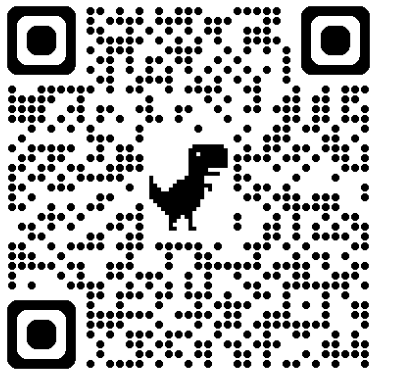


BOUND ENTANGLEMENT IN SYMMETRIC RANDOM INDUCED STATES



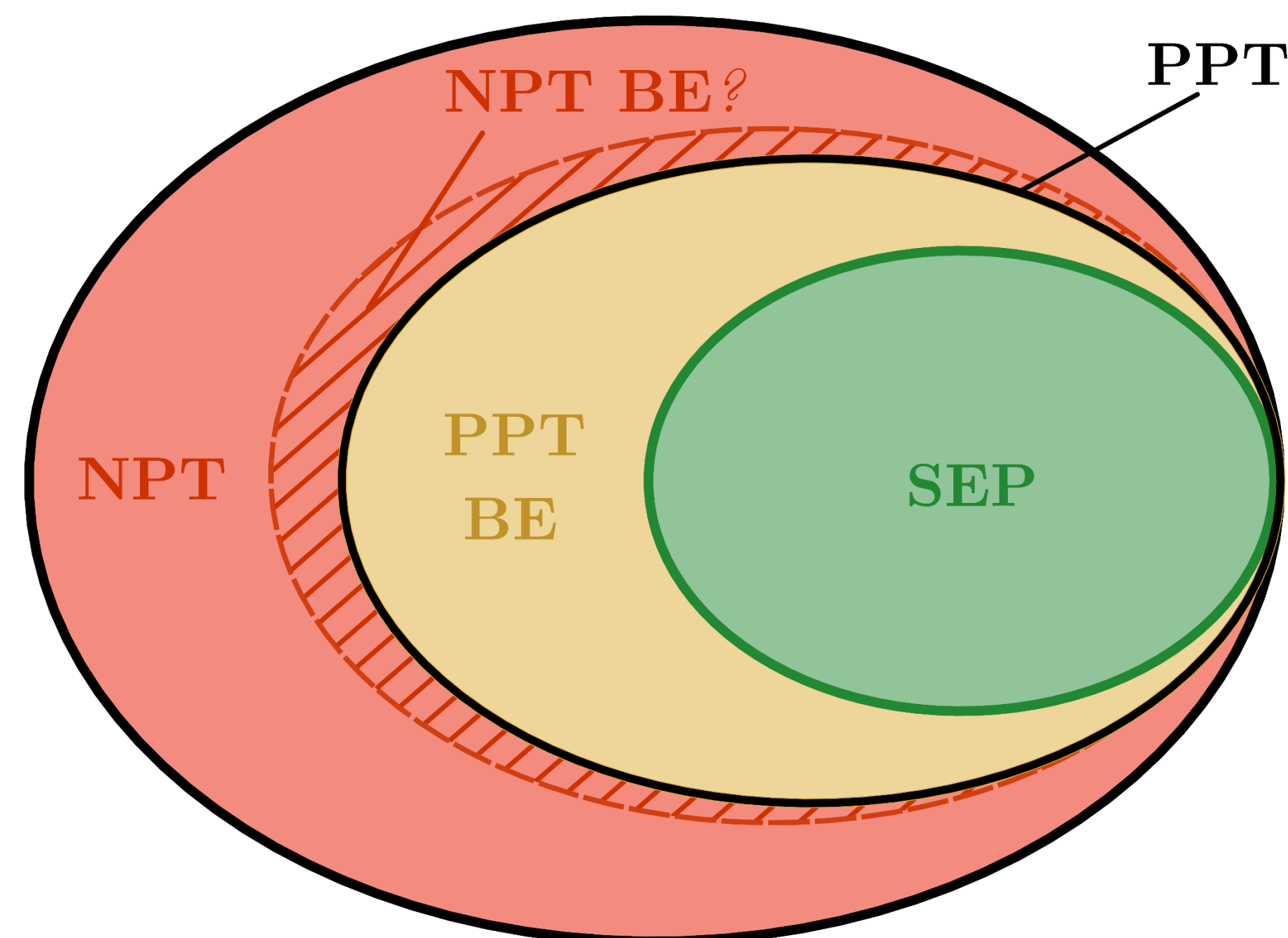
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Bound entanglement, a weak – yet resourceful – form of quantum entanglement, remains notoriously hard to detect and construct. We address this by leveraging symmetric random induced states (RIS), where positive partial transpose (PPT) bound entanglement arises naturally under partial tracing when proper parameters are selected. We investigate the probability of finding PPT bound entanglement in symmetric random induced states constructed via two methods. For $N > 3$ qubits, we demonstrate that bound entanglement naturally emerges under optimal parameters, with a probability of occurrence very close to 1 [1].

