Improved state augmentation method for buffeting analysis of structures subjected to non-stationary wind

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Abstract

Extreme winds such as hurricanes and thunderstorms often present non-stationary characteristics, having time-varying mean wind speeds and non-stationary wind fluctuations. When concerning the wind-induced vibrations under non-stationary wind, the excitation will be a non-stationary process, and the wind-structure coupled system can be represented by a linear time-varying (LTV) system. The aim of this study is to present a state augmentation method to investigate the non-stationary buffeting of a model bridge tower subjected to non-stationary wind with consideration of the aeroelastic damping. Based on the theory of stochastic differential equations and Itô's lemma, the statistical moments of the non-stationary buffeting response are derived through solving a first-order ordinary differential equation system. The proposed method is validated by comparisons with the Monte Carlo method and the pseudo excitation method. The result shows that the state augmentation method has higher accuracy and efficiency than the well-accepted time-frequency techniques.

Keywords: non-stationary wind; aerodynamic damping; buffeting response; state augmentation method; stochastic differential equation; Itô's lemma.

1 Introduction

In contrast with the stationary synoptic wind, extreme wind events such as hurricanes and thunderstorms always exhibit considerable non-stationary and non-gaussian characteristics [1-3], having time-varying mean wind speeds and non-stationary and non-gaussian wind fluctuating components. The rapid changes in the kinematics and dynamics of these flow fields can potentially amplify aerodynamic loads on structures and result in higher non-stationary buffeting responses [4]. Moreover, when considering aeroelastic effects, the aerodynamic damping and stiffness will be time-dependent due to the time-varying mean wind speed, and the wind-structure coupled system can be thus represented as a linear time-varying (LTV) system [5]. Brusco and Solari [6] investigated the aeroelastic effects induced by thunderstorms on the dynamic response of slender structures. These

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facts lead to difficulties in the calculation of the structural response by using the conventional buffeting analysis method [7, 8], which was originally developed for synoptic wind and stationary responses [9]. In view of these non-stationary wind effects, many attempts have been made to develop random vibration theory for non-stationary buffeting, which can be broadly classified into three categories.

The first category of methods is the Monte Carlo method (MCM). It uses multiple deterministic time-history samples generated, for instance, using the evolutionary power spectral density (EPSD) [10, 11] in order to statistically characterize the system response. Huang et al. [12] used this method to evaluate the along-wind responses of tall buildings induced by thunderstorm downbursts and investigated the transient wind load effects on various responses including the extreme value and peak factor. To obtain reliable statistical characteristics of the responses, sufficiently numerous samples should be employed, which results in computationally intensive simulations.

The second category of methods is the generalized frequency-domain method, also named as generalized wind loading chain proposed by Kareem et al. [9], which extends the well-established Davenport's chain [7, 8] from a stationary buffeting analysis in frequency domain to a non-stationary format in time-frequency domain. Based on the Priestley's Evolutionary Spectra theory [10, 11], the time-frequency response spectrum can be evaluated through a sequential product of the time-frequency wind spectrum, the instantaneous aerodynamic admittance function, and the structural transfer function. Chen [13] developed a frequency domain framework for calculating along-wind buffeting responses of tall buildings induced by transient non-stationary wind and studied the effects of wind speed profile, time-varying mean wind speed, and spatial correlations of wind turbulence on structural responses. This framework was later generalized [14] to predict multimodal coupled buffeting responses of long-span bridges under nonstationary wind with consideration of self-excited forces modeled as a frequency-dependent linear time-varying (LTV) system. Note that, to deal with the LTV system [15, 16], the generalized frequency-domain method assumes the system as time-invariant within a moving short window, so that the problem reduces to finding the time-independent frequency response function and stationary response of an equivalent linear time-invariant (LTI) system within each window. This is only appliable to the system with slowly time-varying characteristics.

The third category of methods for investigating the non-stationary buffeting in wind-structure coupled systems is the pseudo excitation method (PEM) developed by Lin et al. [17]. It can deal with the random vibration problem with non-stationary excitations and time-dependent system properties simultaneously. With this method, the nonstationary characteristics of the response can be conveniently obtained, including the EPSD and standard deviation. For instance, Hu et al. [5, 18] considered the self-excited forces characterized by flutter derivatives and computed the non-stationary buffeting response of long-span bridges subjected to typhoon-induced wind loads by means of the PEM. Note that these flutter derivatives are functions of time due to the effects of the time-varying mean wind speeds on the reduced frequencies. He et al. [19] considered the excitations due to stationary track irregularities and non-stationary wind loads simultaneously and calculated the non-stationary responses of the high-speed train-bridge coupled system based on the PEM. In essence, the PEM is a time-frequency technique: the pseudo excitations are expanded in frequency domain, and the responses are integrated in time domain. According to [20], the computation effort of the PEM is mainly taken in calculating the pseudo response. The Precise Integration Method [21] can be combined with this process to improve the efficiency when computing the corresponding time-history responses to the pseudo excitations, yet this is more appliable to the LTI system [22], and does not avoid the operations in the frequency content.

Based on the theory of Itô's stochastic differential equation, Grigoriu [23, 24] proposed the state augmentation method to calculate the stochastic response of linear systems subjected to stationary non-Gaussian excitations expressed in the form of polynomials of Gaussian processes with time-dependent coefficients. With this method, the statistical moments of any order of the transient response can be directly obtained through solving a system of algebraic equations, i.e., moments equation, with high efficiency. This method [25] was then applied to predict the random vibration response of a flexible plate subjected to stationary Gaussian turbulence in which the wind pressure was assumed to be proportional to the square of the wind speed, so that the excitations were stationary non-Gaussian processes. Due to the orthogonality of structural modes, the coupling effect between each mode was not considered in this study, and the plate displacement was restricted to its first structural mode for clarity. Recently, Cui et al. [26] extended this method to study the multimodal buffeting response of long-span bridges subjected to stationary non-Gaussian turbulence, considering the coupling effect of aerodynamic damping between vertical and torsional motions. Although the state augmentation method has been applied in several wind engineering problems, such as quantification of wind loading uncertainties [27, 28] and non-Gaussian turbulence, these studies are primarily based on the stationary case with time-invariant system characteristics. It has not been reported for non-stationary case with time-dependent characteristics of the system, for which the methodology may change substantially, e.g., the statistical moment equations change from algebraic equations in the stationary case to differential equations in the non-stationary case.

The aim of this paper is to extend the state augmentation method to investigate the non-stationary along-wind buffeting of a bridge tower subjected to non-stationary wind loads with consideration of the time-dependent aeroelastic damping. First, the non-stationary wind speed is characterized as a time-varying mean and uniformly modulated wind fluctuations [1]. To formulate the non-stationary buffeting forces, the strip and quasi-steady theories are invoked [6, 29]. The aeroelastic term is included by considering the relative velocity as the combination of the structural velocity and the total wind speed. Due to the time-dependent characteristics of aerodynamic damping, the dynamic equation of the along-wind buffeting can be represented as a linear time-varying (LTV) system. By using the Ornstein–Uhlenbeck process to approximate wind fluctuations, the augmented states of the system and the excitation are written as an Itô-type stochastic differential equation. Based on Itô's lemma, the moments equations of the non-stationary response are derived as a system of first-order ordinary differential equations. The proposed state augmentation method is first validated by comparing the time-varying standard deviation of the responses with those obtained from the Monte Carlo simulations and the pseudo excitation method. Then, the assets of the proposed method as well as the characteristics of non-stationary buffeting responses are discussed by comparing with conventional buffeting analyses. At last, the effect of the aerodynamic damping term on the non-stationary buffeting vibrations is investigated.

