

An efficient multi-scale methodology for the fatigue damage assessment of suspended cables under turbulent wind excitation

Stefano Corazza¹, Francesco Foti¹, Vincent Denoël², Luca Martinelli¹

¹ *Department of Civil and Environmental Engineering, Politecnico di Milano, Italy*

² *Structural and Stochastic Dynamics Division, University of Liège, Belgium*

stefano.corazza@polimi.it, francesco.foti@polimi.it, v.denoel@uliege.be, luca.martinelli@polimi.it

Abstract— The paper presents a computationally efficient multi-scale methodology to address the fatigue life of suspended stranded cables under turbulent wind excitation. The cable is modeled as taut-string and the out-of-plane global dynamic response under buffeting load is obtained via the random vibration theory (macro-scale). Bending stresses are retrieved in a decoupled way by adopting a closed-form perturbation solution for the modal bending curvature within the boundary layers of the cable, leveraging the smallness of the dimensionless bending stiffness parameter (meso-scale). The stress state of each wire is reconstructed by employing a mechanical model of the composite cross-section of the cable (micro-scale). The accumulated fatigue damage of each wire is then computed according to Miner's rule, and a simple expression of the cumulative wires micro-slip is proposed as a potential indicator of susceptibility to fretting fatigue. The influence of some design parameters on both indicators is addressed and the role of the nonlinear moment-curvature relationship on the short-term damage is highlighted.

Keywords— *Stranded cables, Bending stiffness, Fatigue damage, Buffeting, Boundary layers, Micro-slip*

1 Introduction

Metallic cables are key components of large and strategic infrastructures such as suspension or cable-stayed bridges. They are also widely employed in special applications including overhead electrical power lines, cable-roof systems and slender footbridges, thanks to their high strength-to-weight ratio.

However, due to their flexibility and the very low structural damping, metallic cables are largely susceptible to vibrations, mainly induced by traffic and by wind excitation (both of aerodynamic and aeroelastic nature).

As a consequence, fatigue damage induced by oscillations encountered during their service life could become a critical concern, and may ultimately lead to structural failure [1,2].

This paper presents an efficient multi-scale approach which integrates the dynamic analysis of slender structural components with their fatigue life predictions. The proposed methodology is modular and can be adapted to a wide range of structural applications and loading conditions, including stay or suspension cables, Overhead electrical line (OHL) conductors and offshore engineering components such as Tension Leg Platform (TLP) tethers or taut mooring lines for Offshore Wind Turbines (OWT). The proposed approach is here demonstrated considering the case of a suspended cable subject to a buffeting-type excitation, modeled according to the Quasi-Steady aerodynamic Theory (QST) [3]. To keep the presentation as simple as possible, the global (macro-scale) dynamic response of the cable under wind loading is analyzed focusing only on its out-of-plane component and adopting the simple yet effective taut-string model. However, generalization to three dimensional motion and extension to the more refined small-sag cable model [4] is straightforward.

Whenever dealing with slender and flexible cables, bending effects are typically significant only in the regions close to the end sections, where local disturbances arise due to the presence of flexural restraints, hinting the existence of boundary layers. Accordingly, the bending response of the cable is here retrieved in a decoupled manner, as a linear combination of the dynamic response obtained with the global model, under the assumption of perfect flexibility. To this aim, a closed-form perturbation solution of the cable's mode shapes, previously developed by the authors, is adopted. Such expression synthetically accounts for both the degree-of-fixity of the end flexural restraints and the

bending stiffness of the cable, allowing for the reconstruction of the bending curvature within the boundary layers (meso-scale response).

At the micro-scale, the third level of the proposed approach traces back the response of the individual wires of the cable, by adopting the mechanical model formulated in [5]. The stress state in each wire is efficiently reconstructed during the post-processing phase, by solving the cross-sectional response of the cable imposing the curvature time-history previously determined at the meso-scale. The fatigue damage of each wire is then computed according to Miner's rule of damage accumulation, allowing for a quick estimate of the residual life time of the structure.

As it has been recognized in the literature [1,2], however, micro-slip of the outermost wires of the strand play a crucial role in the fatigue performances of suspended cables, therefore limiting the validity of standard high cycle fatigue criteria based on damage accumulation at the material level. In the absence of a unified criterion capable of accounting both effects simultaneously, the cumulative micro-slip experienced by the different wires during their lifetime is therefore introduced in this work as a potential indicator of susceptibility to fretting fatigue, and compared to the results of Miner's damage.

2 Dynamic Response

2.1 Global response (macro-scale)

Let us consider a suspended stranded cable characterized by span length L , diameter D , pretension T_0 , Rated Tensile Strength RTS, and linear mass γ . According to the taut-string model, out-of-plane damped vibrations of the cable are described by the following dimensionless equation:

$$-\frac{\partial^2 \bar{v}}{\partial \xi^2} + \frac{\partial^2 \bar{v}}{\partial \tau^2} + \bar{c}_v \frac{\partial \bar{v}}{\partial \tau} = \bar{f}_D \quad (1)$$

where $\bar{v}(\xi, \tau) = v(x, t)/L$ is the dimensionless out-of-plane displacement, $\xi = x/L$ and $\tau = \omega_c t$ are, respectively a dimensionless space and time variable, $\bar{c}_v$ is a dimensionless viscous damping coefficient and ω_c is the characteristic circular frequency: $\omega_c = \frac{1}{L} \sqrt{\frac{T_0}{\gamma}}$.

