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Magnetic Flux Avalanches in Superconducting Films with Mesoscopic Artificial Patterns

**Motta, A.V. Silhanek, and W.A. Ortiz**

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## **Magnetic Flux Avalanches in Superconducting Films with Mesoscopic Artificial Patterns**

### **Abstract and Keywords**

This article examines the practical problem of thermally driven high-speed flux avalanches occurring in superconducting thin films with mesoscopic artificial patterns. The thin films are synthesized with artificial pins in the form of sub-micrometric antidots (ADs). The article first provides an overview of magnetic flux avalanches in superconductors, with particular emphasis on thermally driven avalanches, before discussing the occurrence and morphology of flux avalanches in superconducting thin films comprised of AD arrays. It analyses the influence of lattice symmetry and different AD geometries on the guidance and consequently the branching of flux avalanches. It also explores how artificial pinning centers inserted in superconducting films affect vortex dynamics.

**Keywords:** superconducting thin films, antidots, magnetic flux avalanches, superconductors, lattice symmetry, artificial pinning centers, vortex dynamics

M. Motta, A.V. Silhanek, and W.A. Ortiz

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During the past few decades, vortex matter in type-II superconductors has been one of the main topics of research in superconductivity. One of the most fascinating issues in the study of vortex matter in superconducting films is the occurrence of abrupt flux invasions, called flux avalanches. They present a startling dendritic morphology in thin films (Leiderer et al. 1993, Durán et al. 1995) which was discovered via magneto-optical imaging (MOI) experiments, a technique allowing for direct observation of the magnetic flux distribution across the sample. When such events are triggered, flux bursts penetrate into the sample at speeds as large as 100 km/s (Leiderer et al. 1993, Bolz et al. 2003), and the otherwise smooth vortex penetration is disrupted. From the applications point of view, this unpredictable phenomenon is certainly undesirable, since it degrades the current-carrying capability of the specimen, being therefore likely to cause a direct impact on the stability of a device. It is thus of practical interest to understand the causes for the occurrence of flux avalanches, as well as the main mechanisms governing these self-propelled events. In some instances, it is crucial to be able to avoid them (Baziljevich et al. 2002, Colauto et al. 2010, 2013), whereas, in some others, it (p. 380) might be desirable to tune—if possible—certain environmental parameters, so as to control their initiation and propagation throughout the film.

A very common strategy employed to enhance the critical current density ( $J_c$ ) consists in inserting structures of pinning centers (PCs), with sizes ranging from nano- to micrometers. Besides promoting an increase of the capacity of carrying currents without losses, artificially lithographed structures in superconducting thin films have also revealed new features of the interaction between vortices and PCs. For instance, superconductors decorated with a regular arrangement of artificial pinning centers exhibit commensurability effects between the periodic pinning landscape and the vortex lattice (Harada et al. 1996), resulting in local maxima in magnetization loops at low fields and high temperatures. These maxima, indicative of an enhanced pinning capability at the corresponding values of the applied field, signify that the arrangement of pinning wells has peaks of efficiency when the “vortices per pinning center” ratio approaches some special numbers.

On the other hand, an experimental fact, known for a long time, is that flux avalanches only occur in films having large enough critical current densities (Gheorghe et al. 2008), so that samples with larger critical current densities are expected to exhibit avalanches. Therefore, the insertion of arrays of pinning centers not only enhances  $J_c$  but also promotes avalanches, in the sense that the instability region in the field-temperature ( $HT$ ) diagram occupies a larger area as compared to the equivalent for a pristine film.

This chapter deals with the occurrence and the morphology of flux avalanches in superconducting thin films with arrays of mesoscopic through holes (antidots, ADs). The influence of different AD geometries, as well as of the lattice symmetry, on the guidance and consequently the branching of flux avalanches, will be discussed.

The chapter is organized as follows. Sec. 13.1 gives a brief introduction to magnetic flux avalanches in superconductors, with special attention to the thermally driven avalanches. The effects of inserting artificial pinning centers in superconducting films on the vortex dynamics, including their features in flux avalanches are treated in Sec. 13.2. Sec. 13.3 offers a rich scenario of avalanche morphologies provoked by their interactions with different AD geometries and lattice symmetries caused by current crowding effects. Finally, a summary is presented in Sec. 13.4.

## 13.1 Avalanches in superconductors

### 13.1.1 Vortex avalanches

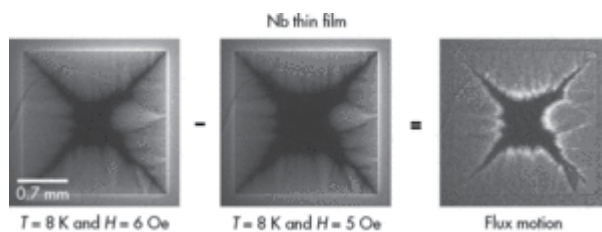
When a hard type-II superconductor is submitted to an external magnetic field, flux penetrates into the sample in a smooth way, establishing a gradual distribution with the maximum on the edge of the superconductor. The so-called critical (p. 381) state models, as firstly presented by Charles Bean in the early 1960s (Bean 1962, 1964), constitute a useful framework to explain the existence of a large hysteresis in the magnetic properties of superconducting materials, as a consequence of the existence of inhomogeneities acting as pinning centers. These models are capable of providing not only qualitative but, in many instances, quantitative descriptions of the critical current of the specimen, which is the central property for applications.

From a macroscopic point of view, currents are induced in the region of the sample where the flux penetrates, to counteract the change in the internal magnetic field, obeying Maxwell's equations. This is true for both superconducting bulk samples and thin films, though for the latter, in the perpendicular geometry, currents flow not only in the flux-penetrated periphery, but rather throughout the whole specimen, due to strong demagnetization effects (see sections 8.4–8.6 in Brandt 1995, Baziljevich et al. 1996). In the flux-penetrated region, the macroscopic current corresponds to the maximum current which the superconductor can carry without any loss—the critical current—which justifies the term “critical state.” Although the critical state model is successful in determining  $J_c$  from magnetic measurements, it is unable to provide a detailed description of the vortex distribution or any indication about the dynamics inside the type-II superconductors.

Microscopically, the critical state models can be described considering that quantum units of flux, so-called vortices, nucleate at the edges and penetrate the sample. Since the vortices are captured by PCs, the pinning force acts—up to its maximum capability, ultimately determined by the nature of the pinning center—against the Lorentz-like force (Chen et al. 1998) generated by the shielding currents. Disregarding other forces, the balance between those two leads to a metastable state: while trying to keep its center free from vortices, the superconducting specimen employs all means—namely, its highest

possible lossless current—to avoid flux penetration, resisting the external magnetic pressure. However, once penetrated, a vortex is pushed toward the center, being possibly anchored by the nearest available PC. Following this reasoning one can easily conclude that the vortex density decreases from the edge to the center of the sample.

In order to build up a picture of the magnetic dynamics of the Bean model, de Gennes (1966) proposed an analogy between critical state models and a sand pile. When sand is added to the tip of a sand pile, the angle between the horizontal plane and the cone generatrix will increase until its critical value—namely, the angle of repose (see Altshuler and Johansen 2004 and references therein). If more grains are added to the pile tip, a readjustment occurs (i.e. an avalanche), so that the angle is maintained at its critical value. In superconductors, small adjustments of the current to maintain it at the critical value lead to vortex movement when the applied magnetic field is ramped slowly. As in sand piles, or even in snowslides, vortex avalanches with small amounts of flux lines occur in order to maintain the critical state in force. (p. 382)



Click to view larger

Fig. 13.1 Direct visualization of the flux distribution as described by the critical state models using MOI technique in a 140 nm thick Nb thin film. The left and center images were taken after a zero field cooling (ZFC) procedure at 8 K and applying a perpendicular field of 6 Oe and 5 Oe, respectively. Brighter regions in left and center panels indicate areas where magnetic flux is stronger. Right panel: differential MO image obtained by subtracting the central image from the left one; although a non-zero difference is seen throughout the whole sample, the highest intensity, close to the center, indicates that the highest difference occurs at the flux front.

Fig. 13.1 presents the magnetic flux landscape recorded by means of MOI in a Nb thin film with area  $2 \times 2 \text{ mm}^2$  and thickness of 140 nm. The left and center panels show magneto-optical (MO) images after cooling the film down to 8 K in zero field—below its critical temperature ( $T_c$ ) of 9 K—and then applying a perpendicular field of 5 Oe (center) and, subsequently, 6 Oe (left). Regions where magnetic flux is absent appear as dark, whereas the maximum local field corresponds to the brightest intensity. Both images show flux distributions as predicted by critical state models for superconducting films

(Jooss et al. 2002): due to the large demagnetization factor, the intensity is the highest near the border of the sample, decreasing toward its center. As expected, the flux front penetrates deeper for larger applied fields. Also seen on the images are fanlike penetration patterns, caused by local defects at the right and bottom edges (Schuster et al. 1994, Vestgård et al. 2007, Brisbois et al. 2016). On the right panel, the differential MO (Soibel

et al. 2000), created by subtracting the central image from the left one, evidences the overall effect of the small vortex avalanches, that occur in response to the field increase, a mechanism through which the flux distribution is self-adjusted in order to maintain the critical state regime. Since the local temperature of the region where such small avalanches occur is essentially constant, we refer to them as isothermal avalanches. The most prominent feature of the image is the brighter rim near the flux front, indicating that flux was reaching regions which thus far had been free from vortices. As a matter of fact, in length scales comparable to the vortex size, the flux penetration process develops in a discontinuous way, through small bursts (p. 383) of vortices invading the sample toward its center. In order to register a map of those randomly distributed isothermal avalanches, one would require an experimental setup with spatial resolution capable of resolving individual vortices (Bending et al. 2004), which is rarely the case in existing facilities throughout the world. In reality, Fig. 13.1 represents the result of a large number of such isothermal avalanches which occurred throughout the film as the applied magnetic field was increased from 5 Oe to 6 Oe.

