
High resolution optical inertial sensors for low frequency seismic isolation

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Abstract

This manuscript presents the design and characterisation of various optical sensors for an active seismic isolation system in 3rd-generation gravitational wave detectors. It is part of the E-TEST and SILENT ERC projects, and contributes to the development of a novel prototype of a passive/active seismic isolation system and Newtonian Noise measurement technique for the Einstein Telescope. The isolation system primarily consists of an inverted pendulum sitting on a 6D active isolation platform. The detector penultimate mirror hangs from a platform on top of the inverted pendulum. The Newtonian Noise measurement technique include a enhanced quantum gravimeter to complement classical seismometer networks. Three variations of sensors have been developed, all based on the same optical technology, and are used in several locations in the prototype.

The core technology is an optical displacement sensor consisting of a quadrature Michelson interferometer. The optical scheme uses polarising elements to demodulate displacements of several wavelengths. The optical scheme also features a double-pass action in the measuring arm to accommodate rotational motions of the target. Normalisation and ellipse correction algorithms have been implemented to minimise nonlinear effects and provide reliable readings of motion of a few mm. The demodulation process, including the ellipse correction, has been implemented in real-time in a Simulink DSpace environment for active control purposes. The optical sensors have been experimentally characterised and a resolution of $10^{-10} \text{ m}/\sqrt{\text{Hz}}$ at 10 mHz, $10^{-12} \text{ m}/\sqrt{\text{Hz}}$ at 1 Hz and $2 \cdot 10^{-13} \text{ m}/\sqrt{\text{Hz}}$ above 10 Hz has been demonstrated. These units are used as stand-alone displacement sensors for monitoring and local control at the inverted pendulum level.

Two types of optical seismometers have been designed, using the above interferometer for reading the motion of the proof-mass. The seismometers are 1D sensors: one vertical (E-VINS) and one horizontal (E-HINS) sensor. The vertical sensor mechanics consists of a 1 DOF pendulum maintained in a horizontal position with a CuBe2 leaf-spring, and connected to the sensor frame with a clamped-blade hinge, achieving a 1.45 Hz resonance frequency. The horizontal sensor consists of a Watt's Linkage, or folded-pendulum, with a 0.51 Hz resonance frequency. They use the above-mentioned optical technology as readout and reach sub-pm resolution above their respective resonance frequency. The sensors have been characterised for cross-axis sensitivity and nonlinearity in the active stage of the HAM-ISI system at LIGO-MIT. 3 pairs of horizontal and vertical sensors have been produced to record the platform's 6 DOFs.

The last instrument is an optical accelerometer (μ VINS) for vibration compensation noise in an absolute quantum gravimeter. The μ VINS is a 1D vertical sensor, whose design is a compact assembly of the E-VINS optical seismometer. The design is light-weight and compact, so as to fit the head of the quantum gravimeter. The sensor bandwidth is widened using a frequency-compensation filter up with a flat response to 100 Hz, for vibration noise compensation performance. A first implementation of the optical accelerometer inside the quantum instrument shows promising results for future hybridisation.

Résumé

Près de 100 ans après leur théorisation par Einstein dans son recueil sur la relativité générale, une onde gravitationnelle a été détectée pour la première fois en 2015 par le détecteur américain LIGO. Depuis, un engouement majeur est né dans la communauté scientifique pour renforcer les instruments servant à leur détection : améliorer la sensibilité des instruments existants, tels que LIGO (USA) et VIRGO (Italie), ou encore développer des instruments complètement nouveaux, libérés des contraintes imposées par l'architecture déjà établie. L'objectif est simple : améliorer la précision, la sensibilité et la bande passante, afin de permettre un plus grand nombre de détections, ou de détecter des événements issus de sources plus variées, dans le but d'améliorer notre compréhension de l'espace et des phénomènes qui s'y produisent.

Ces instruments, baptisés "détecteurs d'ondes gravitationnelles", sont des installations massives de plusieurs kilomètres de long, construites pour détecter des distorsions ultra-faibles de l'espace, dont la dimension caractéristique est plus faible que la taille d'un proton. Pour ce faire, ces installations gigantesques combinent des technologies optiques de pointe, des méthodes d'analyse de données avancées, ainsi que plusieurs couches de systèmes destinés à isoler les éléments sensibles du détecteur des perturbations extérieures.

Ce travail s'inscrit dans le cadre des projets européens E-TEST et SILENT. Ces projets œuvrent à l'exploration de nouvelles technologies pour le futur détecteur "Télescope Einstein". Plus précisément, ils ont développé un système d'isolation sismique, destiné à isoler le détecteur des vibrations provenant de la Terre. Ce système est novateur par son utilisation de mécanismes passifs d'isolation complexes, tels que des pendules inversés, combinés à un système actif : une plateforme pilotée dans les 6 degrés de liberté et isolée activement grâce à un système de contrôle et d'asservissement. Un autre axe du projet vise à renforcer les stratégies traditionnelles de suppression du bruit dit "Newtonien", c'est-à-dire un bruit lié aux fluctuations locales et naturelles du champ de gravité, à l'aide d'un gravimètre quantique amélioré.

Concrètement, ce manuscrit présente trois technologies optiques qui ont été développées dans le cadre de ces projets.

Une première technologie consiste en un interféromètre de Michelson à quadrature de phase. Cet instrument permet de détecter des déplacements inférieurs au picomètre (10^{-12} m, soit un millionième de millièème de millimètre). Contrairement aux interféromètres conventionnels, ce schéma

optique basé sur la quadrature du faisceau lumineux permet de mesurer à la fois ces faibles déplacements, mais également des mouvements beaucoup plus grands, allant jusqu'à plusieurs millimètres. Il s'agit donc d'un capteur combinant une précision extrême et une gamme dynamique de plus de 9 ordres de grandeur (10^9). Cet instrument a été conçu, testé et validé en laboratoire avant d'être implémenté dans le prototype comme capteur de déplacement, utilisé pour l'asservissement et l'alignement.

Ensuite, des capteurs dits "inertiels" ont été développés pour instrumenter la plateforme d'isolation active du prototype, afin de mesurer son mouvement de manière absolue et de fournir cette information au système de contrôle. Ces capteurs consistent en une masse suspendue, dite masse d'épreuve, dont le mouvement est mesuré par un capteur de déplacement. Le capteur optique cité ci-dessus est ici réutilisé pour produire des capteurs inertiels de haute précision. Ces capteurs sont également fabriqués à l'aide de mécanismes de suspension extrêmement souples, dont la fréquence propre est de l'ordre de 1 Hz, afin d'améliorer encore leur précision et leur bande passante. Deux configurations (une verticale et une horizontale) ont été conçues et testées en laboratoire. L'électronique ainsi que les algorithmes de traitement des données ont également été développés et intégrés au système de contrôle temps réel de la plateforme active.

Enfin, un troisième capteur a été développé pour améliorer la précision du gravimètre dit "quantique". Ce capteur consiste en une version compacte du capteur inertiel vertical présenté précédemment, et est installé dans la tête du gravimètre, au plus proche de la partie sensible de l'instrument. Son rôle est de mesurer le niveau de vibration de la tête du gravimètre, avec la plus grande précision possible et sur une large bande passante, afin de soustraire le bruit qui en découle et ainsi améliorer la précision globale du gravimètre. Après le développement du capteur, une première tentative de couplage entre les deux instruments a été réalisée, donnant des résultats prometteurs.

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Introduction

Progress in technology and experimental science has always been tightly bound to progress in measurement. The British engineer Joseph Whitworth captured this idea elegantly in his famous adage:

”You can only make as well as you can measure.”

- Joseph Whitworth, 1840.

This statement is astonishing in its simplicity, yet remarkably relevant to modern technologies.

Over the past half-century, countless innovations have depended on our ability to sense, control, and fabricate with nanometre-scale precision. In medicine, high-precision instruments [1] now assist surgeons in delicate procedures, dramatically improving safety and success rates. Atomic force microscopy [2] has opened a window onto the molecular landscape of materials, fostering the design of composites with unprecedented mechanical and electronic properties. Nanometre-level metrology systems ensure the precise alignment of particle accelerator components [3] over kilometre-scale infrastructures, enabling the discovery of new particles and deepening our understanding of the atomic world. In microelectronics, advances in lithography and nanofabrication [4] have enabled the miniaturisation of mobile and connected devices, whose circuits are assembled with sub-nanometre accuracy — think that the first computers of the mid-20th century were as big as an entire room and were only accessible to a handful of scientists; and nowadays people casually walk with devices with thousands times more processing power in their pocket!

These examples illustrate a broader truth: progress in precision measurement is not merely a technical refinement but also a driver of discovery and improved quality of life. Each step toward higher sensitivity and finer resolution expands both the frontiers of technology and the boundaries of scientific observations. The pursuit of precision has thus become a defining theme across modern physics and engineering.

Science-case

Arguably, one of the most important breakthrough in science for the past decade is the direct experimental observation of a gravitational wave, which was awarded a Nobel Prize in Physics in 2017.

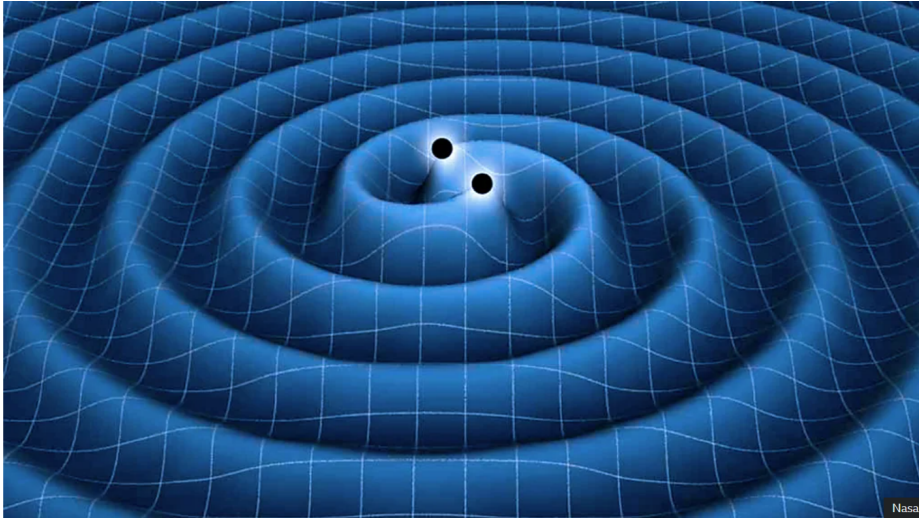


Figure 1: Schematic illustration of a gravitational wave (Copyright: NASA). The spacetime fabric is illustrated as an -initially- flat quadrille tissue. As two black holes inspiral due to their respective gravitational pull, they deform spacetime and create ripples that propagate through the universe.

Gravitational waves are ripples in the fabric of spacetime, schematically illustrated in Figure 1, which travel across the universe at the speed of light. They are curious phenomena that result from Albert Einstein's general relativity (GR) theory of gravity [5]. With his theory, Einstein proposed in 1915 a completely new way of visualising and modelling the universe: theorising that the world we live in is actually a 4-dimensional matrix, where space and time are intimately connected. His theory had major implications. To cite only a few examples:

- Time is no longer seen as an absolute and ineluctable quantity, forever flowing at a constant speed, but is altered by the nearby gravitational field. Simply put, clock ticks are slower when in strong gravitational fields. This has been experientially verified in 1959 by the famous Pound-Rebka experiment [6], and its implication is continuously being applied in GPS satellite clock corrections. Indeed, because the GPS constellation orbits at an altitude of about 20000 km, the on-board atomic clocks tick in a gravitational field far weaker than the same clocks on Earth. This, cumulated with other relativistic effects, leads to the GPS satellites clocks accumulating a lead of approximately 38 microseconds every day with respect to clocks on Earth. This would in turn would cause an error in the GPS system that would accumulate at a rate of 10 km per day, every day (Real-World Relativity: The GPS Navigation System). Without such correction, the GPS system and world-wide communication would certainly be

a lot less accurate than they are nowadays.

- The GR theory predicted the existence of gravitational singularities. Points where the space-time fabric is so curved that nothing can escape. These singularities have been later observed in 1971 [7, 8], and named 'black holes', for that not even light can escape their gravitational pull. Black holes have then fed the imagination of the pop culture, and stimulated innumerable young and old astrophysicists to keep on exploring and discovering the universe.
- The GR theory improved the understanding of planets' and galaxies' motion. GR literally provided the foundation of the Friedmann–Lemaître–Robertson–Walker (FLRW) model, theorising the expansion of the universe [9]. The Hubble telescope recently provided a direct measurement of the redshift-expansion of galaxies, again validating the theory.

Gravitational waves is another of such fancy science facts predicted by Einstein's GR. These ripples are expected to be caused by strong and massive cosmic events, such as black hole mergers or dying stars. However, the expected amplitude of such waves is so weak that many scientists (some say Einstein himself!) long claimed that their detection on Earth would be impossible. Still, scientists kept on developing and improving instruments with extreme sensitivity, with the hope of finally detecting one gravitational wave. It is only one century after Einstein's GR theory that the 2nd generation of instruments, namely the US gravitational wave detector LIGO (Laser Interferometer Gravitational-Wave Observatory), shown in Figure 2, made in 2015 the first ever experimental detection of a gravitational wave, issued by a binary black hole merger (event name GW150914) [10]. This detection has been made possible only because the gravitational wave detector technology has been pushed way beyond what one would have ever thought at Einstein's times. Detectors are nowadays so sensitive and so well-isolated from Earth's noises that they can detect ripples with strain as small as $1 \cdot 10^{-21} \text{ } 1/\sqrt{\text{Hz}}$ [11]. For a detector with a 1 km-long basis, this would correspond to a distortion of space of about 10^{-18} m , a thousand times smaller than a proton diameter!

This detection stimulated the gravitational wave community to improve existing instruments, namely the advanced-LIGO detector in the US [12] and the advanced-VIRGO detector in Italy [13], and also develop next-gen (3rd generation) instruments, namely the European Einstein Telescope (ET) [14] and Japanese KAGRA [15] detectors. Developing new instruments, with ever-increasing sensitivity, capable of detecting a broader spectrum of gravitational waves and, again, improving our understanding of the universe and of the world we live in.



Figure 2: Current 2nd generation of gravitational wave detectors aLIGO [12] (left) and aVIRGO [13] (right).

By their very nature, gravitational waves carry information that are different, and complementary, to the information that astronomers and cosmologists usually gather with classical telescopes, which detect electromagnetic radiations (e.g. optical, X-rays, infrared, etc) emitted in outer-space. Gravitational waves and electromagnetic waves have indeed inherently different emission mechanisms and propagation properties, and are thus fundamentally different messengers with different information content. Such a multi-messenger astronomy has been shown successful with the joint detection in 2017 of a binary Neutron star inspiral (event name GW170817, illustrated in Figure 3) by the advanced LIGO/VIRGO detectors and the Fermi telescope [16]. The detection of a γ -ray burst (GRB 170817A) by Fermi, a few seconds after the coalescence detected by LIGO and VIRGO, corroborates the hypothesis of a neutron star merger and provides the first direct evidence of a link between these mergers and γ -ray bursts. Gravitational wave science, therefore, offers a brand new view of the universe, opening new doors for exploring and understanding our universe.

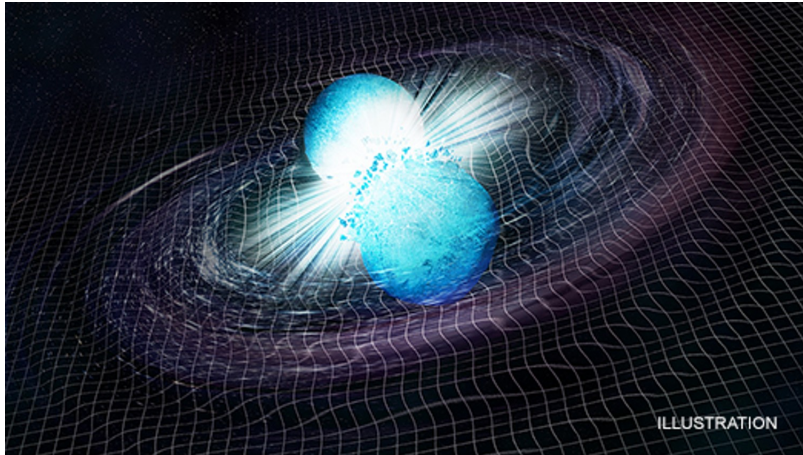


Figure 3: Schematic illustration of the GW170817 event (Copyright: NASA/CXC/M.Weiss. The neutron star merger resulted in the propagation of a gravitational wave and a γ -ray burst. The joint measurement and analysis of both emissions allowed to fully understand this very complex event, and is a great example of multi-messenger astronomy. In fact, this merger also gave birth to a black hole, which is said to be the lowest mass black hole ever found.

The Einstein Telescope

The Einstein Telescope (ET [17]) is a 3rd generation gravitational wave detector under development in EU, illustrated in Figure 4. Using a novel "folded xylophone" design buried underground, longer optical arms, and couple of additional new technologies, the ET aims for a sensitivity that is 10 times better than current 2nd generation detectors (i.e. aLIGO and aVIRGO) as well as expanding the detection bandwidth to lower frequencies (< 10 Hz), that are currently blind to current detectors [14].

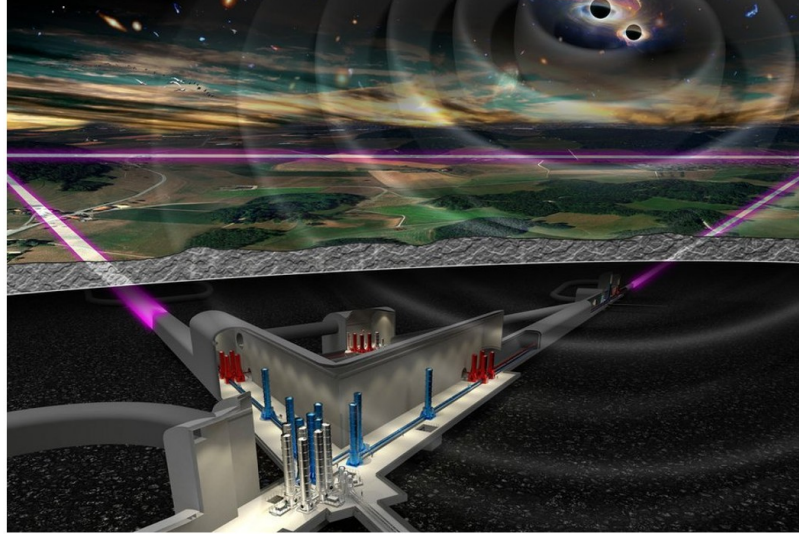


Figure 4: Artistic view of ET (Copyright: NIKHEF).

Without digging too deeply into the scientific details, these ambitious improvements would enormously benefit the gravitational wave science and astronomy communities in multiple ways. Thanks to the higher sensitivity, physicists expect to increase the detection rate by 1000, thereby leading to more detections, a larger statistical population of events, and thus better astronomical studies. The better Signal-to-Noise Ratio (SNR) also allows to detect events with better accuracy, and at a farther distance comparable to the sight distance of classical electromagnetic telescopes [14]. The broadening of the detection bandwidth toward the sub-10 Hz frequencies would allow ET to detect gravitational waves emitted by a broader spectrum of astronomical events, thereby allowing to study events that are simply still undetectable by current detectors. Some examples include lower mass systems, normal modes in Neutron Stars (NS) excited during gravitational wave (GW) events (pulsar glitches, magnetar flares, etc) or non-axisymmetric NS [14, 18]. Figure 5 is a graph that shows the target performance of ET in the lower-frequency band (< 50 Hz), and compares it with that of the 2nd generation instruments. The performance of a GWD is measured with a sensitivity curve as a function of frequency: the lower the curve, the better the sensitivity. The figure shows how much improvement is made between the 2nd and 3rd generation, as well as an illustration of various novel science-cases that will be available to ET and the limitations that need to be addressed. More reading on the ET science cases can be found in [14, 18].

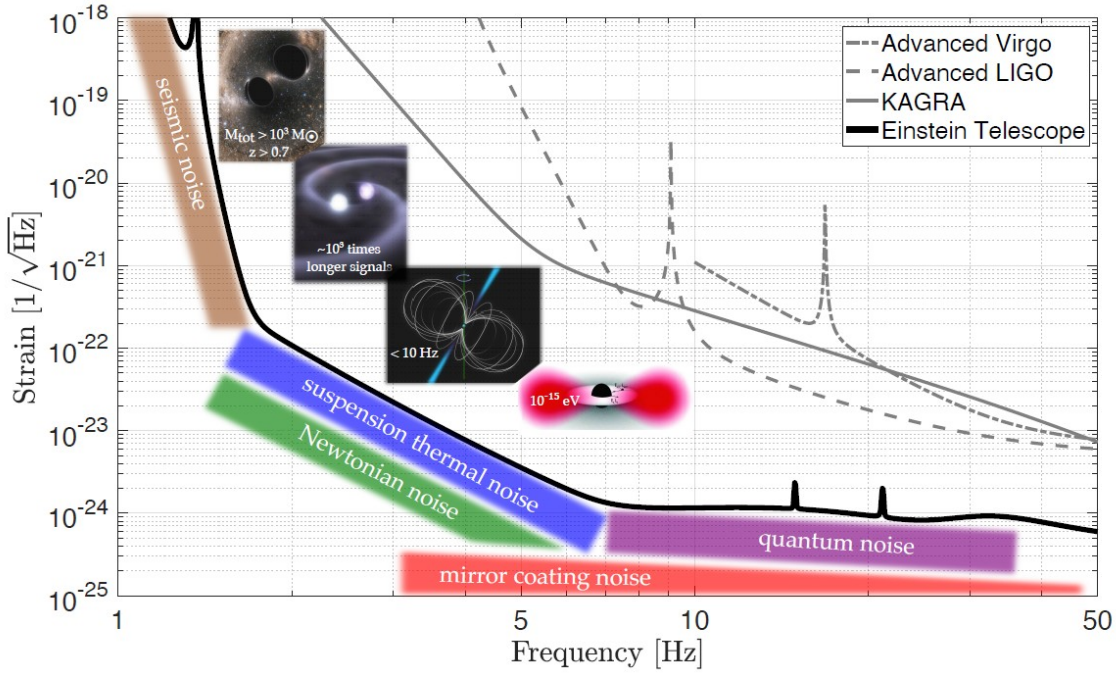


Figure 5: Sensitivity of the ET (low frequency band). Superimpose to the plots are the noise limitations of the telescope in each frequency band, as well as a schematic illustration of astronomical events that can be captured in each bands [17,18].

Obviously, meeting such expectations does not come without overcoming a whole lot of technical challenges. First of all, the ET innovates with its design. The 2nd generation detectors are all based on a classical '*L-shape*' *Michelson interferometer design*. Simply put, they use a single laser beam that is split into two and directed in two orthogonal arms, where they reflect on mirrors, also called test masses, forming an L-shape interferometer. As a gravitational wave passes by, the relative distance ('space') between both test mass vary, so that when the laser beams recombine after they have reflected, they produce an interference pattern, a signature of a gravitational wave. ET, on the other hand, proposes a design based on three independent detectors, each having a 'triangle' shape, with the optical arms forming a 60° angle, and folded in a xylophone configuration. This provides a design with a great spatial coverage in a compact arrangement, shown in Figure 6. In Michelson interferometers, the sensitivity scales proportionally with the length of the optical arms, i.e. how far away are each mirrors. ET will therefore have longer arms, designed for a 10 km baseline, compared to aLIGO and aVIRGO, which are a couple of km long only. On top of that, the ET community figured out that optimising a single interferometer for a wide frequency bandwidth can be extremely challenging, if not hardly possible, due to conflicting requirements. Therefore, they propose to design two distinct detectors, each optimised for a different frequency band, named ET-LF, for 'low-frequency', and ET-HF, for 'high-frequency'. The detectors are then merged together forming a super-detectors, ET-D, which utilises the signal of both detectors where they are most sensitive. Lastly, ET will be buried hundreds of meters underground, in a way to operate in a quiet

environment far away from surface noises. Compactness of the design, therefore, becomes key as burying such a huge facility will lead to enormous engineering work and costs.

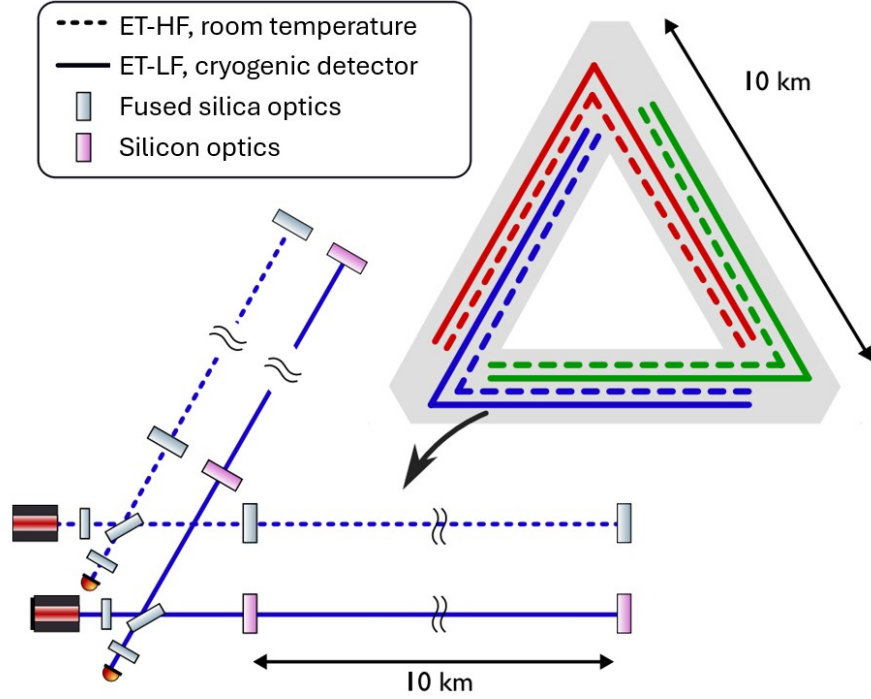


Figure 6: ET optical layout (under development), adapted from [17]. It is made of three different interferometers folded in a 'xylophone' arrangement, each shown in different colours. The dashed traces show the 'high frequency' detectors (ET-HF) and the full traces show the 'low frequency' detectors (ET-LF).

Optimising ET

Now, in order to meet the target sensitivity mentioned above, each of these 6 interferometers (detectors) constituting ET need to be optimised, with all *noise sources* identified and carefully dealt with. Obviously, there is a lot more to such high-performance detectors than simply 'a laser beam, a beam splitter and two test masses (mirrors)'. The detectors include dozens of optical components, such as recycling cavities, all designed for improving the detectors sensitivity. One example and key features are high finesse Fabry-Perot cavities, in which the laser beams bounces back and forth thousands of times, increasing the effective distance travelled by the laser beam. The following section is devoted to explaining the various concepts in which this thesis falls in, without being too scientific nor technical. Curious (and brave!) readers are directed to more specific readings to read more on the exact design and working principle of a GWD.

Noise is a key aspect in this section and in the design phase of a GWD in general, but what exactly do we call noise? Imagine you are sitting in a theatre, listening to classical music. The orchestra plays music that reaches you, and that is *detected* by your ears. Now imagine a phone

ringing in the back seat. The phone bell will also produce a sound that reaches you, and that is detected the same way the music is. But you did not come to listen to a phone ringing, did you? The orchestra music is the *signal* that is of interest, while the phone ringing is noise polluting the play. The same goes for GWDs. The detectors fundamentally captures motions between both test masses. When the motion is 'caused' by dilatation of space-time due to a GW passing by, that's cool, that's your signal of interest. But anything else causing motion of the test masses will also generate signals in the detector, only not those of interest, and are therefore named noises. Noises can be either external, i.e. caused by sources external to the detector (the phone ringing), or internal, e.g. issued by the detector itself (e.g. the music is loud and clear, but due to past trauma, you have poor audition and cannot hear the orchestra properly). In ET, internal noises would be noises caused by, mostly but not limited to, the optical systems (laser emission, optical elements, signal acquisition and processing, etc). External noises would be external factors that effectively cause motion of the test masses (seismic activity, air current, etc). External noises will be of primary interest here.

Noise budgeting consists of carefully investigating all noise sources and identifying whether they represent a *limitation* for the detector. Using the same analogy, it is very likely that the orchestra plays music that is loud enough so that you can still enjoy the play despite the phone bell. The noise in this case is not a limitation and can be safely discarded. In a different context, it could be that some notes are very subtle and low in volume, or the phone bell is really loud, that the notes can be buried in the noise, your ears not capable of hearing them. The noise now becomes a limitation. Recalling the extremely low level of a 'typical' gravitational wave (distortion of space that is the size of a proton, 10^{-18}m , for test masses that are 1 km apart), it is easily understood that anything, even the tiniest perturbation, can be a limitation. And if they are, actions need to be taken in order to reach the target sensitivity.

Figure 7 shows the ET design noise budget [19, 20]. It shows noise budgets for each individual detector, ET-LF and ET-HF as the top figures, and the combined ET-D as the bottom plots. Dashed traces are the noise limitations, and the full traces are the effective sensitivity, i.e. the smallest signals that sit above noise. Each noise source *propagates* through the detectors differently and has different *frequency content*, hence the plot showing noise limitation that are different in different frequency bands. Notice, for example, how ET-HF has significantly lower 'quantum noise' than ET-LF, making its sensitivity better at above 100 Hz. This is done by increasing laser power and minimising optical noises, leading to a high quantum efficiency. On the other hand, ET-LF has better sensitivity below 100 Hz, mostly due to a lower suspension thermal noise. This is achieved by operating ET-LF at a very low, cryogenic temperature of a few tens of Kelvin ($\approx -250^\circ\text{C}$). Notice also how these two requirements are conflicting: high power level means high heat generation, which is hardly compatible with a cryogenic environment. Hence, the smart choice of splitting ET into two detectors.

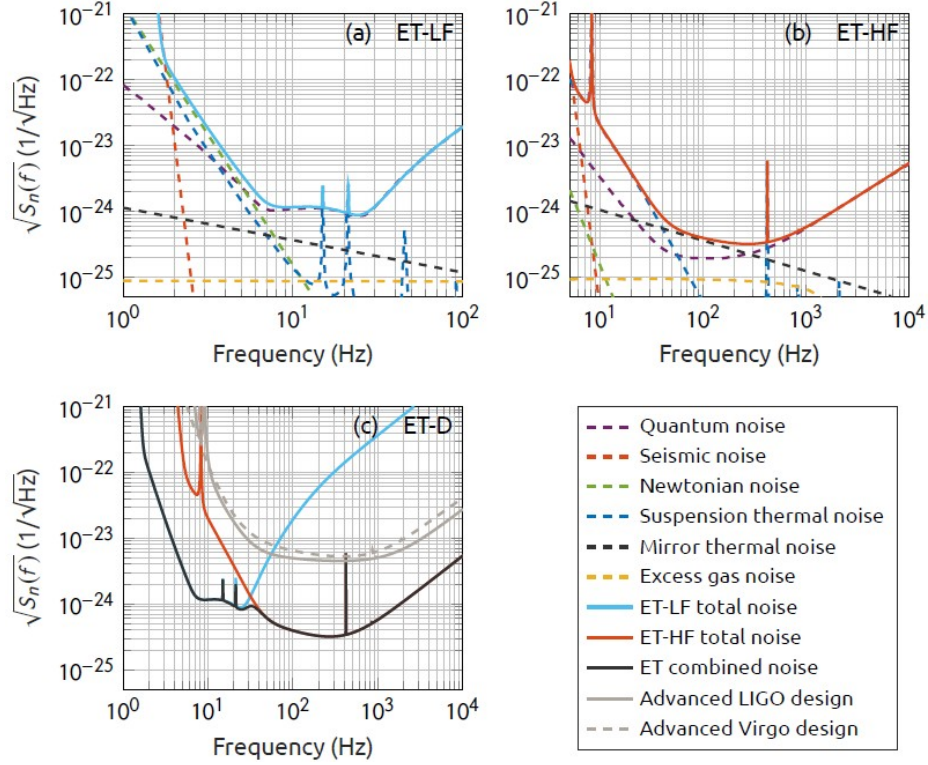


Figure 7: ET Design noise budget [19,20]. The top two graphs shows the noise budgets of ET-LF and ET-HF individually. The bottom graph shows the combined noise budget.

Again, readers are directed to specific literature for an explanation of the various noise sources that appear in the above figure. As far as this work is concerned, let us zoom in on the low-frequency area of ET-LF: sub-10 Hz frequencies. From the above figure, it appears that the sensitivity here hits two hard limitations: *Newtonian Noise* (NN, in the 2 - 7 Hz band) and *seismic noise* (sub-2 Hz). Because of how steep the curves look in the frequency plots, they are also known as the seismic 'wall'. Seismic noise is fairly straightforward to grasp. It consists of ground vibrations, caused by the Earth seismic activity, that propagate to the detector test mass, resulting in a 'not-of-interest' signal. For the seismic noise limit to be as low as it is in the noise figure, the detector must be equipped with a seismic isolation system, i.e. a system that isolates the test mass from ground vibration, avoiding propagation of seismic noise. Newtonian noise refers to a variation in local gravity. Indeed, as per Newton's law of gravity, every two bodies attract each others by a force proportional the masses involved and distance between both bodies: $F = G \frac{Mm}{r^2}$, with $G = 6.67 \cdot 10^{-11} \text{ m}^3/\text{kg}$ the universal gravity constant, M and m the masses involved and r the distance between them. Take one M be the mass of the Earth and r be the Earth radius, and you end up with $g = GM/r^2 \approx 9.81 \text{ m/s}^2$. The formula, however, does not only apply between the Earth and the test masses, but between the masses and any other surrounding bodies, so that the test masses move as they are subjected to a time-varying gravitational acceleration (e.g.: $g(t) = 9.81 + \text{NN}(t) \text{ m/s}^2$). Because of its nature, Newtonian noise cannot be isolated from: one cannot 'avoid' gravity, but it can be measured and

subtracted from the gravitational wave signal. How good is the subtraction process, it depends on how good NN is being measured.

Long story short, this work will focus on novel technologies for both isolating the test masses from seismic noise and measuring Newtonian noise. This work is carried out as part of a broader project aimed at investigating technologies for ET-LF. The project's name is E-TEST, and it will be discussed in the next section.

Context

This work is part of the E-TEST project and is supported by the ERC Consolidator grant "SILENT" (grant agreement number 866259), the Fonds de la Recherche Scientifique grant "INFuSE" (agreement number FNRS PDR T.0049.20), and the European Commission Horizon Europe grant "FIQUgS" (grant agreement number 101080144), which the author gratefully acknowledges.

The E-TEST project (Einstein Telescope EMR Site & Technology, <https://www.etest-emr.eu/>) is an EMR (Euregio Meuse-Rhine, border region between Belgium, the Netherlands and Germany) project gathering multiple partners in the area and aims at developing new technologies to address challenges faced by the 3rd generation gravitational wave detector ET. The partners include University of Liège (ULiège), Liege Space Center (CSL), Fraunhofer-Institut für Lasertechnik (ILT), RWTH Aachen University, Hasselt University (UHasselt), KU Leuven University, National Institute for Subatomic Physics (Nikhef), Rheinische Friedrich-Wilhelms-Universität Bonn, NMWP Management GmbH, Royal Netherlands Meteorological Institute (KNMI), Maastricht University and Université catholique de Louvain (UCLouvain). More particularly, the E-TEST program has three major axes. One axis is the seismology and geology work package, which studies the suitability of the EMR-site underground as a great candidate for the ET. Another one is an optical work package developing optical technologies that are key to ET, such as a high-performance stable 2 μm laser, low noise coating and photodiode. The last axis is a mechanical and thermal engineering work package aiming in developing a prototype of vibration isolation system for the detector test mass, that includes a novel hybrid (active/passive) vibration isolation strategy, a heavy (≈ 100 kg) monocrystalline silicon mirror and a radiative (contactless) system for cooling the test mass to 20 K (-253.15 °C).

More precisely, this work fits within the vibration control & cryogenic cooling work package. The objectives of this work package are manifold:

- Improving seismic isolation of GW detectors test-masses in the 10 mHz to 10 Hz frequency band. This is achieved by proposing a novel concept of a hybrid isolation system composed of a passive suspension stage sitting on top of an active, feedback-controlled, 6D platform. This system will ultimately reduce ET-LF seismic noise and improve sensitivity bandwidth in the low-frequency range.
- Propose a novel cryogenic cooling strategy for ET-LF. The cooling system is purely radiative, using panels and fins covered with high-emissivity painting, to provide a contact-less cooling system down to 20 K that avoids transmission of the cryo-cooler vibrations to the proof-mass.
- The investigation of the use of an absolute quantum gravimeter for complementing traditional methods of Newtonian Noise measurement, which are based on the deployment of an array of seismometers. This aims to improve the measurement of Newtonian Noise, which is one of the most dominant noise sources in GWD below 10 Hz, after seismic noise.

- Reducing thermal and optical noises by proposing a suspension system capable of supporting a 100 kg pure-monocrystalline silica test mass with high surface quality.

The above concepts have been implemented in a thermal and mechanical system named the E-TEST prototype [21–23]. Figure 8 shows an overview of the E-TEST prototype design.

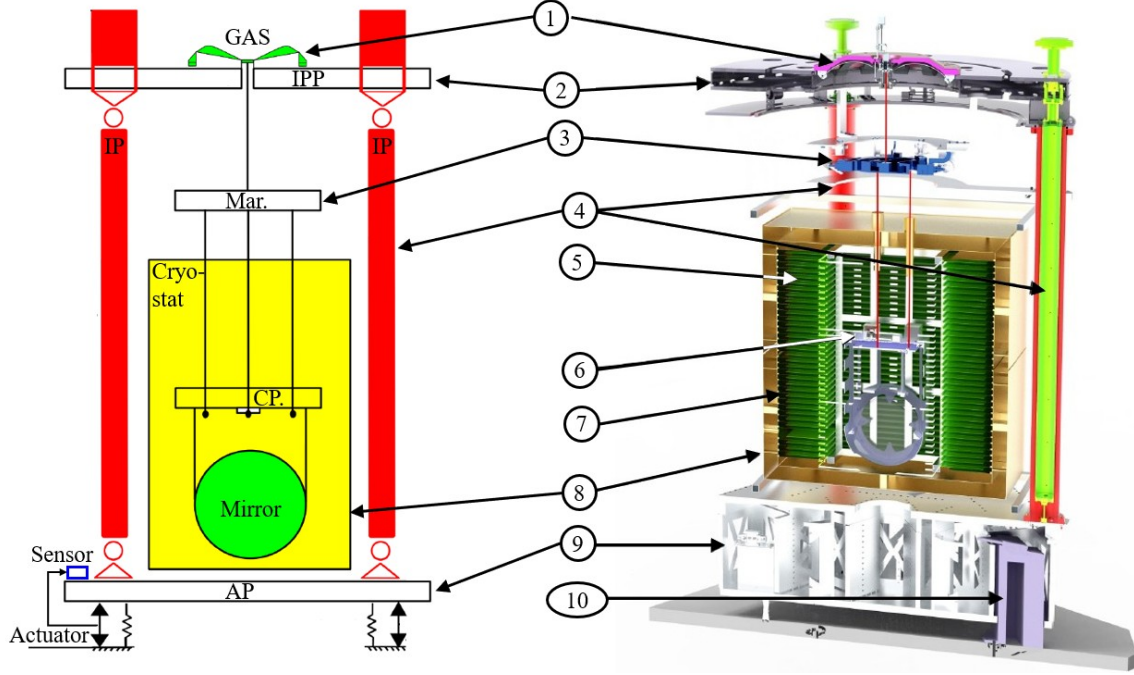


Figure 8: Overview of the E-TEST prototype design (adapted from [21, 23]). (Left) View of the prototype inside the high-vacuum system. (Right) Open view of the prototype. (1) Geometrical Anti-Spring (GAS) system. (2) Inverted Pendulum Platform (IPP). (3) Marionette. (4) Inverted Pendulum (IP) legs. (5) Inner cryostat fins. (6) Cold Platform (CP). (7) Outer cryostat fins. (8) Outer cryostat casing. (9) Active Platform (AP). (10) AP suspensions.

The E-TEST prototype is made of the following key components, from bottom to top. The prototype uses an active stage (AP, 9) that provides active isolation in all 6 dofs to the entire system in the 10 mHz to 10 Hz bandwidth. The AP is instrumented with various high-resolution sensors: 6 custom optical inertial sensors used in feedback active isolation control, 6 BOSEM sensors (Birmingham Optical Sensors and Electro Magnet actuator [24, 25]) used for alignment of the AP alongside a system of springs driven by stepper motors, and 6 voice-coil actuators. The AP is suspended from three leafspring flexures (10) that provide suspension of the AP in the 6 degrees of freedom with approximately 1 Hz modes. On top of the active stage sits the passive suspension chain. It first consists of three inverted pendula (IP, 4), that provides passive isolation in the horizontal degrees of freedom with a few tens of mHz mechanical modes. A platform is mounted on top of the IP (IPP, 2) from which hangs a couple of additional isolation mechanism and ultimately the proof mass. Similarly to the AP, the IPP is also equipped with 4 custom optical displacement sensors that

can be used for closing local damping loops with a voice-coil actuation system, 4 LVDT sensors and another system of stepper motor-driven springs that are used for in-vacuum alignment. The IPP supports a geometrical anti-spring system (GAS, 1), consisting of a set of constrained blades with a very-low effective stiffness [26], from which hangs a marionette (3) used for stirring the mirror during optical experiments, and finally the cold stage (CP, 6) from which the mirror is suspended. The CP is, again, equipped with custom high-resolution cryo-compatible optical inertial sensors [27] for monitoring the proof-mass residual motion. The CP sits inside a cryogenic environment cooled down to 20 K by a radiative cryostat [28]. The cryostat is made of ground-connected panels cooled-down with liquid nitrogen (LN_2) and gaseous helium (GHe) (8) onto which fins covered with high-emissivity painting are attached (7). Then, similar fins with a complementary geometry are attached to the CP (5). Both set of fins (5 and 7) do not make contact and heat is only exchange radiatively. Thanks to the large exchange surface (nearly 80 m^2) and the high-emissivity painting ($\varepsilon = 0.4$), the 100 kg dummy-mirror could be cooled down to 22 K in only 19 days [28]. The Newtonian noise measurement technique is apart from the prototype (not shown in the figure). It proposes to use an Absolute Quantum Gravimeter (AQG [29]) with improved resolution [30] to complement traditional seismometer networks.

For further readings, details on the design of the prototype can be found in [21,22]. The isolation performance has been studied and modelled in [23], and experimental demonstration has been shown in [31–33]. The cryostat design is described in [28]. Details on the CP inertial sensors and cryogenic compatibility can be found in [27,34]. Details on the high-resolution AQG and NN cancellation technique can be found in [30,35,36].

This work focuses on the development and testing of: two designs (horizontal and vertical) of high-resolution optical inertial sensors used for the active inertial feedback control of the AP; optical sensors used for displacement control of the IPP; and an optical accelerometer used for vibration noise compensation in the AQG.

Original contribution

The work presented in this thesis falls in the context of developing new technologies for next-gen gravitational wave detectors, namely A⁺-LIGO and ET (Einstein Telescope). It presents the development of high-resolution optical sensors for the active seismic isolation loop and for instrumenting an AQG (Absolute Quantum Gravimeter, provided by μ Quans/EXail) used for NN measurement. Major original contributions include:

- The development of optical displacement sensors based on quadrature Michelson interferometry. They are characterised with resolution lower than a picometer (10^{-12} m) in the 100 mHz - 1000 Hz bandwidth and over a long measuring range of a few millimetres (2 mm/s maximum velocity), resulting in a dynamic range of 10^9 . Together with the optical device, this work includes the development of its dedicated low-noise electronics and demodulation algorithm (i.e. conversion of the optical signal into physical displacement). An online (real-time) demodulation process has been developed for the use of these sensors in feedback control systems. The sensor's performance stands beyond state-of-the-art commercially available displacement sensors, and the design is vacuum-compatible and meets the standards of GWD isolation systems.
- The development of single-axis optical seismometers that use the above-mentioned optical device as a sensing element. Two designs have been produced: one design with a vertical sensing axis based on a leaf-spring suspension of the sensor's proof mass and one design with a horizontal sensing axis based on a Watt's linkage design. This work covers the entire conception of the seismometers: from the modelling of the suspension/guiding mechanism, design of the sensor mechanics and production of technical drawing, interface with the optical readout, electrical interface, and calibration procedure. The seismometers have been designed for a mechanical resonance of about 0.5 Hz for the horizontal design, and 1.5 Hz for the vertical design. Coupled with the sub-pm resolution optical sensor, the design results in seismometers with resolution better than standard commercial products in the 10 mHz - 100 Hz. Three of each sensor have been produced and are used in the isolation feedback loop of the E-TEST/SILENT active seismic isolation platform. The sensors are named E-VINS and E-HINS (E-TEST Vertical/Horizontal INterferometric INertial Sensor).
- The development of a single-axis (vertical) broadband optical accelerometer that uses the above-mentioned optical device as a sensing element. The design is similar to the above E-VINS sensor, but with dimensions fitting the μ Quans AQG. In addition, a novel strategy based on passive electromagnetic damping of the high-Q mechanical resonance and the use of an online frequency compensation filter is used for broadening and reshaping the bandwidth of the optical accelerometer to meet the requirement of the online vibration compensation technique used in the AQG. The sensor is named μ VINS (Micro (compact) Vertical INterferometric INertial Sensor).

Organization

The manuscript will present the design and characterisation of the novel technologies mentioned above. It is structured as follow:

The high-resolution, long range, optical displacement sensor technology is exposed in Chapter 1. The advantages of optical sensing over traditional (electromagnetic) technologies are briefly discussed, resulting in optical technologies being an ideal fit for this project. Then, the specific optical layout is presented and discussed in section 1.2. A modelling of the interferometric scheme using Jones formalism leads to the development of a demodulation technique a online (real-time) including nonlinearity-correction (ellipse correction). Eventually, the interferometric sensor is experimentally validated and characterised in section 1.3. Various noise sources are identified and investigated, including acquisition (electrical) noises, optical noises, demodulation/nonlinear errors and environmental noises.

Next, Chapter 2 presents the design of the optical seismometer, based on the optical sensing technology exposed in the previous chapter, used for the active isolation of the E-TEST prototype. The fundamental concepts of inertial sensing and inertial control are explained in section 2.1. Section 2.2 shows the mechanical designs of the Vertical (E-VINS) and Horizontal (E-HINS) Interferometric inertial Sensors. Next, section 2.3 details the experimental identification of the sensors responses and resolution. Section 2.4 finally shows isolation performance obtained during a first integration of the inertial sensors in the E-TEST prototype active stage.

The last Chapter 3 discusses the optical accelerometer design for the merging with the Absolute Quantum Gravimeter (AQG). Section 3.1 reminds the working principles of the AQG, from which is derived the need for a high-resolution broadband vibration sensor for vibration noise compensation, as well as design concept specific to broadband inertial sensors. The chapter then follows a structure as above: the mechanical design of the Compact Vertical Interferometric inertial sensor (μ VINS) is shown in section 3.2 and section 3.3 exposes the experimental validation of the broadband optical accelerometer in either force-balanced accelerometer (FBA) mode or frequency-compensated accelerometer (FCA) mode. Section 3.4 then shows results of the integration of the μ VINS inside the AQG for frequency compensation.

Chapter 4 concludes the manuscript by summarising the key features of each technologies and discuss further work and improvements. Succinct summaries may also be found at the end of each chapters.

Additional materials, such as manufacturing and assembly drawings (A), electrical drawings (B), custom signal conditioning design (C) and self-shielded magnetic actuator (D), can be found as appendices.

Chapter 1

Relative displacement sensing in inertial sensors

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Relative motion sensing is crucial in many applications: ranging from mechatronics systems (robots and automated tools), aerospace and defence, and precision metrology. Some applications even require very high resolution, typically sub-nm, to guarantee a proper operation. Notorious examples are precision tools [4] such as photolithography machines, surgical instruments or electron microscopes or active isolation systems of sensitive, high-end, ground-based, scientific instruments such as gravitational wave detectors [37–40], gravimeters [41–43], particle accelerators [44], large telescopes [45] or atomic/optical clocks. Relative motion sensors also play a double role in active isolation systems. They can be used as stand-alone units for monitoring and alignment purposes or for closing local low-authority control loops. Another application is to use them as sensing elements

in inertial sensors. In both cases, contactless technologies, long range and high-resolution are key specifications. In addition, the sensing device should be compact, i.e. with dimensions of a dozen centimetres, so that it can be integrated in the isolation system or in the mechanics of the inertial sensor.

This chapter will present a short review of the current technologies for compact, contact-less, motion sensing, with a deeper emphasis on optical techniques, as they currently provide the highest state-of-the-art sensitivities. Next, section 1.2 will present the optical readout device based on a homodyne, long-range, quadrature Michelson interferometer design. The optical layout, modelling and demodulation process will be discussed. Then, section 1.3 will present the experimental characterisation of the optical device and its noise budget.

1.1 High-resolution, contactless, motion sensing

Relative displacement sensors for use in active isolation systems face two major technological challenges: the need for a high resolution and a contactless sensing technology. Indeed, a contact-less technology is absolutely required to sense the motion of the payload to be isolated without impacting its dynamics, i.e. adding additional stiffness, creating friction, or short-circuiting potential passive isolation mechanisms by creating a direct connection between vibrating ground and payload. This requirement alone already proscribes many displacement sensing technologies that require contact between two elements. For example, potentiometers, which use a wiper sliding on a resistive track to monitor the position, are excluded because of the friction that comes with the guiding and sliding mechanism. Similarly, strain gauges, which are based on the monitoring of the strain of a flexible element and then correlated to a displacement, can not be afforded because they are typically stiff and will transmit ground vibration to the payload. Second, the resolution of the sensor is absolutely key since it sets an ultimate limit to the amount of control that can be achieved. To put it simply, one cannot control better than one can sense. For applications such as gravitational wave detectors that aim at measuring signals as low as $10^{-21} \text{ Hz}^{-1/2}$, sub-nm to sub-pm resolutions are required. This also discards a couple of contactless technologies. For instance, magnetic sensors based on the Hall effect or on the magnetoresistive effect are contact-less but typically only reach μm -like resolution at most. Time-of-flight sensors, based on the measurement of the travelling time of a sound or light wave, also only reach mm-level resolution. One last design requirement is on the size. The footprint should be limited to a dozen centimetres so that the sensor can be fitted in the isolation system and in the mechanics of the inertial units.

This section will overview most of the typical compact contact-less motion sensing technologies with at least a nm-like resolution, updated from [46, 47].

1.1.1 Industry standard solutions

Most well-known and industry standard technologies for compact, contact-less, motion sensing include: Capacitive Position Sensors (CPS), Linear Variable Differential Transformer (LVDT) sensors, Eddy Current Displacement Sensors (ECDS) and interferometric sensors (IFOs). Key specifications are resolution, i.e. the smallest quantity that can be measured and distinguished from sensor noise, and dynamic range, i.e. the ratio between the resolution and the largest quantity that can be measured without saturation or distortion of the signal. Both are typically related in that a sensor can not provide both a very large dynamic range and a very high resolution. Cross-axis sensitivity, i.e. the sensitivity of the sensor to a displacement orthogonal to its main axis, is another important specification that should be minimised. In addition, other practical considerations, such as ease of alignment or compatibility with the operating environment (vacuum, magnetic fields, etc), should be accounted for.

All of these technologies have in common that they are contactless, have compact sensor heads,

sub- μm resolution, and μm -level range, but differ on other key specifications, which will be discussed and compared in the following sections.

Capacitive sensing

Capacitive Position Sensors (CPS) measure the motion between two metallic plates by monitoring the effective capacitance between these plates. Indeed, two charged plates facing each other form a capacitor, whose capacitance value is proportional to the plates' surfaces S and the distance y between them [48]. The sensitivity of a CPS sensor fundamentally scales as

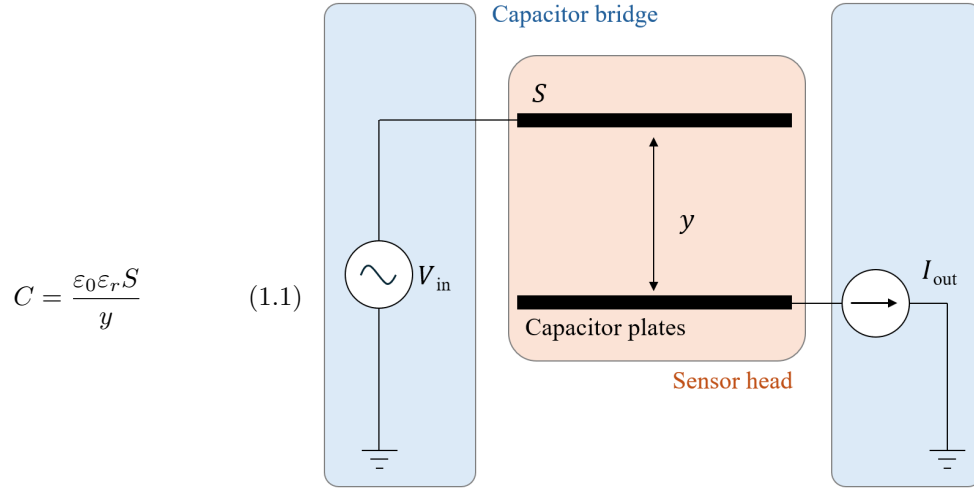


Figure 1.1: Schematic representation of a simple 2 plates Capacitive Position Sensor (CPS). Sensor head (orange). Capacitance reading circuit (blue).

with C the sensor capacitance and $\varepsilon_0 = 8.9 \cdot 10^{-12}$ F/m and ε_r are respectively the absolute electric permittivity in vacuum and the relative permittivity of the medium. The sensor head (plates) necessarily comes with an electric circuit that monitors the capacitance value C . It typically consists of a capacitance bridge (a modified version of the standard Wheatstone bridge for resistors). The design of the capacitance-to-voltage or current circuit has a massive impact on the resolution of the sensor.

CPSs are widely used in precision displacement sensing. They became an almost absolute standard sensing technique in Micro-Electro-Mechanical Systems (MEMS) accelerometers [49] and gyroscopes since it can be "easily" machined, or deposited, onto the wafer material in a way to create a very large (comparatively to the size of the device) surface capacitors, and thus a high sensitivity. The technology is also frequently used in industry standard seismometers, such as the T240 or STS2 [50, 51]. Capacitive sensors typically show some of the highest sensitivities and resolution, commonly reaching $\text{nm}/\sqrt{\text{Hz}}$ -level resolution, mostly limited by the electronic bridge used in the capacitance measurement. High industry standards, capacitive devices have also broken through

the nm-level and can reach a hundred of $\text{pm}/\sqrt{\text{Hz}}$, see Micro-Epsilon ($\mu - \epsilon$) or Physik Instrumente (PI) series [52, 53], also compatible with vacuum environments. Recent research also suggests using advanced CPS designs in accelerometers or gravity sensors [54].

Although recent research and development suggest that CPS are capable of reaching very high resolution, they suffer from several limitations. From a mounting perspective, the performance of CPSs is very sensitive to the alignment of the two sensing plates. CPS will exhibit a strong and increased cross-sensitivity to lateral if the two capacitor plates are not strictly parallel. They are also inherently nonlinear, due to the inverse proportionality of the capacitance on the displacement, leading to limited dynamic range with maximum measurable displacements of typically a couple μm for the most sensitive heads (with nm-resolutions). Exceptions are "large gaps" CPS, which can go up to a mm-range, with a degraded ($> 1 \text{ nm}/\sqrt{\text{Hz}}$) resolution [53]. This makes CPS impractical for monitoring the motion of undamped, high Q -factor, mechanics, namely inertial sensors. The use of CPS as sensing elements in inertial sensors necessarily comes along with a force-balance feedback loop to maintain the proof-mass motion within the linear range [49–51, 55–58].

LVDT sensing

Linear Variable Differential Transformers (LVDTs) are inductive displacement sensors that measure displacement of (typically) a moving ferromagnetic core in a coil assembly, as displayed in Figure 1.2. The coil assembly is made of a winding coil that is surrounded symmetrically by a secondary winding. The primary winding is fed with AC voltage and generates a magnetic flux Φ that is captured at the secondary winding to produce a voltage reading V_{out} . The ferromagnetic core passes through the coils, and its position y in the assembly will affect the mutual inductance between both coils and thus the output voltage [59].

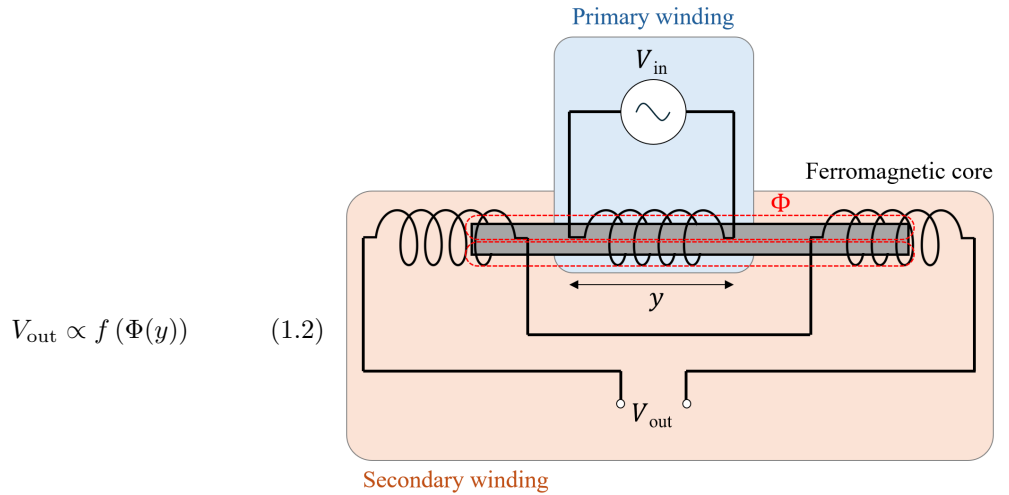


Figure 1.2: Schematic representation of a simple Linear Variable Differential Transformer (LVDT) sensor. Primary winding (blue), secondary winding (orange) and moving core (grey)

LVDT sensors are very reliable and typically have a linear response in a large dynamic range up to several mm. The best resolutions commercially available are, however, limited to the μm range, such as the induSENSOR series from Micro-Epsilon [60]. Only a handful of research papers [61] propose designs with resolution that reach a $10 \text{ nm}/\sqrt{\text{Hz}}$ level. Despite the limited resolution, LVDT sensors still find applications in gravitational wave detection. Because the output reading depends on the static position of the core y in the coil assembly, LVDT sensors are very good DC position sensors. This, combined with the relatively large dynamic range, makes these sensors great for alignment purposes [61].

Eddy current sensing

Eddy Current Displacement Sensors (ECDSs) are another type of induction-based sensors [62, 63]. Similar to LVDT sensors, Eddy current sensors use an assembly of two windings: a primary winding fed with an AC voltage to generate an electromagnetic flux, and a secondary winding for sensing. However, unlike LVDT sensors, the ferromagnetic core is abandoned in the design, and the magnetic flux is left propagating in free space. If a conducting material with sufficient skin depth gets in the proximity of the primary winding, Eddy currents will form. The flux from the primary winding ϕ_p and resulting from the Eddy current Φ_e will set the mutual impedance of the windings. The relationship between the plate distance and the mutual inductance is tabulated in so-called Newman curves. Note that some designs use only one (primary) winding.

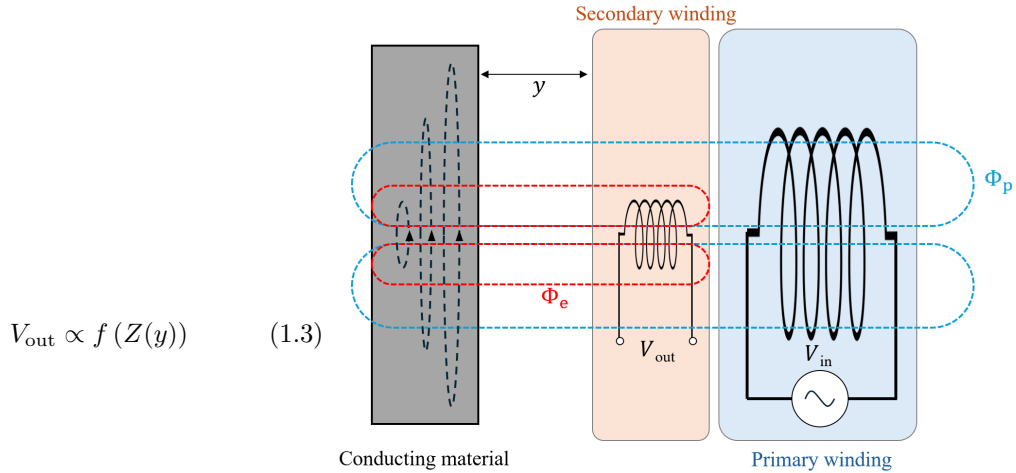


Figure 1.3: Schematic representation of a simple Eddy current sensor. Primary winding (blue), secondary winding (orange) and a moving conducting plate (grey). Dashed lines represent Eddy current (black), primary flux (blue) and Eddy-current induced flux (red).

By giving up on a ferromagnetic core to guide the flux lines, Eddy current sensors cannot reach ranges as high as for long, guided, LVDT sensors, but win in an easier alignment and the ability

to tolerate some degree of angular motion. Lion Precision and Micro-Epsilon ($\mu - \varepsilon$) are leaders in the industry of precision Eddy current devices. Yet, the best resolution is typically at the level of several dozens of nm, with ranges up to a hundred of μm , see the ECLxxx and EddyNCDT series from Lion Precision and $\mu - \varepsilon$ respectively [64, 65]. Recent research suggests designs with sub-nm resolution [66].

Interferometric sensing

Interferometric sensing is a technology that is fundamentally different from the previous electromagnetic devices in that it uses the properties of light to measure displacement or velocity. Displacement sensors based on interferometry come in vary many sizes, resolutions and ranges depending on the specific optical technology or optical layout used [67]. The sensors reviewed in this chapter focus on compact, commercial, sensing technologies, so that laser interferometers with km-long baselines with unprecedented resolution, such that those used in the GW detection, are excluded. For the sake of the demonstration, the working principle of a very simplified Michelson interferometer will be presented. A deeper comparison of the different interferometry technologies will be discussed in a later section.

The optical layout of a Michelson interferometer is shown as in Figure 1.4. In this layout, a laser beam is fed to the sensor from a laser source. The light meets a (typically) 50:50 beam splitter (BS), which transmits half of the light to a reference arm and the other half to a measuring arm (red trace in the figure). The reference arm is equipped with a fixed mirror at a distance L from the BS. Similarly, the measuring arm is equipped with another mirror, but this mirror is now attached to a freely moving target. The light reflects off both mirrors and meets again on BS (dashed blue trace). Any relative motion y between the two arms will cause the laser beam to interfere, resulting in a light power modulated as the interferometric phase Φ_y . This power is measured by a photodetector (PD). It can be calculated that the output signal of the photodiode V_{out} is proportional to the input laser power P_0 and modulated as the interferometric phase Φ_y as

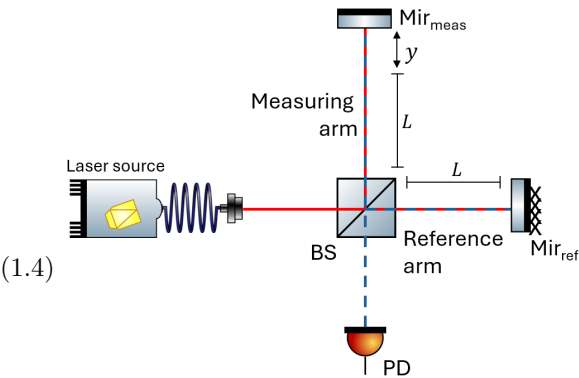
$$\begin{aligned} V_{\text{out}} &\propto \frac{P_0}{2} \left(1 + \cos(2\Phi_y) \right) \\ \Phi_y &= \frac{2\pi}{\lambda} y \end{aligned} \quad (1.4)$$


Figure 1.4: Schematic of a simple Michelson interferometer. Beam path towards the reference and measuring arms (red trace), and toward the detection photodiode (dashed blue trace).

