Public pensions and LTC insurance with family solidarity.*

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Abstract

As income rises, the risk of disability in old age declines, while life expectancy increases. These correlations strengthen the case for public long-term care (LTC) insurance over public pension systems. However, this perspective shifts when considering family solidarity—specifically, the informal care provided by spouses and children to elderly relatives. When viewed through the lens of altruistic caregiving motives, the argument for social LTC insurance becomes more nuanced. The interplay between formal and informal care is a key factor in shaping optimal policy. In this paper, we demonstrate that when family members reliably provide informal care, the design of a comprehensive public LTC system depends on the existence of a private insurance and on the degree of substitutability between informal and formal care.

Keywords: long-term care, mortality risk, disability risk, informal care
JEL classification: H2, H5.

1 Introduction

The provision of long-term care (LTC) has emerged as the defining policy challenges facing modern welfare states. Though the need for such care has always existed, it was long regarded as a private family matter rather than a distinct social risk warranting dedicated public intervention. That consensus has shifted

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dramatically, giving rise to a rich and expanding literature on social insurance for LTC. Two forces principally drive this transformation: the rapid aging of populations — particularly the growth of the very elderly cohort requiring intensive care — and the diminishing capacity of families to shoulder the caregiving burden for disabled older relatives.

The theoretical case for LTC social insurance builds on Rochet's (1991) seminal contribution work, extended by Cremer and Pestieau (1996, 2014), which establishes that social insurance is welfare-improving when the insured risk correlates negatively with income. This paper centers on the distinction between informal and formal care. Informal care is provided primarily by spouses and children, whom we treat as altruistically motivated — while acknowledging that other motives operate alongside altruism, as documented by Klimaviciute et al. (2017) and Alesie et al. (2014) using SHARE data covering roughly twenty countries. Notably, Lefebvre et al. (2025) identify an intergenerational quid pro quo: grandparents who provide childcare are subsequently more likely to receive care from their own children upon becoming disabled.

Formal care, by contrast, is financed through private savings, LTC insurance where markets exist, and public transfers, and may be delivered either at home or in institutional settings. Evidence suggests that the formal care costs incurred by severely disabled elderly individuals living at home are at least as high as — and often exceed — those associated with nursing home residence (OECD, 2020). This apparent paradox reflects the economies of scale that nursing homes derive from spreading fixed infrastructure costs across a large resident population, an efficiency advantage that home-based care cannot readily replicate. In cases of severe dependency, moreover, formal and informal care appear broadly complementary regardless of setting.

Empirical research consistently finds that higher income and wealth predict both lower disability risk and greater longevity in old age. This dual correlation provides a compelling argument for public LTC insurance that is, in certain respects, stronger than the case for traditional pension systems (Nishimura and Pestieau, 2022). The calculus grows more complex, however, when family solidarity enters the picture. The significance of informal care is well documented: Bolin et al. (2008), drawing on SHARE data from ten European countries, find that roughly one-third of single elderly individuals rely on some form of informal support. Assessed in monetary terms, the magnitude is equally striking — Buckner and Yeandle (2011) estimate that informal LTC care amounts to 7.4% of the GDP of the United Kingdom.[1]

When caregiving is altruistically motivated, the optimal design of public LTC insurance requires careful attention to the interplay between formal and informal provision. Our analysis yields three principal findings. First, when private LTC insurance is available and purchased, public LTC policy is qualitatively unaffected by family solidarity: any welfare effect of government provision is offset by households' intertemporal optimization, rendering informal care arrangements irrelevant to the social optimum. Second, this neutrality breaks

[1] See also Norton (2016, Section 3).

down when individuals do not purchase private insurance — whether because public LTC crowds it out or because the market fails to exist, the so-called long-term care insurance puzzle (Pestieau and Ponthiere, 2012). In this case, the optimal level of public LTC spending rises when formal and informal care are complements and falls when they are substitutes. Third, uncertainty about the reliability of informal care reinforces the case for public coverage precisely when the two care types are substitutable.

These results rest on two stylized facts. The first, established by Lefebvre et al. (2018), Connolly et al. (2025), and Lefebvre, Schoenmaeckers and Heymans (2025), documents the joint relationship between income, longevity, and disability risk: income correlates positively with longevity and negatively with old-age disability. The second concerns the interaction between formal and informal care in managing age-related dependency. Following Bonsang (2010), Bolin et al. (2008), and Perelman and Pestieau (2025), we adopt a utility function for dependent individuals that incorporates both care types. Whether their relationship is ultimately substitutive or complementary remains an open empirical question, though preliminary evidence suggests that complementarity may intensify with the severity of disability.

This paper is structured as follows. Section 2 explores how informal care provided by altruistic children influences the optimal level of public LTC insurance. Section 3 examines the relative social desirability of public pensions versus public LTC insurance in the presence of informal care. Section 4 analyzes the impact of uncertainty surrounding informal care on public LTC insurance. Finally, Section 5 concludes.

2 No uncertain lifetime.

The basic model used involves parent-child families, where children's contributions to their elderly parents' care depend on their individual market productivity and their degree of altruism. Families differ in the levels of wages w_i and the disability risk π_i. Individual's lifetime utility can be written as:

$$\begin{aligned} U_i &= \iota_i \beta H(\tilde{a}_i, \tilde{m}_i) + w_i(1-\tau)l_i - s_i - I_i - v(\tilde{a}_i + l_i) \\ &+ (1-\pi_i)\, u(s_i) + \pi_i H\left(a_i, I_i/\pi_i + s_i + g\right), \end{aligned} \tag{1}$$

where ι_i is an indicator function that takes 0 when i's parent is healthy, and 1 when the parent needs care.

In the young period, individuals supply care to the parents $\tilde{a}_i$ in the dependency state ($\tilde{a}_i = 0$ when the parent is healthy). They also earn labor income $w_i l_i$, save s_i and purchase private insurance I_i. We assume that the utility of the first period consumption $c_i = w_i(1-\tau)l_i - s_i - I_i - v(\tilde{a}_i + l_i)$ is linear. $v(\cdot)$ that reflects the disutility of supplying labor and assistance is strictly convex.

In the second period, the individual consume the amount of saving if healthy and his utility is denoted $u(s_i)$. If disabled, his utility is represented by a quasiconcave function $H(a_i, m_i)$ of two arguments: informal care provided by children, a_i, and formal care m_i. The latter is financed by the insurance scheme I_i, saving s_i and public LTC benefit, g. In case his/her parent becomes dependent, the individual utility is augmented by the utility of his disabled parent discounted by $\beta \leq 1$.

The variable $\tilde{a}_i$ refers to the aid that the individual is going to supply to his/her parent in case of diability. We will use the upperscript $\tilde{}$ to denote variables pertaining to the previous generation. We posit that I_i is an actuarially fair insurance scheme. LTC public benefits are financed by a payroll tax paid by the individual when young. The government's formal care g is provided uniformly in the dependent state.