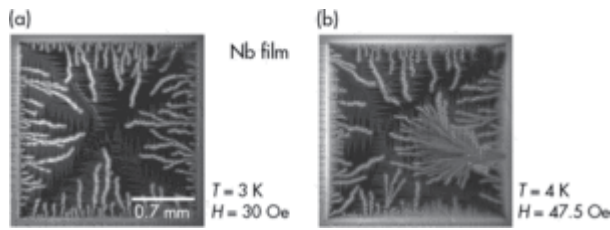
Besides these isothermal avalanches occurring to maintain the critical state, there is a second type of avalanches in superconductors which are the subject of this chapter. These avalanches occur suddenly during nearly adiabatic thermal runaways and, for this reason, are called thermally driven avalanches (Altshuler and Johansen 2004) or, very frequently, simply avalanches. They destroy locally the critical state and sometimes may even be harmful for the maintenance of the superconducting state.

### 13.1.2 Flux jumps and flux avalanches

Since the early 1960s huge jumps have been recorded in the magnetization curves of superconducting bulk samples. Kim and co-workers (1963) have named them flux jumps, highlighting that such odd events can destroy partially or even completely the superconducting state when the magnetic field is ramped. The very first observation of these abrupt penetrations was made seven years earlier by Schawlow (1956), when he was studying the surface energy of the intermediate state of several pure metals with different geometries using the Bitter decoration technique\* (Bitter 1931, 1932).

In 1967, Wertheimer and Gilchrist (1967) reported images of flux jumps in Nb and NbZr disks using MOI. Upon increase of the applied field, the occurrence of non-standard flux jumps in NbZr disks was reported. Bubbles were observed in the helium bath, indicating that a significant amount of heat was released during the abrupt penetration of magnetic flux. Similar penetrations also appear on imaging experiments of thin films, which are manifestations of flux invasions in the form of dendritic morphology<sup>†</sup> and will be discussed later in this chapter.

More recently, J.I. Vestgård and co-authors (2011) have found by numerical simulations that the temperature during the evolution of an avalanche can become momentarily higher than  $T_c$  along the paths of the flux invasion in a thin film. This seems to be in agreement with early observations of Durán et al. (1995) who showed a disordered vortex lattice similar to that obtained in a vortex liquid. Thus, to be on the safe side—considering that vortices do not exist in the normal state—one should replace the term “vortex avalanche” by “flux avalanche” when referring to thermally driven avalanches in thin films. Fig. 13.2 shows flux avalanches for a Nb thin film with thickness of 140 nm, as captured by MOI. At low temperatures and low fields, the flux jumps are generally fingerlike with (p. 384) few ramifications, as illustrated in Fig. 13.2a. Fig. 13.2b presents an image for which the field and the temperature are higher; on the right edge, one can see a huge and multi-branched avalanche, a feature which appears typically at higher temperatures and fields. The fingerlike jumps on the other edges were triggered at lower fields. Also noticeable at the borders is the subtle advance of a front of smooth penetration.



Click to view larger

Fig. 13.2 Flux avalanches observed by MOI in a 140 nm thick Nb thin film. (a) Image of fingerlike dendrites taken at  $T = 3$  K and  $H = 30$  Oe. (b) At a higher temperature and applied field,  $T = 4$  K and  $H = 47.5$  Oe, a huge multi-branched avalanche takes place, after fingerlike dendrites triggered at lower fields. The tooth-saw-like magnetic domain walls are artifacts due to in-plane magnetization domains of the MO layer.

This hypothesis of thermal origin for the flux jumps had already been addressed by Kim et al. (1963). Five years later, a detailed treatment of the flux jump problem was carried out by Swartz and Bean (1968) and, later on, by Mints and Rakhmanov (1981) and by Wipf (1991), all of them considering the stability conditions in type-II superconductors. The mechanism for the occurrence of flux jumps is based on the fact that the heat generated by the viscous movement of

vortices in a small region of the superconductor can result in a thermomagnetic instability (TMI) extending to much larger regions of the superconductor, which could even transit to its normal state, depending on the relative importance between diffusivities of the magnetic flux ( $D_m$ ) and heat ( $D_t$ ) (Mints and Rakhmanov 1981). A dimensionless parameter  $\tau$  can be defined as:

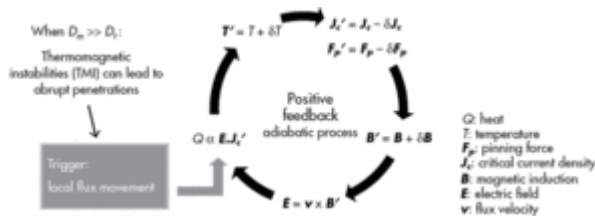
(13.1)

$$\tau = \frac{D_t}{D_m} = \frac{\mu_0 k_0 \sigma}{c}$$

where  $\kappa_0$  is the thermal conductivity,  $\sigma$  is the electrical conductivity, and  $c$  is the specific heat of the bulk superconductor. When  $\tau \gg 1$ , the heat diffusion is faster than the magnetic flux diffusion which means that the specimen has enough time to assimilate the heat generated by the flux motion. Then, the superconductor remains stable and a smooth penetration, as described by the (p. 385) critical state model, takes place. Microscopically speaking, dynamically driven isothermal avalanches build the gradient flux profile as in the critical state-like regime. However, if  $\tau \ll 1$ , that means, when the magnetic flux diffusion is faster than the thermal diffusion, the sample has no time to assimilate the heat generated by the flux motion, like in a nearly adiabatic process. As a consequence, thermomagnetic instabilities can trigger a flux jump due to a positive feedback loop, as illustrated in Fig. 13.3. Then, the Joule heating increases the local temperature  $T$  of the sample by a small amount  $\delta T$ , resulting in a reduction of the critical current density by  $\delta J_c$ , since  $J_c(T)$  is a decreasing function of the temperature. This, in turn, reduces the pinning force locally by  $\delta F_p$  as well. More flux comes into that region due to the decrease of the screening currents and the Faraday law assures that an electric field  $E$  is induced

locally. Thus, more heat is generated, leading to an additional temperature increase and a further reduction of the critical current density, and so on. Depending on the specific circumstances—sample parameters and external conditions—this process can be nearly adiabatic, leading to a local flux jump, or even to an extended jump, embracing the entire volume of the sample.

MOI opened up an opportunity to investigate flux avalanches, including the morphology and dynamics of such catastrophic events. Whereas bulk superconductors have limited morphologies for the flux jumps, thin films present unusual and impressive dendritic morphologies. The occurrence of flux avalanches seems to be a quite general feature of superconductors in the thin film geometry, and has been observed for several materials: YBCO (Leiderer et al. 1993, Baziljevich et al. 2014), Nb (Durán et al. 1995),  $\text{MgB}_2$  (Johansen et al. 2001, 2002), Pb (Menghini et al. 2005),  $\text{Nb}_3\text{Sn}$  (Rudnev et al. 2003),  $\text{YNi}_2\text{B}_2\text{C}$  (Wimbush et al. 2004), NbN (Rudnev et al. 2005), and more recently amorphous  $\text{Mo}_{79}\text{Ge}_{21}$  (a-MoGe) (Motta et al. 2013) and a-MoSi (Colauto et al. 2015). In the next (p. 386) section, MOI, a technique that allows one to explore different regimes of flux penetration, is discussed.



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Fig. 13.3 Schematic diagram for the occurrence of flux jumps. When  $D_t \gg D_m$ , a smooth penetration occurs, whereas for  $D_m \gg D_t$  sudden penetrations take place as a result of thermomagnetic instabilities. Vortex motion generates slowly spreading heat, as in a nearly adiabatic process with a positive feedback, which can be evoked as the main cause for instabilities leading to flux avalanches (after Wipf 1991).

### 13.1.3 Magneto-optical imaging (MOI)

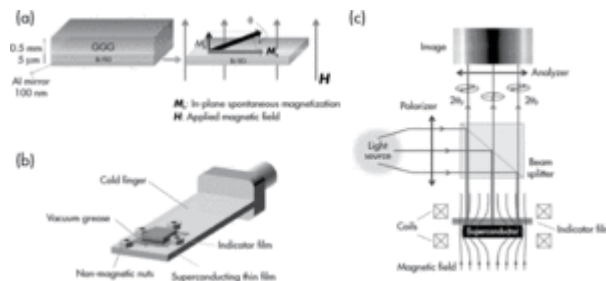
As already cited above, MOI is a quite important technique allowing one to obtain an ample and direct view of the macroscopic flux distribution in superconductors. An equally relevant feature of MOI is the possibility to observe, in real time, events consisting of relocation of magnetic flux. This technique is based on the magneto-optical Faraday effect, in which the polarization plane of a linearly polarized beam of light is rotated when passing through a Faraday-active material under a magnetic field applied parallel to the incident beam (Jooss et al. 2002).