Bipartite bound entanglement



- **PPT criterion:** $NPT \Rightarrow$ entangled [2].
- **Bound entangled (BE) states:** entangled, but not distillable (via LOCC).
- **Undistillability criterion:** $PPT \Rightarrow$ non-distillable [3]
 $\Rightarrow$ PPT BE or separable (SEP).

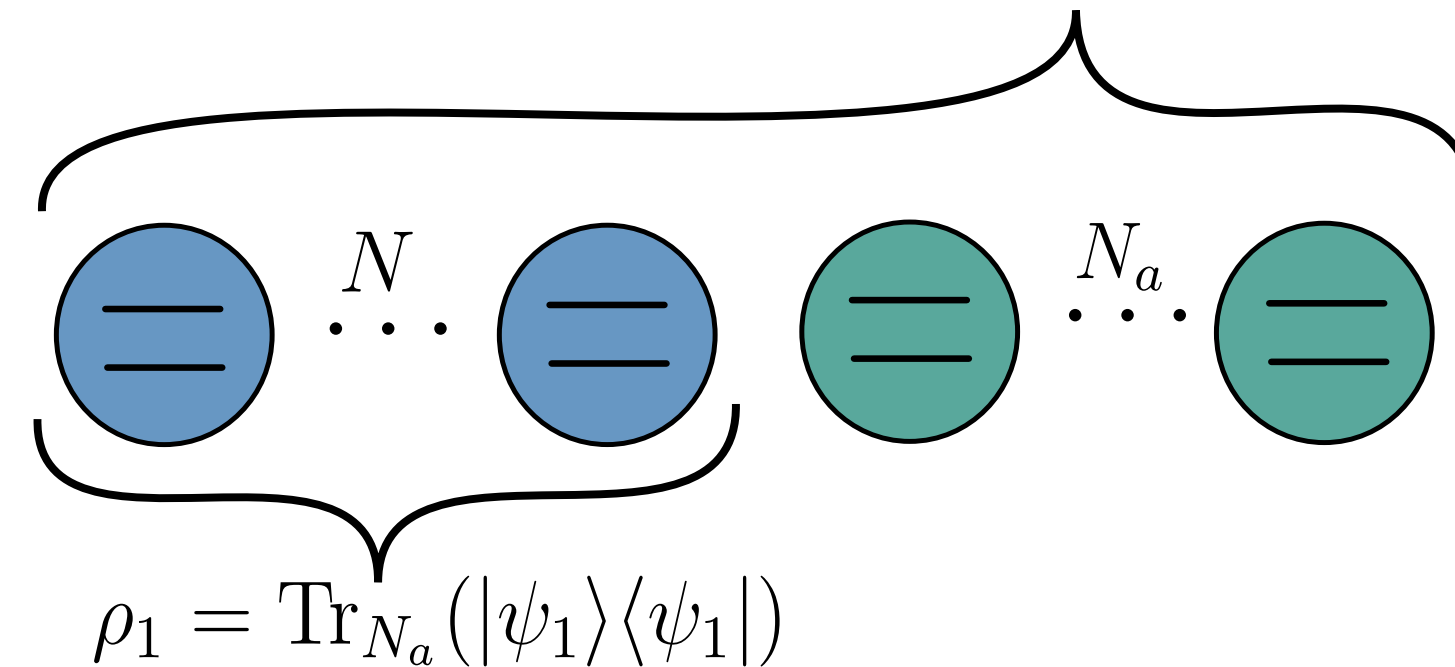
Symmetric states

- States invariant under subsystem state permutations.
- They are either separable or entangled across all bipartitions (fully separable or genuinely entangled, respectively).
- They can be PPT across some or all bipartitions *and* genuinely entangled.
 $\Rightarrow$ BE for specific bipartitions.
- **Classification:**
 - NPT (NPT across all bipartitions).
 - PPT BE (across some or all bipartitions).
 - SEP (PPT across all *and* fully separable).