In Perelman and Pestieau (2025) — see also Bolin et al. (2008) and Bonsang (2009)[2] — a welfare function $H(a, m)$ is implicitly assumed, where a denotes informal care provided by the family, and m denotes formal care. A key finding across these studies is that the degree of substitutability between these two forms of care diminishes as the severity of disability increases.

Regarding the information structure, we adopt a linear tax and a flat-rate public benefit, deliberately departing from a non-linear instrument approach in which individualized tax rates and LTC benefits would be determined by self-selection constraints. We do so because, while we assume the central planner observes the long-term care (LTC) function, individual characteristics remain private information, and the provision of the public LTC is uniform among the eligible individuals.

The provision of formal and informal care is a complex matter, and no single model can do justice to all its dimensions. Our objective here isolates the role of private insurance. Children's informal care will complement or substitute such motives, depending on the functional form $H(a, m)$. As we show, it may affect the design of public long-term care insurance.

Concerning loading costs for the private annuity and insurance, see Nishimura and Pestieau (2022, Appendix A). There, for the loading factors $0 < \lambda^I \leq \lambda^s \leq 1$, they assume that type i's return from saving is $\lambda^s s_i/\phi_i$ for the survival probability ϕ_i introduced in the next section, and the return of the private insurance is $\lambda^I I_i/(\pi_i \phi_i)$. When $\lambda^I < 1$ and $\lambda^s \leq 1$, some individuals may prefer not to have private savings or private insurance. For the informational structure, we consider two polar cases, (i) $\lambda^I = 1 = \lambda^s$ (both annuity markets and the private LTC market operate without loading costs) and (ii) $\lambda^I = 0$ and $\lambda^s = 1$ (the private LTC is not available).

A last word on the LTC utility function $H(a, m)$ is in order. In the theoretical literature, this utility has most often just one argument, which means that formal and informal care are perfect substitutes. Another specification consists in distinguishing between standard consumption c and health status h, in which case we have $H(c, h)$.[3] This specification is often used in health economics. In this paper, we deliberately distinguish between formal and informal care and

[2]The first study covers a large number of European countries. Bonsang uses HRS data.
[3]De Donder and Leroux(2021)

focus on whether these two types of care are substitutes or complements.

The timing of the game is the following:

1. The government chooses the tax rate and the benefit level that maximize the social welfare.
2. The individuals choose their labor supply, their saving and the LTC insurance premium, if any and the level of care $\tilde{a}_i$.

As noted above, our model abstracts from downward altruism and, by extension, from planned bequests. In the case of uncertain lifetimes, accidental bequests are ruled out by assuming the existence of a perfect annuity market. These simplifications are adopted in the interest of tractability. We acknowledge, however, that bequests can serve as a mechanism through which individuals secure care from their children or even grandchildren. A strand of the literature examines quid pro quo arrangements in which attention and support are elicited through the promise of inheritance — arrangements that may be either altruistic or strategic in nature (see Bernheim et al., 1985; Cremer et al., 2016). We provide an analysis that includes the bequest in the Appendix.

2.1 Individuals' problem

At the start of the first period, individuals are aware of health status of their own parent. They chooses $\tilde{a}_i$ l_i, s_i and I_i so as to maximize lifetime expected utility (1). We start with the choice of informal care. In case of healthy parent, there is no care ($\tilde{a}_i = 0$). In case of disabled parent, individuals maximize the following expression:

$$w_i(1-\tau)l_i - s_i - I_i - v(l_i + \tilde{a}_i) + \beta H\left(\tilde{a}_i, \tilde{m}_i\right).$$

The FOC is given by:

$$-v'\left(l_i + \tilde{a}_i\right) + \beta H_1\left(\tilde{a}_i, \tilde{m}_i\right) = 0. \tag{2}$$

To interpret this formula, we need the FOC related to the labor supply:

$$w_i(1-\tau) - v'(l_i + \tilde{a}_i) = 0$$

.

Substituting this standard expression into (2), we obtain:

$$\Delta = -w(1-\tau) + \beta H_1\left(\tilde{a}_i, \tilde{m}_i\right) = 0.$$

From this condition, we obtain a supply function for caring time: $a_i = a\left(m_i; \beta, w_i(1-\tau)\right)$.[4] Differentiating the above FOC we get:

$$\frac{\partial a_i}{\partial m_i} = \frac{\beta H_{12}}{-\beta H_{11}} \lesseqgtr 0 \Leftrightarrow H_{12} \lesseqgtr 0. \tag{3}$$

[4]Note that we implicitly assume a steady state implying that the function $\tilde{a}_i$ is time invariant.

Note that in the separable case where $H_{12} = 0$, elder care would only depend on $w_i (1 - \tau)$ and β. In case of complementarity, elder care increases with m_i and in case of substitutability, we have some crowding out. We might be interested by the effect of w_i on a_i. This can be obtained by differentiating the above FOC:

$$\frac{\partial a_i}{\partial w_i} = \frac{1-\tau}{\beta H_{11}} < 0, \ \frac{\partial a_i}{\partial \tau} = \frac{-w}{\beta H_{11}} > 0.$$

Care and wage are negatively correlated according to the standard opportunity cost argument (Pestieau and Sato (2008)). Children with high wages respond differently to parental care needs than low-wage children.[5]

We now turn to the other FOCs related to s_i and I_i using the supply function $a(m_i, \beta, (1-\tau)w_i)$:

$$-1 + (1-\pi_i)\, u'(d_i) + \pi_i H_2 (a_i, m_i) + \pi_i H_1 (a_i, m_i) \frac{\partial a_i}{\partial s_i} = 0 \qquad (4)$$

$$-1 + H_2 (a_i, m_i) + \pi_i H_1 (a_i, m_i) \frac{\partial a_i}{\partial I_i} \leq 0.$$

where $\frac{\partial a_i}{\partial I_i} = \frac{\partial a_i}{\partial m_i}\frac{1}{\pi_i}$ and $\frac{\partial a_i}{\partial s_i} = \frac{\partial a_i}{\partial g} = \frac{\partial a_i}{\partial m_i}$. Note that granted that the Inada conditions apply to $u(d)$, the FOC with respect to s is aways interior, which is not the case of the FOC with respect to I_i.

In case of interior solution with $I_i > 0$,we have:

$$-1 + H_2 (a_i, m_i) + \pi_i H_1 (a_i, m_i) \frac{\partial a_i}{\partial I_i} = 0; u'(d_i) = 1.$$

Otherwise, when $I_i = 0$, we have

$$-1 + H_2 (a_i, m_i) + \pi_i H_1 (a_i, m_i) \frac{\partial a_i}{\partial I_i} < 0.$$

We will consider those two possibilities. Note that the alternative $I_i = 0$ can result from either an inexistent LTC insurance market[6] or from the occurrence of a corner solution caused by a high loading cost or a generous public program. In the first case, we have $-1 + H_2 (a_i, m_i) \lesseqgtr 0$ and in the second case $-1 + H_2 (a_i, m_i) < 0$.