Since the superconducting materials do not present a significant Faraday effect, Faraday-active crystals have been used to visualize the flux distribution in superconducting materials. The most currently employed indicators are ferrimagnetic films of complex structure—so-called garnet<sup>‡</sup>—exhibiting in-plane spontaneous magnetization. These indicators show a large Faraday effect in a broad temperature range—from cryogenic up to 400 K—with field response nearly temperature-independent below 150 K (Jooss et al. 2002). Since the 1980s, the most used MOI sensor has been the bismuth-substituted yttrium iron garnets (Bi:YIG) with a thickness between 2 and 5  $\mu\text{m}$  (in our case 5  $\mu\text{m}$ ), grown by liquid phase epitaxy on gadolinium gallium garnet ( $\text{Gd}_3\text{Ga}_5\text{O}_{12}$ , GGG) substrates with [100] growing direction. The in-plane spontaneous magnetization ( $M_s$ ) exhibited by these indicators tilts out of the plane by an angle  $\phi$  due to the perpendicular component of the magnetic field which might, for instance, be due to the stray field of a nearby superconductor. The Faraday rotation is proportional to the out-of-plane component ( $M_z$ ). A schematic representation of the indicator film is shown in Fig. 13.4a as well as the stack of layers and their thicknesses for the Bi:YIG films employed in the experiments presented here.

A MO setup is essentially composed by a polarized light microscope, a cryostat with an optical window, electromagnetic coils to apply a magnetic field, and a camera to record the images. The superconducting thin film is carefully positioned on the cold finger with a tiny amount of vacuum grease to assure a good thermal contact between them. The indicator film is directly placed on top of the superconducting film to reveal the magnetic flux distribution. A scheme of the experimental assembly is illustrated in Fig. 13.4b. In our case, non-magnetic nuts are added around the assembly to give mechanical support and to avoid any undesired movement of the indicator film. Once the sample-plus-indicator assembly is mounted on the cold finger, the whole set is placed into the cryostat.

As illustrated in Fig. 13.4c, a light source emits a visible light beam of high intensity, which passes through a polarization filter (polarizer). Then, the (p. 387) polarized beam is deflected by a beam splitter, reaching the indicator film. The light passes through the GGG substrate, which has a low light absorption coefficient, and passes through the magneto-optical layer (MOL), rotating an angle  $\theta_F$  at places where the MOL has a nonzero  $M_z$  component. After that, the mirror layer reflects back the light which returns to the MOL, rotating  $\theta_F$  again, in the same sense. Thus, the total rotation angle of the polarization plane is  $2\theta_F$ , as presented schematically in Fig. 13.4c. Then, the beam passes again through the beam splitter and reaches the analyzer filter. This analyzer is set at an angle close to  $90^\circ$  relative to the polarizer,<sup>§</sup> allowing only rotated light to go through. A CCD

(coupled charged device) camera captures the intensity of the rotated light, producing a map of light intensities proportional to the rotation angle, forming an image of the flux distribution landscape of the superconductor (Jooss et al. 2002, Altshuler and Johansen 2004). The image at the top of Fig. 13.4c illustrates the initial stage of smooth penetration in a superconducting film, where brighter intensities correspond to more intense local fields.



[Click to view larger](#)

Fig. 13.4 (a) Schematic representation of the indicator and the MOL with spontaneous magnetization in plane. A component out-of-plane ( $M_z$ ) arises under the influence of the perpendicular magnetic field. Scheme (b) sketches the grease-superconducting thin film-indicator assembly on the cold finger (after Colauto 2008) and (c) outlines the MO setup. Panels (a) and (c) adapted with permissions from Johansen et al. (1996) and from Altshuler and Johansen (2004), respectively.

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The MOI technique presents a typical magnetic field sensitivity of around  $10^{-5}$  T, and a spatial resolution on the order of  $0.8 \mu\text{m}$ . Among other techniques employed to map magnetic flux distributions, it offers the possibility to go from large fields of view, on the order of centimeters, down to single vortex resolution (Goa et al. 2001). Moreover, the temporal resolution, which reaches 10 ns, is (p. 388) another relevant characteristic of the MOI technique, allowing, for instance, Bolz and co-authors (2003) to measure flux avalanche velocities of about 100 km/s in YBCO thin films.

In view of such characteristic features, dynamical processes can be investigated in a wealth of details by means of MOI, since it is possible to obtain images with a resolution of a few microns in a short time, allowing one to observe in real time changes of the magnetic flux distribution in superconducting materials. These features make MOI a very powerful technique through which flux avalanches of different morphologies are observed in superconducting films. Subsequent thorough studies paved the way for improvement of a theoretical model which is not only capable of describing in detail the general features of this fascinating phenomenon, but even predicting some of its distinctive characteristics.

### 13.1.4 Thermomagnetic model

The thermal origin of the instabilities leading to abrupt penetration of magnetic flux into a superconductor is a well-established idea. Several theoretical studies have been developed in order to map the dynamics of the flux avalanches (Maksimov 1994, Baggio et al. 2005, Denisov et al. 2006). The most successful description has been produced via the Thermomagnetic (TM) model, which allows comparisons with experiments with a high degree of correspondence, as will be discussed later.

Considering a more general description, which includes a superconducting thin film subject to a magnetic field perpendicular to the film plane, the TM model is written using the Ampère and Faraday laws and the heat diffusion equation:

(13.2)

$$c\dot{T} = \kappa_0 \nabla^2 T - d^{-1} h_0 (T - T_0) + d^{-1} j E$$

where  $h_0$  is the coefficient for heat transfer to the substrate held at the temperature  $T_0$ ,  $d$  is the thin film thickness,  $E$  is the electric field, and  $j$  is the sheet current related to the current density  $J$  by  $j = Jd$ . This equation provides the temperature distribution of the specimen and the second term on the right side refers to the thermal link between the superconducting thin film and the substrate. In addition, the nonlinear current-voltage relation of type-II superconductors, which is conventionally approximated by a power law, is also considered:

(13.3)

$$E = \frac{\rho_0}{d} \left( \frac{j}{j_c} \right)^{n-1} j$$

where  $j = |j|$ ,  $n$  is the creep exponent, and  $\rho_0$  is a resistivity constant. The heat diffusion equation is solved together with Eq. 13.3 and Maxwell's equations. The time evolution of the magnetic properties inside the sample is obtained by inverting the Biot-Savart law (Vestgård et al. 2011). In this model, the heat capacity, (p. 389) the thermal conductivity, the thickness, the critical current density, and the heat transfer to the substrate strongly influence the occurrence of thermomagnetic instabilities.

One of the most remarkable predictions of the TM model is the existence of a threshold flux penetration depth ( $l^*$ ) above which a thin superconducting stripe with a width of  $2w$  experiences the onset of dendritic flux avalanches, that is, a condition for the instability onset. This critical value  $l^*$  means that if the flux penetration depth ( $l$ ) from the edge of the sample is smaller than  $l^*$  when magnetic field is applied starting from zero, the sample is stable and a critical state penetration takes place. However, when  $l \geq l^*$ , the model establishes that there is enough penetrated flux for the sample to become unstable, and TMIs trigger flux avalanches (Denisov et al. 2006a). As a consequence of the existence of  $l^*$ , it is possible to identify an onset field to trigger flux avalanches, the so-called threshold magnetic field ( $H^{th}$ ). For a long stripe with width of  $2w$  obeying the Bean model,

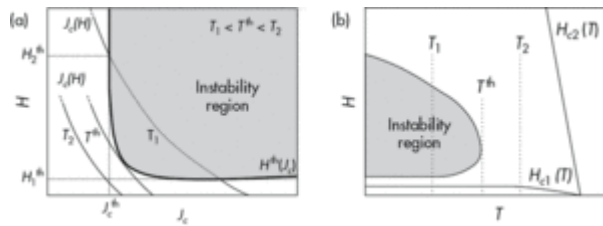
(13.4)

$$H^{th} = \frac{J_c d}{\pi} \operatorname{acosh}\left(\frac{w}{w - l^*}\right)$$

where  $w - l^*$  is the position of the flux front measured from the center of the sample.

The critical penetration depth  $l^*$  also depends on  $J_c$ , which allows one to obtain the stability/instability limit in an applied magnetic field versus critical current density diagram. Fig. 13.5a is an  $H \times J_c$  diagram showing the threshold line (thick continuous),  $H^{th}(J_c)$ , and three isothermal curves (thin),  $J_c(H)$ , representing the typical field dependence of the critical current density at different temperatures. At lower values of  $T$  (e.g.  $T_1$ ),  $J_c(H)$  crosses the threshold curve twice, defining a lower and an upper limit for the occurrence of avalanches,  $H_{1th}$  and  $H_{2th}$ , respectively. As the temperature increases, the difference between these threshold fields decreases, there being a limiting temperature,  $T^{th}$ , above which  $J_c(H)$  and  $H^{th}(J_c)$  do not cross each other ( $T_2$ ), so that no avalanches can occur. One can then envisage that the portion of the  $HT$  diagram where thermomagnetic instabilities occur consists of an enclosed region, as depicted in Fig. 13.5b: at small enough values of  $T$  (e.g.  $T_1$ ), as the field  $H$  is increased from zero in an isothermal process, avalanches start to occur when the threshold penetration depth is achieved; however,  $l^*$  increases with  $H$  [see Eq. 1 in Denisov et al. (2006a)] and eventually exceeds the lateral dimensions of the film so that avalanches become unviable in this upper limit, at which  $l^* = w$ ,  $H^{th}$  diverges (Yurchenko et al. 2007), and  $J_c = J_c^{th}$ , the threshold value of the critical current, below which no avalanches occur. The upper limiting temperature on the instability region,  $T^{th}$ , represents the case for which the curves  $H^{th}(J_c)$  and  $J_c(H)$  (Fig. 13.5a) touch each other in a single point of tangency.

