connections with both reinforcement layouts, namely excessive crack widths under service conditions. To conclude, with this kinematics-based model, dapped ends can be designed for ultimate limit state with or without accounting for fibers. However, the addition of fibers ensures proper cracking behavior at the serviceability limit state, if an appropriate amount of fibers is chosen. The proposed kinematics-based model allows for performing both design calculations in a unified and accurate manner, taking into account all important dapped-end properties.

5 | CONCLUSION

This study addressed the challenge of designing reinforced concrete (RC) dapped ends to achieve adequate crack control by investigating the use of steel-fiber reinforced concrete (SFRC) in addition to conventional reinforcement. To this end, an analytical and experimental

investigation was carried out to (i) quantify the beneficial effects of fibers, and (ii) develop and validate a model to design SFRC dapped-end connections both at service and ultimate limit states.

The results from an experimental campaign consisting of eight large-scale dapped-end tests with varying fiber contents showed that the addition of steel fibers enhances the ductility and increases the strength by up to 30% when the fiber amount is at least 70 kg/m^3 . At this fiber amount, the crack width for a given load is reduced by approximately 40%.

A kinematics-based model was proposed to estimate the behavior of SFRC dapped ends along the re-entrant corner. The model extends an existing model for conventional RC dapped ends. The deformations along the corner crack and in the compression zone are idealized through the rotation around the crack tip and the depth of the compression zone. These kinematic variables are related to the corresponding resisting forces (reinforcement and SFRC in compression and tension) through appropriate constitutive relationships. Enforcing equilibrium among these resisting forces provides the shear force for a given deformation, while the crack width is obtained from the assumed rotation. The validation against the experimental results demonstrated that the model can accurately capture the opening of the corner crack, the evolution of the resistance mechanisms, and the ultimate strength.

For new dapped ends, steel fibers are recommended to be considered only for crack control purposes. Accordingly, the design should first be carried at the ultimate limit state without considering the effect of steel fibers. The crack width at the serviceability limit state is then evaluated for this initial design (i.e., without fibers). If the resulting crack width exceeds the code limits, as is typically the case, the required fiber dosage can be determined to reduce the crack width to within the prescribed regulatory threshold. The proposed kinematics-based models allows to conduct all these design steps.

NOMENCLATURE

Δ	distance between locations of the crack and strain gauge on the reinforcement
$\emptyset$	diameter of reinforcing bars
$\emptyset_f$	diameter of steel fiber
α	inclination of compression damage zone
α_f	fiber orientation factor
β	inclination of the diagonal reinforcement
β_f, β_{eh}	coefficients to consider the effect of fiber slip on longer embedment side (frictional behavior, mechanical anchorage effect)
$\delta_{c,top}$	shortening of the top concrete fiber

ρ_f	fiber volumetric ratio	f_y	yield strength of the reinforcement
σ_c	stresses in compression zone	F_{c0}	force of undamaged compression zone
σ_f	tensile stress of SFRC	F_{CDZ}	force of compression damage zone
σ_s	stresses in the reinforcement	f_{ct}	tensile strength of concrete
σ_t	stresses in the reinforcement at the crack	$F_{j,i}$	force in the i th layer of the reinforcement with $j = \{h, v, d\}$ for horizontal, vertical and diagonal reinforcement
$\sigma_{c,top}$	stress at the top fiber	F_{sfV}	vertical force of the steel fibers
σ_{eh}	tensile stress of SFRC due to the mechanical anchorage of end-hooked steel fibers	F_{svi}	vertical force in the stirrup layer $i = \{1, 2, 3\}$
σ_{st}	tensile stress of SFRC due to the frictional bond behavior of steel fibers	H	horizontal support load
$\tau_b, \tau_{b0}, \tau_{b1}$	bond shear stresses (general, elastic and plastic region)	h	height of reduced cross section at re-entrant corner
$\tau_{eh,max}$	maximum bond strength due to the mechanical anchorage of end-hooked steel fibers	$K_{eh,i}$	K_{eh} at $w_k = \frac{(l_f - l_i)}{2}$
$\tau_{f,max}$	frictional bond strength of steel fibers	K_{st}, K_{eh}	factors to represent average pullout stresses of fiber due to frictional bond behavior and mechanical anchorage of end-hooked fiber, respectively
θ	crack angle	l_c	length along the top fiber influenced by the rotation around the crack tip
$\varepsilon_{c,top}$	strain at the top fiber	l_i	distance between the mechanical anchorages of steel fibers
ε_{c0}	strain at peak stress f_c	l_x	length of different parts of the strain and stress distribution at the corner crack, $x = 0, 1, 2, a, p$
ε_c	strain in compression zone	l_{CDZ}	length of the compression depth zone
$\varepsilon_{s,exp}$	measured strain of the reinforcement	l_{cr}	length of crack
$\varepsilon_s, \varepsilon_{s,y}, \varepsilon_{s,u}$	strain of the reinforcement (general, at yielding, ultimate)	l_f	length of steel fiber
ε_t	strain of the reinforcement at the crack	O	point of the crack tip
φ	rotation around tip of the crack	Q	experimental load of the three-point bending test
a	horizontal distance between re-entrant corner and load introduction	S_{eh}	slip at maximum tensile force due to mechanical anchorage of fiber perpendicular to the crack surface
A, B, RI	variables for the model for SFRC under compression	S_f	slip at frictional bond strength for fiber perpendicular to the crack surface
A_{sj}	steel reinforcement amount for $j = \{H, V\}$	V	vertical support load
c	minimum concrete cover of the considered layer of reinforcement	V_f	fiber amount
c_f	lever arm to the resultant force of the fiber tensile stresses across the corner crack, from the crack tip	V_c	vertical force carried by Strut 2
c_{c0}	lever arm to the force of undamaged compression zone from the crack tip	w	corner crack width
c_{CDZ}	lever arm to the force of compression damage zone from the crack tip	w_j	crack width at the reinforcement in its direction with $j = \{h, v, d\}$ for horizontal, vertical and diagonal reinforcement
$c_{d,i}$	distance along the crack from the re-entrant corner to the i th layer of diagonal reinforcement	w_k	crack width at a general point of the corner crack located at a distance $l_{cr,k}$ from the crack tip
$c_{j,i}$	perpendicular distance from the re-entrant corner to the i th layer of type j reinforcement with $j = \{h, v\}$ for horizontal and vertical reinforcement	x	compression zone depth
d	effective depth of the reduced cross section at re-entrant corner		
E_c	Young's modulus of concrete		
E_s	Young's modulus of reinforcement		
f_c	compressive strength of concrete		
F_f	resultant force of fiber tensile stresses across the corner crack		
f_u	ultimate strength of the reinforcement		

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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