2.2 Government's problem

2.2.1 Interior solution.

We now turn to the social welfare maximization when $I_i > 0$. We posit that the government is only concerned by the welfare of the members of a generation.

[5]Wang (2022) shows that higher-wage women are more likely to purchase long-term care for their parents while lower wage women are more likely to drop out of the labor force to provide informal long-term care (see also Wang and Youderian 2025).

[6]This is what is called the LTC insurance puzzle. See on this Pestieau and Ponthiere (2012) and Pestieau (2025).

Further, given the quasilinearity of the individual utility function, we have to concavify the lifetime utility with a strictly concave transfotmation $V(.)$ such that the transformed utility is:

$$V_i = V\left[w_i(1-\tau)l_i - s_i - I_i - v(\iota_i\tilde{a}_i + l_i) + (1-\pi_i)\,u(s_i) + \pi_i H\left(a_i, I_i/\pi_i + s_i + g\right)\right],$$

with $V' > 0$ and $V'' < 0$. We write the following Lagrangian to be maximized:

$$\mathcal{L} = E\{V[w(1-\tau)l - s - I - v(\iota\tilde{a} + l) + (1-\pi)\,u(s) + \pi H\left(a\left(.\right), I/\pi + s + g\right)]$$
$$+\mu\left[\tau wl - \pi g\right]\},$$

where $Ez = \sum n_i z_i$ and μ denotes the multiplier associated with the revenue constraint. Note that the operator E includes the different types of individuals but also whether they have a disabled parent or not.

Here we are laundering out $\iota_i\beta H(\tilde{a}_i, \tilde{m}_i)$ in (1). $H(\tilde{a}_i, \tilde{m}_i)$ was counted in the welfare-maximization problem of the previous generation. The following way of welfare evaluation is advocated by Harsanyi (1995) and Hammond (1987): "excluding all external preferences, even benevolent ones, from our social utility function (Hammond (1987, p.87))."

We obtain the FOCs:

$$\frac{\partial\mathcal{L}}{\partial g} = E\left[V'[(-1 + (1-\pi)\,u'(d) + \pi H_2)\frac{\partial s}{\partial g} + (-1 + H_2)\frac{\partial I}{\partial g} + \pi H_2 + \pi H_1\frac{da}{dg}] - \mu\pi\right] = 0$$

$$\frac{\partial\mathcal{L}}{\partial\tau} = -E\left[V'wl + \pi V'w(1-\tau)\frac{\partial a}{\partial\tau} - \mu\left(wl + \tau w\frac{\partial l}{\partial\tau}\right)\right] = 0.$$

where π in the second term of $\frac{\partial\mathcal{L}}{\partial\tau}$ is the probability of the dependency state of the parent, which we assume to be the same as that of the present generation. Using the FOCs of the individual, this can be rewritten as:

$$\frac{\partial\mathcal{L}}{\partial g} = E\left[V'[-\pi H_1[\frac{\partial a}{\partial s}\frac{\partial s}{\partial g} + \frac{\partial a}{\partial I}\frac{\partial I}{\partial g}] + \pi H_2 + \pi H_1\frac{da}{dg}] - \mu\pi\right] = 0$$

or, since $\frac{\partial a}{\partial g} = \frac{\partial a}{\partial m}$ and $\frac{\partial a}{\partial I} = \frac{\partial a}{\partial m}\frac{1}{\pi}$ and $\frac{da_i}{dg} = \frac{\partial a_i}{\partial m_i}\left[1 + \frac{\partial m_i}{\partial s_i}\frac{\partial s_i}{\partial g} + \frac{\partial m_i}{\partial I_i}\frac{\partial I_i}{\partial g}\right]$,

$$\frac{\partial\mathcal{L}}{\partial g} = E\left[V'[\pi H_2 + \pi H_1\frac{\partial a}{\partial g}] - \mu\pi\right] = 0$$

The above equation shows that the government's optimal provision which equates its benefit and the cost depends on $\frac{\partial a}{\partial g} = \frac{\partial a}{\partial m}$.

Combining those two FOCs, we obtain the compensated effect of the tax denoted $\frac{\partial\mathcal{L}^{\mathcal{C}}}{\partial\tau} = \frac{\partial\mathcal{L}}{\partial\tau} + \frac{dg}{d\tau}\frac{\partial\mathcal{L}}{\partial g}$. In other words, we look at the effect of the tax combined with that of public LTC, keeping a balanced budget.

$$\frac{\partial\mathcal{L}^{\mathcal{C}}}{\partial\tau} = -EV'wl - E\pi V'w(1-\tau)\frac{\partial a}{\partial\tau} + E\pi V'H_2\frac{Ewl}{E\pi} + E\pi V'H_1\frac{\partial a}{\partial m}\frac{Ewl}{E\pi} + \mu\tau Ew\frac{\partial l}{\partial\tau} = 0$$

This can be also written as, for $\Lambda = E\pi V' w(1-\tau)\frac{\partial a}{\partial \tau}$:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -EV'wl - \Lambda + E\pi V' H_2 \frac{Ewl}{E\pi} + E\pi V' \left(1-H_2\right) \frac{Ewl}{E\pi} + \mu\tau Ew \frac{\partial l}{\partial \tau} = 0, \quad (5)$$

or

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -cov\left(V', wl\right) + cov\left(V', \pi\right) \frac{Ewl}{E\pi} - \Lambda + \mu\tau Ew \frac{\partial l}{\partial \tau} = 0.$$

And thus, we have :

$$\tau = \frac{-cov\left(V', wl\right) + cov\left(V', \pi\right) \frac{Ewl}{E\pi} - \Lambda}{-\mu Ew \frac{\partial l}{\partial \tau}} \quad (6)$$

In the numerator of formula (6), the first two terms reflect the redistributive role of this LTC scheme. The first covariance is clearly negative. Given that it is generally assumed that the risk of disability is negatively correlated with income, the second covariance is positive. Finally, the last term represents the negative effect that the tax has on the supply of informal care by the individuals whose parents are disabled. The denominator is standard. It represents the efficiency effect of the tax: if the labor supply is inelastic this effect will be small and the tax will be relatively high. We thus have that in case of interior solutions, family solidarity does not affect public LTC insurance. More importantly, in this case, private insurance neutralizes the insurance role of the family in the design of the LTC program.

2.2.2 LTC insurance unavailable

When LTC insurance is not available, the choice of the individual is thus reduced to $\tilde{a}_i$, l_i, and s_i that are used to maximize lifetime expected utility U_i subject to the caring function $a\left(m_i; \beta, w_i(1-\tau)\right)$ that is obtained.