The dimensionless linearized drag force $\bar{f}_D$ can be defined as the superposition of a dynamic excitation term in the fluctuating part of the wind velocity and a dissipative term in the structural velocity, according to the standard QST:

$$\bar{f}_D = \frac{\varrho L^2 C_D D \omega_c U}{T_0} \bar{u}(\xi, \tau) - \frac{\varrho L^2 C_D D \omega_c U}{T_0} \frac{\partial \bar{v}}{\partial \tau} \quad (2)$$

where ϱ is the air density, C_D is the drag coefficient of the cable, U is the mean wind velocity and $\bar{u}$ is the dimensionless fluctuating part of the wind velocity. According to the Eurocode 1 Part 4 [6], the mean wind velocity at the considered site can be expressed as:

$$U(z) = U_b c_0 c_r(z) \quad (3)$$

where U_b is the basic wind velocity, while c_0 and c_r respectively denote the orography factor and the roughness factor, whose values are completely defined by the terrain category and the reference height of the structure z , which is herein conventionally defined as $z = h - \frac{2}{3}d$, where h is the distance of the cable end sections from the ground and d is the midspan sag. The latter is computed according to the classical parabola approximation (see [4]) as $d = \frac{\gamma g L^2}{8 T_0}$, g being the gravity acceleration. For the sake of simplicity, in this work, the effect of the mean wind speed on the static equilibrium configuration, i.e. the swing angle and the tension variation are both neglected.

The undamped eigenvalue problem of the taut string delivers the following orthogonal mode shape functions:

$$\phi_n(\xi) = \sqrt{2} \sin(n\pi\xi) \quad (4)$$

The corresponding dimensionless natural circular frequencies are expressed by $\bar{\omega}_n = n\pi$ and their dimensional counterpart simply reads $\omega_n = \omega_c n\pi$. The natural frequencies are instead given by $f_n = \omega_c n/2$. Solution of Eq. (1) is sought in the following form, by employing the separation of variables:

$$\bar{v}(\xi, \tau) = \sum_{n=1}^{N_M} \bar{q}_n(\tau) \phi_n(\xi) \quad (5)$$

where $\bar{q}_n(\tau)$ are the dimensionless time-varying principal coordinates of the system and N_M is a truncation mode number. Substitution of Eq. (5) into Eq. (1), projection onto the linear modal basis and integration over $\xi \in [0,1]$ delivers the following uncoupled system of second-order ordinary differential equations in the modal coordinates:

$$\ddot{\bar{q}}_n + 2 \bar{\omega}_n (\xi_n^s + \xi_n^a) \dot{\bar{q}}_n + \bar{\omega}_n^2 \bar{q}_n = K \int_0^1 \bar{u}(\xi, \tau) \phi_n(\xi) d\xi \quad n = 1, \dots, N_M \quad (6)$$

where the dot symbol has been used to indicate the derivative with respect to the dimensionless time variable and ξ_n^s is the n -th modal structural viscous damping coefficient of the system. Moreover:

$$K = \frac{\rho L^2 C_D D \omega_c U}{T_0} \quad (7)$$

and the following frequency-dependent aerodynamic modal viscous damping coefficient has been defined:

$$\xi_n^a = \frac{C_D}{4\pi} \frac{\rho D^2}{\gamma} \frac{U}{D f_n} \quad (8)$$

Due to the linearity of both the structural system and the loading condition, Eqs. (6) can be conveniently solved in the frequency domain. To this aim, let us define the n -th dimensionless Frequency Response Function (FRF) of the system:

$$\bar{H}_n(\bar{\omega}) = [-\bar{\omega}^2 + 2i\bar{\omega}\bar{\omega}_n(\xi_n^s + \xi_n^a) + \bar{\omega}_n^2]^{-1} \quad (9)$$

where i denote the imaginary unit and $\bar{\omega}$ is the dimensionless circular frequency. The entries of the dimensionless Cross Power Spectral Density (CPSD) matrix of the buffeting load are easily expressed as:

$$\bar{S}_{\bar{f}_{D,nm}}(\bar{\omega}) = K^2 \rho_{nm}(\bar{\omega}) \bar{G}_{\bar{u}}(\bar{\omega}) \quad (10)$$

where the frequency-dependent correlation coefficient $\rho_{nm}(\bar{\omega})$ has been defined as:

$$\rho_{nm}(\bar{\omega}) = \iint_0^1 \text{Coh}(\xi_1, \xi_2, \bar{\omega}) \phi_n(\xi_1) \phi_m(\xi_2) d\xi_1 d\xi_2 \quad (11)$$

and $\text{Coh}(\xi_1, \xi_2, \bar{\omega})$ is the coherence function of the turbulence, which is here expressed according to Davenport's exponential function model:

$$\text{Coh}(\xi_1, \xi_2, \bar{\omega}) = \exp\left[-\frac{|\xi_1 - \xi_2|}{\beta}\right] \quad \beta = \frac{2\pi U}{\tilde{C} L \bar{\omega} \omega_c} \quad (12)$$

where $\tilde{C}$ is the decay factor, which is set equal to 10, as recommended in [7]. For the special of sinusoidal modal shape functions, Eq. (11) admits a simple closed-form solution, which is here omitted for the sake of conciseness.