In order to shed light on the morphology of flux avalanches, Denisov et al. (2006) have shown that the dendritic avalanches are more likely to occur in thin films than in bulks as a consequence of the demagnetization effects, requiring (p. 390) treatment using non-local electrodynamics. For bulk samples, the flux jump shape depends on the electric field background, there existing a threshold value ( $E_c$ ) separating uniform (below  $E_c$ ) from fingering jumps (see above). In the case of thin films, the heat transfer coefficient between the superconducting film and the substrate is also important, in the sense that, above a certain critical value, avalanches appear in dendritic patterns, independent of  $E$ . Moreover, temperature plays a crucial role on the morphology of the flux avalanches in films. At lower temperatures, the flux patterns are small, narrow, with few branches, and sometimes occur as in fingerlike shapes. However, at higher temperatures, close to  $T^{th}$ , the avalanches are larger and highly branched (Johansen et al. 2002).



Click to view larger

Fig. 13.5 Schematic diagrams of the region at which flux avalanches occur. Panel (a):  $H \times J_c$  (after Yurchenko et al. 2007). Panel (b):  $H \times T$  (after Colauto et al. 2007).

Recently, Vestgård et al. (2013, 2015) have found by numerical simulations that varying some parameters, such as the thickness, the Joule heating, the thermal diffusion, and the heat transfer to the substrate, one can also modify the branching and size of the flux avalanches. For a thicker sample, the field to

trigger the very first jump is higher, and the jumps become larger and with fewer ramifications than for thin samples. When the Joule heating, related to the positive feedback, is increased the avalanches are larger and more branched. For larger thermal diffusion, the dendritic behavior smears and the avalanches adopt a thick fingerlike morphology. Improving the heat transfer to the substrate provokes a decrease in the avalanche size, with fewer and thinner branches. Nevertheless, an experimental confirmation of these findings is still lacking, since it is very hard to control most of those parameters in a systematic way.

Once again, it is worth mentioning that simulations have shown that the temperature at the core of the flux avalanche can be higher than  $T_c$  during its evolution through the superconducting material. Besides that, Vestgård et al. (2011) have found values for the velocity of an avalanche which are compatible with those so far measured in real experiments (Leiderer et al. 1993, Bolz et al. 2003), reaching tens of km/s. Also, the avalanche velocity is not constant, being larger at the initial steps of the propagation process, and decreasing markedly at later stages. Early investigations of non-dendritic flux jumps in thick Nb samples by Wertheimer and Gilchrist (1967) show more modest average flux front velocities strongly dependent on the thickness of the Nb (for instance, around 50 m/s for (p. 391) thickness of 100  $\mu\text{m}$ ). The authors also report a fast exponential increase of the flux velocity at the early stages of the avalanche formation, followed by a progressive slowing down as it further evolves toward the center of the sample.

The TM model has provided consistent information about the flux avalanche regime and has also been an important aid to the understanding of some experimental data. Since the purpose of this chapter is to describe avalanches in decorated superconducting films, the following section discusses the influence of arrays of artificial pinning centers in the superconducting properties of thin films.

## **13.2 Artificial pinning centers in superconducting films**

### 13.2.1 Enhancement of the critical current density

The pinning force is caused by virtually any kind of inhomogeneity or defect distributed throughout the material. These inhomogeneities, such as impurities, vacancies, stoichiometric defects, grain boundaries, dislocations, voids, thickness variation of a film, etc., suppress the superconducting properties at that point or region. They reduce the condensation energy related to the nucleation of the normal core of the vortex and act as attractive potential wells holding the vortices in place and avoiding their viscous motion, thus minimizing the free energy of the system. Such naturally created pinning centers, which are randomly distributed, cause distortions in the Abrikosov lattice (Abrikosov 1957) due to the landscape of forces acting on the vortex system and destroying the translational long-range order. This mechanism of pinning, called core pinning and first described by P.W. Anderson (1962), is more effective when the dimensions of the inhomogeneities approximately match with the coherence length  $\xi$  of the superconducting material.

Besides the intrinsic pinning, artificial pinning centers can be introduced in type-II superconductors to optimize and trap the vortex lines. This strategy may increase the critical current density and consequently, the potential technological functionality of the material. One way to insert these defects is by means of irradiating the specimen with high-energy heavy ions (Civale et al. 1991), creating randomly distributed cylinders—known as columnar defects—of non-superconducting material of diameter  $\sim \xi$ , having a distribution of pinning strengths.\*\* This approach has been largely used in the study of vortex matter (p. 392) in the high-temperature superconductors where  $\xi$  is typically of the order of a few nanometers, similar to the typical size of defects that can be created by such irradiation schemes (Banerjee et al. 2003).

Although the understanding of the pinning strength of individual imperfections for single vortices is important, the collective effect involving many pinning centers and many vortices is essential to immobilize the vortices efficiently (Campbell and Evetts 1972). Bearing this in mind, fabrication of periodic arrays of pinning centers in superconducting films became a goal to be achieved and, indeed, its realization brought to the scene a rich diversity of new vortex phenomena such as commensurate pinning effects (Harada et al. 1996), vortex rectification (see Silhanek et al. 2010 and references therein), manifestation of multiquanta vortices (Metlushko et al. 1994, Moshchalkov et al. 1996), and composite flux-line lattices (Baert et al. 1995, Silhanek et al. 2005).

In fact, recent advances in micro- and nanofabrication have allowed the preparation of superconducting thin films decorated with arrays of artificial pinning centers. Among the fabrication procedures, lithography techniques, such as electron beam lithography, have made it possible to attain distribution of pinning sites with sizes and separations comparable to the superconducting characteristic length scales, that is, the temperature dependent magnetic penetration depth  $\lambda(T)$  and coherence length  $\xi(T)$ , ranging from micro- to nanometers. As a consequence, there is a two-dimensional (2D) lateral

modulation—due to the presence of the defect array—which leads to a confinement of the flux lines and, as is well known in quantum mechanics, quantization effects (Moshchalkov and Fritzsche 2011).

Besides the size and the distance among the imperfections, micro- and nanofabrication also permit one to control the distribution, the geometry, as well as the type of the defect. There are basically three different types of defects: the antidots<sup>††</sup> (Baert et al. 1995, Moshchalkov et al. 1996), the blind holes (partially drilled holes) (Bezryadin et al. 1996, Raedts et al. 2004), and the magnetic dots (magnetic material either on top of or below the superconductor) (Vélez et al. 2008).

Another mechanism allowing one to trap vortices is so-called electromagnetic pinning which takes place due to the perturbation of screening currents of the vortex near the edge superconductor/defect—there they must flow parallel to the hole edge—as imposed by the boundary conditions. Buzdin and Feinberg (1993) have studied this interaction by the method of images, in which a vortex interacts with an antivortex in the hole. Therefore, the empty AD always acts as an attractive potential for the vortices. Although a multiquanta vortex is energetically unfavorable in continuous media, and even in randomly distributed defects, relatively large artificial pinning centers placed in periodic arrays can favor its appearance. The saturation number  $n_s$ , that is, the maximum (p. 393) number of vortices that a cylindrical antidot in a lattice can pin, can be roughly estimated<sup>‡‡</sup> by:

(13.5)

$$n_s \simeq \frac{r_{AD}}{2\xi(T)}$$

where  $r_{AD}$  is the radius of the AD. When  $1 < n < n_s$ ,  $n$  is the number of flux quanta, the flux  $n\phi_0$  is captured by the defect and creates a potential barrier for other vortices far from the AD. Increasing the magnetic pressure, for instance, vortices close to the hole can overcome this barrier and enter the hole. For  $n > n_s$ , the additional vortices start to occupy interstitial positions in the superconducting region. The interesting problem of optimum AD size for maximizing the pinning was addressed theoretically by Takezawa and Fukushima (1994) and experimentally by Moshchalkov et al. (1998).

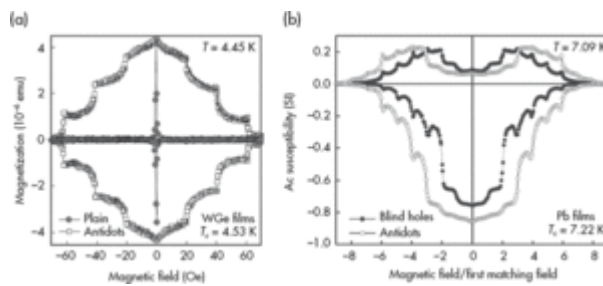
The interplay between artificial pinning centers arranged periodically, resulting in periodic pinning forces, and the repulsive interaction between vortices leads to commensurability effects (Daldini et al. 1974). It means that, when the lattice constant of the pinning center mesh and that of the vortex array are identical, a local maximum in pinning force is reached and, consequently, a local maximum in the critical current density occurs. The magnetic fields at which this matching occurs are called matching fields, being defined as (Moshchalkov and Fritzsche 2011):

(13.6)

$$H_n = n \frac{\phi_0}{A}$$

where  $A$  is the unit-cell area of the lattice of ADs. At the first matching field  $H_1$ , for instance, the equilibrium configuration is one in which each hole is, on average, occupied by a single flux quantum. Fig. 13.6 shows the matching fields obtained by dc magnetization and ac susceptibility measurements for different samples. Fig. 13.6a shows magnetization versus magnetic field curves for a  $W_{0.77}Ge_{0.33}$  thin film with holes of radius  $0.17\text{ }\mu\text{m}$  in a square lattice of  $1\text{ }\mu\text{m}$ , prepared by electron beam lithography; the matching fields are multiples of  $20.7\text{ Oe}$ . In the same graph one can see the absence of matching fields in the magnetic response of a reference plain film. Fig. 13.6b shows both components of the ac susceptibility for a Pb thin film with square holes of  $0.8\text{ }\mu\text{m}$  in a square lattice of  $1.5\text{ }\mu\text{m}$  and blind holes with the same characteristics, both with matching fields being multiples of  $9.2\text{ Oe}$ .