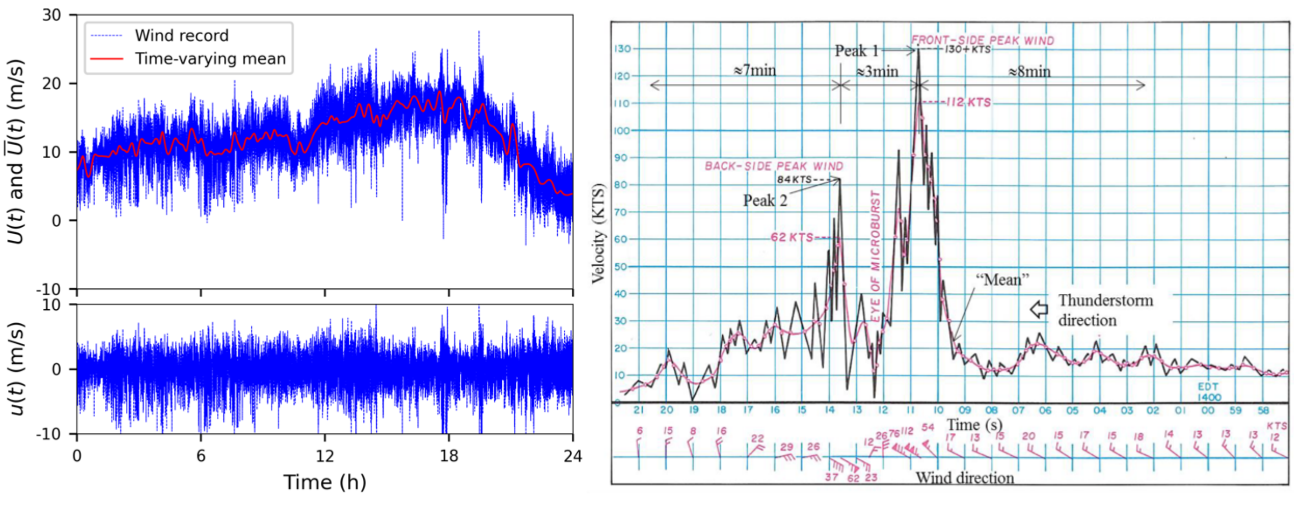
2 Dynamics of structures subjected to non-stationary wind

2.1 Non-stationary wind model

Analogously to the stationary wind model, which is comprised of the mean and fluctuating components, the velocity of a non-stationary extreme wind $U(t)$ can be usually characterized as the summation of a deterministic time-varying mean $\bar{U}(t)$ and a zero-mean non-stationary fluctuating wind component $u(t)$

$$U(t) = \bar{U}(t) + u(t) \tag{1}$$

Note that the variation of $\bar{U}(t)$ is much slower than that of $u(t)$. Therefore, when subjected to a time-varying mean wind, the structure will present a static response so that the quasi-static analysis can be employed. As for the fluctuating component $u(t)$, it can be represented either as a stationary or non-stationary random fluctuation. For example, the wind fluctuation of typhoons can be regarded as a stationary process, whereas that of the thunderstorm downbursts may exhibit stronger non-stationarity [1], as shown in Fig. 1 (Fig. 1 (b) of the downburst is reproduced from [30]).



(a) (b)

Fig. 1. Time histories of non-stationary extreme winds and their wind fluctuations: (a) typhoon "Bolavan" [31]; (b) downburst "Andrews" - reproduced from [30].

Related study [1] suggests that the frequency contents in the EPSD of these non-stationary fluctuations evolve rather slightly with time. In view of this, a simplified uniformly modulated model [1] is adopted to reprceesent the non-stationary wind fluctuations, so that

$$u(t) = \beta(t) u_s(t) \tag{2}$$

in which $\beta(t)$ is the deterministic time-modulation function, and $u_s(t)$ is a stationary Gaussian process with zero mean. Note that $u(t)$ will degrade into a stationary wind fluctuation when $\beta(t)$ is a constant. It is proved by Huang et al. [12] that this treatment can provide a good simulation for estimating the along-wind load effect on high-rise buildings.

2.2 Steady-state equation of wind-induced vibrations

As an example of application of the present method, the study considers the bridge tower of a suspension bridge during construction stage, that can be usually represented by a cantilever Euler beam with distributed mass along the height, as shown in Fig. 2. Only the vibration in the along-wind direction is considered in this study for the sake of simplicity. The lateral motion can be included by using the same method.

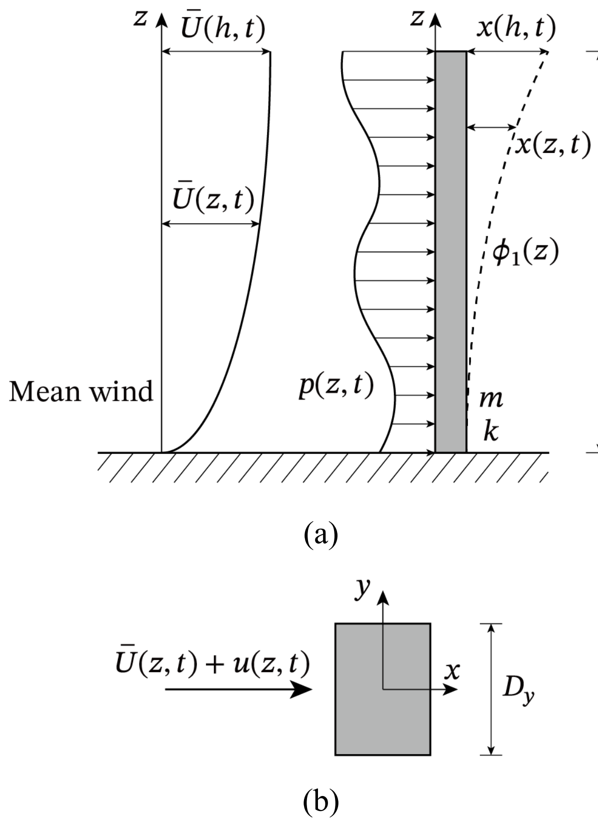


Fig. 2. Schematic of along-wind buffeting of bridge tower (a) lateral view; (b) horizontal plane.

In this study, the quasi-steady assumption is adopted to formulate the buffeting forces and buffeting responses, and the aeroelastic effect is considered under the quasi-steady assumption as well, by taking the relative velocity as a combination of the structural velocity and the total wind speed [6]. Following the strip theory, the along-wind buffeting force per unit height of the bridge tower at the elevation z is obtained as

$$p(z,t) = \frac{1}{2}\rho C_D D_y[\bar{U}(z,t) + u(z,t) - \dot{x}(z,t)]^2 \tag{3}$$

in which $\bar{U}(z,t)$ and $u(z,t)$ are the time-varying mean and turbulence components of the non-stationary wind velocity as a function of time t at the height z; $x(z,t)$ is the along-wind displacement of the tower; $\rho = 1.25\ \mathrm{kg/m^3}$ is the air density, D_y is the width of the vertical surface orthogonal to the wind direction, and C_D is the drag coefficient at the mean static angle of wind attack $\alpha_0 = 0$.

Considering the relatively small magnitude of the system response and the non-stationary wind fluctuations compared to the mean wind speed, the second order quantities in the expansion of Eq. (3) are neglected, and the total buffeting force is divided into the time-varying mean wind load component and the fluctuating component

$$\bar{p}(z,t) = \frac{1}{2}\rho C_D D_y \bar{U}(z,t)^2 \tag{4}$$

$$p^*(z,t) = \rho C_D D_y \bar{U}(z,t) u(z,t) - \rho C_D D_y \bar{U}(z,t)\dot{x}(z,t) \tag{5}$$

As it is well-known, in contrast with the conventional formula of buffeting forces [12], an additional term, i.e., motion-induced force $\rho C_D D_y \bar{U}(z,t)\dot{x}(z,t)$, is found in the right hand of Eq. (5). This term is regarded as the aerodynamic damping force since it is proportional to the structure velocity.