The wind turbulence is modeled as a stationary Gaussian random process, characterized by the uni-lateral PSD:

$$\bar{G}_{\bar{u}}(\bar{\omega}) = \frac{1}{2\pi} \frac{S_u(f)}{\omega_c L^2} \quad (13)$$

$S_u(f)$ being specified by the Italian regulation CNR-DT 207 [7,8] as it follows:

$$S_u(f) = \frac{6.868 \sigma_u^2(z) L_u(z)/U(z)}{[1 + 10.302 f L_u(z)/U(z)]^{5/3}} \quad (14)$$

where $L_u(z)$ is the alongwind integral length scale, fully defined by the terrain category and the reference height, while $\sigma_u^2(z)$ is the variance of the alongwind turbulence, which is related to the turbulence intensity $I_u(z)$ as it follows:

$$I_u(z) = \frac{1}{c_0 \log(z/z_0)} \quad \sigma_u(z) = I_u(z)U(z) \quad (15)$$

z_0 being the roughness length. The CPSD matrix of the modal coordinates finally read:

$$\bar{S}_{\bar{q}_n \bar{q}_m}(\bar{\omega}) = K^2 \rho_{nm}(\bar{\omega}) \bar{H}_n(\bar{\omega}) \bar{G}_{\bar{u}}(\bar{\omega}) \bar{H}_m^*(\bar{\omega}) \quad (16)$$

where the symbol $*$ has been used to indicate the complex conjugate operator.

2.2 Solution in the bending boundary layers (meso-scale)

Leveraging the smallness of the dimensionless bending stiffness, it is possible to approximately decouple the local bending effects from the global dynamic response of the system. The bending response of the cable is hence retrieved as a linear combination of the dynamic response obtained with the global model, under the assumption of perfect flexibility. To this aim, basing on the work originally presented by the authors in [9], the following asymptotic expression of the n -th dimensionless modal curvature is adopted:

$$\bar{\kappa}_n(\xi) = \frac{n\pi}{\epsilon} [\rho_0 e^{-\xi/\epsilon} - \rho_1 (-1)^n e^{-(1+\xi)/\epsilon}] - \epsilon (n\pi)^3 [(-1 + \xi)\rho_0 + \xi\rho_1] \cos(n\pi\xi) - \sin(n\pi\xi) (n\pi)^2 [1 + \epsilon(\rho_0 + \rho_1)] \quad (17)$$

where $\epsilon = \sqrt{\frac{EI}{T_0 L^2}} \ll 1$ is the dimensionless bending stiffness of the cable. It is worth noting that ϵ also provides a measure of the extent of the cable boundary layers. The coefficients ρ_0 and ρ_1 are the dimensionless degree-of-fixity parameters, associated to the boundary conditions in $\xi = 0$ and $\xi = 1$, respectively, which are defined as:

$$\rho_i = \frac{k_i}{T_0 L \epsilon + k_i} \quad i = 0, 1 \quad (18)$$

where k_i is the rotational stiffness of the i -th restraint. If $\rho_0 = \rho_1 = 0$, the doubly hinged configuration is retrieved, while if $\rho_0 = \rho_1 = 1$, the doubly clamped configuration is obtained. Once the global dynamics of the system has been solved, the knowledge of the modal response is used to compute the dimensionless PSD of the strand bending curvature at the generic dimensionless abscissa ξ under the associated wind buffeting load as:

$$\bar{S}_{\bar{\chi}_s}(\bar{\omega}; \xi) = \sum_{n=1}^{N_M} \sum_{m=1}^{N_M} \bar{\kappa}_n(\xi) \text{Re}[\bar{S}_{\bar{q}_n \bar{q}_m}(\bar{\omega})] \bar{\kappa}_m(\xi) \quad (19)$$

It is worth noticing that, in the field of transmission line engineering, fatigue checks are usually carried out at a conventional distance corresponding to $x_p = 89$ mm from the supporting clamp edge [1,2], i.e. at $\bar{\xi} = x_p/L$.

2.3 Cross-sectional response of the cable (micro-scale)

Let us make reference to a generic cable cross-section and denote with the symbols $n_j, R_j, \alpha_j, \nu_j, r_{w,j}, E_{w,j}, A_{w,j}, I_{w,j}$ the number of wires, the helix radius, the lay angle, the Poisson's coefficient, the radius, the Young's modulus, the area and the moment of inertia of the wires belonging to the j -th layer of the strand, where $j = 1, \dots, N_L$, starting from the innermost layer of wires which surrounds the core.

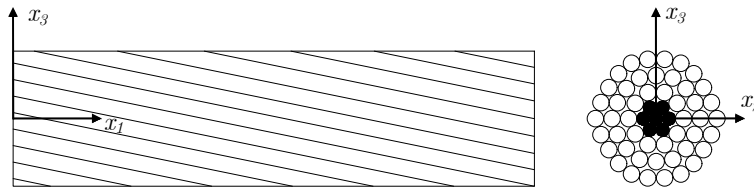


Fig. 1. ACSR 48/7 Bersfort: side view and cross-section.

Let us further denote with θ the swept angle, i.e. the angle which the projection of the position vector defines with the direction of the axis x_3 , measured in the clockwise direction (see Fig. 1).