By observing Fig. 13.6a, one can see that the effectiveness of vortex anchorage by the lattice of ADs appears as a substantial enhancement of the critical current at temperatures close to  $T_c$ . It is evidenced by a wider and taller loop for the sample decorated with ADs as compared to the plain film. An important variable for an array of PCs is its spatial arrangement. Recently, Misko and Nori (2012) (p. 394) have predicted that a graded distribution of ADs would increase the pinning capacity of the sample. Such gradual arrangement presents a maximum density on the edges, decreasing toward the center of the sample, mimicking the vortex distribution described by critical state models for a partially penetrated sample started from the virgin state. This has been experimentally confirmed by several studies: Motta et al. (2013), Wang et al. (2013, 2016), and Guénon et al. (2013), and further theoretical efforts have been done by Ray et al. (2013, 2014) and by Reichhardt et al. (2015). In a particular case, Motta et al. (2013) verified that at low fields, a linearly graded array of ADs provides more efficient pinning than a uniform distribution, that is, the critical current density for the sample patterned with a graded distribution is enhanced as compared to a homogeneous distribution of ADs.



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Fig. 13.6 Matching fields measured using different techniques. (a)  $M(H)$  curve for a patterned  $W_{0.77}Ge_{0.33}$  and a plain film and (b) in-phase and out-of-phase components of the ac susceptibility for Pb thin films with arrays of through holes (open symbols) and blind holes (filled symbols) (after Raedts et al. 2004). Panel (a) adapted with permission from Moshchalkov et al. (1996).

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Concerning the improvement of the critical current density, nanostructured superconducting films have made it possible to attain the depairing current density,  $J_{dp}$ . This is the ultimate limit, that is, the highest dissipationless current achievable, being reached when the kinetic energy due to the supercurrents becomes larger than the binding energy of the Cooper pairs. In practice, the critical current is limited by the vortex depinning process in most of the samples and consequently,  $J_c$  is orders of magnitude smaller than

$J_{dp}$ . Conceivably,  $J_c = J_{dp}$  can be reached in a superconducting strip of width smaller than  $4.4 \xi(T)$ , so that vortices cannot fit inside the sample (Likharev 1979). Xu et al. (2010) claim to have achieved  $J_{dp}$  in superconducting nanomesh films with dimensions smaller than both  $\lambda$  and  $\xi$ . The authors argue that the supercurrents in such mesh can find alternative paths, avoiding resistive regions caused by thermal fluctuations and thus keeping  $J_{dp}$  as the upper current limit for a larger temperature window. By using resistivity measurements, Córdoba et al. (2012) have described a reentrance of the superconducting state for low temperatures and higher fields in thin films decorated with a square lattice of holes; according to the authors, the sample supports applied currents as high as  $J_{dp}$ . Arguably, these findings are of (p. 395) practical importance for technological applications, allowing for development of more efficient devices.

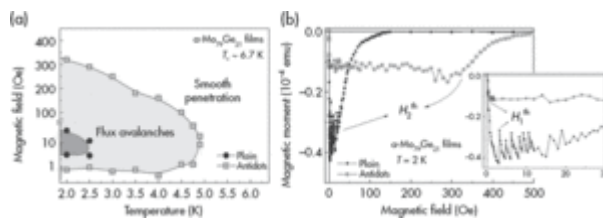
To summarize this section, we have reviewed that nanostructured superconducting films have attracted much attention as a test bench for improving some superconducting critical parameters, most specially the current carrying capacity. A variety of new vortex phenomena has arisen due to the introduction of neatly fabricated and well-controlled arrays of artificial pinning centers in films, enabling one to understand the interaction of the vortex lattice with such arrangements of pinning centers at the microscopic level. The effects of underlying arrays of antidots on the abrupt flux penetration due to

thermomagnetic instabilities, as well as the guidance of flux penetration, are discussed in the next section.

### 13.2.2 Guidance and flux avalanches

Apart from the above-described effects related to interactions among vortices and arrays of PCs, flux avalanches are also influenced by the existence of an underlying landscape of pinning centers. Such interaction is quite strong, resulting in the occurrence of flux avalanches in a larger window of temperatures and magnetic fields and, furthermore, in the existence of guidance of the intriguing ramifications in preferential directions, which is described below.

Firstly, the existence of the antidot lattice increases the instability region in the magnetic field versus temperature diagram. Fig. 13.7a shows the region where the avalanches take place for 25 nm thick amorphous  $\text{Mo}_{79}\text{Ge}_{21}$  films (p. 396) with and without a square lattice of square ADs having a lattice parameter of 1  $\mu\text{m}$  and AD side of 0.5  $\mu\text{m}$ . It is evident from the figure that the area in the diagram referring to avalanche activity is larger for the patterned film (light gray) as compared to the plain film (dark gray), with upper limiting temperatures of 2.5 K and 4.9 K, respectively. For Pb films with and without ADs, such difference in  $T^{\text{th}}$  is smaller, around 0.5 K (Menghini et al. 2005), probably due to the fact that the intrinsic pinning is stronger in Pb than in a-MoGe and hence, the presence of AD arrays impacts the latter more considerably (Motta et al. 2014).



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Fig. 13.7 Occurrence of flux avalanches in a-MoGe films. (a)  $HT$  diagram showing the region dominated by thermomagnetic instabilities for a plain 25 nm thick a-MoGe and for a sister film decorated with a square array of square ADs. (b) Flux jumps in the magnetic moment against field curves for each of the samples, taken at 2 K, where one can identify the upper threshold limit ( $H_{2th}$ ). The inset shows the lower threshold field ( $H_{1th}$ ) needed to trigger the first avalanche. Panel (a) adapted with permission from Motta et al. (2013).

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In order to obtain the boundary between smooth entrance and abrupt penetration illustrated in Fig. 13.7a, the field-dependence of the magnetic moment [ $m(H)$ ] was measured at different temperatures. Fig. 13.7b shows  $m(H)$  taken at 2 K for those a-MoGe samples, where one can see that both curves present a jumpy behavior, which is a fingerprint of flux avalanches when observed using this technique. The boundaries are traced by taking the first jump, that is,  $H_{1th}$ , highlighted in the inset, and the last flux jump ( $H_{2th}$ ) for several temperatures (Colauto et

al. 2007), as illustrated in the graphs. As already mentioned above, the magnetic field range of such jumps at this temperature is larger for the patterned sample, evidencing that arrays of antidots are inducers of flux avalanches (Hébert et al. 2003, Silhanek et al. 2004, Menghini et al. 2005, Motta et al. 2011, 2013). Another aspect seen in the inset to Fig. 13.7b is the fact that the jump heights are smaller in the decorated film as compared to the equivalent feature for the plain specimen.

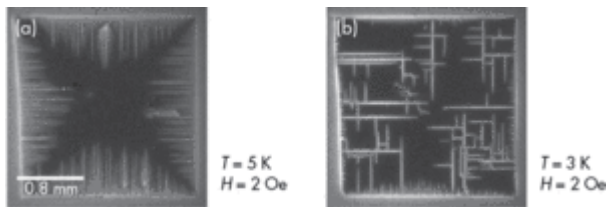
The experimental fact that the region in the  $HT$  diagram corresponding to avalanches occasioned by thermomagnetic instabilities is larger for the patterned film can also be interpreted in the light of the TM model. The plain film has a lower critical current density and its  $J_c(H)$  curve in Fig. 13.5, even at low temperatures, crosses the  $H^{th}(J_c)$  limiting curve relatively close to its vertex. For  $T > T^{th}$ ,  $J_c(H)$  does not cross the  $H^{th}(J_c)$  line and, consequently, avalanches do not happen anymore. When an array of ADs is inserted in the film, the critical current density is enhanced and then the  $J_c(H)$  curves shift to the right, as also illustrated in Fig. 13.5a. Thus, the temperature needed to bring  $J_c(H)$  outside the instability region has to be higher than for a plain film, resulting in an increase of the region area in the  $HT$  diagram where the avalanches occur. Such an increase of the avalanche activity has been also observed when arrays of magnetic dots in superconducting films are magnetized parallel to the applied magnetic field, increasing the

pinning force on the vortices and, consequently, leading to higher critical currents (Gheorghe et al. 2008).

Moreover, arrays of antidots induce easy directions, which depend on the lattice symmetry and the AD geometry, as demonstrated in the next section. Flux entering the sample experiences guided motion, regardless of the invasion being smooth or abrupt. Also known as vortex channeling, it has been observed by Pannetier et al. (2003), using MOI in an  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  thin film disk with a square array of circular ADs. Under conditions of smooth penetration, the (p. 397) authors observed that flux fronts form a macroscopic square, which is nothing but an imprint of the fourfold symmetry of the lattice of ADs. It occurs since the vortices are guided preferentially along the channels parallel to the principal axis of the array, that is, along the rows of ADs, leading to an anisotropic penetration into the disk. Besides that, a square sample patterned with a square array of ADs whose main axes are parallel to the sample edges exhibits channels and a deeper penetration as compared to a sister sample with rows of ADs aligned along the sample diagonal—that is, rotated  $45^\circ$  from the edges—for which no channels are seen. Therefore, the vortex channeling reduces the shielding capacity of patterned thin films. Fig. 13.8a illustrates the smooth penetration regime taken at  $T = 5 \text{ K}$  and  $H = 2 \text{ Oe}$  for a Nb thin film with thickness of  $20 \text{ nm}$  decorated with a square array of ADs with lattice parameter of  $1.5 \mu\text{m}$  and AD size of  $0.4 \mu\text{m}$ . Both panels (Fig. 13.8a–b) are fair examples of vortex channeling along the AD rows perpendicular to the edges of the sample, both in the critical state regime (a) and in the avalanche regime (b).