The displacement y of the measuring arm can be demodulated by taking the arc-cosine of the above equation. The sensitivity of the interferometer depends on the laser wavelength λ and on the processing electronics, and can easily break through the $\text{pm}/\sqrt{\text{Hz}}$ resolution. The range of interferometric sensors is inherently limited to the capability to demodulate the phase of the interferometer. Indeed, in the case of the simple Michelson interferometer above for example, the arc-cosine operation allows for demodulating the phase Φ_y only in a $[0; +\pi/2]$ range, or a range limited to motion of maximum $\lambda/4$. However, several methods, such as phase counting or modulated interferometry, allow to bypass this limitation. They will be discussed in a later section.

The high resolution, combined with the capability to extend the range to multiple laser wavelengths, makes interferometry one of the best displacement sensing technologies. Many industries, either already well established in the sensing industry, such as $\mu\text{-}\epsilon$ or new emerging companies specialised in optical sensing, such as SmartAct or AttoCube, propose industrial-grade optical sensors that are compact, have pm-level resolution and dynamic range as high as a mm [68–71]. Extensive research has also been made during the past decades, resulting in long-range devices with pm to fm/ $\sqrt{\text{Hz}}$ resolution levels [72–85].

Contactless high-resolution technology comparison

Figure 1.5 shows a comparison of the different displacement sensing technologies discussed above. They are compared in terms of range (maximum measurable displacement) and resolution (minimum measurable displacement). The figures superimpose specs of industry-standards products in high resolution displacement sensing [52, 53, 60, 64, 65, 68–71], as well as a handful of recent lab-grade research [54, 61, 66, 72–85].

The figure clearly outlines specific operating regions for the different technologies. LVDT sensor stands out by their capability to read motion up to dozens of mm, depending on the length of the core and of the wiring assembly, but hardly reach a hundred nm resolution for the most sensitive devices. On the opposite, high resolution CPSs manage to break the nm-RMS resolution, with a couple of devices reaching levels of several dozens of pm. They can, however, only operate in the mm-range for most sensors, and are limited to dozens of μm for the most sensitive ones. ECDSs stand somewhere in between both technologies, bridging the gap between the longer measurement range of LVDT sensors and high-resolution CPSs. In all of these three technologies, the compromise between resolution and range is obvious: none can provide simultaneously pm-nm resolution and mm-level range. On the other hand, IFO sensors seem to break the rule by providing both unmatched pm-like resolution and mm range for most commercial products. Research has also reached sub-pm to dozens of fm-resolution, while maintaining high range, proving that the technology still has margins for pushing the boundaries of the state-of-the-art high-resolution long-range displacement sensing. The combination of very high resolution and long-range makes interferometric sensors very promising for applications in active vibration isolation.

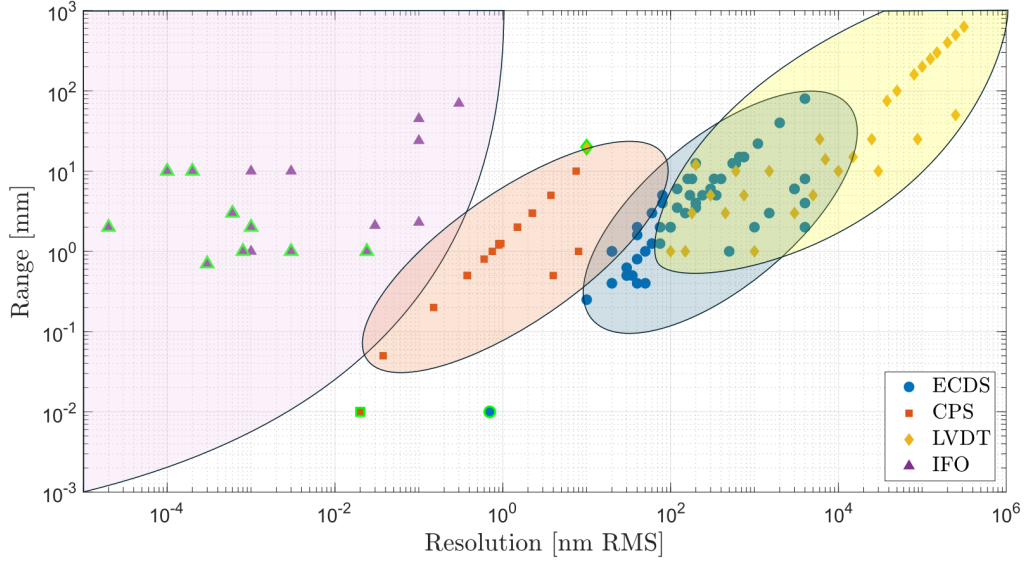


Figure 1.5: Comparison of the main compact, contact-less, displacement sensors technologies. Coloured symbols represent commercially available devices. Green symbols represent laboratory-grade devices.

1.1.2 Long range optical sensing

The previous overview of top-class industry standards and lab research clearly established interferometric sensing as the most promising technology for compact, contactless, high-resolution displacement sensing. However, as explained above, interferometry is inherently a very short-range sensing technology as it fundamentally consists of using light as a ruler for measuring distance. Depending on the wavelength used, the length of this "ruler" therefore ranges from hundreds of nm (for visible light) to a couple of μm at most for infrared sensors. On top of that, the periodic nature of a light wave from an interferometer further limits the detection to only a portion of the full wavelength. As exposed previously, a simple Michelson interferometer only range of a quarter of the wavelength $\lambda/4$, resulting in a range of 400 nm for a Near-InfraRed (NIR, $\lambda = 1600$ nm) device.

Several techniques exist to bypass the limitation related to the demodulation of the periodic interferometric phase. A detailed comparison can be found in [86]. This section will discuss the main strategies of fringe counting or phase tracking used in long-range interferometry, and mention the performance of a handful of recent devices designed using each technique. Because most of these concept already require a good amount of knowledge and understanding in optics, and because a same technique can be implemented in many different ways, it almost impossible to list and describe in details all variations of each technology. Only a global, low-level, overview of the main concept will be presented, and detailed material will be referenced to the appreciation of the readers.

Homodyne interferometry

Homodyne interferometry refers to devices that only use a single laser frequency in the optical scheme. This is, for example, the case of the Michelson interferometer presented above, but also for many other conventional interferometer designs (Mach-Zehnder, Sagnac) [87].

Because only one laser frequency is used, the interferometer signals will necessarily be periodic over distances of a wavelength. To allow for a demodulation of the phase over multiple wavelengths, one trick is to demodulate the phase from two signals in phase-quadrature. This strategy has been applied to multiple optical schemes, ranging from conventional Michelson designs [73–76,88] to Mach-Zehnder types [72,84,89]. Polarising elements are commonly used to split the light beam into two different polarisations, and retarders, such as waveplates, to create a $\pi/2$ phase shift and multiple photodetectors to measure the different quadrature signals. These elements need to be integrated into the design, which sensibly complexifies the optical layout. Once the two quadrature signals have been generated, the demodulation process is, on the other hand, fairly light and straightforward. Various designs have been produced over the years, using different adjustments of polarising elements and waveplates, and all commonly reach resolutions that range from $10 \text{ fm}/\sqrt{\text{Hz}}$ to $0.1 \text{ pm}/\sqrt{\text{Hz}}$. The resolution is commonly limited by the processing electronics and quantum efficiency of the photodetectors.

Another way to create quadrature is to use different Transverse Electromagnetic Modes (TEMs) of light [90]. A well-aligned light beam is composed of a superposition of TEMs, which can be spatially separated with tilted optics. As they travel, they will accumulate different phases, named Gouy phase. However, this technique is very sensitive to precise alignment and is not practical for robust, all-around sensors.

Lastly, another way to use homodyne interferometry for long-range motion is to use a motorised reference mirror [77–79]. Here, a single signal is used, but the reference mirror is mounted on an actuated element (typically a piezo actuator), and a servo-loop is used so that the reference mirror tracks the motion in the measuring arm. This way, the interferometer is locked in a linear operating point and never exceeds its range. The dynamic range requirement is shifted to the piezo actuator, which can be of several dozen μm . In these designs, the optical layout is simplified as it no longer needs to create quadrature, but a servo-loop needs a careful design for proper operation. Demodulation is not needed here, as the control signal is proportional to the measuring motion, is the processing is very simple and light. Devices have shown to reach fm-range resolution, reaching quantum efficiency of the photodiodes and control noise [77–79].

Heterodyne interferometry

As opposed to homodyne interferometers, heterodyne devices use simultaneously multiple laser frequencies [87]. The different frequencies can be generated with another phase-locked laser or with acoustic-optic modulators. Examples can be found in [91–94].

Because there are two different frequencies in the optical layout, they create a beating signal at the frequency difference Δf between the two signals, and the interferometric signals are encoded on this phase of the beat signal. The phase can be demodulated for any displacement, provided that a phase tracking algorithm is used. Similarly to homodyne quadrature interferometers, heterodyne devices need the use of additional optical elements (AOMs, etc...), leading to complex assemblies. Heterodyne setups are also often fairly expensive due to the use of AOMs or multiple laser systems. The demodulation process is fairly simple, even though more complex due to the fact that multiple frequencies need to be processed [86]. One advantage is that the beat-note can be used to modulate the signals at frequencies (typically in the kHz to MHz regime) where noise is lower, leading to better resolution [95].

Again, many devices have been produced using heterodyne detection schemes [91–94, 96]. The modulation allows to reach resolution as low as $70 \text{ fm}/\sqrt{\text{Hz}}$ for recent devices [96].

Modulated interferometry

Deep Frequency Modulation (DFM) and Deep Phase Modulation (DPM) are two sophisticated techniques used to extend the dynamic range of a conventional interferometer. Unlike homodyne or heterodyne systems, which often rely on creating quadrature signals by altering the optical layout (e.g., adding local oscillators or phase shifts), modulated interferometry increases the dynamic range by applying intentional, high-depth modulation to the laser’s phase or frequency [97]. This additional modulation transforms the displacement-dependent phase signal into a non-periodic function of the interferometric phase, typically involving a sum of multiple harmonics (i.e. higher-order Bessel functions in the case of deep modulation). As a result, the interferometer’s output is no longer a simple cosine of phase, but rather a richer signal (e.g. a cosine of cosine) from which the absolute phase, and thus the displacement, can be unambiguously demodulated over many wavelengths using signal processing techniques.

Several pm-resolution sensors have been designed with this technology. They have shown to be capable of reaching $1 \text{ pm}/\sqrt{\text{Hz}}$ to $0.3 \text{ pm}/\sqrt{\text{Hz}}$ resolution [80, 81].

This technique is elegant in that it does not fundamentally modify the optical layout, only a modulation of the input is required. This allows to use simple, and therefore inexpensive and compact, optical scheme. However, they need quite a sophisticated demodulation process, making these devices less appropriate for real-time applications where fast and light demodulation is desired.

Digitally enhanced interferometry

Digitally enhanced interferometry is a fairly recent technique in optical sensing. Similarly to modulated interferometry, it does not need major modifications of the original, conventional optical layout. It even takes the concept one step further and takes any adaptations required for extending the dynamic range in the digital domains. Most digitally enhanced interferometers are based on a

pseudo-random code phase modulated onto an optical carrier [82, 98, 99]. Such devices have been demonstrated to reach sub $\text{pm}/\sqrt{\text{Hz}}$ sensitivity. Digitally enhanced interferometers have a simple optical layout, but require even more advanced backend processing than modulated interferometers, often involving a pseudo-random algorithm and multiplexing.

Phase tracking and fringe counting technique comparison

The above global overview and literature review of recent, lab-grade, long range interferometer showed that similar performances, in the $10 \text{ fm}/\sqrt{\text{Hz}}$ to $1 \text{ pm}/\sqrt{\text{Hz}}$ range, can be achieved using any techniques from homodyne quadrature interferometry, mirror tracking, modulated interferometry and digitally enhanced interferometry.

While they all share similar resolution, the techniques differ massively in their implementation. It has been shown that conventional homodyne and heterodyne interferometry require a complexification of the optical layout to create unambiguous demodulation, but use a light and simple demodulation process. On the opposite, modulated interferometry and digitally enhanced interferometry preserve the original, conventional, optical layout at the cost of an expensive demodulation scheme and algorithm. For real-time applications where the interferometer phase needs to be demodulated online, schemes with simpler algorithms, such as homodyne and heterodyne interferometry, would be appropriate. Then, homodyne (free or locked) interferometers are significantly cheaper than their heterodyne counterparts due to the optical elements they require being very inexpensive: polarising elements and waveplates typically cost hundreds of EUR, compared to phase-locked laser sources and AOMs, which scale in thousands.

For this reason, it is believed that homodyne quadrature interferometry is an appropriate technology for seismic isolation, vibration control and inertial sensing applications. These applications indeed call for sensors that need to be in-loop (with online, low-latency, demodulation) and also require numerous sensors to sense various degrees of freedoms, so that several inexpensive instruments are more valuable than fewer high-cost ones.

1.2 The Compact Quadrature Michelson Interferometer

To meet the stringent requirements of gravitational-wave instrumentation, the inertial sensors are equipped with a compact design of a long-range, quadrature, Michelson interferometer because such devices currently allow reaching some of the highest state-of-the-art sensitivities [72, 74, 75, 80, 83, 84, 86]. Similar readouts have demonstrated sub-picometre resolution when used as sensing elements in inertial devices [73, 76, 100–107]. In addition, the contact-less measuring technique, long dynamic range and vacuum compatibility makes this technology ideal for this application.

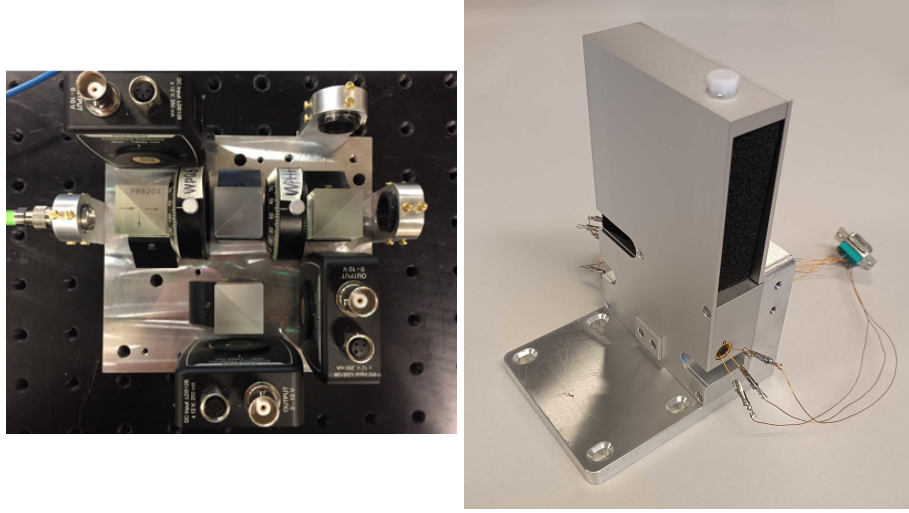


Figure 1.6: Pictures of the compact quadrature Michelson interferometer. (Left) Benchtop prototype mounted on a breadboard for proof-of-concept [46, 108]. (Right) Final device produced at EXail, mounted vertically equipped with a protection cover for the optical fibre.

Building upon the experience gained from previous prototypes [46, 108], a compact interferometer has been designed for reading the proof-mass motion of the inertial sensors. Figure 1.6 shows photos of a benchtop assembly of the interferometer for prototyping and of the final optical device produced at EXail. The interferometer is enclosed in a dark box, which only allows an opening for the output laser beam. The opening is located under the protection cap at the bottom of the device in the left figure. The laser beam is fed to the interferometer via a fibre-coupled input that can be seen under the white hood, and the fibre is also protected under an aluminium cover. The output signals of the interferometer consist of the photodiodes seen in the left figure in the golden casing. The device is compact, and has dimensions of $85 \times 60 \times 27 \text{ mm}^3$ (excluding the fibre and fibre cover), so that it can be fairly easily fitted into the mechanics of an inertial sensor and of the isolation system.

This section will present details on the design of the compact quadrature Michelson interferometer. The optical layout will be described first, and the working principle will be supported by an analytic modelling using Jones calculus for polarisation-based optical system. The demodulation of the reading of the interferometer will then be discussed. Finally, the final device and the associated hardware of the entire acquisition chain will be presented.

1.2.1 Optical layout

The optical layout of the interferometer is shown in Figure 1.7. It inherits from previous development of Michelson interferometers based on quadrature polarisation states of light [46, 72, 74–76, 83, 108]. The layout is fundamentally similar to that of a classical Michelson interferometer: relative motion between two orthogonal arms is encoded in the interferometric phase as the beams recombine at the beam splitter. In this design, the light in the Z-arm reflects off a fixed mirror (Mir_z) serving as a reference arm and the Y-arm, or measuring arm, targets a movable mirror (CC_y). Any motion in the Y-arm will cause the laser beams reflected from CC_y to exhibit a phase difference compared to the one reflected from Mir_z . This phase shift will imprint in the interferometric phase and can be demodulated to recover displacement along the Y-arm.

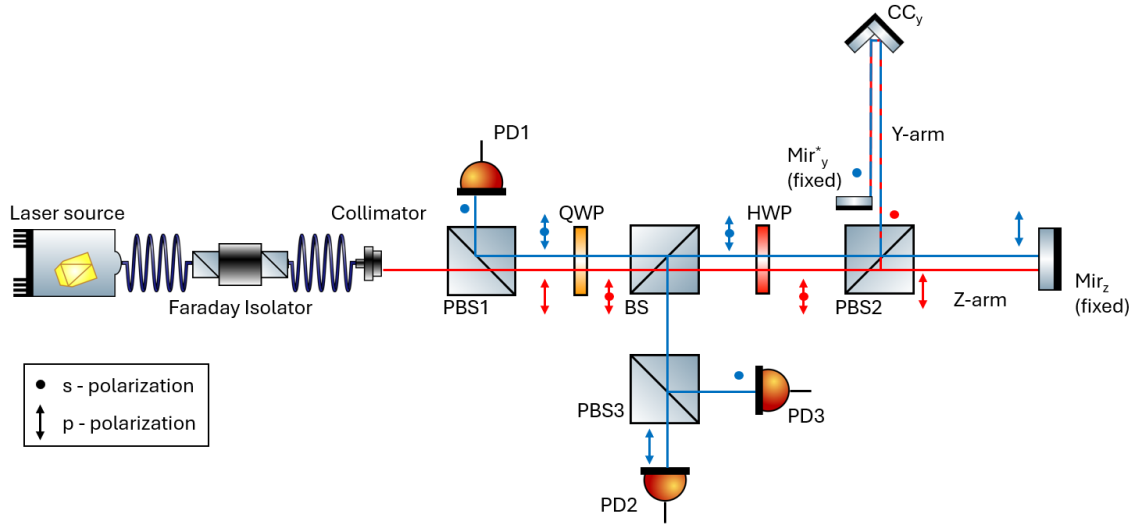


Figure 1.7: Quadrature Michelson interferometer optical layout. The forward beam path toward the two arms of the interferometer is shown in red, while the return path is shown in blue. The Z-arm is the reference arm and reflects off a fixed mirror. The Y-arm is the measuring arm and targets of moving corner cube reflector. The forward and return paths are shown off-centre for visualisation purposes.

The laser beam is fibre-coupled into the interferometer via a polarisation-maintaining FC/APC optical fibre and then propagates in free space. A Faraday isolator is placed at the input to suppress back reflection to the laser source. The beam first encounters a polarising beam splitter (PBS1) oriented to transmit the p-polarised component. If the collimator axis is well aligned, no power loss should be observed, but in case of any misalignment PBS1 effectively acts as a filter that ensures a clean p-polarisation state of the laser at the entrance of the interferometer. The laser then passes through a quarter wave-plate (QWP) with its fast axis rotated 45° anticlockwise. The QWP introduces a phase shift of exactly $\lambda/4$ and its orientation converts the linearly polarised light into circular polarisation. Zero-order wave-plates are used, as they provide a single-wavelength retardance and lower temperature sensitivity compared to their low-order or multi-order alternatives.

The laser beam then reaches a 50:50 non-polarising beam splitter (BS), where half of the input power is transmitted. The transmitted beam passes through a half wave-plate (HWP) rotated 22.5° anticlockwise, further rotating the polarisation state, and finally reaches another polarising beam splitter (PBS2). Here, the p-polarisation is transmitted in the Z-arm, while the s-polarisation is sent to the orthogonal Y-arm. The Y-arm is equipped with a corner cube retroreflector (CC_y) in a double-pass configuration. This choice accommodates angular motion of the Y-arm which would eventually be aimed to a moving target as it will be further discussed in a subsequent section. After reflection from both arms, the two arms then meet at PBS2 with orthogonal polarisation states and therefore do not interfere. They both pass again through HWP so that the polarisation states are mixed again. On the way back to the photodiodes, a portion of the beam is transmitted at the BS and arrives at the QWP with the p-polarisation aligned with the slow axis of the wave-plate while the s-polarisation aligns with the fast axis. At PBS1, the s-polarisation is transmitted to the first photodiode (PD1) and optical power is measured. Meanwhile, the reflected beam from BS encounters a third polarising beam splitter (PBS3), and the mixed polarisation is decomposed into p- and s-polarised components, which are detected at photodiodes 2 (PD2) and 3 (PD3), respectively.

Below is a table summarising the list of elements and orientation.

Component	Description	Ref., Qty.	
		Prototype	Final device
Laser source	DFB benchtop source, 1550.12 nm, active wavelength control	NKT Koheras Adjustik X15, 1	
Faraday isolator	1550 nm isolator with FC/APC PM fibre	IO-G-1550-APC, 1	
Collimator	Fixed focus collimator, 1550 nm, p-axis aligned vertically	F240APC-1550, 1	EXail, 1
PBS	Polarizing beam splitter cube, 1200 - 1600 nm	PBS204, 3	EXail, 3
QWP	Zero-order quarter wave-plate, fast axis at 45° anticlockwise, 1550 nm	WPQ10E-1550, 1	EXail, 1
BS	50:50 Non-polarizing beam splitter cube, 1100-1600 nm	BS018, 1	EXail, 1
HWP	Zero-order half wave-plate, fast axis at 22.5° anticlockwise, 1550 nm	WPH10E-1550, 1	EXail, 1
Mirror	Flat-mirror 1550 nm	None	EXail, 2
Corner cube	TIR retroreflector, 1050 - 1700 nm	PS974M-C, 2	EXail, 1
PD	Photodetectors, Prototype: Ge amplified, Final device: InGaAs	PDA50B2, 3	FGA21, 3

Table 1.1: List of optical components for the quadrature Michelson interferometer, prototype version and final version. Most items of the prototype assembly are Thorlabs items.

Double-pass reflection in the Y-arm

Since the optical device will eventually target the proof mass of an inertial sensor, accommodating the target's angular trajectory is a major concern.

This mostly drives the choice for a corner cube in the Y-arm. The geometrical properties of retroreflectors indeed ensure that the reflected laser beam will remain parallel to the incident beam, regardless of the reflector's angle. However, retroreflectors also displace the beam away from its incoming axis. The non-overlapping incident and reflected beams cause the interference fringes to lose contrast and visibility, which ultimately translates into additional noise on the displacement

reading and an enhanced nonlinear behaviour. To compensate for that, a second flat mirror is placed in the Y-arm to send the laser beam back to the retroreflector. The second pass in the retroreflector causes the laser beam to undergo the same geometrical reflections in a reverse operation so that the reflected beam remains co-aligned with the initial incident beam and minimises misalignment, even for large excursions of the Y-arm target. The corner cube retroreflector in combination with a double-pass strategy also ease the alignment process. Finally, this double-pass strategy doubles the distance travelled by the laser in the Y-arm, providing an optical amplification factor of 2, which also doubles the sensitivity of the interferometer [109]. The multi-pass reflection also helps minimise the spurious polarisation effect induced by solid TIR (total Internal Reflection) corner cube, which is harmful to the sensitivity of any polarisation-based interferometer [110, 111].

1.2.2 Modelling

The equations relating motion along the Y-arm to the interferometric phase, and ultimately to the signals sensed at the photodiodes, can be derived using Jones calculations [112]. In this formalism, light is expressed as a 2D vector with the first element being the electric field of the p-polarisation state and the second element being the electric field of the s-polarisation state. A Jones matrix is associated with each optical component of Figure 1.7. They are 2×2 matrices representing the light field transformation when passing through the component. The list of Jones matrices is given in Table 1.2. For the sake of the demonstration, the Jones Matrices considered in this section are the academic expressions and neglect material/manufacturing properties. The exact Jones matrices for a specific component depend on the material optical properties and manufacturing process, and are often identified experimentally. They are usually specified by the manufacturer. The polarising properties typically differ from the textbook expressions by only a few per cent and can be assumed negligible in these calculations. Similarly, the polarising properties of the retroreflectors are listed in Table 1.2, but the target in the Y-arm is assumed to be an ideal flat mirror in this calculation. The retroreflector matrices are derived from a geometrical modelling of the reflections inside a TIR-based corner cube. They have been calculated and verified experimentally for each of the six propagation trips in the corner cube, depending on the incident sextant of the beam [110]. These matrices will be used in a later analysis on the effect of the polarising properties of retroreflectors on the quadrature Michelson interferometer.

The light power captured at the photodiodes is obtained by multiplying the input electric field at the collimator output with the Jones matrices of all optical elements that the beam meets on its path to the photodiode. Assuming that the input collimator axis is correctly aligned and outputs a linearly polarised light aligned with the p-polarisation state, the light field at the input of the interferometer writes

$$E_{\text{in}} = \sqrt{P_0} e^{i(\omega t + \Phi_0)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (1.5)$$

Polarising element	Jones Matrix	
	Transmission	Reflection
Non-polarising 50:50 beam splitter	$BS_t = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$BS_r = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Polarising beam splitter	$PBS_t = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$PBS_r = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Rotation matrix for any angle θ	$R(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$	
Quarter wave-plate at 45° angle	$QWP_{45^\circ} = R(-45^\circ) \cdot e^{i\frac{\pi}{4}} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \cdot R(45^\circ) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$	
Half wave-plate at 22.5° angle	$HWP_{22.5^\circ} = R(-22.5^\circ) \cdot \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \cdot R(22.5^\circ) = \frac{i}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	
Reflective elements	Jones Matrix	
	Reflection	
Flat mirror	$Mir = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	
TIR corner cube Trip 1: ACO $\rightarrow$ ABO $\rightarrow$ BCO	$CC_1 = \begin{bmatrix} -0.0266 + 0.9586i & 0.2282 + 0.1685i \\ 0.2282 + 0.1685i & -0.9083 + 0.3075i \end{bmatrix}$	
TIR corner cube Trip 1r: BCO $\rightarrow$ ABO $\rightarrow$ ACO	$CC_{1,r} = \begin{bmatrix} CC_1(1,1) & -CC_1(2,1) \\ -CC_1(1,2) & CC_1(2,2) \end{bmatrix}$	
TIR corner cube Trip 2: ABO $\rightarrow$ BCO $\rightarrow$ ACO	$CC_2 = \begin{bmatrix} 0.6746 + 0.0090i & -0.1766 + 0.7167i \\ 0.6330 - 0.3797i & -0.2071 - 0.6421i \end{bmatrix}$	
TIR corner cube Trip 2r: ACO $\rightarrow$ BCO $\rightarrow$ ABO	$CC_{2,r} = \begin{bmatrix} CC_2(1,1) & -CC_2(2,1) \\ -CC_2(1,2) & CC_2(2,2) \end{bmatrix}$	
TIR corner cube Trip 3: BCO $\rightarrow$ ACO $\rightarrow$ ABO	$CC_3 = \begin{bmatrix} 0.6746 + 0.0090i & 0.6330 - 0.3797i \\ -0.1766 + 0.7167i & -0.2071 - 0.6421i \end{bmatrix}$	
TIR corner cube Trip 3r: ABO $\rightarrow$ ACO $\rightarrow$ BCO	$CC_{3,r} = \begin{bmatrix} CC_3(1,1) & -CC_3(2,1) \\ -CC_3(1,2) & CC_3(2,2) \end{bmatrix}$	

Table 1.2: Jones matrices for the components of the interferometer layout. These are ideal matrices found in optics textbooks, and neglect the manufacturing and material properties of the specific elements of the interferometer. The matrices for the TIR corner cube are derived from a geometrical modelling of the light propagation in the retroreflector [110].

with P_0 the input laser power, Φ_0 the laser phase and $\omega = 2\pi\frac{\lambda}{c}$ the laser frequency with $\lambda = 1550.12$ nm the wavelength and $c \approx 3 \cdot 10^8$ m/s the speed of light in free space. Note that the initial laser phase Φ_0 is set when turning the laser source on, and will not impact the reading, as only the phase difference between the two arms will result in interferences. Note also that in case of a misaligned collimator key, the first polarising beam splitter PBS1 will act as a filter and remove any spurious s-polarisation component of the input light field. This translates into an effective power in the interferometer being less than the input, nominal, power. For example, considering a spurious amount of s-polarisation light α caused by a non-proper alignment of the collimator axis, the light field at the output of PBS1 is

$$\text{PBS1}_t \cdot E'_{\text{in}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \sqrt{P_0} e^{i(\omega t + \Phi_0)} \cdot \begin{bmatrix} 1 - \alpha \\ \alpha \end{bmatrix} = (1 - \alpha) E_{\text{in}} \quad (1.6)$$

with E'_{in} the light input with misaligned polarisation state. The light at the output of PBS1 is equivalent to a well-aligned interferometer fed with a lesser input power $P'_0 = \left| \text{PBS}_t \cdot E'_{\text{in}} \right|^2 = (1 - \alpha)^2 P_0$.

Propagating the light beam through the layout until PBS2, the light electric field entering the reference Z-arm of the interferometer is given by

$$\begin{aligned} E_{\rightarrow \text{Z-arm}} &= \text{PBS2}_t \cdot \text{HWP}_{22.5^\circ} \cdot \text{BS}_t \cdot \text{QWP}_{45^\circ} \cdot \text{PBS1}_t \cdot E_{\text{in}} \\ &= \frac{1}{2\sqrt{2}} \begin{bmatrix} -1 + i & 0 \\ 0 & 0 \end{bmatrix} \cdot E_{\text{in}} \end{aligned} \quad (1.7)$$

The light in the Z-arm reflects off a fixed mirror placed at a distance L from the beam splitter and accumulates twice a phase shift $\Phi_L = \frac{2\pi L}{\lambda}$.

$$\begin{aligned} E_{\leftarrow \text{Z-arm}} &= e^{i\Phi_L} \cdot \text{Mir}_z \cdot e^{i\Phi_L} \cdot E_{\rightarrow \text{Z-arm}} \\ &= \frac{1}{2\sqrt{2}} e^{i2\Phi_L} \begin{bmatrix} -1 + i & 0 \\ 0 & 0 \end{bmatrix} \cdot E_{\text{in}} \end{aligned} \quad (1.8)$$

Similarly, the light electric field entering the measuring Y-arm of the interferometer is

$$\begin{aligned} E_{\rightarrow \text{Y-arm}} &= \text{PBS2}_r \cdot \text{HWP}_{22.5^\circ} \cdot \text{BS}_t \cdot \text{QWP}_{45^\circ} \cdot \text{PBS1}_t \cdot E_{\text{in}} \\ &= \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 & 0 \\ 1 + i & 0 \end{bmatrix} \cdot E_{\text{in}} \end{aligned} \quad (1.9)$$

The light in the Y-arm reflects off a movable target. Assuming that the target moves as y , the light accumulates a phase shift as $\Phi_y = \frac{2\pi y}{\lambda}$, in addition to the static phase shift Φ_L due to the static arm-length. Because of the double-pass strategy, the phase shifts are accumulated 4 times, as pictured in Figure 1.8. For the sake of the demonstration, the target in the Y-arm is assumed to be

a flat mirror; the case with a TIR retroreflector will be covered in a later section.

$$\begin{aligned}
 E_{\leftarrow Y\text{-arm}} &= e^{i(\Phi_L + \Phi_y)} \cdot \text{Mir}_y \cdot e^{i(\Phi_L + \Phi_y)} \cdot \text{Mir}_y^* \cdot e^{i(\Phi_L + \Phi_y)} \\
 &\quad \cdot \text{Mir}_y \cdot e^{i(\Phi_L + \Phi_y)} \cdot E_{\rightarrow Y\text{-arm}} \\
 &= \frac{1}{2\sqrt{2}} e^{i(4\Phi_L + 4\Phi_y)} \begin{bmatrix} 0 & 0 \\ -1 - i & 0 \end{bmatrix} \cdot E_{\text{in}}
 \end{aligned} \tag{1.10}$$

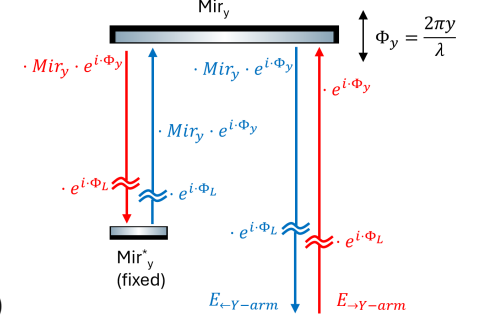


Figure 1.8: Schematic of the reflections in the Y-arm. The moving target in the Y-arm is assumed to be a flat mirror.

As the light beams recombine at PBS2, the total light electric field is

$$\begin{aligned}
 E_{\leftarrow \text{PBS2}} &= \text{PBS2}_{\text{t}} \cdot E_{\leftarrow Z\text{-arm}} + \text{PBS2}_{\text{r}} \cdot E_{\leftarrow Y\text{-arm}} \\
 &= \frac{1}{2\sqrt{2}} e^{i2\Phi_L} \begin{bmatrix} -1 + i & 0 \\ e^{i(2\Phi_L + 4\Phi_y)}(-1 - i) & 0 \end{bmatrix} \cdot E_{\text{in}}
 \end{aligned} \tag{1.11}$$

The signal measured at PD1 is obtained by propagating the light from PBS2 back to PBS1

$$\begin{aligned}
 E_{\text{PD1}} &= \text{PBS1}_{\text{r}} \cdot \text{QWP}_{-45^\circ} \cdot \text{BS}_{\text{t}} \cdot \text{HWP}_{-22.5^\circ} \cdot E_{\leftarrow \text{PBS2}} \\
 &= \frac{i}{4} e^{i2\Phi_L} \begin{bmatrix} 0 & 0 \\ 1 + e^{i(4\Phi_y + 2\Phi_L)} & 0 \end{bmatrix} \sqrt{P_0} e^{i(\omega t + \Phi_0)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
 &= \frac{i\sqrt{P_0}}{4} e^{i(\omega t + \Phi_0 + 2\Phi_L)} \begin{bmatrix} 0 \\ 1 + e^{i(4\Phi_y + 2\Phi_L)} \end{bmatrix}
 \end{aligned} \tag{1.12}$$

Note that on the return path from PBS2 the light beam sees the wave-plate rotated clockwise, instead of counter-clockwise. Then, the measured light power on PD1 is

$$\begin{aligned}
 P_{\text{PD1}} &= |E_{\text{PD1}}|^2 = \left| \frac{i\sqrt{P_0}}{4} \right|^2 \cdot \left| e^{i(\omega t + \Phi_0 + 2\Phi_L)} \right|^2 \cdot \left(0 + \left| 1 + e^{i(4\Phi_y + 2\Phi_L)} \right|^2 \right) \\
 &= \frac{P_0}{16} \cdot 1 \cdot \left((1 + \cos(4\Phi_y + 2\Phi_L))^2 + \sin^2(4\Phi_y + 2\Phi_L) \right) \\
 &= \frac{P_0}{16} \left(1 + \cos^2(4\Phi_y + 2\Phi_L) + 2\cos(4\Phi_y + 2\Phi_L) + \sin^2(4\Phi_y + 2\Phi_L) \right) \\
 &= \frac{P_0}{8} \left(1 + \cos(4\Phi_y + 2\Phi_L) \right)
 \end{aligned} \tag{1.13}$$

Similarly, the signal measured at PD2 and PD3 are obtained by propagating the light from PBS2

to PBS3

$$\begin{aligned}
E_{\text{PD2}} &= \text{PBS3}_t \cdot \text{BS}_t \cdot \text{HWP}_{-22.5^\circ} \cdot E_{\leftarrow \text{PBS2}} \\
&= \frac{1}{4\sqrt{2}} e^{i2\Phi_L} \begin{bmatrix} -1+i + (-1+i)e^{i(4\Phi_y+2\Phi_L)} & 0 \\ 0 & 0 \end{bmatrix} \sqrt{P_0} e^{i(\omega t + \Phi_0)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&= \frac{\sqrt{P_0}}{4\sqrt{2}} e^{i(\omega t + \Phi_0 + 2\Phi_L)} \begin{bmatrix} (-1+i)(1 + e^{i(4\Phi_y+2\Phi_L)}) \\ 0 \end{bmatrix}
\end{aligned} \tag{1.14}$$

And the light power captured at PD2 is

$$\begin{aligned}
P_{\text{PD2}} &= |E_{\text{PD2}}|^2 = \left| \frac{\sqrt{P_0}}{4\sqrt{2}} \right|^2 \cdot \left| e^{i(\omega t + \Phi_0 + 2\Phi_L)} \right|^2 \cdot \left(\left| (-1+i)(1 + e^{i(4\Phi_y+2\Phi_L)}) \right|^2 + 0 \right) \\
&= \frac{P_0}{8} \left(1 + \sin(4\Phi_y + 2\Phi_L) \right)
\end{aligned} \tag{1.15}$$

Similarly for PD3,

$$\begin{aligned}
E_{\text{PD3}} &= \text{PBS3}_r \cdot \text{BS}_t \cdot \text{HWP}_{-22.5^\circ} \cdot E_{\leftarrow \text{PBS2}} \\
&= \frac{1}{4\sqrt{2}} e^{i2\Phi_L} \begin{bmatrix} 0 & 0 \\ 1+i - (1+i)e^{i(4\Phi_y+2\Phi_L)} & 0 \end{bmatrix} \sqrt{P_0} e^{i(\omega t + \Phi_0)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&= \frac{\sqrt{P_0}}{4\sqrt{2}} e^{i(\omega t + \Phi_0 + 2\Phi_L)} \begin{bmatrix} 0 \\ (1+i)(1 - e^{i(4\Phi_y+2\Phi_L)}) \end{bmatrix}
\end{aligned} \tag{1.16}$$

And the light power measured on PD3 is

$$\begin{aligned}
P_{\text{PD3}} &= |E_{\text{PD3}}|^2 = \left| \frac{\sqrt{P_0}}{4\sqrt{2}} \right|^2 \cdot \left| e^{i(\omega t + \Phi_0 + 2\Phi_L)} \right|^2 \cdot \left(0 + \left| (1+i)(1 - e^{i(4\Phi_y+2\Phi_L)}) \right|^2 \right) \\
&= \frac{P_0}{8} \left(1 - \sin(4\Phi_y + 2\Phi_L) \right)
\end{aligned} \tag{1.17}$$

With $\Phi_y = \frac{2\pi y}{\lambda}$, the displacement y along the Y-arm is effectively modulated in the signal measured at the three photodiodes. In addition to Φ_y , a spurious phase Φ_L also appears in Equation 1.13, 1.15 and 1.17. This phase corresponds to the phase accumulated along the static arm-length L of the Y- and Z-arm. In a balanced Michelson interferometer, the static phase is equal on both arms and cancels out when recombined at PBS2. However, the double reflection in the Y-arm causes the effective static length to be twice as large as that in the Z-arm, even if both arms are mechanically balanced. In any case, since Φ_L is a static component (under the assumption of negligible thermal expansion or mechanical noise in the optical assembly), it will only contribute to the reading as a DC offset and will therefore not affect performance.

1.2.3 Demodulation

From subsection 1.2.2, it has been established that the motion y along the Y-arm is imprinted in the interferometer phase and is read at the photodiodes as

$$\begin{cases} \Phi_y = \frac{2\pi}{\lambda} y \\ P_{PD1} = \frac{P_0}{8} (1 + \cos(4\Phi_y)) \\ P_{PD2} = \frac{P_0}{8} (1 + \sin(4\Phi_y)) \\ P_{PD3} = \frac{P_0}{8} (1 - \sin(4\Phi_y)) \end{cases} \quad (1.18)$$

The most straightforward calculation would then be to demodulate the signal of one photodiode to obtain the interferometer phase Φ_y , and then estimate the displacement using the appropriate optical factor $\lambda/(2\pi)$ [m/rad].

Since demodulating the signal from one photodiode will cause ambiguity due to the periodicity of the sine or cosine function, the interferometer phase is extracted from two signals in phase quadrature, $Q_{\cos}$ and $Q_{\sin}$, so that an arctangent (aperiodic) function can be used. First, two quadrature signals are reconstructed by subtracting pairs of photodiodes. This operation also helps to minimise common-mode noise on the photodiode signals, such as laser intensity noise (see section 1.3).

$$\begin{aligned} Q_{\sin} &= P_{PD1} - P_{PD3} = \frac{P_0}{8} (\cos(4\Phi_y) + \sin(4\Phi_y)) \\ &= \frac{P_0}{8} (\cos(4\Phi_y) - \cos(4\Phi_y + \frac{\pi}{2})) \\ &= \frac{P_0}{8} (-2 \sin(\frac{4\Phi_y - 4\Phi_y - \frac{\pi}{2}}{2}) \sin(\frac{4\Phi_y + 4\Phi_y + \frac{\pi}{2}}{2})) \\ &= \frac{P_0}{8} (-2 \sin(\frac{-\pi}{4}) \sin(4\Phi_y + \frac{\pi}{4})) \\ &= \frac{\sqrt{2}P_0}{8} \sin(4\Phi_y + \frac{\pi}{4}) \end{aligned} \quad (1.19)$$

And similarly,

$$\begin{aligned} Q_{\cos} &= P_{PD1} - P_{PD2} = \frac{P_0}{8} (\cos(4\Phi_y) - \sin(4\Phi_y)) \\ &= \frac{\sqrt{2}P_0}{8} \cos(4\Phi_y + \frac{\pi}{4}) \end{aligned} \quad (1.20)$$

Graphically, these two signals, if plotted on a Lissajous figure, will create a circular figure centred on the origin. The interferometric phase is then represented as the angle of a point in this figure. A

ratio of these two signals creates the tangent modulation.

$$\begin{aligned} \frac{Q_{\sin}}{Q_{\cos}} &= \frac{\frac{\sqrt{2}P_0}{8} \sin\left(4\Phi_y + \frac{\pi}{4}\right)}{\frac{\sqrt{2}P_0}{8} \cos\left(4\Phi_y + \frac{\pi}{4}\right)} \\ &= \tan\left(4\Phi_y + \frac{\pi}{4}\right) \end{aligned} \quad (1.21)$$

So that the estimation of the motion $\hat{y}$ along the Y-arm is calculated as

$$\begin{aligned} \hat{y} &= \frac{\lambda}{2\pi} (\hat{\Phi}_y) \\ &= \frac{\lambda}{2\pi} \left(\frac{1}{4} \tan^{-1}\left(\frac{Q_{\sin}}{Q_{\cos}}\right) - \frac{\pi}{16} \right) \\ &= \frac{\lambda}{8\pi} \tan^{-1}\left(\frac{Q_{\sin}}{Q_{\cos}}\right) - \frac{\lambda}{32} \end{aligned} \quad (1.22)$$

Where the second term is a DC component, which can be safely ignored since only the dynamic value of the reading is of interest. This simple demodulation process can be summarised as shown in the flowchart in Figure 1.9.

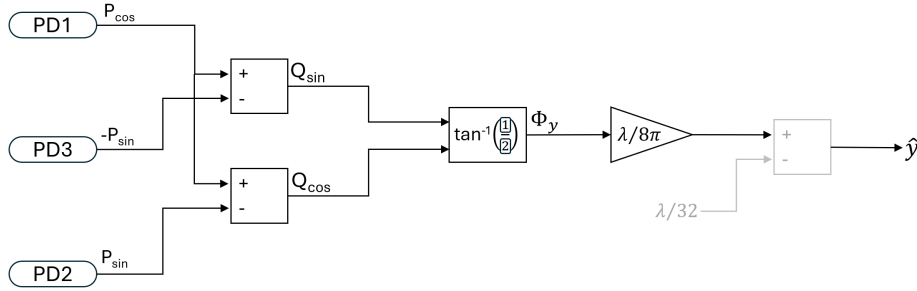


Figure 1.9: Demodulation process flowchart for the quadrature Michelson interferometer. PD_i are the photodiode output signals and $\hat{y}$ is the demodulated displacement estimation.

However, the above calculation is only meaningful for photodiode signals as they are depicted in Equation 1.18. In practice, imperfections will cause the actual signals to differ from the theoretical case, leading in a general case to an elliptic Lissajous figure. In such a case, the arctangent operation will fail and introduce errors and nonlinearities. A normalisation and an ellipse correction algorithm are implemented in the processing of the signal, before demodulating the interferometer phase [46, 76, 113, 114]. These two algorithms compensate for these imperfections by identifying the ellipse parameters of the actual Lissajous figure, obtained experimentally, and then converting the ellipse back to a unit circle solving a least-squares problem.

From a global perspective, practical imperfections in the interferometric readout will cause alterations of the photodiode reading that can be categorised in three components: (i) offset errors, (ii) gain errors and (iii) phase error. i: Offset errors (b_i) correspond to a DC value read at the

photodiodes, and may be different for the three photodiodes, mainly in the case of a bad alignment of the optics, or a sub-optimal interferometer contrast. Indeed, a bad contrast implies that the laser beams do not completely overlap when recombining at PBS2 and therefore do not interfere. This translates to a static amount of power being measured at a photodiode. ii: Gain errors (a_i) can arise from bad alignment and discrepancies in the signal conditioning electronics or in characteristics of each photodiode. Indeed, the laser power as expressed in Equation 1.18 represents the amount of light power that lands on each photodiode in units of [W]. The signal measured at the output of the photodiode is a current that depends on the responsivity of the photodiode active area $\mathfrak{R}_i$ [A/W]. In addition, this photocurrent is usually very small due to the low responsivity values and input power, and cannot be processed as it. The photodiode signals are then fed to a signal conditioning unit which consists of a pre-amp analogue circuits with a certain transimpedance gain value G_i [V/A] to convert the photocurrent signal into a useful voltage. In addition, fringe contrast I_i , misalignment α_i and even power loss in optical components η_i , will degrade the amount of light perceived at the photodiode. Overall, the voltage output for a photodiode writes $V_{\text{PD}i} = G_i \mathfrak{R}_i \cdot \alpha_i I_i \eta_i \cdot P_{\text{PD}i}$. iii: Phase error (ϕ_i) can arise due to imperfect material optical properties or misalignment. More discussion on these noise sources and their implication in the performance of the readout in section 1.3. In the end, a more realistic expression of the photodiode signals (in [V]), would be

$$\begin{cases} V_{\text{PD}1} = a_1 \cdot \left(1 + \cos(4\Phi_y + \phi_1)\right) + b_1 \\ V_{\text{PD}2} = a_2 \cdot \left(1 + \sin(4\Phi_y + \phi_2)\right) + b_2 \\ V_{\text{PD}3} = a_3 \cdot \left(1 - \sin(4\Phi_y + \phi_3)\right) + b_3 \end{cases} \quad (1.23)$$

where $a_i = G_i \mathfrak{R}_i \alpha_i I_i \eta_i \cdot \frac{P_0}{8}$ represents the global gain of the photodiode, including the electronic and optical gains.

It can easily be understood that the gain, offset and phase errors may be different for all three photodiodes. In this case, it appears from Equation 1.23 that the Lissajous figure of the quadrature signals will have the general shape of an ellipse rather than a circle. A normalisation process and an ellipse correction of the signal can help convert the figure back to a circle, which will minimise demodulation errors and nonlinearities.

Normalisation

First, a normalisation process is applied to the photodiode voltage to compensate for the gain and offset errors. From a subset of a measurement of each photodiode data, which will be referred to as the calibration dataset $V_{\text{PD}i}^*$, the gains $\tilde{a}_i$ and offset $\tilde{b}_i$ are calculated as

$$\begin{aligned} \tilde{a}_i &= \frac{\max(V_{\text{PD}i}^*) - \min(V_{\text{PD}i}^*)}{2} \\ \tilde{b}_i &= \frac{\max(V_{\text{PD}i}^*) + \min(V_{\text{PD}i}^*)}{2} \end{aligned} \quad (1.24)$$

The photodiode voltages are then normalised as $\bar{V}_{\text{PD}i} = \frac{V_{\text{PD}i} - \tilde{b}_i}{\tilde{a}_i}$, leading to

$$\begin{cases} \bar{V}_{\text{PD}1} = \frac{a_1}{\tilde{a}_1} \cdot \left(1 + \cos(4\Phi_y + \phi_1)\right) + \frac{b_1 - \tilde{b}_1}{\tilde{a}_1} \\ \bar{V}_{\text{PD}2} = \frac{a_2}{\tilde{a}_2} \cdot \left(1 + \sin(4\Phi_y + \phi_2)\right) + \frac{b_2 - \tilde{b}_2}{\tilde{a}_2} \\ \bar{V}_{\text{PD}3} = \frac{a_3}{\tilde{a}_3} \cdot \left(1 - \sin(4\Phi_y + \phi_3)\right) + \frac{b_3 - \tilde{b}_3}{\tilde{a}_3} \end{cases} \quad (1.25)$$

Provided that at least a full fringe is captured in the calibration datasets $V_{\text{PD}i}^*$, the offsets and gain values should be well approximated, so that $a_i/\tilde{a}_i = 1$ and $(b_i - \tilde{b}_i)/\tilde{a}_i = 0$, and the voltage readings are effectively normalised. Since the gain and offset values of the voltage reading may depend on ambient conditions, namely on temperature, a frequent re-calibration of the $\tilde{a}_i$ and $\tilde{b}_i$ values is recommended.

Ellipse correction

The above normalisation step effectively removes the offset component and equalises the gain to 1 for all voltage readings. This normalises the radius of the Lissajous figure to 1 and centres the figure at the origin, but it does not correct for the ellipticity. The ellipticity is corrected by fitting the ellipse parameters of the calibration dataset, after normalisation, by solving a least-squares minimisation problem [115]. Next, these values are used to fit the ellipse back to a circle [46, 76, 113, 114].

The quadrature signals are calculated from the normalised calibration dataset

$$\begin{cases} Q_{\sin}^* = \bar{V}_{\text{PD}1}^* - \bar{V}_{\text{PD}3}^* \\ \quad = \cos(4\Phi_y + \phi_1) + \sin(4\Phi_y + \phi_3) \\ \quad = -2 \sin\left(\frac{8\Phi_y + \phi_1 + \phi_3 + \frac{\pi}{2}}{2}\right) \sin\left(\frac{\phi_1 - \phi_3 - \frac{\pi}{2}}{2}\right) \\ \quad = -2 \sin\left(\frac{\phi_1 - \phi_3}{2} - \frac{\pi}{4}\right) \sin\left(4\Phi_y + \frac{\phi_1 + \phi_3}{2} + \frac{\pi}{4}\right) \\ Q_{\cos}^* = \bar{V}_{\text{PD}1}^* - \bar{V}_{\text{PD}2}^* \\ \quad = 2 \cos\left(\frac{\phi_1 - \phi_2}{2} - \frac{\pi}{4}\right) \cos\left(4\Phi_y + \frac{\phi_1 + \phi_2}{2} + \frac{\pi}{4}\right) \end{cases} \quad (1.26)$$

For ease of notation, let us rename

$$\begin{cases} Q_{\sin}^* = k_{\sin} \sin\left(4\Phi_y + \frac{\pi}{4} + \phi_0\right) + Q_{\sin,0} \\ Q_{\cos}^* = k_{\cos} \cos\left(4\Phi_y + \frac{\pi}{4}\right) + Q_{\cos,0} \end{cases} \quad (1.27)$$

Where $k_{\sin} = 2 \sin\left(\frac{\phi_1 - \phi_3}{2} - \frac{\pi}{4}\right)$ and $k_{\cos} = 2 \cos\left(\frac{\phi_1 - \phi_2}{2} - \frac{\pi}{4}\right)$ represent the gains of the quadrature signals, which are different despite the normalisation step, due to the phase mismatch. $\phi_0 = \frac{\phi_1 + \phi_3}{2} - \frac{\phi_1 + \phi_2}{2}$

represents the overall phase difference between the quasi-quadrature signals. $Q_{\sin,0}$ and $Q_{\cos,0}$ may account for residual offsets despite the normalisation step, but are usually very small.

Calling the trigonometric identity and substituting in Equation 1.27, the two equations combine as

$$\begin{aligned}
& \frac{1}{k_{\sin}^2} Q_{\sin}^{*2} + \frac{1}{k_{\cos}^2} Q_{\cos}^{*2} - \frac{2 \sin(\phi_0)}{k_{\sin} k_{\cos}} Q_{\sin}^* Q_{\cos}^* - 2 \left(\frac{Q_{\sin,0}}{k_{\sin}^2} - \frac{Q_{\cos,0} \sin(4\Phi_y + \frac{\pi}{4})}{k_{\sin} k_{\cos}} \right) Q_{\sin}^* \\
& - 2 \left(\frac{Q_{\cos,0}}{k_{\cos}^2} - \frac{Q_{\sin,0} \sin(4\Phi_y + \frac{\pi}{4})}{k_{\sin} k_{\cos}} \right) Q_{\cos}^* \\
& = \cos^2(\phi_0) - \left(\frac{Q_{\sin,0}^2}{k_{\sin}^2} + \frac{Q_{\cos,0}^2}{k_{\cos}^2} - \frac{2 Q_{\sin,0} Q_{\cos,0} \sin(\phi_0)}{k_{\sin} k_{\cos}} \right)
\end{aligned} \tag{1.28}$$

The above equation can indeed be written under the form of a general conic equation, confirming the elliptical nature of the quasi-quadrature signals,

$$Q_{\sin}^{*2} = -A_1 Q_{\cos}^{*2} - A_2 Q_{\sin}^* Q_{\cos}^* - A_3 Q_{\sin}^* - A_4 Q_{\cos}^* + A_5 \tag{1.29}$$

The A_i coefficients can be calculated by solving Equation 1.29 using the calibration dataset and a least-squares solving method. Then, the ellipse parameters are found by identifying terms from Equation 1.28 and Equation 1.29

$$\begin{cases} \frac{k_{\sin}}{k_{\cos}} = \sqrt{A_1} \\ \phi_0 = \sin^{-1} \left(\frac{A_2}{2\sqrt{A_1}} \right) \\ Q_{\cos,0} = \frac{-A_3 \sqrt{A_1} \sin(\phi_0) - A_4}{2A_1 \cos^2(\phi_0)} \\ Q_{\sin,0} = \frac{-A_3}{2} + \sqrt{A_1} \sin(\phi_0) Q_{\cos,0} \\ k_{\sin} = \frac{\left(A_5 + Q_{\sin,0}^2 + A_1 Q_{\cos,0}^2 - 2 Q_{\sin,0} Q_{\cos,0} \sqrt{A_1} \sin(\phi_0) \right)^{1/2}}{\cos(\phi_0)} \end{cases} \tag{1.30}$$

Eventually, this set of parameters is used to correct the ellipticity of the quasi-quadrature signals

as

$$\begin{aligned}
 Q_{\sin} &= \frac{\bar{Q}_{\sin} - Q_{\sin,0}}{k_{\sin} \cos(\phi_0)} - \frac{\bar{Q}_{\cos} - Q_{\cos,0}}{k_{\cos}} \tan(\phi_0) \\
 &= \sin\left(4\Phi_y + \frac{\pi}{4}\right) \\
 Q_{\cos} &= \frac{\bar{Q}_{\cos} - Q_{\cos,0}}{k_{\cos}} \\
 &= \cos\left(4\Phi_y + \frac{\pi}{4}\right)
 \end{aligned} \tag{1.31}$$

where $\bar{Q}_{\sin} = \bar{V}_{PD1} - \bar{V}_{PD3}$ and $\bar{Q}_{\cos} = \bar{V}_{PD1} - \bar{V}_{PD2}$ are the quasi-quadrature signals obtained after subtraction of pairs of normalised photodiode signals.

As for the normalisation step, a sufficient portion of the ellipse need to be spanned in the calibration dataset for the least-square fitting to work efficiently. Also, it is worth noting that the ellipticity of the quadrature signal may depend on ambient conditions, especially temperature condition, so that a frequent re-calibration is advised.

Practical demodulation process

In the end, the practical demodulation process follows the same steps as told in the beginning of the section, but includes the normalisation and ellipse correction algorithms presented above. Figure 1.10 shows the flowchart of the more practical demodulation process.

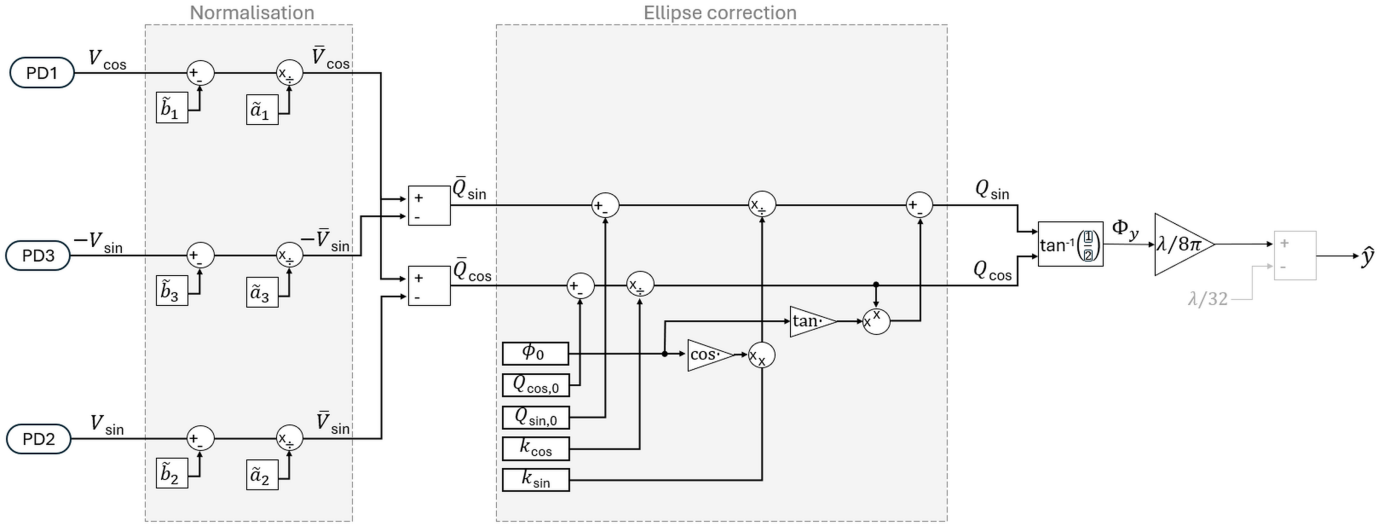


Figure 1.10: Practical demodulation process flowchart for the quadrature Michelson interferometer. PD_i are the photodiode voltage readings. The $\tilde{a}_i$ and $\tilde{b}_i$ coefficients are the gains and offset values obtained after calibration. Similarly, the ϕ_0 , $Q_{\cos,0}$, $Q_{\sin,0}$, $k_{\cos}$ and $k_{\sin}$ values are the ellipse parameters obtained after calibration. $\hat{y}$ is the demodulated displacement value.

The most general expression for the reading of the photodiodes, accounting for optical misalign-

ment, material properties, electronics imbalance and other practical limitations, has been demonstrated to write

$$\begin{cases} V_{PD1} = a_1 \cdot \left(1 + \cos(4\Phi_y + \phi_1)\right) + b_1 \\ V_{PD2} = a_2 \cdot \left(1 + \sin(4\Phi_y + \phi_2)\right) + b_2 \\ V_{PD3} = a_3 \cdot \left(1 - \sin(4\Phi_y + \phi_3)\right) + b_3 \end{cases} \quad (1.32)$$

A subset of measurement data, or calibration data set V_{PDi}^* is used to estimate the gains and offset in each signal. They are then compensated following Equation 1.25, leading to normalised signals $\bar{V}_{PDi}$.

The normalised voltage is used to calculate two quasi-quadrature signals that will be used for demodulating the interferometric signals

$$\begin{cases} \bar{Q}_{\sin} = \bar{V}_{PD1} - \bar{V}_{PD3} \\ \bar{Q}_{\cos} = \bar{V}_{PD1} - \bar{V}_{PD2} \end{cases} \quad (1.33)$$

The ellipticity of the quasi-quadrature signals is next corrected following Equation 1.31, using the estimation of the ellipse parameters in Equation 1.30 obtained by solving the conic equation of the quasi-quadrature signals from the calibration dataset. This eventually leads to

$$\begin{aligned} Q_{\sin} &= \sin\left(4\Phi_y + \frac{\pi}{4}\right) \\ Q_{\cos} &= \cos\left(4\Phi_y + \frac{\pi}{4}\right) \end{aligned} \quad (1.34)$$

The interferometric phase can finally be demodulated from the corrected quadrature signals as

$$4\Phi_y + \frac{\pi}{4} = \tan^{-1}\left(\frac{Q_{\sin}}{Q_{\cos}}\right) \quad (1.35)$$

And eventually, the motion along the Y-arm of the interferometer is

$$\begin{aligned} \hat{y} &= \frac{\lambda}{2\pi}(\hat{\Phi}_y) \\ &= \frac{\lambda}{8\pi} \tan^{-1}\left(\frac{Q_{\sin}}{Q_{\cos}}\right) - \frac{\lambda}{32} \end{aligned} \quad (1.36)$$

A model of the interferometric readout has been built in MATLAB . The model uses the Jones matrices listed in Table 1.2 to simulate the propagation of an arbitrary input y to the reading of the interferometer photodiodes following the optical layout in Figure 1.7. The model can be used both as a benchmark for validating the working principle of the interferometer and as a 'digital twin' to estimate the performance of the readout in particular conditions, such as specific material properties

or alignment.

An input mimicking the motion of the proof-mass of an inertia sensor with a resonance frequency of 1.4 Hz and a Q -factor of 200 has been fed to the MATLAB model to generate photodiode signals. The electronics is first modelled to be ideal with a nominal gain of $1 \cdot 10^3$ [V/mA]. Then, imbalances are introduced to generate gains, offsets and phase errors resulting in an ellipticity of the quasi-quadrature signals. In this example, the gains and offset errors are modelled as random number which value is within 10% of the nominal fringe amplitude. The ellipticity is generated by misaligning the quarter wave-plate by a couple of degrees around the nominal position $[40^\circ - 50^\circ]$. Various misalignment angles are used to generate different level of ellipticity, which is defined as

$$\zeta = \sqrt{1 - \left(\frac{a}{b}\right)^2} \quad (1.37)$$

where a and b represents the major and minor axis of the ellipse. It was observed that a misalignment of the waveplate by 2° typically lead to an ellipticity of 10%.

The photodiode signals are then processed using normalisation and ellipse correction as depicted in Figure 1.10. A similar, but not identical, input is used to generate the calibration dataset. For the sake of the demonstration, the gains and ellipse parameters are assumed constant. Figure 1.11(a) shows the Lissajous figure of the quasi-quadrature signal that would be obtained with no correction algorithm, where the ellipticity is clearly visible. Figure 1.11(b) shows the Lissajous figure of the quasi-quadrature signals after normalisation and after ellipse correction. It can be seen that the ellipse correction algorithm is indeed effective in compensating for the ellipticity of the Lissajous figure.