The cross-sectional response of the strand under a combination of axial force and bending in the plane (x_1, x_3) is described according to the model proposed in [5]. A two-step approach is adopted, and the bending response is superimposed to the effect of the axial force. Within this context, the total strain (ε) of a generic point of a wire cross-section is expressed as $\varepsilon = \varepsilon_a + \varepsilon_b$. The first term, ε_a , is due to the axial force and can be calculated as:

$$\varepsilon_a = \frac{T_0 \cos^2(\alpha)}{\sum_{j=1}^{N_L} n_j \cos^3(\alpha_j) EA_{w,j}} = \varepsilon_0 \cos^2(\alpha) \quad (20)$$

where $EA = \sum_{j=1}^{N_L} n_j \cos^3(\alpha_j) EA_{w,j}$ and ε_0 respectively denote the axial stiffness and the axial strain of the cable cross-section. The second term, ε_b , is due to the bending of the cross-section of the strand and its determination requires the solution of a nonlinear problem. Indeed, whenever the strand is bent, an axial force gradient is generated along the length of the wires. Such gradient makes the wires prone to sliding with respect to the neighboring ones and is counteracted by the tangential friction forces acting on the internal contact surfaces between the wires.

The possible activation of these sticking/sliding frictional interfaces makes the bending behavior of strands inherently non-linear and non-holonomic. Two limit cases may be distinguished. Whenever the friction forces are large enough to overcome the axial force gradient along the wire, and all the wires are ideally welded together such that any relative displacement among them is prevented (*full-stick* state). The strand cross section can be modeled as a planar rigid body, leading the bending stiffness to reach its maximum theoretical value [5]:

$$EI_{max} = EI_{min} + \sum_{j=1}^{N_L} \frac{n_j}{2} \cos^3(\alpha_j) EA_{w,j} R_j^2 \quad (21)$$

On the other hand, if the external surfaces of the wires are assumed perfectly lubricated, the strand behaves as a bundle of independent curved thin rods (*full-slip* state). The kinematics of each individual wire is described by means of the planar rigid body assumption, and the minimum theoretical value of the flexural stiffness is attained [5]:

$$EI_{min} = \sum_{j=0}^{N_L} \frac{n_j}{2} \cos(\alpha_j) EI_{w,j} \left(1 + \cos^2(\alpha_j) + \frac{\sin^2(\alpha_j)}{1+\nu_j} \right) \quad (22)$$

On a practical ground, the *full-stick* and *full-slip* limit cases deliver bending stiffness values that can be considered as representative of the strand cross-sectional response in the small and large curvature regime, respectively.

The nonlinear transition between these two limit cases is mainly governed by the value of the radial contact forces that clench the wires together (which is, in turn, linearly proportional to the axial force T_0) and of the internal friction coefficient. In the analysis of the bending response of stranded cables, one of the two limit cases of *full-stick* or *full-slip* is often pragmatically assumed. Focusing on the fiber of the wire cross-section furthest from the plane of bending, the bending strain turns out be linearly related to linear curvature of the strand, $\chi_s(t)$, according to the following expressions [5]:

$$\varepsilon_b(t) = r_w \cos^2(\alpha) \chi_s(t) \quad \text{or} \quad \varepsilon_b(t) = (r_w + R \cos \theta) \cos^2(\alpha) \chi_s(t) \quad (23)$$

for the *full-slip* state and the *full-stick* state, respectively, which translate in the following PSDs:

$$S_{\varepsilon_b}(\omega) = r_w^2 \cos^4(\alpha) S_{\chi_s}(\omega) \quad \text{or} \quad S_{\varepsilon_b}(\omega) = (r_w + R \cos \theta)^2 \cos^4(\alpha) S_{\chi_s}(\omega) \quad (24)$$

where $S_{\chi_s}(\omega) = \bar{S}_{\chi_s}(\bar{\omega}) / (L^2 \omega_c)$. In the more general case of nonlinear bending response of the strand cross-section, the bending strain $\varepsilon_b(t)$ is computed at each time instant through the stick-slip model proposed in [5]. Once the total strain of the wire is known, the corresponding normal stress is simply obtained as $\sigma(t) = E_w(\varepsilon_a + \varepsilon_b(t))$.

3 Fatigue Analyses

3.1 Fatigue damage accumulation

Standard approaches to estimate the fatigue life of complex structural system under random loading conditions rely on time-domain simulations. Once the structural response is known, the rain flow counting method is used to decompose

the stress time signal into an equivalent set of constant amplitude stress reversals. The damage for each cycle is then computed utilizing the S-N curve, which is obtained from fatigue tests of the material under constant amplitude loading. The accumulated damage over the entire time signal is finally obtained according to a certain damage accumulation rule, such as the Palmgren-Miner one, which consider linear damage accumulation over the stress cycles. The generic form of the S-N curve is given by the following base-10 logarithmic law:

$$\log_{10} N = \log_{10} A - m \log_{10} S \quad (25)$$

where N is the number of cycles to failure, S is the stress range, m is the negative slope of S-N curve, and $\log_{10} A$ is the intercept of the S-N curve. The short-term yearly damage of the generic k -th simulation is then computed according to Miner's rule, rearranging Eq. (25), as:

$$d_{y,k} = T_{ref} \sum_{i=1}^{K_s} \frac{n_{i,k}}{N_{i,k}} = \frac{T_{ref}}{A} \sum_{i=1}^{K_s} n_{i,k} S_{i,k}^m \quad (26)$$