Similar results were obtained by Silhanek et al. (2003) by using magnetotransport measurements. By controlling the orientation of the current in-plane, they have concluded that the vortex velocity follows the principal axis of the lattice even when the Lorentz force, induced by the applied current, is misaligned up to  $15^\circ$ . Along the easy direction, the vortex velocity is higher than along the diagonals (hard direction) and hence the mobility shows an angular dependence. Furthermore, the anisotropic penetration disappears gradually at temperatures close to  $T_c$  since the pinning efficiency vanishes.

In a rectangular array of PCs, guided vortex motion is facilitated along the short side of the array, whereas pinning is stronger along the long side. Therefore, a critical current anisotropy is observed, which leads to a lower limit value along the short side of the lattice, thus resulting in anisotropic flux penetration (Welling et al. 2004, Vélez et al. 2008). Interestingly, a similar result, that is, anisotropic (p. 398) critical current density, was found for a square array of rectangular ADs, in which the critical current is higher along the long side of the hole, as compared to the equivalent short side (Gheorghe et al. 2006).



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Fig. 13.8 MO images of the flux penetration in a 20 nm thick Nb thin film patterned with a square array of ADs with lattice parameter of  $1.5\ \mu\text{m}$  and AD size of  $0.4\ \mu\text{m}$ . (a) Channeling in the smooth penetration regime, along the main axes of the array, taken at 5 K and 2 Oe; and (b) flux avalanches along the main axis of the array, taken at 3 K and 2 Oe.

Superconducting thin films decorated with arrays of magnetic dots have also presented channeling and an anisotropic critical current density, depending on the magnetic state of the dots as well as on the orientation of the array relative to the edges of the sample. Gheorghe et al. (2008) have studied a variety of Pb films patterned with square arrays of Co/Pt magnetic dots. Of particular interest here are two decorated

samples of square shape, one with rows parallel to the sample borders ( $S_s$ ) and the other with rows aligned to the diagonals ( $S_d$ ). A clear contrast in the flux penetration behavior appears when the magnetization of the dots is aligned with the applied field: sample  $S_s$  exhibits channeling, whereas sample  $S_d$  does not. Such difference occurs due to the fact that, in the latter case, vortex motion is inhibited, since the Lorentz forces are perpendicular to the edges. Besides that, an anisotropic distribution of currents was observed in the flux front penetration of a circular sample. When the dots were demagnetized, there was no influence of the pinning centers and the flux fronts were circular. However, when the dots were fully magnetized, that is, parallel to the applied field, the flux front exhibited a fourfold symmetry. Thus, the lattice symmetry and magnetic state of the underlying array of magnetic dots influence the flux penetration in such patterned superconducting films.

In the flux avalanche regime, penetration along preferential directions has been observed by several authors using MOI (Vlasko-Vlasov et al. 2000, Menghini et al. 2005, Gheorghe et al. 2006, 2008, Motta et al. 2011, Zadorosny et al. 2013). Vlasko-Vlasov et al. (2000) have pointed out that in a Nb film patterned with a lattice of holes, the main trunk and the secondary branches are guided along the AD rows for increasing and decreasing applied fields. Different experiments conducted under the same conditions presented the same overall patterns, that is, dendritic formations along the easy axes of the AD lattice, although randomly distributed across the sample area. Thus, even though guidance results from the presence of the array of PCs, the stochastic nature of the process is maintained, just as in the case of plain films. Analogous results were also obtained in Pb and also Nb samples with different AD sizes by Menghini et al. (2005), Gheorghe et al. (2006), Motta et al. (2011), and Zadorosny et al. (2013). Fig. 13.8b presents an example of AD-guided dendrites following the principal axis of the lattice taken at  $T = 3\ \text{K}$  and  $H = 2\ \text{Oe}$  for a Nb

thin film decorated with a square array of ADs with lattice parameter of  $1.5\ \mu\text{m}$  and AD size of  $0.4\ \mu\text{m}$ .

Menghini et al. (2005) have described the evolution with applied field and temperature of the dendrite morphology in Pb films decorated with square patterns of ADs. At isothermal conditions and increasing fields, once an avalanche is triggered, its shape is frozen while new avalanches may appear elsewhere. Such a feature has already been discussed in plain films by Johansen et al. (2001). At low temperatures, the flux avalanche morphology is essentially fingerlike (not branched) and following the direction toward the center of the sample. At higher temperatures, the dendrites become larger and treelike, that is, more branched. (p. 399) At the upper border of the instability regime (see Fig. 13.5b),  $T \approx T^{th}$ , a single, large, treelike avalanche takes place. Therefore, even when penetration occurs in the form of sudden invasions of flux bundles—which is a far more complex scenario than for a single vortex motion—the underlying pinning landscape also plays a non-trivial role in the morphology of the penetrated flux distribution.

Although the guidance of flux avalanches was reported more than a decade ago, the essential ingredient to understand the resulting peculiar morphology, which accounts for the subjacent AD distribution, has not been discussed in depth. This ingredient, the current crowding effect, is presented in more detail in the next section.

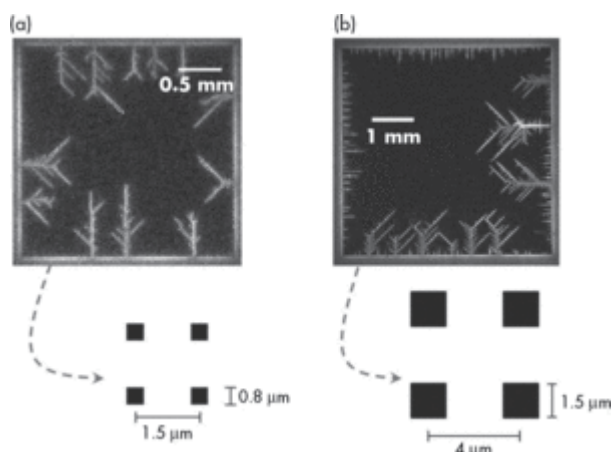
### **13.3 Effects of the antidot geometry and lattice symmetry in flux avalanches**

A typical event in nature exhibiting dendritic morphology is an atmospheric electric discharge, which leads the system cloud-air-ground to decrease its electrostatic energy. In a sense, flux avalanches in superconducting thin films belong to a similar type of complex systems and can be interpreted as its magnetic equivalent, that is, a flux avalanche corresponds to a magnetic discharge. This section compiles a comprehensive set of experiments involving magnetic avalanches in sharply patterned films. It is worth noting that the implications of the experimental findings reported in this chapter could reach domains beyond flux avalanches in superconductors, such as electric discharges in controlled non-uniform media (Kayaba and Kikkawa 2008).

Even though introducing arrays of ADs is beneficial to increase the critical current density, it is detrimental for applications of superconducting thin films at low temperatures and low fields, since it favors the occurrence of stronger and more abundant flux avalanches. This is a problem of fundamental impact on the designing of devices using superconducting films. It is thus highly desirable to be able to detect and understand magnetic flux avalanches in superconducting films decorated with different arrays of antidots which might, in turn, be influential for the triggering and guidance of the abrupt magnetic events.

Of particular interest is the role of lattice symmetry and AD geometry on the morphology of the avalanches.

Let us start by discussing the intriguing format assumed by the branches of avalanches occurring in patterned films (Motta et al. 2011a, 2014). Fig. 13.9a presents a MO image captured at  $T = 3$  K for a patterned a-MoGe film submitted to a perpendicular applied dc field of 0.7 Oe. The lattice parameter is  $1.5\ \mu\text{m}$  and the lateral dimension of the square ADs is  $0.8\ \mu\text{m}$ , with the principal axes completely aligned with the edges of the sample. Noticeably, the main trunk of each avalanche is perpendicular to the sample edges where it was rooted, following, (p. 400) as discussed earlier, the main axis determined by the AD rows. However, the secondary ramifications always emerge at  $45^\circ$  relative to the main trunk, which confers to the avalanche an aspect similar to a Christmas tree. Fig. 13.9b shows that avalanches occurring in a Nb thin film also decorated with a square array of square ADs exhibit exactly the same morphology, with the secondary branches growing at  $45^\circ$ . Although the dimensions of lattice parameters and ADs are different from one film to the other, the overall morphology of the dendrites is reproduced. Moreover, Nb is well known for having strong intrinsic pinning, whereas MoGe, contrarily, is quite weak in this sense (Van Baarle et al. 2003). Therefore, such similarity in the avalanche morphology is not material-related, being thus directly attributable to the geometric features of the underlying pinning landscape (Motta et al. 2014).



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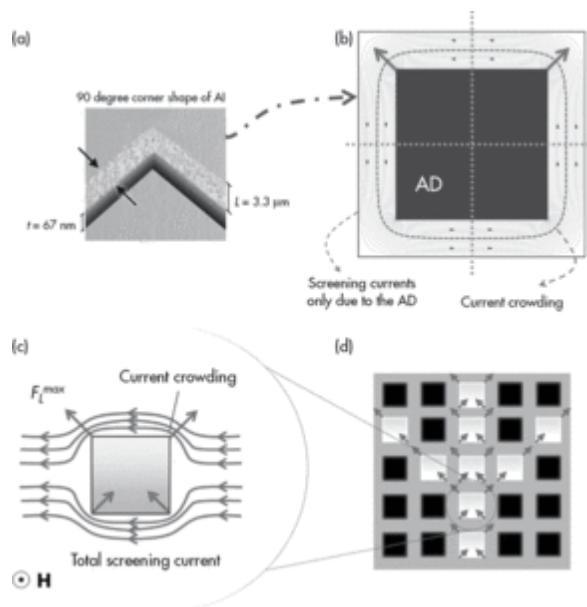
Fig. 13.9 MO images of flux avalanches in (a) a-MoGe film patterned with a square array of  $1.5\ \mu\text{m}$  with square ADs of  $0.8\ \mu\text{m}$  taken at  $T = 3$  K and  $H = 0.7$  Oe, and in (b) Nb film decorated with a square array of  $4\ \mu\text{m}$  with square ADs of  $1.5\ \mu\text{m}$  taken at  $T = 4.5$  K and  $H = 1.7$  Oe.