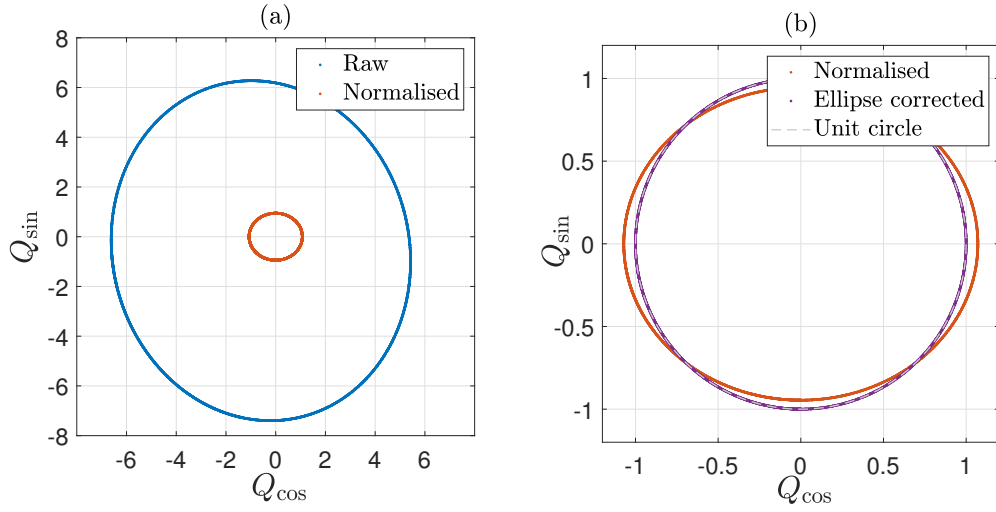


Figure 1.11: Lissajous figures of the quasi-quadrature signals obtained by feeding a realistic input motion y to a MATLAB model of the quadrature Interferometer. An ellipticity of $\zeta = 0.48$ has been generated in the model. (a) Comparison between the raw, unprocessed figure and the figure after normalisation. (b) Comparison between the figure after normalisation and after ellipse correction.

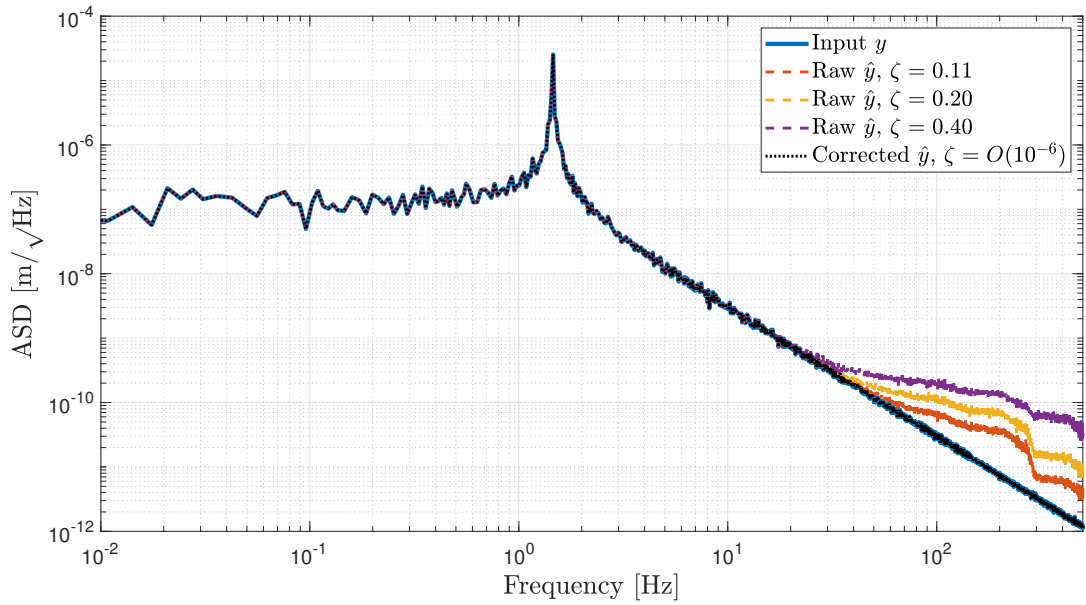


Figure 1.12: ASD of the input motion y fed to the MATLAB model, and of the estimated motion $\hat{y}$ from the demodulation of the raw (unprocessed) signals and from the signals processed with normalisation and ellipse correction. Larger eccentricity values ζ lead to greater demodulation noise level. The signal corrected with ellipse fitting effectively mitigate this error for any ζ values.

Figure 1.12 shows the amplitude spectral density (ASD) of input motion y , and of the estimated motion $\hat{y}$ demodulated from the raw (unprocessed) photodiode signals and from the signals processed with normalisation and ellipse correction. Here, the effect of the failure of the arctangent operation on the quasi-quadrature signals is clearly visible, as it translates into a broadband error dominating the signal at higher frequencies, typically above 10 Hz. The figure also shows that the level of noise is greater for larger ellipticity values. On the other hand, the estimated motion using normalisation and ellipse fitting shows a perfect match with the input motion, regardless of the amount of ellipticity introduced in the model. A calculation of the residual ellipticity in the corrected quadrature signals indicates values that are of the order of 10^{-6} in all cases. This confirms the working principle of the interferometer, the correct implementation of its model and of the associated demodulation.

1.2.4 IFO design and data acquisition chain

After validation of the benchtop prototype [46,108], a compact assembly of the quadrature Michelson interferometer has been produced by EXail, shown in Figure 1.13. Figure 1.14 shows the optical layout overlayed to the monolithic assembly. The interferometer is enclosed in a dark box, which only allows an opening for the output laser beam. The opening is located under the protection cap at the bottom of the device in the left figure. The laser beam is fed to the interferometer via a fibre-coupled input that can be seen under the white hood, and the fibre is also protected under an aluminium cover. The output signals of the interferometer consist of the photodiodes seen in the right figure in the golden casing. The device dimensions are $85 \times 60 \times 27 \text{ mm}^3$ (excluding the fibre and fibre cover), so it can be fairly easily fitted into the mechanics of an inertial sensor and of the isolation system.

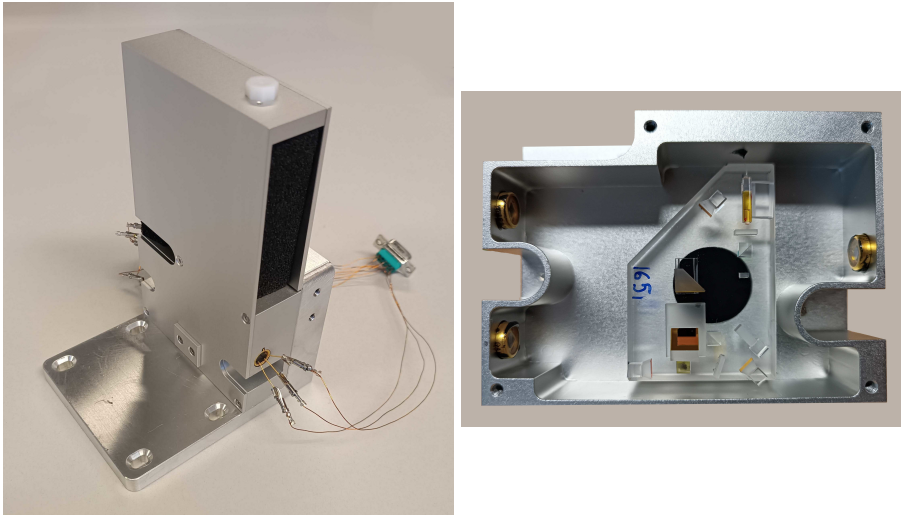


Figure 1.13: Photos of the compact quadrature Michelson interferometer. (left) Device mounted in a vertical position with a protection cover for the optical fibre. (right) Inside of the device, assembly of the interferometer on an optical breadboard.

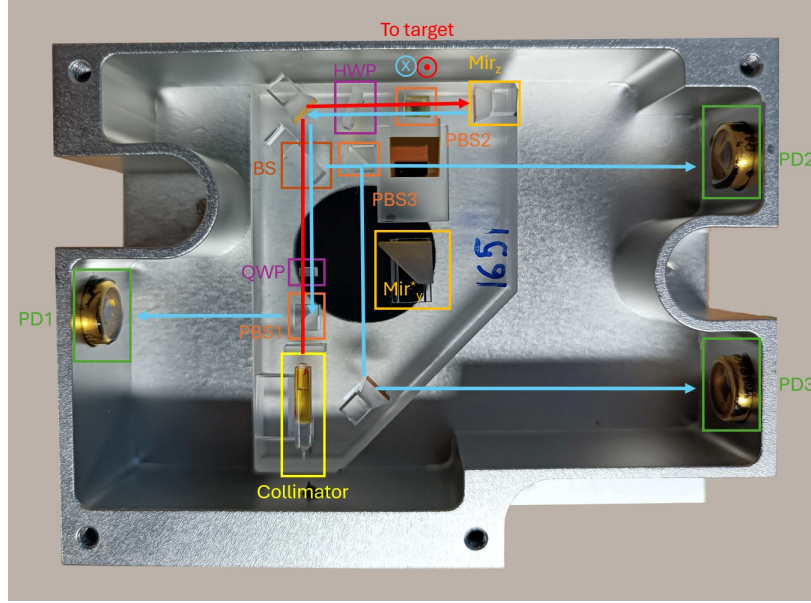


Figure 1.14: Optical assembly overlaid with the optical scheme.

Figure 1.15 schematises the acquisition chain from the interferometer to the displacement output. The interferometer is fed light by a commercial benchtop laser source. The photocurrents generated at the photodiodes are then processed in a signal conditioning unit and the voltages are read on a real-time target to demodulate the displacement. The laser source is a Koheras ADJUSTIK X15 Distributed Feedback (DFB) benchtop laser source. It is a highly stable laser source using active wavelength control. It features a $2 \mu\text{rad}/\sqrt{\text{Hz}}$ phase noise @1 Hz, for a 1 m optical path difference (OPD) and a Relative Intensity Noise (RIN) peak $< -100 \text{ dBc/Hz}$ @0.3 MHz. Note that the low phase/frequency noise is especially relevant for this device since the subtraction of pairs of photodiodes in the calculation of the quasi-quadrature signals cancels the RIN noise and other common-mode noises. The laser source feeds the interferometer with 1550.12 nm linearly p-polarised light with a nominal power of 4 mW via an FC/APC polarisation maintaining optical fibre. The photodiodes are Thorlabs FGA21 InGaAs photodiodes, characterised by a responsivity $\mathfrak{R} = 1.04 \text{ A/W}$, a dark current $I_d = 50 \text{ nA}$ for a 1V bias, and a Noise Equivalent Power (NEP) of $6 \cdot 10^{-14} \text{ W}/\sqrt{\text{Hz}}$ at the nominal wavelength.

The output signals of the photodiodes are photocurrents typically of the order of 0.5 mA, given the nominal input power of 4mW, the photodiode responsivity $\mathfrak{R} \approx 1 \text{ A/W}$ and Equation 1.18. A conditioning unit is therefore required to convert the photocurrent to a useful voltage that can be processed in the data acquisition system. Low-noise transimpedance amplifiers have been designed for that purpose. Figure 1.16 shows the schematics of a signal conditioner including 3 transimpedance circuits. The core of the design is an operational amplifier (Op-amp) placed in an inverting configuration with a $10 \text{ k}\Omega$ feedback resistor to produce the desired gain. The photodiode is also powered with a 2.5 V bias voltage to maximise the response and dynamic range. Noise from such

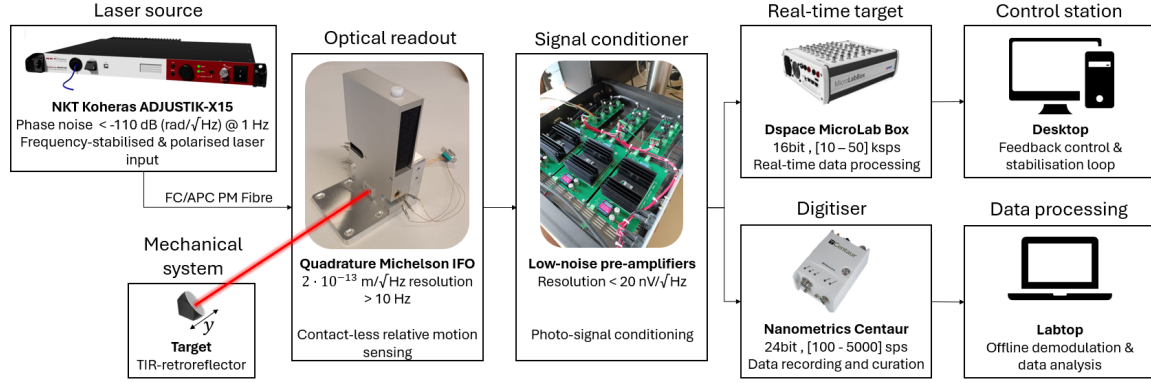


Figure 1.15: Schematics of the acquisition chain, from the laser input to the desktop station.

an electronic circuit primarily arises from the active component, i.e. the Op-amp. The LT1792 low-noise, precision, JFET Op-amp was chosen for its high performance at lower frequencies, typically $8.3 \text{ nV}/\sqrt{\text{Hz}}$ and $10 \text{ fA}/\sqrt{\text{Hz}}$ at 10 Hz when supplied with $\pm 15 \text{ V}$ (online datasheet). In addition, the $1/f$ Flicker-noise can become a severe limitation at sub-Hz frequencies. To limit the contribution from resistive components, especially from the feedback resistor, a Vishay Z201 ultra-high precision Z-Foil resistor with a -40 dB Noise Index (NI) is used (online datasheet). A 1.6 kHz low-pass filter is also used to avoid high-frequency noise propagation, and decoupling capacitors at the Op-amp and photodiode levels are also used to isolate from power supply noise. A more quantitative discussion on noise analysis will be presented in section 1.3. The circuitry shown in Figure 1.16 corresponds to one physical acquisition card, capable of processing one optical readout. Three of the same circuitry are grouped in one rack unit (i.e. signal conditioner unit) for processing 3 devices. The pinout map of one signal conditioner unit can be found in Appendix B, and additional details can be found in Appendix C.

Eventually, the voltage readings are recorded on data acquisition systems for processing and data analysis. Two recorders can be used depending on the application. When online (real-time) demodulation and synchronous acquisition of multiple devices are required, a MicroLab Box system from DSpace (online datasheet) is used. It is a real-time target system equipped with 24 16-bit analogue ADCs and 16 14-bit analogue DACs, allowing the processing of up to 7 optical devices simultaneously. It is equipped with a real-time processor and a programmable FPGA1, which allows the offloading of the processing power from the desktop station to the embedded processor. The system works under the MATLAB SIMULINK environment so that the demodulation process described in subsection 1.2.3, especially Figure 1.10, has been transcribed in SIMULINK language, and is then compiled to a C++ script which is loaded in the MicroLab Box system. The displacement demodulation is therefore performed in real-time using a sampling frequency of typically 10 kHz (up to 50 kHz). Another dSpace system (Scalexio LabBox equipped with DS6221 ADC and DS6241 DAC boards, online datasheet) with similar characteristics is used for driving the active platform of the ETEST prototype. The displacement data can then directly be recorded for data analysis and

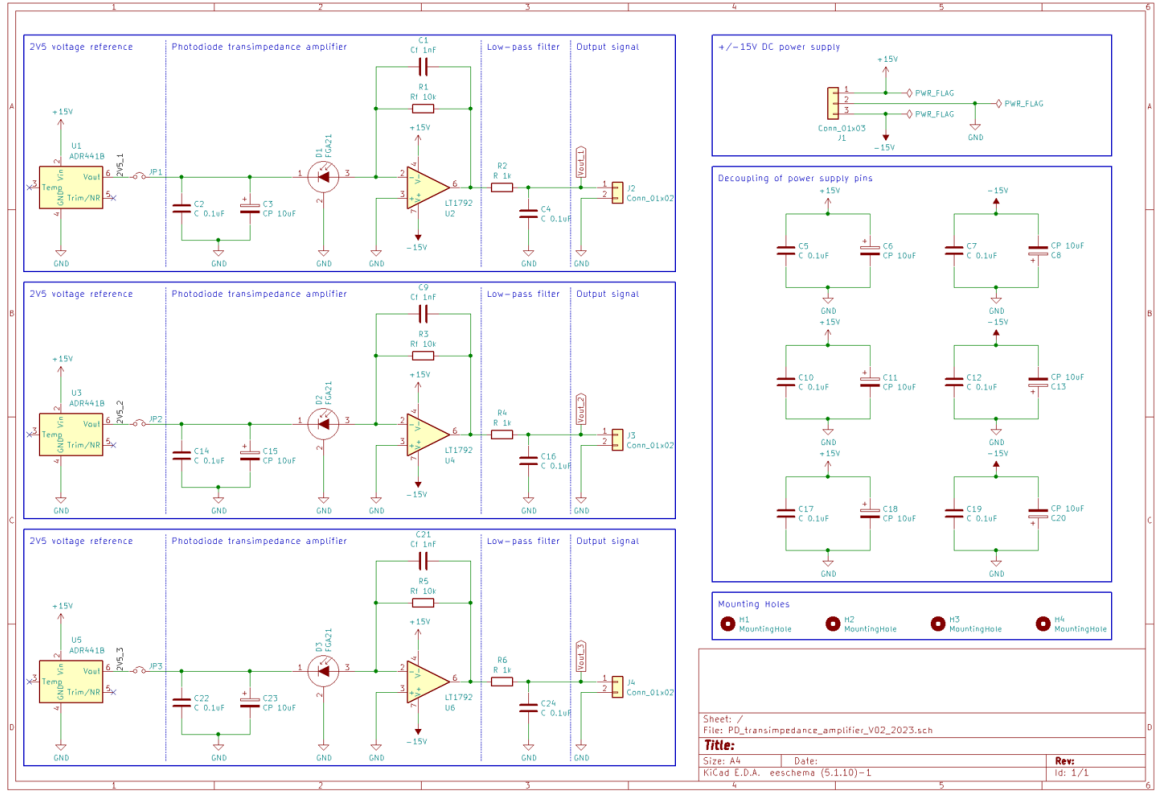


Figure 1.16: KiCad drawing of the pre-amp signal conditioner unit. The circuit primarily consists of a transimpedance amplifier using an LT1792 in inverting configuration, a low pass filtering at ≈ 1.6 kHz, and a set of decoupling capacitors for the Op-amp and photodiode voltage supply.

post-processing, or transferred to the control station via an Ethernet connection for data viewing or closing control loops. For other applications where online demodulation is not needed, a Nanometrics Centaur digitiser (online datasheet) can be used instead. This digitiser is equipped with 6 24-bits ADC channels with a sampling rate up to 2000 kHz, therefore offering a better resolution than the dSpace systems. However, it does not offer processing power, so that the raw interferometer signals need to be recorded, demodulated and processed offline. Since 3 voltage readings per optical sensor are required, the digitiser can only record up to 2 interferometers simultaneously. The Centaur is therefore only suited for noise measurement where high resolution is required and when only up to 2 devices need to be connected.

1.3 Experimental validation

The resolution of a device represents the smallest quantity that can be measured and distinguished from noise. It is an essential characteristic of any measuring device and is especially relevant in applications where very small signals need to be captured. In GW instrumentation or active seismic isolation in general, the sensor's resolution sets the ultimate limit of the residual vibration level of the system. This has a direct impact on the performance of the GW detectors at < 10 Hz frequencies. Therefore, the high-performance requirements of 3rd generation GW detectors necessarily call for low-noise sensors. For this reason, the various sources of noise that may limit the resolution of the Michelson interferometer need to be understood, quantified, and mitigated through careful design of the devices or signal processing. The sources of disturbances that will create a spurious signal in the reading of the interferometer are many [86, 116]. They can be of an internal origin, such as the acquisition chain (e.g. cabling, electronics, ADCs), the optical assembly (e.g. imperfect optical properties or alignment) or from the laser system (e.g. intensity or frequency fluctuations). They can also originate from external sources, such as pressure gradients, scattered light, or thermal effects. Hunting these various noise sources is a crucial step in the design of a low-noise instrument.

1.3.1 IFO resolution

The resolution of the optical device can be obtained from a null measurement. This procedure consists in mechanically blocking the measuring arm, to a length matching the length of the reference arm, so that the output reading contains primarily the contribution of all noise sources present in the system. This is achieved by solidly clamping the corner cube in the measuring arm to the casing of the optical readout, right in front of the opening for the laser beam, making the beam travelling in a totally enclosed and quasi air-tight space. This mounting of the optical readout is named the null configuration. On top of that, the setup is placed in a dark and sealed box to further minimise noise from ambient light and air fluctuation. The inner walls of the box are also covered in polystyrene foam providing passive thermal insulation for minimising thermal and acoustic noises. The IFO in null configuration is placed on the floor of the box, which is padded with foam to decrease transmission of ground vibrations. The acquisition chain is the same as the one exposed in section 1.2: a NKT Koheras Adjustik X15 laser source at 4 mW power is used to feed light to the device. The voltage outputs at the signal conditioner unit are recorded on either the MicroLab Box real-time target with 16-bit ADC channels, or on the Centaur digitiser with 24-bit ADC channels. The null measurements are a 30 min-long recording sampled at 10 kHz when using the MicroLab Box, and a 1 hour-long recording sampled at 2 kHz when using the Centaur. Both measurement are taken overnight for lowest ambient vibration noise and better thermal stability. The results from the null measurements are shown in Figure 1.17. The optical device shows a noise floors of 10^{-11} m/ $\sqrt{\text{Hz}}$ at 100 mHz, 10^{-12} m/ $\sqrt{\text{Hz}}$ at 1 Hz and $2 \cdot 10^{-13}$ m/ $\sqrt{\text{Hz}}$ at 10 Hz and above when using the MicroLab Box real-time target. With the Centaur recorder, the resolution drops down to $8 \cdot 10^{-14}$ m/ $\sqrt{\text{Hz}}$ at higher frequencies. Superimposed in the figure are also measurements of the data

acquisition chain noises and the predicted noise models using both recorders. The data acquisition chain noise is measured by recoding the output signal when the laser emission is disabled, and is found to be the major limitation to the IFO resolution for frequencies of 1 Hz onwards with the MicroLab Box. The Centaur recorder theoretically allows to reach a shot-noise limited resolution, modelled at $2 \cdot 10^{-14} \text{ m}/\sqrt{\text{Hz}}$, but lands slightly above due to more difficulties in obtaining good calibration parameters using the Centaur since there is no real-time visualisation of the measured signals. The frequency band between 100 mHz and 3 Hz is probably residual motion in the null setup. The excess noise contribution below few hundreds of mHz that appear in both measurements will be hunted down in the subsequent sections.

This section will discuss the resolution of the optical device. First, a noise model is established based on the modelling in section 1.2. Next, the different noise sources will be hunted and projected in the IFO resolution in order to estimate their contribution to the null measurement.

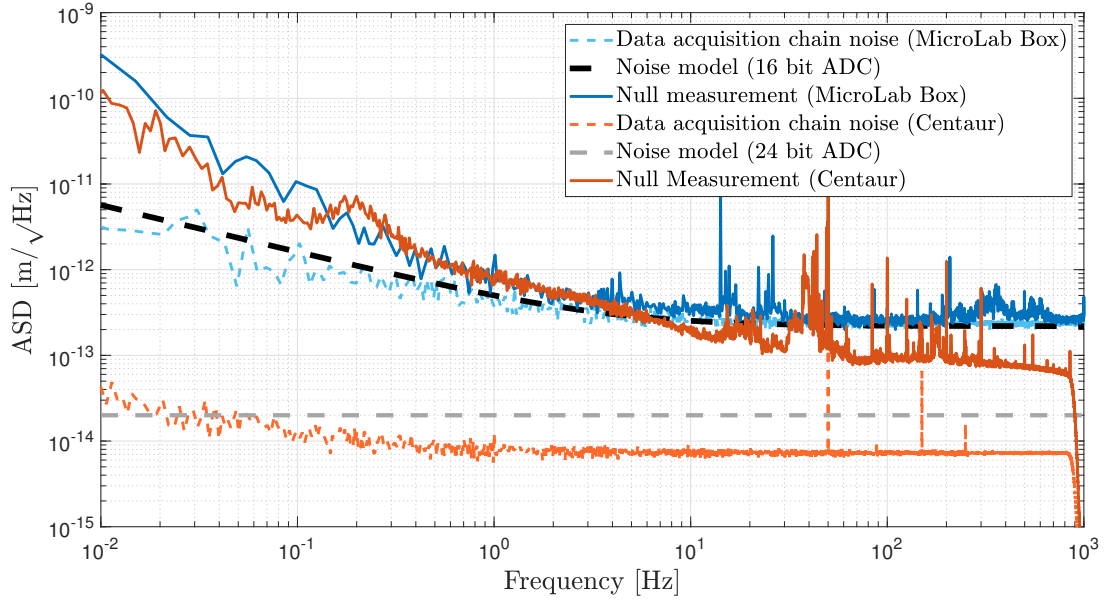


Figure 1.17: Amplitude spectrum of the resolution of the optical device obtained from a null measurement, using the MicroLab Box real-time target and the Centaur recorder. The acquisition chain noise corresponds to a measurement of the IFO signal when the laser emission is disabled.

1.3.2 Noise hunting

This section will pin out the main limitations in the resolution of the interferometric device. A noise model is built and used to propagate each individual noise source to the displacement output of the optical readout. The major noise sources will be quantified and their contribution to the noise figure will be calculated. When applicable, strategies for mitigating a specific noise source will be discussed.

1.3.2.1 IFO Noise model

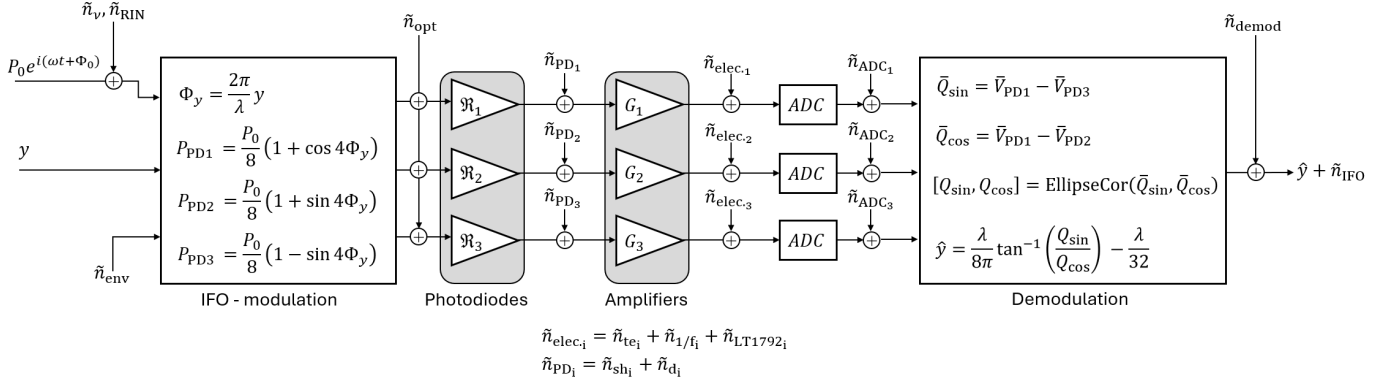


Figure 1.18: Diagram of the interferometer noise model. The diagram schematises how the main noise sources combine from the input y to the demodulated displacement $\hat{y}$. $P_0 e^{i(\omega t + \Phi_0)}$ is the input laser power tainted by the laser noises $\tilde{n}_\nu$ and $\tilde{n}_{RIN}$. $\tilde{n}_{env}$ represents the environment noises, $\tilde{n}_{opt}$ the optical noises, $\tilde{n}_{sh}$ and $\tilde{n}_d$ the photodiode noises, $\tilde{n}_{te}$, $\tilde{n}_{1/f}$ and $\tilde{n}_{LT1792}$ the electronics noises, $\tilde{n}_{ADC}$ the ADC noise and $\tilde{n}_{demod}$ the demodulation noises.

The contribution of a noise source to the displacement output reading depends on where the noise is added in the sensor acquisition chain. Figure 1.18 shows a schematic of the noise propagation process for the major sources of noise. The optical readout converts the input motion y into an interferometric phase Φ_y , and the optical layout modulates this phase with sine and cosine functions read at 3 photodiodes, as demonstrated in subsection 1.2.2. However, alongside the interferometric phase related to the physical displacement y in the Y-arm, spurious optical power and phase appear in the signal due to environmental effects $\tilde{n}_{env}$ (such as temperature, pressure fluctuation or vibration in the reference arm) and to the laser source frequency and intensity noise $\tilde{n}_\nu$ and $\tilde{n}_{RIN}$. In addition, imperfections in the optical layout such as bad alignment or material properties, also alter the interferometer signals as $\tilde{n}_{opt}$. These noise sources, all cause the power output of the interferometer to contain a spurious amount of light and phase. Because they all originate from constituents of the optical system, they are categorised as "optical" noises. Environmental noises, although also altering the power output of the interferometer, deserve to be discussed in a separate section as "ambient" noises. The optical power is then read on the three photodiodes and converted to photocurrents according to the photodiode responsivity $\mathfrak{R}_i$. The photodiodes however add shot and dark current noises to the photocurrent readings $\tilde{n}_{sh,i}$ and $\tilde{n}_{d,i}$. The current readings are converted to voltage readings according to the pre-amp gains G_i , on top of which adds thermo-electrical noise $\tilde{n}_{te,i}$, Flicker noise $\tilde{n}_{1/f,i}$ and Op-amp noise $\tilde{n}_{LT1792,i}$. Finally, the voltage readings are recorded on the real-time target, but discretisation and quantisation noises add up as $\tilde{n}_{ADC,i}$. The above noise sources, from the photodiode outputs to the reading at the real-time target, all add up on the optical output along the data acquisition chain and are referred to as "data acquisition chain" noise. Ultimately, all three voltage readings are processed as explained in subsection 1.2.3 to demodulate the displacement signal $\hat{y}$. Error in the calibration parameters and the nonlinearity of the arctangent operation will translate

into noise in the demodulated signals, which will be named as "demodulation" noise $\tilde{n}_{\text{demod}}$.

Accounting for the noise sources, the power output [W] of the optical device is written as

$$\begin{cases} P_{\text{PD1}} = \frac{P_0}{8} \left(1 + \cos(4\Phi_y + \tilde{n}_\nu + \tilde{n}_{\text{env}}) \right) + \tilde{n}_{\text{RIN}} \\ P_{\text{PD2}} = \frac{P_0}{8} \left(1 + \sin(4\Phi_y + \tilde{n}_\nu + \tilde{n}_{\text{env}}) \right) + \tilde{n}_{\text{RIN}} \\ P_{\text{PD3}} = \frac{P_0}{8} \left(1 - \sin(4\Phi_y + \tilde{n}_\nu + \tilde{n}_{\text{env}}) \right) + \tilde{n}_{\text{RIN}} \end{cases} \quad (1.38)$$

where $\tilde{n}_{\text{RIN}}$ is the laser intensity noise, common to all outputs. The laser frequency and ambient noises $\tilde{n}_\nu$ and $\tilde{n}_{\text{env}}$ are also common to the power output but with different modulation, which complicates the mitigation process.

The power signals are read at the three photodiodes to output photocurrent signals [A], to which add the photodiode shot noise $\tilde{n}_{\text{sh},i}$ and dark-current noise $\tilde{n}_{\text{d},i}$

$$I_{\text{PD},i} = \Re_i(P_{\text{PD},i}) + \tilde{n}_{\text{sh},i} + \tilde{n}_{\text{d},i} \quad (1.39)$$

where the photodiode noises are uncorrelated for the three different signals.

Similarly, the photocurrent signals are processed by the signal conditioner to output voltage readings [V], and pick up thermoelectric $\tilde{n}_{\text{te},i}$, 1/f Flicker noise $\tilde{n}_{1/\text{f},i}$, Op-amp noise $\tilde{n}_{\text{LT1792},i}$. Voltage readings are eventually recorder at the MicroLab Box which adds up ADC noise $\tilde{n}_{\text{ADC},i}$.

$$V_{\text{PD},i} = G_i \Re_i(P_{\text{PD},i}) + G_i(\tilde{n}_{\text{sh},i} + \tilde{n}_{\text{d},i}) + \tilde{n}_{\text{te},i} + \tilde{n}_{1/\text{f},i} + \tilde{n}_{\text{LT1792},i} + \tilde{n}_{\text{ADC},i} \quad (1.40)$$

The above equations show how the major noise source combines in the optical device and translates to voltage readings, and can be used for a quick estimation of how noise propagates to the device output voltage depending on the photodiode responsivities $\Re_i$ and the signal conditioner gains G_i . However, the voltage readings need to be propagated through the demodulation process exposed in subsection 1.2.3 to calculate the exact impact of the noise sources in the demodulated

displacement value in [m]. Each individual voltage reading would be written as

$$\left\{ \begin{array}{l} V_{PD1} = G_1 \left(\Re_1 \left(\frac{P_0}{8} \left(1 + \cos(4\Phi_y + \tilde{n}_\nu + \tilde{n}_{\text{env}}) \right) + \tilde{n}_{\text{RIN}} \right) + \tilde{n}_{\text{sh1}} + \tilde{n}_{\text{d1}} \right) \\ \quad + \tilde{n}_{\text{te1}} + \tilde{n}_{1/\text{f1}} + \tilde{n}_{\text{LT1792,1}} + \tilde{n}_{\text{ADC1}} \\ V_{PD2} = G_2 \left(\Re_2 \left(\frac{P_0}{8} \left(1 + \sin(4\Phi_y + \tilde{n}_\nu + \tilde{n}_{\text{env}}) \right) + \tilde{n}_{\text{RIN}} \right) + \tilde{n}_{\text{sh2}} + \tilde{n}_{\text{d2}} \right) \\ \quad + \tilde{n}_{\text{te2}} + \tilde{n}_{1/\text{f2}} + \tilde{n}_{\text{LT1792,2}} + \tilde{n}_{\text{ADC2}} \\ V_{PD3} = G_3 \left(\Re_3 \left(\frac{P_0}{8} \left(1 + \sin(4\Phi_y + \tilde{n}_\nu + \tilde{n}_{\text{env}}) \right) + \tilde{n}_{\text{RIN}} \right) + \tilde{n}_{\text{sh3}} + \tilde{n}_{\text{d3}} \right) \\ \quad + \tilde{n}_{\text{te3}} + \tilde{n}_{1/\text{f3}} + \tilde{n}_{\text{LT1792,3}} + \tilde{n}_{\text{ADC3}} \end{array} \right. \quad (1.41)$$

Because of the normalisation, ellipse correction and arctangent operation, analytically propagating the above equation in the demodulation process becomes very heavy. The interferometer model can be used to propagate a noise source through the Jones matrices modelling, and then through the demodulation, to estimate its noise contribution to the demodulated displacement.

Alternatively, it was found that, in the case a good optical alignment, using the optical gain of the interferometer, i.e. $\frac{\lambda}{8\pi}$ ([m/V]), as a conversion factor is often fairly accurate and can be used as a rough approximation. Exceptions include the laser frequency noise $\tilde{n}_\nu$ and the environmental noises $\tilde{n}_{\text{env}}$, since these noise sources originate before the modulation signal. Eventually, a rough approximation of the spectrum of the demodulated displacement can be obtained by combining the noise budgeting in Equation 1.40 and the idealised demodulation process in Equation 1.22, leading to

$$\Phi_{\hat{y}} = \Phi_y + \left(G_i \frac{\lambda}{8\pi} \right)^2 \left(\Phi_{\text{sh},i} + \Phi_{\text{d},i} \right) + \left(\frac{\lambda}{8\pi} \right)^2 \left(\Phi_{\text{te},i} + \Phi_{1/\text{f},i} + \Phi_{\text{ADC},i} \right) \quad (1.42)$$

where $\Phi \cdot$ here refers to power spectrums.

1.3.2.2 Data acquisition chain noise

The data acquisition chain refers to the electrical components involved in the reading of the output signals of the optical readout. It includes of the photodiodes for detecting the power outputs, the signal conditioning unit for pre-amplification of the photocurrents and the ADC of the real-time target or digitiser for data processing and recording. Each of these components can be characterised by a noise level, which adds up to the signal as the signal propagates along the acquisition chain. The photodiodes photocurrent outputs are spoiled with shot and dark current noises cause by the manufacturing process of the p-n junction and its interaction with light [117, 118]. The electronics that composes the signal conditioning unit suffers from thermo-electrical and $1/f$ noises polluting the voltage readings [86, 117, 119, 120]. The data acquisition system also digitise the analogue signals

to discrete values using only a limited computation power, leading to discretisation error of the Analogue-to-Digital Converters (ADC) [86, 121, 122].

Below are quantified the ADC, signal conditioner and photodiode noises.

ADC noise

Data acquisition systems use Analogue-to-Digital Converters (ADCs) to digitise and record an analogue voltage readings to a digital value. The ADCs however only discretise a voltage reading using on a limited number of bits, leading to errors. This is referred to as the quantisation noise [86, 121, 122]. The quantisation noise spectrum Φ_q ($[V^2/Hz]$) writes

$$\Phi_q = \frac{(\Delta V / 2^{n+1})^2}{12 f_n} \quad (1.43)$$

with ΔV the voltage range of the ADC ($[V]$), n the number of bits used for the discretisation and f_n the Nyquist frequency ($[Hz]$), i.e. half of the sampling rate. In addition, the acquisition system also suffers from thermo-electrical and $1/f$ noises [117, 119, 120] due to the electrical circuitry, often specified by the manufacturer. Since the input impedance of DAQ devices are usually designed very large, typically in the $M\Omega$ range in order to minimise its impact on the voltage being measured, thermo-electrical and $1/f$ noises can be significant. The overall ADC noise Φ_{ADC} ($[V^2/Hz]$) therefore writes

$$\Phi_{ADC} = \Phi_q + \Phi_{te, ADC} + \Phi_{1/f, ADC} \quad (1.44)$$

The data acquisition systems used for operating the optical device are a MicroLab Box real-time target system from DSpace or a Centaur digitiser from Nanometrics. The MicroLab Box is equipped with 16-bits ADCs with a $[-10 ; +10]$ voltage range, and that are run typically with a 10 kHz sampling frequency, setting the quantisation noise limit to $\Phi_{q, \mu Box} = 1.24 \cdot 10^{-6} V/\sqrt{Hz}$. Measurement of the Microlab Box ADC noise indicated that the thermo-electrical noise is slightly above quantisation noise, which should be the case for a well-designed ADC. The thermo-electrical noise has been measured as $\Phi_{te, \mu Box} + \Phi_{1/f, \mu Box} = 3.91 \cdot 10^{-6} \sqrt{(1 + 5f^{-1})} V/\sqrt{Hz}$, i.e. a white noise of $3.91 \cdot 10^{-6} V/\sqrt{Hz}$ with a $1/f$ contribution crossing at 5 Hz. So that this last contribution is dominating over the quantisation noise. On the other hand, the Centaur recorder has 24 bits ADC channels with variable gains and sampling frequencies. Typically, the Centaur is operated on a 20 V_{pp} range and 2 kHz sampling rate, resulting in a quantisation noise limit of $\Phi_{q, Cent.} = 5.44 \cdot 10^{-9} V/\sqrt{Hz}$. Its thermo-electrical and $1/f$ noises for this range setting has been measured as $\Phi_{te, Cent.} + \Phi_{1/f, Cent.} = 1.16 \cdot 10^{-7} \sqrt{(1 + 0.2f^{-1})} V/\sqrt{Hz}$.

$$\begin{cases} \Phi_{ADC, \mu Box} = 3.91 \cdot 10^{-6} \sqrt{(1 + 5f^{-1})} & [V/\sqrt{Hz}] \\ \Phi_{ADC, Cent.} = 1.16 \cdot 10^{-7} \sqrt{(1 + 0.2f^{-1})} & [V/\sqrt{Hz}] \end{cases} \quad (1.45)$$

The noise floor of both systems can be measured by simply shorting an ADC input channel with a resistor. A resistor with an impedance equivalent to that of the signal conditioner output is used to best represent the operating condition of the ADC. Figure 1.19 shows the spectrum of a 15 min-long measurement with the MicroLab box and Centaur ADCs running at a 10 kHz and 2 kHz sampling frequency, respectively. The measurement are in very good agreement with the modelling of $\Phi_{ADC, \mu Box}$ and $\Phi_{ADC, Cent.}$, setting a noise floor limit for the acquisition chain to approximately $3.9 \mu V/\sqrt{Hz}$ above 10 Hz with an f^{-1} contribution below 5 Hz for the MicroLab Box, and of 100 nV/ $\sqrt{Hz}$ above 1 Hz with an f^{-1} contribution below 0.2 Hz for the Centaur.

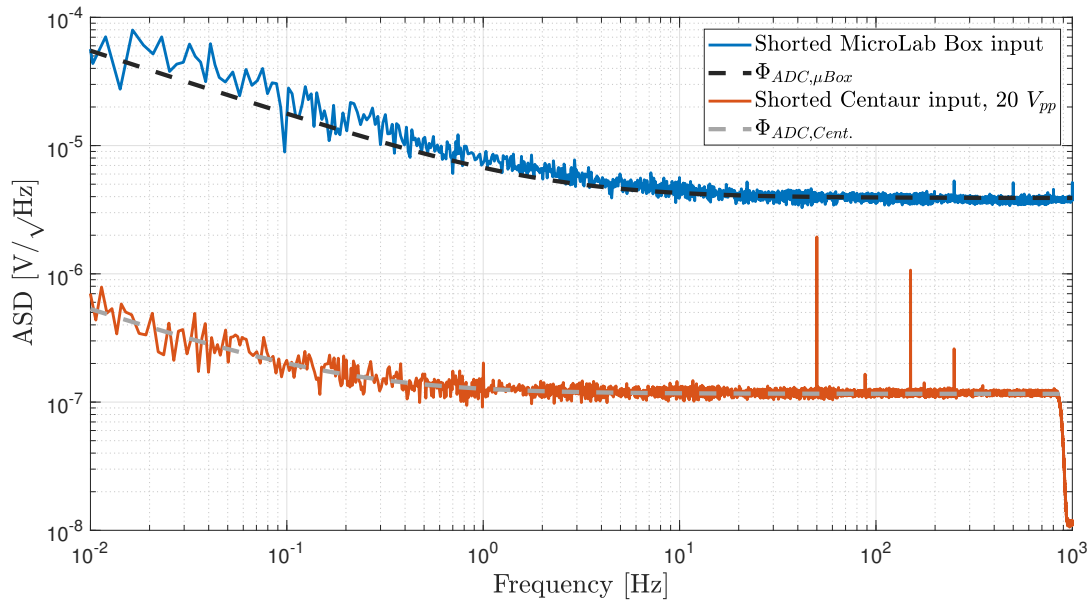


Figure 1.19: ADC noise spectrum of the DSpace MicroLab box real time target. The noise is measured by shorting the ADC input of the MicroLab box with an equivalent impedance to the signal conditioner output.

Conditioning unit noise

The signal conditioning unit includes pre-amp circuits for converting the photocurrents to voltage readings, maximising the ADC range. The electronic drawing is shown in Figure 1.16 and primarily consists of an Op-amp in inverting configuration with a 10 k Ω resistor in feedback to produce the desired gain, decoupling capacitors at the voltage input to the Op-amp and for the photodiode bias voltage and a low pass filtering. Noise in such electronics primarily arises from thermo-electrical and $1/f$ noises in the resistors and current and voltage noises of from the Op-amp [86, 117, 119, 120].

The thermo-electrical, or Johnson, noise is caused by random agitation of charge carriers in

resistive components [120, 123]. The thermo-electrical noise spectrum Φ_{te} ($[V^2/Hz]$) writes

$$\Phi_{te} = 4k_B T Z \quad (1.46)$$

with $k_B = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant, T the temperature of the component ($[K]$) and Z the impedance of the component ($[\Omega]$).

The $1/f$, or Flicker, noise is often attributed to impurities or imperfections in semiconductor components [119]. In this case, the $1/f$ noise in active components such as Op-amps is usually specified by manufacturers. However passive elements such as resistors, have also been shown to produce a $1/f$ noise [124]. This noise is often called current noise [125]. In both cases, the $1/f$ noise has the characteristic of being a coloured noise: its power spectrum grows as f^{-1} at low frequencies, as opposed to white noises which have flat frequency spectrums. This makes this noise source especially relevant for low-frequency applications. The $1/f$ noise spectrum $\Phi_{1/f}$ ($[V^2/Hz]$) writes

$$\Phi_{1/f} = \frac{K}{f^\alpha} \quad (1.47)$$

where K and α are parameters that are specific to a given component material and manufacturing process. $\alpha \approx 1$ is a very common and widely accepted assumption. A range of α values for various devices can be found in [123]. In the case of the $1/f$ noise for a resistor, the K parameter has been found to have a quadratic dependence on the value of the average voltage V_{DC} applied to the component [124–126],

$$\Phi_{1/f} = \frac{C V_{DC}^2}{f^\alpha} \quad (1.48)$$

In addition, the Hooge parameter C for a resistor can be calculated from its Noise Index (NI ($[dB]$)), defined as a ratio of the RMS value of the $1/f$ noise in μV over the potential applied to the resistor.

$$NI = \frac{10^6 \sqrt{K \ln(10)}}{V_{DC}} \quad (1.49)$$

The NI is a characteristic of a resistor, provided by the manufacturer, that depends on the material and manufacturing. A table of the NI for common types of resistor can be found in [125]. It can range from typically ≈ -5 dB for the cheaper carbon film resistors, to ≈ -15 dB for deposited carbon resistor and to ≈ -40 dB for very low-noise wire-wound and metal foil resistors.

The thermo-electrical and $1/f$ noise of the various elements of the pre-amp circuit, and the current and voltage noise of the Op-amp combine in an output voltage noise at the output of the signal conditioner. Figure 1.20 shows the equivalent noise diagram of the pre-amp circuit used in

the signal conditioner unit. I_{in} is the circuit input current (i.e. photocurrent) and V_{out} is the output voltage (i.e. IFO voltage reading). V_{te} and $V_{1/f}$ represent the circuit equivalent thermo-electrical and $1/f$ noise for the various resistive components and the I_{LT1791} and V_{LT1791} represent the current and voltage noise of the Op-amp.

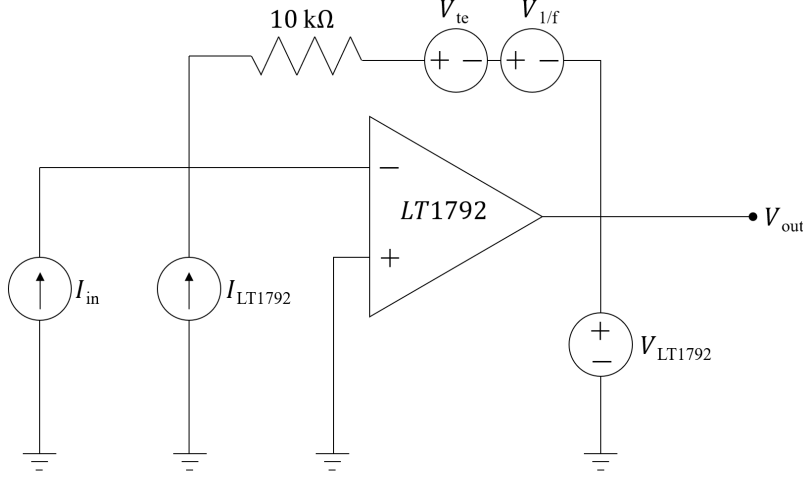


Figure 1.20: Equivalent noise diagram of the signal conditioner unit. I_{in} is the circuit input current (i.e. the photodiode photocurrent), V_{out} is the output voltage (i.e. IFO voltage reading). V_{te} and $V_{1/f}$ represent the thermo-electrical and $1/f$ noise from the various resistive components and the I_{LT1791} and V_{LT1791} represent the current and voltage noise of the Op-amp.

The LT1792 low-noise, precision, JFET Op-amp was chosen for its high performance at lower frequencies, typically $8.3 \text{ nV}/\sqrt{\text{Hz}}$ and $10 \text{ fA}/\sqrt{\text{Hz}}$ at 10 Hz with $\pm 15 \text{ V}$ supply. Since the feedback resistor is suspected to be the main contributor to thermo-electrical and $1/f$ noise due to its large impedance and the voltage that it will carry, a Vishay ultra-high precision Z-Foil resistor with a -40 dB Noise Index (NI) is used. A 1.6 kHz low-pass filter is used to avoid high-frequency noise propagation, and decoupling capacitors at the Op-amp and photodiode levels are included to provide isolation from power supply noise. A model of the noise diagram using the specifications for the resistor and Op-amp mentioned above has been implemented in LTSpice for running noise calculations of the pre-amp circuit. Noise simulations estimated the total output thermo-electrical noise from the various resistors at $\Phi_{te} = 1.28 \cdot 10^{-8} \text{ V}/\sqrt{\text{Hz}}$ and the total $1/f$ noise at $\Phi_{1/f} = 6.79 \cdot 10^{-8} \cdot f^{-0.5} \text{ V}/\sqrt{\text{Hz}}$. The output Op-amp noise is estimated as $\Phi_{LT1792} = 4.21 \cdot 10^{-9} \sqrt{1 + 40.22 f^{-1}} \text{ V}/\sqrt{\text{Hz}}$, i.e. a white noise at $4.21 \cdot 10^{-9} \text{ V}/\sqrt{\text{Hz}}$ with a $1/f$ contribution with a 40.22 Hz crossing frequency.

The different contributions of noise from the condition unit are summarised as

$$\begin{cases} \Phi_{te} = 1.28 \cdot 10^{-8} & [\text{V}/\sqrt{\text{Hz}}] \\ \Phi_{1/f} = 6.79 \cdot 10^{-8} \cdot f^{-0.5} & [\text{V}/\sqrt{\text{Hz}}] \\ \Phi_{LT1792} = 4.21 \cdot 10^{-9} \sqrt{1 + 40.22 f^{-1}} & [\text{V}/\sqrt{\text{Hz}}] \end{cases} \quad (1.50)$$

Similarly to the ADC noise measurement, the signal conditioner noise can be experimentally measured by shorting its input channels and recoding the output signals. Again, an equivalent impedance to the FGA21 photodiodes are used to best represent the operating conditions. Because the electrical noises of the signal conditioner unit are expected to be significantly lower than the ADC noise of the MicroLab Box real-time target, this measurement is performed using a different, low noise, acquisition unit. The recorder used for this measurement is a Centaur 24-bits recorder from Nanometrics. When set on the lowest range setting (1 V_{pp}) and at a 2 kHz sampling frequency, the Centaur has an ADC noise level measured at $9.7 \cdot 10^{-9} \text{ V}/\sqrt{\text{Hz}}$ above 10 Hz, using an identical process procedure as for the MicroLab Box ADC noise. The Centaur noise floor is more than 2 orders of magnitude lower than that of the MicroLab Box and should therefore be sufficient to capture the signal conditioner noise. The recorder is however not equipped with real-time processing capabilities and can therefore not be used for operating the IFO in active control. Figure 1.21 shows the measurement of the signal conditioner unit with shorted input. The raw output of one channel (blue trace) shows a great match with the noise predictions, especially with the calculated $1/f$ noise, but shows an increase in the noise level at frequencies above 3 Hz. It has been observed that this increase is similar and coherent for all three channels of the signal conditioner unit. A coherent subtraction allows to remove this contribution and obtain a noise level of $2 \cdot 10^{-8} \text{ V}/\sqrt{\text{Hz}}$, corresponding to the thermo-electrical noise prediction. The coherent noise in all channels is assumed to be caused by the ground being common for all 3 channels. Since this is a common-mode noise on all channels, it will be rejected in the demodulation process as explained in subsection 1.2.3.

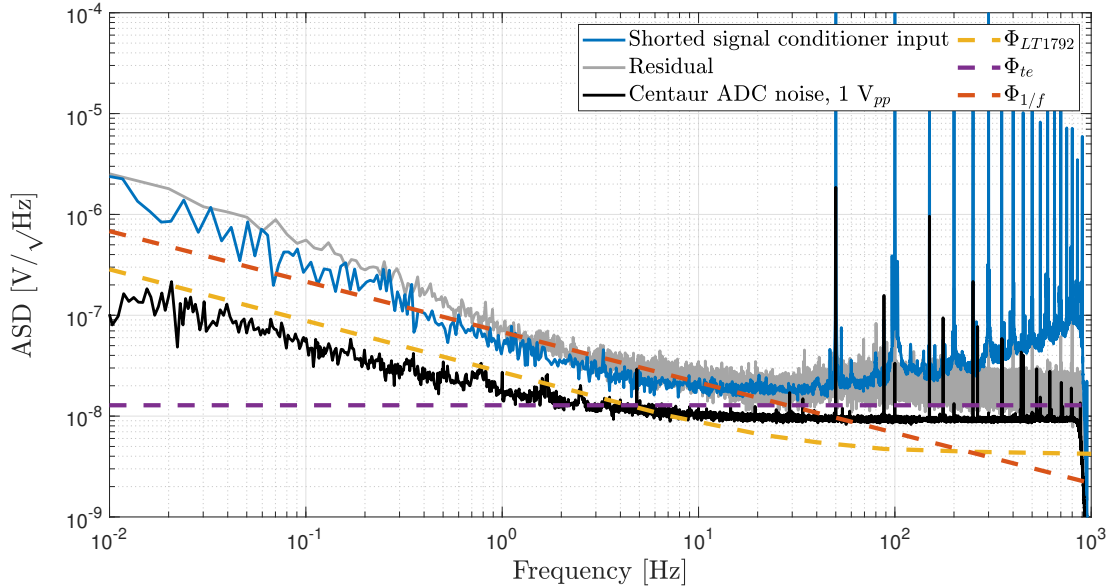


Figure 1.21: Conditioning unit noise spectrum. The noise is measured by shorting the signal conditioning input channels with an equivalent impedance to the photodiodes.

Photodiode noises

Photodiodes are semiconductor components equipped with a photo-sensitive element that generates a signal proportional to the light power landing on its active area. It most often consists of a p-n junction that uses the photoelectric effect to absorb photons with sufficient energy to create an electron-hole pair, and thus a photocurrent [127]. When the photodiode operates with no bias voltage, a load placed between the anode and the cathode prevents the current flow and a voltage builds up, according to the photovoltaic effect. Instead, when a reverse bias voltage is applied, the photodiode outputs a current that is linearly proportional to the input optical power. This is the photoconductive mode. The photoconductive mode has several advantages: faster response (increased bandwidth), higher responsivity $\mathfrak{R}$ ([A/W]), and most importantly far better linearity than the photovoltaic mode. While the photovoltaic mode shows better noise performance, especially in the dark current noise which is minimal [118], the better linearity makes the photoconductive mode more relevant for this application. There are primarily two noise sources responsible for polluting the output signal of photoconductive photodiodes: shot noise and dark current.

The shot noise Φ_{sh} ([A²/Hz]), often times called Schottky noise, is related to the quantum properties of light and its interaction with potential barriers [86, 117]. The discrete nature of light implies that charges in the p-n junction are generated one at a time, as photons land individually on the active area, leading to random fluctuations in the signal output. Higher laser intensity means a larger number of photons so that the intensity noise scales with the laser intensity and writes

$$\Phi_{\text{sh}} = 2qI_{\text{DC}} \quad (1.51)$$

with $q = 1.6 \cdot 10^{-19}$ C the electron charge and I_{DC} ([A]) the average photocurrent produced at the photodetector.

On the other hand, the dark current corresponds to a leakage of current caused by the bias voltage applied on the photodiode [86, 118]. The photodiode therefore continuously outputs a random photocurrent, even in the absence of light, hence the name of dark current. The dark current level of a photodiode is usually very small, ranging from pA to nA, depending on the active material and surface area. However, the dark current scales with the bias voltage and can ultimately lead to sensible values. The Dark current RMS amplitude I_{d} ([A]) for a given photodiode active element and bias voltage can be calculated as can be calculated as

$$I_{\text{d}} = I_{\text{sat.}} \left(e^{\frac{qV}{k_{\text{B}}T}} - 1 \right) \quad (1.52)$$

with $I_{\text{sat.}}$ the saturation current([A]) of the active element, V the bias voltage ([V]), $q = 1.6 \cdot 10^{-19}$ C the electron charge, $k_{\text{B}} = 1.38 \cdot 10^{-23}$ J/K the Boltzmann's constant and T the temperature ([K]).

This dark current then couples into the mechanism of shot noise with the same equation as above,

but using the dark current I_d as the current driving the shot noise formula.

$$\Phi_d = 2qI_d \quad (1.53)$$

The photodiodes used in the IFO are Thorlabs FGA21 InGaAs photodiodes. They have a wavelength range from 800 - 1700 nm with a peak sensitivity of $\mathfrak{R} = 1.04$ A/W at 1590 nm. They are characterised by a Noise Equivalent Power (NEP) of $6 \cdot 10^{-14}$ W/ $\sqrt{\text{Hz}}$ and a saturation current of 10 mA. The dark current is also specified by the manufacturer for a 1 V bias as 50 nA (typ). The signal conditioning unit applies a 2.5 V bias voltage, which results in a dark current of approximately $I_d = 70$ nA at room temperature, also according to manufacturer datasheet. This results in a dark current noise of $\Phi_d = 1.50 \cdot 10^{-13}$ A/ $\sqrt{\text{Hz}}$. Given that the IFO is typically fed with an input laser power $P_0 = 4$ mW and Equation 1.18 from the modelling of the interferometer, the average current output should be of $\mathfrak{R} \cdot \frac{P_0}{4} \approx 1$ mA. Therefore, a theoretical calculation of the shot noise leads to $\Phi_{\text{sh}} = 1.82 \cdot 10^{-11}$ A/ $\sqrt{\text{Hz}}$.

In fine, the photodiode noises are calculated as

$$\begin{cases} \Phi_d = 1.50 \cdot 10^{-13} & [\text{A}/\sqrt{\text{Hz}}] \\ \Phi_{\text{sh}} = 1.82 \cdot 10^{-11} & [\text{A}/\sqrt{\text{Hz}}] \end{cases} \quad (1.54)$$

The photodiode noises are measured placing 3 photodiodes in a dark environment. Again, because the expected noise level are lower than the MicroLab box ADC noise, the same Centaur 24-bit recorder is used. The photodiode are also connected to the signal conditioner unit, which also imposes its noise level on the measurement. The Centaur ADC noise has been down-converted to units of Ampere by dividing the voltage noise by the pre-amp gain signal conditioner unit (10^4 V/A). The photodiode dark noise is measured by simply recording the output signals of the 3 photodiodes. The Centaur ADC is set to the lowest range setting, 1 V_{pp}, with 2 kHz sampling rate for the best resolution. In order to cancel out any remains of ambient light or other external inputs on the photodiode readings, a coherent subtraction of the 3 photodiodes signals is calculated. Results are shown in Figure 1.22. The prediction of the dark current noise spectrum Φ_d lies below the Centaur ADC noise level, so that a direct measurement is not possible with the current experimental setup. The residual of the 3 photodiode signals in dark condition (blue traces) exposes a noise level at approximately 2 pA/ $\sqrt{\text{Hz}}$ above 5 Hz, approximately sitting on the ADC noise as expected. The excess noise with an $f^{-0.5}$ scaling below 5 Hz is expected to be caused by the ambient environment and leftovers of the coherent subtraction. On the other hand, the measurement of the shot noise requires the photodiode to be illuminated with an input power of the same magnitude as what they will experience in the operation of the optical device. The average optical power output is $P_0/4$ as demonstrated in subsection 1.2.2, so that 1 mW average optical power is read at the photodiodes when the IFO is used with nominal power rating of $P_0 = 4$ mW and assuming perfect alignment. In order to discard the laser source noises, especially the laser intensity noise (RIN) as discussed later, 4 mW are used to illuminate 3 photodiodes with the same laser beam through a set of splitters.

This way, the laser RIN noise appears as a common noise and can be rejected with a coherent subtraction. Results are plotted in the same Figure 1.22. For this measurement, the Centaur gain setting had to be increased to a larger $10 V_{pp}$ range, resulting in a larger ADC noise. The ADC noise is also superimposed and is not limiting the measurement. The photodiode shot noise (orange trace), shows a great match with the theoretical estimation of Φ_{sh} , reaching a noise level of approximately $2 \cdot 10^{-11} A/\sqrt{Hz}$. The 10 Hz to 50 Hz frequency range most certainly shows interferences resulting from vibrations in the optical setup (beam splitter and photodiode holders), excess low frequency noise below 10 Hz is explained the same way as above.

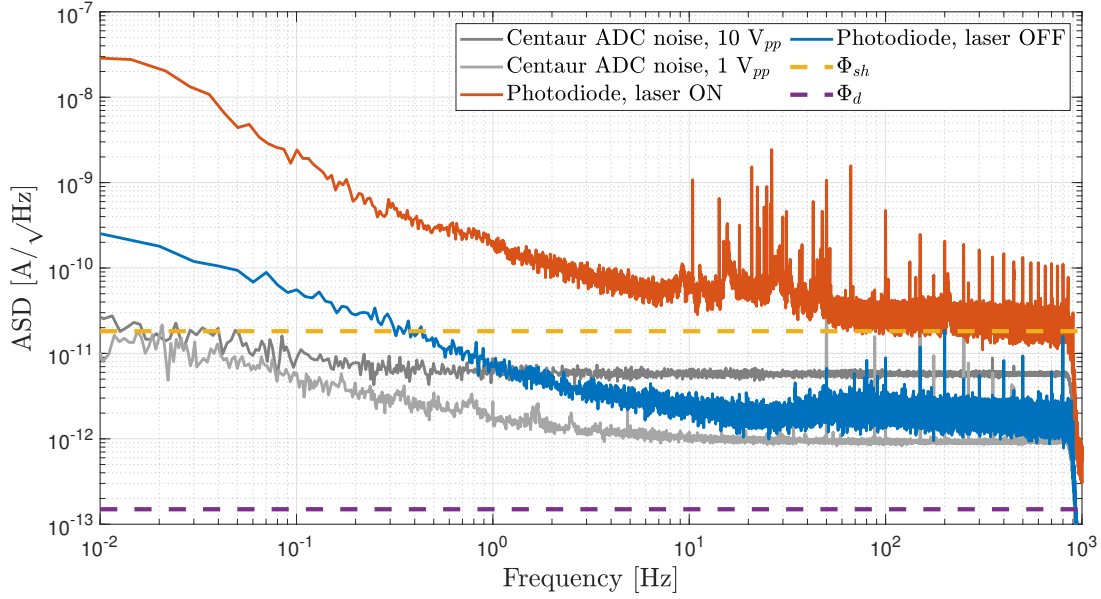


Figure 1.22: FGA21 photodiode dark current and shot noise spectrums. The dark current noise spectrum is measured as the output signal of the photodiode in a dark environment (blue trace), and the shot noise spectrum is measured as the residual of 3 photodiode signals illuminated with 4 mW (orange trace). The Centaur acquisition noise is converted to Ampere units by dividing with the pre-amp gain value.