where T_{ref} is a reference time period, K_s is the total number of stress cycles, $n_{i,k}$ is the number of stress cycles in the i -th stress block of the k -th simulation (obtained from the rain flow counting method), and $N_{i,k}$ is the number of cycles to failure of the i -th stress block of the k -th simulation. Moreover, $S_{i,k}$ is the i -th stress range of the k -th simulation, which is corrected for the mean stress of the considered cycle by applying the well-known Goodman formula [10]. The long-term yearly damage is then retrieved in a discrete fashion as:

$$D_y = \sum_{k=1}^{N_w} d_{y,k} p_k \quad (27)$$

where N_w is the total number of basic wind velocities considered in the fatigue analysis and p_k is the probability of occurrence of the k -th basic wind velocity, which is computed as $p_k = f_{U_b} \Delta U_b$, where f_{U_b} is the probability density function of the basic wind velocity at the specific site (see Fig. 2(a)) and ΔU_b is the spacing of the mono-dimensional grid adopted for the analysis.

In this work, three different wires of the outermost layer of the strand are selected to perform the fatigue analyses. For what concerns the *full-stick* and *full-slip* limit states, the maximum (ϵ_{max}) and minimum (ϵ_{min}) dimensionless bending stiffness are respectively used to compute the PSD of the strand bending curvature according to Eq. (19) and consequently the PSDs of the wire bending strains for the two limit cases (see Eq. (24)). In both cases, 10-minutes synthetic time histories (corresponding to an analysis duration $t_f = 600$ s and a reference period $T_{ref} = 52650$ s) of the bending strain are then generated from the PSDs, and the stress state in the wires is straightforwardly retrieved.

For what concerns the analysis carried out with the nonlinear model, the generated time history of the dimensional strand curvature computed with ϵ_{min} is directly imparted as an input in the solution algorithm of the nonlinear bending problem of the cable, and the normal stress of the selected wires is directly obtained as an output.

3.2 Wires micro-slip

The importance of fretting and wear on the fatigue performance of stranded cables has long been recognized in the literature, motivating assessment procedures that consistently connect the dynamic response of the cable with the local contact conditions. Basing on the work originally presented in [11], it is possible to show that the micro-slip (u_{MS}) due to the tangential compliance of the radial contact patch of a wire in the outermost layer of a strand subject to uniform bending satisfies the following second order differential equation:

$$\frac{d^2 \bar{u}_{MS}}{d\theta^2} - \Lambda^2 \bar{u}_{MS}(\theta) = -R \cos \alpha \cot \alpha \chi_s \cos \theta \quad (28)$$

where $\bar{u}_{MS} = u_{MS}/R$, and:

$$\Lambda^2 = \frac{R^2}{EA_w \sin^2(\alpha)} \frac{2(1-\nu)}{C_n(f_r)} \quad (29)$$

In the above equation, $C_n(f_r)$ denotes the normal contact compliance of the radial contact patch, which depends upon both the mechanical and geometrical properties of the wires as well as on the radial contact forces per unit of length (f_r) acting on the internal contact helix. In this work, the expression found by Godreau et al. for $C_n(f_r)$ is adopted (see [11]). Eq. (28) can be solved under the following periodic boundary conditions, which translate the cylindrical symmetry and periodicity of the mechanical problem: $\bar{u}_{MS} \left[(2z + 1) \frac{\pi}{2} \right] = 0$, with $z \in \mathbb{N}^+$.

General solution of Eq. (28), hence, reads:

$$\bar{u}_{MS}(\theta, t) = \frac{R \cos \alpha \cot \alpha}{1 + \Lambda^2} \chi_s(t) \cos \theta \quad (30)$$

As a potential indicator of the susceptibility to fretting fatigue, the yearly cumulative dimensionless micro-slip $\bar{u}_{MS}^*(t; \theta)$ is then defined for a generic wire identified by the swept angle θ as:

$$\bar{u}_{y,MS,k}^*(t; \theta) = T_{ref} \int_0^{t_f} \left| \frac{\partial \bar{u}_{MS,k}}{\partial t} \right| dt \quad (31)$$

Finally, the long-term yearly cumulative micro-slip is retrieved as (cf. Eq. (27)):

$$\bar{U}_{y,MS}^* = \sum_{k=1}^{N_w} \bar{u}_{y,MS,k}^* p_k \quad (32)$$

4 Application example

In this section, the proposed multi-scale methodology is applied to a well-documented benchmark overhead electrical line span already studied in [12]. The considered stranded cable is an Aluminium Conductor Steel Reinforced (ACSR) 48/7 Bersfort, which is constituted of a core and an inner layer of 6 steel wires, and by 48 aluminium wires organized in 3 outer layers (see Fig. 1). The cable cross-section is characterized by the following properties: $D = 35.6$ mm, $\gamma = 2.375$ kg/m, $RTS = 180$ kN, $C_D = 1.1$, $EA = 56.1$ MN, $EI_{max} = 3749$ Nm² and $EI_{min} = 60.8$ Nm².

The steel (aluminium) wires of the cable are characterized by the following properties: $E_w = 200$ GPa ($E_w = 69$ GPa), $\nu = 0.3$ ($\nu = 0.3$) and $r_w = 1.66$ mm ($r_w = 2.14$ mm). The internal friction coefficient is assumed to be constant between the layers of wires, and it is assumed equal to 0.5, while the lay angle of the outermost layer of wires is given by $\alpha = 14.11^\circ$. The considered benchmark line has an overall span length $L = 450$ m and it is located at $h = 25$ m from the ground. The conductor is strung at a tension $T_0 = 36$ kN, leading to a dimensionless pretension $\eta_0 = T_0/RTS = 20\%$, and the degree-of fixity parameters are selected as $\rho_0 = \rho_1 = 1$. The air density is set to $\rho = 1.25$ kg/m³, the orography coefficient is selected as $c_0 = 1$, while a category terrain II is considered for the purpose of the present work, which implies $z_0 = 0.05$ m [6,7], leading to $c_r = 1.07$, $I_u = 18\%$, and $L_u = 75.5$ m.