The corresponding mean wind response component $\bar{x}(z,t)$ can be directly calculated by employing the quasi-static analysis [9, 12], e.g., $k\bar{x}(z,t) = \bar{p}(z,t)$. The along-wind buffeting of the bridge tower subjected to non-stationary wind fluctuating loads can be determined by solving the dynamic equation

$$m\ddot{x} + \left[c + \rho C_D D_y \bar{U}(z,t)\right]\dot{x} + kx = \rho C_D D_y \bar{U}(z,t) u(z,t) \tag{6}$$

in which m, c and k are the mass, damping coefficient and bending stiffness per unit height of the tower, respectively. It is seen that, when the bridge tower is subjected to non-stationary wind, the dynamic system presents time-dependent characteristic due to the time-varying mean wind velocity in the aerodynamic damping term, and the buffeting response is non-stationary even though the wind fluctuation component is stationary. This leads to difficulties in dealing with the calculation by using conventional techniques, which can be alleviated by using the proposed method.

With the application of the modal-superposition method, the displacement response of the bridge tower can be transformed to the modal-amplitude coordinates, i.e., $x(z,t) = q_1(t)\phi_1(z)$, where the contribution of high-order modes is neglected in the present case for the sake of simplicity. Moreover, to describe the variation of the mean wind velocity along the height, $\bar{U}(z,t)$ is referenced to the value at the top of the tower, $\bar{U}_h(t)$, and $\bar{U}(z,t) = \bar{U}_h(t)\Delta_{\bar{U}}(z)$ [12, 27, 28], where $\Delta_{\bar{U}}(z)$ represents the wind profile and the subscript h is the height of the tower. In this case, the mean wind response is obtained as

$$\bar{q}_1(t) = \frac{0.5\rho C_D D_y \int_0^h \Delta_{\bar{U}}^2(z)\phi_1(z)\mathrm{d}z}{M_1\omega_1^2}\bar{U}_h^2(t) \tag{7}$$

and the generalized equation for the buffeting response is derived as

$$\ddot{q}_1+\left[2\omega_1\xi_1+\frac{\rho C_D D_y \bar{U}_h(t)}{M_1}\int_0^h \Delta_{\bar{U}}(z)\phi_1^2(z)\mathrm{d}z\right]\dot{q}_1+\omega_1^2 q_1=\frac{\rho C_D D_y \bar{U}_h(t)}{M_1}\int_0^h \Delta_{\bar{U}}(z)u(z,t)\phi_1(z)\mathrm{d}z \quad (8)$$

in which $\bar{q}_1(t)$ and $q_1(t)$ are the generalized coordinates for the mean wind response and fluctuating response, respectively; $\phi_1(z)$ is the first bending modal shape of the tower in the along-wind direction; ω_1 is the first modal circular frequency, and $\xi_1=\int_0^h c\phi_1^2(z)\mathrm{d}z\,/(2M_1\omega_1)$ is the first modal damping ratio, where $M_1=\int_0^h m\phi_1^2(z)\mathrm{d}z$ is the corresponding generalized mass.

It is noticed that the wind fluctuation $u(z,t)$ appearing in the buffeting load is partially correlated at different heights z, which makes the integral on the right-hand side of Eq. (8) cumbersome. The concept of Modal Correlation Length (MCL) [27, 32] has been developed and utilized for decades to evaluate wind loads on vertical structures, which can transform the partially correlated turbulence-induced forces to an equivalent fully correlated wind load that acts on a reduced portion of the building height. By using this concept, the buffeting force is represented in terms of its MCL, so Eq. (8) becomes

$$\ddot{q}_1+\left[2\xi_1\omega_1+\frac{\rho C_D D_y \bar{U}_h(t)}{M_1}\int_0^h \Delta_{\bar{U}}(z)\phi_1^2(z)\mathrm{d}z\right]\dot{q}_1+\omega_1^2 q_1=\frac{\rho C_D D_y \bar{U}_h(t)}{M_1}h\Lambda_{1u}u_{0.6h}(t) \quad (9)$$

in which $u_{0.6h}(t)$ is the reference turbulence components at $z=0.6h\approx 2h/3$, and Λ_{1u} is the along-wind MCL corresponding to the first bending mode, which can be found from [27, 32]

$$|\Lambda_{1u}|^2=\int_0^h\int_0^h \Delta_{\bar{U}}(z_1)\Delta_{\bar{U}}(z_2)\phi_1(z_1)\phi_1(z_2)\exp\left[-\frac{2c_{z,u}n_1}{\bar{U}_h(t)}\frac{|z_1-z_2|}{\Delta_{\bar{U}}(z_1)+\Delta_{\bar{U}}(z_2)}\right]\frac{\mathrm{d}z_1\mathrm{d}z_2}{h^2} \quad (10)$$

where $c_{z,u}=10$ is the standard coherence decay coefficient, and $n_1=\omega_1/(2\pi)$ is the first modal frequency in Hz.

As mentioned in Section 2.1, the non-stationary wind fluctuation can be expressed as a uniformly modulated Gaussian process, i.e., $u_{0.6h}(t)=\beta(t)u_{0.6h}^s(t)$, in which $\beta(t)$ is the modulation function, and $u_{0.6h}^s(t)$ is a zero-mean stationary Gaussian process that can be seen as the corresponding stationary wind fluctuation. In this case, Eq. (9) is given in its state-space form

$$\mathrm{d}\begin{bmatrix}q_1\\ \dot{q}_1\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\omega_1^2 & -2\xi_1\omega_1-M_1^{-1}\rho C_D D_y\gamma_1\bar{U}_h\end{bmatrix}\begin{bmatrix}q_1\\ \dot{q}_1\end{bmatrix}\mathrm{d}t+\begin{bmatrix}0\\ M_1^{-1}\rho C_D D_y\bar{U}_h h\Lambda_{1u}\beta\end{bmatrix}u_{0.6h}^s\mathrm{d}t \quad (11)$$

in which $\gamma_1=\int_0^h \Delta_{\bar{U}}(z)\phi_1^2(z)\mathrm{d}z$.

3 Moments equation of the non-stationary responses

3.1 Augmented states of the system and the excitation

According to [23, 24], the stationary Gaussian process $u_{0.6h}^s(t)$ can be approximated by an Ornstein-Uhlenbeck (OU) process, i.e., $Z(t)\approx u_{0.6h}^s(t)$, which satisfies the stochastic differential equation

$$\mathrm{d}Z(t)=-\alpha Z(t)\mathrm{d}t+\sigma\sqrt{2\alpha}\mathrm{d}W(t) \quad (12)$$

in which $1/\alpha$ is the time relaxing coefficient, σ is the standard deviation of $Z(t)$, and $W(t)$ is a standard Wiener process satisfying $\mathrm{E}[\mathrm{d}W(t)\mathrm{d}W(t)] = \mathrm{d}t$ where $\mathrm{E}[\cdot]$ indicates the expectation operator. The parameters α and σ can be found through fitting the auto-correlation function $R_{ZZ}(\tau) = \sigma^2 \mathrm{e}^{-\alpha|\tau|}$ or the single-side power spectral density function $S_{ZZ}(\omega) = 4\alpha\sigma^2/(\alpha^2 + \omega^2)$ with the ones of $u^s_{0.6h}(t)$, of which the fitting procedure will be shown in Section 4.