How to generate random PPT BE states

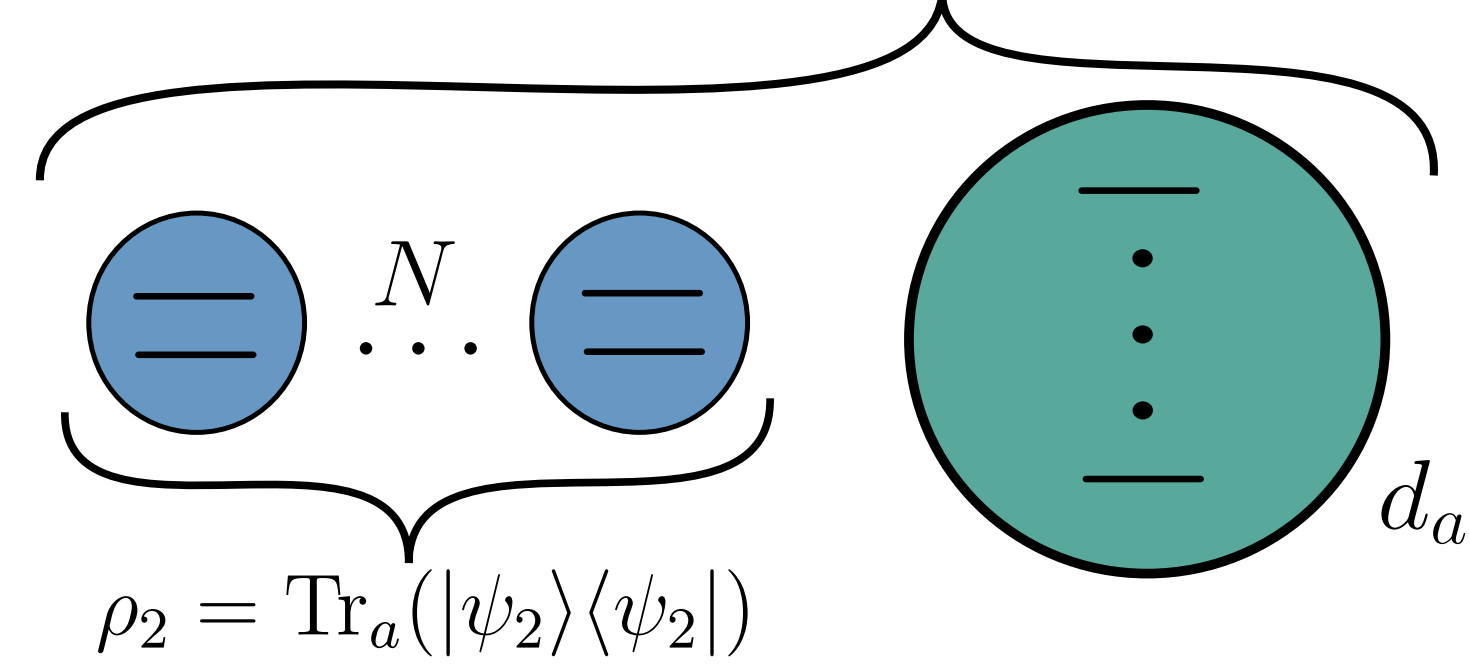
Method I

$$|\psi_1\rangle \in \mathcal{H}_S^{N+N_a+1}$$



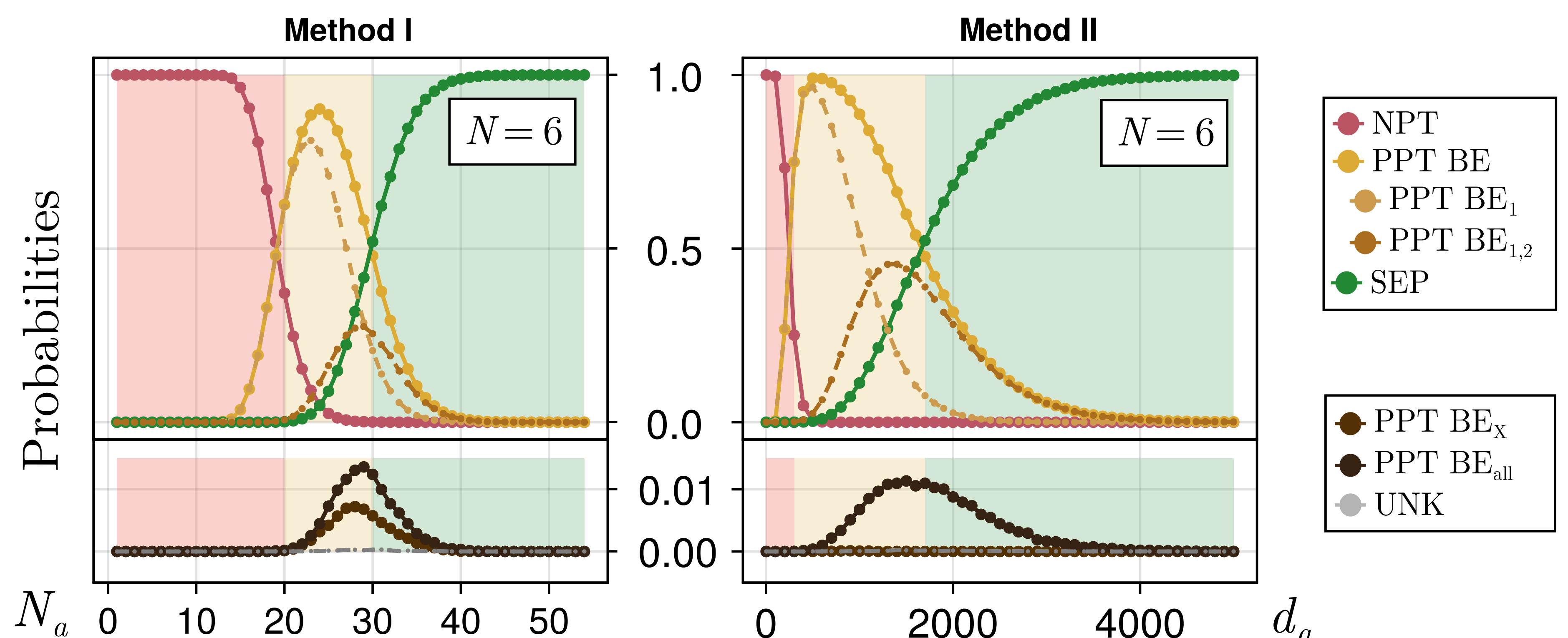
Method II

$$|\psi_2\rangle \in \mathcal{H}_S^{N+1} \otimes \mathcal{H}_a^{d_a}$$



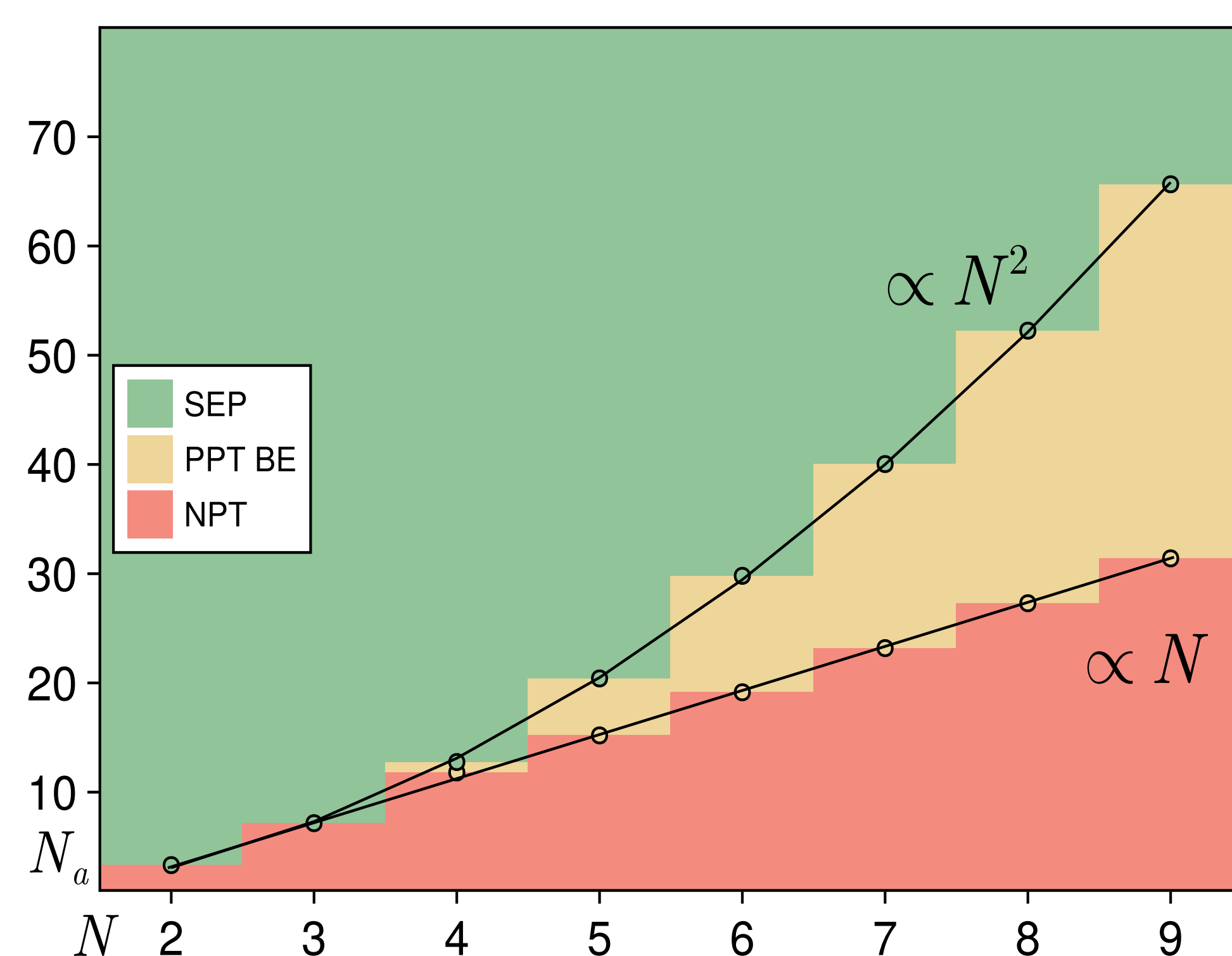
- **Random induced state (RIS):** a mixed state obtained by partially tracing a random pure state in an enlarged space [4].
- **Method I:** a RIS ρ_1 is created from an initial pure state $|\psi_1\rangle$ randomly generated on the symmetric subspace $\mathcal{H}_S^{N+N_a+1}$ of $N + N_a$ qubits after tracing out N_a qubits.
- **Method II:** a RIS ρ_2 is created from an initial pure state $|\psi_2\rangle$ randomly generated on the tensor product space $\mathcal{H}_S^{N+1} \otimes \mathcal{H}_a^{d_a}$ of the symmetric subspace of N qubits and a d_a -dimensional qudit after tracing out the ancilla qudit.
- **Detection:** $\rightarrow$ PPT test of ρ_1 and ρ_2 across all bipartitions.
 $\rightarrow$ Separability test (SDP) of states that are PPT across all bipartitions.