Data acquisition chain noise budget

Figure 1.23 shows the noise budget of the optical device by compiling the noise measured for the different components of the acquisition chain. Equation 1.42 from the noise model in section 1.3 is used to transpose voltage and current noises into displacement noises. Assuming perfect alignment of the interferometer, the ADC and signal conditioner noises scales into displacement reading as $\lambda/(8\pi)$ ([m/V]), and the photodiode noises scales as $G\lambda/(8\pi)$ ([m/A]), with $\lambda = 1550$ nm the laser wavelength and $G = 10$ kV/A the signal conditioner pre-amp gain. The figure shows that the data acquisition chain sets the resolution of the optical device to $\Phi_{\hat{g}} = 2 \cdot 10^{-13} \sqrt{(1 + 5f^{-1})}$ m/ $\sqrt{Hz}$, dominated by the MicroLab Box ADC noise when using the real-time target system. When using

the Centaur digitiser, the optical readout can be shot noise limited and can reach a resolution $\Phi_{\dot{y}} = 2 \cdot 10^{-14} \text{ m}/\sqrt{\text{Hz}}$ at above 10 Hz. The photodiodes have been measured to contribute to the noise budget with a $10 \text{ fm}/\sqrt{\text{Hz}}$ white noise level dominated by shot noise, while dark current noise was found lower than the recorder used in the dark current noise measurement. The signal conditioner has been evaluated to a $1 \text{ fm}/\sqrt{\text{Hz}}$ contribution, set by the resistor thermo-electrical and $1/f$ noise.

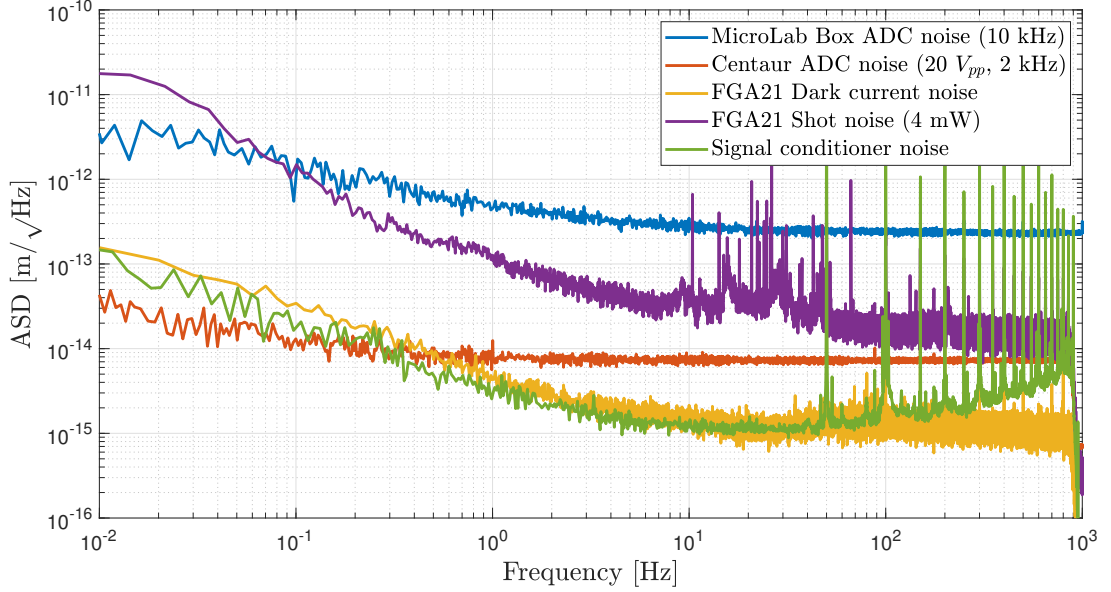


Figure 1.23: Data acquisition chain noise. When using the MicroLab Box system, ADC noise dominates the noise and set a resolution limit to approximately $2 \cdot 10^{-13} \text{ m}/\sqrt{\text{Hz}}$ above 10 Hz. When using the Centaur recorder, the optical readout is shot-noise limited and reaches $2 \cdot 10^{-14} \text{ m}/\sqrt{\text{Hz}}$ above 10 Hz.

1.3.2.3 Optical noises

Optical noises regroup all noise sources that cause the optical output power of the interferometer to differ from the textbook expressions from Equation 1.18. They may arise from multiple origins, ranging from laser source noise, optics alignment or optics material properties. Laser source intensity and fluctuation noises are caused by instability of the laser cavity and its sensitivity mainly to vibrations and temperature fluctuations [122, 128–130]. While the laser intensity noise is in principle coherent to all output power of the interferometer and can be mitigated with coherent subtraction [46], frequency noise directly translates into phase noise, and as a result displacement noise [86, 130]. Misalignment of the optomechanical assembly and material properties lead to power loss or imperfect polarisation state, leading to ellipticity in the quasi-quadrature signals.

In particular, next section will cover noise caused by the polarising properties of TIR retroreflectors [110, 111], wave-plate misalignment, and laser frequency and intensity noises.

Retroreflector polarising properties

The measuring arm of the optical device uses a TIR retroreflector to accommodate the angular trajectory of the target. Indeed, the geometrical properties of a corner cube guarantee the reflected beam to remain parallel to the incident beam, for any angle of the retroreflector. However, Total Internal Reflection (TIR) based retroreflectors, such as corner cubes, have been shown to have polarising properties, which alter the polarisation state of a light beam reflecting into its surfaces [110, 111]. For example, Figure 1.24 adapted from Thorlabs Labs Facts shows the effect of the polarisation properties of an uncoated TIR retroreflector on an input p-polarised light beam. Light entering a retroreflector can undergo 6 propagation trips depending on the input sextant: 3 trips representing different reflections within the corner cube, and 3 reverse trips that are identical to the first 3 but in reverse order. Experiments and calculations show that the polarisation properties are different for each trip. Because the optical device uses the polarisation state of light to create quadrature signals, the polarisation properties of the retroreflector will affect the reading by introducing a spurious polarisation change in the measuring arm.

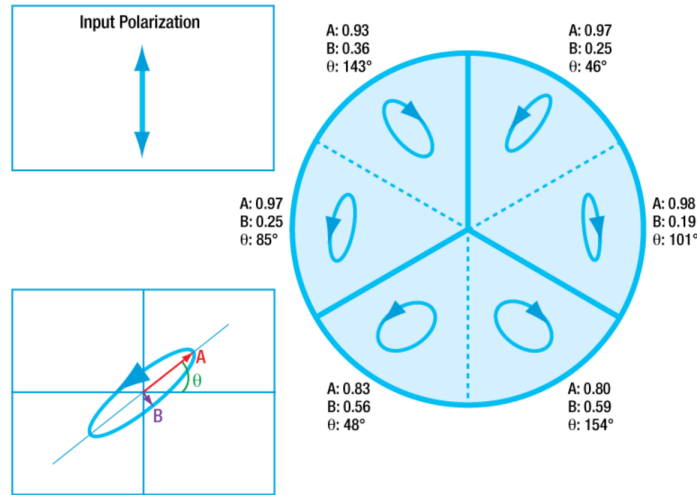


Figure 1.24: Schematic of the output polarisation properties for a p-polarised input light in a TIR retroreflector. From Thorlabs Labs Facts.

Jones matrices representing the polarisation transformation of a light electric field when reflecting onto the corner cube have been derived based on solid analytic geometry and light interaction with the corner cube [110]. The Jones matrices for the six propagation trips of light inside the corner cube are listed in Table 1.2. They assume a light beam entering the base of the corner cube with a normal incident angle and a glass body. The optical properties of the retroreflector material indeed affect the polarisation properties via the material refractive index, and the angle of incidence also impacts the geometrical reflections on the different sextants. The calculations are also supported by an experimental verification of the linear eigenpolarisation of a BK7 corner cube.

To quantify the effect of the corner cube polarisation properties, the Jones matrices have been

included in the IFO model built in subsection 1.2.2. More precisely, Equation 1.10 has been adapted with the corner cube Jones matrices to calculate the light polarisation state in the measuring (Y-) arm. Three different cases are compared: the ideal case using a flat mirror (no polarisation change), a second case using a retroreflector in the Y-arm, and a third case using a retroreflector with second-pass in the Y-arm. The three cases are illustrated in Figure 1.25. The last case corresponds to the optical layout of the IFO.

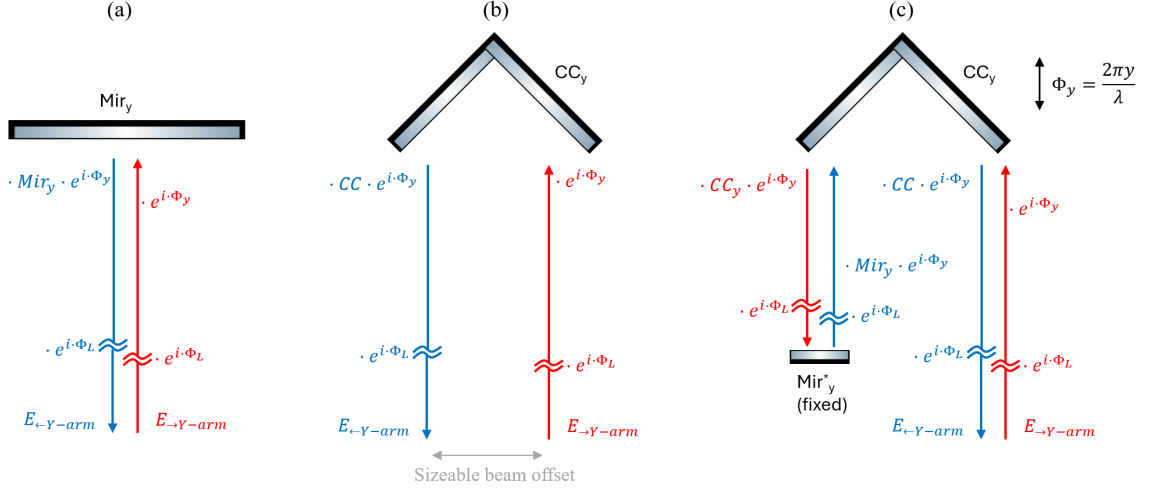


Figure 1.25: Schematics of the three different cases of reflections in the Y-arm to investigate the effect of the polarisation properties of a corner cube in the measuring arm. (a) Using a flat mirror (no polarisation change). (b) Using a TIR BK7 corner cube. (c) Using a TIR BK7 corner cube in double-pass configuration.

Figure 1.26 shows a simulation of the IFO output signals modelled for the three reflection cases in the measuring arm described above. The right plot shows the non-calibrated Lissajous figure (no normalisation nor ellipse correction) for the quadrature signals in each case. The Lissajous figures show differences in the figure radii, but no ellipticity. It can indeed be shown by expanding the mathematics in subsection 1.2.2 that the polarisation properties of the corner cube will affect equally the power output at each photodiode port. Any amount of spurious light power in the p-polarisation state is indeed discarded when reflecting on PBS2, as seen in the optical layout in Figure 1.7, resulting in power loss. This then translates into an effective power being less than the nominal power, but not into ellipticity. This can be verified in the left plot in Figure 1.26, showing the raw power outputs (in [W]) for the three cases. In the nominal case, i.e. with a flat mirror, the output signal should range from 0 to 0.5 mW given the nominal input power of 4 mW. The other two cases show an effective signal amplitude that is lesser than nominal, due to the optical power lost in the Y-arm. Note that all signals are generated using the same input displacement y in the measuring arm and that the fringes are twice as fast in the case of the corner cube with double-pass because of the optical amplification. Also, the effective power is larger in the case of the corner cube with double-pass than with no double-pass. The double-reflection on Mir_y^{*} effectively sends the reflected beam along the same geometrical reflections in CC_y but in reverse order, and generally

helps recovering the initial polarisation state, but does not totally cancel the spurious polarisation.

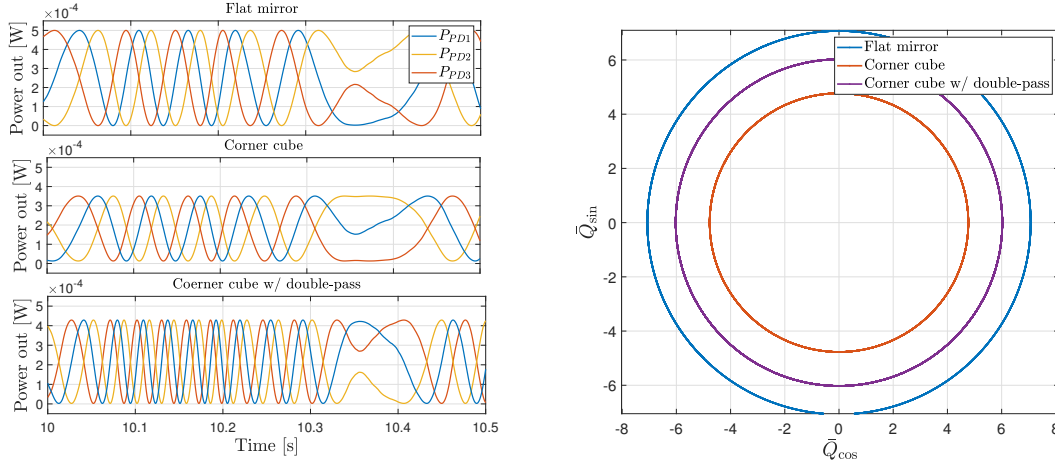


Figure 1.26: IFO output signals modelled for the three reflection cases in the Y-arm: with a flat mirror, a corner cube and a corner cube with double-pass. (Left) Raw fringes in [W], no normalisation. (Right) Raw Lissajous figure of the quadrature signals, no normalisation. The trip in the corner cube is assumed $ABO > BCO > ACO$, as referred to the polarisation properties in Table 1.2.

In these simulations, a trip $ABO > BCO > ACO$, as referred to the polarisation properties in Table 1.2, is assumed in the corner cube. The amount of polarisation change in the retroreflector depends on the exact trips due to the polarisation properties being different for each trip. The table shows that all propagation trips are not equivalent and that there is a preferred trip with lesser power lost in the Y-arm. This is also reflected in the above Figure 1.24 where it is clearly seen that 2 trips (one trip and the corresponding reverse trip) show significantly less polarisation change than the other 4. The table also shows that the double-pass does generally not totally compensate for the polarisation change, and in fact, worsens the effect for 2 of the 4 trips. However, 4 trips appear with significantly lower power loss, compared to the case with a simple corner cube where only 2 out of 6 trips show low power loss. In practice, orienting the corner cube for one specific trip is close to impossible, so a manual adjustment is made by rotating the corner cube around its axis. The adjustment is then a lot easier in the case of the corner cube with double reflection, since 4 trips out of 6 show good performance. In the above discussion, the reference arm of the interferometer is equipped with a flat mirror in all cases, for consistency with the layout in subsection 1.2.1. The model shows that replacing the reference mirror with another corner-cube does not help in recovering the correct polarisation when both arms recombine, but rather double the effective power loss.

It has been shown that the corner cube polarisation properties only convert into power loss, and consequently a smaller output signal amplitude. This effect should therefore be well-mitigated by the normalisation step in the demodulation process, as explained in subsection 1.2.3. Indeed, the calibration signals will contain the amplitude of the fringes, accounting for the effective power at the output of the IFO, which will then be normalised. Figure 1.27 shows the spectrum of the demodulated displacement $\hat{y}$ simulated for the 3 different propagation trips in the corner cube with

	ACO > ABO > BCO	ABO > BCO > ACO	BCO > ACO > ABO
Flat mirror	0.00 %	0.00 %	0.00 %
Corner cube	4.18 %	32.53 %	32.53%
Corner cube with double-pass	10.52 %	14.80 %	75.74 %
	BCO > ABO > ACO	ACO > BCO > ABO	ABO > ACO > BCO
Flat mirror	0.00 %	0.00 %	0.00 %
Corner cube	4.18 %	32.53%	32.53%
Corner cube with double-pass	10.52 %	75.74 %	14.80 %

Table 1.3: Power loss (in [%] of the nominal 0.5 mW output) in the IFO when the target in the Y-arm is equipped with a flat mirror, a corner cube and a corner cube with double-pass.

double-pass. The 3 reverse propagation trips are omitted for clarity of the figure, and because they lead to the same results due to symmetry. The figure shows that the demodulation process is successful in estimating the input y fed to the model, for all propagation trips regardless of the power loss. In the same figure are also shown the spectrum of the demodulation of the 'null input'. The null input corresponds to an input noise y with a magnitude corresponding to the resolution of the interferometer obtained from the null measurement. It consists of a white noise with an amplitude of $2 \cdot 10^{-13} \text{ m}/\sqrt{\text{Hz}}$, which reflects the amplitude of the cumulated quadratic sum of the noise sources in the device. The spectrum of the demodulated null $\hat{y}$ shows that the optical power loss induced by the corner-cube has little impact on the scaling of the noise level. Only for the worse two propagation trips (i.e. BCO > ACO > ABO and ACO > BCO > ABO), the power loss of 75 % leads to a sensible increase in the acquisition chain noise by the same amount, due to the scaling of the device resolution with the gain of the interferometer.

The consequence of the polarisation properties of the corner cube on the resolution is experimentally measured by comparing spectrums of a null measurement when the blocked target in the Y-arm is equipped with a retroreflector with double-pass, and a flat mirror. The test setup is identical to the one described in subsection 1.3.1. The null measurements are shown in Figure 1.28. In the same figure are superimposed the acquisition (ADC) noises for both measurements. It can be seen that the acquisition noise appears more significant in the case of the corner cube with double-pass (CC), as a result of the lower signal amplitude due to the power loss, which translates into a limitation in the 10 Hz and above frequency band. The ADC noise appears twice larger in the case of the use of the corner-cube than when using a flat mirror, which is more significant than predicted in the model. This can be explained by a sub-optimal alignment of the corner-cube, leading to extra power loss, as it will be discussed in the next section.

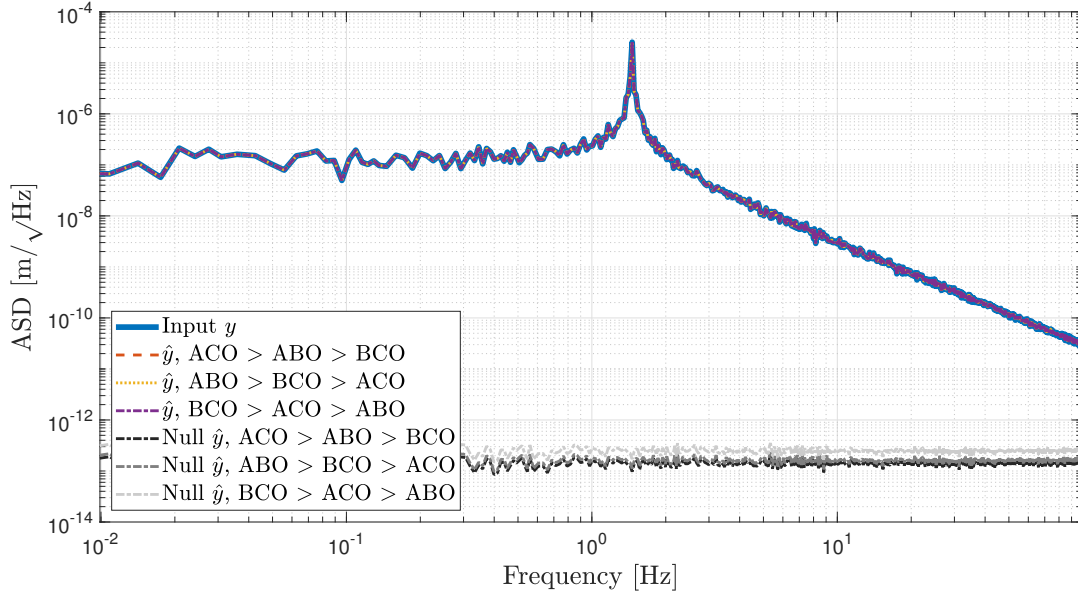


Figure 1.27: simulated amplitude spectrum of the demodulated displacement $\hat{y}$ and null input. Null $\hat{y}$, for the 3 different propagation trips in the case of a Y-arm equipped with a corner cube with double-pass.

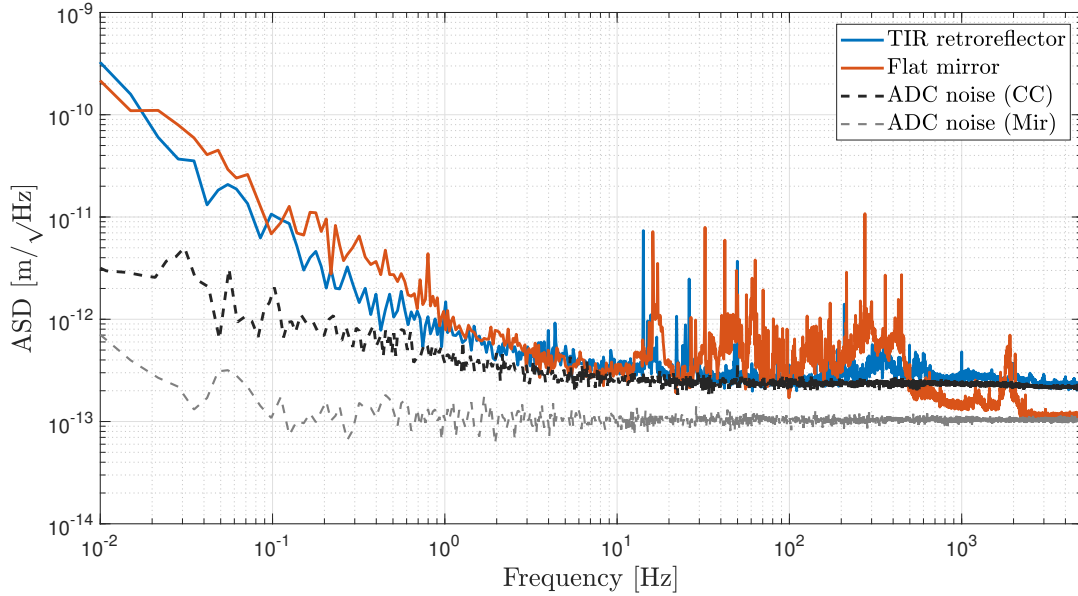


Figure 1.28: Amplitude spectrum of the resolution of the optical device using a TIR retroreflector with double-pass in the measurement arm (blue) and using a flat mirror in the measurement arm (orange). The dashed black and grey lines corresponds to measurements of the ADC noise of the device using the retroreflector and the flat mirror, respectively.

Misalignment

The IFO can suffer from misalignment in two ways. First, the optics inside the interferometer can be badly aligned. Second, the corner cube attached to the moving target might be misaligned. Both lead to a loss of fringe contrast and visibility. The first case can be assessed from the modelling of the IFO, and the second case is best investigated experimentally with a retroreflector mounted on translation stages.

The consequence of a bad alignment in the optical layout of the IFO is investigated by purposely misaligning components in the IFO MATLAB model exposed in subsection 1.2.2. Optical misalignment causes a decrease in fringe contrast and visibility, which translates into offsets and amplitude mismatches in the power output of the device. These then cause the signals of the interferometer to be in quasi-quadrature, and therefore ellipticity in the Lissajous figure of these two signals. A case of this has already been exposed in subsection 1.2.3, where misalignment of wave-plate has been shown to introduce large ellipticities. The ellipse correction algorithm has also been demonstrated to be efficient in compensating for this ellipticity and allows to demodulate the Lissajous figure without introducing errors, c.f. Figure 1.12. In addition, the IFO optical assembly is outsourced to EXail. With the precise assembly of all optical elements, in combination with the ellipse correction algorithm, it can be assumed that noise from internal misalignment of optical components is very negligible.

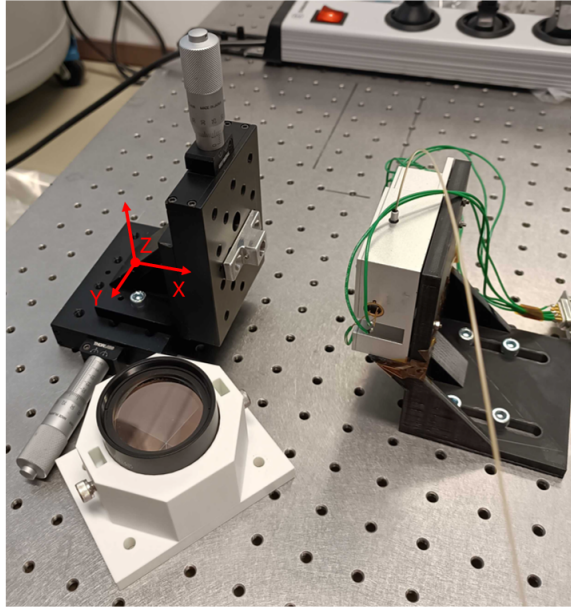


Figure 1.29: Setup for measuring the sensitivity of the IFO to retroreflector misalignment. The interferometer is mounted on a rigid frame, and targets a retroreflector mounted on a Y,Z translation stage.

The alignment of the IFO with the corner cube mounted on the target, on the other hand, is a lot

more difficult to manage in practice. The sensitivity of the optical device to corner cube misalignment is investigated by monitoring the change in fringe visibility when changing the relative alignment between an interferometer and a corner cube. A picture of the setup is shown in Figure 1.29. The interferometer is mounted on a rigid frame clamped to an optical table. The retroreflector in the measuring arm is mounted on a $\{Z,Y\}$ translation stage (X being the axis of the interferometer beam), clamped on the same optical table. The retroreflector is first placed in nominal position accordingly to the IFO according to the manufacturer (EXail) specifications, which should provide the best contrast. The relative positioning is performed manually so that 0.5 mm of initial misalignment can be expected.

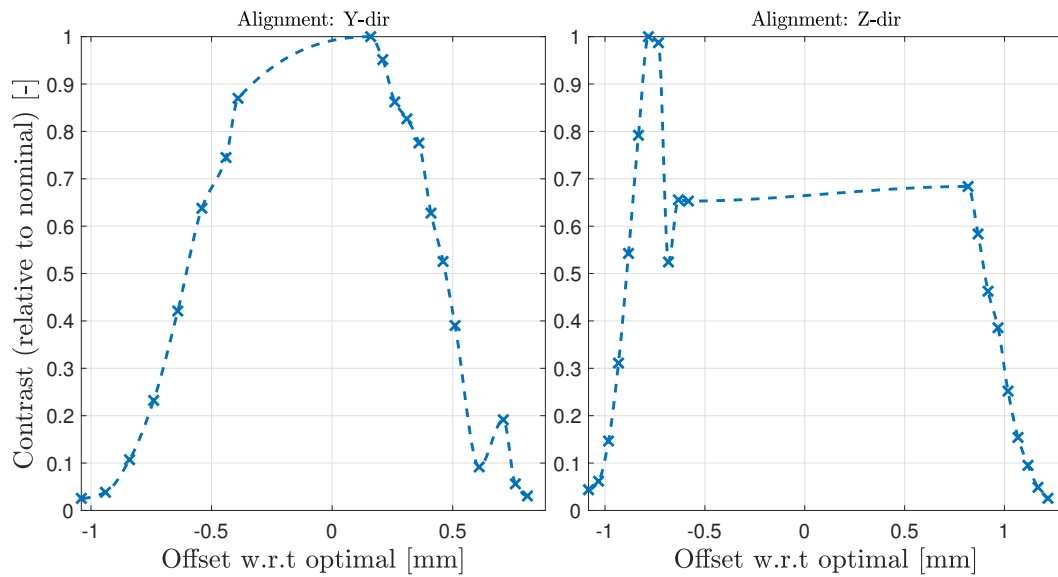


Figure 1.30: Interferometer contrast (relative to nominal position) when displacing the retroreflector from the nominal position.

The translation stages are then used to offset the corner cube from the nominal position in the Y and Z axis. The axes are moved successively, i.e. the corner cube is maintained in nominal Z position when moved along Y and vice-versa. Figure 1.30 shows the loss of visibility compared to nominal contrast when the corner cube is moved from the nominal position. The contrast peaking at a -0.75 mm offset from nominal position when aligning the Z-direction (right plot) probably results from the manual alignment procedure and from the experimental setup. The graphs show that there is approximately a 1 mm range in both Y and Z directions where the corner cube maintains a decent contrast (less than 30% loss with respect to nominal). The 1 mm range corresponds to the size of the laser beam (0.87 mm at the output of the collimator) relative to the size of the corner cube faces (10 mm diameter corner cube).

Table 1.4 shows the contrast drop when moving the corner cube along both axes simultaneously. The figure shows that the conclusion on the uniaxial tests can be extended to in-plane offsets with

a radius of 0.2 mm from the nominal position. The measurements have been made on a separate setup from the previous test, explaining the discrepancy between the two graphs.

		Z-axis [mm]						
		-0.6	-0.4	-0.2	0	0.2	0.4	0.6
Y-axis [mm]	-0.6	0.01	0.04	0.04	0.06	0.04	0.02	0.01
	-0.4	0.04	0.08	0.13	0.17	0.12	0.08	0.05
	-0.2	0.08	0.13	0.33	0.58	0.36	0.17	0.08
	0.0	0.06	0.25	0.45	1.00	0.67	0.20	0.05
	0.2	0.13	0.26	0.33	0.61	0.51	0.15	0.09
	0.4	0.05	0.10	0.14	0.16	0.21	0.21	0.06
	0.6	0.04	0.07	0.07	0.03	0.07	0.04	0.01

Table 1.4: Interferometer contrast (relative to nominal position) when offsetting the retroreflector in both Y and Z axis with steps of 0.2 mm.

Since the corner cube misalignment leads to a loss of fringe visibility, this will translate into smaller signal amplitude and interferometer gain. Ultimately, this will result in larger ADC noise since the output signals will not use the full ADC voltage range. The effective signal amplitude related to the alignment of the corner cube in the measuring arm is therefore accounted for in the calculation of the ADC noise.

Laser intensity noise

The intensity noise refers to random fluctuation in the output power of a laser source around the setpoint value [128]. The Relative Intensity Noise (RIN, [dB/Hz]) is often specified by laser manufacturers and relates to the intensity noise Φ_I [W^2/Hz] as

$$\text{RIN} = \frac{\Phi_I}{P_0^2} \quad (1.55)$$

where P_0 ([W]) is the output power setpoint.

Laser intensity noise therefore corresponds to a spurious intensity fluctuation, proportional to the power intensity, that enters the optical layout of the IFO. Assuming ideal alignment of the optical components, the laser intensity noise will propagate equally to the three power outputs of the interferometer and will appear as a common-mode noise on the photodiode readings. Therefore, the subtraction of pairs of signals in the calculation of the quadrature signals $Q_{\cos}$ and $Q_{\sin}$ will discard any intensity noise. Indeed, from subsection 1.2.2, the voltage readings V_{PDi} ([V]) at the three photodiodes including laser intensity noise (but neglecting all other noise contribution), would

write

$$\begin{cases} V_{PD1} = G_1 \mathfrak{R}_1 \frac{P_0 + n_I}{8} \cdot (1 + \cos(4\Phi_y)) \\ V_{PD2} = G_2 \mathfrak{R}_2 \frac{P_0 + n_I}{8} \cdot (1 + \sin(4\Phi_y)) \\ V_{PD3} = G_3 \mathfrak{R}_3 \frac{P_0 + n_I}{8} \cdot (1 - \sin(4\Phi_y)) \end{cases} \quad (1.56)$$

with n_I ([W]) the laser intensity noise, G_i the pre-amp electronics gain ([V/A]) and $\mathfrak{R}_i$ the photodiode responsivities ([A/W]). Following the demodulation process exposed in subsection 1.2.3, the voltage readings are normalised with gains $\tilde{a}_i = G_i \mathfrak{R}_i \frac{P_0}{8}$ and offset $\tilde{b}_i = G_i \mathfrak{R}_i \frac{P_0}{8}$ values calculated on a calibration dataset, resulting in

$$\begin{cases} \bar{V}_{PD1} = \frac{V_{PD1} - \tilde{b}_1}{\tilde{a}_1} \\ \quad = \cos(4\Phi_y) \left(1 + \frac{n_I}{P_0}\right) + G_1 \mathfrak{R}_1 \frac{n_I}{P_0} \\ \bar{V}_{PD2} = \sin(4\Phi_y) \left(1 + \frac{n_I}{P_0}\right) + G_2 \mathfrak{R}_2 \frac{n_I}{P_0} \\ \bar{V}_{PD3} = -\sin(4\Phi_y) \left(1 + \frac{n_I}{P_0}\right) + G_3 \mathfrak{R}_3 \frac{n_I}{P_0} \end{cases} \quad (1.57)$$

Assuming equal characteristics for the three photodiodes and pre-amp circuits, i.e. $G_1 = G_2 = G_3$ and $\mathfrak{R}_1 = \mathfrak{R}_2 = \mathfrak{R}_3$, the subtraction of pairs of signals to form the quadrature signals will lead to

$$\begin{cases} Q_{\sin} = \bar{V}_{PD1} - \bar{V}_{PD3} \\ \quad = \sqrt{2} \left(1 + \frac{n_I}{P_0}\right) \sin(4\Phi_y + \frac{\pi}{4}) \\ Q_{\cos} = \bar{V}_{PD1} - \bar{V}_{PD2} \\ \quad = \sqrt{2} \left(1 + \frac{n_I}{P_0}\right) \cos(4\Phi_y + \frac{\pi}{4}) \end{cases} \quad (1.58)$$

where it can be seen that the offset component related to the intensity noise $G_i \mathfrak{R}_i \frac{n_I}{P_0}$ cancels out. Similarly, the gain component is suppressed when demodulating the interferometric phase $\hat{\Phi}_y$ in the arctangent operation.

$$\begin{aligned} \hat{\Phi}_y &= \frac{1}{4} \left(\tan^{-1} \left(\frac{Q_{\sin}}{Q_{\cos}} \right) - \frac{\pi}{4} \right) \\ &= \frac{1}{4} \left(\tan^{-1} \left(\frac{\sin(4\Phi_y + \pi/4)}{\cos(4\Phi_y + \pi/4)} \right) - \frac{\pi}{16} \right) \end{aligned} \quad (1.59)$$

In the end, the intensity noise is entirely discarded from the demodulation process, first in the calculation of the quadrature signals and second in the ratio of the quadrature signals prior to taking the arctangent. This however assumes that laser intensity noise is equal in all photodiodes, i.e. no power output imbalance due to optical alignment and that the electronics (photodiode

sensitivities and signal conditioner gains) are strictly identical. While the latter can be safely assumed, as discrepancies between similar electronic circuits are typically less than a few percent, the first condition is more complex to satisfy in practice (see above section). For this reason, an experimental validation of the RIN noise rejection is carried out.

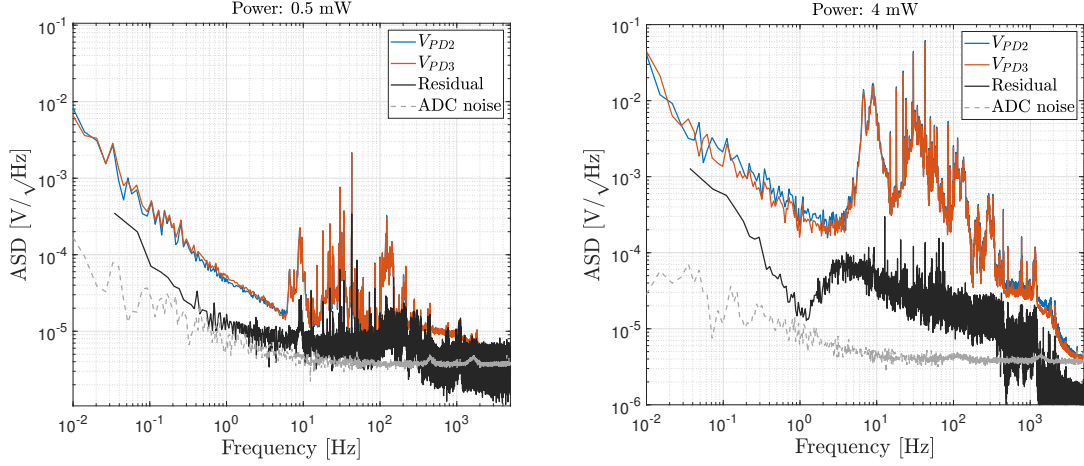


Figure 1.31: Spectrum of the the IFO sine and minus sine voltages readings. The black line represent the residual from a coherent subtraction between the Sin and mSin signals. (Left) 0.5 mW laser power setpoint. (Right) 4 mW laser power setpoint.

The RIN noise of a laser source is independent of laser power but varies with frequency. The NKT Koheras laser source is characterised by a RIN noise < -100 dBc/Hz at peak frequency (0.3 MHz) and < -135 dBc/Hz at 10 MHz. The RIN noise level in the [0.1 - 10] MHz bandwidth is provided by the manufacturer. However, the frequency range of interest for this optical readout is located at much lower frequencies, i.e. 10 mHz - 100 Hz. A direct measurement of the Koheras X15 RIN noise in this frequency band has already been performed in [46]. The measurement process consisted in feeding two photodiodes with the same beam input, using a beam splitter, and at constant laser power. In this condition, RIN noise will be a common-mode signal and is isolated in a multiple-channel coherent analysis. The measurement was successful in validating the mitigation of RIN noise when subtracting the two photodiodes. In order to transpose the conclusion to the optical device, the experiment is repeated but now using the full optical layout instead of illuminating only 2 photodiodes. The optical layout is mounted on an optical bench with both the reference and measuring arms blocked to identical arm-length for a null measurement, and the laser is fed from the Koheras Adjustik X15. In this condition, assuming no motion in the Y- and Z-arms and negligible ambient effects, laser intensity noise should be a major contribution to the power output. The 3 output voltages are recorded on the MicroLab Box ADCs. Figure 1.31 shows the spectrum of the raw output voltages V_{PDi} prior normalisation. A coherent subtraction is made to reject common-mode noise, i.e. RIN noise. The definition of the RIN noise shows that the laser intensity noise scales with the power setpoint P_0 , so that the test is performed using a low output power of 0.5 mW, and the maximum output power of 4 mW (set by the pre-amp electronics saturation), in order to magnify

the contribution of laser intensity noise in the signal. The residual from a coherent subtraction of the sine and minus sine readings (V_{PD2} and V_{PD3}) with a 0.5 mW input power shows that the intensity noise is effectively discarded in the [0.5 - 1000] Hz frequency band. Below 0.5 Hz, photodiode signals are probably limited by external noises. However, the measurement at 4 mW shows residual noises in the coherent subtraction. At 1 Hz and above, it can be seen that two orders of magnitude of coherent signals is being removed in the subtraction, the residual therefore presumably consists in the nonlinear response of the photodetector or other external factors. Below that frequency, the residual at 4 mW shows the same amplitude as for the 0.5 mW measurement, implying that both curves show a common limitation, which is not related to intensity noise.

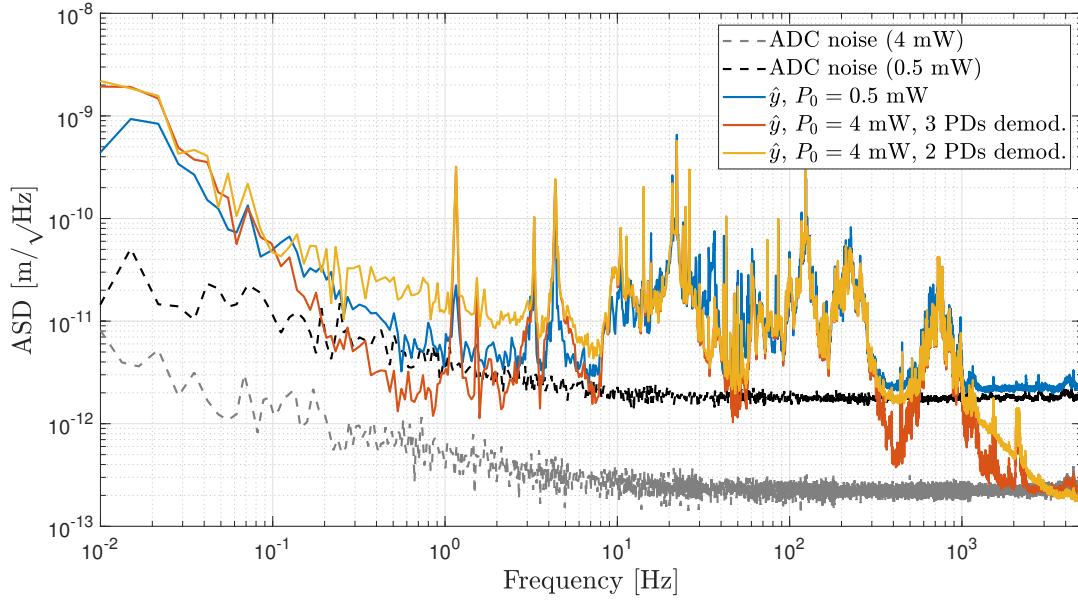


Figure 1.32: Spectrum of the output displacement $\hat{y}$ for a 0.5 mW and 4 mW input laser power. The measurement at 4 mW is demodulated using the full demodulation process (3 PDs demod.), or using a 2 photodiodes process with no subtractions in the calculation of the quadrature signals (2 PDs demod.).

Figure 1.32 shows the spectrum of the demodulated displacement for both tests at 0.5 mW and 4 mW input laser power. Both measurements (blue and orange traces) are equally good in measuring the residual vibrations in the test setup, and show overlapping spectrums in most of the frequency band. The 0.5 mW trace only show limitation in the [0.1 - 3] Hz band due to larger ADC noise caused by the smaller fringes amplitude. Since RIN is a fixed property of a laser source independent of the power setting, the intensity noise power spectrum is expected to be approximately 64 times larger in the 4 mW measurement than in the 0.5 mW measurement. That both measurements show identical spectrums for significantly different intensity noise input is an indirect validation of the successful intensity noise rejection. Also, superimpose in Figure 1.32 is the displacement obtained by demodulating the interferences of the 4 mW measurement using only the reading from V_{PD1} and

V_{PD2} (yellow trace). Indeed, these two photodiodes read sine and cosine signals which can be used as quadrature signals, and the displacement can be demodulated without subtraction of signals prior to taking the arctangent. Since the yellow and orange traces are calculated from the exact same dataset, they should lead to the same spectrum. The discrepancies between the two traces therefore clearly highlights the intensity noise (and other common-mode noise), in $[0.1 - 10]$ bandwidth, that is not rejected with the partial demodulation process using only 2 photodiodes. This conclusion on noise rejection is not limited to laser intensity noise but extends to any common-mode noises. Below 0.5 Hz, all curves meet and seem to sit on another noise source independent of laser power.

Laser frequency noise

Similarly to laser intensity noise, laser frequency noise represents fluctuations in the frequency of the output laser beam around the laser central wavelength [122,129,130]. It is commonly attributed to vibrations or thermal effects in the laser cavity. Frequency noise is a severe limitation in fringe-counting interferometers as it directly translates into phase noise, which is indistinguishable from the interferometric phase due to target motion. The laser phase Φ_p ($[\text{rad}/\sqrt{\text{Hz}}]$) issued by the frequency noise writes

$$\Phi_p = \left(2\pi \frac{\Delta L_0}{c}\right) \Phi_\nu \quad (1.60)$$

with Φ_ν the laser frequency noise ($[\text{Hz}/\sqrt{\text{Hz}}]$), c the speed of light and ΔL_0 the Optical Path Difference, i.e. the length mismatch between the reference and the measuring arms. Since the displacement along the measuring arm y relates to the interferometric phase Φ_y as $y = \frac{\lambda}{2\pi} \Phi_y$, the frequency noise will contribute to the readout displacement noise Φ_d ($[\text{m}/\sqrt{\text{Hz}}]$) as

$$\Phi_d = \frac{\lambda}{2\pi} \cdot \Phi_p = \frac{\Delta L_0}{\nu} \Phi_\nu \quad (1.61)$$

with Φ_ν the laser central wavelength.

The NKT Koheras Adjustik X15 is a DFB laser source that comes with an active wavelength control mechanism using a piezo-crystal to act on the laser cavity. This technology allows to reach sub $5 \text{ Hz}/\sqrt{\text{Hz}}$ frequency noise at 100 Hz and above. The frequency noise however increases with a typical slope of f^{-1} at lower frequencies down to 1 Hz. As for the laser intensity noise, the bandwidth of the IFO device lies at a lower frequency band, 10 mHz - 100 Hz, where laser specifications are no longer guaranteed by the manufacturer. The laser frequency noise has therefore been measured using an Optical Frequency Discriminator (OFD) from the Silentsys company, France. The OFD is a system that takes a laser power input (1510 -1590 nm, 200 μW max.) and delivers a reading voltage that is proportional to laser frequency fluctuation and is therefore suited for frequency noise measurement with a noise floor limit $< 0.1 \text{ Hz}^2/\text{Hz}$ down to Hz-like frequencies. A first experiment is carried out to measure the frequency noise along with an IFO null measurement. The experimental

layout is shown in Figure 1.33. To obtain a simultaneous measurement of the laser frequency noise and IFO noise, the laser output from the NKT laser source is split to the IFO in a blocked setup and to the OFD. Because of the different power requirement between the IFO (max. 4 mW) and the OFD (max. 200 μ W), the 20 mW laser output is first directed to a 1:4 PM fibre splitter to output approximately 4 mW, accounting for loss in the splitter and fibres. One output of the splitter is directed to the IFO null measurement, and another output is directed to the OFD. A power attenuator with a 10:1 extinction ratio and another 1:4 PM fibre splitter is used to feed approximately 80 μ W to the OFD. The IFO null measurement, the frequency noise, as well as room temperature from a temperature sensor are all recorded on MicroLab Box 16-bit ADCs.

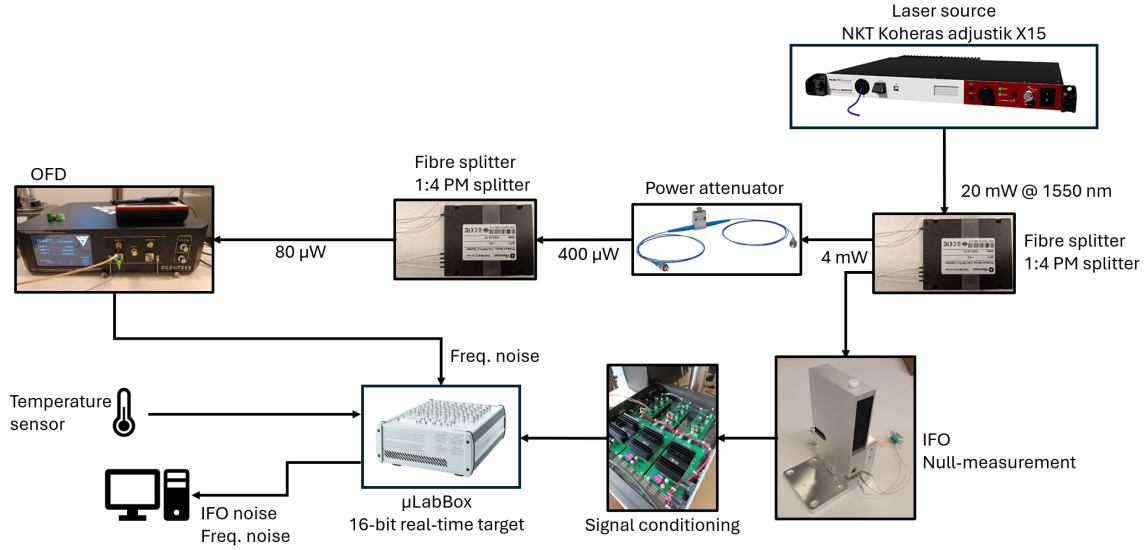


Figure 1.33: Frequency noise measurement setup. The laser output of the NKT Koheras X15 is fed to the IFO in the null-measurement configuration and to the OFD for measuring the frequency noise.

Figure 1.34 shows a power spectrum of the laser source frequency noise, in $[\text{Hz}^2/\text{Hz}]$, measured by the OFD (left axis). Measurements were taken during 14 to 16 hours at a 500 Hz sampling rate in order to capture frequency noise down to 10^{-4} Hz. Overnight measurements showed much better and more consistent results due to lower ambient external load on the setup (temperature fluctuation and lab vibrations). The frequency noise spectrum has next been projected into displacement noise and superimposed with the spectrum of the IFO null measurement (right axis). Note the bump appearing at 150 mHz probably being caused by residual vibrations induced by the micro-seismic peak. Similarly, the IFO spectrum is transposed into frequency noise units so that all curves can be plotted on the same graph for easier reading and comparison. In the conversion factor, an optical arm-length difference ΔL_0 of 4 cm is used, as it has been measured for the optical layout to be the typical arm-length difference that will be experienced in the mountings of the optical readout. The OFD acquisition noise and IFO ADC noise are also measured by turning off the laser emission.

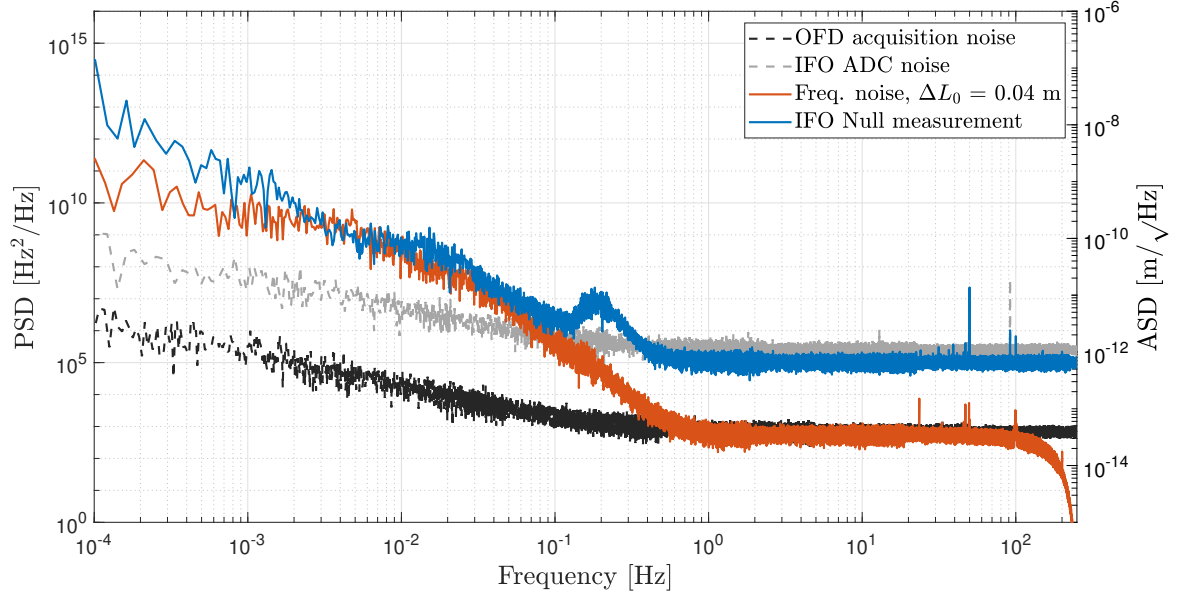


Figure 1.34: Measurements of the laser frequency noise measured with the OFD and a IFO null-measurement. The frequency noise is projected into displacement noise using a $\Delta L_0/\nu$ conversion factor assuming a 4 cm optical path length difference, and vice-versa. The left axis shows the traces in units of $[\text{Hz}^2/\text{Hz}]$. The right axis shows the traces in units of $[\text{m}/\sqrt{\text{Hz}}]$.

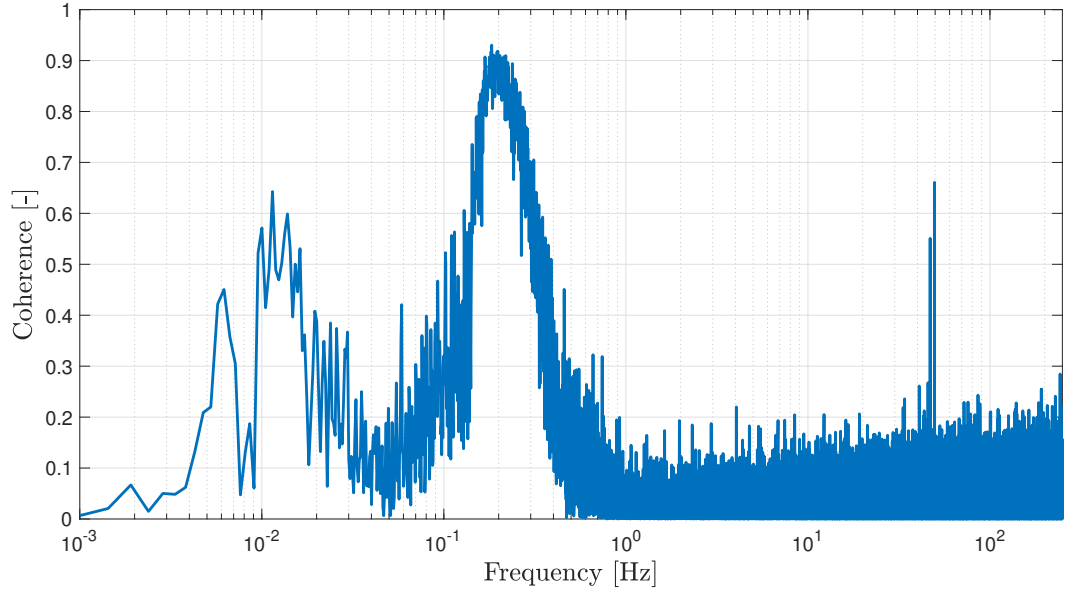


Figure 1.35: Coherence between the laser frequency noise and the IFO null measurement.

From Figure 1.34, both the measurement of the frequency noise with the OFD and the inter-

ferometer null measurement show similar spectrums in the 5 mHz to 100 mHz frequency range. However, the simultaneous measurement of frequency noise and the IFO null measurement allows to calculate the coherence between both signals. The coherence plot is shown in Figure 1.35 and indicates in facts a relatively low coherence in this bandwidth. This implies that both the IFO and the OFD may suffer from a similar noise source. A temperature sensor installed near the setup indicates a 2 to 3 °C temperature drift throughout the measurement. The laser cavity inside the OFD, used in the frequency measurement, is indeed very sensitive to temperature fluctuation, and support from the Silentsys company pointed out that a 3 °C is significant and could degrade the performance at such low frequencies. Unfortunately, the temperature sensor used for this measurement did not have a sufficiently good resolution for performing a coherent subtraction analysis.

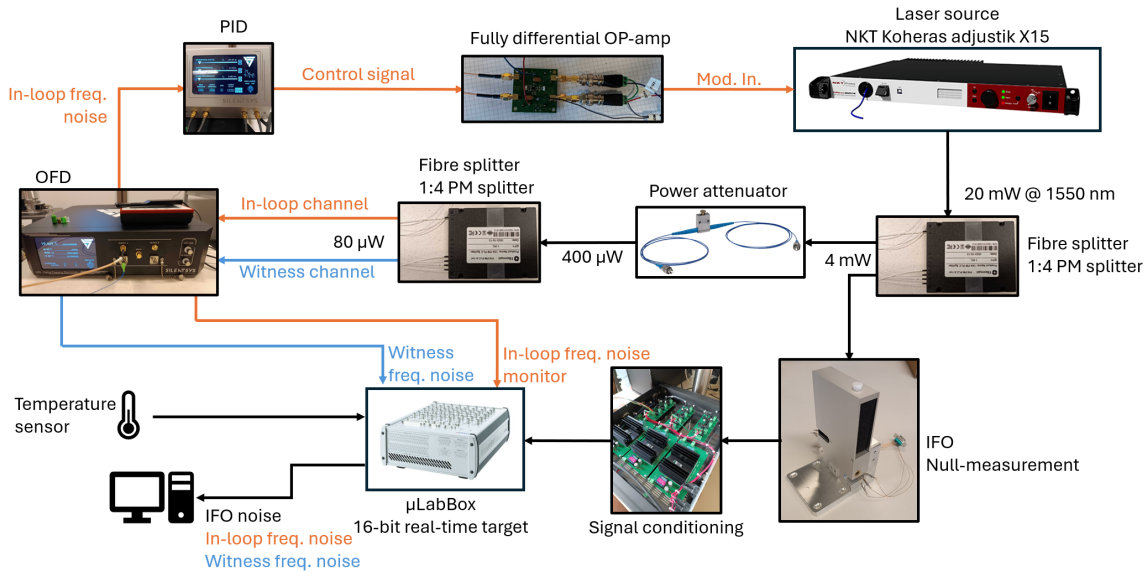


Figure 1.36: Frequency stabilisation setup. The laser output of the NKT koheras X15 is fed to the IFO in the null-measurement configuration and to the OFD for active frequency stabilisation. One channel of the OFD is used to drive the modulation input (Mod. In.) of the laser source through a PID controller for stabilising the laser frequency noise. The other channel of the OFD is used as an out-of-loop setup.

Despite the potential limitation of the OFD due to the thermal environment, an attempt to reinforce actively the wavelength control in the NKT laser source has been performed. The NKT Koheras laser sources are equipped with an external modulation input (Mod. In.) which can be used to drive the servo-controlled in the laser cavity. Therefore, the frequency noise measurement of the OFD can be used to drive the wave-length control of the laser source to actively stabilise the frequency noise. A description of the frequency stabilisation setup is shown in Figure 1.36. 2 fibres from the second 1:4 PM fibre splitter are used to feed the 2 input channels of the OFD. One channel is used to drive a PID controller (also provided by Silentsys), which drives the modulation input of the laser source. A Fully differential Op-amp is used to convert the single-ended output of the PID to the differential Mod. In. input in order to use the full wavelength modulation range. On

the other hand, the second channel is used as a witness, out-of-loop, measurement of the frequency noise. Everything else is identical to the previous experiment.

Figure 1.37 shows the open-loop transfer function measured by driving the laser modulation input with a low-passed white noise signal and measuring the frequency noise at the output channel for designing the PID control. The measurement shows great coherence down to 100 mHz, so stabilisation performance can be expected in this bandwidth.

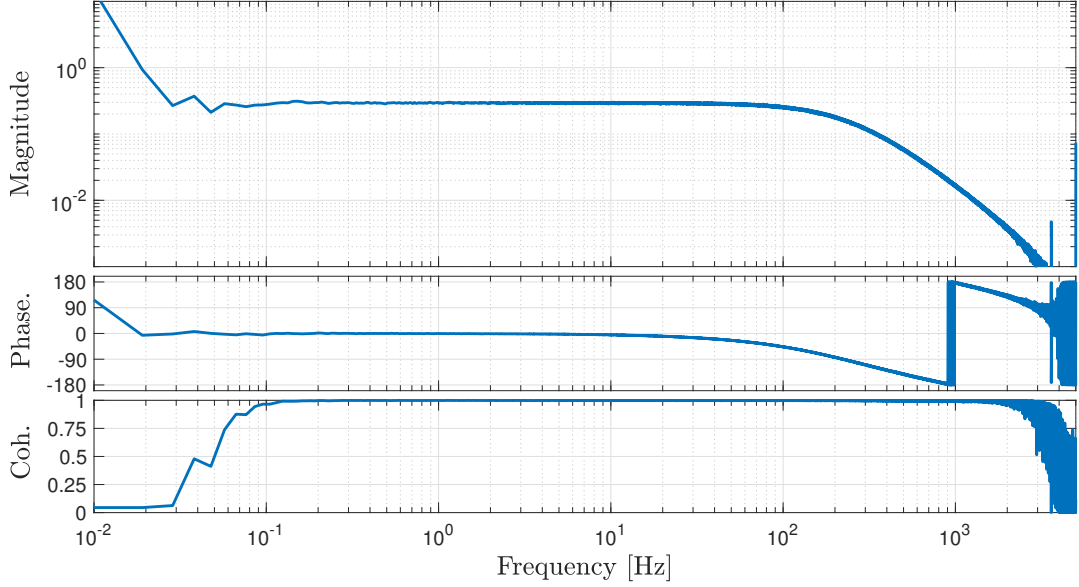


Figure 1.37: Open-loop transfer function obtained by driving the laser external modulation input with white noise and recording the OFD in-loop channel output.

Figure 1.38 shows spectrums of the laser source frequency noise ($[\text{Hz}^2/\text{Hz}]$) with the stabilisation loop enabled (plain lines) and disabled (dashed lines). The blue traces show spectrums of the in-loop channel and the orange traces show the spectrum of the witness channel. The figure shows that, while the PID action is successful in stabilising the in-loop channel, the improvement in the witness channel is very marginal and represents a factor of 10 at frequencies below 100 mHz.

Eventually, Figure 1.39 shows the spectra of the IFO null measurement with the frequency stabilisation loop ON and OFF. The green trace shows the null measurement measured in the thermal box as a reference. The traces show no difference between the IFO resolution with the stabilisation loop enabled or not. As mentioned above, the low coherence between frequency noise and IFO noise, presumably due to the bad thermal environment and the OFD resolution partially explains why the stabilisation loop is not effective in lowering the noise level below 100 mHz. In addition, a measurement of the control noise has been pointed out as a limitation of the setup. Figure 1.40 shows the control noise converted to units of frequency noise and superimposed onto the measurement of the frequency noise with the stabilisation loop ON. It is measured as the spectrum

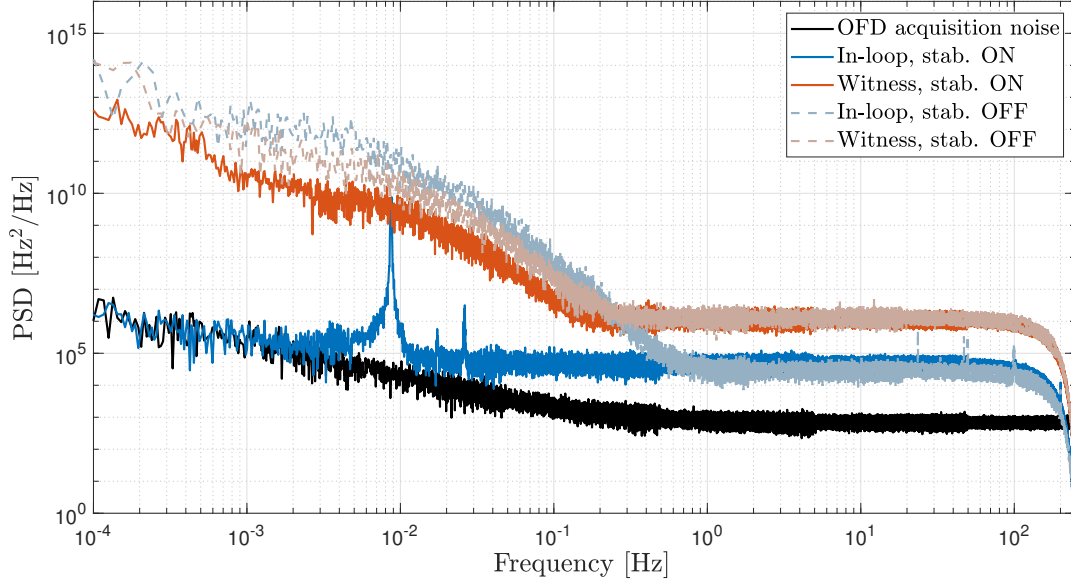


Figure 1.38: Spectrum of the laser frequency noise with the OFD stabilisation loop ON and OFF. The blue traces show spectrum of the in-loop channel and orange traces show the spectrum of the witness channel. Dashed lines are for stabilisation OFF measurements and plain lines are stabilisation ON measurements.

of the voltage at the wavelength modulation input with the laser emission OFF. It therefore includes the OFD acquisition noise, the PID servo electrical noise and the fully differential Op-amp noise. The figure shows that the spectrum of control noise effectively overlaps with the witness measurement of the frequency noise.

In conclusion, the Optical Frequency Discriminator allowed to flag the frequency noise as a potential major contribution to the IFO resolution in the 5 mHz to 100 mHz frequency band. However, the limitation of the OFD due to the poor thermal environment and the high control noise prevented to successful use of the OFD in a wavelength stabilisation loop to lower the frequency noise contribution. Performance is expected to improve with better thermal insulation of the OFD, of the laser source and of the IFO device, and with better analogue electronics in the control loop.