By substituting Eq. (12) into Eq. (11), the states of the system and the excitation can be written as

$$\mathrm{d}\begin{bmatrix} q_1 \\ \dot{q}_1 \\ Z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_1^2 & -2\xi_1\omega_1 - M_1^{-1}\rho C_D D_y \gamma_1 \bar{U}_h & M_1^{-1}\rho C_D D_y \bar{U}_h h \Lambda_{1u}\beta \\ 0 & 0 & -\alpha \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \\ Z \end{bmatrix} \mathrm{d}t + \begin{bmatrix} 0 \\ 0 \\ \sigma\sqrt{2\alpha} \end{bmatrix} \mathrm{d}W \quad (13)$$

Eq. (13) can be recognized as an Itô-type stochastic differential equation [33] of the form

$$\mathrm{d}\mathbf{Y}(\mathrm{t}) = \mathbf{g}(\mathbf{Y}(\mathrm{t}), t)\mathrm{d}t + \mathbf{h}(\mathbf{Y}(\mathrm{t}), t)\mathrm{d}W(\mathrm{t}) \quad (14)$$

in which $\mathbf{Y}(\mathrm{t})$ is the state vector, $\mathbf{g}(\mathbf{Y}(\mathrm{t}), t)$ and $\mathbf{h}(\mathbf{Y}(\mathrm{t}), t)$ are explicit functions of the states and time, of which the forms are given in Appendix. A.

3.2 Statistical moments of the response

Assume $\xi(\mathbf{Y}(\mathrm{t}), t)$ to be a scalar function of the state vector $\mathbf{Y}(\mathrm{t})$ and time t. According to Itô's lemma [34]

$$\mathrm{d}\xi = \left\{\frac{\partial \xi}{\partial t} + [\nabla_{\mathbf{Y}}(\xi)]^{\mathrm{T}}\mathbf{g} + \frac{1}{2}\mathbf{h}^{\mathrm{T}}[\mathrm{H}_{\mathbf{Y}}(\xi)]\mathbf{h}\right\}\mathrm{d}t + [\nabla_{\mathbf{Y}}(\xi)]^{\mathrm{T}}\mathbf{h}\mathrm{d}W \quad (15)$$

in which the superscript T denotes the transpose operation, $\nabla_{\mathbf{Y}}(\cdot)$ is the gradient operator with respect to $\mathbf{Y}$ and $\mathrm{H}_{\mathbf{Y}}(\cdot)$ is the Hessian matrix. According to [23, 24], the expectation of Eq. (15) is derived as

$$\frac{\mathrm{dE}[\xi]}{\mathrm{d}t} = \mathrm{E}\left[\frac{\partial \xi}{\partial t}\right] + \sum_{i}^{3} \mathrm{E}\left[\frac{\partial \xi}{\partial Y_i} g_i\right] + \frac{1}{2}\sum_{i,j}^{3} \mathrm{E}\left[h_i h_j \frac{\partial^2 \xi}{\partial Y_i \partial Y_j}\right] \quad (16)$$

in which g_i is the i-th element of the vector $\mathbf{g}(\mathbf{Y}(\mathrm{t}), t)$, and h_j is the j-th element of the vector $\mathbf{h}(\mathbf{Y}(\mathrm{t}), t)$. It is noted that $\mathrm{E}[\partial\xi/\partial t]$ is zero if ξ does not depend on t explicitly, and $\mathrm{dE}[\xi]/\mathrm{d}t$ is zero if $\mathbf{Y}$ is a stationary process [23, 26].

Now let ju be the moments function of $\mathbf{Y}(\mathrm{t})$

$$\xi(\mathbf{Y}(\mathrm{t})) = q_1^a \dot{q}_1^{\,b} Z^f \quad (17)$$

in which the superscripts a, b and f are the non-negative integer power indices. Substituting Eq. (17) into Eq. (16) and noting that $\mathrm{E}[\partial\xi/\partial t] = 0$, Eq. (16) can be written as

$$\begin{aligned} \dot{\mathrm{m}}(a,b,f) &= a \times \mathrm{m}(a-1,b+1,f) - \omega_1^2 b \times \mathrm{m}(a+1,b-1,f) \\ &\quad - \left(2\xi_1\omega_1 b + M_1^{-1}\rho C_D D_y \gamma_1 \bar{U}_h b + \alpha f\right) \times \mathrm{m}(a,b,f) \\ &\quad + M_1^{-1}\rho C_D D_y \bar{U}_h h \Lambda_{1u}\beta b \times \mathrm{m}(a,b-1,f+1) \\ &\quad + \sigma^2 \alpha f(f-1) \times \mathrm{m}(a,b,f-2) \end{aligned} \quad (18)$$

in which $\mathrm{m}(a,b,f) = \mathrm{E}\left[q_1^a \dot{q}_1^{\,b} Z^f\right]$ denotes the statistical moments of the state vector $\mathbf{Y}$, and

$\dot{\mathrm{m}}(a,b,f)$ is the derivative of $\mathrm{m}(a,b,f)$ with respect to t. In particular, the moments are zero if any power indexes, i.e., a, b or f, takes on negative values.

In fact, the above formula is the basis of the state augmentation method (SAM) [23]. It is seen that the statistical moments $\mathrm{m}(a,b,f)$ satisfy the first-order ordinary differential equations reported in Eq. (18), and for each order $s = a + b + f$, moments can be directly obtained by solving them with prescribed initial conditions, without going through intermediate step, e.g., the response spectra. Note that before evaluating the moments for the order s, the calculation of the moments for the previous order $s' < s$ is required in advance, e.g., $\mathrm{m}(1,0,2)$ depends on $\mathrm{m}(1,0,0)$. There is however one exception: the second moments of the response can be obtained without solving $s = 1$ but only using the first-order differential equations system

$$\dot{\mathbf{m}} = \mathbf{P}\mathbf{m} + \mathbf{Q} \tag{19}$$

in which $\mathbf{m} = [m(2,0,0) \quad m(0,2,0) \quad m(0,1,1) \quad m(1,0,1) \quad m(1,1,0)]^{\mathrm{T}}$; $\mathbf{P}$ and $\mathbf{Q}$ are given in Appendix. B.

For clarity, the analytical formula derived in this study only involves one mode. When considering multimodal vibrations, the dimension of the moments equation will increase exponentially due to coupling effects, which may lead to complexity in the derivation. However, the presented methodology is not substantially changed if additional degrees of freedom are considered.

4 Validation

To illustrate the reliability and computation efficiency of the state augmentation method (SAM), the proposed method is first applied to calculate the buffeting response of a bridge tower subject to a non-stationary wind consisting of a time-varying mean and a stationary wind fluctuation. Notice that non-stationary fluctuations will be considered in Section 5. The obtained results are validated by comparing them with those obtained using the Monte Carlo method (MCM) and pseudo excitation method (PEM). Table 1 lists the main parameters used to describe the bridge tower, considering only its first mode of vibration. The bridge tower model with these parameters is also used in Section 5.

Table 1. Parameters for the first vertical mode of the bridge tower.