Efficiency of the Methods



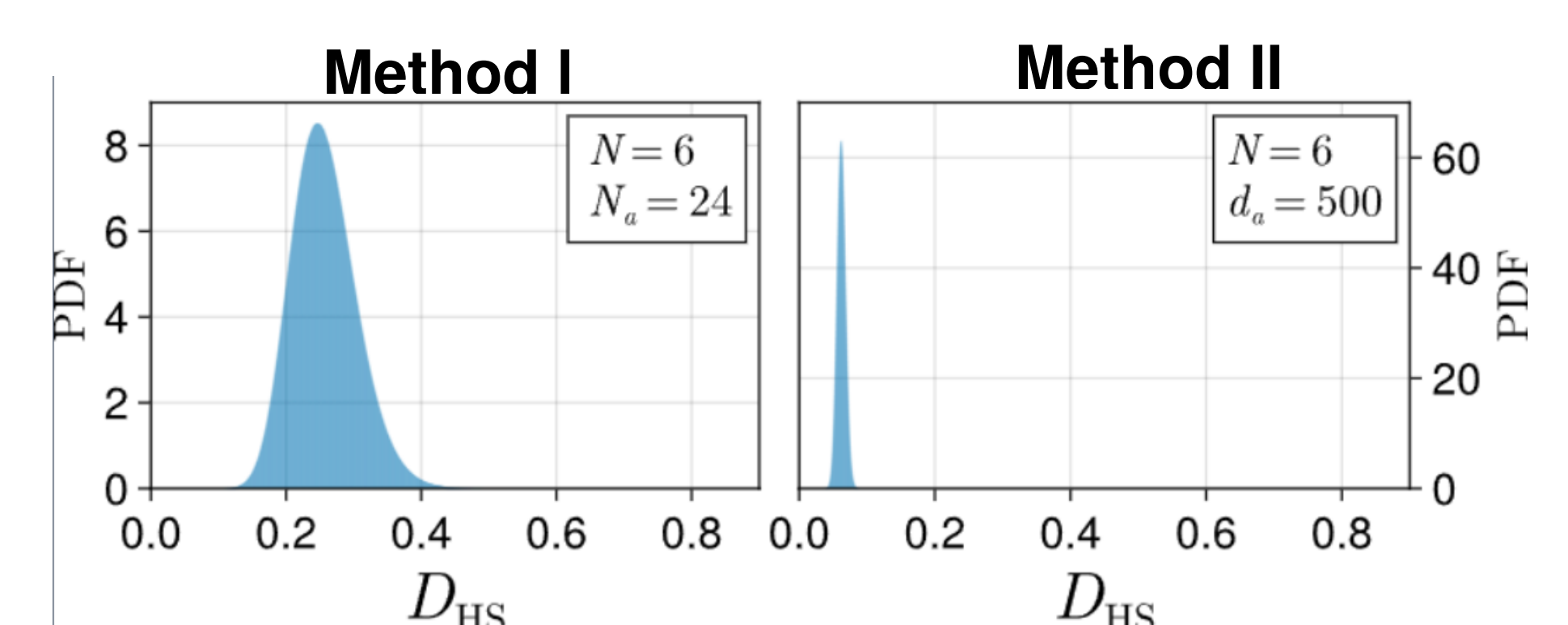
- **Phases:** Increasing ancilla parameters yields smooth transitions $NPT \rightarrow PPT\ BE \rightarrow SEP$.
- **PPT BE_{k₁,...,k_J}** $\equiv$ PPT BE across all bipartitions $N - k_j | k_j$ ($j = 1, \dots, J$).
- **Hierarchy:** PPT BE appears progressively across bipartitions ($PPT\ BE_1 \rightarrow PPT\ BE_{1,2} \rightarrow \dots \rightarrow PPT\ BE_{all}$). **Exception:** PPT BE_X ($X \equiv$ all bipartitions but $\lceil N/2 \rceil + 1 | \lfloor N/2 \rfloor - 1$).

Scaling of Method I



- **Scaling:** The PPT BE region grows with N , with linear ($NPT \rightarrow PPT\ BE$) and quadratic ($PPT\ BE \rightarrow SEP$) transitions.
- **Odd Even Dichotomy:** Sawtooth scaling of maximal $PPT\ BE_{all}$ probability with N (increase for odd N , decrease for even N).

Comparison of the Methods



- **Method I:** lower sensitivity to structure, yielding a more diverse set of PPT BE states.
- **Method II:** higher maximal probability and wider parameter range (more robust).
- Method II produces a narrower distribution in state space, while Method I yields a broader one.

References

- [1] J. Louvet, F. Damanet, and T. Bastin, Phys. Rev. A **113**, 042440 (2026)
- [2] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).
- [3] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **80**, 5239 (1998)
- [4] G. Aubrun, S. J. Szarek, and D. Ye, Communications on Pure and Applied Mathematics **67**, 129 (2013).