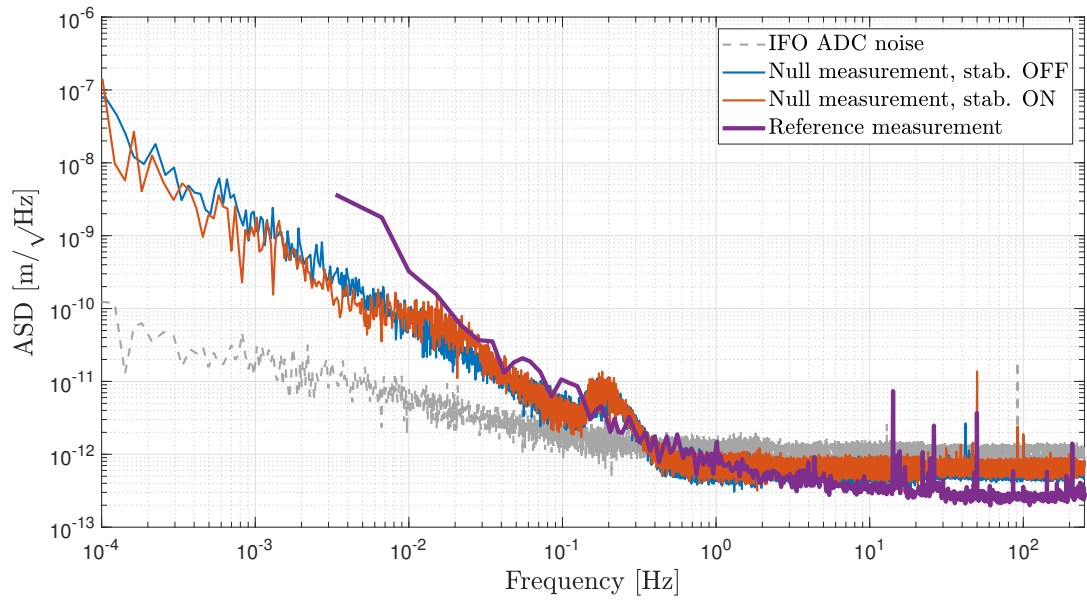


Figure 1.39: Spectrum of the IFO null measurement with the OFD frequency stabilisation loop enabled (orange) and disabled (blue). The purple trace shows the null measurement measured in the thermal box as a reference.

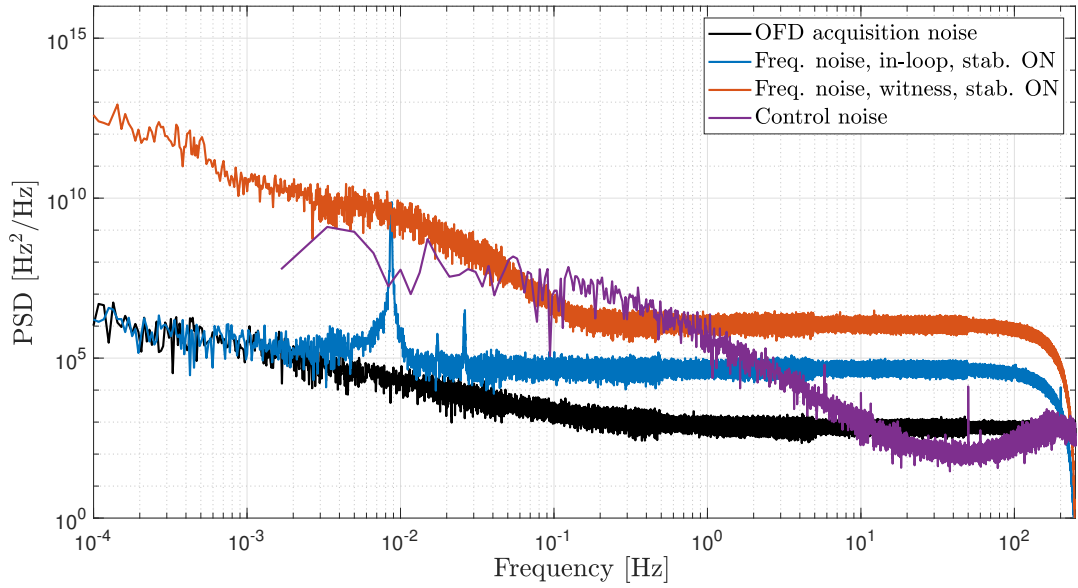


Figure 1.40: Spectrum of the control noise superimposed with measurement of the frequency noise (in-loop and witness channels), when the stabilisation loop is enabled.

Optical noise budget

To summarise, it was found that most optical noise sources can be safely discarded either thanks to the demodulation process or with a careful alignment of the optics.

First, a modelling of the polarisation properties of TIR retroreflectors such as the one used in the measuring arm of the IFO have been found to cause loss of contrast (lesser amplitude) in interferometer fringes. This is partially mitigated in the design of the optical layout by the use of a double-pass process in the Y-arm: while a single-pass in the corner-cube would lead to a contrast drop by more than 30% for 4 out of the 6 propagation trips in the corner-cube, the contrast loss is limited to 10 - 15 % using a double-pass strategy. Only 2 sextants, resulting in propagation trips leading to contrast loss greater than 75%, are to be avoided. Then, the normalisation and ellipse correction algorithms have been modelled to correctly accommodate for any remains of contrast loss, provided accurate calibration. The contrast drop however translates into a lower interferometer gain ($[m/V]$), leading to a larger acquisition (ADC) noise level compared to the nominal case. This can be mitigated in careful alignment of the corner-cube, selecting the input sextant with lowest contrast drop.

Second, misalignment of components in the optical layout has been investigated from the Jones modelling of the interferometer. It has been seen that, while significant ellipticity in the quasi-quadrature signals can be caused by bad wave-plate orientation, this is again efficiently managed in the ellipse correction algorithm so that the interferometric phase can be demodulated with no error. On top of that, the precise assembly of the optical components by EXail,, limits any alignment errors to few mrad, so that this can be assumed negligible. The alignment of the corner-cube in the measuring arm is however still an alignment process and is subject to larger errors. An experimental investigation showed that the corner-cube can be positioned with an region of approximately 0.5 mm from the nominal position while preserving acceptable contrast (less than 30% visibility loss). This is due to the geometrical properties of the corner-cube, making the alignment insensitivity to angular position, and to the double-pass reflection that aims to send the reflected beam exactly on the same path as the incoming beam. Therefore, a careful positioning of the retroreflector can mitigate any misalignment noise.

Third, subtraction of pairs of photodiode in demodulation process was proven successful to mitigate laser intensity noise. Since the RIN noise is coherent to the 3 photodiodes of the optical layout, subtraction of pairs of photodiodes to calculate the quadrature signals efficiently cancels the RIN noise. A comparison between the demodulation of the same measurement using the 3 available photodiodes, or using only 2 quadrature photodiodes, showed that the RIN noise that is appearing the "2-photodiode signals" is correctly discarded in the "3-photodiode process".

Lastly, laser frequency noise has been measured using an Optical Frequency Discriminator (OFD) from Silentsys, France. The OFD measured a large frequency noise contribution of the NKT Adjustik X15 laser source at sub-Hz frequencies. It can be seen from Figure 1.34 that the laser frequency noise matches the IFO null measurement in the [5 - 100] mHz frequency range. However, the

low coherence between the frequency noise spectrum and a simultaneous measurement of the IFO resolution indicates that the frequency noise may not be the limitation, but instead that both the OFD and the IFO are impacted by a similar noise source, such as the lab thermal and mechanical environments. This is comforted by the fact that the attempt on stabilising the laser frequency noise using the reading of the OFD to drive the wavelength modulation input of the laser source was unsuccessful.

1.3.2.4 Demodulation noise

An algorithm [86, 113] is used for the demodulation of the real-time interferometric phase. The algorithm includes software correction methods [76, 114, 115, 131] in order to mitigate ellipticity in the signal caused by optical alignment [132], ghost fringes [133] or electronics imbalance. However, errors in the calibration of the gains and ellipse parameters, as well as the nonlinearity of the arctangent demodulation, both lead to error in the demodulation of the displacement which are referred to as the demodulation noise [134]. In addition, the sampling frequency of the acquisition system should be properly selected to avoid losing the counting of the phase.

Non-linearity of the demodulation process

The optical layout of the interferometer allows a long-range (larger than $\lambda/4$) reading of the displacement thanks to a phase-unwrapping algorithm. Such fringe interferometers, often referred to as "fringe counting" interferometers, are however notoriously known for suffering from a nonlinear response [134]. Because of its nonlinear nature, the noise level grows as the sensed motion gets larger, and is usually found to be in the nm-range for most practical applications [135]. The sources of nonlinearity are multiple and may be viewed in two categories: those that arise from the physical properties of the device (such as imperfect polarising properties or geometrical mismeasurement of the optical components [132], electronic gain imbalance, cross-talk between orthogonal polarisation or ghost signals [133]), and those that arise from the demodulation process. The sources from the first category can be mitigated by the ellipse correction algorithm [86, 113]. Indeed, these effects generally translate into quasi-quadrature of the interferometer signals, and thus ellipticity, which has been demonstrated to be effectively compensated in subsection 1.2.3. Note that, while some sources of ellipticity may be constant for a fixed design, such as ellipticity resulting from a poor alignment, some other sources may cause ellipticity that vary in time. This is for example the case of the nonlinearity resulting from the thermal sensitivity of the optical chain (including laser source, fibre and optical assembly), which shows ellipse parameters that slowly fluctuate with time. For this reason, a frequent recalibration of the device is performed. It has been typically observed that recalibrating on a 2-3-day basis is a good practice. On the other hand, the nonlinearity resulting from the demodulation process cannot easily be mitigated. The arctangent operation indeed makes the calculation of the displacement inherently nonlinear. For small displacement of the target in the measuring arm, the arctangent operates within its linear regime, and this noise therefore does

not show up in a null measurement. In order to highlight this noise contribution, the motion of the target needs to be allowed.

The nonlinearity noise of the interferometer can be assessed by using two similar IFO to sense the displacement of two targets submitted to the same motion and extracting the residual between the two sensors with a coherent subtraction. The residual for two linear devices should lie on the resolution of the device (null measurement), any discrepancies between the residual and the null measurement may therefore be attributed to nonlinear noise. Such a setup has been realised at the LIGO-MIT lab, USA, where two pendulums have been mounted on an actuated platform. The pendulums consisted of the mechanics of horizontal inertial sensors, i.e. a Watt's Linkage with a 0.6 Hz natural frequency. More details on the mechanics of the sensors can be found in section 2.2. The target (corner-cube) is attached to the horizontal linkage and the IFO is clamped to the rigid frame so that it measures the horizontal displacement of the pendulum relative to the frame. Two of such assemblies are placed side-by-side, fixed to an actuated platform. The platform moves under ground motion with an RMS motion of roughly 1 μm amplitude in the 100 mHz to 10 Hz bandwidth, measured using a reference sensor on the platform (T240). Various feedback and feedforward control loops can be activated, reducing the RMS motion down to 0.5 μm for a low authority (damping) loop, and to a couple of nm for a higher authority (isolation) loop. The setup is placed in a large vacuum tank pumped down to 10^{-2} mbar to minimise air pressure, acoustic and thermal noise. More details on the setup will be given in section 2.3.

Figure 1.41 shows spectrums of the pendulum motions (plain lines) and residual from a coherent subtraction (dashed lines). The residuals have been measured for three different horizontal motions of the actuated platform. Because the level of nonlinearity scales with the signal level, the RMS value of the interferometer signal is also calculated for the different platform residual motion, in the same 100 mHz - 10 Hz bandwidth. Since the pendulum that the IFO is targeting has a dynamic that filters motion above 0.6 Hz, the sensor RMS is slightly below platform RMS motion. The sensor signal level is 0.3 μm for the open-loop configuration (1 μm platform residual motion), 0.2 μm for the low-authority control configuration (0.5 μm platform residual motion), and 5.5 nm for the high-authority control (2 nm platform residual motion).

The residual of the coherence subtraction of the two pendulums motion for 1 μm and 0.5 μm platform RMS shows a flat white noise spectrum of $1 \text{ nm}/\sqrt{\text{Hz}}$ in the 10 mHz to 10 Hz bandwidth, consistent with the typical nonlinear response of a fringe counting interferometer [135]. As the platform motion decreases to 2 nm RMS, the residual effectively decreases in the 100 mHz to 10 Hz bandwidth, i.e. the bandwidth where active control is effective. The residual reaches some $10^{-10} \text{ m}/\sqrt{\text{Hz}}$ in the [0.1 - 1] Hz band, and sub- $10^{-10} \text{ m}/\sqrt{\text{Hz}}$ above. The coherent subtraction rejects a little more than 1 order of magnitude of coherent signal in the [0.5 - 30] Hz band, meaning that the residual could still be limited to contributions other than a purely nonlinear optical readout. Coupling of the pendulum to the transverse platform motion and tilt, could be a limitation. Below 0.1 Hz, the platform motion is in fact amplified by tilt-couplings in the control loops, which explains the spectrum being larger than in the other measurements with smaller control authority. The isolation

system is limited below 100 mHz most probably by the tilt-horizontal coupling of the sensor/actuator plant, which limits the reduction of the residual in his bandwidth. The residual spectrums for the different RMS platform motions converge toward the interferometer null measurement as the RMS motion lowers. Therefore, the use of these interferometers as sensing elements in high-authority control systems will naturally lead to a reduction of the contribution of this nonlinear noise in the noise budget. However, applications where the IFO is measuring the mass of a free-moving device should consider the nonlinearity of the demodulation process as a limitation of the resolution.

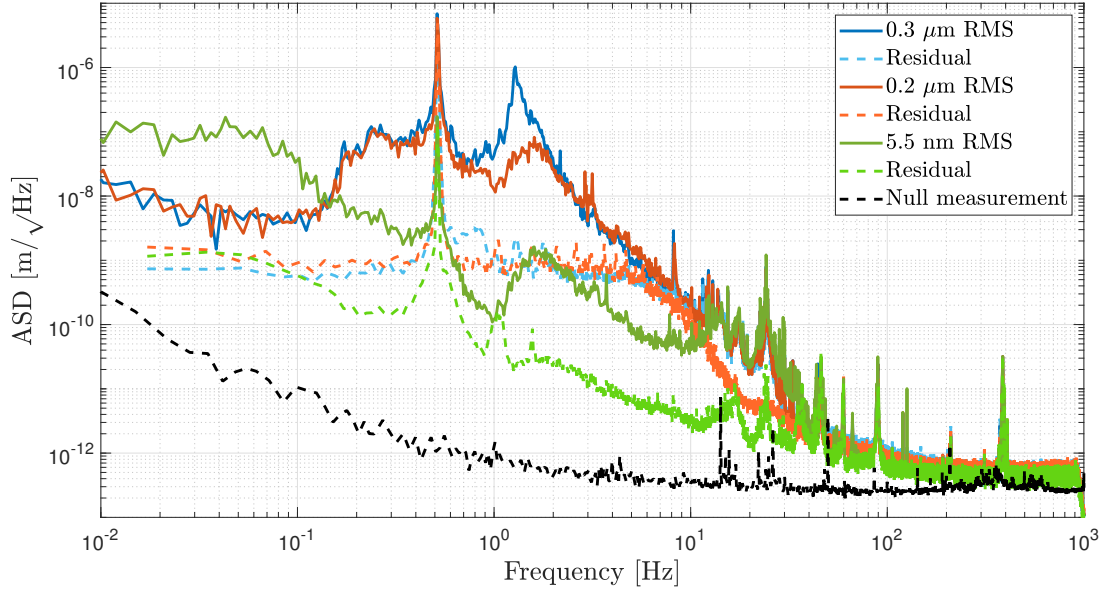


Figure 1.41: Spectrum of the horizontal displacement reading of the interferometer (plain lines) and residual from a coherent subtraction (dashed lines). Spectrum for RMS of the interferometer signal of 0.3 μm , 0.2 μm and 5.5 nm.

Note that there are a couple of factors spoiling the above measurement. First, Watt's Linkages do not guide the target corner cube along a perfect translation, but rather a slightly curved trajectory with an inflection point at the linkage centre position. This mechanical nonlinear response might have a non-negligible contribution to the calculated residual. This is also exacerbated by the fact that the polarising properties of the corner-cube retroreflector vary with the angle of the light beam incident to its face, as discussed above, leading to varying ellipse parameters for the large motion of the target along a curved trajectory. Second, the residuals also include any noises that are uncoherent between the two interferometers. The pendulums and interferometers are placed in a sealed, vacuumed, chamber providing excellent isolation from acoustic, air pressure and temperature noises. The two devices are also powered with the same laser source so laser noise should also be coherent. However, both devices are fed light using a few m-long optical fibres that are out of the vacuum vessel. The optical fibre length might decorrelate laser noise from the two devices, and all pick up temperature and mechanical (vibration) noise along the way, contributing to a non-

negligible laser intensity/frequency noise at sub-Hz frequency (see previous section). All of these external factors will contribute to de-correlate the signals between the two similar pendulum, which will limit the coherent subtraction and lead to an overestimation of the demodulation nonlinear noise.

Ellipse fitting error

The normalisation and ellipse fitting algorithm have been demonstrated in subsection 1.2.3 to efficiently correct the ellipticity induced by optics alignment or signal conditioner gain imbalance, and as a result avoid error in the demodulation of the interferometric phase.

This assumes a perfect identification of the ellipse parameters used in the correction algorithm. In practice, it has been measured that an excellent fitting of the ellipse parameters can be obtained by solving a least-square optimisation problem, provided that the subset of measurement data, i.e. the calibration dataset, covers a decent portion of the ellipse. The residual ellipticity ζ in the quadrature signals after ellipse correction is typically of the order of magnitude of a thousand of a percent: $\zeta = O(10^{-6})$. However, the ellipse parameters have shown dependency on ambient conditions, causing a slow drift of the ellipse shape in time. One notable example is the intensity fluctuation of the laser source caused by fluctuating lab temperature. The lab temperature has been measured to fluctuate by 3°C on a daily basis, leading to noticeable breathing of the ellipse amplitude over the course of a day.

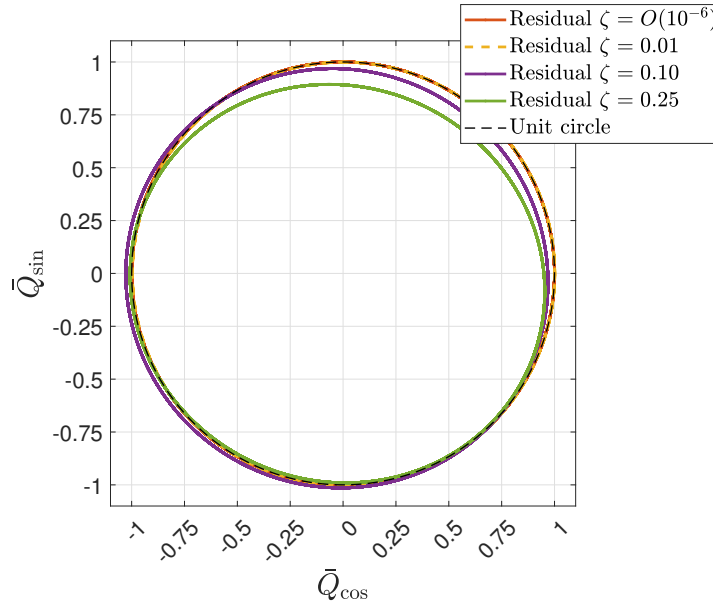


Figure 1.42: Lissajous figures after ellipse correction for an ideal calibration (no residual ellipticity), and for imperfect calibration with 1%, 10% and 25% residual ellipticity.

The IFO MATLAB model is used to characterise the error caused by the residual ellipticity of the

interferometer signals due to imperfect ellipse correction. The model is modified with an imbalance of 1% in the pre-amp gains and offsets of signal conditioner unit and a 2° misalignment of the quarter wave-plate, leading to quasi-quadrature signals with an ellipticity $\zeta \approx 10\%$. An input motion y is fed to the model of the interferometer to output photodiode readings. The ellipse parameters are characterised using a sample of the photodiodes and are stored. The IFO MATLAB model is then modified in a way to further increase the ellipticity of the quasi-quadrature signals, but the output readings are still processed using the first set of ellipse parameters, leaving a residual ellipticity in the signals. Figure 1.42 shows the Lissajous figures of the quadrature signals of the interferometer for the initial calibration and for the subsequent imperfect calibrations. It can be seen that the IFO signals processed with the correct ellipse parameter outputs quasi-quadrature signals $\bar{Q}_{\sin}$ and $\bar{Q}_{\cos}$ with negligible residual ellipticity, $\zeta = O(10^{-6})$. As the quarter wave-plate angle is further misaligned by a couple of degrees in the model, the corrected signals show residuals of ellipticity. Three cases have been simulated: one case with a minor residual ellipticity $\zeta = 1\%$, another case with a significant residual ellipticity $\zeta = 10\%$ and one last case with $\zeta = 25\%$ ellipticity.

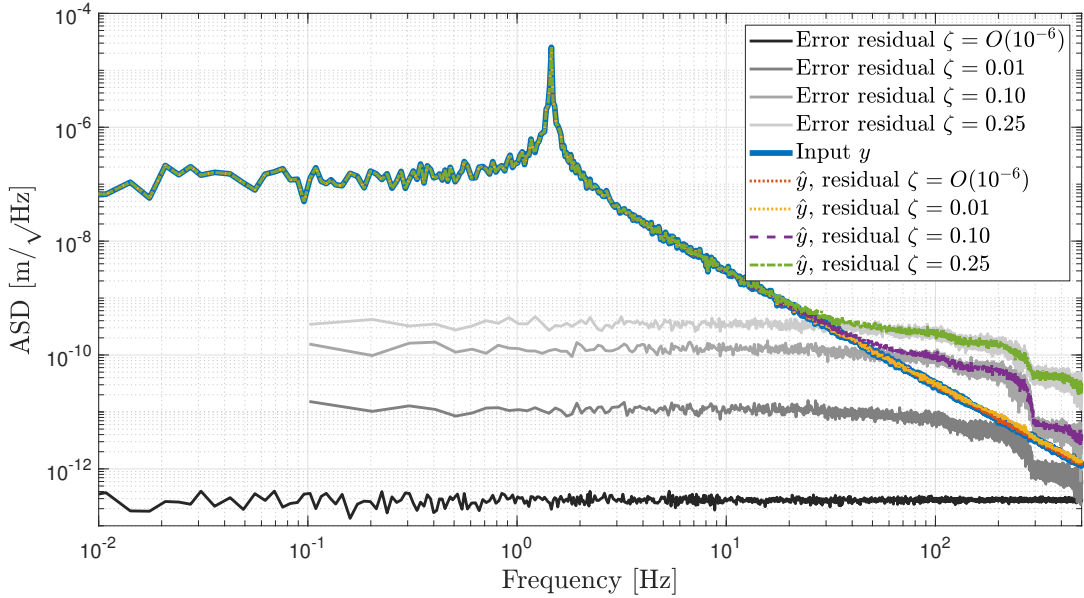


Figure 1.43: Spectrum of the input motion to the IFO model (y) and demodulated displacement $\hat{y}$ for an ideal calibration (no residual ellipticity), and for imperfect calibration with 1%, 10% and 25% residual ellipticity (coloured traces). Spectrum of the error from a coherent subtraction between the input y and the different demodulated signals $\hat{y}$ (black-and-white traces).

The Lissajous figures are then demodulated to calculate the displacement $\hat{y}$ read by the optical device. Figure 1.43 shows spectrums of the input y fed to the model, and of the demodulated $\hat{y}$ after ellipse correction. The displacement $\hat{y}$ demodulated from the well-calibrated Lissajous figure with no residual ellipticity shows a perfect match with the input y . As the residual ellipticity ζ increases, an error in the demodulation shows as a broadband noise dominating the signals at higher frequencies.

This is particularly visible for the cases of residual $\zeta = 10\%$ and residual $\zeta = 25\%$, where the output $\hat{y}$ appears sitting on white noise signals at frequencies >60 Hz and >30 Hz, respectively. The noise level associated with this demodulation error is calculated as the residual from the coherent subtraction between the input signal y and the demodulated signals $\hat{y}$. The demodulation noises are superimposed in Figure 1.43 shown as the black-and-white lines. The noise level related to the demodulation with no residual ellipticity (black trace) lies on the data acquisition chain noise, which has been measured at $2 \cdot 10^{-13}$ m/ $\sqrt{\text{Hz}}$ and has been included in the model. The residual for the other cases however predicts demodulation noise of $1 \cdot 10^{-11}$ m/ $\sqrt{\text{Hz}}$, $1.2 \cdot 10^{-10}$ m/ $\sqrt{\text{Hz}}$ and $3.8 \cdot 10^{-10}$ m/ $\sqrt{\text{Hz}}$ for residual ellipticities of $\zeta = 1\%$, $\zeta = 10\%$ and $\zeta = 25\%$ respectively. The demodulation noise therefore appears as a broadband white noise whose amplitude increases with the residual ellipticity, and that can be nulled with a proper calibration of the ellipse parameters.

Given the operating environment of the interferometer, 1% is the typical residual ellipticity that can be observed in the IFO quasi-quadrature signals for calibration parameters older than a day. This is mostly caused by: the release of mechanical stresses in the opto-mechanical assembly, thermal dilatation and temperature sensitivity of the optical chain (laser source and fibres). This implies that the resolution of the optical readout can be expected to degrade from $2 \cdot 10^{-13}$ m/ $\sqrt{\text{Hz}}$ to a couple of nm/ $\sqrt{\text{Hz}}$ over the course of a day if the calibration parameters are not re-assessed. A re-calibration on a periodic daily basis is recommended for guaranteeing optimal readout resolution.

Sampling frequency

In phase unwrapping interferometer, there exists a direct relationship between the ADC sampling frequency and the maximum target velocity which can be demodulated [76, 136, 137]. Indeed, for a Michelson-like interferometer using a double pass in the measuring arm, one full fringe corresponds to a displacement of $\lambda/4$, with λ the laser wavelength. Therefore, as the target moves with a velocity v , the fringe rate f_{IFO} writes

$$f_{\text{IFO}} = \frac{4}{\lambda}v \quad (1.62)$$

To avoid aliasing, the sampling frequency F_s should be at least twice the maximum expected fringe frequency, so that

$$F_s \geq \frac{8}{\lambda}v_{\text{max}} \quad (1.63)$$

The reciprocal of this is that the highest velocity that can be demodulated is $v_{\text{max}} = (\lambda/8)F_s$. Any motion of the target faster than this value implies a change in interferometric phase greater than a quadrant of the Lissajous figure, and the counting of the phase it lost. The optical readout uses $\lambda = 1550$ nm wavelength laser and is typically recorded at 10 kHz, leading to a maximum velocity of approximately 2 mm/s. This is assumed largely sufficient for its application in active

seismic isolation, where maximum ground motion is typically in the order of magnitude of the $\mu\text{m/s}$ at the micro-seismic peak (150 mHz) [104, 138, 139].

Demodulation noise budget

The algorithm used for demodulating the interferometer signals into a displacement value can be the source of nonlinearities or systematic errors, referred to as the demodulation noise. The nonlinearity of the demodulation process, a poor calibration of the ellipse correction parameters and the sampling rate have all been investigated as main sources of demodulation noise.

First, the ellipse correction algorithm implemented in the demodulation process has proven successful in mitigating errors from an imperfect quadrature of the interferometer signals due to optical or electrical properties. The arctangent operation used for demodulating the interferometer phase downstream of the ellipse correction however makes the optical readout an inherent nonlinear device. A measurement of the residual from the coherent subtraction of two similar IFO devices demonstrated a broadband nonlinear noise typically at a level $1 \text{ nm}/\sqrt{\text{Hz}}$, in operating condition when measuring μm -scale motions. This is typically the order of magnitude for the nonlinear response of fringe counting interferometers [134, 135], even though other sources of nonlinearity also inevitably couple in the measurement. This noise however lowers as the level of measured motion decreases, and has been measured as low as $1 \cdot 10^{-11} \text{ m}/\sqrt{\text{Hz}}$ at frequency above 1 Hz, for targets motions of a couple of nm RMS. Therefore, for the application of the optical device in active seismic isolation, it can be expected that the nonlinear demodulation noise will converge to the null measurement floor, as the payload motion decreases, and this contribution should not be a limitation in the noise budget. For other applications where the optical device is shooting a free-moving target, a nonlinear noise may be non-negligible.

Next, the demodulation error caused by a sub-optimal ellipse correction has been investigated from the MATLAB model of the interferometer. Calculations showed that residual ellipticities in the quasi-quadrature signals translate into broadband white noise. The demodulation noise is estimated at $1 \cdot 10^{-11} \text{ m}/\sqrt{\text{Hz}}$, $1.2 \cdot 10^{-10} \text{ m}/\sqrt{\text{Hz}}$ and $3.8 \cdot 10^{-10} \text{ m}/\sqrt{\text{Hz}}$ for residual ellipticities of $\zeta = 1 \%$, $\zeta = 10 \%$ and $\zeta = 25 \%$, and is negligible in the case of a good calibration with no residual ellipticity. Practice showed that the ellipse parameters drift over time mostly due to stress release in the opto-mechanics, thermal dilatation and temperature sensitivity of the optical chain (laser source and fibres). Over the course of a day, it can be expected that the calibration parameters might be off and cause of 1% residual ellipticity in the signal, leading to a $10 \text{ pm}/\sqrt{\text{Hz}}$ noise level. Re-calibrating on a daily basis, or in any case after any manipulation of the opto-mechanics, allows to mitigate this demodulation noise so that it does not become a limitation.

Finally, the acquisition chain is run at a typically 10 to 50 kHz sampling rate, due to the limitation of the recorder to perform real-time processing on a number of input channels. The sampling rate however imposes an upper limit to the velocity of the target that can be measured. Indeed, if the target moves by more than half a fringe between two successive samples, the phase counting is lost

and the demodulation will issue a massive error. Calculations for a sampling rate of 10 kHz yielded a maximum velocity rating of 2 mm/s. This is assumed far sufficient for seismic activity monitoring, since ground motion is typically in the μm -range at the microseismic peak [104, 138, 139].

1.3.2.5 Environmental noise

A number of environmental factors may interact with the optical readout device to produce unwanted signals. Notorious examples are: (i) temperature fluctuations, which couple with the laser source noise due to the laser cavity temperature sensitivity, the optical fibre temperature sensitivity and thermal dilatation in the optomechanical assembly. (ii) Ambient light, which also obviously spoils the light power signals read by the photodiodes. (iii) Scattered light caused by unwanted reflections of the laser beam leads to spurious, or ghost interferences. (iv) Environmental air conditions also affect the reading through changes in the medium refractive index with pressure and relative humidity fluctuations, and through acoustic noise. All of these lead to unwanted signals or periodic errors which fall under the umbrella of environmental or external noise, as they do not originate neither from the optical system, nor from the acquisition chain nor from the demodulation process. Some can be effectively mitigated with hardware solutions. A careful design can for instance mitigate ambient light contribution and air fluctuation by enclosing the optical assembly, and acoustic noise and humidity fluctuation can be avoided by operating the system in a vacuum vessel. More recently, software solutions have also been found for compensating other periodic noise sources that can be complex or not practical to isolate. Online Wiener filtering has been applied in the context of Gravitational-Wave detection to suppress vibration and temperature noise in suspended interferometers [140, 141]. Particle swarm methods and other learning algorithms have also been used for compensating refractive index changes [142, 143]. These methods rely on the use of a set of sensors, often named Environmental Compensation Unit (ECU), to measure relevant environmental conditions and apply an online or offline correction.

Below are discussed noise and periodic errors caused by ambient light, ghost interference, temperature fluctuation and air fluctuation.

Ambient light

It has been measured that ambient light can heavily spoil the optical power measurement of the photodiodes in free-air optical experiments, and dominate the spectrum at frequencies below 10 Hz [46]. The interferometric device can however effectively be shielded from ambient light with the use of a casing around the optical assembly. The optical assembly made by EXail is enclosed in a box which only allows a small opening aperture for the laser beam to shoot the target. This massively decreases the amount of ambient light that can reach the photodiode active area. On top of that, for the main application of these optical devices as sensing elements in inertial sensors, the mechanics of the inertial sensors also include a box cover that adds another layer of protection from ambient light.

Ghost interference

Enclosing the optical assembly is very effective in rejecting ambient light, but it comes with the side effect of an increase of light scattering. Scattered, or stray, lights are unwanted reflections of the laser beam on surfaces (such as the enclosure walls), or even back-reflections on surfaces of the optical elements themselves. The stray light beams may end up aligned with the "main" interferometer and cause spurious interferences, often named ghost interferences [144–146]. These interferences lead to quasi-quadrature or complex mixing and end up contributing to a nonlinear noise in the readout. Ghost reflections can also lead to nonlinearity errors [133]. Because ghost interferences do not translate into simple ellipticity, correcting them with a software solution is not an easy task. For example, Figure 1.44 shows the effects of ghost interferences on the Lissajous figure of the raw (not corrected) quadrature signals of the interferometer. The plots are obtained by injecting scattered light in the MATLAB model of the interferometer, as it will be explained below. The amplitude of the scattered light is vastly exaggerated for visualisation purposes. It can be seen that the Lissajous figure is distorted into a non-elliptical shape, making the ellipse correction algorithm inefficient. Effective solutions against light scattering consist of physically avoiding unwanted reflections: curving or inclining the nearby walls to avoid reflection in the plane of the interferometer, texturing the wall to promote diffuse rather than specular reflections or using black painting to absorb light. If a part of the main beam is reflected in a way that it leaves the interferometer path, a beam dump can be used to successfully avoid any further undesired scattering.

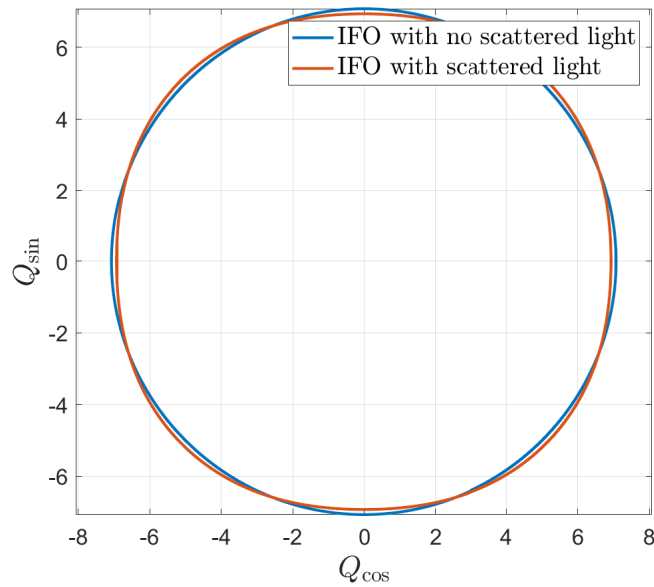


Figure 1.44: Example of the raw Lissajous figure (no ellipse correction) of an IFO with no scattered light model (blue trace) and with a scattered light model (orange trace). The effect of scattered light are vastly exaggerated for visualisation.

Most paintings are often not compatible with vacuum operation, and machining angled or curved walls is expensive. Therefore, the enclosure of the optical interferometer designed by EXail is machined with a rough, textured, surface that avoids specular reflection. Despite this, ghost interferences can not be fully avoided and they have been measured to contribute to the noise budget with a nonlinear noise level of 10^{-11} m/ $\sqrt{\text{Hz}}$.

Predicting ghost interferences can be very difficult as it implies a modelling of scattered and stray light reflections on mechanical surfaces using an optomechanical simulation tool such as Ansys Zemax. Alternatively, one fairly simple way to highlight experimentally evidence of ghost interferences is to block the optical path to the reference arm of the optical device and monitor the photodiode signals. In this case, any interferences captured by the photodiodes necessarily result from interactions between the main laser reflected from the measuring arm and scattered light beams. Because the ghost interferences only show if there is a moving target along the measuring arm, the optical readout is mounted to target a corner cube attached to the bulb of a vertical pendulum. Figure 1.45 shows a sample of the time series of the photodiodes reading voltages for the main IFO, i.e. both arm or "open", and for the ghost IFO, i.e. when the reference arm is blocked with an angled textured black tissue. The main IFO (coloured traces) shows fringes with an amplitude of approximately 1.4 V whereas the ghost IFO (black-and-white traces) shows fringes with an amplitude of roughly 0.01 V. The ghost interferences are therefore measured to contribute to the main IFO with an amplitude of 0.7% of the main fringe amplitude.

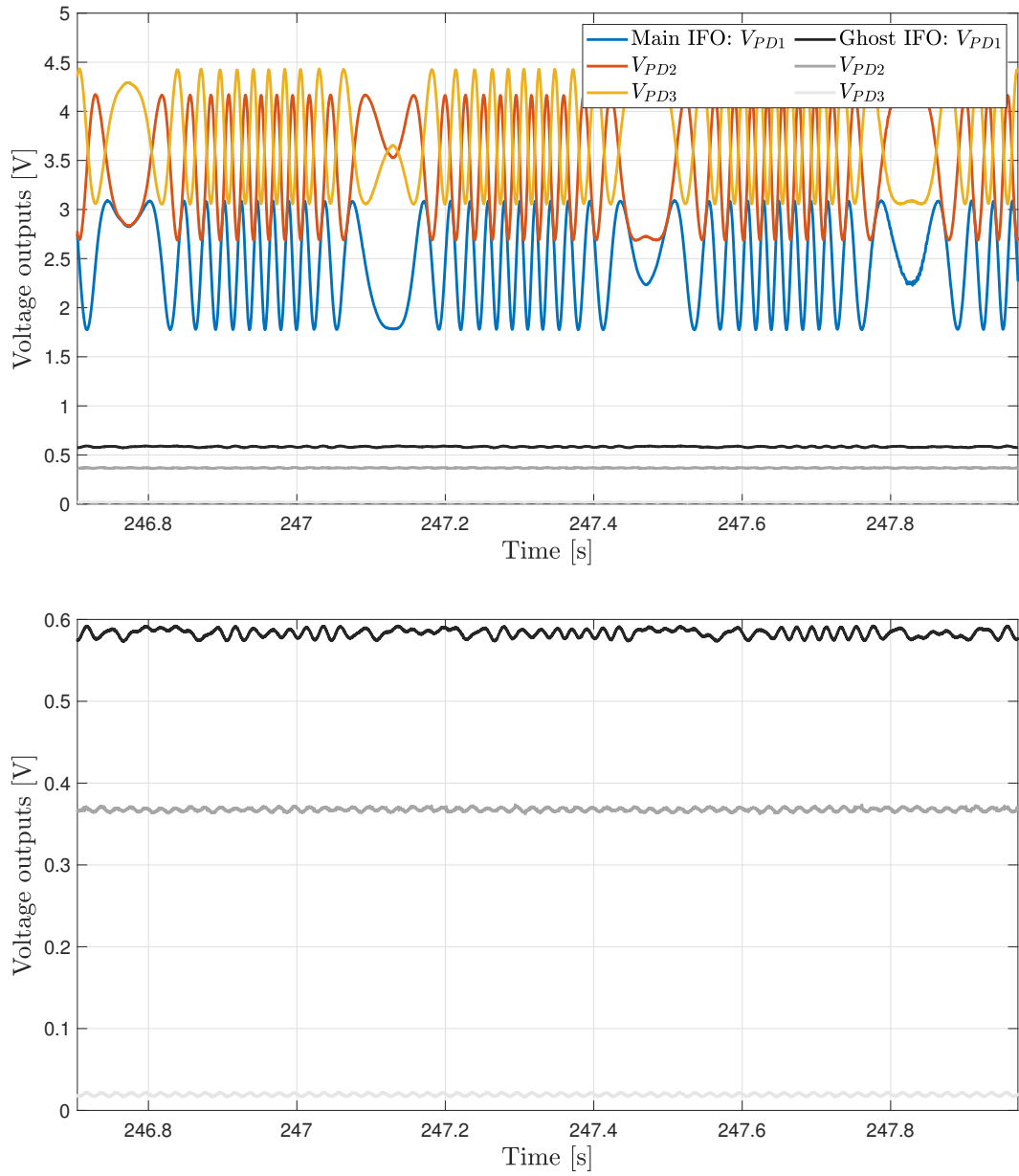


Figure 1.45: Time series of the voltage outputs of photodiodes 1 to 3. (coloured traces) Main interferences. (black-and-white traces) ghost interferences. (Bottom) Zoom-in on the ghost interferences.

This parameter is used to add the contribution of scattered light to the MATLAB model of the interferometer. The ghost interferences are modelled as output power signals, as written in Equation 1.18, whose amplitude equals 0.7% of the main interferometer fringe amplitude. These are then added on top of the main interferences, resulting from the measuring arm and the reference arm. Figure 1.46 shows spectrums of the motion measured by the optical readout considering the main interferometer and the ghost interferometer. The blue trace shows the free motion of the

vertical pendulum with a 1.5 Hz resonance frequency clamped on an optical bench, calculated by demodulating the signal of the main interferometer (main IFO and ghost fringes). The orange trace shows the contribution of the ghost fringes obtained by demodulating the voltage reading measured when the optical path is blocked in the reference arm so that only the ghost fringes are measured. This measurement shows that ghost fringes contribute to the noise budget with a nonlinear noise level of $10^{-11} \text{ m}/\sqrt{\text{Hz}}$. Superimposed in the figure is the spectrum of the modelled ghost fringes (dashed yellow trace), in good agreement with the experimental measurement.

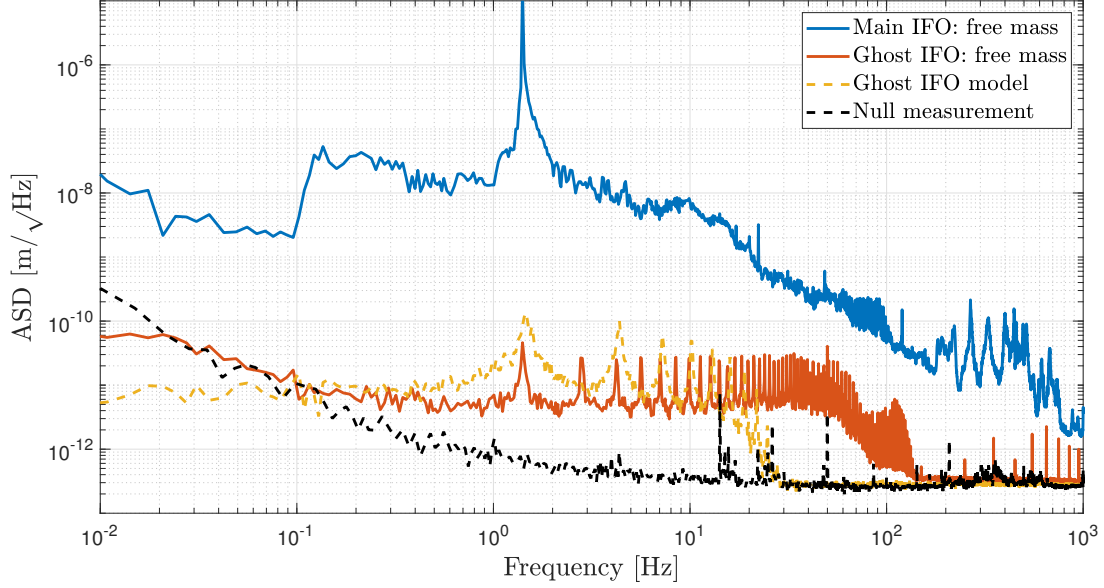


Figure 1.46: Spectrums of the displacement of the free vertical pendulum measured with the optical readout. Using the main interferometer (blue trace), and with the ghost interferometers only (orange trace). (yellow) Model of the ghost interferences inferred from the measurement, with scattered amplitude of 0.6% of the main IFO amplitude. (black) IFO resolution, measured with a null measurement.

Because the ghost interference amplitudes are proportional to the target motion along the measuring arm, its noise contribution is expected to scale down with a decreasing motion amplitude. Figure 1.47 shows spectrums of the ghost IFO noise calculated for various amplitude of motion in the measuring arm. They are generated by injecting an input motion in the MATLAB model with the same order of magnitude as in Figure 1.46 (blue trace), and then decreasing the amplitude by a factor 10 and 100 successively (orange and yellow traces), and finally inputting a null motion (black trace). The model shows that the ghost interferometer noise decreases all the way to the null noise floor. When the pendulum motion is 100 times less than its response to ground motion, the ghost IFO noise is roughly equivalent to the optical readout resolution.

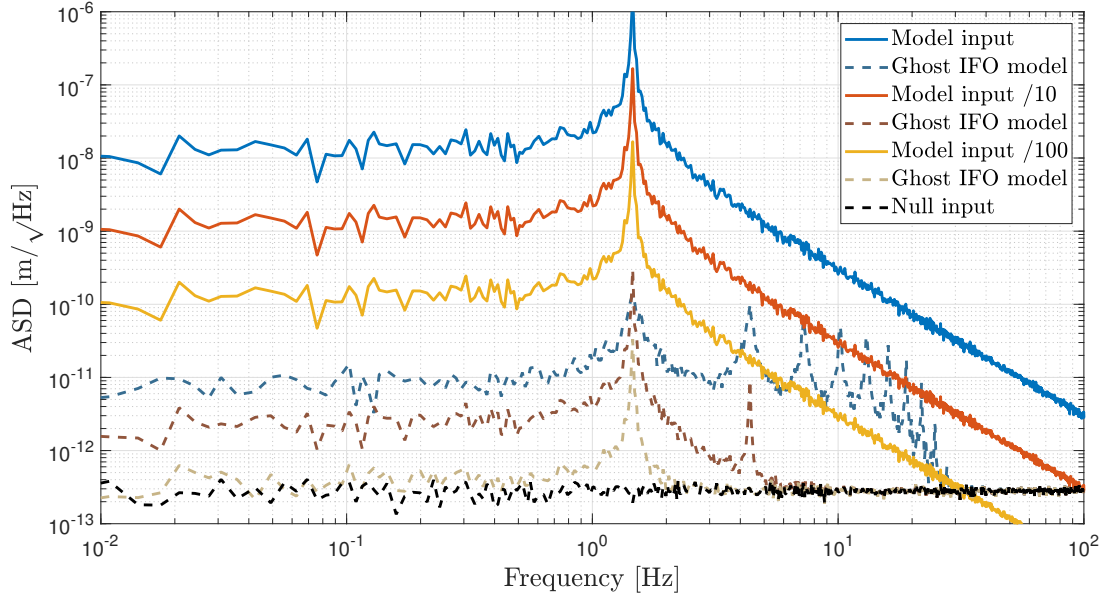


Figure 1.47: Spectrums of the modelled ghost IFO for a decreasing amplitude of the target motion.

In summary, the textured walls of the optical device have demonstrated a fairly good capability in reducing scattered light. The ghost IFO noise resulting from the light scattering has been measured to contribute to the noise budget with a nonlinear noise level of $10^{-11} \text{ m}/\sqrt{\text{Hz}}$ under target motion with a μm motion amplitude. However, since it decreases as the motion along the measuring arm decreases, the noise contribution will converge to the null measurement noise floor when the device is used in active seismic isolation systems.

Temperature fluctuation

Long-term temperature drifts are detrimental to optical setups in numerous ways and can be a severe limitation in optical experiments in the very low-frequency regime, typically sub-100 mHz. Laser cavities in laser sources are notoriously known for being sensitive to ambient temperature [122, 128–130], leading to laser frequency and intensity noise as discussed above. Similarly, optical fibres may experience power loss and polarisation change caused by thermal stresses, dilatation and refractive index changes of the fibre core under temperature fluctuations. Thermal dilatation of the optical assembly leads to misalignment, and thermal stresses might impact the optical properties of various elements, notably for mounted wave plates or polarising splitters [147]. the optomechanical assembly is particularly at risk, due to the assembly of materials with different thermal expansion coefficients.

The temperature sensitivity of the optical assembly is assumed to be low thanks to the monolithic assembly of all components on a glass breadboard, minimising thermal stresses and asymmetric

dilatations. In addition, the optomechanical assembly will operate in a vacuum, where thermal fluctuation is limited. Only the laser source and fibres feeding the optical readouts are left operating in normal lab conditions. As noticed previously, the temperature in a non-thermatised lab fluctuates by 2 to 3° over the course of a night, and a few degrees more during daytime. This is expected to bring noticeable noise in the mHz frequency range. Unfortunately, no measurements could accurately correlate a null measurement with a temperature measurement due to the lack of high-resolution, low-frequency, temperature sensors. Operating the laser source in a box thermatised using active and/or passive temperature control can help attenuate the temperature sensitivity of the setup. The sensitivity of the optical fibre, however, would require a temperature control of the entire lab.

Air fluctuation

It is often acknowledged that operating an optical experiment in a vacuum environment reduces air refractive index noise and acoustic noise. Since the optical readouts will ultimately be operated in a vacuum, they have been tested in vacuum conditions. The setup for testing the in-vacuum test is shown in Figure 1.48. An optical readout is mounted in a null measurement configuration on a breadboard, and placed in a Pfeiffer high vacuum chamber. Vacuum is made using a Leybold Turbolab 250i system, which is a tabletop system using a turbomolecular pump (250 l/s) backed up by a scroll pump to create a high-vacuum in chambers of small to medium volumes. Because the clamping mechanism used in the null configuration is not vacuum compatible, the corner cube in the measuring arm is mounted on a Thorlabs kinematic mount fixed to a breadboard, on which the IFO is also screwed.

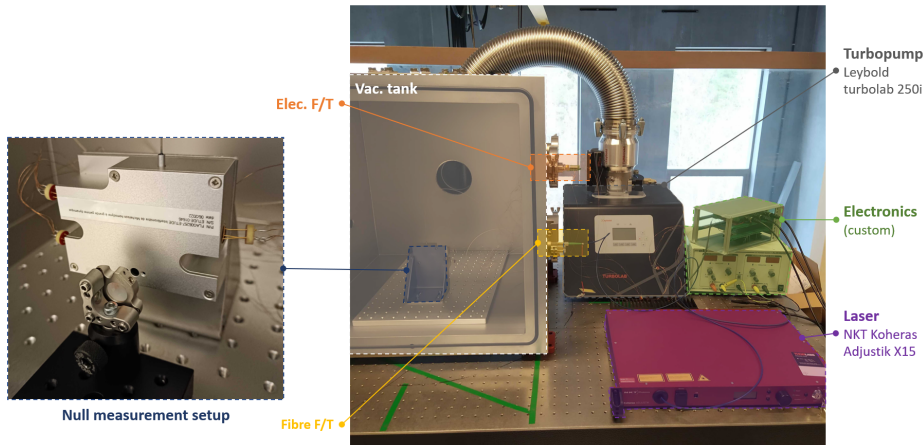


Figure 1.48: Setup of the optical readout null measurement in a vacuum environment.

Figure 1.49 shows spectrums and time series of the null measurement in the vacuum chamber. They consist of 10 min-long measurements recorded at 10 kHz. The 1 bar measurement was taken with the chamber closed before running the vacuum system. The 1 mbar measurement was taken during the pump down. The 10^{-3} mbar measurement was taken with the vacuum pump off. The

vacuum system reaches a level of 10^{-6} mbar in a couple of hours and has been left running a couple of extra days to vacuum away outgassing. When turning off the vacuum pumps, the pressure slowly rises and stabilises at a level of $1 \cdot 10^{-3}$ mbar due to residual outgas. The 10^{-3} mbar measurement corresponds to a null measurement taken in this stable condition. The spectrum of for the ambient pressure (1 bar) and vacuum condition (10^{-3} mbar) do not show a significant difference, especially at a frequency of 5 Hz and below. Above 5 Hz, the setup captures the vibrations of the mechanical mounts of the corner cube and optical device. The measurement made during the pump down (1 mbar) shows a larger spectrum at 10 Hz and below, caused by the slow mechanical drift in the setup caused by the decrease of the Buoyancy force as the air is pumped out, which also shows in the time series of that measurement. The resolution spectrum of the IFO in null configuration measured in the thermalised box (black trace) shows a similar noise level below 5 Hz, showing that the device is not limited by air conditions.

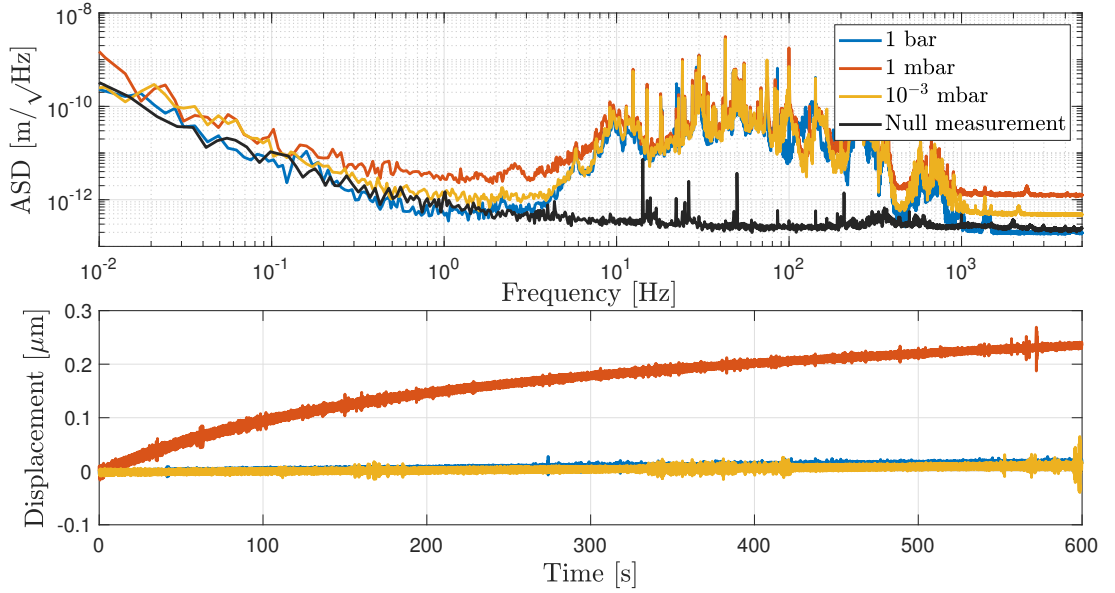


Figure 1.49: Spectrums and time series of optical readout resolution measured at various vacuum levels. The null measurement in the thermal box (black) is shown as a reference.

Environmental noise budget

Ambient light, scattered light, temperature and air pressure have been investigated as sources of external noise.

The design of the enclosure of the optical device has been proven to successfully shield the interferometer from ambient light and air pressure. The textured and rough walls of the enclosure promote diffusive scattering of light, leading to ghost interferences that have been measured to contribute to only 0.6% of the main interferometer fringes. This has been measured to contribute

to the noise budget with a nonlinear noise level of $1 \cdot 10^{-11} \text{ m}/\sqrt{\text{Hz}}$ for a target motion at a μm -scale motion. Temperature sensitivity could unfortunately not be quantified. A high-resolution, low-frequency, temperature sensor coupled with a temperature-controlled environment could help assess the contribution of temperature fluctuation to the readout noise.

1.3.3 Total noise budget

Figure 1.50 shows the resolution of the IFO measured in the null configuration in the setup described in subsection 1.3.1. The black trace shows the resolution measured with the Centaur recorder and the grey trace shows the resolution measured with the MicroLab Box real-time target. Superimposed in the plot are measurements of various noise sources that have been shown to contribute to the noise budget (coloured traces). They are the acquisition chain noise, including photodiode noises (Thorlabs FGA21 dark current and shot noise @ nominal 4mW laser power), signal conditioner noise (thermoelectrical, $1/f$ Flicker and Op-amp) and the MicroLab Box/Centaur acquisition noise (discretisation and electrical); and the laser frequency noise with an assumed optical path length difference ΔL_0 of 4 cm. It can be seen that the main limitation in the resolution of the IFO when using the MicroLab Box recorder is the ADC noise at frequencies of 4 Hz and above, setting a noise limit at $2 \cdot 10^{-13} \text{ m}/\sqrt{\text{Hz}}$ above 10 Hz. When using the Centaur recorder, the ADC noise is low enough for the optical device to be shot noise-limited at $2 \cdot 10^{-14} \text{ m}/\sqrt{\text{Hz}}$ in the entire 10 mHz - 1000 Hz frequency band. The IFO null measurement however is measured at $8 \cdot 10^{-14} \text{ m}/\sqrt{\text{Hz}}$ at 100 Hz above, caused by a sub-optimal alignment in this test ($\approx 70\%$ fringe visibility) and larger uncertainties in the calibration parameters. Indeed, since calibration and demodulation cannot be run online using this digitiser, a calibration data need to be recorded before-hand and stored for post-processing. The frequency range from 0.1 Hz to 60 Hz is probably dominated by residual motion. At sub-100 mHz frequencies, the null measurement spectrums match with a measurement of the laser source (NKT Koheras X15) frequency noise spectrum, measured with a Silentsys Optical Frequency Discriminator (OFD). However, a simultaneous measurement of the IFO in the null configuration and the laser frequency noise showed a low coherence in the same frequency range, indicating that the low-frequency noise cannot with certainty be attributed to frequency noise. A possible explanation could be that an unmeasured external noise source could be limiting both the optical readout and the OFD (since the OFD is also, at its core, an interferometer).

Other noise contributions not plotted in the figure have been shown to either have a negligible contribution or can be mitigated with a careful alignment and operation of the interferometer. The polarisation error induced by the properties of the TIR corner cube has been shown to be minimal using a double-pass in the corner cube and shooting at the appropriate sextant. A careful alignment of the retroreflector with good fringe visibility (contrast) value also leads to a very minor increase in the acquisition noise. It has been measured experimentally that acceptable contrast values (> 0.7) can be obtained within a range of 0.2 mm around the nominal position of the retroreflector. Laser intensity noise is successfully managed in the demodulation process and does not lead to a measurable noise. Ellipse fitting errors can similarly be avoided by correctly calibrating the IFO,

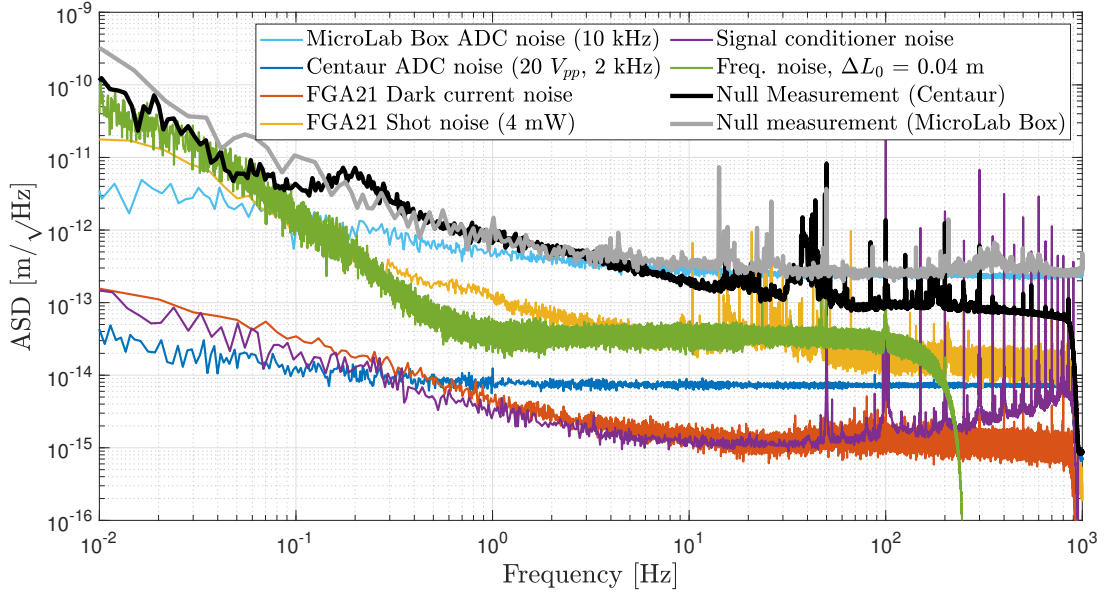


Figure 1.50: Global noise budget of the IFO in null configuration in the thematised box. (Black line) Resolution spectrum measured with the Centaur digitiser. (Grey line) Resolution spectrum measured with the MicroLab Box real-time target. (coloured lines) Individual noise contributions from the signal acquisition and optical chain measured individually on separate setups.

and recalibrating the ellipse parameters on a daily basis, avoiding the residual ellipticity in the quadrature signals to exceed 1%. The sampling frequency, typically 10 kHz, has been calculated to be sufficient for the demodulation algorithm not to lose fringe counting for most applications of the IFO. The optical readout design has also been shown to be immune to ambient light and air fluctuations thanks to the enclosure of the optical assembly.

The optical readout has also been shown to be subjected to various nonlinear noise sources. Because of their nonlinear nature, these noises only bring a sensible limitation when the optical readout shoots a moving target. Figure 1.51 shows the contribution of the nonlinear noise sources to the IFO noise budget. The blue trace shows a typical motion of the target on the measuring arm of 1 μm , measured when the corner cube is mounted on a simple pendulum. The nonlinearity of the demodulation process, vastly attributed to the arctangent operation (orange trace), shows a nonlinear noise contribution with a 1 $\text{nm}/\sqrt{\text{Hz}}$ level for the typical motion. The ghost interferences caused by stray and scattered light have been measured to contribute with a 10 $\text{pm}/\sqrt{\text{Hz}}$ nonlinear noise level, again for a similar μm -scale motion of the target. Because the main application for the optical readout is to be used as a sensing element in isolation systems, it is expected that the nonlinear noise contributions will decrease as the level of residual motion of the isolated payload decreases, until they reach the null (locked) noise floor. Note that the measurement of the nonlinearity of the demodulation process is certainly overestimated since it was calculated as the residual motion of two similar pendulums placed in the same environment after a coherent subtraction. However, external

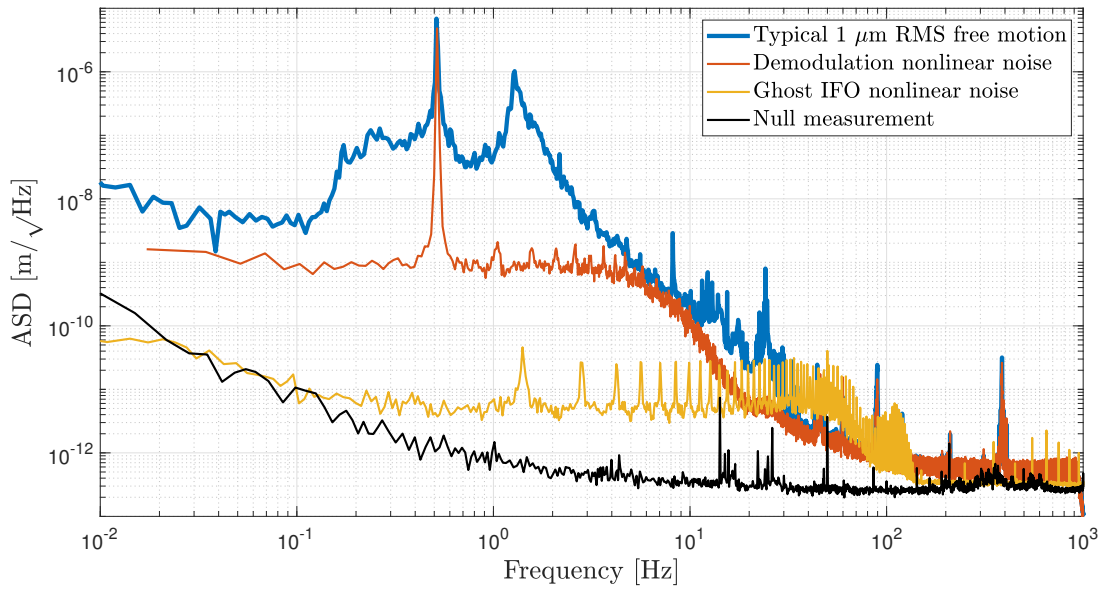


Figure 1.51: Global noise budget of the IFO in a free configuration. Typical μm -scale motion of a target in the measuring arm (blue trace). Nonlinear noises measured when the optical readout operates on a free moving target (orange and yellow traces). Resolution of the IFO on a locked target (black trace).

factors, such as temperature and vibration sensitivity of the optical fibres, will contribute to de-correlate the signals between the two similar pendulums, which will limit the coherent subtraction.

