

Arclength Control-Based Continuation of Systems Featuring Internally-Resonant Modes

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Abstract

Arclength control-based continuation (ACBC) enables the experimental identification of frequency response curves (FRC) in nonlinear systems, including their unstable branches. So far, its application has been limited to systems exhibiting isolated nonlinear modes. This work extends the ACBC methodology to nonlinear systems featuring internally-resonant modes. Specifically, we consider the case where an internal resonance induces a Neimark–Sacker bifurcation, leading to an unstable branch of periodic responses and quasiperiodic oscillations. We demonstrate that ACBC can stabilize the unstable branch through two examples, namely a system of two coupled Duffing oscillators exhibiting a 1:1 internal resonance and a nonlinear cantilever beam exhibiting a 3:1 internal resonance between its first two modes.

Introduction

Bifurcation diagrams are a central tool in nonlinear dynamics, but their experimental construction is challenging due to unstable branches in open-loop sweeps. Control-based continuation methods address this by combining feedback control with parameter continuation. CBC stabilizes and traces branches that before were unstable [1, 2]. Among the derivative-free variants, arclength control-based continuation traces an FRC at a target forcing amplitude by stepping along an arc in the reference (amplitude, frequency) input plane, which lets the procedure easily navigate through fold bifurcations. Up until now, ACBC has been applied around single, isolated nonlinear resonant modes. This work extends it to systems featuring internally-resonant modes, where the frequency (or one of its harmonics) of one mode approaches another modal frequency. The internal resonance induces a Neimark–Sacker (NS) bifurcation: a pair of complex-conjugate Floquet multipliers leaves the unit circle, the response becomes quasi-periodic, and the periodic branch becomes unstable.

Results and Discussion

Here, the stabilization of branches exhibiting NS bifurcations is demonstrated on two benchmark systems with hardening nonlinearities. The first corresponds to a 1:1 internal resonance in the coupled Duffing oscillators shown in Fig. 1a, while the second involves a 3:1 internal resonance between the first and second modes of the nonlinear beam in Fig. 1b, taken from [3]. Their respective FRCs, shown for two forcing levels in Figs. 1c and 1d, display the characteristic right-bending hardening resonance curves with an unstable branch. At higher forcing levels, part of the branch becomes unstable due to a NS bifurcation (denoted by N on the FRC). The ACBC scheme stabilizes these branches using a derivative controller with sufficiently high gain. The identification results are presented both numerically and experimentally.

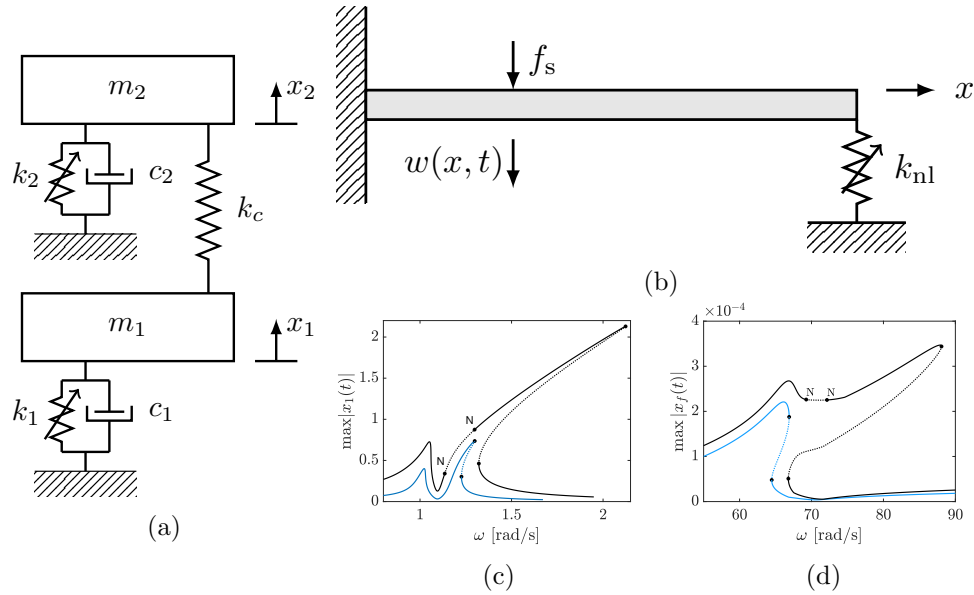


Figure 1: (a,c) Coupled Duffing oscillators. (b,d) Nonlinear beam.

References

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