The problem of the government is expressed by the following Lagrangian to be maximized:

$$\mathcal{L} = E\{V[w(1-\tau)l - s - v(\iota\tilde{a} + l) + (1-\pi)\,u(s) + \pi H\left(a\left(.\right), s+g\right)]$$

$$+\mu\left[\tau wl - \pi g\right]\}$$

Interior solutions are obvious here and thus, we obtain the FOCs:

$$\frac{\partial \mathcal{L}}{\partial g} = E\left[V'[\pi H_1 \frac{\partial a}{\partial m} + \pi H_2] - \mu\pi\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\left[V'wl + \pi V' w(1-\tau)\frac{\partial a}{\partial \tau} - \mu\left(wl + \tau w \frac{\partial l}{\partial \tau}\right)\right] = 0.$$

Combining those two FOCs as before, we obtain the compensated effect of the tax

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -E\left[V'[wl - \pi H_2 \frac{Ewl}{E\pi} - \pi H_1 \frac{\partial a}{\partial m}\frac{Ewl}{E\pi}] - \mu\tau w \frac{\partial l}{\partial \tau}\right] - \Lambda =$$

$$= -cov\left(V', wl\right) + cov\left(\pi, V'H_2\right)\frac{Ewl}{E\pi} + EwlEV'\left[H_2 - 1\right] - \Lambda + E\pi H_1 \frac{\partial a}{\partial m}\frac{Ewl}{E\pi} - \mu\tau Ew \frac{\partial l}{\partial \tau}$$
$$= 0. \tag{7}$$

And thus :

$$\tau = \frac{-cov\left(V', wl\right) + cov\left(\pi, V'H_2\right)\frac{Ewl}{E\pi} + EwlEV'\left[H_2 - 1\right] - \Lambda + E\pi V'H_1 \frac{\partial a}{\partial m}\frac{Ewl}{E\pi}}{-\mu Ew \frac{\partial l}{\partial \tau}} \tag{8}$$

As seen above, the first covariance is clearly negative and the second positive. The sign of the third term of the numerator calls for some discussion. In the First Best, we have $H_2 - 1 = 0$, which can be implemented with an actuarial fair private insurance. If public benefits are high enough, one can have a corner solution, in which case $H_2 - 1 < 0$. In the absence of such insurance, the level of m is likely to be suboptimal, in which case one has $H_2 - 1 > 0$. As to the last term of the numerator, it is positive as long as $H_{12} > 0$. We thus have that family solidarity fosters public spending in case of complementarity between a and m. This finding is summarized in the following proposition:

Proposition 1

Complementarity (substitutability) between formal and informal care fosters (reduces) public LTC spending when private LTC insurance is not available. Otherwise, informal care only impacts public LTC insurance though the effect of the tax on the caring supply.

In the Appendix, we provide an analysis to show that our main effects are robust to the assumption of joy-of-giving bequests. The private saving and private long-term care insurance (in case the latter is available) are strategically used by the parent for the mutual dependence between the bequest and the child's informal care. In case of interior solutions of I_i, family solidarity does not affect the two covariance terms of (5). When private LTC insurance is not available, in addition to complementarity/substitutability between formal and informal care, the parents' bequest motive fosters public LTC spending.

3 Uncertain lifetime.

We now introduce another risk, namely the mortality risk that makes the length of life uncertain and calls for an annuitization of savings.. We want to see the role of family solidarity in the choice of public pensions and LTC insurance.

The basic model used involves parent-child families, which differ in the levels of wages w_i, the survival probability ϕ_i and the disability risk π_i. Individuals' lifetime utility can be written as:

$$U_i = \iota_i \beta H(\tilde{a}_i, \tilde{m}_i) + w_i(1-\tau)l_i - s_i - I_i - v(\tilde{a}_i + l_i)$$
$$+\phi_i (1-\pi_i) u(s_i/\phi_i + P) + \phi_i \pi_i H(a_i, I_i/\pi_i + s_i/\phi_i + P + g), \quad (9)$$

where the indicator function ι_i for the parent's dependency status is the same as before. Formal cares m_i is financed by the insurance scheme I_i, annuitized saving s_i, public pensions P, and public LTC benefit, g. LTC public benefits and pensions are financed by a payroll tax paid by the individuals when young.

The pension P supplements s_i/ϕ_i. We uncover whether there is a role of public pension under the observed assumptions of the correlations between longevity or disability risk and income.

3.1 Individuals' problem

Before knowing whether they survive the first period and if so, their health status, individuals choose $\tilde{a}_i$, l_i, s_i and I_i so as to maximize their lifetime expected utility (9).

The first-order conditions (FOCs) with respect to $\tilde{a}_i$ (in case of disabled parent), l_i, s_i, and I_i are as follows:

$$w_i(1-\tau) - v'(\tilde{a}_i + l_i) = 0$$
$$-1 + (1-\pi_i) u'(s_i/\phi_i) + \pi_i H_2(a_i, m_i) + \pi H_1(a_i, m_i) \frac{\partial a_i}{\partial m_i} = 0 \quad (10)$$
$$-1 + H_2(a_i, m_i) + H_1(a_i, m_i) \frac{\partial a_i}{\partial m_i} \leqslant 0.$$

and $(1-\tau)w_i = \beta H_1(\tilde{a}_i, \tilde{m}_i)$ when the parent is disabled.

In case $I_i > 0$. we have that $u'(d_i) = 1$.

3.2 Government's problem

We now turn to the social welfare maximization. We write the following Lagrangian:

$$\mathcal{L} = E\{V[w(1-\tau)l - s - I - v(\iota\tilde{a} + l) + \phi(1-\pi) u(s/\phi + P) + \phi\pi H(a(.), m)]$$
$$+\mu[\tau wl - \phi\pi g - \phi P]\}.$$

Note that $\tilde{a}_i > 0$ when the parents are alive and disabled. The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial g} = E\{V'[(-1 + (1-\pi)\, u'(d) + \pi H_2)\, \frac{\partial s}{\partial g} + (-1 + H_2)\, \frac{\partial I}{\partial g} + \phi\pi H_2 + \phi\pi H_1 \frac{da}{dg}] - \mu\phi\pi\} = 0$$

$$\frac{\partial \mathcal{L}}{\partial P} = E\{V'[(-1 + (1-\pi)\, u'(d) + \pi H_2)\, \frac{\partial s}{\partial P} + (-1 + H_2)\, \frac{\partial I}{\partial P}$$

$$+\phi\,(1-\pi)\, u'(d) + \phi\pi H_2 + \phi\pi H_1 \frac{da}{dP}] - \mu\phi\}$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\{V'\left[wl + (1-\tau) w \frac{\partial \tilde{a}}{\partial \tau}\right] - \mu\left(wl + \tau w \frac{\partial l}{\partial \tau}\right) = 0.$$

Using the FOCs of the individual, we can consider the following fiscal reform; we consider that the tax is given and not necessarily optimal, and we examine the welfare incidence of increasing g at the expense of P while keeping a balanced budget.