1.4 Conclusion and future work

This chapter presented the design of a sub-pm resolution, vacuum-compatible, optical readout device based on a homodyne quadrature Michelson interferometry [46, 72, 74–76, 83, 108].

The optical layout has been presented in subsection 1.2.1. Modelling of the layout based on Jones calculations has been performed in subsection 1.2.2, and has been used to derive its demodulation process for calculating the displacement along the measuring arm based on a recording of the photodiode readings. The demodulation process is exposed in subsection 1.2.3 and uses ellipse fitting [115] and online ellipse correction [46, 76, 113, 114] algorithms for mitigating most of the nonlinearities issued from the optical systems, such as misalignment, components imperfections or electronics imbalance. The assembly of the optical device by eXail and the custom-made signals acquisition chains are presented in subsection 1.2.4.

An experimental validation and calibration of the optical device has been presented in section 1.3. A measurement of the resolution of the IFO by mechanically blocking the measuring arm of the device showed a resolution of $2 \cdot 10^{-13} \text{ m}/\sqrt{\text{Hz}}$ at frequencies above 10 Hz when connected to a real-time target, or $8 \cdot 10^{-14} \text{ m}/\sqrt{\text{Hz}}$ using a high-resolution recorder. A characterisation of various noise sources has been extensively presented in subsection 1.3.2. The noise contributions of the entire signal acquisition chain have been experimentally measured on separate setups [86, 117–122, 124], leaving the real-time target ADC noise as the major contribution. Noise sources from the optical chain have similarly been assessed. Most of these have been seen as either having a negligible contribution or can be null-ed with a careful alignment and operation of the IFO. Polarising properties of corner-cube [110, 111] are mitigated with the double-pass in the retroreflector, and can be avoided by targeting one of the appropriate sextants. Misalignment of the optics is also found to have a very minimal impact thanks to the ellipse correction algorithm, and the precise alignment of the optical components by eXail. The alignment of the corner cube can be managed to an acceptable contrast (> 0.70) within a 0.2 mm range from the nominal position, which leads to a minor increase in the noise level. Laser intensity noise [128] has been found correctly cancelled in the demodulation process. Laser frequency noise [122, 129, 130] has been thoroughly investigated as potentially a major contribution at sub-1 Hz frequencies. While the null measurement spectrum matches with a measurement of the laser source frequency noise, a low coherence between both measurements in the same frequency range indicates that the low-frequency noise cannot with certainty be attributed to frequency noise. The demodulation process has been investigated as a source of nonlinear noise [134, 136, 137]. It was found that the arctangent operation may be responsible for a nonlinear noise with a $1 \text{ nm}/\sqrt{\text{Hz}}$ level for typical target motion of 1 μm amplitude. Error due to poor ellipse fitting can also be significant but can be simply mitigated by recalibrating the ellipse parameter on a daily basis, avoiding the residual ellipticity in the quasi-quadrature signals to exceed 1%. The real-time recorder sampling frequency, typically 10 - 50 kHz, has been calculated sufficient for the arctangent operation to not fail and lose the counting of the fringes. Lastly, various environmental noise sources have been investigated. The eXail enclosure has been demonstrated successful in rejecting ambient light and air current noise. Ghost fringes [144–146] caused scattered light have been measured to

have a limited impact on the resolution due to the rough surface of the enclosure, and only contribute with a $10 \text{ pm}/\sqrt{\text{Hz}}$ nonlinear noise level. Temperature noise [122, 128–130], however, is expected to contribute significantly to the noise budget.

The global noise budget has been laid out in subsection 1.3.3. The resolution of the optical readout was measured as $10^{-10} \text{ m}/\sqrt{\text{Hz}}$ @ 10 mHz, $10^{-11} \text{ m}/\sqrt{\text{Hz}}$ @ 100 mHz, $10^{-12} \text{ m}/\sqrt{\text{Hz}}$ @ 1 Hz and $2 \cdot 10^{-3} \text{ m}/\sqrt{\text{Hz}}$ @ 10 Hz and above. The noise hunting showed that the 1 Hz and above frequencies are dominated by the signal acquisition chain noise, especially by the real-time target ADC noise, and the sub-1 Hz frequencies may be limited by the laser frequency noise. When the IFO shoots at a free-moving target, the demodulation and the ghost fringes nonlinear noises have been estimated to contribute with noise levels of $1 \text{ nm}/\sqrt{\text{Hz}}$ and $10 \text{ pm}/\sqrt{\text{Hz}}$, respectively, for motion that is in the μm -range. Because they are both nonlinear noises, their noise floors will decrease in the application of the optical device in isolation control loops.

Further work is needed for a better temperature sensitivity calibration of the optical readout and laser source. A temperature-controlled enclosure, equipped with a high-resolution and low-frequency temperature sensor should help in measuring the temperature sensitivity of the setup.

Chapter 2

Optical seismometers for active isolation in GWD

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Gravitational wave detectors (GWDs) are high-end scientific instruments with unprecedented resolution, aiming to capture ripples in the space-time fabric with strain levels upon reaching the Earth as small as $1 \cdot 10^{-21} \text{ } 1/\sqrt{\text{Hz}}$ [11]. For a detector with a 1 km-long basis, this would correspond to a distortion of about 10^{-18} m , which is a thousand times smaller than a proton diameter. Interferometric GWDs is currently the most promising technology and has demonstrated to meet the requirements of GW detection with sensitivities of $10^{-20} \text{ } 1/\sqrt{\text{Hz}}$ in the [0.1 - 1000] Hz band [148, 149], resulting in the first detection of a binary black hole merger in 2015 [10]. This observation has been made possible by optimising every component of the detector with state-of-the-art performance: from the control electronics, the laser and optical system and detector mechanics. In particular, transmission of ground vibration to the detector test-masses is a major limitation in the low-frequency regime ($< 20 \text{ Hz}$) for ground-based interferometric detectors [150].

The Einstein Telescope (ET) 3rd generation GWD aims for a sensitivity of a factor of 10 better than current detectors over a wide frequency band [10 - 10000] Hz, and of a factor 100 to 1000 at lower frequencies [14]. Over the past decade, it has become clear that such improvement, especially in the seismic frequency band (< 10 Hz), will definitely require a subtle combination of active inertial isolation and passive mechanical isolation [17]. The E-TEST project [21,22] proposed a novel hybrid isolation system approach: consisting of a VIRGO/KAGRA-style cascaded of pendulums and inverted pendulums with local control loops [38,39], which sits on top of an active LIGO-like active isolation platform [37, 40]. The entire system is operated in a UHV-vessel at 10^{-6} mbar. The active isolation platform (AP) is instrumented with high-resolution seismometers to drive inertial feedback loops and boost isolation in the [0.1 - 10] Hz frequency band, whereas the isolation at higher frequencies is brought by the mechanical suspension.

Because the performance of the inertial feedback control loops is ultimately set by the resolution of the in-loop sensors, custom inertial sensors with resolution better than commercial sensors in the [0.1 - 10] Hz band have been developed for this project. These sensors combine the sub-pm resolution, long-range, optical readout presented in chapter 1 and long-period mechanical suspension for the proof-mass. Two single-axis designs have been produced: the E-TEST Horizontal Interferometric inertial Sensor (E-HINS) senses horizontal inertial motion of the platform, and the E-TEST Vertical Interferometric inertial Sensor (E-VINS) senses its vertical motion. The platform is equipped with 3 of each sensor, in order to reconstruct and drive all 6 degrees of freedom.

This chapter will present the design and performance of the E-VINS and E-HINS sensors. It starts with a short overview of the general design aspects of inertial sensors and active inertial control. Next, section 2.2 will present the mechanical design of both sensors. Then, performance and characterisation will be discussed in section 2.3. Integration of the sensors in the E-TEST prototype will eventually be shown in section 2.4.

2.1 Inertial sensors: Concept and mechanical design

An inertial sensor is a device capable of measuring the absolute, inertial, motion of the support on which it is placed. Although traditionally issued from the seismology community, which requested the need for sensors capable of measuring inertial ground motion and seismic activity, inertial sensors are nowadays found in many various applications. Depending on the specific requirements and performance, they come in different shapes, sizes and weights: from large units weighting few kilos for navigation, seismology or civil engineering, to cm-scale sensors weighting few hundred grams for borehole installation, to mm-scale devices barely weighting a couple of grams commonly used in smartphones and other smart devices, automotive or medicine. More recently, inertial sensors have also found application in seismic isolation, where they are used to drive an inertial feedback loop [40, 151]. These inertial sensors are commonly named differently based on their size and/or application, even though this naming is somewhat arbitrary: geophones, VBB (Very-BroadBand) seismometers, (force-balanced) accelerometers, MEMS (Micro-ElectroMechanical Systems) accelerometers. Yet, all these devices fall under the umbrella of inertial sensors and, regardless of their size, specific design or application, are all based on the same fundamental working principle.

The underlying physics behind inertial sensors is discussed in several references [58, 152, 153]. A categorisation of inertial sensors and their related specific modelling is also discussed in [154]. The following section proposes a global overview of the fundamental working principle of an (open-loop) inertial sensor: from its basic mathematical modelling, to the calculation of the sensor's performance (sensitivity response function and resolution). Readers are referred to the latter reference for the specificity of the modelling of particular types of inertial sensors (e.g. geophones), not included in this global overview. The use of inertial sensors in active isolation is also briefly explained. Eventually, an overview of traditional and more modern, key mechanical aspects of the inertial sensor is presented.

2.1.1 Inertial sensor working principle

Inertial sensors are devices capable of measuring the absolute motion of the support on which they are placed. Obviously, an *absolute* motion measurement is, strictly speaking, impossible, since a measurement is always made with respect to some sort of reference. Inertial sensors achieve this by measuring relative motion with respect to a reference mass, often named the sensor proof-mass, that sits on a soft suspension. Depending on the dynamics of the suspension, the proof-mass effectively acts as an absolute inertial reference, within a certain frequency bandwidth. Then, the absolute support motion is inferred from the (presumably) known mechanical behaviour of the proof-mass suspension.

Modelling

Figure 2.1 shows the most simplistic dynamic modelling of an inertial sensor [58, 154]. In this modelling, the sensor proof-mass is modelled as a point mass m ([kg]). The proof-mass sits on a mechanical suspension and guiding mechanisms, which only allows the proof-mass to move along the direction of the measurement. This mechanics is modelled as a 1 degree of freedom spring-damper system: k ([N/m]) represents the suspension stiffness and c ([Ns/m]) represents the damping and losses in the mechanics. Note that the 1 degree of freedom assumption is very reasonable and not only applies to single-axis sensors, but also extends to tri-axial sensors, as the latter are commonly made of multiple (3) proof-masses, each guided along orthogonal directions. The motion to be measured w ([m]) transmits through the mechanics and results in a proof-mass motion x ([m]). w could represents any inertial motion: from ground to a payload motion. For the of simplicity, w will be referred here as ground motion, but the discussion does not only limits to seismometers. A relative motion device g ([V/m]) is then used for measuring the motion $y = x - w$ and to output the inertial u ([V]), from which w can be inferred. The sensor output u is however tainted with various noises that will limit the sensor resolution: i.e. the smallest measurable ground motion. They are modelled as internal noises $\tilde{n}_i$ ([V]) and external noises $\tilde{n}_e$ ([N]). These will be discussed further in a subsequent section.

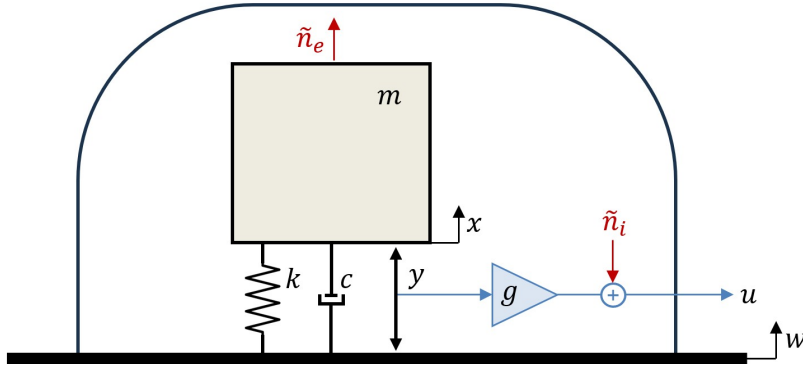


Figure 2.1: Symbolic schematic diagram of an open-loop inertial sensor. k ([N/m]), c ([Ns/m]) and m ([kg]) are the sensor's mechanical properties: suspension stiffness, damping coefficient and proof-mass, respectively. Ground motion w ([m]) translates into proof-mass motion x ([m]) with relative motion y ([m]). (Blue) Relative displacement sensor: readout gain g ([V/m]) and inertial sensor output u ([V]). (Red) Noise sources: internal noises $\tilde{n}_i$ ([V]) and external noises $\tilde{n}_e$ ([N]).

The sensor output is modelled by solving a set of two equations: consisting of the sensor's mechanics equation and the sensor's readout equation. The first equation is simply the Newton's Law applied to the proof-mass suspension, and the latter corresponds to the metre to voltage conversion of the relative displacement readout.

$$\begin{aligned}
 \text{Mechanics equation:} \quad & m\ddot{x} + c(\dot{x} - \dot{w}) + k(x - w) = \tilde{n}_e \\
 \text{Readout equation:} \quad & u = g \cdot y + \tilde{n}_i
 \end{aligned} \tag{2.1}$$

Solving the mechanical equation for the relative motion $y = x - w$,

$$m\ddot{y} + c\dot{y} + ky = -m\ddot{w} + \tilde{n}_e \quad (2.2)$$

And using the Laplace transform to write in the frequency domain,

$$Y = \frac{-ms^2}{ms^2 + cs + k}W + \frac{1}{ms^2 + cs + k}N_e \quad (2.3)$$

Finally, substituting in the readout equation yields the modelling of the sensor's output U

$$U = \frac{-gms^2}{ms^2 + cs + k}W + N_i + \frac{g}{ms^2 + cs + k}N_e \quad (2.4)$$

The latter equation provides a direct relationship between the sensor output U and ground motion w . Normalising the transfer functions with m makes the suspension resonance frequency $\omega_0^2 = k/m$ and Q -factor $c/m = \omega_0/Q$ explicitly appear in the relationship, which are parameters much simpler to identify in practice than k and c . So that the inertial sensor equation writes

$$\text{Sensor equation:} \quad U = \frac{-gs^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}W + N_i + \frac{1}{m} \frac{g}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}N_e \quad (2.5)$$

Sensitivity response function

The sensitivity of a sensor corresponds to the conversion, or scaling factor, between the actual physical quantity being measured and the sensor's voltage output. For example, the sensitivity of the relative displacement readout would simply be g , in units of Volts per metres.

Equation 2.5 shows that there indeed exists a relationship between the inertial sensor output U and the physical ground motion W . However, the equation also shows that the dynamics of the proof-mass suspension contributes to the sensitivity. Therefore, the sensor sensitivity cannot be described with a simple scalar conversion value from motion to voltage reading, e.g. [V/m], but rather a frequency-dependant scaling factor that consists in the combination of the readout sensitivity (in [V/m] - typically frequency independent), and the proof-mass suspension mechanical response to ground motion (in [m/m] - frequency dependant). This scaling factor is often named the sensitivity response function.

Solving Equation 2.5 for U/W leads to the inertial sensor's sensitivity response function S ([V/m])

$$S = \frac{U}{W} = g \cdot \frac{-s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad [\text{V/m}] \quad (2.6)$$

Figure 2.2 (blue trace) shows an example of the sensitivity response function $S = U/W$ of a 1 Hz inertial sensor, with a Q -factor of 200 and equipped with a relative displacement readout with gain

$g = 1 \text{ V/m}$. The trace shows that the sensitivity of the inertial sensor to ground displacement is a second-order mechanical high-pass filter. The inertial sensor indeed is a perfect estimator of ground motion about the sensor resonance, with a scaling of g , and therefore has a sensitivity bandwidth from f_0 up to the first flexible mode of the mechanical suspension. The sensitivity then drops below resonance with a f^{-2} slope. This results from the mechanical suspension that decouples the proof-mass from ground motion above the suspension resonance, so that it can act as an inertial reference.

For the sake of completeness, superimposed to the figure are also the sensitivity response function of a 1 Hz geophone/seismometer and inertial accelerometer, also equipped with a relative displacement readout $g = 1 \text{ V/m}$. Geophones and seismometers are typically designed to have an output proportional to ground velocity $U/\dot{W}$ (dashed red trace). It can be seen that the sensitivity response function is flat to ground frequency only if the mechanics is significantly damped (solid red trace). This is commonly done either by closing a force-balance control loop [55, 56, 155] or by using a voice-coil with a shunted coil in the design [153]. The advantage of this approach is a broader sensitivity bandwidth, i.e. a larger frequency bandwidth in which the response is flat, and centred on f_0 . The sensitivity is, however, lower in magnitude compared to the first case. This translates into a lower resolution, as it will be shown later. The sensitivity response of an inertial accelerometer is shown as the yellow trace. These devices are typically designed for a flat response to ground acceleration $U/\ddot{W}$. The sensitivity is fundamentally flat to acceleration only from DC to the sensor mechanical resonance, but can be increased significantly using a force-balance feedback loop on the proof-mass [55, 57, 155], resulting in a bandwidth typically even larger than seismometers. However, the sensitivity shows a $1/k$ scaling factor at DC, which again degrades resolution. These remarks are very generic, and a specific design for a particular application may result in specifications that contradict these general guidelines. For example, very broadband seismometers designed for a particular application can have bandwidths that compete with long-period inertial "accelerometers" designed for high resolution and low-frequency bandwidth. Yet, the general conclusion is that, regardless of the type of inertial sensor, there exists a trade-off between the sensor bandwidth and its resolution, due to the very nature of the suspension.

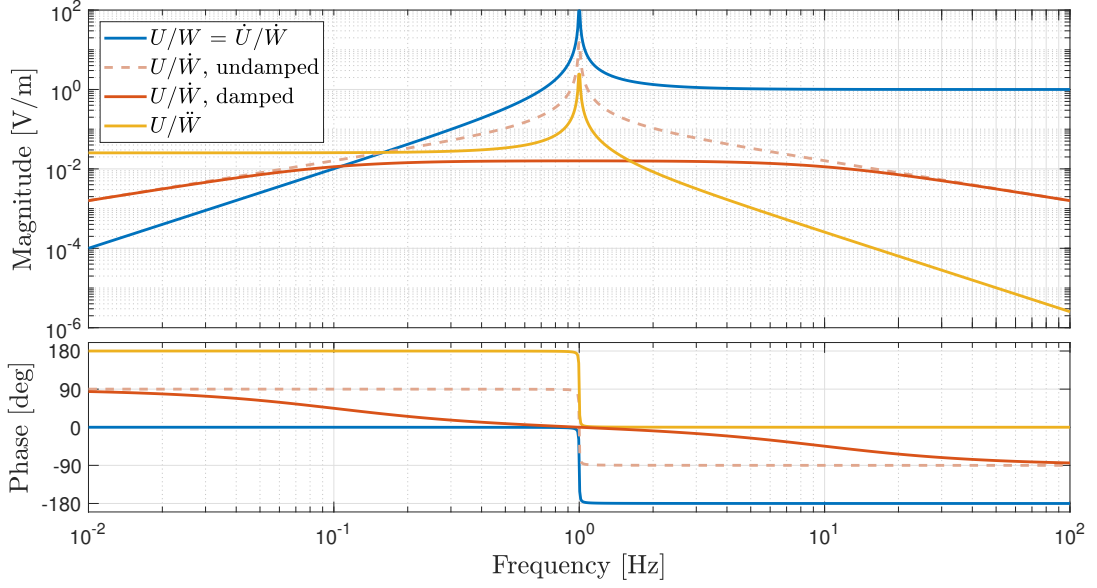


Figure 2.2: Typical sensitivity response functions of a 1 Hz sensor with a 200 Q -factor and a relative readout gain of 1 V/m. (Blue) U/W or $\dot{U}/\dot{W}$ sensitivity response of an inertial sensor. (Solid and dashed reds) $U/\dot{W}$ damped and undamped sensitivity response of a seismometer. (Yellow) $U/\ddot{W}$ sensitivity response of an inertial accelerometer.

Resolution and thermal noise

The inertial sensor resolution is the smallest motion that can be measured and distinguished from noise. From a global point of view, noises can be of two origins: internal to the sensor and external to the sensor. Internal noises are all noise sources that originate from within the sensor design and are not related to external factors. They are voltage noises that pollute the sensor output voltage U . They can usually not be shielded from, and for that sets an ultimate limitation to the sensor resolution. A major contribution is the relative displacement readout intrinsic noise, and commonly includes the sensor electronics, acquisition noise, etc. It is accounted for in the modelling as $\tilde{n}_i$. On the other hand, external noises are noise that directly relates to environmental conditions and directly act on the proof-mass as an external force. They are modelled as $\tilde{n}_e$. Thermal noise is a notorious example of such noise [156], but external noise does however not only limit to thermal noise: magnetic sensitivity, or air pressure are other examples. These noises, but thermal noise, may often be minimised with a careful design and shielding of the sensor. Because the readout and thermal noises often largely dominates other contributions of in their respective category, the discussions will specifically focus on these two: $\tilde{n}_r$ ([V]) and $\tilde{n}_{th}$ ([N]) representing respectively readout voltage noise and thermal noise. Other noise sources can easily be added in the modelling by adding them along-side either noise.

The sensor equation for U shows how noises typically transfer to the sensor output. Now, to

know how they then translate into a limitation of the sensor to estimate ground motion, Equation 2.5 should be corrected by the sensor's sensitivity response Equation 2.6 to obtain a modelling of the estimated ground motion $\hat{W}$:

$$\begin{aligned}\hat{W} &= S^{-1} \cdot U \\ &= W + \left(\frac{-gs^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \right)^{-1} N_r + \frac{-1}{ms^2} \cdot N_{th}\end{aligned}\quad (2.7)$$

Because of their random nature, noises are best described with their spectral densities, leading to

$$\Phi_{\hat{W}} = \Phi_W + \underbrace{\left(\frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{gs^2} \right)^2 \Phi_r + \left(\frac{1}{ms^2} \right)^2 \Phi_{th}}_{\text{Inertial sensor noise}} \quad (2.8)$$

where $\Phi_{(\cdot)}$ denotes power spectral densities in $[(\cdot)^2/\text{Hz}]$.

The equation shows that the readout noise scales with the inverse of the sensor's sensitivity response S^{-1} , and external noises scale with the inverse of the sensor's proof-mass and readout gain g .

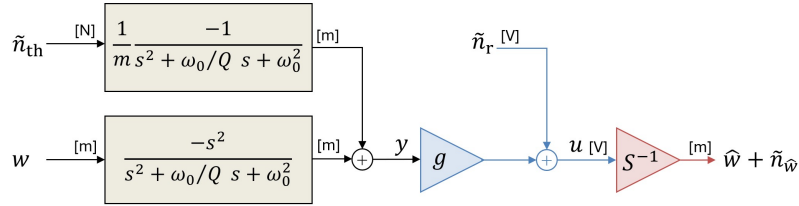


Figure 2.3: Typical noise diagram for an inertial sensor. w and $\hat{w}$ are respectively the ground motion and sensor-measured ground motion. y and u are respectively the proof-mass relative motion and the readout measured motion. The brown blocks represent the sensor's mechanical response to inputs. The blue block represents the readout g . The S^{-1} red block represents the digital inversion of the sensor's sensitivity response. $\tilde{n}_{th}$ represents thermal noise(s), $\tilde{n}_r$ the readout noise and $\tilde{n}_{\hat{w}}$ ([m]) the global inertial sensor noise.

Equivalently, Figure 2.3 shows a block diagram representation of how the thermal noises (and other external noises) and readout noise (or other internal noises) propagate in inertial sensor to the sensor output noise $\tilde{n}_{\hat{w}}$.

The proof mass can be shielded from many external noises with a careful design of the casing: thermal insulation, air-tight enclosure, magnetic shielding, etc. Thermal noise, however, is a fundamental noise present in any mechanism, unless it is operated in a 0 K environment. It is notoriously known for being a severe limitation to inertial sensors' resolution at low frequencies [153, 155–157]. This is why its modelling will be briefly discussed in the following.

The fluctuation-dissipation theorem sets a solid foundation to calculate thermal noise in mechanical experiments [158, 159]. The theorem suggests that any dissipation of energy in a mechanical system leads to noise that can be expressed as thermal noise. These noises can be of two origins: either external or internal to the mechanical systems. The external category includes velocity-dependent damping effects and is commonly modelled with a damping coefficient $c = \frac{\omega_0 m}{Q_B}$ where Q_B represents the Q -factor related to the velocity-dependent damping. This contribution is often named Brownian thermal noise since it is often dominated by gaseous damping caused by Brownian molecular agitation, but also contains any other velocity-dependent damping forces such as Eddy-current or magnetic damping [160]. The internal category is attributed to the dissipation of energy within the mechanics of the suspension itself. It is mostly characterised by thermoelastic losses in flexures, but also includes internal friction loss, clamping loss, heat flow and bulk and surface effects [161–164]. All these effects are often modelled as an imaginary part of the suspension stiffness: $k(1 + i\phi_S)$ where ϕ_S is the suspension mechanical loss factor. This contribution is often named structural thermal noise.

The fluctuation-dissipation theorem suggests calculating the thermal noise spectrum from the real part of the mechanical impedance, i.e. $F/\dot{x}$. The equation of motion for the sensor proof mass suspension, including the loss mechanisms discussed above, writes

$$(ms^2 + cs + k(1 + i\phi_S)) X = (-ms^2 + cs + k(1 + i\phi_S)) W + F \quad (2.9)$$

So that the mechanical impedance is

$$\begin{aligned} \frac{F}{sX} &= ms + c + k/s(1 + i\phi_S) \\ &= i\omega m + c - ik/\omega + k\phi_S/\omega \end{aligned} \quad (2.10)$$

And thermal noise Φ_{th} ($[N^2/Hz]$) is then calculated as

$$\begin{aligned} \Phi_{th} &= 4k_B T \Re \left\{ \frac{F}{sX} \right\} \\ &= 4k_B T \left(c + \frac{k\phi_S}{\omega} \right) \\ &= \underbrace{4k_B T \left(\frac{m\omega_0}{Q_B} \right)}_{\Phi_{th,B}} + \underbrace{4k_B T \left(\frac{\omega_0^2 m}{\omega Q_S} \right)}_{\Phi_{th,S}} \end{aligned} \quad (2.11)$$

where both contributions of the Brownian $\Phi_{th,B}$ and structural $\Phi_{th,S}$ thermal noises explicitly appear. In the above equation, $k_B = 1.38 \cdot 10^{-23} \text{ m}^2\text{kg/s}^2/\text{K}$ is the Boltzmann constant, T ([K]) the temperature, $\omega = 2\pi f$ the frequency and Q_B and $Q_S = \phi_S^{-1}$ respectively the Brownian and structural Q -factors. Note that the structural thermal noise is a coloured noise with a f^{-1} frequency-dependent spectrum, while the Brownian thermal noise is a white noise. This formula can then be plugged into

Equation 2.6 to model the impact of thermal noises on the inertial sensor resolution.

Figure 2.4 shows a typical noise budget for a 1 Hz inertial sensor equipped with a relative displacement sensor with a $1 \text{ pm}/\sqrt{\text{Hz}}$ resolution spectrum Φ_r . The thermal noise is calculated assuming equal contribution of structural and Brownian Q -factors $Q_B = Q_S = 200$, $T = 297 \text{ K}$, and a proof-mass $m = 1 \text{ kg}$. The noise budget is derived for an inertial displacement sensor (left plot), i.e. in units of $[\text{m}/\sqrt{\text{Hz}}]$, and for an inertial accelerometer (right plot), i.e. in units of $[\text{m}/\text{s}^2/\sqrt{\text{Hz}}]$. It can easily be seen that the accelerometer plot is simply the other plot, but derived twice. The geophone/seismometer plot would lie somewhere in between, but is not shown for clarity of the figure. In these plots, the blue trace shows the readout noise contribution to the noise budget. It has the same $10^{-12} \text{ m}/\sqrt{\text{Hz}}$ resolution as the readout self-noise, but is polluted by the sensor's mechanical response, and shows a typical increase of f^{-2} below resonance (resp. f^2 above resonance for the accelerometer case). Note that in practice, the spectrum typically rises faster than f^{-2} at low frequencies due to the readout noise not being perfectly white, but showing some $f^{-0.5}$ contribution in the 100 mHz and below range, as discussed in section 1.3. The red and yellow traces show, respectively, the contribution of Brownian and structural thermal noises to the noise budget, with the structural thermal noise being more dominant at lower frequencies due to its coloured spectrum. The inertial sensor resolution therefore consists of the quadrature sum of all contributions in the noise budget.

Note that an inertial sensor is often considered "well-designed" if both the readout noise and thermal noises have similar contributions at lower frequencies. The $\omega_0 m Q$ -product is a great indicator of that [56]. Both the readout and thermal noises are balanced for an $\omega_0 m Q$ -product in the range of a thousand (1250 in this example). For very low $\omega_0 m Q$ -product, thermal noises significantly dominates and the sensor resolution will improve by increasing the sensor mass m , or increasing the Q -factor even though one typically have lesser control on this parameter. For very large $\omega_0 m Q$ -products, the sensor readout dominates. The global resolution would improve by lowering the sensor resonance frequency.

The resolution of an inertial sensor is a very good indicator of its performance. However, to know if a sensor's characteristics meet a specific application requirement, the Signal-to-Noise Ratio (SNR) is a better-suited quantity. The SNR is the ratio between the physical signal to measure and the sensor resolution. One often wishes for an SNR of at least 20 dB (in amplitude ratios, i.e. a factor 10) for a good measurement. Because both the signal and the resolution are frequency dependent, the SNR also indicates the frequency bandwidth that will be measured with high fidelity. For an inertial sensor intended to measure ground motion, the SNR is therefore obtained by comparing the noise and the seismic spectrums. The Peterson New Low and High Noise Models (PNLNM and PNHNM) are models that represent the lower and higher bounds of seismic spectrum at about any places on Earth [138]. They are shown as the dashed and solid black traces in Figure 2.4. Even using the NLNM as a baseline, the resolution of a 1 Hz inertial sensor with a 1 pm-like readout resolution, a 200 Q -factor and a 1 kg proof-mass is capable of capturing ground motion in the 10 mHz to 10 Hz bandwidth, and with at least 40 dB of SNR in the 100 mHz to 1 Hz bandwidth.

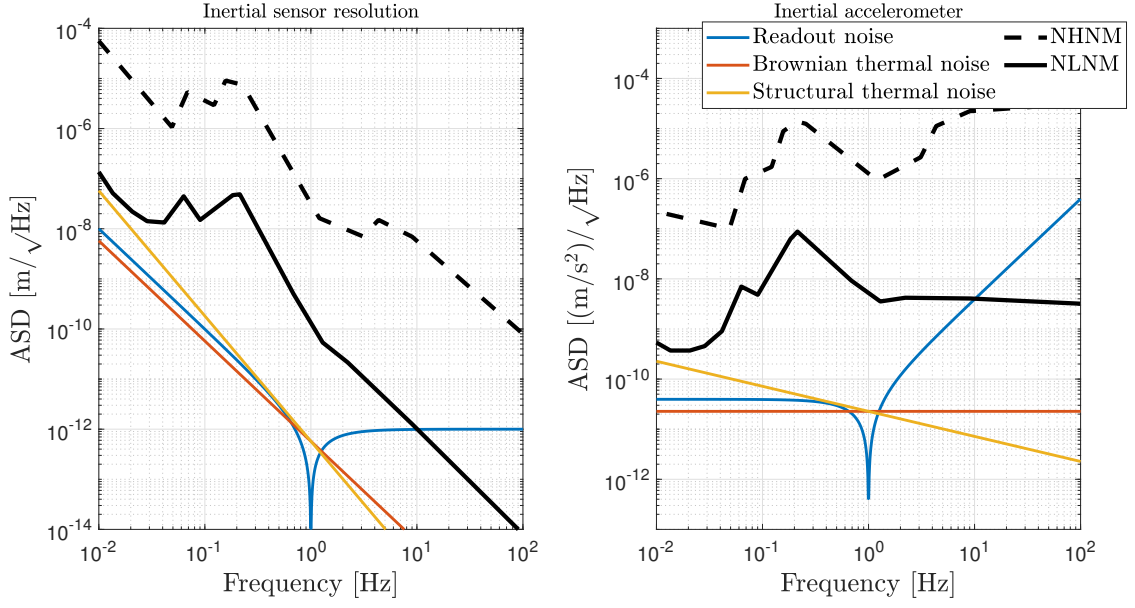


Figure 2.4: (Left) Typical noise budget of an inertial sensor and (Right) inertial accelerometer. (Blue) Relative displacement readout noise for a device with $1 \text{ pm}/\sqrt{\text{Hz}}$ resolution. (Red and Yellow) Brownian and structural thermal noise spectrums for a Brownian Q -factor $Q_B = 200$ and a structural loss angle $\Phi_S = 1/200$. (Solid and dashed black) Peterson New High Noise Model (NHNM) and New Low Noise Model (NLNM).

Shunting

As shown previously, an inversion of the sensor sensitivity response function is required for a correct interpretation of the sensor output and to estimate the ground motion. This inversion can be implemented physically on the sensor output by means of an analogue circuitry, or left for post-processing with a digital filter. For both cases, this inversion mostly consists in inverting the sensor mechanical response. If the sensor has a high Q -factor, which is often desired for thermal noise reasons as exposed above, the inversion will result in trying to notch a sharp resonance peak. This is often not desired since this operation is not robust, and strongly depends on the quality of the characterisation of the sensor dynamics and is sensitive to time fluctuation, such as ageing or thermal drift.

One way around this is to equip the inertial sensor with a shunted voice-coil actuator [153, 154]. A schematic representation of the shunt operation is pictured in Figure 2.5. Shunting makes the coil a closed circuit, which therefore allows current to circulate in the coil. As the magnet moves, the varying magnetic field induces an electric current in the coil proportional to the magnet velocity, as per the Faraday law. In turns, this current circulation generates a magnetic field, accordingly to the Biot-Savart law, which opposes the motion of the magnet, therefore inducing a velocity-dependent force, i.e. damping. Another way to understand shunting, from an energy point of view, is to consider that mechanical energy is transformed into electric energy by induction via the coil/magnet system.

It is then dissipated in the resistor, creating damping. Shunting therefore provides damping, with minimal modification to the sensor design, but at the expense of thermal noise since the voice-coil damping will inevitably translate into the same kind of noise as Brownian noise.

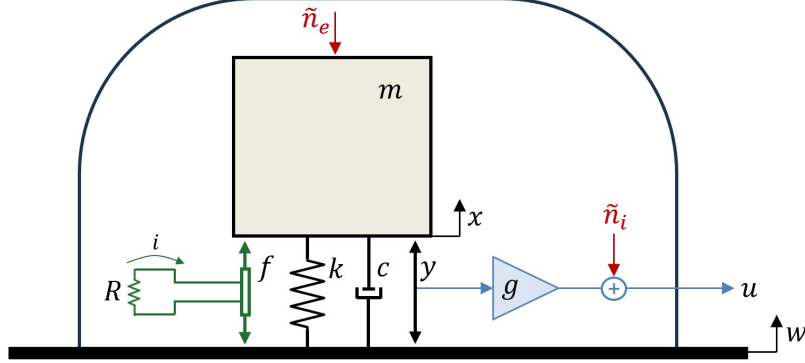


Figure 2.5: Symbolic schematic diagram of an inertial sensor with resistive shunt. Same parameter description as Figure 2.1. (Green) Shunt operation: f ([N/A]) voice-coil actuator, R ([Ω]) shunt resistor, i ([A]) shunt current.

The sensor equation for an inertial sensor with a resistive shunt is obtained by solving the same two equations as above, but also coupling the actuator equation and Kirchhoff's law for the voice-coil actuator and shunt circuit.

$$\begin{aligned}
 \text{Mechanics equation:} \quad & m\ddot{x} + c(\dot{x} - \dot{w}) + k(x - w) = f_A + \tilde{n}_e \\
 \text{Readout equation:} \quad & u = g \cdot y + \tilde{n}_i \\
 \text{Actuator equation:} \quad & f_A = -Ti \\
 \text{Shunt equation:} \quad & Ri = T(\dot{x} - \dot{w})
 \end{aligned} \tag{2.12}$$

with f_A ([N]) the force produced by the voice-coil actuator, i ([A]) the current flowing in the coil, T ([N/A]) the voice-coil constant and R ([Ω]) the shunt resistor.

The shunt equation easily combines with the actuator equation to write

$$f_A = -Ti = -\frac{T^2}{R}(\dot{x} - \dot{w}) \tag{2.13}$$

Substituting in the mechanics equation yields

$$m\ddot{x} + \left(c + \frac{T^2}{R}\right)(\dot{x} - \dot{w}) + k(x - w) = \tilde{n}_e \tag{2.14}$$

Proceeding the same way as before, the equation is re-written for the relative motion $y = x - w$

and turned to the Laplace domain

$$\left(ms^2 + \left(c + \frac{T^2}{R} \right) s + k \right) Y = -ms^2 W + N_e \quad (2.15)$$

Finally substituting in the readout equation, the shunted inertial sensor outputs equation writes

$$\text{Sensor equation:} \quad U = \frac{-gs^2}{s^2 + \frac{c + \frac{T^2}{R}}{m} s + \omega_0^2} W + N_i + \frac{1}{m} \frac{1}{s^2 + \frac{c + \frac{T^2}{R}}{m} s + \omega_0^2} N_e \quad (2.16)$$

It can be seen from the above equation that the sensitivity is now changed for

$$S_{\text{shunt}} = \frac{-gs^2}{s^2 + \frac{c + \frac{T^2}{R}}{m} s + \omega_0^2} \quad (2.17)$$

and that damping is effectively increased by the shunted voice-coil as T^2/R . The amount of damping can be controlled by either the voice-coil constant T or the shunt resistor R . Because the voice-coil constant is often set by a specific voice-coil design (magnet strength, coil number of turns, etc), the shunt resistor is often easier to tune to the desired damping level.

By identification with the canonical expression of a 2 degree of freedom system, one finds the relationship between the shunt resistor and Q -factor of the suspension:

$$\begin{aligned} \frac{c + \frac{T^2}{R}}{m} &= \frac{\omega_0}{Q} \\ \iff R &= \frac{T^2}{\frac{m\omega_0}{Q} - c} \end{aligned} \quad (2.18)$$

where Q is the effective Q -factor after shunt, and c is the structural/gaseous damping natively present in the sensor. To obtain a critically damped resonance peak, the Q -factor is set to $Q^{\text{crit}} = \sqrt{2}/2$, so that the appropriate shunt resistor R^{crit} would be

$$R^{\text{crit}} = \frac{T^2}{\sqrt{2}m\omega_0 - c} \quad (2.19)$$

and depends mostly on the voice-coil constant T , the initial amount of damping c , and the suspension properties ω_0 and m .

Figure 2.6 shows examples of the same inertial sensor shown above, but with various shunt operations. The shunt uses a voice-coil actuator with a constant of 5 N/A. This value is comparatively strong, in comparison with the voice-coil of sizes that can be found in inertial sensors, but needs to be compensated for the high native Q -factor, and large mass. The plot shows the sensitivity response function for the optimal shunt R^{crit} leading to critical damping, calculated as 2.82 Ω for the sensors' characteristics, as well as two other shunts with sub-optimal resistors of 10 Ω and 100

Ω . In comparison with the passive sensitivity response (blue line), it can be seen that the ideal shunt indeed brings critical damping with no amplification at the resonance frequency. The other two sensitivities still show amplification. The residual Q -factor is caused by the resistance being too large, and limits the current that can flow in the coil as a response to the magnet motion. As a result, the actuator force is lesser and so is the damping effect.

Additionally, it has also been shown that a slightly more exotic, yet still very simple, version of the shunt including capacitive element can slightly increase the damping effect and bandwidth of the inertial sensor [165].

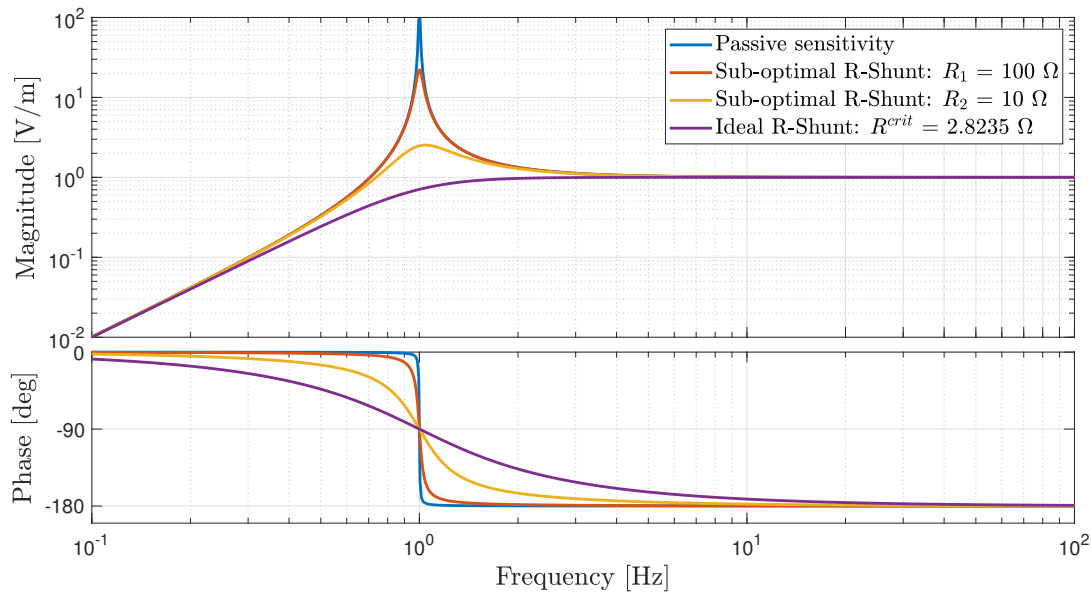


Figure 2.6: Sensitivity response functions of a 1 Hz inertial sensor for various resistive shunt. (Blue) Passive (open-coil) sensitivity response: Q -factor = 200. (Red and Yellow) Sub-optimal resistive shunt with $R_1 = 100 \, \Omega$ and $R_2 = 10 \, \Omega$. (Purple) Critically damped sensitivity with $R^{crit} = 2.82 \, \Omega$.

Inertial sensors in active control

Thanks to their capability to measure absolute motion, within a certain bandwidth, inertial sensors used in feedback isolation loops can result in performance better than other feedback strategies, such as relative motion or force feedback [166].

Let us consider a 1 degree of freedom payload M ([kg]) sitting a passive suspension k ([N/m]) and c ([Ns/m]) depicted in Figure 2.7. The payload is equipped with an inertial sensor of mass m and mechanical characteristics k_s and c_s . The relative motion y ([m]) between the sensor proof-mass m and the payload mass M is measured by a relative displacement readout g_s ([V/m]) and provide the inertial sensor output u ([V]), tainted with noise $\tilde{n}_s$ ([V]). The sensor noise includes both the internal (e.g. readout) and external (e.g. thermal) noise sources. The sensor output U drives an

actuator f ([N]) mounted on the payload through the feedback control filter $H(s)$ ([V/V]) and a control electronics g_a ([N/V]). The actuation force also contains some amount of noise, arising, for example, from the control electronics, Digital-Analogue-Converters (DAC), or actuator device. It is accounted for as $\tilde{n}_a$ ([N]). Such an inertial feedback loop aims at decreasing both the transmission of ground motion w ([m]) to the payload and sensitivity to external forces f_E ([N]).

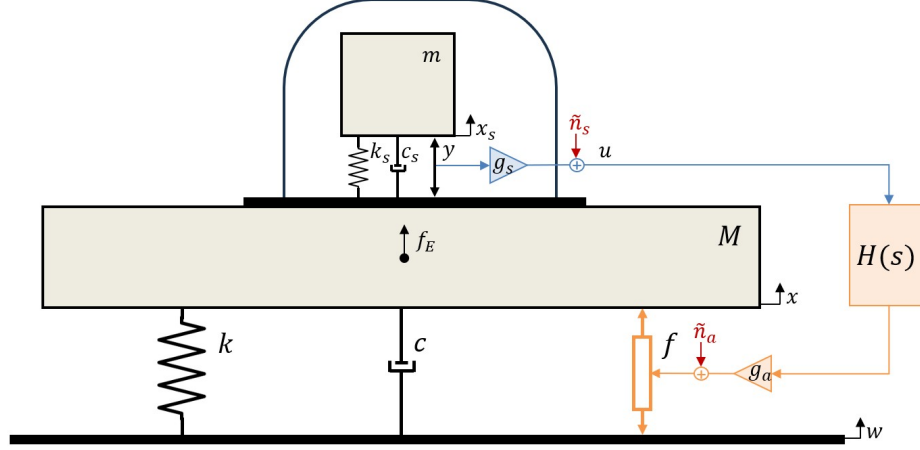


Figure 2.7: Schematic representation of payload active inertial feedback control. M ([kg]) is the payload with passive isolation k ([N/m]) and c ([Ns/m]). m is the inertial sensor with mechanical properties k_s and c_s . u ([V]) is the sensor output with noise $\tilde{n}_s$ ([V]). $H(s)$ ([N/V]) is the control filter and f ([N/V]) the force actuator. The control system isolates the payload motion x ([m]) from ground motion w ([m]) and external forces f_E ([N]).

The equation of motion for the payload M writes

$$M\ddot{x} + c(\dot{x} - \dot{w}) + k(x - w) = c_s(\dot{x}_s - \dot{x}) + k_s(x_s - x) + f_E + f \quad (2.20)$$

Solving the above equation would require coupling this with the sensor's proof-mass equation of motion. However, the inertial sensor proof-mass m is in practice several orders of magnitude smaller than the payload mass M : the ratio is typically several tens to hundreds of kg for M , versus a few hundreds of grams for m . It is therefore very safe to assume that the sensor dynamics do not significantly affect the payload. The right-hand-side terms can be dropped, leading to a simplified equation of motion for the payload

$$M\ddot{x} + c(\dot{x} - \dot{w}) + k(x - w) = f_E + f \quad (2.21)$$

The actuator force is the sensor output U propagated through the control filter H . Recalling the

inertial sensor equation Equation 2.5, it gives

$$\begin{aligned}
 f &= -g_a H(s) u + N_a \\
 &= -g_a H(s) \left(\frac{-g_s s^2}{s^2 + (\omega_{0,s}/Q_s)s + \omega_{0,s}^2} X + N_s \right) + N_a \\
 &= -g_a H(s) S_s X - g_a H(s) N_s + N_a
 \end{aligned} \tag{2.22}$$

naming the sensor sensitivity response $S_s = \frac{-g_s s^2}{s^2 + (\omega_{0,s}/Q_s)s + \omega_{0,s}^2}$ for convenience.

Writing the payload equation of motion in the Laplace domain and substituting the feedback equation, it yields

$$\begin{aligned}
 (Ms^2 + cs + k + g_a H(s) S_s) X &= (cs + k)W + F_E - g_a H(s) N_s + N_a \\
 \iff X &= \frac{cs + k}{Ms^2 + cs + k + g_a H(s) S_s} W + \frac{1}{Ms^2 + cs + k + g_a H(s) S_s} F_E \\
 &\quad - \frac{g_a H(s)}{Ms^2 + cs + k + g_a H(s) S_s} N_s + \frac{1}{Ms^2 + cs + k + g_a H(s) S_s} N_a
 \end{aligned} \tag{2.23}$$

This last equation represents the dynamics of the payload M with an active inertial feedback loop. From this equation, the system performance indexes are both the transmissibility of ground motion to payload X/W and the compliance to external forces X/F_E , and write

$$\begin{aligned}
 \frac{X}{W} &= \frac{cs + k}{Ms^2 + cs + k + g_a H(s) S_s} = \frac{(\omega_{0,p}/Q_p)s + \omega_{0,p}^2}{s^2 + (\omega_{0,p}/Q_p)s + \omega_{0,p}^2 + g_a H(s) S_s/M} \\
 \frac{X}{F_E} &= \frac{1}{Ms^2 + cs + k + g_a H(s) S_s} = \frac{1}{s^2 + (\omega_{0,p}/Q_p)s + \omega_{0,p}^2 + g_a H(s) S_s/M}
 \end{aligned} \tag{2.24}$$

with $\omega_{0,p}$ and Q_p the payload resonance frequency and Q -factor.

For a large control gain $H(s)$, the performance indexes goes to 0

$$\begin{aligned}
 \lim_{H(s) \rightarrow \infty} \frac{X}{W} &= 0 \\
 \lim_{H(s) \rightarrow \infty} \frac{X}{F_E} &= 0
 \end{aligned} \tag{2.25}$$

showing that inertial control can effectively isolate the payload from both ground motion and external loads. Such performance cannot be reached with other conventional passive or active approaches, such as force feedback or relative motion feedback [166].

However, even for a large control gain $H(s)$, a residual payload motion will remain and is ulti-

mately set by the inertial sensor noise N_s :

$$\lim_{H(s) \rightarrow \infty} X = S_s^{-1} N_s \quad (2.26)$$

This justifies the need for high-resolution inertial sensors in high-performance active inertial isolation. Figure 2.8 illustrates isolation performance on the transmissibility X/W that can be achieved for a 1 degree of freedom payload as shown above. The payload is modelled with a mass $M = 100$ kg and a suspension mode $\omega_{0,p} = 2\pi \cdot 2$ rad/s and $Q_p = 100$. The inertial sensor is modelled with a mass $m = 1$ kg and a suspension mode $\omega_{0,s} = 2\pi \cdot 0.2$ rad/s and $Q_s = 200$. The sensor and actuator electronics have gains of $g_s = 1$ V/m and $g_a = 1$ N/V respectively. A control filter $H(s)$ is designed based on the open-loop transfer function from actuation force f to the inertial sensor output u

$$\frac{U}{f} = \frac{U}{X} \cdot \frac{X}{f} = S_s \cdot \frac{1}{M} \frac{1}{s^2 + (\omega_{0,p}/Q_p)s + \omega_{0,p}^2} \quad (2.27)$$

Resulting in a lead-lag controller $H(s)$

$$H(s) = -3 \cdot 10^4 \frac{(s + 2\pi 0.04)(s + 2\pi 8)}{(s + 2\pi 0.0025)(s + 2\pi 2000)} \quad (2.28)$$

The open-loop gain $H(s) \cdot \frac{U}{f}$ is shown as the dashed black trace. It shows the lead and lag actions at high and low frequencies, respectively. The loop gain also shows the resonance frequency of the sensor at 0.2 Hz and the roll-off at frequencies below due to the sensor dynamics. The transmissibility with no control loop is shown as the blue trace. It can be seen that ground motion is fully transmitted to the payload and only frequencies above the payload suspension mode at 2 Hz are passively isolated. When engaging the inertial feedback loop, the transmissibility lowers to the red trace. The system shows nearly two orders of magnitude of isolation in the sensitivity bandwidth of the sensor (above 0.2 Hz), and isolation down to 10 mHz with slightly less performance due to the sensor dynamics.

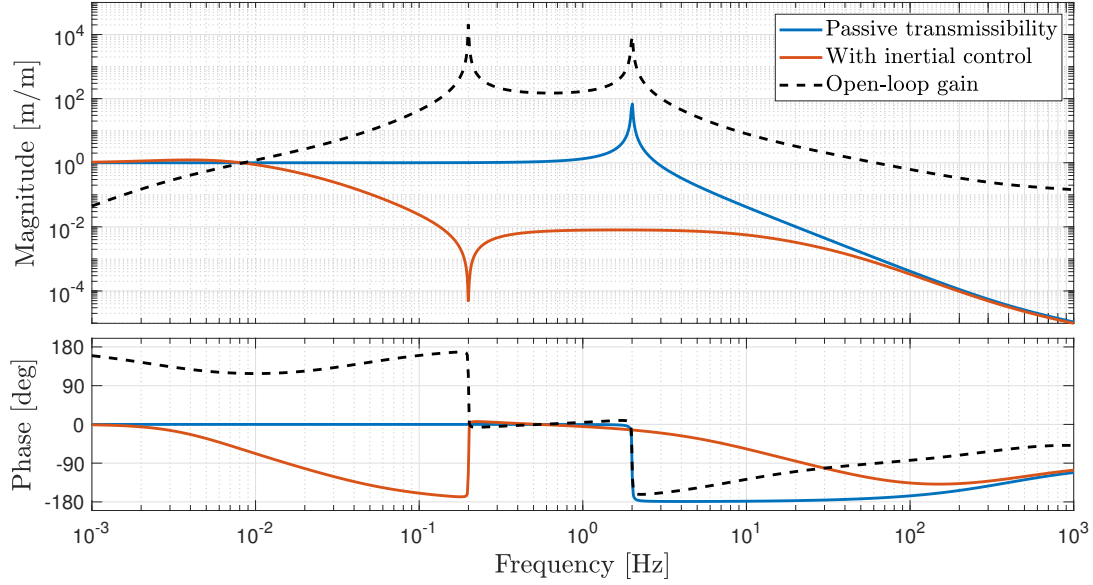


Figure 2.8: Performance of a 2 Hz 1 degree of freedom system with active inertial feedback using a 0.2 Hz inertial sensor. (Solid blue) Passive transmissibility of ground motion to payload motion. (Solid red) Transmissibility of ground motion to payload with active inertial control engaged. (Dashed black) Open-loop gain: open-loop transfer function $\times$ controller.

In fine, this shows that high-performance active inertial isolation calls for high-resolution inertial sensors. As discussed above, the modelling of an inertial sensor shows that its noise scales with both the readout self-noise and the sensor mechanics, so that the requirement for a good inertial sensor is both a high-resolution relative readout device and a low resonance frequency and high Q -factor suspension mechanism for the proof-mass.

2.1.2 Compact long period suspensions

The previous section reviewed the basic working principle common to most inertial sensors. In practice, several suspension mechanisms and design variants have been developed, each offering advantages depending on the instrument's size, target application, or measured degree of freedom. Currently, most very-long-period seismometers use pendulum-type suspensions [58, 167], in which the inertial mass is constrained to rotate about a single axis. Pendulum designs, combined with low-stiffness flexural or hinge pivots, offer the best capability to provide a soft suspension in one degree of freedom while mechanically constraining other directions and minimising losses (e.g. friction) compared to guided spring-mass systems. The pendulum can be oriented to oscillate in a horizontal plane for sensing horizontal motion, in a vertical plane for vertical motion, or in an inclined plane for multi-component sensing. In all cases, a dedicated suspension mechanism is required to balance the proof mass and set the natural frequency. A number of well-established, industry-standard

suspension types exist, which will be reviewed in the following.

This section therefore provides an overview of the main suspension mechanisms and pivot designs used in standard long-period seismometers, complemented by a brief survey of recent literature. The focus is on medium-sized and portable instruments: i.e. with characteristic dimensions of a few tens of centimetres, proof-mass weighting up to a kg, and resonance frequencies on the order of 1 Hz.

Vertical suspension mechanism

Achieving a soft suspension with oscillation periods of several seconds, while simultaneously providing sufficient elastic force to balance gravity, is challenging. This difficulty is due to the static deflection under gravitational load, which increases as the natural frequency decreases ($\Delta x \propto g/\omega_0^2$). This makes compact, dozens of cm-scale, designs with long periods impractical when using traditional spring suspensions.

To overcome this limitation, more advanced mechanisms have been developed. Standard solutions include the LaCoste suspension [168] and leafspring suspensions [56]. More exotic approaches based on Euler springs [169, 170] or anti-spring concepts [26] have also been proposed. A comprehensive comparison of these suspension types, including their associated thermal noise, is provided in [171].

LaCoste suspension

Lucien J.B. LaCoste is historically a pioneer in high-resolution seismology, and proposed in 1934 the design of a vertical seismometer with a theoretical infinite period (zero natural frequency) [168]. The key innovation was the use of a “zero-length” spring, i.e. a spring whose free length is zero, realized in practice by prestressing the coils of an ordinary spring.

In LaCoste’s design, the zero-length spring supports a nearly horizontal pendulum carrying the proof mass, as illustrated in Figure 2.9(a). An infinite period is obtained only under ideal conditions: a perfect zero-length spring, and exact alignment of the pendulum’s centre of mass (m) with its pivot point (O'). In practice, the natural frequency depends on small offsets in the pendulum angle θ , the positioning of the spring, and deviations from the zero-length condition. Achieving these requirements is feasible with large, meter-scale pendulums, but becomes increasingly difficult in compact designs. Furthermore, the large prestress in the spring favors creep and long-term drift in mechanical properties [170], although solutions using a compensation spring has been suggested.

Despite these challenges, LaCoste demonstrated periods on the order of 30 s (≈ 30 mHz natural frequency) with a 2.5 kg proof mass suspended from a 45 cm long pendulum [168]. The concept later became foundational not only in broadband seismometers but also in high-precision gravimeters [172].

Leafspring suspension

Leafspring suspensions were later introduced as a simpler, more robust, and more compact alternative to the LaCoste suspension [56]. They rely on a metallic blade, initially machined straight and then curved into position. One end of the blade is attached to a static frame, while the other supports a pendulum-like proof mass. The loading of the blade generates a net restoring torque that can be tuned to counteract gravity, maintaining the proof mass in near-horizontal equilibrium, as depicted in Figure 2.9(b). The choice of blade material is critical: it must have a high elastic limit to sustain large deformations, and a sufficiently high Young's modulus to generate the required restoring torque. Alloys such as NiSpan-C or Copper-Beryllium (CuBe2) are commonly employed.

Wielandt [56] demonstrated that properly designed leafspring suspensions can achieve very long effective periods, approaching tens of seconds. In their experiments, a 30 s period was achieved with a compact $18 \times 16 \times 12$ cm instrument (STS-1). The instrument has been a standard in high resolution seismology for many years. The calculation of stresses in the curved blade and its restoring torque requires solving a large-deflection problem, typically expressed via elliptic integrals. In practice, finite-element software with nonlinear mechanical packages is often used to optimise the design. Studies such as [171] show that the effective resonance frequency depends not only on the blade geometry (curvature, width, length, thickness), but also on clamping geometry (position of the left and right clamping points, orientation angles) and the proof mass position relative to the pivot. Under certain clamping configurations, negative-stiffness effects can be introduced at the pivot, further lowering the resonance frequency.

Compared to zero-length coil springs, leafspring suspensions operate at lower stress levels, reducing creep and long-term drift. Their simple geometry, a single metal blade, makes them easier to manufacture, while still providing near-infinite effective periods. Leafsprings suffer from similar thermal issues as coil springs, but it is believed that using pairs of leafspring with compensating material thermal properties may help in minimising thermal drift. For these reasons, leafspring suspensions have become a standard choice for many commercial instruments [51, 173–176]. Several research-grade instruments also rely on similar suspensions [76, 100, 102, 103, 106, 177, 178].

Euler spring suspension

Euler springs consist of slender blades loaded in compression until they approach the critical buckling load. In the post-buckling regime, the blade behaves as a very soft spring, with an effective stiffness that can be estimated from classical Euler buckling theory. This property allows the design of compact suspensions with low vertical resonance frequencies.

Vibration isolation systems employing Euler springs have been proposed in the literature [169]. Using pairs of such springs and coupling them with a pendulum-style proof mass, leads to more "exotic" designs for vertical seismometers, as illustrated in Figure 2.9(c). For example, Hosain et al. [170] demonstrated a research prototype combining Euler springs with a traditional LaCoste suspension, achieving a vertical resonance frequency of 0.15 Hz.

Anti-spring suspension

The anti-spring concept consists in having an effective negative stiffness, caused by a net 'restoring' force that acts in the same direction as the spring deformation. While a single spring cannot exhibit true negative stiffness, the effect can be synthesised through a specific arrangement of loaded springs [179], as illustrated in Figure 2.9(d). In practice, a primary vertical suspension provides the restoring force opposing gravity (F_z), while additional springs in transverse directions generate forces ($F_{x,l}$ and $F_{x,r}$) that partially cancel the vertical stiffness of the main suspension, thereby lowering the effective stiffness.

Originally proposed by Molyneux [179] for ultra-low frequency passive isolation, anti-spring systems have since been applied in gravitational-wave detectors [38] and other precision instruments [180]. Negative stiffness can be realized through various methods, including: flexures compressed via adjustable screws [181], sets of magnets arranged to produce opposing forces [182] or geometrically constrained blade springs [183]. The ability of anti-spring arrangements to produce very soft vertical suspensions makes them highly relevant for vertical seismometers [26, 181]. By tuning the load on the transverse springs, the vertical resonance frequency can be adjusted, allowing sub-Hz operation in compact, high-performance instruments.

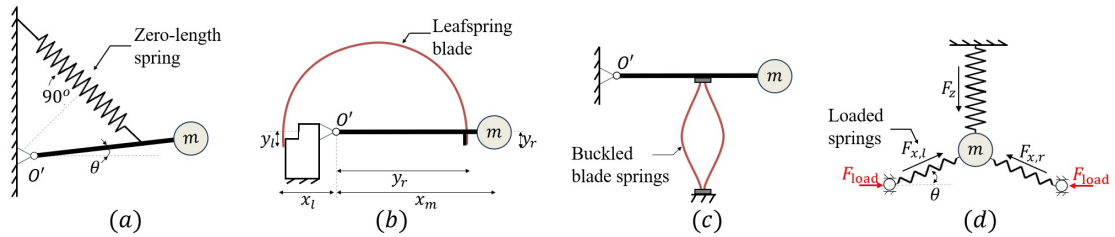


Figure 2.9: Schematic representation of standard suspension mechanisms for vertical seismometers. m represents to proof-mass centre of mass and the thicker rod the pendulum body. O' is the pivot join and allows rotation θ along the sensing axis. (a) LaCoste Suspension [168]. (b) Leafspring suspension [56]. (c) Euler suspension [169]. (d) (Geometric) anti-spring suspension.

Horizontal suspension mechanism

Horizontal suspensions have different design constraints than vertical ones. Because the vertical motion is constrained, horizontal suspensions do not need to balance the proof mass against gravity, allowing more freedom for very soft and compact suspension mechanisms.

Horizontal seismometers can of course re-use the same concept as exposed above for vertical suspension. In addition, few other suspensions designs such as the Lehman (or 'garden-gate') pendulum [184, 185] and inverted or folded pendulums designs [186, 187] have been developed.

Lehman pendulum

James D. Lehman proposed in 1977 a simple yet effective horizontal seismograph design capable of achieving periods of several tens of seconds [184]. The configuration, illustrated in Figure 2.10(a), consists of a pendulum allowed to rotate around a nearly vertical axis (ψ). Because vertical motion is constrained, no additional suspension elements are required; the stability of the proof mass about its equilibrium position is provided solely by the low stiffness of the hinge pivot (O'). For a pendulum of length L inclined at an angle ψ , the natural period is given by $T \propto 2\pi\sqrt{L/g \sin \psi}$. This makes the resonance frequency easily tunable by adjusting the inclination angle. Notably, a slight upward tilt of the pendulum introduces negative gravitational stiffness, which can be tuned against the stiffness of the pivot joint. This yields extremely long period mechanisms, although at the cost of a marginally stable equilibrium.

Due to its simplicity and effectiveness, the Lehman pendulum has been widely used not only in educational purposes and amateur instruments, but also in research and industrial-grade seismometers [185]. A notable example is the Streckeisen STS-1H, which achieves a nominal 20-second period using a pendulum of about 10 cm length [173]. More recent research instruments demonstrate comparable or improved performance [100].

Inverted and folded pendulum

Because the natural period of a simple pendulum scales to its length ($T \propto \sqrt{L/g}$), compact simple pendulum-like designs may merely achieve periods of 1 s. Instead, Emil Wiechert proposed in the early 20th century the use of an inverted pendulum, i.e. a pendulum operating in an upside-down configuration [56, 186, 188].