Parameter	Definition	Unit	Value
M_1	Generalized mass of the first mode	$\mathrm{kg} \cdot \mathrm{m}^2$	0.5
ξ_1	The first modal damping ratio	/	0.01
n_1	The first modal frequency	Hz	0.084
h	Height of the bridge tower	m	267.4
D_y	Width of the tower section	m	8
C_D	Drag coefficient at zero attack angle	/	2

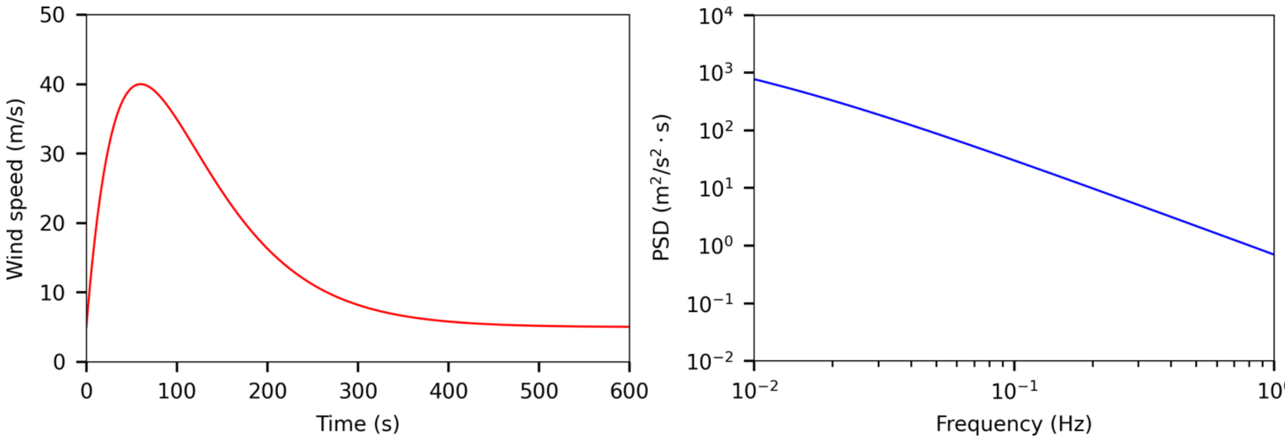
The time-varying wind speed model is based on the method adopted by Chen [13]. The time-varying mean wind speed at $z = h$ is given as a three-parameter function

$$\bar{U}_h(t) = (\bar{U}_{\max} - \bar{U}_{\min})\left(\frac{t}{t_0}\right)e^{1-\frac{t}{t_0}} + \bar{U}_{\min} \tag{20}$$

in which $\overline{U}_{\max}$ and $\overline{U}_{\min}$ are the maximum and the minimum values of the time-varying mean wind speed, and t_0 is the time instant corresponding to $\overline{U}_{\max}$, where $\overline{U}_{\max} = 40\ \mathrm{m/s}$, $\overline{U}_{\min} = 5\ \mathrm{m/s}$ and $t_0 = 600\ \mathrm{s}$ are used in this section. For sake of simplicity, the wind profile $\Delta_{\overline{U}}(z)$ with respect to the tower top is described by power law with the exponent 0.12. Note that different wind profiles do not substantially change the calculation of the buffeting response since this term is interpreted only as a coefficient γ_1, which is reported in [13]. The PSD of the stationary along-wind fluctuating component at $z = 0.6h$, i.e., $u_{0.6h}^{s}(t)$, is expressed as the form of Simiu's spectrum

$$S_{u_{0.6h}^{s}}(n) = \frac{800u_*^2}{(1+200n)^{5/3}} \tag{21}$$

in which $u_* = 2.45\ \mathrm{m/s}$ is the friction velocity corresponding to the turbulence intensity $I_u = \sigma_u/\overline{U}_{\max} = 15\%$, and n is the frequency in Hertz. Fig. 3 (a) and (b) show the time-varying mean wind speed and the Simiu's spectrum for the stationary wind fluctuations, respectively.



(a) (b)

Fig. 3. Adopted non-stationary wind speed model: (a) time-varying mean wind speed [13]; (b) PSD of the stationary fluctuating wind component based on Simiu's spectrum.

To apply the proposed state augmentation method, the first step is to determine the values of α and σ of the OU process so that the corresponding stationary wind fluctuation, i.e., $u_{0.6h}^{s}(t)$, can be approximated by the OU process, i.e., $Z(t)$. Note that the total buffeting response is dominated by the background and resonant components, due to the narrow-band characteristic of the mechanical transfer function around the system natural frequency. Therefore, when finding the values of α and σ, the variance σ^2 of $Z(t)$ is supposed to be the same as that of $u_{0.6h}^{s}(t)$, and then α could be found through minimizing the difference between $S_{u_{0.6h}^{s}}(n)$ and $S_{ZZ}(\omega)$ at the natural frequency of the first vertical mode of the bridge tower. In this case, α and σ are obtained as $\alpha = 0.18$ and $\sigma = 4.13\ \mathrm{m/s}$. The comparison between the Simiu's spectrum according to Eq. (21) and the PSD of the fitted OU process is plotted in Fig. 4.

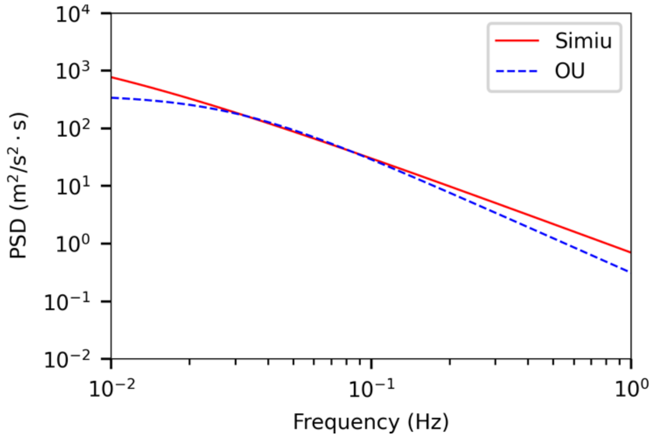


Fig. 4. PSD Comparison between the stationary fluctuating wind component $u^s_{0.6h}(t)$ and the fitted OU process $Z(t)$.

Both the stationary wind fluctuation samples for the MCM and the pseudo excitations for the PEM are generated from the Simiu's spectrum given by Eq. (21), with the lower cut-off frequency at 0.01 Hz and the upper one at 0.2 Hz which is sufficient to cover the dominant frequencies of the system response. For comparison, 50 and 100 frequency points are selected respectively for generating the pseudo excitations. Considering the time-dependent characteristics of the dynamic equation, a fourth order Runge-Kutta method with an error estimator of fifth order is used to calculate the time-history responses, with integration step $\Delta t = 0.6$ s. The response statistics obtained by the MCM are estimated over 1000 random samples, and that of the PEM are calculated by integrating the response EPSD over 50 and 100 frequency points, respectively.

Figure 5 shows the RMS at each instant of time for the along-wind displacement and velocity at the top of the bridge tower, given by the proposed state augmentation method (SAM), the Monte Carlo method (MCM), and by the pseudo excitation method with 50 frequency points (PEM-50) and 100 frequency points (PEM-100). Meanwhile, the mean response component based on the quasi-static analysis is plotted as black lines. These results are calculated from 0 s to 600 s.