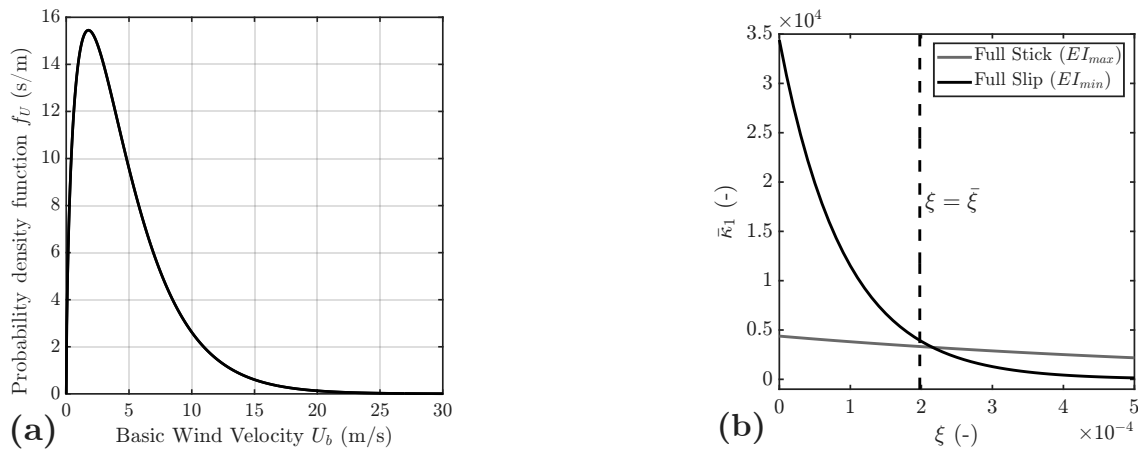


Fig. 2. (a) Probability density function of the basic wind velocity in the city of Palermo [14] (b) Trend of the first dimensionless modal curvature inside the boundary layer obtained under the assumption of *full-stick* and *full-slip* limit cases (see Eq. (17)).

Moreover, the parameters of a S-N curve obtained for the aluminium wires of an ACSR conductor tested in [13] are selected to conduct the fatigue analyses. They correspond to $m = 3.425$ and $\log_{10} A = 12.21$. Additionally, a Weibull distribution for the Probability Density Function (PDF) of the basic wind velocity (f_{U_b}) is adopted. The shape and scale parameters of the Weibull distribution have been fitted from wind measurements in the city of Palermo, in Italy, and the results have been published in [14]. The selected PDF is depicted in Fig. 2(a).

4.1 Short-term damage and cumulative micro-slip

In this section, the fatigue damage simulations are conducted as a function of the basic wind velocity U_b , whose values are selected to be uniformly spaced between 1 and 30 m/s (with $\Delta U_b = 1$ m/s). To limit the seed variability, 10 realizations are performed for each value of basic wind velocity, and the mean of the different results is computed.

Moreover, 20 modes of the cable are retained for the global dynamic response (i.e. $N_M = 20$), after a preliminary convergence analysis. Figs. 3 and 4 show the results of both yearly short-term damage and cumulative slip as a function of the basic wind velocity, computed for the selected wires of the cable cross-section located at the suspension clamp (i.e. at $\xi = 0$, see Fig. 3) and at a distance of 89 mm from the supporting clamp (i.e. $\xi = \bar{\xi}$, see Fig. 4).

While the qualitative behaviour of the cumulative micro-slip is the same for both the considered cable cross-sections (compare Fig. 3(b) and Fig. 4(b)), the different modelling approaches to the bending response of the cable deliver significantly different damage predictions. Fig. 3(a) shows that damage values computed with the different models tend to the ones obtained under the *full-slip* state, with the only exception of the curve computed for wire C under the assumption of *full-stick* state. Fig. 4(a) instead allows to clearly appreciate the damage transition obtained for the different wires with the nonlinear model. In fact, the damage curves computed for both wire A and C tend to be close to their respective *full-stick* predictions at low wind velocities, while progressively bending towards the *full-slip* ones as U_b increases. The reason for the difference in the results attained at different cable cross-sections is deemed to be related to the rapid evolution of the modal bending curvature within the boundary layers, as depicted in Fig. 2(b).

In fact, at the suspension clamp, the values of modal bending curvature associated with the *full-slip* case are much higher than those attained with the *full-stick* case, while being comparable at $\xi = \bar{\xi}$. Moreover, it is worth noting that the micro-slip of wire A is strictly zero, its swept angle being 90° (Eq. (30)). As a significant conclusion, it is noted that the most critical wire in terms of Miner's damage is the furthest from the bending plane, while being the one closest to the same plane if the cumulative slip is considered as indicator of fretting fatigue. This consideration reveals the importance of developing a combined criterion which accounts for both aspects in a concise and reliable manner.