$$\frac{\partial \mathcal{L}}{\partial g} = E\{V'[-\phi\pi H_1 \frac{\partial a}{\partial s}\frac{\partial s}{\partial g} - \phi\pi H_1 \frac{\partial a}{\partial I}\frac{\partial I}{\partial g} + \phi\pi H_2 + \phi\pi H_1 \frac{da}{dg}] - \mu\phi\pi\} = 0$$

$$\frac{\partial \mathcal{L}}{\partial P} = E\{V'[-\phi\pi H_1 \frac{\partial a}{\partial s}\frac{\partial s}{\partial P} - \phi\pi H_1 \frac{\partial a}{\partial I}\frac{\partial I}{\partial P} + \phi\,(1-\pi)\, u'(d)$$

$$+\phi\pi H_2 + \phi\pi H_1 \frac{da}{dP}] - \mu\phi\}$$

Using the fact that $\frac{\partial a}{\partial g} = \frac{\partial a}{\partial m}\left[1 + \frac{\partial s}{\partial g}\frac{1}{\phi} + \frac{\partial I}{\partial g}\frac{1}{\pi\phi}\right]$, this can be rewritten as:

$$\frac{\partial \mathcal{L}}{\partial g} = E\{V'\left[\phi\pi H_2 + \phi\pi H_1 \frac{\partial a}{\partial g}\right] - \mu\phi\pi\} = 0$$

$$\frac{\partial \mathcal{L}}{\partial P} = E\{V'[\phi\,(1-\pi)\, u'(d) + \phi\pi H_2 + \phi\pi H_1 \frac{\partial a}{\partial P}] - \mu\phi\}$$

Public pension is universally provided. We want to know whether for given revenue, increasing P at the expense of g is socially desirable when family solidarity intervenes: $\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial P} = \frac{\partial \mathcal{L}}{\partial P} + \frac{dg}{dP}\frac{\partial \mathcal{L}}{\partial g}$. We thus have to sign the following expression:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial P} = EV'\left[\phi\,(1-\pi)\, u'(d) + \phi\pi H_2 - \phi\pi H_2 \frac{E\phi}{E\phi\pi} + \phi\pi H_1 \frac{\partial a^C}{\partial P}\right] \tag{11}$$

where $\frac{\partial a^C}{\partial P} = \frac{\partial a}{\partial P} - \frac{E\phi}{E\phi\pi}\frac{\partial a}{\partial g} = \frac{\partial a}{\partial m}\frac{E\phi(\pi-1)}{E\phi\pi} < 0$.

Expression (11) can be rewritten as:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial P} = cov\left(\phi\,(1-\pi)\,, V'u'(d)\right) \tag{12}$$

$$-E(\phi(1-\pi))E[cov(\frac{\phi\pi}{E\phi\pi}, V'H_2) + V'(H_2 - u'(d)) + \frac{\phi\pi}{E\phi\pi}V'H_1\frac{\partial a}{\partial m}]$$

As in the previous section, we distinguish between the case where LTC insurance is available or not. In the case $I > 0$, the Euler equation yields:

$$\left(H_2 + H_1 \frac{\partial a}{\partial m}\right) \frac{\phi\pi}{E\phi\pi} = u'(d) \frac{\phi\pi}{E\phi\pi}.$$

Thus, (11) can be rewritten to:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial P} = cov\left(\phi\left(1-\pi\right), V'u'(d)\right) - \frac{E(\phi(1-\pi))}{E\phi\pi} cov\left(V'u'(d), \phi\pi\right)$$

The first covariance is positive and the second negative (Nishimura and Pestieau, 2022, eq. (8)), which implies that it is always desirable to have a balanced budget decrease in pension benefits with the increase of the LTC benefits (the priority in the government's budget should be given to LTC over pensions).

If, however, we have $I = 0$, these simplifications are not possible and we are back to equation (10). Then we have:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial P} = cov\left(\phi\left(1-\pi\right), V'u'(d)\right) - cov\left(\phi\pi, V'u'(d)\right)\right) \frac{E\phi\left(1-\pi\right)}{E\phi\pi}$$

$$+E\phi\left(1-\pi\right) E\left[V'\left(u'(d) - H_2\right)\right] - E\phi\left(1-\pi\right) E\frac{\phi\pi}{E\phi\pi} H_1 \frac{\partial a}{\partial m}$$

The first covariance is negative and the second positive. Given that $H_2 - u'(d) > 0$, the third term is also negative. The sign of the fourth one depends on that of H_{12}. These results can be summarized in the following proposition.

Proposition 2

The negative effect on social welfare of a budget-balancing increase in public pension offset by a decrease in public LTC is not influenced by family solidarity when private LTC insurance is available. When it is not available, family solidarity strengthens (weakens) this effect in case of complementarity (substitutability).

Owing to the correlation between income and disability risk on the one hand, and between income and longevity on the other, LTC schemes carry a particular advantage over pension programs (Nishimura and Pestieau, 2022). When private LTC is available, family assistance does not play any role in the first-order condition. When it is not available, the additional effect of family solidarity depends on complementarity or substitutability. Note that we can obtain a tax formula similar to (6) with the tax financing either public pensions or public LTC benefits.

4 Uncertain child altruism.

When disabled parents rely on family members for care and assistance, various life events can disrupt these support systems without warning. Let us cite the most notable of these disruptions. Death of caregivers creates an immediate and often permanent gap in support. When an adult child who provides daily assistance passes away, the disabled parent may be left without their primary caregiver and emotional support. Illness can affect caregivers limiting their ability to provide care. Migration for employment, education, or personal reasons can physically separate caregivers from those who need care. Economic hardship can reduce the caregiver's availability. Finally, family disputes and relationship breakdowns can sever caregiving arrangements.

These disruptions modify the design of public policy as we now see. The choice of $a_i = a\left(m_i, \tilde{\beta}, (1-\tau)\, w_i\right)$ is unchanged.

4.1 Individuals' problem.

Let us denote the uniform probability of informal care by ζ. This means that with a probability $1-\zeta$, the disabled parent does not receive any informal care ($a=0$). This implies that only a fraction $\zeta\pi$ of individuals will provide assistance to their parents.We assume certain lifetime.

The lifetime utility of the individual facing a probability π of becoming disabled and a probability of getting aid from the child ζ is the following.

$$U_i = \iota_i \beta H(\tilde{a}_i, \tilde{m}_i) + w_i(1-\tau)l_i - s_i - I_i - v(l_i + \tilde{a}_i) + (1-\pi)\, u(s_i)$$

$$+\pi_i (1-\zeta)\, H\left(0, I_i/\pi_i + s_i + g\right) + \pi_i \zeta H\left(a_i, I_i/\pi_i + s_i + g\right)$$

Here, $\iota_i = 1$ if the individual is altruistic and the parent is disabled.