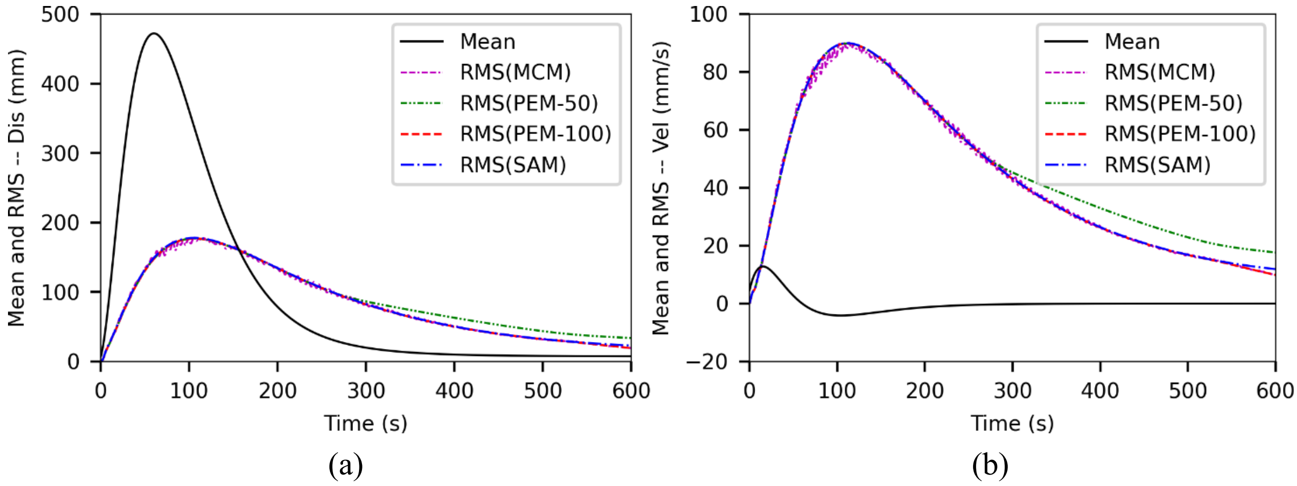


Fig. 5. Time-varying mean and RMS of the along-wind buffeting responses at the top of the bridge tower: (a) displacement; (b) velocity.

As shown in Fig. 5, the magnitudes of the mean and RMS responses vary greatly with time, showing remarkable non-stationary characteristics. It is seen that the results calculated by the SAM, the MCM and the PEM with 100 frequency points are all in very good agreement, which shows the reliability of the proposed method for determining the non-stationary buffeting response. Moreover, the computational efficiency of the proposed method is much higher than that of the MCM and the PEM. In this study, an Intel Core i7 4-Core computer with main frequency 3.4 GHz and 16 GB RAM is used. The computation time cost by the proposed method is about 0.93 s, whereas the time taken by the MCM is about 761 s, and by the PEM with 50 frequency points is about 35 s and with 100 frequency points is about 70 s (without parallel computing). Note that, according to [20], the computation effort of the PEM is mainly taken in calculating the pseudo responses, and the total time is approximately proportional to the number of frequencies. In contrast, when using the SAM, the statistical moments are directly obtained by solving the moments equation once only.

5 Case study

5.1 Non-stationary wind field

In this section, the proposed state augmentation method is employed to investigate the along-wind buffeting responses of the bridge tower subjected to a more complex non-stationary wind, that consists of a time-varying mean and a uniformly modulated non-stationary fluctuation. This non-stationary wind speed model is obtained from a wind record measured in a mountainous area in China, of which the time history is shown in Fig. 6, with a sampling frequency of 10 Hz.

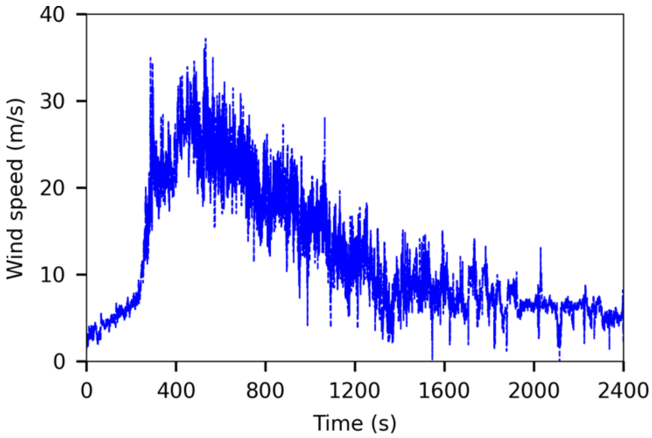
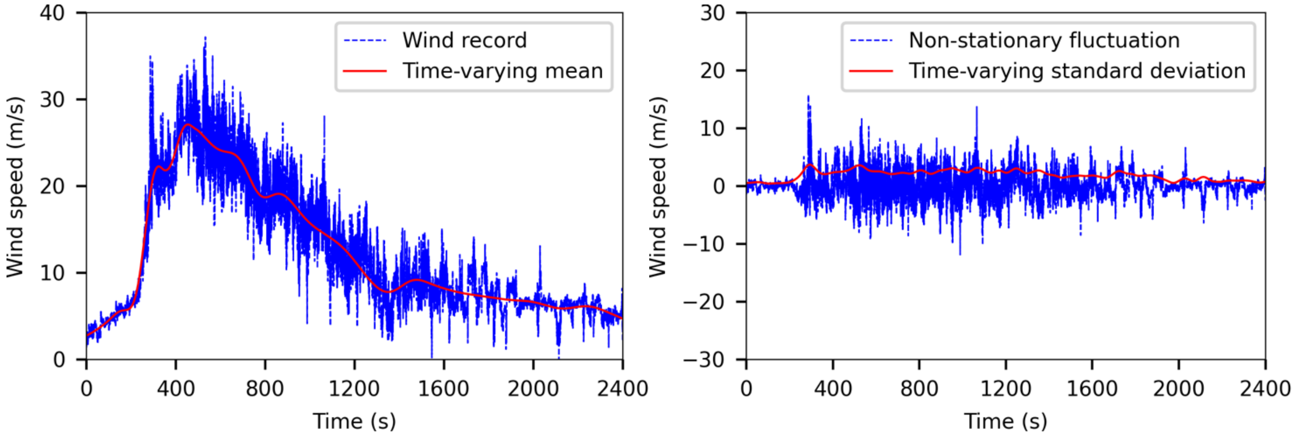


Fig. 6. Time history of the measured non-stationary wind speed.

The time-varying mean wind speed, i.e., $\overline{U}(t)$, and the zero-mean non-stationary wind fluctuation, i.e., $u(t)$, are derived by applying the discrete wavelet transform (DWT) technique [1, 35], in which the analyzing wavelet is chosen as an Daubechies wavelet of order 20 (db20) with a level 10 of decomposition (equivalent to a window size of 102.4 s). Fig. 7 (a) and (b) show the time-varying mean wind speed and time histories of the zero-mean non-stationary wind fluctuating component, respectively. The upper cut-off frequency of the time-varying mean wind speed is around 0.005 Hz, which is much less than the fundamental frequency of the bridge tower (0.084 Hz) and larger than the frequency of sudden pulse period of wind record (0.0038 Hz), so that the quasi-static analysis can be employed to calculate the corresponding mean wind response component. The solid red line in Fig. 7 (b) is the time-varying standard deviation of the non-stationary fluctuation, $\sigma_u(t)$, for which the estimate will be elaborated in the following.



(a) (b)

Fig. 7. Non-stationary wind speed model obtained from the field measurement in a mountainous area in China: (a) time-varying mean wind speed; (b) zero-mean non-stationary fluctuating wind component.