In order to explain this tendency of the ramifications to follow the diagonal direction of the square AD array, it is sufficient to rely on recent papers on the current crowding effect by Clem and Berggren (2011), Hortensius et al. (2012), Henrich et al. (2012), Clem et al. (2012), Adami et al. (2013), and Semenov et al. (2015). Together they provide theoretical predictions and experimental results which allow one to recognize the main ingredient of this peculiar morphology. In

fact, the current crowding effect, anticipated by Clem and Berggren, was observed experimentally by Adami and coauthors (2013) in superconducting corner-shaped microstrips of aluminum, that is, an Al film in the format of a  $90^\circ$  angle, in which an asymmetry on the vortex dynamics was detected. It means that flux penetration from the inner concave angle is easier than from the outer convex angle. Since the Al sample

consists of a  $3.3\text{ }\mu\text{m}$  wide transport bridge embracing (p. 401) a  $90^\circ$  angle, as presented in Fig. 13.10a, comparable to the distance between the ADs in the samples presented in Fig. 13.9, each of our ADs could perfectly be envisaged as a square void surrounded by four such samples, positioned in such a way as to form a closed superconducting loop. Fig. 13.10b depicts the case of an AD surrounded by superconducting microstrips, composed by four  $90^\circ$  bridges placed in each quadrant. Apart from the screening current component which circulates around the whole sample, the streamlines are closed loops of screening currents around the AD. The dashed thick line in the superconducting microloop represents the middle current streamline and emphasizes the current crowding occurring close to the inner corners.

If magnetic flux reaches such a central portion of the sample coming from an edge, entering the AD as a consequence of the local magnetic pressure, it will leave the AD as in the current crowding effect experiment reported by Adami and co-authors (Adami et al. 2013), that is, entering the superconducting microstrip preferentially through the inner concave angle. Moreover, in a superconducting film with an array of ADs, the total current flow splits around the AD, leading to (p. 402) current crowding effects on the AD corners. In other words, the current crowding effect can be understood as a conglomeration of the current streamlines around an obstacle, in this case, the AD corners.



Click to view larger  
 Fig. 13.10 (a) AFM image of a 90° microstrip of aluminum. (b) Combination of four 90° strips to form a square AD, showing only the screening currents around it. (c) Scheme of the total current around a square AD and the current crowding effect in its corners. (d) Evolution of a flux avalanche in a sample with a square array of square ADs. The white ADs are those already penetrated by flux and the arrows on the corners indicate where the Lorentz force is maximum (after Motta 2013). Panel (a) adapted with permission from Adami et al. (2013).

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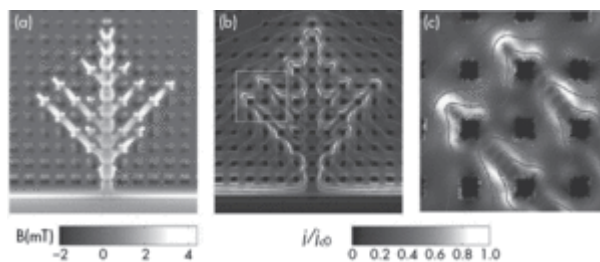
Vortices subject to the maximum Lorentz force ( $F_L^{\max}$ ) are those in regions of maximum density of current streamlines, that is, the tip of the ADs, as depicted by thick arrows in Fig. 13.10c. Regarding the avalanche progression, Fig. 13.10d illustrates how the main trunk and its 45° ramifications evolve in a sample with a square array of square ADs. The main trunk develops mainly due to screening currents parallel to the sample edge, which circulate throughout the film and push the flux toward the center. In turn, the secondary branches grow as a consequence of the current crowding effect, resulting in a maximum Lorentz force at the corners of the square AD.

The 45° branching in a superconducting film patterned with a square array of square ADs has also been reproduced by numerical simulations

based on the TM model (discussed in Sec. 13.1.4). Fig. 13.11 illustrates a Christmas tree-like avalanche as captured by the distribution of the magnetic flux density (a) and the sheet current (b). Panel (a) shows that flux rushes into the sample preferentially in the direction along the diagonal of the ADs. One can notice that flux is pushed away from the tip even in the main trunk, where brighter tones are present, maintaining clearly the habit to propagate at 45°. Shown in panel (b) are the current streamlines, which present the same tendency: paths around the square AD reveal the sources of flux/antiflux in that neighborhood and, thus, the preferential direction of the Lorentz force pushing flux along the direction of the AD diagonals. Panel (c) shows a zoomed-in image of the region marked

in (b), in which one can see the streamlines agglomerated on the tip of the AD in the forefront of the avalanche branches.

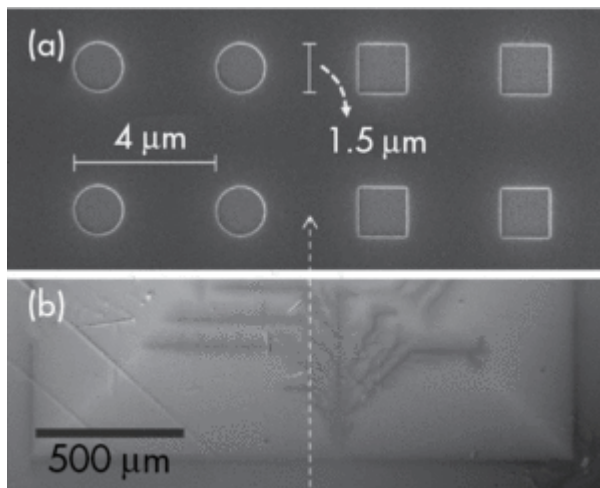
Another suggestive observation, which also emphasizes the role of the AD geometry on the morphology of avalanches, has been reported by Motta et al. (2014). Fig. 13.12 depicts results for a Nb film patterned with circular ADs on the left side, and square ADs on the right, as illustrated in panel (a). Panel (b) presents a dendrite triggered on the right side (square AD half) of the sample, (p. 403) with  $45^\circ$  branches. When branches reach the left half of the film (circular ADs), the ramifications modify their directions due to the different AD geometry and follow a fingerlike pattern (on the left of the white dashed line), with some small  $90^\circ$  branches. The image was captured at a temperature of 6 K, after increasing the field up to 48 Oe and then decreasing it down to 14 Oe. The fact that one had to “prepare the state” in order to see both habits in a single image was attributed by the authors as due to the importance of intrinsic, randomly distributed pinning centers in Nb, which play a far more important role in this sample than in a-MoGe films. The current crowding effect for square ADs displayed in a square lattice, introduces a second set of main axes, rotated by  $45^\circ$  from the primary axes dictated by lattice symmetry. For circular ADs, the main axes for flux guidance remain those originally set by the lattice symmetry, and the avalanches are essentially fingerlike sticks with minor ramifications perpendicular to the main branch.



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Fig. 13.11 Flux avalanche in a square array of square ADs reproducing the Christmas tree at the reduced temperature ( $T/T_c$ ) of 0.4 and applied field of 5.3 Oe. (a) Distribution of the magnetic flux density, and (b) the normalized induced sheet current. (c) Zoom-up of the region marked on panel (b). Figure adapted with permission from Motta et al. (2014).

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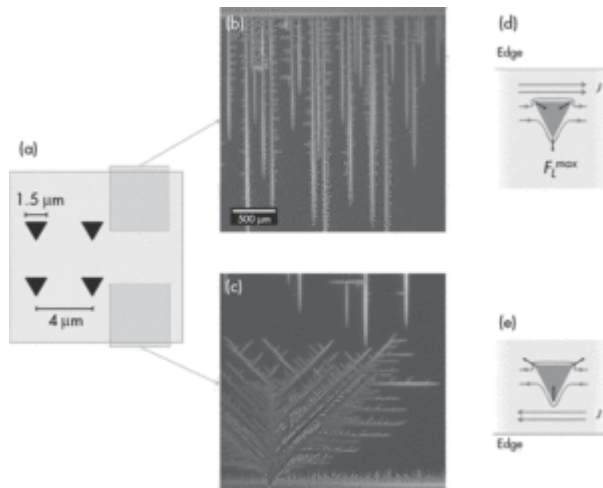
[Click to view larger](#)

Fig. 13.12 Nb sample with circular ADs on the left side and square ADs on the right. (a) SEM image with the AD size and the lattice parameter. (b) MO image taken at  $T = 6$  K and  $H = 14$  Oe after increasing the magnetic field up to 48 Oe. Figure adapted with permission from Motta et al. (2014).

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To further emphasize the effects of the antidot geometry and of the lattice symmetry on the flux avalanche morphology, a Nb film patterned with a square lattice of triangular ADs was prepared, and results are presented in Fig. 13.13. Panel (a) shows the dimensions of the square lattice ( $4 \mu\text{m}$ ) and of the triangular ADs ( $1.5 \mu\text{m}$ ) placed in the sample. As the lateral dimensions are larger than the field of view of the microscope, partial MO images were captured in different portions of the sample, two of which are illustrated in (a). Panel (b) presents the flux distribution on the top edge of the sample, which faces the base of the

triangular ADs, leading to straight fingerlike trunks with  $90^\circ$  secondary branches induced by current crowding effects at the tips of the ADs. When the flux avalanches invade the sample from the opposite edge, as shown in panel (c), their morphology is in the form of a Christmas tree, as in a square array of square ADs. Again, the orientation of the tips of the triangular AD relative to the flux front induces this pattern, due to current crowding, as illustrated in Fig. 13.13c and discussed above for square ADs. The effect of the lattice symmetry on the avalanche morphology is evident by the  $45^\circ$  secondary branches on avalanches rooted at the bottom edge. After entering the AD, flux has the tendency of leaving it from the triangle base at  $30^\circ$  from the sample edge (maximum Lorentz force), as presented in Fig. 13.13d. However, the triangle tip acting as its first attractive neighbor is located along the diagonal of the lattice and, consequently,  $45^\circ$  branches are (p. 404) the natural solution of the branching. This case emphasizes that the avalanche morphology results from compromise between the AD shape and the lattice symmetry. (Menghini et al. 2007).