The FOCs are:

$$w_i(1-\tau) - v'(l_i) = 0$$

$$-1 + (1-\pi_i)\, u'(s_i) + \pi_i \zeta \left[H_2\left(a_i, m_i\right) + H_1\left(a_i, m_i\right) \frac{\partial a_i}{\partial m_i} \right]$$

$$+\pi_i (1-\zeta)\, H_2\left(0, m_i\right) = 0$$

$$-1 + \zeta \left[H_2\left(a_i, m_i\right) + H_1\left(a_i, m_i\right) \frac{\partial a_i}{\partial m_i} \right] + (1-\zeta)\left[H_2\left(0, m_i\right)\right] \leq 0,$$

and $(1-\tau)w_i = \beta H_1(\tilde{a}_i, \tilde{m}_i)$ when the parent is disabled and when the individual is altruistic.

4.2 Government's problem

The problem of the government is to maximize the following Lagrangian:

$$\mathcal{L} = E\{V[u(w(1-\tau)l - s - I - v(\iota\tilde{a} + l) + (1-\pi)\, u(s) + \pi(1-\zeta)\, H\left(0, m\right)$$

$$+\pi\zeta H(a,m)] + \mu[\tau wl - \pi g]\}$$

Given $a = a(m,\beta,(1-\tau)w)$, we have the following FOCs:

$$\frac{\partial \mathcal{L}}{\partial g} = E\{V'[(-1+(1-\pi)u'(d) + \pi \varXi H_2)\frac{\partial s}{\partial g} + (-1+\pi \varXi H_2)\frac{\partial I}{\partial g}$$

$$+\pi \varXi H_2 + \zeta\pi H_1(a,m)\frac{da}{dg} - \mu\pi\} = 0,$$

where $\varXi H_2 = \zeta H_2(a,m) + (1-\zeta)H_2(0,m)$.
Using the FOCs of the individual, it can be rewritten as:

$$\frac{\partial \mathcal{L}}{\partial g} = E\{V'[\zeta\pi H_1(a,m)\frac{\partial a}{\partial g} + \pi V' \varXi H_2] - \mu\pi\} = 0$$

The FOC related to the tax rate is:

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\{V'[wl + \pi\zeta w(1-\tau)\frac{\partial a}{\partial \tau}] - \mu\left(wl + \tau w\frac{\partial l}{\partial \tau}\right)\} = 0.$$

We now combine those two equations to obtain, with $\Lambda = E\pi V' w(1-\tau)\frac{\partial a}{\partial \tau}$:

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -E\left[V'wl - \pi V' \varXi H_2\frac{Ewl}{E\pi} - \pi\zeta V' H_1\frac{\partial a}{\partial m}\frac{Ewl}{E\pi} - \mu\tau w\frac{\partial l}{\partial \tau}\right] - \zeta\Lambda \quad (13)$$

$$= -cov(V', wl) + \varXi cov(\pi, V'H_2)\frac{Ewl}{E\pi} - \zeta\Lambda$$

$$-Ewl[EV' - E\varXi V'H_2] + \zeta E\pi H_1(a,m)\frac{\partial a}{\partial m}\frac{Ewl}{E\pi} - \mu\tau Ew\frac{\partial l}{\partial \tau} = 0,$$

where $\varXi cov(\pi, V'H_2) = \zeta cov(\pi, V'H_2(a,m)) + (1-\zeta)cov(\pi, V'H_2(0,m))$

We again distinguish between two cases depending on whether or not parents purchase private insurance. If $I > 0$, from the FOC of the individual, we can rewrite equation (13) as :

$$\frac{\partial \mathcal{L}^{\mathcal{C}}}{\partial \tau} = -cov(V', wl) + \varXi cov(V'H_2, \pi)\frac{Ewl}{E\pi} - \zeta\Lambda + \mu\tau Ew\frac{\partial l}{\partial \tau} = 0$$

.

And thus :

$$\tau = \frac{-cov(V', wl) + \varXi cov(V'H_2, \pi)\frac{Ewl}{E\pi} - \zeta\Lambda}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

If instead, $I = 0$, we have

$$\tau = \frac{-cov\left(V', wl\right) + \Xi cov(\pi, V'H_2) \frac{Ewl}{E\pi} + Ewl\left[EV' - E\Xi V'H_2\right] - \zeta\Lambda + \zeta E\pi V'H_1 \frac{\partial a}{\partial m}\frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}} \tag{14}$$

To see the impact of ζ on the level taxation $[\tau(\zeta)]$,we look at the two extreme cases:

$$\tau(0) = \frac{-cov\left(V', wl\right) + cov(\pi, V'H_2\left(0, m\right)) \frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

$$\tau(1) = \frac{-cov\left(V', wl\right) + cov(\pi, V'H_2\left(a, m\right)) \frac{Ewl}{E\pi} - \Lambda + E\pi H_1 \frac{\partial a}{\partial m}\frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

Notice that, since $-w_i - v''(\frac{\partial \tilde{a}_i}{\partial \tau} + \frac{\partial l_i}{\partial \tau}) = 0$ and $\frac{\partial \tilde{a}_i}{\partial \tau} > 0$, individuals' labor supply decrease due to taxation is *greater* when they take care of both labor supply and informal care (i.e., the case when individuals are more altruistic). In addition, it appears that when $H_{12} = 0$, the two formulas, $\tau(0)$ and $\tau(1)$, differ in the negative externality $\Lambda = -E\pi V'w(1-\tau)\frac{\partial a}{\partial \tau}$. When $H_{12} > 0$, the level of public spending seems to benefit from an increase in ζ, whereas when $H_{12} < 0$, we observe the opposite effect. This is pretty intuitive. In case of separability, there is no interaction between g and a. Family solidarity thus has a no direct impact on the desired level of public spending. In case of strong complementarity between g and a, there is a clear link between formal and informal care. An increase in a makes public spending more attractive. Finally, in case of strong substitutability between a and g, an increase in family solidarity crowds out formal care and thus discourages public spending. We can summarize these findings in a proposition.

Proposition 3

When every individual purchases private insurance, the risk of default of informal care has impact on the level of social LTC by a lower labor-supply elasticity. Additionally, in case private insurance is not available, the incidence depends on the sign of $H_{12.}$ In case of complementarity, family solidarity fosters public spending. In case of substitutability, it discourages it.

5 Conclusions

Before concluding, two remarks are in order. Throughout this paper, we have examined two polar cases regarding private insurance: either it was non-existent or it was actuarially fair. We could have introduced intermediate scenarios with loading factors that make insurance less attractive. In such cases, private insurance would not fully neutralize the impact of informal care on the desirability of public long-term care spending. In other words, family assistance would either positively or negatively influence the level of public spending.