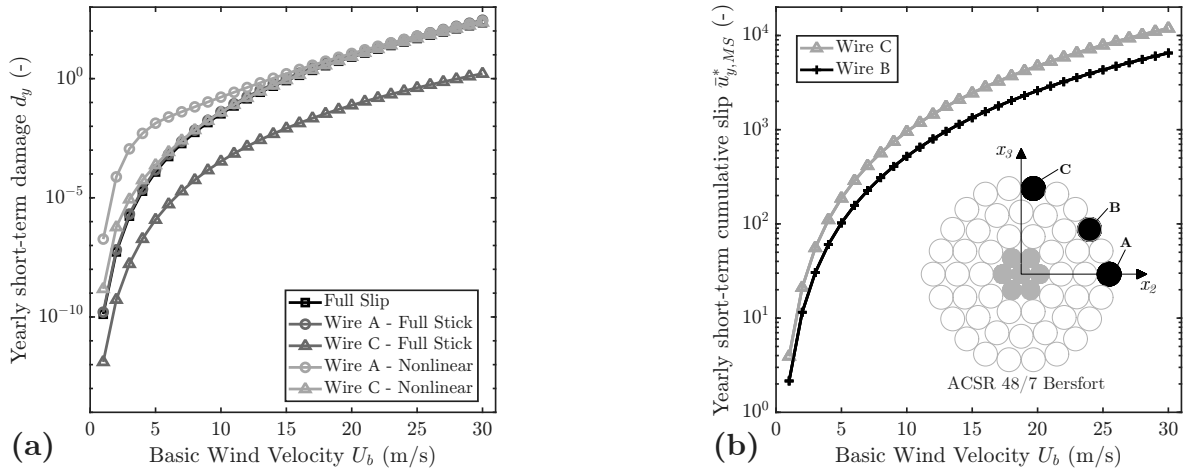


Fig. 3. Yearly short-term damage (a) and cumulative slip (b) for different wires of the ACSR Bersfort, as a function of the basic wind velocity. Results are computed at the suspension clamp position (i.e. at $\xi = 0$). Please note that $\bar{u}_{y,MS}^* = 0$ for wire A.

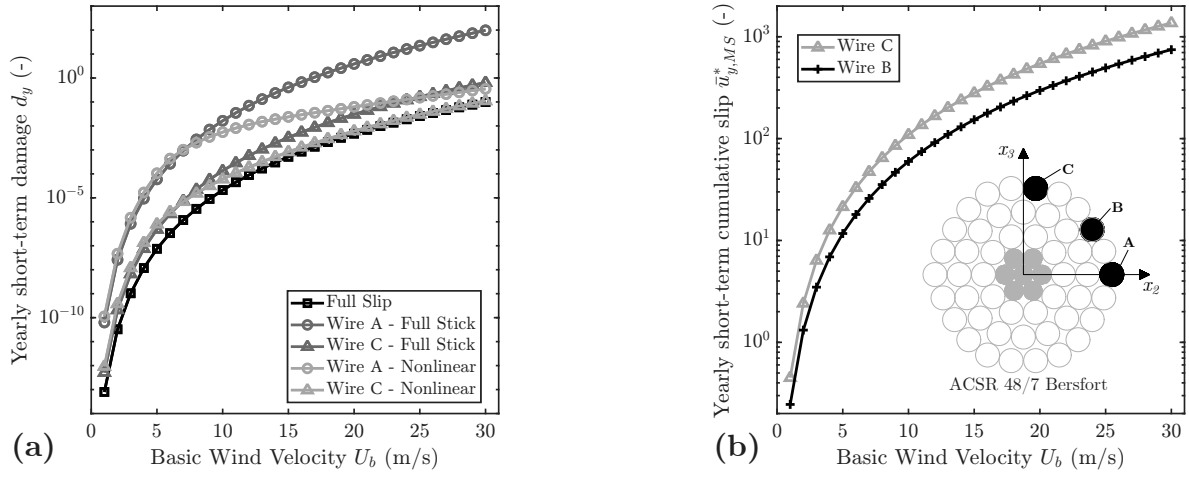


Fig. 4. Yearly short-term damage (a) and cumulative slip (b) for different wires of the ACSR Bersfort, as a function of the basic wind velocity. Results are computed at 89 mm from the suspension clamp (i.e. at $\xi = \bar{\xi}$). Please note that $\bar{u}_{y,MS}^* = 0$ for wire A.

4.2 Long-term damage and cumulative micro-slip

This section investigates the effect of both the dimensionless pretension (η_0) and the span length (L) of the cable on the long-term yearly damage and cumulative slip of the selected wires. Results are computed for the cross-section located at 89 mm from the suspension clamp position (i.e. at $\xi = \bar{\xi}$). Each value of long-term damage is computed based on the results of 200 analyses associated with different values of basic wind velocity, adaptively spaced to better capture the shape of the PDF represented in Fig. 2(a) ($N_w = 200$). For each simulation, 10 realizations are performed. As depicted in Fig. 5, the increase of the cable pretension is associated with a strong reduction of both the long-term yearly damage and the cumulative dimensionless micro-slip. While the long-term damage for wire C calculated with the nonlinear model closely follows the curve computed under the hypothesis of *full-slip*, wire A remains the most critical in terms of Miner's damage accumulation. The *full-stick* hypothesis, however, appears quite conservative, especially at low values of pretension in the cable. Wire C is still the most critical in terms of cumulative micro-slip indicator. On the contrary, as it is shown in Fig. 6, the increase of span length is associated with an increase of the long-term yearly damage and a slight reduction of the wire cumulative micro-slip. Wire A is the most critical in terms of accumulated damage, and the hypothesis of *full-stick* state confirms to be excessively conservative, especially for long span lengths.