Several instruments have since been built following Wiechert's concept, schematized in Figure 2.10(b.1). For the inverted pendulum to remain in a stable equilibrium, additional restoring elements must be included alongside the pivot hinge (O'). These can take the form of elastic springs, strings, or magnetic levitation-based systems [189], or stability can be maintained using servo feedback loops [190, 191]. With such arrangements, compact instruments with 10-cm pendulums and proof masses of a few hundred grams can reach natural periods in the 1–10 s range.

The inverted pendulum principle was later refined into the so-called 'folded' pendulum. The idea is to eliminate the need for auxiliary restoring elements by coupling the inverted leg to a standard pendulum through a rigid link, forming a Watt's linkage mechanism [192], illustrated in Figure 2.10(b.2). In this configuration, the negative gravitational stiffness of the inverted leg is balanced by the positive stiffness of the normal leg, resulting in an overall horizontal suspension of very low effective stiffness. Fine tuning of the resonance frequency can be achieved by shifting the centre of mass (m) toward either leg. With ideal tuning, the system can in principle reach infinite periods within a compact geometry [193, 194].

Folded pendulum designs have inspired many research-grade instruments. For example, Blair et al. [187] demonstrated a resonance frequency of 17 mHz using a 5.5 kg proof mass and pendulum legs

of about 15 cm. Modern implementations often employ monolithic flexures machined from a single block using Electrical Discharge Machining (EDM), minimizing mechanical losses and achieving very high quality factors [195]. Numerous subsequent research instruments report resonance frequencies in the 100 mHz range with Q -factors of several thousand [78, 79, 107, 195–201]. Note also that the folded-pendulum concept can be extended to the vertical direction using magnets to pre-load the Watt’s linkage [202].

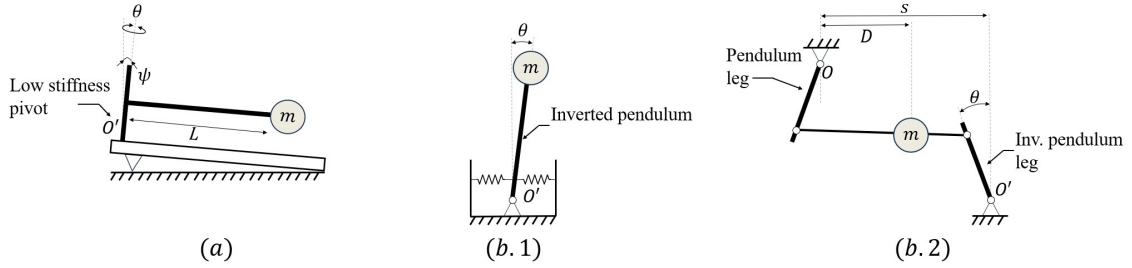


Figure 2.10: Schematic representation of standard suspension mechanisms for horizontal seismometers. m represents to proof-mass centre of mass and the thicker rod the pendulum body. O' is the pivot joint and allows rotation θ along the sensing axis. (a) Lehman pendulum [168]. (b.1) Inverted pendulum [186]. Folded pendulum or Watt’s linkage [187, 192].

Multi-axis suspension mechanism

Although this work focuses on single-axis instruments, the traditional conceptual design of a triaxial sensor is briefly discussed below as it is used in most traditional seismometers found in the industry.

In practice, one often wishes to measure seismic motion in all three translations: X , Y and Z (or often named 'East-West' (EW), 'North-South' (NS) and Vertical (V) in seismology). Combining three independent single-axis sensors, one aligned with gravity and two in the horizontal plane, is of course an option. However, this solution is often inconvenient because horizontal and vertical seismometers generally rely on different suspension mechanisms, leading to mismatched response functions and calibration challenges.

A more elegant solution to produce a unit with multi-component sensing is to use three pendulum-type proof-masses arranged in a so-called Galperin configuration [50, 58, 203]. The Galperin configuration uses 3 similar sensing elements inclined upward to about 35° ($\arctan(1/\sqrt{2})$) to the horizontal plane and, when seen from a top-view, make an angle of 120° with respect to each other. A schematic of *one* Galperin-type arrangement is shown in Figure 2.11(a). Note that this arrangement is not unique [50]. A simple geometric projection of the sensing axes $\{U, V, W\}$ then provides the Cartesian

$\{X, Y, Z\}$ components:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 & 1 & 1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \begin{pmatrix} U \\ V \\ W \end{pmatrix} \quad (2.29)$$

Note that the angular degree of freedom may also be extracted in this configuration, but is often omitted in seismology due to tilt-to-translation coupling, rendering tilt estimation impractical.

This arrangement is also named a "homogeneous" or "symmetric" triaxial geometry, since it uses three identical sensing elements. Therefore, one does not need to account for different axes having different sensitivities for triaxial units utilising sets of horizontal and vertical suspensions. In the Galperin configurations, the proof-mass suspensions can either be of the LaCoste types or of the leafspring types, as shown in Figure 2.11(b) and Figure 2.11(c) [58].

The simplicity and symmetry of this configuration make the Galperin triaxial seismometer one of the most popular in the industry. Notorious examples are the Streckeisen STS-2 seismometer or the Nanometrics Trillium seismometer [51, 175]. A research unit also proposed to use three folded pendulums arranged along a radius and with their sensing axis inclined at an angle to create a FP-based Galperin triaxial seismometer [204].

In the framework of this thesis, the choice of having physically different horizontal and vertical units has been selected to allow more freedom in locating the sensors in the actively controlled platform.

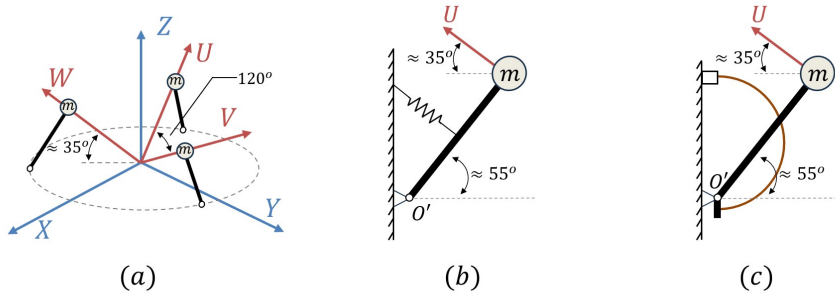


Figure 2.11: Schematic representation of a triaxial seismometer. (a) One Galperin configuration [50, 203]. (b) Inclined LaCoste suspension [58, 205]. (c) Inclined leafspring suspension [56, 58].

Pivot design

One last key aspect in inertial sensors' mechanical design is the choice of the pivot, or hinge, joint. The pivot's purpose is to allow the free rotation of the proof-mass along the sensing axis while constraining the other degrees of freedom. It is denoted as O' in the previous schematics. An

ideal pivot should: have minimal stiffness, so that it does not add to the stiffness of the suspension mechanism; have a well-defined axis of rotation; have an effective rejection of the joint spurious modes (e.g. torsion mode); and finally be frictionless for minimal losses.

The latter constraint is particularly relevant, since it has been demonstrated through the fluctuation-dissipation theorem that friction and contact translate into thermal noise and ultimately limit the sensor's resolution [159]. This alone already excludes the use of many mechanical hinges and bearings, so that nowadays virtually any seismometer design uses flexures, i.e. components using the elastic properties of flexible material, as pivot joints. Traditional flexure elements include the clamped-blade hinge [206], the cross-spring hinge [207, 208] and the notch hinge [209]. A detailed review of each flexure can be found in [46]. The material choice will not be extensively discussed, although it is very relevant for structural loss and thermal noise reasons. Most flexures conventionally use low dissipation, low temperature dependence, metals (CuBe2, titanium, Ni-Span-C), but recent research also suggests using glass (e.g. fused-silica) flexures in inertial sensors for even lower structural thermal noise [210, 211].

Clamped-blade flexure

A clamped-blade hinge is probably the simplest design of flexure joint [206]. A schematic is shown in Figure 2.12(a). It uses the elastic properties of a thin blade of metal clamped on one side on the static mounting frame of the proof-mass, and on the other side on the pendulum. Mechanics of material therefore teaches that the stiffness of the blade in flexion around an axis θ at mid-span will be much less than other flexible modes, such as translations X , Y or Z and rotations RY (torsion) and RZ . Thereby producing an effective flexural pivot allowing RX rotation. The joint stiffness is calculated from basic beam theory:

$$\kappa = \frac{EI_x}{L} \quad (2.30)$$

with $I_x = wt^3/12$.

The stiffness therefore reduces with a longer blade L , thinner thickness t and width w . However, these conditions are in contradiction with maintaining the spurious modes at higher frequencies. Notably, the torsion (RY) mode requires a large width w to be effectively rejected, and excessive blade length L will favour the Z -translation mode. Additionally, the exact location of the rotation axis θ may be difficult to anticipate as it depends on the exact assembly of the blade in its clamps and alignment. The flexure is also more prone to plastic deformation, damage and long-term creep, since the stresses in such a single, slender, blade can be large.

For these reasons, the simple clamped-blade hinge is often not preferred in traditional devices. Its simplicity and space efficiency still make this joint a great candidate, providing a careful design and assembly procedure.

Cross-spring flexure

Cross-spring flexures are a logical improvement to the basic clamped-blade hinge [207]. A simplified schematic is proposed in Figure 2.12(b). The hinge consists of pairs of clamped-blades that cross each other at mid-span with at right angles, but other angles are possible. The schematic shows a pair of 2 blades, but the design can be extended to any (profitably) symmetric arrangement. Stiffness characterisation is not straightforward, and a procedure is given in [208]. It is, however, typically larger than a clamped-blade since it uses several flexures.

This hinge structure is much more efficient in rejecting spurious modes, is more robust, and has a better-defined axis of rotation than a simple clamped-blade. For these reasons, this joint has been used in most industry-grade seismometers and other applications with similar needs. The Riverhawk Free-Flex pivot is a notorious industrial design derived from the cross-spring principle [212]. Streckeisen STS-1H&V, STS-2, Guralp 6T or Nanometrics Trillium all use similar cross-spring flexures [51, 173, 175, 176]. It however comes with a bulkier and usually stiffer design.

Notch flexure

Notch flexures are fairly new flexure types that have been made possible by advanced machining processes [209]. The idea is to machine a monolithic piece of material with a very thin notch, as illustrated in Figure 2.12(c), making the material very flexible along the axis θ passing through the notch waist, while remaining stiff in all other directions. Using processes such as Electrical-Discharge Machining (EDM), the notch can be machined with characteristic sub-mm thickness. Notch flexures are traditionally circular, and in this case the Tseytlin formula can be used to estimate their stiffness [200, 213], depending on the notch thickness t , width w and radius R :

$$\kappa = \frac{Ewt^2}{16 \left[1 + \sqrt{1 + 0.215(2R/t)} \right]} \quad (2.31)$$

where it can be seen that very small thickness and large radius can lead to extremely low stiffness. For notches of a more general, i.e. elliptical, shape, empirical formulas may be used instead [209].

Aside from low stiffness and great rejection of spurious modes, another advantage of the notch flexure lies in its monolithic design. Because they can be machined from a single bulk of material, notch flexures typically have no clamping or contact losses, and may result in a very high Q -factor design. A couple of research prototype proposes monolithic seismometers, with the proof-mass and notch flexures machined by EDM from the same bulk of metal, resulting in Q -factor of several thousands [107, 195, 196, 199, 201].

Such flexures are, however, fairly fragile and are often limited to research and laboratory-grade instruments, rather than field seismometers.

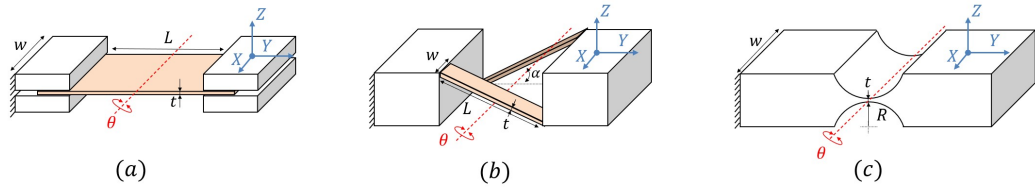


Figure 2.12: Schematic representation traditional flexures hinge used in seismometer. Left side is clamped while the right side connected to sensor the proof-mass. (a) Clamped-blade hinge [206]. (b) Cross-spring hinge [207, 208]. (c) Notch hinge [209].

2.2 Optical inertial sensors mechanical design

The target frequency band of the active isolation system in E-TEST lies in the 10 mHz - 10 Hz range. For the inertial feedback loop to be efficient in this bandwidth, the sensors need to be optimised for low-frequency measurement. This is achieved by coupling a high-resolution, μm -range, readout technology with long-period suspension mechanisms of the sensor's proof mass. Also, the isolation of the payload in all 6 degrees of freedom requires (at least) 6 sensing axes. It has been decided to use 3 inertial units, each consisting of a combination of two single-axis inertial sensors: one for sensing the vertical motion and one for sensing the horizontal motion. The vertical sensor is designed using a leafspring suspended pendulum [56] and clamped blade hinge, providing a second-like period in a compact volume, whose design is inherited from [46, 73, 100, 108, 151, 214]. The horizontal sensor uses a Watt's linkage [187, 195] guiding the proof-mass in a quasi-straight horizontal translation with sub-Hz resonance frequency, with a design inherited from [78, 79, 199–201]. Both sensors use the quadrature Michelson interferometer showcased in chapter 1 for reading out the proof-mass motion.

This section presents the mechanical design of the 2 single-axis (Vertical / Horizontal) Interferometric inertial Sensors designed for the E-TEST project (E-VINS & E-HINS). The designs is broken down into 4 major sub-assemblies: the static frame assembly, the proof-mass assembly, the readout assembly and the actuator assembly. The static frame assembly consists of the main body of the sensor, it is the frame onto which the different sub-assemblies are attached. The proof-mass assembly includes the proof-mass, its suspension and/or guiding mechanism and the retroreflector assembly. The readout assembly includes the optical device, the mechanical interferences with the sensor static frame and the alignment mechanism for pointing the retroreflector attached to the proof-mass. The actuator assembly includes the voice-coil and its interface on the proof-mass and sensor frame.

2.2.1 E-VINS Mechanical design

The E-TEST Vertical INterferometric inertial Sensor (E-VINS) is designed combining a long-period, compact, leafspring suspension [56] of the proof-mass and the high resolution optical readout showcased in section 1.2. Figure 2.13 shows a picture of the assembled sensor in a clean room (right) and a CAD view of the sensor with the different sub-assemblies highlighted in various colours (left). Details and technical drawings of most of the mechanical parts can be found in [210]. Table 2.1 contains references to the various components in the assembly. For a UHV vacuum-compatibility, all components are vacuum-certified down to 10^{-6} mbar, and care is taken in the mechanical design to use venting holes for the threaded holes that are not drilled through the body. If venting of a hole is not possible, vented screws are used instead.

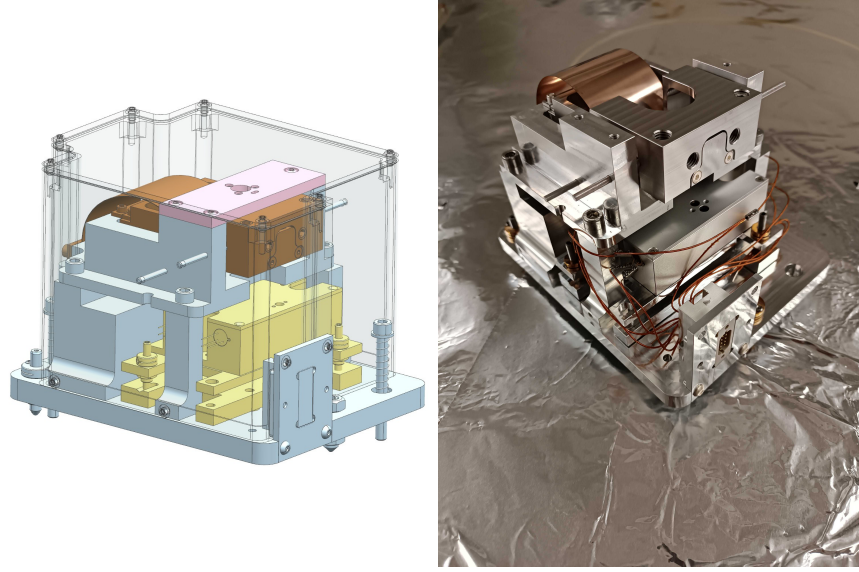


Figure 2.13: Assembled E-VINS picture in cleanroom (right). E-VINS CAD view (left). Sensor body (light blue), proof-mass assembly (brown), readout assembly (yellow) and actuator assembly (pink).

Sub assembly	ID	Component	Manufacturer	Ref.
Static frame	1a	Sensor body (Al.)	-	L. Améz-Droz [210]
Static frame	1b	Connector interface D-sub 9 Male UHV Connector UHV Compatible wire UHV Wiring Accessories	- Arun Microelectronics Arun Microelectronics Arun Microelectronics	L. Améz-Droz PhD [210] VC9DM KW6 BC1.5
Static frame	1c	Hex Adjustment screw, vacuum	MKS Newport	AJS100-0.5H-V6
Static frame	1d	Spring-loaded screws	LeeSpring	LHP038B01S
Static frame	1e	Blocking rods (Al.)	-	L. Améz-Droz [210]
Static case	1f	Casing (Al.)	-	L. Améz-Droz [210]
Proof-mass	2a	Pendulum bob (SS.)	-	L. Améz-Droz [210]
Proof-mass	2b	CuBe2 suspension leafspring	Laminerie MATTHEY	CuBe2 190/C17200/CW101C
Proof-mass	2c	CuBe2 campled-blade hinge joint	Laminerie MATTHEY	CuBe2 190/C17200/CW101C
Proof-mass	2d	Retroreflector interface (SS.)	-	L. Améz-Droz [210]
Proof-mass	2e	Tuning mass (SS.)	-	Headless hex socket M8x12
Readout	3a	Quadrature Michelson interferometer	EXail	Section 1.2
Readout	3b	Ball pivot (Al.)	-	L. Améz-Droz [210]
Readout	3c	Interface (Al.)	-	L. Améz-Droz [210]
Readout	3d	Miniature adjustment screw	MKS Newport	AJS8-100-0.5H-V6
Readout	3e	Clamp blades (Al.)	-	L. Améz-Droz [210]
Voice-coil	4a	Vacuum compatible VC actuator	H2W	NCC03-07-001-1V (Coil only)
Voice-coil	4b	Magnet assembly Neodymium N45 magnets	- Supermagnets	Appendix D S-12-03N
Voice-coil	4c	Static frame interface (Al.)	-	L. Améz-Droz [210]

Table 2.1: List of components in the E-VINS sensor assembly.

Static frame assembly

The static frame assembly is shown in Figure 2.14. Technical drawings of the main body can be found in [210]. The static frame (1a) is the main body of the sensor, which is the element that connects all other sub-assemblies. It should therefore not exhibit obvious flexible behaviour, which justifies its bulky design. It also sets the overall dimensions of the full assembly, which has been designed in accordance with the design of the ETEST active platform SILENT. The sensor footprint is $180 \times 140 \text{ mm}^2$, with a height of 160 mm. The readout photodiode pins are wired to a D-sub 9 connector mounted on a connector interface (1b). The connector pinout map can be found in Appendix B. The sensor is mounted on 3-pin feet (1c), two of which are adjustable to level the sensor axis with the vertical axis. This is a delicate operation, since the sensitivity of the vertical sensor to transverse horizontal motion is proportional to this misalignment. Spring-loaded screws (1d) are used to guarantee good contact between the sensor body and its interface in the active platform, while preserving adjustment capabilities with the levelling feet. A blocking mechanism using two rigid rods (1e) can block the proof-mass when manipulating the sensor. The sensor also comes with a casing for isolation from ambient light and air currents (shown in Figure 2.13).

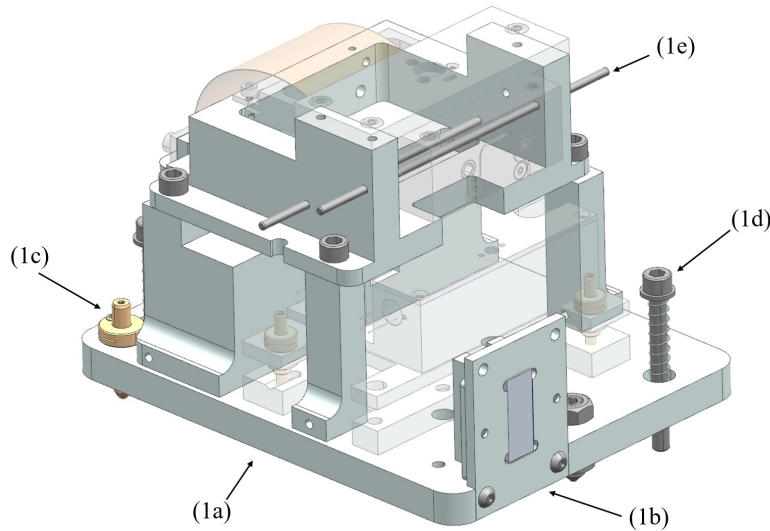


Figure 2.14: E-VINS frame assembly. Sensor body (1a), connector interface (1b), levelling feet (1c), spring-loaded screws (1d), blocking mechanism (1e).

Proof-mass assembly

Figure 2.15 pictures the proof-mass assembly. Technical drawings for the proof-mass and interfaces can be found in [210]. The proof-mass is a leafspring suspended pendulum (2a). It weights approximately 0.4 kg, accounting for the 0.306 kg stainless steel body of the pendulum, the additional tuning masses (2e) and the actuator magnet (4b, see Figure 2.19). It is maintained in a static horizontal position by a CuBe2 leafspring suspension clamped at one end on the static frame, and

at the other end on the pendulum bob (2b). The proof-mass rotates on a horizontal axis defined by a CuBe2 clamped-blade hinge joint (2c). The interferometer shoots at a retroreflector mounted on the proof-mass end (2d). A set of horizontal and vertical headless screws is available on the proof-mass end to allow a fine-tuning of the pendulum inertia and static position (2e). This is crucial for aligning the proof-mass centre of mass in the same horizontal plane as the hinge joint, and thus minimising the sensitivity of the suspension to horizontal motion.

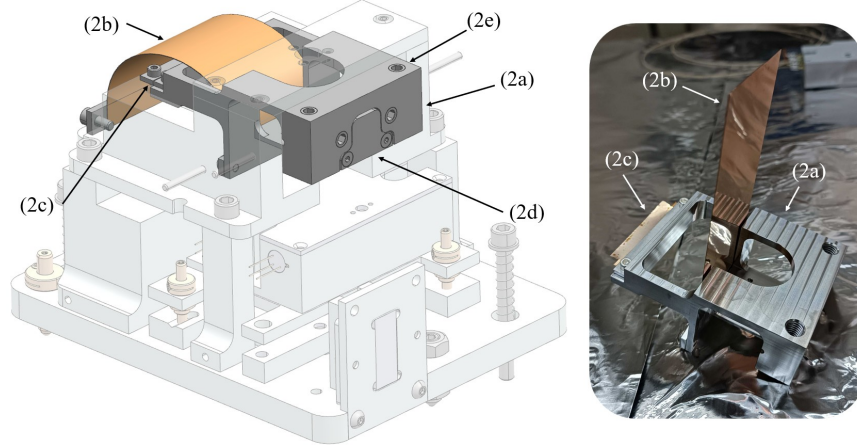


Figure 2.15: E-VINS proof-mass assembly. Proof-mass (≈ 0.4 kg) (2a), CuBe2 leafspring suspension (2b), clamped-blade hinge joint (2c), retroreflector and interface (2d), tuning mass (2e).

The key mechanism in the suspension of the proof-mass lies in the suspension leafspring. The leafspring suspension was chosen for its potential to mimic an actual anelastic, zero stiffness, LaCoste suspension [168], but with a much simpler design [56]. The LaCoste suspensions indeed rely on the use of zero-length springs, which are difficult to produce and sensitive to mounting and imperfections. On the other hand, leafspring suspensions have been demonstrated capable of producing suspensions with periods of several seconds in a much more compact, robust and simpler design and became a very widespread suspension mechanism in vertical and tri-axial industry-grade and research-grade seismometers [46, 51, 73, 76, 100, 103, 105, 108, 151, 173, 175–177, 214]. The CuBe2 blade is initially straight and bent into position. The stress distribution in the curved blade creates a restoring torque that opposes the gravitational torque and maintains the proof-mass in a static horizontal position. CuBe2 (Copper-Beryllium alloy) is chosen for its high elastic limit compared to other similar flexible metal sheets, allowing it to sustain the large deformation without undergoing plastic deformation.

The restoring torque in the leafspring is largely determined by the blade geometry (length, width and thickness), its curvature in bent position and the location of the clamping points [76, 103, 106]. The geometry of the suspension was defined from an optimisation process discussed in [171, 210]. First, the blade is chosen to be bent into an almost perfect semi-circle, for ease of assembly and better repeatability. Indeed, it has been shown in [171] that a non-uniform curvature can lead to a lower resonance frequency due to the stress distribution in a non-circular leafspring creating a negative stiffness contribution in the flexure of the pivot joint, and thus a suspension mechanism with a lower

effective stiffness. This can for example be achieved by lowering the clamping position of the blade on the frame or clamping the blade at an angle from the vertical. However, this also leads to a smaller stability margin and makes the assembly of such a blade sensitive to alignment [171, 210], resulting in a poor sensor-to-sensor repeatability. Numerical modelling in METAFOR [171, 215], a FE simulation tool for large and nonlinear deformations, then defined a combination of clamping point positions and blade geometry that leads to a suspension of the proof-mass with its centre of mass in the same horizontal plane as the hinge joint. Among the various possible configurations that lead to similar suspension resonance, the one with shorter blade length and larger width is preferred as it better maintains the other flexible modes of the blade at higher frequencies. The optimisation resulted in a suspension blade of length $L_s = 114$ mm, width $w_s = 52$ mm and thickness $t_s = 0.24$ mm and the suspension geometry shown in Figure 2.16.

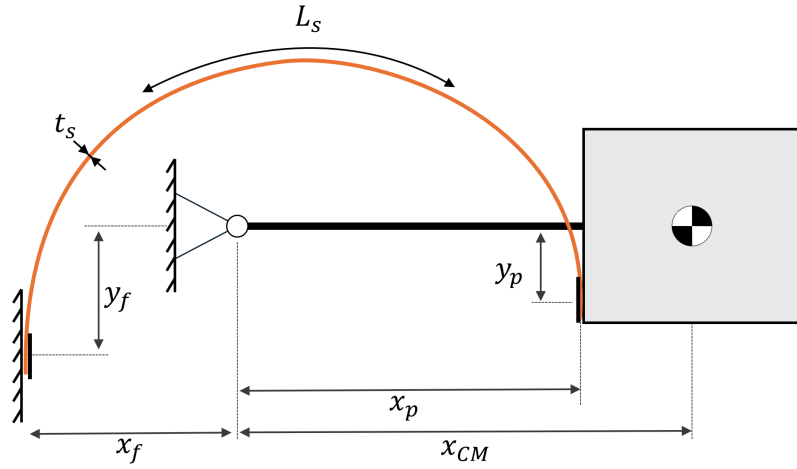


Figure 2.16: Geometry of the E-VINS leafspring suspension and positions with respect to the hinge joint. Blade geometry $\{L_s, w_s, t_s\} = \{114, 52, 0.24\}$ mm. Center of mass position $\{x_{CM}, y_{CM}\} = \{62, 0\}$ mm, frame clamping point $\{x_f, y_f\} = \{24.5, 19\}$ mm, pendulum clamping point $\{x_p, y_p\} = \{53, 18.5\}$ mm.

Assuming that the pendulum centre of mass and the hinge joint are perfectly level and a semi-circular shape for the blade, M. Zeoli [171] showed that the leafspring suspension stiffness k_s can be calculated from a balance of the gravitational and restoring torques and the energy theorem

$$k_s = \frac{\pi^2}{6} \frac{E w_s t_s^3}{L_s^3} = 99.76 \quad [\text{N/m}] \quad (2.32)$$

with $E \approx 125$ GPa the CuBe2 Young modulus, and L_s , w_s and t_s respectively the suspension leafspring length, width and thickness.

The hinge used in this assembly is a clamped blade hinge selected for its compactness and ease of assembly. The blade used in this hinge is a CuBe2 blade of similar width $w_h = 52$ mm for large torsion mode frequency, a smaller thickness $t_h = 0.05$ mm to minimise the stiffness, and the free

length of the hinge L_h , i.e. length between the two clamps, is manually adjusted to approximately 1 mm. The rotational stiffness of such a clamped blade can be calculated using usual beam theory. The hinge stiffness κ_h is then estimated as

$$\kappa_h = \frac{1}{12} \frac{E w_h t_h^3}{L_h} = 0.0677 \quad [\text{Nm}] \quad (2.33)$$

The global effective stiffness of the suspension then results from the combination of the 'linear' stiffness provided by the leafspring and the angular rotation stiffness provided by the hinge pivot. It can be fairly easily calculated from a kinematic diagram of the E-VINS suspension shown in Figure 2.17. Choosing the θ angle as the pendulum degree of freedom and the trigonometric constraint, using the small angle assumption, $x = x_p \theta$, the total restoring torque calculated at the pivot is

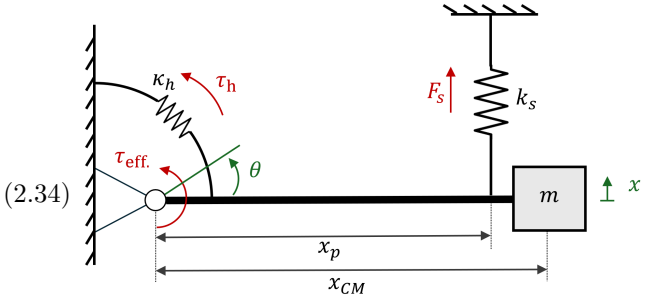
$$\begin{aligned} \tau_{\text{eff.}} &= \tau_h + F_s x_p \\ &= \kappa_h \theta + k_s x x_p \\ &= \kappa_h \theta + k_s x_p^2 \theta \end{aligned} \quad (2.34)$$


Figure 2.17: Kinematic diagram of the E-VINS leafspring suspension.

It results that the equivalent stiffness for the global E-VINS suspension is

$$\kappa_{\text{eff.}} = \frac{\tau_{\text{eff.}}}{\theta} = \kappa_h + k_s x_p^2 = 0.348 \quad [\text{Nm}] \quad (2.35)$$

Eventually, the resonance frequency of the vertical suspension mode of the proof mass is calculated as

$$f_{0,\text{E-VINS}} = \frac{1}{2\pi} \sqrt{\frac{\kappa_{\text{eff.}}}{m x_{\text{CM}}^2}} = 2.39 \quad [\text{Hz}] \quad (2.36)$$

This configuration has been verified experimentally on a testboard with adjustable blades parameters, clamping positions and proof mass centre of mass, confirming the numerical and analytical design [171, 210].

Readout assembly

The readout assembly is shown in Figure 2.18. Technical drawings can be found in [210]. The optical readout in the quadrature Michelson interferometer presented in section 1.2 (3a). Note that the cover for the optical fibre is removed in this assembly. The interferometer is mounted on a tip/tilt

stage used for aligning the laser output with the retroreflector. The interferometer is mounted on a ball pivot (3b) that sits on an interface with the static frame (3c). Two miniature adjustment screws are mounted on the interferometer plate to control the tip and tilt angles (3e). Two blade clamps are used to press on the interferometer cover and guarantee good contact with the frame (3e). Two angular degrees of freedom have been found sufficient to maximise fringe visibility and contrast. The separate interface with the static frame makes it convenient for the readout system to be assembled as a stand-alone unit, and then be integrated into the global sensor assembly.

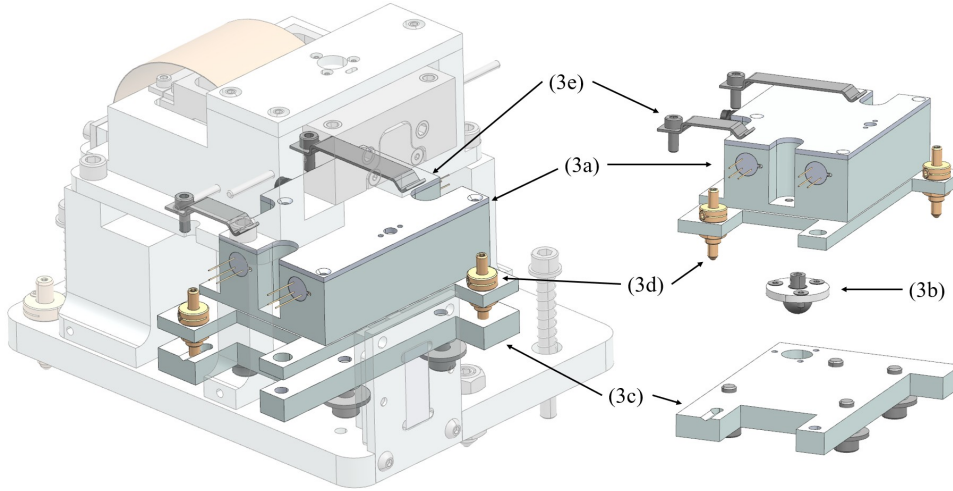


Figure 2.18: E-VINS readout assembly. Optical readout (3a), interface with the static frame (3b), adjustment feet (3c), ball pivot (3d), clamps (3e).

Actuator assembly

A contactless voice-coil actuator has also been included in the sensor design. Figure 2.19 shows the voice-coil assembly. The actuator consists of a moving magnet voice coil with the coil (4a) fixed to the sensor frame via an interface (4c) and a magnet assembly (4b) attached to the proof mass. The magnet is a custom assembly of two smaller magnets in a quadrupole configuration so as to minimise sensitivity to stray magnetic field and Eddy-current damping caused by the moving magnetic field [216–218]. The design of the quadrupole magnet is detailed in Appendix D. The voice-coil actuator has been designed to allow for closing force-balance loops or to be used in a shunt operation to provide passive damping. The main interest for closing a force-balance loop is to linearise the sensor reading by locking the proof-mass in a reference position, therefore shifting the linearity requirement from the mechanics and the optical sensor to the actuator [55–58,152,157]. This can be an asset for a sensor with an optical reading since the quadrature Michelson interferometer has been measured in section 1.3 to show a nonlinear noise with a couple of $\text{nm}/\sqrt{\text{Hz}}$ level under normal seismic activity conditions. However, since the E-VINS is primarily used in a 6D platform actively isolated with inertial feedback loops, the residual platform motion is assumed sufficiently low for the nonlinear response of the sensor to be negligible, so that the force-balance loop would

only add control noise in the sensor with no significant added value. Another application of the voice-coil actuator is to apply a shunt operation on the coil to increase passive damping. Shunting indeed consists in closing the coil wire with a resistor, therefore allowing current to circulate in the coil as the magnet moves, which is then dissipated under Joule effect [154]. This allows to dampen the sharp mechanical resonance of the proof-mass suspension, which in turn can ease the control filters design process, as inverting a sharp resonance peak is often undesired for robustness. This however comes with the trade-off of an increased mechanical thermal noise according to the fluctuation-dissipation theorem [159]. However, as it will be discussed in subsection 2.3.2, the sensor mechanical response is assumed stable enough (i.e. no significant drift in the resonance frequency nor Q -factor under eternal loads) for the characterisation and inversion process to be robust still. For these reasons, the actuator is currently unused in the operation of the inertial sensor, but is still included in the design so that either configuration can be readily enabled if necessary.

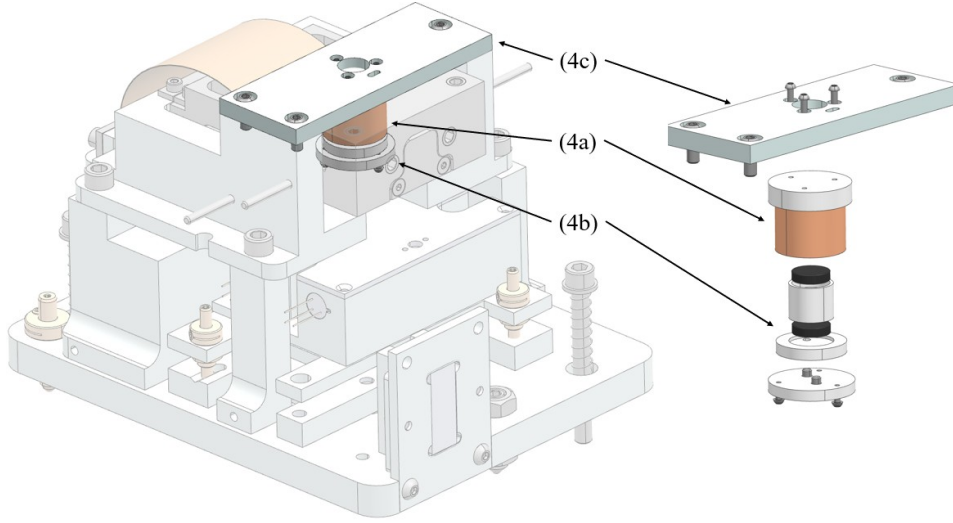


Figure 2.19: E-VINS actuator assembly. Static coil (4a), moving magnet assembly (4b), static frame interface (4c).

2.2.2 E-HINS Mechanical design

A picture of the ETEST Horizontal Interferometric inertial Sensor (E-HINS) is shown in Figure 2.20 (right), and a CAD view of the sensor with the different sub-assemblies highlighted in various colours (left). The sensor is designed combining a long-period Watt's Linkage suspension [187, 195] for the proof-mass and the high-resolution optical readout showcased in section 1.2. Table 2.2 contains references to the various components in the assembly, and technical drawings can be found in Appendix A. Similarly to the E-VINS sensor, E-HINS is designed UHV-compatible (10^{-6} mbar) and thus all sensor components comply with the same regulation (UHV-certified components and through/vented holes).

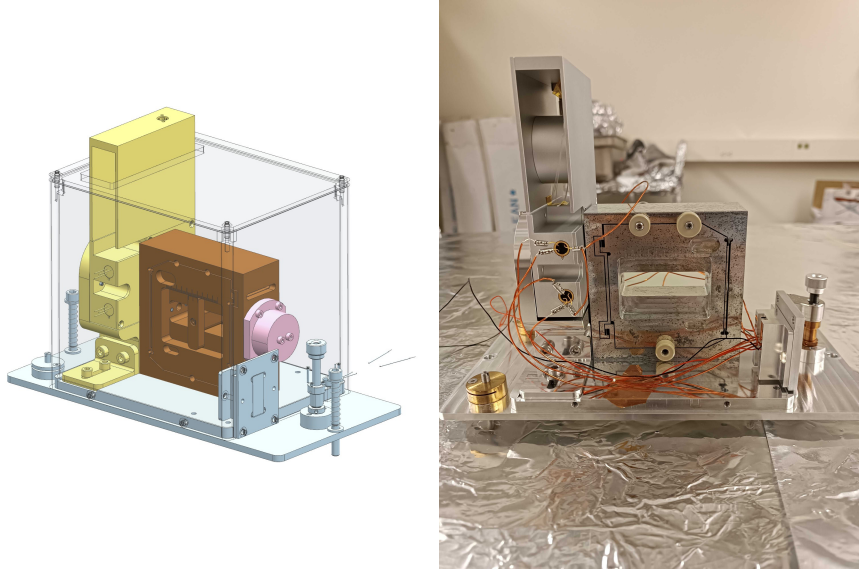


Figure 2.20: Assembled E-HINS picture in cleanroom (right). E-HINS CAD view (left). Sensor body (light blue), proof-mass assembly (dark brown), readout assembly (yellow) and actuator assembly (light pink).

Sub assembly	ID	Component	Manufacturer	Ref.
Static frame	1a	Sensor baseplate (Al.)	-	Appendix A
Static frame	1b	Connector interface	-	Appendix A
		D-sub 9 Male UHV Connector	Arun Microelectronics	VC9DM
		UHV Compatible wire	Arun Microelectronics	KW6
		UHV Wiring Accessories	Arun Microelectronics	BC1.5
Static frame	1c	Manual levelling foot	VU Amsterdam, NIKHEF	-
Static frame	1d	Motorised levelling foot	Thorlabs	PIAK10VF
		Piezo Inertia Motor Controller	Thorlabs	KIM101
Static frame	1e	Spring-loaded screws	LeeSpring	LHP038B01S
Static case	1f	Casing (Al.)	-	Appendix A
Proof-mass	2a	Watt's linkage body (Al. 7075)	VU Amsterdam, NIKHEF	-
Proof-mass	2b	Watt's linkage legs		
Proof-mass	2c	Watt's linkage flexure joints		
Proof-mass	2d	Tuning mass		
Proof-mass	2e	Retroreflector Interface (Al.)	-	Appendix A
Readout	3a	Quadrature Michelson Interferometer	EXail	section 1.2
Readout	3b	Interface (Al.)	-	Appendix A
Readout	3c	Alignment brackets (Al.)	-	Appendix A
Voice-coil	4a	Vacuum compatible VC actuator	H2W	NCC03-07-001-1V (Coil only)
Voice-coil	4b	Magnet assembly	-	Appendix D
		Neodymium N45 magnets	Supermagnets	S-12-03N
Voice-coil	4c	Static frame interface (Al.)	-	Appendix A
Voice-coil	4d	Moving mass interface (Al.)	-	Appendix A

Table 2.2: List of components in the E-HINS sensor assembly.

Static frame assembly

Figure 2.21 shows the static frame assembly. Technical drawings can be found in Appendix A. The static frame mostly consists of a baseplate on which the different sub-assemblies are attached (1a). The plate footprint is $260 \times 140 \text{ mm}^2$ and the box, not shown in the figure, is 270 mm high (including a couple of cm of free-length for the optical fibre). An interface for a D-sub 9 connector (1b) can also be fixed on the plate to ease the cabling and release stresses in the wires to the photodiodes. The connector pinout map can be found in Appendix B. The plate is mounted on a set of 3 levelling feet (1c & 1d). These are especially important in this design for adjusting the Watt's Linkage. Indeed, because of the very small gap of $0.5 (\pm 0.01) \text{ mm}$ between the linkage legs and the sensor body, a fine adjustment of the horizontality of the plate is needed to properly balance the proof-mass and prevent it from falling on either side of the suspension. Among these 3 feet, the two back ones can be adjusted manually to provide a rough alignment (1c), and the front one is motorised (1d). The latter foot is a vacuum-compatible piezoelectric inertia actuator from Thorlabs that uses a "slip-stick" motion to drive a lead screw clamped between two jaws and a piezoelectric actuator. This foot is used to provide a remote fine-levelling of the sensor when placed in a sealed vacuum tank. Finally, the sensor plate is equipped with two spring-loaded screws (1e) to guarantee good contact between the sensor and the interface on the active platform. A casing for isolation can be seen in Figure 2.20.

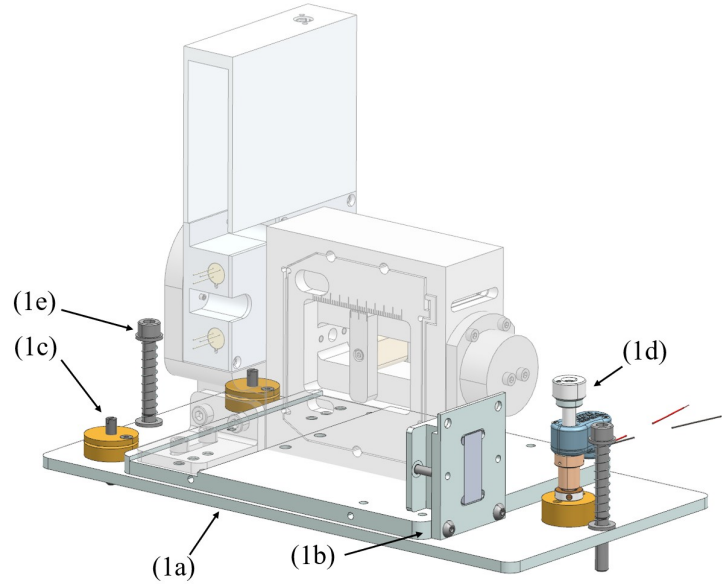


Figure 2.21: E-HINS frame assembly. Sensor baseplate (1a), connector interface (1b), manual levelling feet (1c), motorised levelling foot (1d), spring-loaded screws (1e).

Proof-mass assembly

The proof-mass assembly is shown in Figure 2.22. The core of the guiding mechanism lies in a Watt's linkage (2a), whose design is inherited from [78, 79, 199–201]. The linkage is machined out of a solid block of aluminium (alloy 7075) using precision Electrical-Discharge Machining (EDM), which allows for machining small tolerances and shapes that would not be feasible with traditional tools. The monolithic design eliminates shear effects and contact losses due to assembly of components, leading to low dissipation, high Q-factor mechanics, which is good for performance and Brownian noise reasons. It also minimises temperature sensitivity since the linkage is made of one material. The moving element of the linkage (proof-mass) sits on two pendulums (2b), one traditional pendulum (P, left leg in the figure) and one inverted pendulum (IP, right leg in the figure), which combination of both gives a horizontal translation guiding with low natural frequency. The pivot joints of the pendulums are notch joints (2c) with a very small thickness of $0.1 (\pm 0.01)$ mm, which is made possible with EDM, and a length of 5 mm. The flexures design, especially the "S-shape" design for the IP leg (2c'), is such that all joints operate in tension, allowing the design of thin flexures that would bulk if used in compression. A tuning mass (2d) can be used to tune the linkage resonance frequency by shifting the centre of mass to either side of the pendulum or the inverted pendulum. The retroreflector is mounted on the moving body of the linkage (2e).

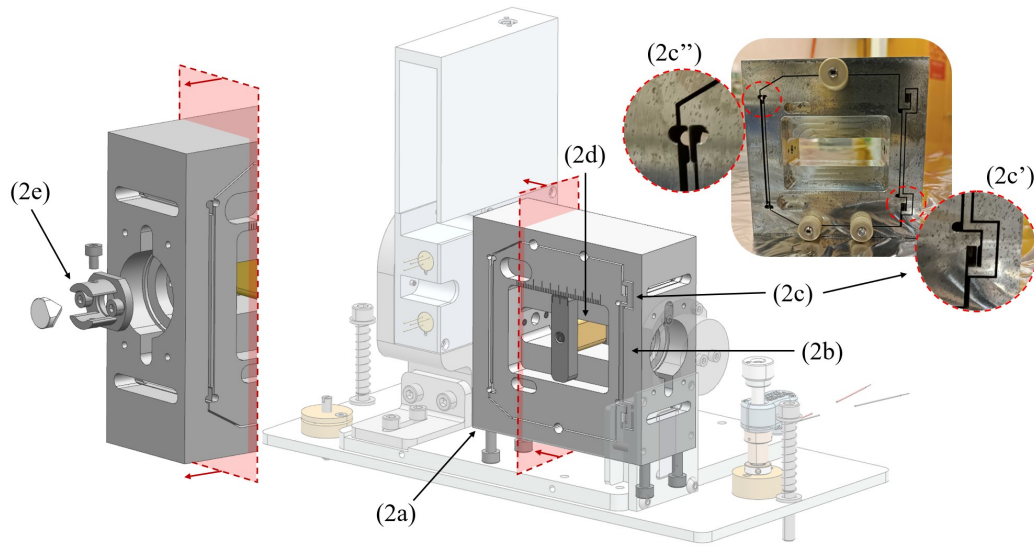


Figure 2.22: E-HINS proof-mass assembly. Watt's Linkage (2a), Watt's Linkage leg (inverted pendulum) (2b), notch flexure joint (2c), tuning mass (2d), retroreflector and interface (2e).

Watt's linkages are well-established mechanisms that have been known for decades [192]. They have however only recently been used as suspension/guiding mechanisms in seismometers and accelerometers, especially in the field of gravitational wave detection [78, 79, 107, 187, 195, 196, 199–201]. Figure 2.23 shows a kinematic model of a Watt's linkage. The linkage is represented as a rigid link (thick black horizontal link) that connects a pendulum (O) and an inverted pendulum (O'). The

proof mass weight is distributed on both pendula as m_P^0 and m_{IP}^0 , and M represents the tuning mass that can be tuned to shift more load on either legs. D defines the position of the mass with respect to the pendulum leg O , S being the length of the linkage. The mass and inertia of each leg is also taken into consideration m_P^c and m_{IP}^c , and I_P^c and I_{IP}^c , which are not necessarily the same since the design of the inverted pendulum leg involve more material to use its flexure joint in flexion. Both pendulums are assumed to have similar dimensions, with L being the total length of the leg and L_P the effective length from flexure-to-flexure.

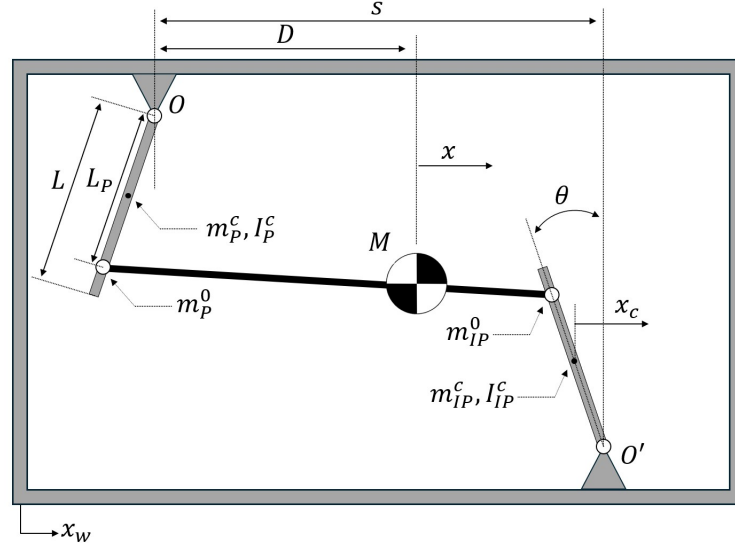


Figure 2.23: Simplified kinematic model of a Watt's linkage.

The kinematics of the mechanism was studied by solving the Lagrangian equations. Below is a synthetic procedure, the full calculations may be found in [193,194,201]. First, the effective load on each pendulum for a given position of the tuning mass M is

$$\begin{aligned} m_P &= m_P^0 + M \left(1 - \frac{D}{S}\right) \\ m_{IP} &= m_{IP}^0 + M \left(\frac{D}{S}\right) \end{aligned} \quad (2.37)$$

The linkage potential and kinetic energy V and T , respectively, can be evaluated as

$$\begin{aligned} \mathcal{T} &= \frac{1}{2} (I_P + I_{IP}) \dot{\theta}^2 + \frac{1}{2} (m_P^c + m_{IP}^c) \dot{x}_c^2 + \frac{1}{2} (m_P + m_{IP}) \dot{x}^2 \\ \mathcal{V} &= \frac{1}{2} \left(\frac{1}{2} (m_P^c + m_{IP}^c) g L + (m_P + m_{IP}) g L_P + \kappa_{\text{flex.}} \right) \theta^2 \end{aligned} \quad (2.38)$$

with $\kappa_{\text{flex.}}$ the effective stiffness of the flexures (notch joints). θ , x and x_c are the degrees of freedom of the linkage, i.e. the legs' angular position w.r.t. vertical, the legs' centre of mass

translation and the proof-mass translation. The linkage kinematic constraints however write

$$\begin{aligned} x_c &= x_w - \frac{1}{2}L\theta \\ \theta &= \frac{x_w - c}{L_P} \end{aligned} \quad (2.39)$$

with x_w the frame motion, i.e. seismic motion. This leaves only 1 degree of freedom for the linkage, which can be chosen as the proof-mass translation x . Solving the Euler-Lagrange equation, with the Lagrangian $\mathcal{L} = \mathcal{T} - \mathcal{V}$ for x allows to extract the equation of motion from x_w to x .

From the equation of motion, the linkage resonance frequency can be estimated as

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{(m_P^c - m_{IP}^c) \frac{gL}{2L_P^2} + (m_P - m_{IP}) \frac{g}{L_P} + \frac{\kappa_{flex.}}{L_P^2}}{(m_P^c + m_{IP}^c) \frac{L^2}{3L_P^2} + (m_P + m_{IP})}} \quad (2.40)$$

It can be seen that the natural frequency is set by the balance of the positive gravitational stiffness of the pendulum, the negative gravitational stiffness of the inverted pendulum (hence the negative sign for the IP terms), and the restoring stiffness of the flexures. Theoretically, the tuning mass could be tuned such that the pendulum and inverted pendulum mass cancel, and the effective stiffness can be as low as the notch joints' stiffness. In practice this is however difficult to reach as the linkage would operate very close to an instability regime, i.e. global negative stiffness, reached for $(m_P^c - m_{IP}^c) \frac{gL}{2L_P^2} + (m_P - m_{IP}) \frac{g}{L_P} < \frac{\kappa_{flex.}}{L_P^2}$.

The calculation of the E-HINS Watt's linkage natural frequency would therefore require knowledge of the legs' mass and inertia, and the exact distribution of the weight on each leg. A rough estimation can be obtained assuming equal weight distribution and massless legs. On the other hand, the effective flexures stiffness can be calculated using beam theory. Note that the notch flexure is assimilated to a clamped-blade due to its very large length-to-radius ratio, making it resemble a 'monolithic clamped-blade' more than a circular notch. Otherwise, the Tseytlin formula would be more appropriate [200, 213].

$$\begin{aligned} \kappa_{flex.} &= 8 \frac{Ewt^3}{12l} \\ &\approx 0.047 \quad [\text{Nm}] \end{aligned} \quad (2.41)$$

with $E = 71.7$ GPa Al 7075 Young modulus, $w = 5$ mm the flexures width, the thickness $t = 0.1$ mm stiffness and the length $l = 5$ mm. The coefficient 8 reflects the eight flexures in the mechanism, i.e. 2 flexures per joint, 2 joints per leg and 2 legs.

The resonance frequency of the E-HINS Watt's linkage can therefore be calculated as

$$f_{0,E-HINS} = \frac{1}{2\pi} \sqrt{\frac{\kappa_{flex.}}{L_P^2 m_{tot.}}} = 0.609 \quad [\text{Hz}] \quad (2.42)$$

with the pendulum length $L_P = 65$ mm and the total proof-mass weight $m_{\text{tot.}} = 765$ g, calculated from the proof-mass volume and the Al 7075 density.

Readout assembly

Figure 2.24 pictures the readout assembly. It consists of the optical readout showcased in section 1.2 (3a), a mechanical interface with the static frame (3b) and a translation mechanism for alignment (3c). As exposed previously, 2 degrees of freedom are sufficient for guaranteeing proper alignment of the interferometric device with respect to the retroreflector. The mechanical interface onto which the readout is screwed is therefore drilled with vertically aligned oblong holes, providing a manual vertical adjustment capability. The readout and interface assembly then sits in a slot machined in the sensor baseplate (1a) on which it can glide, so as to provide a horizontal translation adjustment capability.

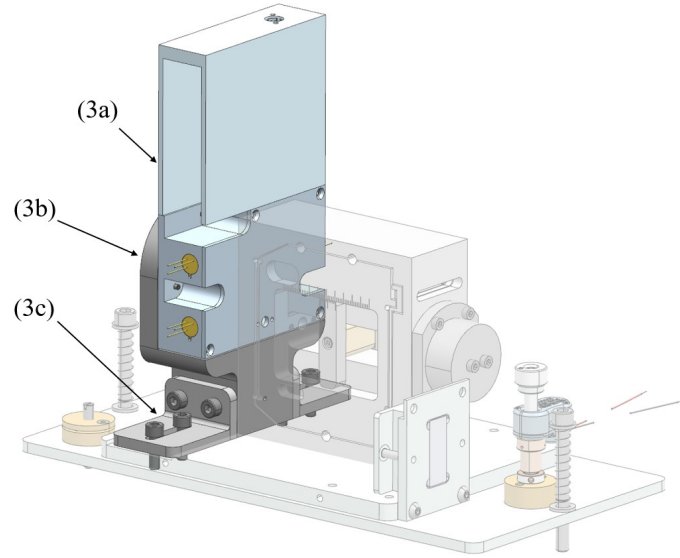


Figure 2.24: E-HINS readout assembly. Optical readout (3a), mechanical interface (3b), alignment brackets (3c).

Actuator assembly

Similarly to the E-VINS sensor, an interface for a contactless, moving-magnet, voice-coil actuator has been included in the design. It is shown in Figure 2.25 below. The voice-coil actuator uses a commercial vacuum-compatible coil from H2W (2a) coupled with a custom quadrupole assembly of Neodymium magnets (4b). The coil is static and interfaced with the frame of the Watt's linkage (4c), and the magnet is interfaced with the moving proof-mass (4d). The design of the voice-coil actuator is the same as in the E-VINS sensor. Details can be found in Appendix D. The same reasons mentioned for the E-VINS sensor drove the integration of a voice-coil actuator: i.e. having

the possibility to operate the E-HINS sensor in a force balance accelerometer mode for a larger bandwidth and better linearity [55–58, 152, 157], or shunting the coil for higher passive damping. But again, the balance between control noise injection due to the feedback loop and the seismically quiet operating environment of the sensor made the use of force-balance architecture unnecessary. The voice-coil actuator therefore is not used in the current version of the sensor.

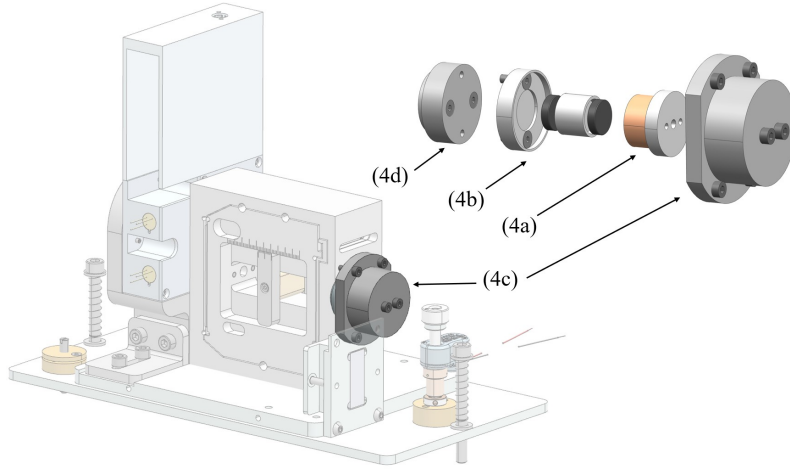


Figure 2.25: E-HINS voice-coil assembly. Static coil (4a), moving magnet assembly (4b), static frame interface (4c), moving mass interface (4d).

2.3 Characterisation

Inertial sensors are characterised by two main specifications: their sensitivity response function and their resolution. Properly characterising the sensor is of primary importance for a correct processing and interpretation of the seismic data. Additionally, the sensor can show sensitivity to external environmental factors, such as temperature, acoustics or magnetic field, which should also be anticipated, and ideally shielded or accounted for when processing raw data. Characterisation of seismometers is discussed in several references [56–58,155]. While these well-established methods have been shown to be successful in identifying most standard seismometers’ specifications, characterising low-frequency sensors with sub-pm resolution readout systems brings additional challenges. Indeed, the sensitivity of the sensor to various external loads or cross-directions is typically enhanced for sensors with long mechanical periods [154], and the nonlinear response of the interferometric reading also often spoils the quality of the measurements. For this reason, testing the sensor in a stabilised environment and ideally isolated from external loads (e.g. in a vacuum vessel) is best for a good quality characterisation.

This section will present first the testing environment for characterising the E-VINS and E-HINS inertial sensors. The sensor’s sensitivity response functions will then be identified, and sensitivities to other transverse directions and tilts are also assessed. The sensor’s resolution is next quantified from a coherent subtraction and compared with a model of the sensor’s theoretical resolution and the sensor’s sensitivity to external factors is also investigated.

2.3.1 Test environment

Because of the high sensitivity of the optical readout and of the long-period mechanical suspension of the inertial sensors, characterisation is best performed in an actively stabilised environment isolated from external loads. Most of the E-VINS and E-HINS characterisation has therefore been made in one of the HAM-ISI systems at the LIGO-MIT Lab. The tests took place in summer (July–September) 2024, discussion and plots on the performance of the HAM-ISI refers to the status of the system in this time window.

The LIGO detectors require exceptional isolation and positioning accuracy of the suspended optics to meet the stringent requirements of gravitational-wave detection. Requirements on the isolator dimensions, loading capacity (up to 800 kg), structural stiffness (flexible modes above 100 Hz) and isolation performance have been set in early 2001 [219]. In-vacuum isolation systems have been developed accordingly [40,220]. For isolation from external factors, all isolators and optics are sealed in HAM (Horizontal Access Modules) or BSC (Basic Symmetric Chambers) vacuum chambers. The core optic is supported by two-stage active ISI (Internal Seismic Isolation) systems placed in BSC chambers, named BSC-ISI. The auxiliary optics are supported by single-stage active ISI systems placed in HAM chambers, named HAM-ISI. Both the active platforms of the BSC-ISI and HAM-ISI systems sit on passive HEPI (Hydraulic External Pre-Isolator) stages for additional isolation. The

LIGO-MIT Lab in Boston owns a replica of the LIGO detectors, with shorter arm-length (≈ 10 m) but similar isolation systems, used for research and development purposes. For ease of access and mounting, 2 E-VINS and 2 E-HINS seismometers have been placed on top of the HAMX-ISI table supporting the auxiliary optics in the X-arm.

Installation in the HAM-ISI system

The HAM-ISI is a six-axis passive-active UHV-compatible isolator, capable of carrying more than 500 kg of payload and stabilising it with nanometre resolution [40, 220]. 2 E-VINS and 2 E-HINS sensors have been mounted in the top platform of the HAMX-ISI system in the LIGO-MIT Lab. Figure 2.26 shows pictures of the sensors screwed tight on the HAMX-ISI top platform. The sensors' sub-assemblies have been individually UHV-cleaned with isopropyl baths and baked before being reassembled and installed. The vertical inertial sensors have been centred as best as possible to align with the Z-axis of the HAM-ISI table, and the horizontal sensors have been aligned with the Y-axis.

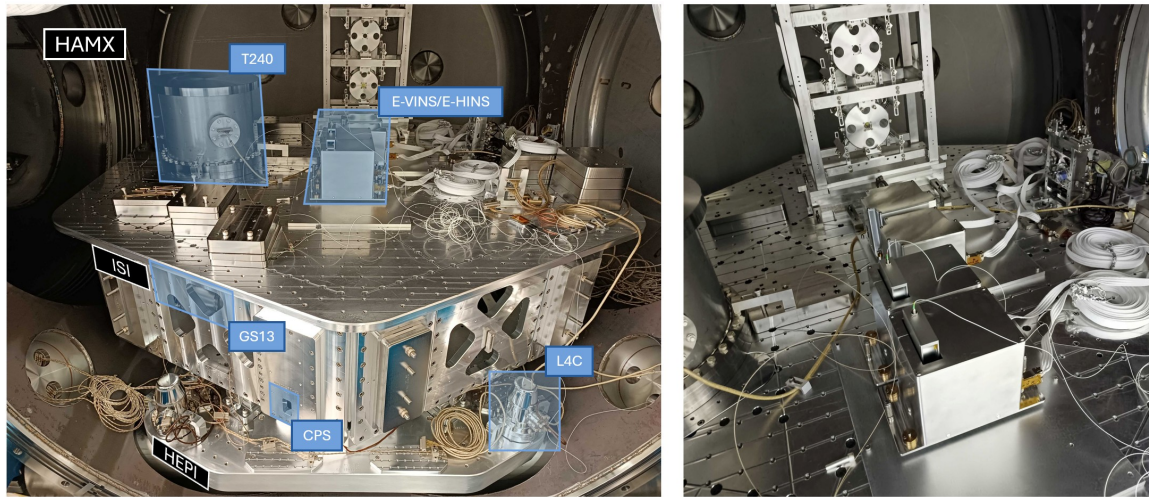


Figure 2.26: Test environment for characterising the E-VINS and E-HINS seismometers. (Left) Picture of the ISI platform in the HAMX vacuum chamber. (Right) Close-up view of the E-VINS and E-HINS sensors installed.