Regarding the concept of uncertain altruism, there is a substantial body of literature on the topic[7]. However, these studies differ from our approach in three key ways. First, they explore two methods by which public spending can be associated with private spending: topping up and opting out. In contrast, our paper adopts the former, assuming from the outset that public spending can supplement other sources of financing for formal care. Second, these studies focus solely on formal care, whereas our paper examines the substitutability between formal and informal care. Finally, our paper addresses the issue of triple heterogeneity (wages, disability risk, and longevity), while most of the other studies consider identical individuals.

It is well established that the risk of disability in old age decreases with higher income and wealth, while longevity tends to increase with these factors. One key implication of these correlations, as shown by Nishimura and Pestieau (2020), is that the case for a public long-term care (LTC) scheme is stronger than that for public pensions. In this paper, we demonstrate that this dynamic may shift when family solidarity—specifically informal care provided by a spouse or children—is taken into account.

Assuming altruism as the primary motivator for caregiving, we demonstrate that the generosity of a social long-term care insurance is highly sensitive to both the degree of substitutability between formal and informal care, as well as the availability of private insurance. When everyone purchases actuarially fair insurance, family solidarity does not significantly impact public LTC spending. However, if private insurance is not universally adopted, the presence of informal care can either support or reduce public LTC spending, depending on whether formal and informal care are seen as complementary or substitutive. Given the limited availability of LTC insurance in many countries, it is likely that public LTC expenditure will be shaped by the strength of family solidarity.

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Appendix

Consider the benchmark analysis with $\phi_i = 1$ and $\zeta = 1$. Individual i's circumstance is characterized by:

- Her parent's dependency status, being healthy or dependent
- The level of $\tilde{m}_i$ of the parent (in case of dependency)
- the levels of wages w_i and their own disability risk π_i

Here we drop the assumption of quasi-linearity. Individual's lifetime utility can be written as:

$$U_i = \iota_i \beta H(\tilde{a}_i, \tilde{m}_i) + u(\iota_i \tilde{B}_i + (1-\iota_i)\tilde{b}_i + w_i(1-\tau)l_i - s_i - I_i - v(\tilde{a}_i + l_i))$$

$$+ (1-\pi_i)\left(u(s_i - b_i) + \gamma k(b_i)\right) + \pi_i(H\left(a_i, I_i/\pi_i + s_i + g - B_i\right) + \gamma k(B_i))$$

where the tilde denotes the variables with respect to i's parents and the informal care to them. ι_i is an indicator function that takes 0 when i's parent is healthy, and 1 when the parent needs care. $H(\tilde{a}, \tilde{m})$ is the parent's LTC utility. Formal care $\tilde{m}_i$ is, as in the basic model, partly financed by the insurance scheme and saving of the parent. The public LTC benefit of the parent is financed by a payroll tax as in the text.

The parent leaves the bequest b_i and B_i for the healthy and dependent states, respectively. $\gamma k(\cdot)$ is a strictly concave utility with respect to the bequest, with $\gamma > 0$ being the parent's willingness to leave the bequest.

The timing of the game in each period is the following:

1. The government chooses the tax rate and the benefit level that maximize social welfare.
2. The individuals chooses their labor supply, their saving and the LTC insurance premium. At the same time, he/she chooses to help his/her parent in case of disability, and parents choose the amount of the bequest.

The children expect the bequest as parents' Nash reaction to $\tilde{B}_i$. As such, $\tilde{B}_i$ represents both joy-of-giving and exchange motives.

The interaction between informal care and bequest

At the start of the first period, individual chooses informal care:

- in case of healthy parent: $\tilde{a}_i = 0$.
- in case of disabled parent: $\tilde{a}_i = \arg\max_{a_i} u\left(\tilde{B}_i + w_i(1-\tau)l_i - v(\tilde{a}_i + l_i) - s_i - I_i\right) + \beta H\left(\tilde{a}_i, \tilde{m}_i - \tilde{B}_i\right)$,

where β denotes the degree of altruism and $\tilde{m}_i = \tilde{I}_i/\pi_i + \tilde{s}_i + \tilde{g}$ denotes formal care.

In case of loss of autonomy, he/she devotes a fraction of time $\tilde{a}_i$ to caring. The FOC that defines the optimal $\tilde{a}_i$ and $\tilde{b}_i$ and $\tilde{B}_i$ are simply

$$\Delta = -u' \cdot (1-\tau)w_i + \beta H_1\left(\tilde{a}_i, \tilde{m}_i - \tilde{B}_i\right) = -u' \cdot v' + \beta H_1\left(\tilde{a}_i, \tilde{m}_i - \tilde{B}_i\right) = 0,$$

$$-u'(\tilde{s}_i - \tilde{b}_i) + \gamma k'(\tilde{b}_i) = 0, \quad -H_2 + \gamma k'(\tilde{B}_i) = 0.$$

From these conditions, we obtain a supply function for caring time: $\tilde{a}_i = \tilde{a}\left(\tilde{m}_i; \beta, (1-\tau)w_i\right)$. In order to clarify the timing, we maintain the variable tilde here. Differentiating the above FOCs we get:

$$\frac{\partial \tilde{a}_i}{\partial \tilde{m}_i} = -\frac{\beta H_{12}(1 - \frac{\partial \tilde{B}_i}{\partial \tilde{m}_i}) - u'' \cdot (1-\tau)w_i \frac{\partial \tilde{B}_i}{\partial \tilde{m}_i}}{\beta H_{11} + u''((1-\tau)w_i)^2}, \tag{A-1}$$

and

$$-\frac{\beta(H_{12})^2(1 - \frac{\partial \tilde{B}_i}{\partial \tilde{m}_i}) - H_{12}u'' \cdot (1-\tau)w_i \frac{\partial \tilde{B}_i}{\partial m_i}}{\beta H_{11} + u''((1-\tau)w_i)^2} + H_{22}(1 - \frac{\partial \tilde{B}_i}{\partial \tilde{m}_i}) = \gamma k'' \frac{\partial \tilde{B}_i}{\partial \tilde{m}_i} \tag{A-2}$$

Differentiating $\Delta = 0$ and rearranging, we obtain (A-1) where the informal care and the bequest in the dependent state have interactions. In turn, (A-1) is substituted to the differentiation of $-H_2 + \gamma k'(\tilde{B}_i) = 0$ to have (A-2) with respect to $\partial \tilde{B}_i/\partial \tilde{m}_i$.

From the negative definiteness $H_{11}H_{22} - H_{12}^2 > 0$, we have, from (A-2),

$$0 < \frac{\partial \tilde{B}_i}{\partial \tilde{m}_i} < 1 \ if \ \ -\gamma k''(\beta H_{11} + u''(v')^2) + H_{12}u''v' < 0$$

Intuitively, when $\frac{\partial \tilde{a}_i}{\partial \tilde{m}_i} H_{12} < 0$ as in the case of the quasi-linearity, the relationship $-H_2 + \gamma k'(\tilde{B}_i) = 0$ by the parent will cause some reaction by the parent such that $\frac{\partial \tilde{B}_i}{\partial \tilde{m}_i} > 0$. As such, the care giving and the bequest has some interactions. Parent (the previous generation to i) allocates formal care $\tilde{m}_i$ by being aware of these reactions.