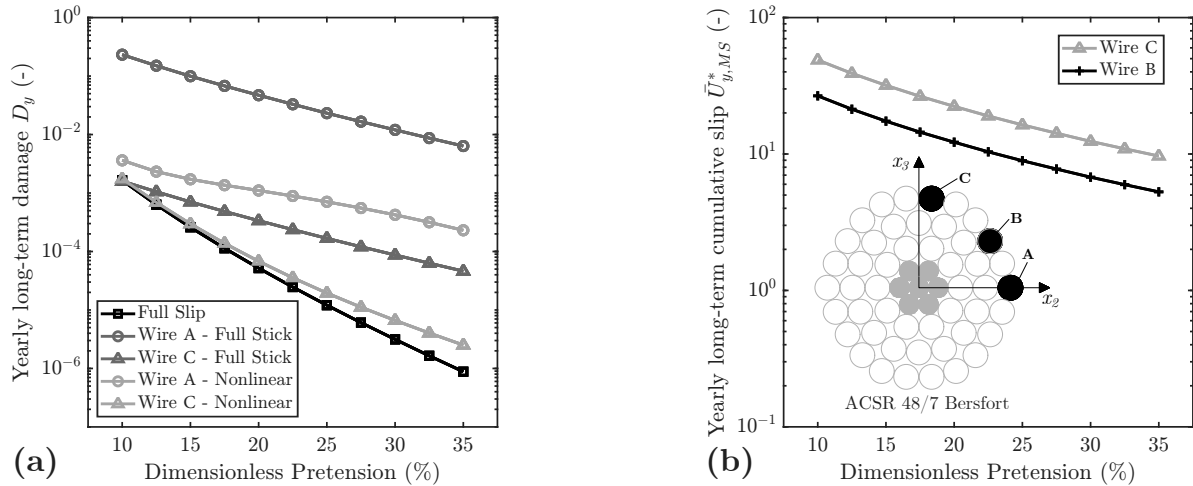


Fig. 5. Yearly long-term damage (a) and cumulative slip (b) for different wires of the ACSR Bersfort, as a function of the dimensionless pretension. Please note that $\bar{U}_{y,MS}^* = 0$ for wire A.

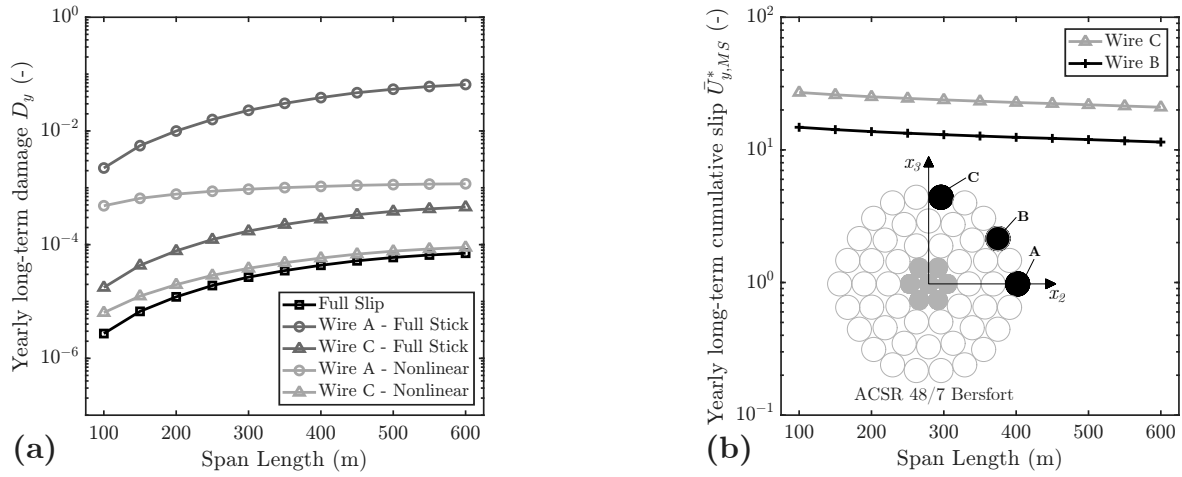


Fig. 6. Yearly long-term damage (a) and cumulative slip (b) for different wires of the ACSR Bersfort, as a function of the cable span length. Please note that $\bar{U}_{y,MS}^* = 0$ for wire A.

5 Conclusion

In this work, an efficient multi-scale methodology for the assessment of the fatigue life of suspended stranded cables under turbulent wind excitation has been presented. The preliminary results aimed to demonstrate the effectiveness of the proposed methodology when applied to a simple case study; however, its potential is considerably broader, allowing for extension to more complex structural models and loading conditions.

Results highlight the great variability of the nonlinear bending stiffness inside the boundary layers of the cable, questioning the conventional but arbitrary position at which fatigue checks for OHL conductors are usually carried out. In addition, the contrasting outcomes of the Miner's damage and the cumulative micro-slip observed in the wires of the cable's outermost layer are emphasized, underscoring the need for a novel indicator that integrates both effects in a comprehensive and reliable manner.

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