The uniformly modulated model is adopted here to describe the non-stationary fluctuating component, in which the time-modulation function $\beta(t)$ is represented by a normalized time-varying standard deviation, i.e., $\sigma_u(t)/\sigma_{u,\max}$, where $\sigma_u(t)$ is the standard deviation of the non-stationary fluctuation. $\sigma_u(t)$ can be obtained by using the kernel regression method based on the Nadaraya–Watson estimator [1, 36]

$$\sigma_u^2(t) = \frac{\sum_{i=1}^{N}[u(t_i)]^2 K_\delta(t-t_i)}{\sum_{i=1}^{N} K_\delta(t-t_i)} \tag{22}$$

in which $K_\delta(x) = \frac{1}{\sqrt{2\pi}\delta}\exp\left(-\frac{x^2}{2\delta^2}\right)$ is a scaled Gaussian kernel function with a bandwidth δ, and N is the number of data points of the wind speed time history. The estimated standard deviation $\sigma_u(t)$ is plotted in Fig. 7 (b) as the solid red curve. Meanwhile, the corresponding stationary wind fluctuation can be obtained by dividing the non-stationary wind fluctuating component $u(t)$ with the normalized time-varying standard deviation

$$u^s(t) = \frac{u(t)}{\beta(t)} \tag{23}$$

The PSD of $u^s(t)$ is obtained by using Welch's method with overlapping time segments, resulting in a frequency resolution of 0.0024 Hz. Considering that the PSD of the derived stationary wind fluctuation $u^s(t)$ may not follow the widely used wind spectra, therefore, the PSD estimate is fitted as a general wind spectrum formula form [1, 37]

$$S_{u^s_{0.6h}}(n) = \frac{6u_*^2 A n^{d_3}}{(1+Bn^{d_1})^{d_2}} \tag{24}$$

in which $u_* = 1.464\ \mathrm{m/s}$ is corresponding to the variance of the stationary wind fluctuations. A, B, d_1, d_2, d_3 are the parameters to be determined, and n is the frequency in Hertz. The parameters in Eq. (24) can be estimated through nonlinear fitting and are given as $A = 14.91$, $B = 20.64$, $d_1 = 1.041$, $d_2 = 1.714$, $d_3 \approx 0$.

The time history of the derived stationary wind fluctuations is shown in Fig. 8 (a), and the fitted general wind spectrum together with the PSD estimated from $u^s(t)$ are shown in Fig. 8 (b). It is seen that the slope of the spectrum in the high-frequency region, i.e., $d_3 - d_1 d_2 = -1.784$, falls in the vicinity of Kolmogorov's slope $-5/3$, which complies with the analysis in [1].

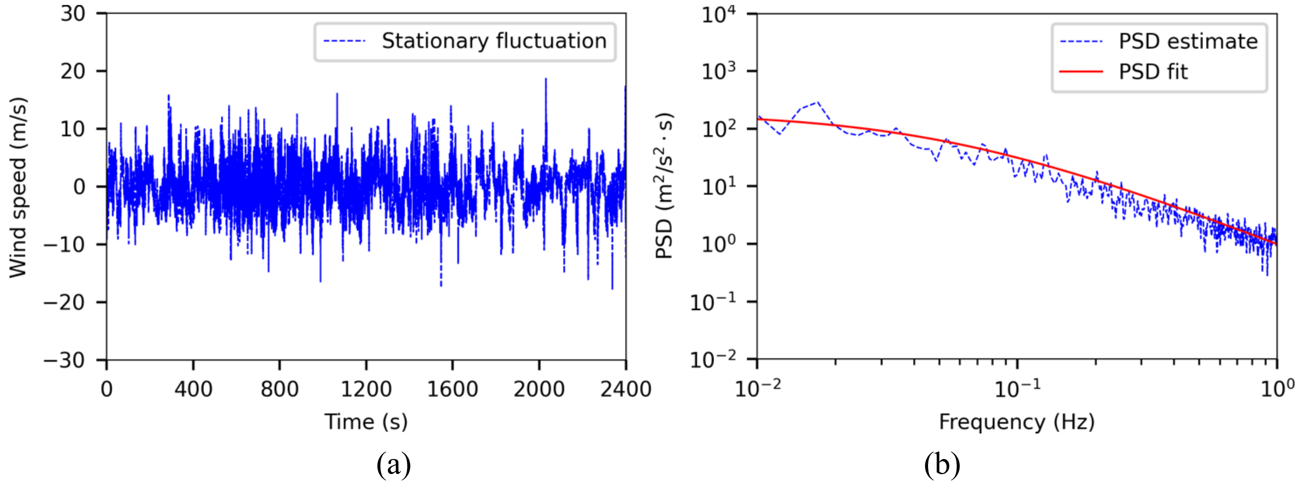


Fig. 8. The derived stationary wind fluctuation based on the uniformly modulated model with the time-modulation function following the normalized time-varying standard deviation: (a) time history; (b) fitted general wind spectrum and PSD estimate.

As performed in the previous case, α and σ shall be calibrated to match the expected wind spectrum, and in this case, their values are found to be $\alpha = 0.36$ and $\sigma = 3.29\ \mathrm{m/s}$. The comparison between the fitted general wind spectrum according to Eq. (24) and the PSD of the fitted OU process is plotted in Fig. 9.

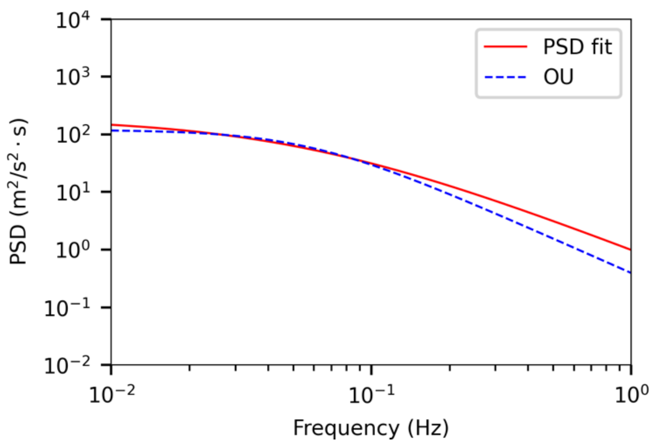


Fig. 9. PSD Comparison between the derived stationary wind fluctuations $u^s_{0.6h}(t)$ and fitted OU process $Z(t)$.

5.2 Comparison between proposed method with time-frequency method

Figure 10 shows the time-varying RMS for the along-wind displacement response at the top of the bridge tower obtained by the proposed state augmentation method (SAM), and the mean response component based on the quasi-static analysis plotted as the black line. To demonstrate the advantages of the proposed method in investigating non-stationary buffeting, the conventional time-frequency analysis method is adopted as a comparison by assuming the wind speed as stationary within a short moving window, and the frequency domain method (FDM) is employed to obtain the stationary response within each window. The results are calculated from 0 s to 2400 s.

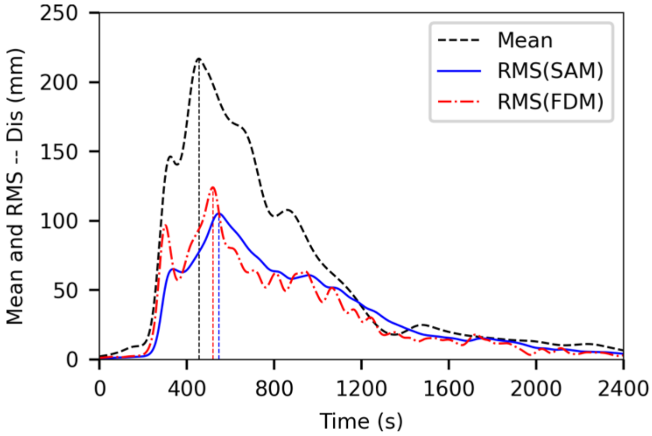


Fig. 10. Time-varying mean and RMS of the along-wind displacement response at the top of the bridge tower obtained by the proposed state augmentation method (SAM) and the conventional time-frequency analysis method, i.e., frequency domain method (FDM).