In turn, the sign of $\frac{\partial \tilde{a}_i}{\partial \tilde{m}_i}$ in (A-1) rests not only on H_{12} if $u'' < 0$. The expectation of the bequest would make a positive effect on $\tilde{a}_i$ when $u'' < 0$. In addition to the substitutability/complementary channel discussed in the text, anticipating the bequest increases the informal care. As another effect, $\frac{\partial \tilde{a}_i}{\partial w_i} < 0$ in the basic model (the opportunity cost argument) remains operative with some modification.

Saving, private insurance and labor supply

Before knowing his/her health status, the child chooses l_i, s_i and I_i so as to maximize lifetime expected utility U_i subject to the caring function $a\left(m_i; \beta, \omega_i\right)$.

The FOCs of the child with respect to l_i, s_i and l_i respectively are:

$$w_i(1-\tau) - v'(l_i + \tilde{a}_i) = 0$$

$$-u'(c_i) + (1-\pi_i)\, u'(s_i - b_i) + \pi_i H_2\left(a_i, m_i - B_i\right) + \pi_i H_1\left(a_i, m_i - B_i\right) \frac{\partial a_i}{\partial s_i} = 0$$

$$-u'(c_i) + H_2\left(a_i, m_i - B_i\right) + \pi_i H_1\left(a_i, m_i - B_i\right) \frac{\partial a_i}{\partial I_i} \leq 0.$$

where $\frac{\partial a_i}{\partial I_i} = \frac{\partial a_i}{\partial m_i}\frac{1}{\pi_i}$ and $\frac{\partial a_i}{\partial s_i} = \frac{\partial a_i}{\partial g} = \frac{\partial a_i}{\partial m_i}$. Notice that, from the FOCs of the bequests that mentioned above, $u'(s_i - b_i)(1 - \frac{\partial b_i}{\partial s_i}) + \gamma k'(b_i)\frac{\partial b_i}{\partial s_i} = u'(s_i - b_i)$ and $H_2(1 - \frac{\partial b_i}{\partial s_i}) + \gamma k'(B_i)\frac{\partial b_i}{\partial s_i} = H_2$ and $H_2(1 - \frac{\partial B_i}{\partial I_i}) + \gamma k'(B_i)\frac{\partial B_i}{\partial I_i} = H_2$. Granted that the Inada conditions applies to $u(d)$ and $k(\cdot)$, the FOCs with respect to s, b aand B are always interior. A wealthy parent may save and purchase private insurance more than poorer parents.

In case of interior solution with $I_i > 0$,we have:

$$-1 + H_2\left(a_i, m_i\right) + \pi_i H_1\left(a_i, m_i\right) \frac{\partial a_i}{\partial I_i} = 0; u'(c_i) = u'(s_i - b_i).$$

Otherwise, when $I_i = 0$, we have

$$-u'(c_i) + H_2\left(a_i, m_i\right) + \pi_i H_1\left(a_i, m_i\right) \frac{\partial a_i}{\partial I_i} < 0.$$

Government's Problem

We now turn to the social welfare maximization. As in the text, we posit that the government is only concerned by the welfare of the members of a generation.We write the following Lagrangian to be maximized:

$$\mathcal{L} = E\{u(\iota\tilde{B} + (1-\iota)\tilde{b} + w(1-\tau)l - s - I - v(l + \iota\tilde{a})) + (1-\pi)\, u(s-b) + \pi H\left(a\left(.\right), I/\pi + s + g - B\right)$$

$$+\mu\left[\tau w l - \pi g\right]$$

where $Ez = \sum n_i z_i$.

Here we omit $\iota_i \beta H(\tilde{a}_i, \tilde{m}_i)$, $\gamma k(b_i)$ and $\gamma k(B_i)$, as in the same reason mentioned in the text.

The government's FOCs, taking account of the above FOCs of the individual, are:

$$\frac{\partial \mathcal{L}}{\partial g} = E\left[-\pi H_1[\frac{\partial a}{\partial s}\frac{\partial s}{\partial g} + \frac{\partial a}{\partial I}\frac{\partial I}{\partial g}] + \pi H_2[\frac{\partial B}{\partial s}\frac{\partial s}{\partial g} + \frac{\partial B}{\partial I}\frac{\partial I}{\partial g}] + \pi H_2(1 - \frac{dB}{dg}) + \pi H_1\frac{da}{dg} - \mu\pi\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -E\left[u'(1-\tau)w\frac{\partial a}{\partial \tau} + u'(c)wl - \mu\left(wl + \tau w\frac{\partial l}{\partial \tau}\right)\right] = 0.$$

where we have used the equality: $\frac{da_i}{dg} = \frac{\partial a_i}{\partial m_i}\left[1 + \frac{\partial m_i}{\partial s_i}\frac{\partial s_i}{\partial g} + \frac{\partial m_i}{\partial I_i}\frac{\partial I_i}{\partial g}\right]$ and $\frac{dB_i}{dg} = \frac{\partial B_i}{\partial m_i}\left[1 + \frac{\partial m_i}{\partial s_i}\frac{\partial s_i}{\partial g} + \frac{\partial m_i}{\partial I_i}\frac{\partial I_i}{\partial g}\right]$.

The FOC on τ is unchanged from the one in the text, so we have, assuming that each individual i has the same probability of dependency as her parent, we have, for $\Lambda = E\pi u'(c)\frac{(1-\tau)w_i\partial\tilde{a}}{\partial\tau} + E\pi H_2\frac{Ewl}{E\pi}\frac{\partial B}{\partial m}$:

$$\tau = \frac{-cov\left(u'(c), wl\right) + cov\left(u'(c), \pi\right)\frac{Ewl}{E\pi} - \Lambda}{-\mu Ew\frac{\partial l}{\partial \tau}} \tag{A-3}$$

In case of private LTC insurance unavailable, he similar manipulation to yield the one related to

$$\tau = \frac{-cov\left(u'(c), wl\right) + cov\left(\pi, H_2\right)\frac{Ewl}{E\pi} + EwlE\left[H_2 - u'(c)\right)\right] - \Lambda + E\pi H_1\frac{\partial a}{\partial m}\frac{Ewl}{E\pi}}{-\mu Ew\frac{\partial l}{\partial \tau}}$$

With the formulation developed in this section, we obtain that the FOC related to (5) and (7):

- In case of interior solutions of I_i, family solidarity does not affect the two covariance terms of public LTC insurance.
- In case of corner solutions (private LTC insurance is not available), in addition to complementarity/substitutability between formal and informal care, the parents' bequest motive fosters public LTC spending through (A-1).
- Lastly, the last term in (A-3) contains the welfare decrease of the individual by the income effect of the bequest by public assistance.

The second point signify the interaction of the bequest as an exchange and the increase of individual care provision in (A-1).