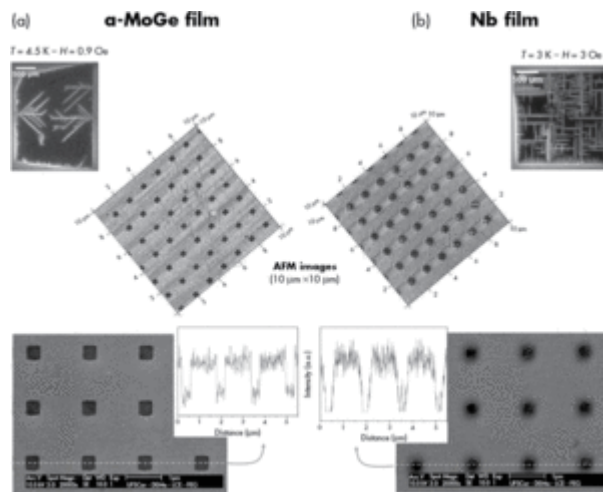


Click to view larger

Fig. 13.13 Flux avalanche in a Nb film patterned with a square array of triangular ADs. (a) A scheme of the decorated sample (not to scale) and the regions where the MO images presented in (b) and (c) were taken at  $T = 3$  K and  $H = 3$  Oe. Panels (d) and (e) show the total screening current and the maximum Lorentz force (boldface arrows) around triangular ADs placed close to the top and bottom edges (after Motta 2013).

It is worth noticing that the avalanche morphology is strongly influenced by the geometric quality of the AD, including the sharpness of the interface between the AD and superconductor. Fig. 13.14 shows microstructural images, obtained by AFM (atomic force microscopy) and by SEM (scanning electron microscopy), and MO images for two films decorated with ADs of  $0.4 \mu\text{m}$  displayed in a square array of  $1.5 \mu\text{m}$ , for (a) a-MoGe and (b) Nb films. AFM images illustrate the difference between both samples: whereas the AD geometry for the a-MoGe film is sharply square, the ADs on the right (Nb film) are clearly rounded. This is

also confirmed by the SEM images, in which the profiles taken along one row of ADs, as indicated in the bottom panel, show clearly a sharp AD/superconductor interface for the a-MoGe sample, different from the gradual crossover for the Nb film. Therefore, these representative images, taken by SEM and AFM techniques, provide a consistent picture reinforcing that the  $45^\circ$  ramifications are due to the sharp corners of the square ADs in a-MoGe, presented on the top left of Fig. 13.14a, while the  $90^\circ$  branched avalanches, shown on the top right of Fig. 13.14b, are byproducts of the round ADs in Nb. This result reinforces the notion that the avalanche morphology is not material-dependent, but intimately related to an interplay among the AD geometry and lattice symmetry. (p. 405)

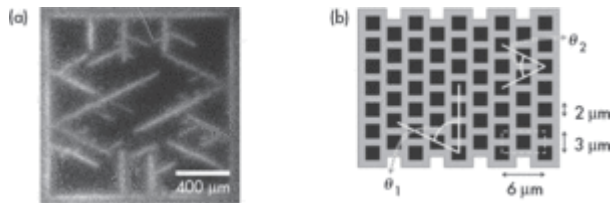


Click to view larger

Fig. 13.14 MO, AFM, and SEM images for two structured films with ADs of  $0.4 \mu\text{m}$  in square arrays of  $1.5 \mu\text{m}$ . Left side (a): a-MoGe film; right side (b): Nb film. Upper left and right panels show avalanches taken at  $T = 4.5 \text{ K}$  and  $H = 0.9 \text{ Oe}$  (a-MoGe film) and at  $T = 3 \text{ K}$  and  $H = 3 \text{ Oe}$  (Nb). Central pictures are AFM images and, just below, SEM profiles taken along the rows of ADs marked by the dashed lines in the very bottom images (after Motta 2013).

As a final example of how symmetry and geometry combine to guide the avalanche dendrites, Fig. 13.15a shows sudden penetrations on a specially patterned a-MoGe film. Its AD lattice consists of a centered rectangular 2D Bravais lattice, prepared by shifting every row by one half of the repetition length in its own direction and by a full length in the orthogonal direction, as detailed in Fig. 13.15b. As expected, the avalanches follow the angles  $\theta_1 \approx 63^\circ$  and  $\theta_2 \approx 53^\circ$  along the window of temperatures and fields in which avalanches take place. Besides that, avalanches that occur in the top and bottom edges have main trunks perpendicular to the

border, which are, however, absent in the left and right edges. This difference in morphology is a consequence of the fact that every vertical AD row is perpendicular to the horizontal borders (and guides the avalanches into the sample with [\(p. 406\)](#) main trunk and inclined branches), whereas there is no row at all perpendicular to the side borders, giving rise to a “doubly degenerate” main trunk. This remarkable result emphasizes that the avalanche morphology depends on the geometric features of the ADs and on how they are structured in the film.



Click to view larger

Fig. 13.15 (a) MO image captured at  $T = 4$  K and  $H = 1.1$  Oe for an a-MoGe film decorated with a centered rectangular lattice depicted (not to scale) in (b), with  $\theta_1 \approx 63^\circ$  and  $\theta_2 \approx 53^\circ$ .

## 13.4 Summary and outlook

Superconducting thin films submitted to magnetic fields in the perpendicular geometry may become unstable when temperature and field

values are within certain restricted intervals. Such instabilities, of thermomagnetic origin, generate practical conditions for the triggering of flux avalanches, which may invade the film at extremely high speeds. These avalanches represent a problem of multiple importance, given the richness of the physics involved in their nucleation and propagation and, not less significant, their negative impact on the current-carrying capability of the specimen. Furthermore, inserting arrays of submicrometric antidots (ADs) is a customary strategy in the pursuit of increasing critical currents but, as a counterpart, the presence of such arrays induces avalanches. The focus of the present chapter is on the effects on flux avalanches—triggering conditions, propagation, and morphology—caused by the presence of arrays of artificial pinning centers in the superconducting media, particularly in thin film geometry. At temperatures close to  $T_c$ , effects related to the insertion of lattices of artificial pinning centers become evident, such as the matching fields and the accompanying enhancement of the critical current which, in some cases, can reach the depairing current. At lower temperatures, the presence of arrays of ADs increases the avalanche activity, enlarging the temperature and magnetic field ranges within which dendrites are triggered. The effects of lattice symmetry and AD geometry on the morphology of flux avalanches can be thoroughly understood in terms of the so-called current crowding effect, which takes place at the corners of the ADs. Such ADs, when displayed in 2D arrangements are, in fact, the building blocks of a toy-model with which one can understand—and even anticipate—the specific morphology adopted by avalanches of magnetic flux, occurring in superconducting films with mesoscopic artificial patterns.

A feature of the problem still deserving further consideration is that, for 2D arrangements of ADs, the preferential axes for flux propagation—and, as a consequence, the avalanche morphology—are ultimately defined by the AD geometry and the lattice symmetry. Conceivably, for fixed AD format and lattice symmetry, one can vary the ratio between AD size and lattice parameter, so as to tune the relative importance of the current crowding effect at the corners of the ADs for the avalanche morphology. Thus, for large enough values of the lattice parameter, the avalanches would predominantly be fingerlike, with small secondary branches orthogonal to the main trunk, regardless of the AD geometry.

(p. 407) Another aspect to be stressed here is the noticeable similarity between the problem treated in this chapter, that is, the overall features of magnetic avalanches in patterned superconducting films, and the equally important problem related to the occurrence of electric discharges in controlled non-uniform media: both studies are likely to benefit from advances on the counterpart problem.

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## Notes:

(\*) The Bitter decoration technique consists of spreading out a ferromagnetic or a superconducting powder on the specimen in the state (temperature and magnetic field) that one wants to observe. After that, the images of the powder distribution are generally taken by means of a scanning electron microscope or optical microscope.

(†) The term “dendritic” comes from the Greek word for tree (“déndron”) and means having a branching structure similar to a tree or having dendrites.

(‡) The general stoichiometric formula of garnets is  $X_3Y_2(ZO_4)_3$ .

(§) As a matter of fact, the field sense (up/down) can be discriminated when the analyzer and polarizer are slightly uncrossed.

(\*\*) Columnar defects running across the full thickness of a sample would be able to fully pin flux lines; on the other hand, blind columns, that is, defects not going through the sample, provide less efficiency in vortex pinning. Thus, a set of columns presenting a range of depths corresponds in practice to a collection of trapping centers with a distribution in the pinning strength (see Niebieskikwiat et al. 2001).

(††) The term “antidot” has been taken from similarly structured thin films of semiconductors (Moshchalkov et al. 1996).

(##) This equation deduced by Mkrtchyan and Shmidt (1972) was later corrected by Nordborg and Vinokur (2000). The number of vortices trapped by a pinning center can also depend on magnetic field, density of holes, and symmetry of the lattice and the defect geometry. See for instance, Doria and Zebende (2002), Bezryadin et al. (1996), Buzdin (1993), and Kramer et al. (2009).

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