It is seen that the maximum values of the RMS obtained by the proposed SAM and conventional FDM are both found to occur delayed in time than that of the mean response, i.e., occurrence of the highest mean wind speed. Besides, there are large discrepancies between the two RMS results during $0-800$ s. Especially, the maximum RMS value of the conventional time-frequency analysis occurs slightly earlier, and also much higher than that of the proposed method. These facts have also been reported by several papers [12, 13] and attributed to the lack of "build-up" time for the non-stationary response to reach a stationary state.

Note that, in essence, the conventional time-frequency analysis method regards the system as time-invariant and the excitations as stationary within a moving short window [16], and assumes that the transient response will rapidly attenuate to zero and the response will rapidly reach a stationary state within each window. This assumption, however, is only applicable to the system and the excitations with slowly time-varying characteristics. For a transient extreme wind event, the wind non-stationary characteristics may change rapidly, so there is not enough time for the structural vibration to reach a stationary state. This may lead to errors in the calculation of the non-stationary responses when using the conventional time-frequency domain method, as shown in Fig. 10 during $0-800$ s.

In contrast, unlike the conventional time-frequency analysis, there is no tending stationary state hypothesis for non-stationary vibrations when using the proposed state augmentation method, which allows the mean wind speed to be fast time-varying and, more importantly, allows abrupt variations of the wind speed to occur. In this case, the structure might present a sluggish reaction to the excitation, i.e., the time delay and a smaller amplitude as shown in Fig. 10. Obviously, this difference may be reduced when the system has a larger damping or higher natural frequency, which imply a shorter "build-up" time. It can be proved that the time delay mainly depends on the value of the product $\xi_1\omega_1$ [13], which represents the decay rate to reduce the transient output to zero by the coefficient $e^{-2\xi_1\omega_1 t}$.

5.3 Effects of aerodynamic damping on buffeting

The aerodynamic damping term is usually considered as time-invariant and incorporated in the structural damping. When dealing with the non-stationary wind events, the aerodynamic damping will vary with time due to the time-dependent characteristics of the mean wind speed, and the system will be time-varying if taking this term into account. This leads to difficulties in the calculation by using the conventional buffeting analysis method [5, 15]. As already discussed, with the help of the proposed state augmentation method, it is possible to consider the time-dependent aerodynamic damping term and study the non-stationary vibration of the linear time-varying system subjected to non-stationary excitations.

To illustrate the effect of the aerodynamic damping term on non-stationary buffeting vibrations, Fig. 11 shows the RMS result of the along-wind displacement at the top of the bridge tower for the case when the aerodynamic damping term, i.e., motion-induced force in Eq. (5), is not considered (referred to as "Case 2"), together with the previous result considering the aerodynamic damping term referred to as "Case 1" correspondingly, both obtained by the proposed augmentation method. The investigation is based on the parameters of Table 1.

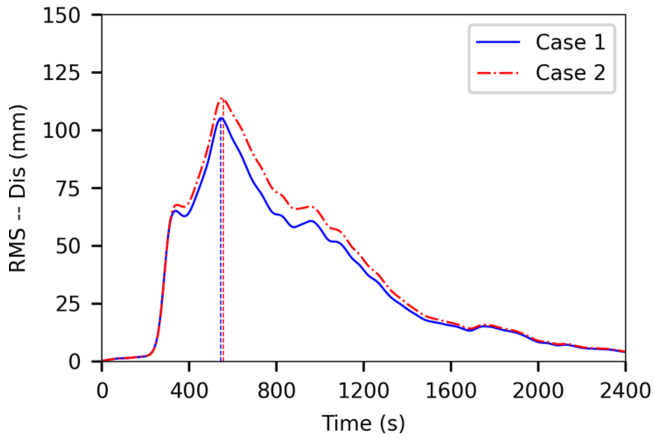


Fig. 11. Time-varying RMS responses of the along-wind displacement at the top of the bridge tower for cases whether the aerodynamic damping term is considered or not, obtained by the proposed state augmentation method.

As can be seen from Fig. 11, the RMS result of "Case 2" is higher than that of "Case 1", nearly 8% higher at the peak value. This is reasonable because the total damping is smaller when the aerodynamic damping term is not considered, resulting in a larger response of the structure. Moreover, the maximum RMS value of "Case 2" is observed to occur slightly later than that of "Case 1". This is because the smaller system damping will preclude the structure from reaching a stationary response in a shorter "build-up" time, as discussed in Section 5.2.

6 Conclusion

This study has investigated the non-stationary buffeting of a bridge tower subjected to non-stationary wind loads. The strip and quasi-steady assumptions are adopted to formulate the buffeting forces and taking the motion-induced force into account. Due to the time-varying mean wind speed, the dynamic equation of the along-wind buffeting presents time-dependent characteristics, and thus the responses are non-stationary even if the wind fluctuating components are stationary. Based on the stochastic differential equation theory and Itô's lemma, a state augmentation method has been presented to calculate the statistical moments of the non-stationary buffeting response. This method involves three steps. First, the Ornstein–Uhlenbeck process is used to approximate the time sequence of non-stationary wind fluctuations. Second, the augmented states of the system and the excitation are written as an Itô-type stochastic differential equation. Third, based on Itô's lemma, the moments equation of the response is derived, expressed as a first-order ordinary differential equation system. The proposed state augmentation method is validated by comparisons with the Monte Carlo method and the pseudo excitation method. In the well-accepted methods, much computation effort is needed due to the non-stationary characteristics of the response, whereas the proposed method is far more efficient.

As expected, the results show that the magnitudes of the mean and RMS responses vary a lot with time, presenting similar trends as the one found in the time-varying mean wind speed. However, the maximum RMS value occurs slightly delayed in time than that of the highest mean wind speed, rather than simultaneously. Unlike the conventional time-frequency analysis, there is no tending stationary state hypothesis in the proposed state augmentation method, so that the structure presents a sluggish reaction to the excitation. Additionally, it is found that the aerodynamic damping plays a significant role in non-stationary buffeting vibrations, resulting in lower structural responses and rapidly reaching a stationary state.

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Appendix A. Variables in Itô-type Equation

$$\mathbf{Y}(\mathrm{t}) = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ Z \end{bmatrix} \tag{A1}$$

$$\mathbf{g}(\mathbf{Y}(\mathrm{t}), t) = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_1^2 & -2\xi_1\omega_1 - M_1^{-1}\rho C_D D_y \gamma_1 \bar{U}_h & M_1^{-1}\rho C_D D_y \bar{U}_h h \Lambda_{1u}\beta \\ 0 & 0 & -\alpha \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \\ Z \end{bmatrix} \tag{A2}$$

$$\mathbf{h}(\mathbf{Y}(\mathrm{t}), t)=\begin{bmatrix} 0 \\ 0 \\ \sigma\sqrt{2\alpha} \end{bmatrix} \tag{A3}$$

Appendix B. Coefficients of Eq. (19)

$$\mathbf{P}=\begin{bmatrix} 0 & 0 & 0 & 0 & 2 \\ 0 & -2p_1 & 2p_2 & 0 & -2\omega_1^2 \\ 0 & 0 & -\alpha-p_1 & -\omega_1^2 & 0 \\ 0 & 0 & 1 & -\alpha & 0 \\ -\omega_1^2 & 1 & 0 & p_2 & -p_1 \end{bmatrix} \tag{B1}$$

$$\mathbf{Q}=\begin{bmatrix} 0 \\ 0 \\ p_2\sigma^2 \\ 0 \\ 0 \end{bmatrix} \tag{B2}$$

where $p_1=2\xi_1\omega_1+M_1^{-1}\rho C_D D_y \gamma_1 \bar{U}_h$ and $p_2=M_1^{-1}\rho C_D D_y \bar{U}_h h \Lambda_{1u}\beta$.

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