Isolation performance

The HAM-ISI system uses several isolation mechanisms. A simplified schematic of the HAM-ISI system and control architecture is shown in Figure 2.27 [40]. First, the HAM-ISI system is equipped with a passive hydraulic stage (HEPI) connected to ground via bellows through the HAM chamber. The HEPI stage provides passive isolation above typically 10 Hz. Hydraulic damping can be enabled to damp the stage resonances. The active ISI stage is suspended on the HEPI stage using blade springs and flexures, with suspension modes at typically 1 Hz. The ISI is instrumented with various sensors. GS13 seismometers are mounted inside the stage to measure the ISI motion in all 6 degrees of

freedom and drive feedback control loops to voice-coil actuators. High resolution CPS (Capacitive Position Sensors) measure the relative motion between the ISI and the HEPI stage, and can be blended with the signals of the GS13s to minimise noise magnification at low frequency. L4C seismometers are mounted on the HEPI stage and can drive feedforward isolation loops. Eventually, an STS2 sensor placed outside of the vacuum system is used to apply sensor correction on the CPSs to further boost performance at sub-Hz frequencies. A T240 long-period seismometer is mounted on top of the ISI platform and is used as an out-of-loop witness of the ISI residual motion.

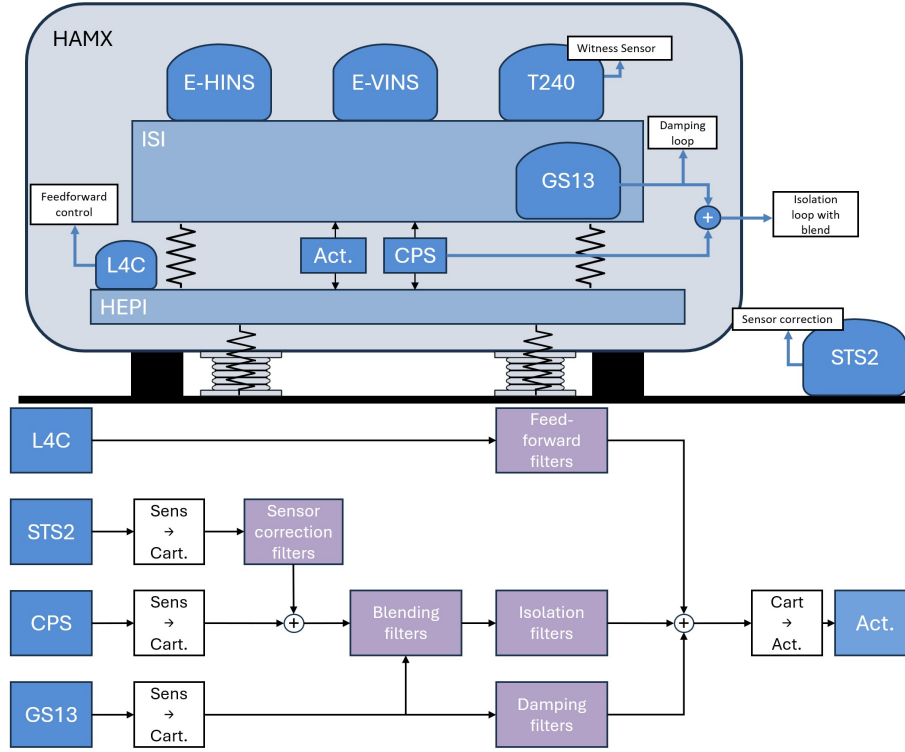


Figure 2.27: Schematic of the HAMX-ISI system and control architecture [40].

The various isolation mechanisms can, and should, be engaged sequentially, resulting in 4 different isolation levels: no control engaged ("Cont off"), damping loop engaged only ("Damp"), damping and isolation loop engaged with CPS blend (" +ISO loop") and isolation loop enhanced with STS2 sensor correction (" +Sens cor"). Unfortunately, the feedforward loop was not operational for these tests. This is presumably because the HEPI hydraulic damping was not available, resulting in sharp, undamped HEPI modes that complicate the design of the feedforward filters. Isolation performance of the HAM-ISI system is quantified as the transmission of ground motion, measured by the STS2 sensor placed on the floor, and the residual motion of the HAM-ISI table, measured by the T240 witness sensor. Figure 2.28 shows the transmissibility transfer functions of the HAMX-ISI in the Z- and Y-directions with the different control loops engaged. The transmissibility plot with no control loop engaged (blue trace) shows the f^{-2} passive isolation above the HAM-ISI suspension modes

at 1.8 Hz in the Z-direction and 1.3 Hz in the Y-direction, and an additional decay above 16 Hz corresponding to the HEPI mode. The damping control loop (orange trace) is effective at killing the amplification at the HAM-ISI resonance modes. Engaging the isolation loop with the CPS blend (yellow trace) provides two orders of magnitude of isolation in the [1 - 10] Hz bandwidth. Isolation performance degrades below 1 Hz due to the loss of resolution of the GS13 sensors. Adding sensor correction from the STS2 sensor (purple trace) allows extending the isolation bandwidth down to a couple of hundred mHz. Below 100 mHz, the HAM-ISI motion is amplified by control noise and couplings.

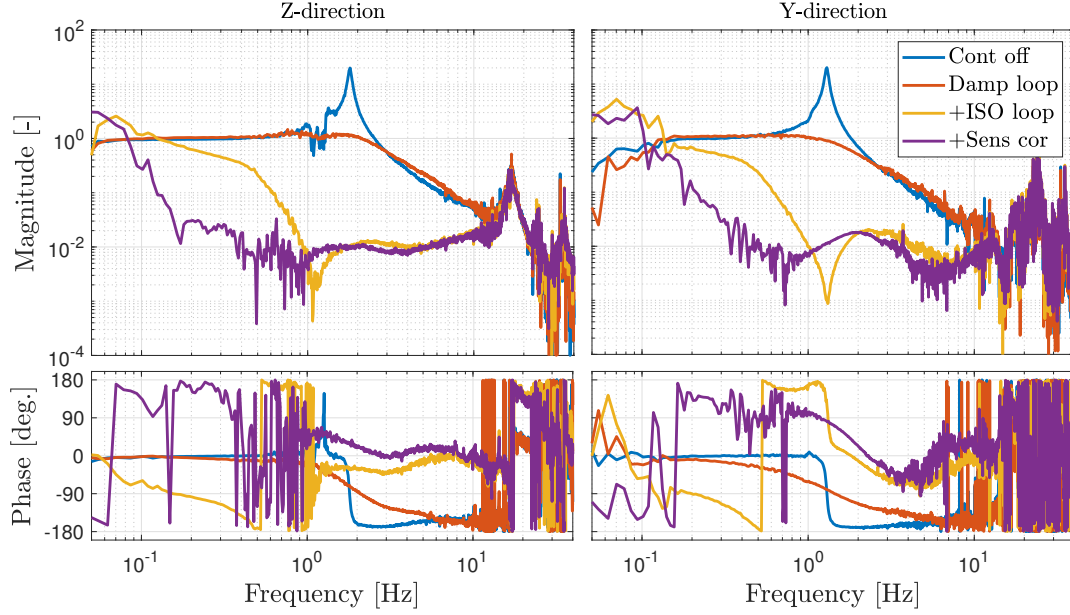


Figure 2.28: Isolation performance of the HAMX-ISI platform. Transmissibility transfer functions in the Z- and Y-directions from the STS2 sensor placed on ground and the witness T240 sensor placed on the HAMX-ISI platform, for increasing isolation levels. (Blue) No control loop engaged. (Red) Damping loops engaged. (Yellow) Isolation loops engaged. (Purple) Sensor correction filters engaged.

Figure 2.29 shows spectrums of the residual motion of the HAMX-ISI platform measured by the witness T240 sensor, for the various isolation levels. Ground motion spectrum (black trace) is shown as a reference. The plots show a similar conclusion as previously, i.e. approximately 2 orders of magnitude of isolation in the [0.1 - 16] Hz band, and even larger isolation above HEPI suspension mode at 16 Hz. The spectrum also allows for calculating the residual RMS platform motion. The RMS is calculated by integrating the spectrum in the [0.1 - 10] Hz bandwidth, as this is the bandwidth of interest for characterising the optical inertial sensors. The RMS motion is evaluated at approximately 0.8 μm when no control loops are engaged, dominated by the microseismic peak and/or the HAM-ISI resonance, 0.5 μm with the damping loop engaged, 100 nm with the isolation loop, and 10 nm with the isolation loop augmented with sensor correction.

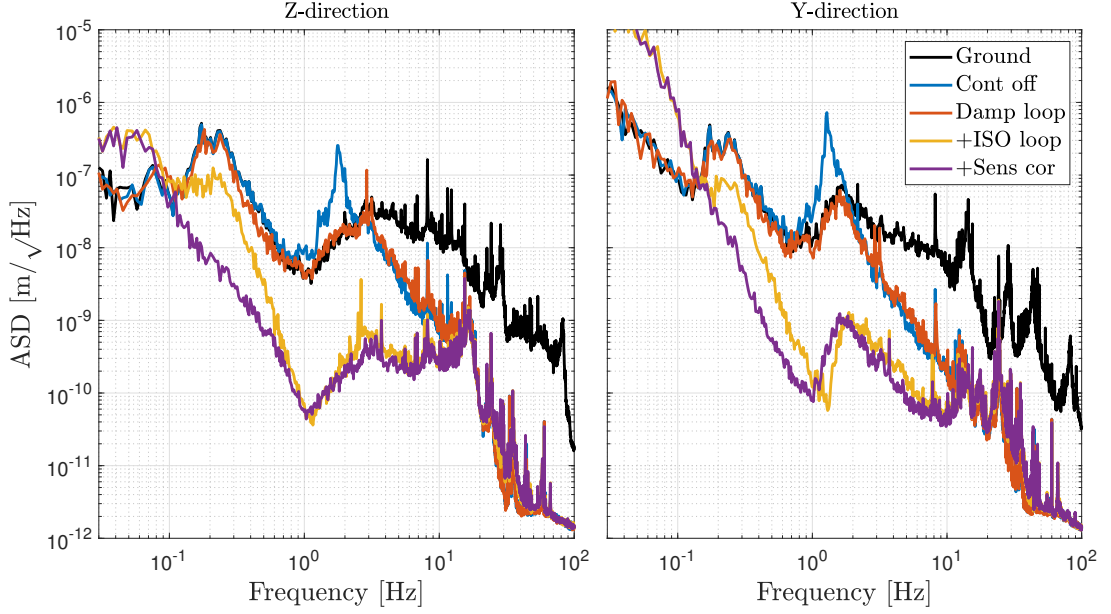


Figure 2.29: (Coloured lines) HAMX-ISI platform residual motion spectrum measured by the witness T240 sensor placed on the ISI platform in the Z- and Y-directions, for various isolation levels. (Blue) No control loop engaged. (Red) Damping loops engaged. (Yellow) Isolation loops engaged. (Purple) Sensor correction filters engaged. (Black) Ground motion spectrum measured with the STS2 sensor.

The above specifications defined the testing environment used for characterising the sensitivity function and resolution of the E-VINS and E-HINS sensor. Depending on the calibration test, control loops can be decided to be engaged or left disabled. When the vacuum system is turned on, the vacuum level in the HAMX-ISI platform is 10^{-2} mTorr ($\approx 10^{-5}$ mbar).

2.3.2 Sensitivity

As discussed in the first section, an inertial sensor is essentially a device that uses the dynamics of a 1 degree of freedom system (proof-mass suspension) to convert absolute seismic motion to a measurable relative displacement. Therefore, the sensor sensitivity cannot be described with a simple scalar conversion value from motion to voltage reading, e.g. [V/m], but rather a frequency-dependant scaling factor that represent the combination of readout sensitivity (in [V/m] - typically frequency independent), and the proof-mass suspension mechanical response to ground motion (in [m/m] - frequency dependant). This scaling factor is often named sensitivity response function.

The gain of the optical readout used in the E-VINS and E-HINS sensors is however not trivial to determine. Indeed, since the interferometer requires the demodulation of 3 voltage signals to produce 1 displacement reading, the optical gain cannot be expressed as simple linear metre to voltage conversion. Instead, the demodulation process exposed in subsection 1.2.3 can be considered as part of the optical readout so that the demodulated output is readily in unit of [m], and the

sensor sensitivity function solely consists in the mechanical response of the sensor. For the sake of the explanation, the interferometer gain can be understood as being made of two contributions. The first one is the conversion of the photodiode voltages to interferometric phase ([rad/V]), and the second is the conversion of interferometric phase to displacement ([m/rad]). The later scaling factor is simple to determine and depends on the optical layout. It has been shown in section 1.2 to equal $8\pi/\lambda \approx 1.62 \cdot 10^7$ rad/m, with $\lambda = 1550$ nm. The former however is more complex to identify as it involves the calculation (i.e. various summations) and demodulation (i.e. trigonometric operations) on multiple signals. It also depends on the quality of the alignment and fringe visibility, and therefore is specific to each individual device and opto-mechanical assembly. These are accounted for in the calibration, i.e. normalisation and ellipse correction, of the interferometer signals.

Assimilating the sensor to a 1 degree of freedom system, the theoretical Laplace formulation of the sensitivity response function of the E-VINS and E-HINS sensors is expressed as the proof-mass suspension mechanical response to ground motion ([m/m])

$$S_{\text{E-XINS}} = g_{\text{IFO}} \cdot \frac{-s^2}{s^2 + \frac{2\pi f_{0,\text{E-XINS}}}{Q_{\text{E-XINS}}} s + (2\pi f_{0,\text{E-XINS}})^2} \quad (2.43)$$

with $s = i2\pi f$ the Laplace variable, g_{IFO} the optical readout gain ($= 1$ if the demodulation is included in the pre-processing), f_0 the mechanical resonance frequency of the proof-mass suspension ([Hz]), Q the mechanical quality factor ([-]) and E-XINS refers for either E-VINS or E-HINS.

A simple experimental identification of a seismometer sensitivity consists in recording the voltage reading of the seismometer placed side-by-side with another reference seismometer of known sensitivity response. Calculating the transfer function between both seismometer and correcting for the sensitivity of the reference instrument therefore yields the sensitivity response of the first seismometer. The E-VINS and E-HINS sensitivities have been measured on the HAMX-ISI, in vacuum, using the T240 witness sensor as a reference. The HAMX-ISI damping loops have been engaged to minimise nonlinear response due to the undamped ISI motion, but the isolation loops were disabled to maintain high SNR on both sensors. Sensitivity response functions are shown in Figure 2.30, and are calculated as the transfer function from each individual sensor to the co-aligned T240 sensor axis, and corrected for the T240 sensitivity response. The plot also shows the coherence between each sensors and the corresponding T240 axis. The plot shows great coherence in the 70 mHz to 20 Hz bandwidth which mostly corresponds to the target frequency band of the sensors. Above 20 Hz, the T240 sensor has been measured to touch its intrinsic noise level, and below 70 mHz environment noise is suspected to be a limitation. Within this bandwidth, the measurements show very clean sensitivity functions with no spurious poles and great repeatability, for both E-VINS and E-HINS sensors. 1 DOF model fits to the measured sensitivities lead to a resonance frequency $f_{0,\text{E-VINS}} = 1.453$ Hz and a Q -factor of 200 for the E-VINS sensor, and $f_{0,\text{E-HINS}} = 0.515$ Hz and a Q -factor of 50 for the E-HINS sensor. These values are in very good agreement with the modelling in section 2.2, with relative errors of 3.5% for the E-VINS calculation and of 6.5% for the E-HINS calculation.

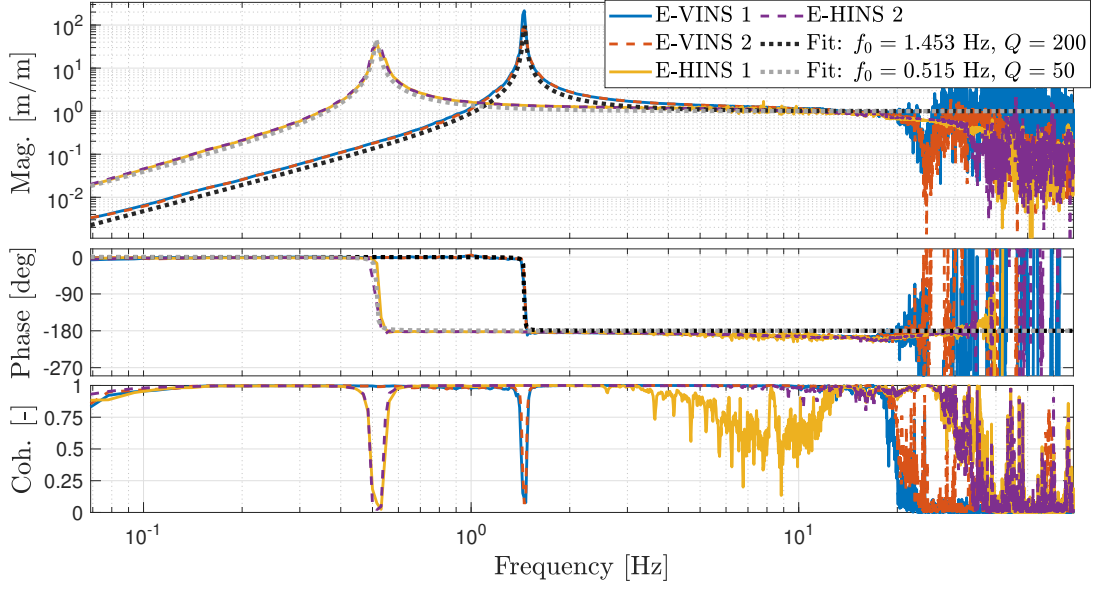


Figure 2.30: E-VINS and E-HINS sensitivity response function measured with respect to a reference T240 seismometer. (Solid blue and dashed red) Measured E-VINS's sensitivity function. (Solid yellow and dashed purple) Measured E-HINS's sensitivity function. (Dotted black and grey) 1 DOF fit model for both sensors.

In fine, the sensitivity response function for the E-VINS and E-HINS sensors write

$$\begin{aligned}
 S_{\text{E-VINS}} &= g_{\text{IFO}} \cdot \frac{-s^2}{s^2 + 0.0456s + 83.347} & [\text{rad/m}] \\
 S_{\text{E-HINS}} &= g_{\text{IFO}} \cdot \frac{-s^2}{s^2 + 0.0647s + 10.471} & [\text{rad/m}]
 \end{aligned} \tag{2.44}$$

with the gain of the interferometer optical gain $g_{\text{IFO}} = 8\pi/\lambda \approx 1.62 \cdot 10^7 \text{ rad/m}$, or $g_{\text{IFO}} = 1 \text{ rad/m}$ if the optical gain is included in the real-time demodulation of the fringes.

An alternative experimental characterisation would consist in measuring the ringdown response of the sensor to an impulse either inputted with a shake-table, or applied on the proof-mass directly, and extract the sensor resonance frequency and Q -factor. This method has the advantage of not relying on the use of an external reference instrument and provide arguably better estimation of the mechanical characteristics. However, this method assumes a purely 1 DOF system while the above measurements provide characterisation over the full bandwidth. Providing that the measurement is long enough and that the resonance peak is not too sharp, the spectral resolution can be assumed high enough for an accurate fit of a frequency response.

Cross-axis sensitivity

Cross-axis sensitivity represents the sensitivity of a seismometer to motions in transverse directions. Minimising cross-axis sensitivity is crucial for the correct interpretation of the reading of the sensor. In active inertial isolation especially, cross-sensitivity to tilt, or tilt-to-translation coupling, in inertial sensors leads to a misinterpretation of tilt payload motion as a translation motion, which propagates throughout the control loops. This commonly leads to magnification of the payload motion, especially in the lower (100 mHz-like) frequency range [221, 222].

Minimal sensitivity to most cross-directions can be achieved with a good design and alignment of the inertial sensors' suspension and guiding mechanism. For ease of visualisation, Figure 2.31 shows the E-VINS and E-HINS sensor CAD onto which a Cartesian coordinate system has been overlaid. The main sensing axis is shown in red. The clamped-blade hinge joint used in the E-VINS sensor makes the proof-mass mainly sensitive to Z-translation, and mechanically insensitive to X-translation along the direction of the clamping of the hinge and to RZ- and RY-rotation. Provided that the sensor is correctly levelled, i.e. that the proof-mass centre of mass lies in the same horizontal plane as the hinge joint, the E-VINS sensor is also mechanically insensitive to the Y-translation. Similarly, the E-HINS mechanics is mainly sensitive to Y-translation, and insensitive to X-translation, RX- and RY-rotation, and to translation Z if the sensor is well levelled, i.e. if both notch flexures of the same leg are in the same vertical plane. For both sensor mechanics, there exists however one rotation motion that still couples in the direction of the reading, being RX (i.e. rotation along the joints/flexures). This is known as the tilt-to-translation coupling corresponding to translation motion of the proof-mass, when the sensor support tilts in a gravitational field [153–155].

In this regard, the E-HINS and E-VINS sensors should be treated separately. The E-HINS sensor cannot distinguish linear acceleration to tilt with respect to the gravity direction. This is due to the Principle of Equivalence. This is fundamental physics, and no mechanism or guiding can prevent this coupling. Instead, the single-blade and joint combination used in the E-VINS sensor causes direct mechanical coupling to rotation. There also exists a residual sensitivity to RX associated to the pivot stiffness. These can be mitigated by design, for example using parallel guidance mechanism which claims to only show 2nd order sensitivity to tilt. In addition, the E-VINS sensor also show mechanical coupling to horizontal direction Y in case of misalignment between the proof-mass centre of mass with the pivot joint.

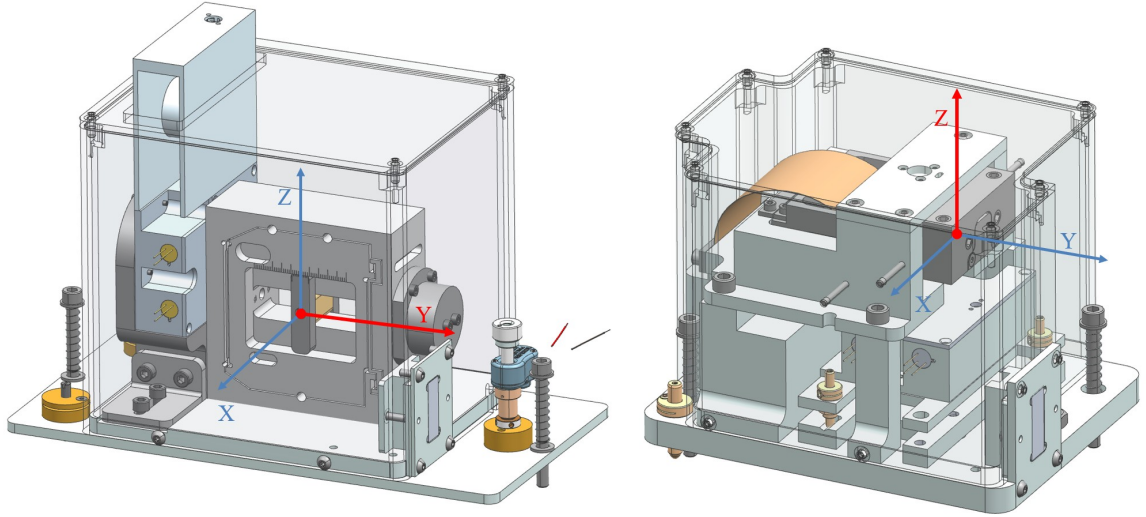


Figure 2.31: (Right) E-VINS and (Left) E-HINS sensors' sensitivity axes. (Red arrows) Main sensing axis and (Blue arrows) cross-axes.

In order to have a sensible measure of coupling, transfer function and coherence between the optical inertial sensors and the damped HAMX-ISI motion is monitored. They are shown in Figure 2.32. First, the E-VINS and E-HINS sensors indicated fairly negligible coherence with the Y and Z motion (green and light blue traces), respectively, indicating a good quality of the levelling of the inertial mass. Only below some 100 mHz frequencies can some residual coupling be seen from the coherence. Then, transfer functions from RX to the E-HINSs and E-VINSs reading directly expose tilt to horizontal coupling. The transfer functions are corrected for the sensitivity functions of the optical seismometers, so that they represent the ratio of output displacement to tilt. The tilt-translation transfer functions show a coupling that increases towards lower frequencies, as expected from the nature of the coupling. At frequencies of 300 mHz and below, the optical sensor's response to tilt nearly equals the response to its nominal axis. At higher frequencies, the E-VINS's sensors surprisingly measured a significantly larger tilt contribution to the reading than the E-HINSs, as indicated by the coherence and magnitude of the transfer functions.

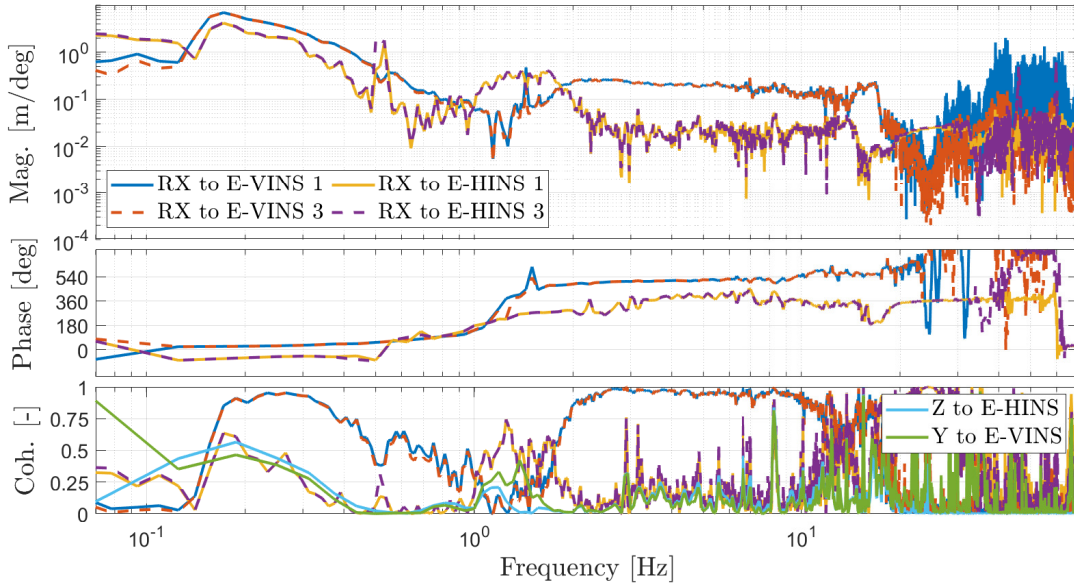


Figure 2.32: E-VINS and E-HINS tilt-to-translation couplings assessed as the transfer functions and coherences from the HAMX-ISI tilt motion (RX) to the reading of the optical inertial sensors. (Solid blue and dashed red) Sensitivity of E-VINSs to rotation around the pendulum hinge axis. (Solid yellow and dashed purple) Sensitivity of E-HINSs to rotation around their transverse axis direction. (Solid green) Coherence between Z-translation and E-HINS reading. (Solid light blue) Coherence between Y-translation and E-VINS reading.

Note that these results need to be taken with a degree of caution. Indeed, the HAMX-ISI motion is measured using GS13 seismometers. The parallel guidance design of the GS-13 seismometers helps in minimising cross-axes coupling, making it a second order effect. The ISI platform, however, still suffers from mechanical couplings due to non-collocated centre of actuation, centre of mass and centre of rotation. Figure 2.32 therefore contains a mix of several coupling mechanisms, and most certainly overestimates the couplings of the E-VINS and E-HINS sensors. Best characterisation of the cross-axis sensitivities would be obtained using a high-resolution 6D shake table, which would be capable of driving each transverse direction individually with minimal couplings to other degrees of freedom.

Dampening the resonance peak

As it can be seen in Figure 2.30, the E-VINSs and E-HINSs have been measured to be characterised by a fairly sharp resonance peak with Q -factors of 200 and 50, respectively. In order to reconstruct actual motion and drive the control system, an inversion of the sensor dynamics is often required. However, inverting high- Q modes is not a robust operation and is often not desired, as a correct inversion needs an accurate characterisation of the resonance frequency and Q -factor, which depend on the mechanical alignment of the sensor, and may drift in time due to vacuum, temperature and

mechanical levelling. In order to ease the design of the control filters, the E-VINS and E-HINS sensors are equipped with voice-coil actuators that can be used to damp the short resonance peaks with a shunt operation.

Shunting consists in closing the wires of the actuator coil with a resistive element. A schematic of a resistive (and capacitive) shunt is shown in Figure 2.33 (a), and a picture of the shunt operation on a E-VINS sensor is shown in (b). Shunting makes the coil a closed circuit, which therefore allows current to circulate in the coil. As the magnet moves, the varying magnetic field induces an electric current in the coil proportional to the magnet velocity, as per the Faraday law. In turns, this current circulation generates a magnetic field, accordingly to the Biot-Savart law, which opposes the motion of the magnet, therefore inducing a velocity-dependent force, i.e. damping. Another way to understand shunting, from an energy point of view, is to consider that mechanical energy is transformed into electric energy by induction via the coil/magnet system. It is then dissipated in the resistor, creating damping. This strategy has been implemented on an E-VINS optical sensor.

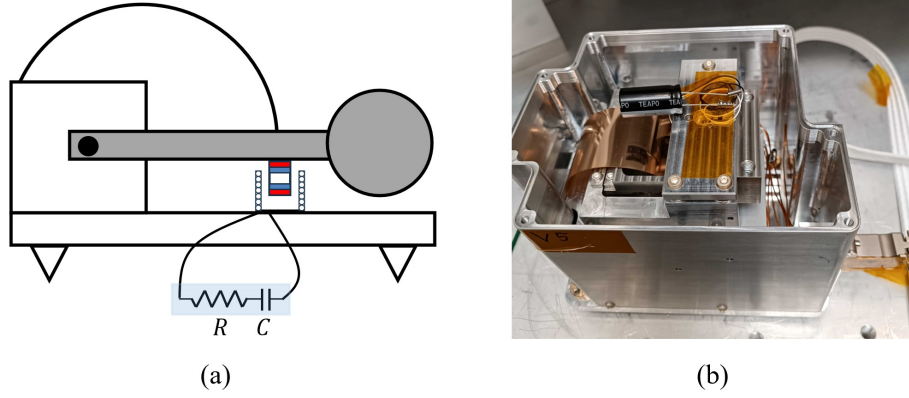


Figure 2.33: Passive shunt of the E-VINS sensor. (a) Schematic diagram of the RC shunt. (b) Picture of a shunted E-VINS.

For best damping effects, the resistor value should be chosen low to allow current to circulate. It has been established in section 2.1 that the resistor value needed to reach critical damping R^{crit} ($[\Omega]$) depends on the voice-coil constant T ($[\text{N/A}]$) and on the proof-mass mass suspension properties (mass m , stiffness k , natural frequency ω_0 and Q -factor), and writes

$$R^{\text{crit}} = \frac{T^2}{\sqrt{2km} - c} = \frac{T^2}{\omega_0 (\sqrt{2}m - 1/Q)} \quad (2.45)$$

Using the characteristics of the E-VINS sensor identified above: $\omega_{0,\text{E-VINS}} = 2\pi \cdot 1.453 \text{ rad/s}$, $Q_{\text{E-VINS}} = 200$, $m = 0.4 \text{ kg}$ and the quadrupole actuator constant $T = 0.6465 \text{ N/A}$ (see Appendix D),

the required resistor for critical damping is

$$R^{\text{crit}} = 0.082 \quad [\Omega] \quad (2.46)$$

Such a very low value results from the voice-coil constant T being fairly small, due to the quadrupole action decreasing the actuator strength, and the Q -factor being comparatively large. Unfortunately, this resistor value cannot be reached in practice, as the impedance of the coil (NCC03-07-001-1V) itself has already been measured at $R_{\text{coil}} = 9.6 \Omega$. This limits the amount of passive damping that can be achieved by simply shorting the coil wires.

To balance this, a capacitor C has been placed in series with the lowest shunt resistor, as capacitors has been shown to be a simple method to increase damping and bandwidth in inertial sensors [165]. The sensitivity response function of an inertial sensor with RC-shunt can be obtained by solving the equation of motion of the inertial mass and the Kirchhoff law of the RC circuit. The sensitivity response function of the E-VINS seismometer in RC-shunt writes as

$$S_{\text{E-VINS}}^{\text{RC}} = \frac{-s^2(R_{\text{coil}}Cs + 1)}{(R_{\text{coil}}Cs + 1)(s^2 + \omega_{\text{E-VINS}}/Q_{0,\text{E-VINS}}s + \omega_0^2) + \frac{T^2C}{m}s^2} \quad (2.47)$$

It can be seen that adding a capacitor with a large C value can increase the passive damping brought by the shunt operation. In practice, the largest capacitor that was available was a 4.7 mF capacitor (non-vacuum compatible). Figure 2.34 shows the calculated sensitivity response functions for the E-VINS sensor in the different shunt configurations. The open coil curve shows the 1-DOF fit to the E-VINS sensitivity as identified in the previous section with a Q -factor of approximately 200. The shunted sensitivity with the calculated R^{crit} resistor of 0.082Ω effectively leads to critical damping. Realistically, a shunt operation of 9.6Ω , corresponding to the resistance R_{coil} of the coil wire obtained by shorting the coil, however, only allows a reduction of the resonance amplitude by a factor of roughly 5. Adding a 4.7 mF capacitor C to the shunt can reduce it by an additional factor of 2.

Figure 2.35 shows measured spectrums of an E-VINS sensor for the different shunt conditions. The measurements have not been corrected for the E-VINS sensitivity response so that the resonance peak is clearly visible. The E-VINS in the shorted-coil configuration, corresponding to an actual shunt resistor of 9.6Ω , indeed shows a tenfold reduction in the resonance amplitude compared to the open coil case. And another factor 2 reduction is achieved with the RC-shunt. While these measurements are ASDs, not transfer functions, and thus the ASD amplitude clearly depends on the seismic activity during the measurement, these results seem to correlate with the calculated behaviour of the shunted E-VINS.

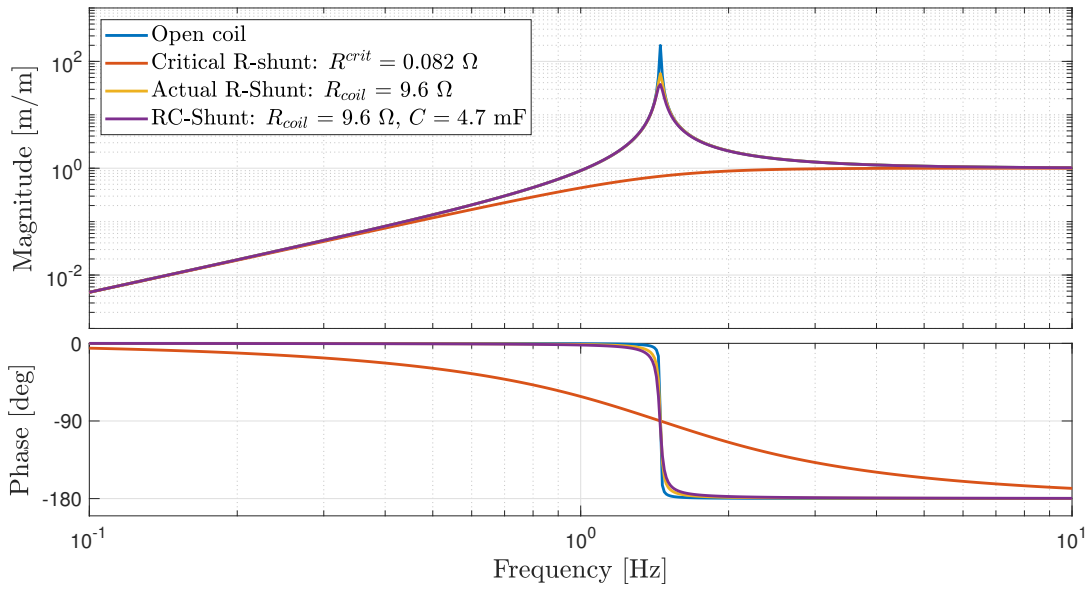


Figure 2.34: Modelling of the E-VINS sensitivity transfer function for different shunt configuration. (Blue) Open coil: Q -factor ≈ 200 . (Red) Critical R-shunt: $R = 0.082 \Omega$, Q -factor $= \sqrt{2}/2 = 0.71$. (Yellow) Practical R-shunt: $R_{coil} = 9.6 \Omega$. (Purple line) RC-shunt: $R = 9.6 \Omega$ and $C = 4.7 \text{ mF}$.

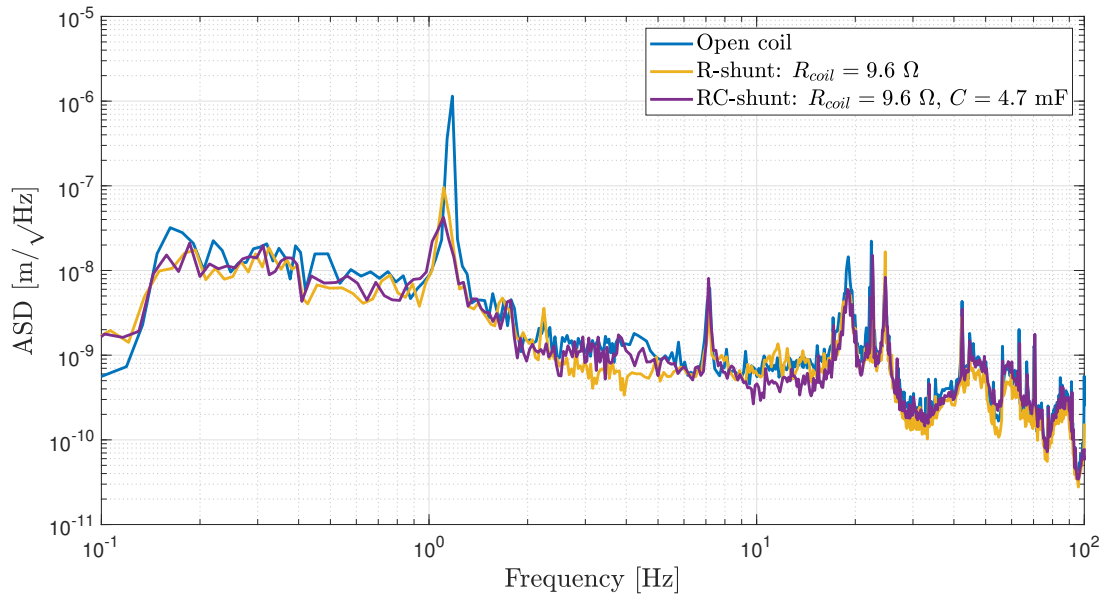


Figure 2.35: E-VINS measured displacement spectrum for different shunt configuration. (Blue) Open coil: Q -factor ≈ 200 . (Yellow) R-shunt (shorted coil wires): $R_{coil} = 9.6 \Omega$. (Purple) RC-shunt: $R = 9.6 \Omega$ and $C = 4.7 \text{ mF}$.

In fine, the sensors' response Q -factors can be reduced by a factor of 10 or 20, using a R-shunt or an RC-shunt, respectively. This results in Q -factors of 20 or 10 for the E-VINS sensor, as opposed to the mechanical Q -factor of 200. Such flattening of the sensors' sensitivity response might ease the design of the control filters, although the sensitivity still shows an amplification peak, as the sensors cannot be critically damped with the current actuator design due to its low transducer constant value. Obviously, such a shunt operation comes with a trade-off of an increased low-frequency noise level, since such velocity-dependent damping translates into mechanical thermal noise from the fluctuation-dissipation theorem [159]. At the time of this writing, it is thought that the sensors' sensitivity response is stable enough, i.e. no significant fluctuation of the sensors' resonance frequency nor Q -factor under ambient loads (thermal, residual pressure, etc), and the sensitivity fit accurate enough for the shunt operation not to be necessary. The voice-coil actuators are left open-ended, but both sensor designs still include the possibility to enable RC-shunt damping.

2.3.3 Resolution

The sensor's resolution is the smallest motion that can be sensed and distinguished from noise. In the case of an inertial sensor, the sensor resolution is set by the combination of the relative displacement readout resolution and the mechanical response of the proof-mass suspension. In addition, external factors such as temperature, air pressure or magnetic sensitivity contribute to the resolution.

Noise model and mechanical thermal noise

The E-VINS and E-HINS sensors noise diagram is shown in Figure 2.36. In the following, $\omega_0 = 2\pi f_0$, m and Q are respectively the suspension resonance frequency, proof-mass weight and suspension Q -factor of either sensor. The orange blocks represent the inertial mass sensitivity to ground motion w and to external forces (mostly thermal noise), and are respectively the sensor's sensitivity response function and compliance transfer function. The interferometer block represents the optical readout and acquisition chain. It includes the interferometer modulation of the mechanical proof-mass motion y into interferometric phase, and its demodulation into readout displacement $\hat{y} + \tilde{n}_{\text{IFO}}$, and all internal noise contributions as referred to Figure 1.18. Note that it also includes digitalisation of the signal in either the MicrolabBox system or the Centaur recorder. The red S^{-1} blocks represents the digital inversion filter that reconstruct ground motion $\hat{w} + \tilde{n}_{\hat{w}}$.

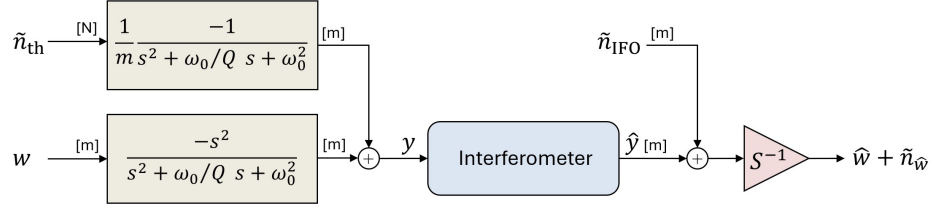


Figure 2.36: Noise diagram for the E-HINS and E-VINS sensors. w and $\hat{w}$ are respectively the ground motion and sensor-measured ground motion. y and $\hat{y}$ are respectively the proof-mass motion and the optical readout measured motion. The brown blocks represent the sensor's mechanical response to inputs. The blue block represents the optical readout and includes the acquisition chain (including recording). The S^{-1} red block represents the digital filters that invert the sensor's sensitivity response. $\tilde{n}_{th}$ ([N]) represents thermal noise(s), $\tilde{n}_{IFO}$ ([m]) the interferometer noise and $\tilde{n}_{\hat{w}}$ ([m]) the sensor noise.

The total sensor resolution $\tilde{n}_{\hat{w}}$ therefore mostly depends on the optical readout noise $\tilde{n}_{IFO}$ and on thermal noise $\tilde{n}_{th}$ and scales as the inverse sensor's sensitivity response S^{-1} . For a well-calibrated inertial sensor, where $S^{-1} = \left| \frac{s^2 + (\omega_0/Q)s + \omega_0^2}{-s^2} \right|$ exactly matches the sensor sensitivity response, the sensor output spectrum $\Phi_{\hat{w}}$ ([m²/Hz]) writes

$$\begin{aligned} \Phi_{\hat{w}} &= S^{-2} \cdot \left[\left| \frac{-s^2}{s^2 + (\omega_0/Q)s + \omega_0^2} \right|^2 \Phi_w + \Phi_{IFO} + \left| \frac{1}{m} \frac{-1}{s^2 + (\omega_0/Q)s + \omega_0^2} \right|^2 \Phi_{th} \right] \\ &= \Phi_w + S^{-2} \Phi_{IFO} + \left| \frac{1}{ms^2} \right|^2 \Phi_{th} \end{aligned} \quad (2.48)$$

The above equation allows to model the E-HINS and E-VINS sensors' resolution. The filter S has been experimentally identified in the characterisation of the inertial sensors in subsection 2.3.2. Likewise, the optical readout noise Φ_{IFO} has been measured from a null measurement in section 1.3. A direct measurement of the thermal noise spectrum Φ_{th} is however a lot more complex, so that is is often estimated from a model.

Following the fluctuation-dissipation theorem exposed in section 2.1, the global thermal noise is calculated as a combined contribution of Brownian thermal noise $\Phi_{th,B}$ and structural thermal noise $\Phi_{th,S}$:

$$\Phi_{th} = \underbrace{4k_B T \left(\frac{m\omega_0}{Q_B} \right)}_{\Phi_{th,B}} + \underbrace{4k_B T \left(\frac{\omega_0^2 m}{\omega Q_S} \right)}_{\Phi_{th,S}} \quad [N^2/Hz] \quad (2.49)$$

with $k_B = 1.38 \cdot 10^{-23}$ m²kg/s²/K is the Boltzmann constant, $T = 297$ K the temperature, $\omega = 2\pi f$ the frequency and Q_B and $Q_S = \phi_S^{-1}$ respectively the Brownian and structural Q -factors. The Q -factors that have been characterised in the previous section however correspond to a combination

of both loss mechanisms and do not allow for individual characterisation of both Q -factors. While gaseous damping largely decreases in vacuum, and is often considered negligible below a couple of mbar for most mechanical experiments (i.e. when leaving the viscous flow regime) [161], the presence of the quadrupole magnet of the voice-coil actuator and the overall magnetic sensitivity of the sensor does not allow to entirely rule out Brownian contribution. Also, structural loss angle is usually of the order of magnitude of 10^{-4} to 10^{-5} for most metals [163,210], resulting in Q -factors in the range of $Q_S \approx [10000 - 100000]$. Clamping loss due to the assembly of non-monolithic suspensions however typically significantly decreases the structural Q -factors to values of a couple of hundreds. In the end, due to the lack of additional means to discriminate between which of the structural or Brownian thermal noise dominates the E-VINS and E-HINS assembly, both thermal noises are modelled using the Q -factor value measured in the characterisation.

Although not performed in the framework of this thesis, a protocol proposed in [20] for characterising thermal noise in MEMS accelerometers allows to distinguish the Brownian Q -factor from the structural Q -factor using the scaling of Q_B with vacuum and the quadratic scaling of Q_S with resonance frequency. Indeed, for a mechanism with tunable stiffness, a measurement of the global Q -factor for different values of ω_0 and at different vacuum levels allows for an unambiguous discrimination of Q_B and Q_S .

Measured resolution

A direct measurement of the resolution of seismometers can be obtained from a Huddle test procedure [223,224]. The procedure consists of measuring simultaneously the output of multiple similar sensors subjected to the same seismic input and performing a subtraction of the coherent signal from all sensors. The residual from the coherent subtraction therefore corresponds to the intrinsic noise of the seismometers. The method requires at least two devices, but the quality of the coherent subtraction improves with the number of channels (sensors) used in the calculation.

A Huddle test measurement has been performed on the two E-VINS and E-HINS inertial sensors installed in the HAMX-ISI system. The test was performed in a sealed chamber and at 10^{-5} mbar vacuum. The HAMX-ISI control system was set with the highest isolation level engaged (isolation loops and sensor correction), with a residual iRMS ISI motion of 10 nm in the [0.1 - 10] Hz bandwidth, in order to minimise nonlinear noises. These will be discussed in a subsequent section. Figure 2.37 shows the results of this test for the E-HINS sensors (left) and the E-VINS sensors (right). The blue line shows the residual ISI motion measured by either of the inertial sensors, and the orange line shows the residual from the coherent subtraction. The calculation of the residual includes any other channels that would be coherent with the inertial sensors, which also accounts for cross-sensitivities. Specifically, the residual for the E-VINS signal uses, obviously, the signal of the other E-VINS placed side-by-side, but also the Z-motion signal measured by the T240 witness sensor placed nearby, the reconstructed HAMX-ISI Z-motion measured by the GS13 seismometer and the HAMX-ISI RX-rotation to account for tilt-to-translation coupling. Similarly, the E-HINS residual is calculated using the other E-HINS signal, T240 Y-translation and HAMX-ISI

Y-translation and RX-rotation. X-translation signals are also included in the calculation to account for non-perfect alignment of the E-HINS axis with the HAMX-ISI Y-axis. Also, superimposed in the figure are the calculated readout noise (dashed black) and thermal noises (dashed pink and purple). The noises have been calculated according to the modelling exposed in the previous section. In particular, the readout noise corresponds to the optical readout measured from a null measurement using the 16-bit acquisition system (see section 1.3), scaled with the inverse sensitivity response of the corresponding sensor. Thermal noises are calculated for room temperature using the Q -factor measured in the characterisation process (see subsection 2.3.2), and assuming equal Brownian and structural contributions to the global Q -factor, as discussed in the previous section.

Figure 2.37 shows that the sensor resolution (residual) lands above 10 Hz on the theoretical resolution of approximately $2 \cdot 10^{-13} \text{ m}/\sqrt{\text{Hz}}$, depending on the quality of the interferometer alignment, calculated from the optical readout noise. At frequencies around the sensors' suspension resonance, both sensors are supposed to be limited by thermal noise, and the readout noise takes over again as $f^{-2.5}$ slope at sub-Hz frequencies. Note the -2.5 slope that differs from the expected, mechanical, -2 slope related to the sensors' sensitivity response, due to the excess low-frequency noise in the interferometer attributed to potentially temperature effects. The residual however lands a little less than a factor 10 above the theoretical calculations in this low-frequency range, attributed to either residual of coherent signal that is not correctly subtracted, sensitivity to cross-directions, or nonlinear effects.

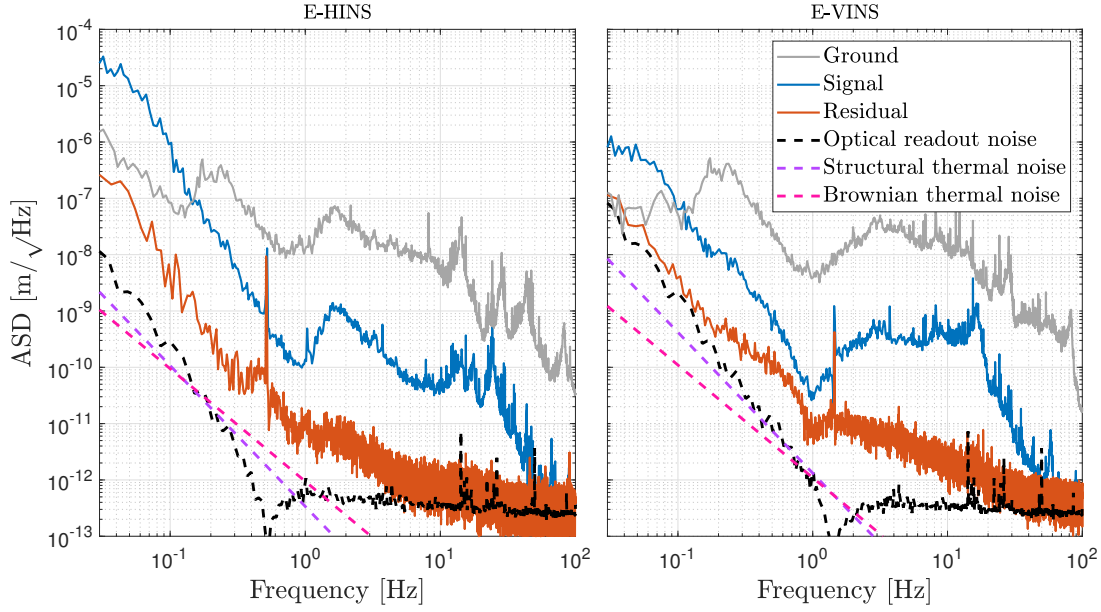


Figure 2.37: (Right) E-VINS and (Left) E-HINS resolution measured as the residual from a coherent subtraction. (Blue) Signal measured. (Red) Residual from the coherent subtraction between the sensor and other coherent channels on the HAMX-ISI platform. (Dashed black) Optical readout noise calculated from the measured noise of a null measurement. (Dashed purple and dashed pink) Calculated Brownian and structural thermal noises, respectively. (Grey) Ground spectrum measured from an STS2 seismometer is shown as a reference.

Overall, the noise figures show a resolution of $1 \cdot 10^{-8} \text{ m}/\sqrt{\text{Hz}}$ @ 100 mHz, $1 \cdot 10^{-11} \text{ m}/\sqrt{\text{Hz}}$ @ 1 Hz, $1 \cdot 10^{-12} \text{ m}/\sqrt{\text{Hz}}$ @ 10 Hz and $3 \cdot 10^{-13} \text{ m}/\sqrt{\text{Hz}} > 10 \text{ Hz}$ for the E-HINS optical inertial sensors; and $4 \cdot 10^{-9} \text{ m}/\sqrt{\text{Hz}}$ @ 100 mHz, $4 \cdot 10^{-12} \text{ m}/\sqrt{\text{Hz}}$ @ 1 Hz, $2 \cdot 10^{-12} \text{ m}/\sqrt{\text{Hz}}$ @ 10 Hz and $3 \cdot 10^{-13} \text{ m}/\sqrt{\text{Hz}} > 10 \text{ Hz}$ for the E-VINS optical inertial sensor. This validates the production of sub-pm resolution inertial sensors using optical reading technology and a low-resonance frequency mechanism at higher frequencies. There are however still margins for improvements at lower frequencies as it can be seen from the discrepancy between the theoretical noise budget and the measured resolution, which is assumed to be caused by residual nonlinearities and temperature effects.

Non-linearities

Nonlinearities are a notorious limitation in high-resolution seismometers [225]. As discussed in section 1.3, the optical readout is suspected to be the main contributor to the nonlinear response of the E-VINS and E-HINS sensor. The nonlinear noise level of the interferometer has been measured at $1 \text{ nm}/\sqrt{\text{Hz}}$ for relative displacement of the μm -scale, and decreasing to $10^{-11} \text{ m}/\sqrt{\text{Hz}}$ for relative displacement of a couple of nm-range. Because the E-VINS and E-HINS inertial sensors operate with a free-moving proof mass, i.e. with no force-feedback action, the interferometer nonlinearities fully transmit to the seismometer output.

Figure 2.38 shows the resolution of the E-VINS and E-HINS sensors resolution for various residual motions of the HAMX-ISI. The HAMX-ISI residual motion is controlled by sequentially engaging the different control layers: no control engaged (Cont off, blue lines), damping loops only (Damp loops, red lines), damping and isolation loops (+ISO loops, yellow lines), and damping and isolation loops with sensor correction (+Sens cor, purple lines). The residual platform motion is calculated as the RMS value of the motion measured by the T240 witness sensor integrated in the [0.1 - 10] Hz frequency band, as explained above in subsection 2.3.1. The residual motion is estimated at 0.8 μm , 0.5 μm , 100 nm and 10 nm for the different isolation levels. For each, the figure shows the seismometer signals (solid traces) and resolution (dashed traces), calculated the same way as above, i.e. as the residual from a coherent subtraction between each sensor and the multiple coherent channels available in the HAMX-ISI. As for the interferometer nonlinearities, the figure shows that the E-VINS and E-HINS noise level decreases with the HAMX-ISI residual motion. With motions in the μm -scale (no control and damping control only), the sensors are measured with a resolution of $10^{-10} \text{ m}/\sqrt{\text{Hz}}$ at 1 Hz, and the residual lies approximately two orders of magnitude below the signal level. As the motion level decreases by nearly an order of magnitude, a significant decrease in the resolution is observed for both sensors, although more unpronounced for the E-VINSs, and the residual lowers to $10^{-11} \text{ m}/\sqrt{\text{Hz}}$. The additional control layer decreases the motion, especially at sub-Hz frequencies, and improves the noise level marginally for the E-VINS sensor, and by a factor of 2 to 3 above 1 Hz for the horizontal sensor. These numbers correspond to the measurement of the nonlinear response of the optical readout. In addition, the sensor's mechanics may be held responsible for some degree of nonlinearity due to the fact that the proof mass is not guided along a purely linear trajectory. However, even for μm -scale motion, the transmitted ground motion only generates μrad rotation at the proof mass level, so that the small angle assumption might still be considered valid.

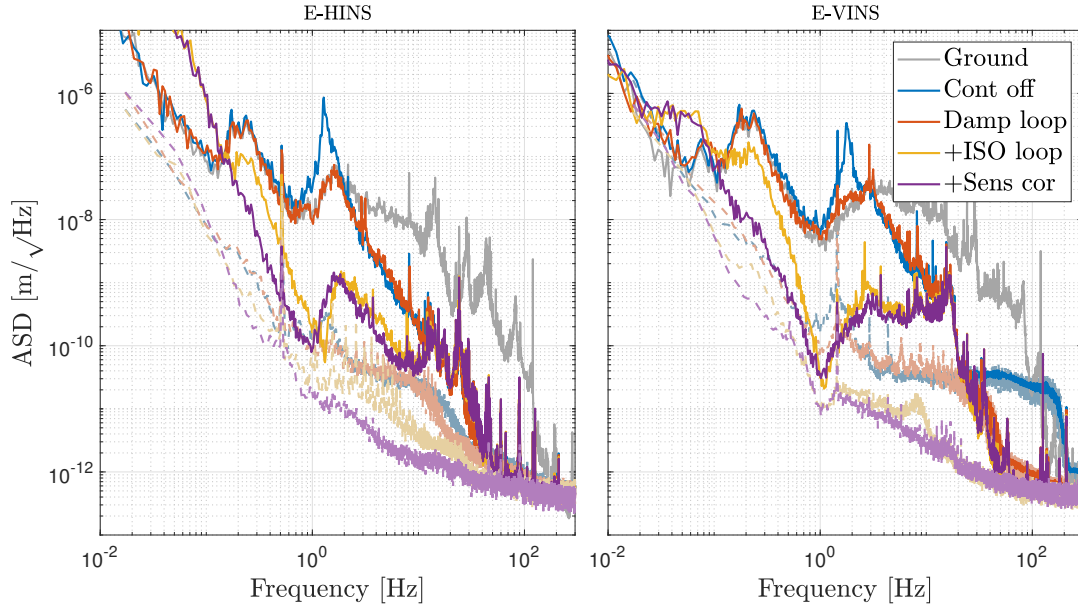


Figure 2.38: (Right) E-VINS and (Left) E-HINS resolution measured as the residual from a coherent subtraction, for various residual motions of the HAMX-ISI platform. (Solid and dashed blue) Signal and residual measured with no control loop engaged: $iRMS = 0.8 \mu m$. (Solid and dashed red) Signal and residual measured with damping loops engaged: $iRMS = 0.5 \mu m$. (Solid and dashed yellow) Signal and residual measured with isolation loops engaged: $iRMS = 100 nm$. (Solid and dashed purple) Signal and residual measured with sensor correction filters engaged: $iRMS = 10 nm$. (Grey) Ground spectrum measured from an STS2 seismometer, shown as a reference.

In order to resolve nonlinearities in the seismometer, most modern commercial instruments operate using a force-balance feedback loop. The servo loop uses the proof mass relative motion signal to drive a magnetic or electrostatic actuator to lock the proof mass on the frame motion [55, 57, 157]. This forces the sensor mechanics and readout to operate at a null position, and shifts the linearity requirements to the actuator and control system. In theory, this action allows for linearising the sensor and extending the bandwidth without degrading the sensor signal-to-noise ratio. However, the actuation and control electronics necessarily come with some noise characteristics, which propagate in the servo loop as control noise and degrade the sensor resolution. Because the E-VINS and E-HINS sensors are intended to be used for feedback inertial control of an active platform, which will eventually reach nm-like residual motion, the use of a force-balance feedback loop is believed unnecessary.

Another method for measuring nonlinearities in force-balanced seismometers is to inject a two-tone signal in the feedback actuator and record the low-frequency intermodulation at the sensor output [226, 227]. Although the method is traditionally applied to a force-balanced seismometer, it is believed that it can also be applied to the E-VINS and E-HINS open-loop sensors using the quadrupole actuator as a calibration coil. Data processing might however be more complex due to the undamped response of the mechanics, which interact with the two-tone signal. Alternatively, an

identification technique using a shake table has also recently been proposed [225], provided that the shake table is accurate enough: i.e. is as linear as the seismometer or whose motion can be measured with higher linearity and resolution than the seismometer.

Acoustic/air pressure noise

In order to estimate the effects of air pressure and acoustic noise on the reading of the optical inertial sensors, sensor noises have been measured with the HAMX chamber at ambient pressure (and vacuum vessel open) and in high-vacuum conditions ($\approx 10^{-5}$ mbar). The measurements are taken with no isolation loop engaged, since the control loops require the platform to be in-vacuum, or at least with the chamber sealed. This has been illustrated on an E-VINS sensor. Results are shown in Figure 2.39. The residual measured in high-vacuum (purple trace) is lower than in the in-air residual (yellow trace) by nearly one order of magnitude in the $[0.7 - 3]$ Hz range, which typically corresponds to the acoustic noise range. Outside of this bandwidth, both the in-vacuum and in-air measurements show very similar spectrums, validating that the sensors' cover efficiently shields the proof-mass and interferometer from air pressure fluctuation noise.

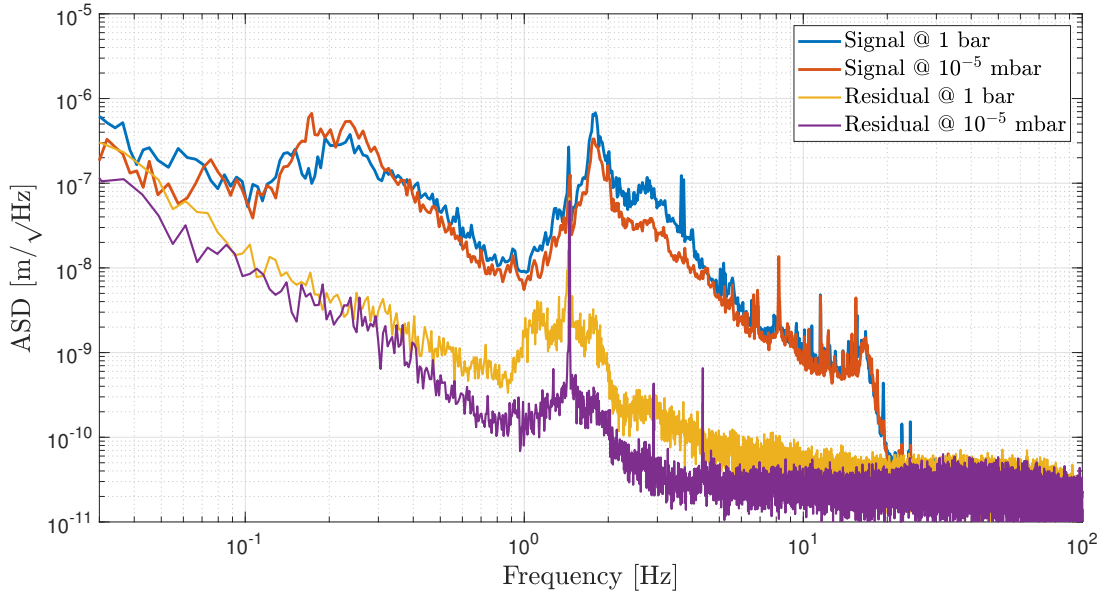


Figure 2.39: Comparison of the E-VINS resolution at ambient pressure and in high-vacuum. (Blue and red) Signals levels at ambient pressure and at 10^{-5} mbar. (Yellow and purple) Residual from a coherent subtraction at ambient pressure and 10^{-5} mbar.

2.4 Installation in E-TEST

After designing, producing and thoroughly characterising the optical seismometers, the E-VINS and E-HINS sensors were ready for integration in the E-TEST prototype.

E-TEST is prototype developed for testing a novel hybrid isolation strategy with cryogenic cooling for the test-mass of future gravitational-wave detectors [21–23]. The E-VINS and E-HINS inertial sensors have been developed to meet the stringent isolation performance required from the active stage in the [0.01 - 10] Hz band.

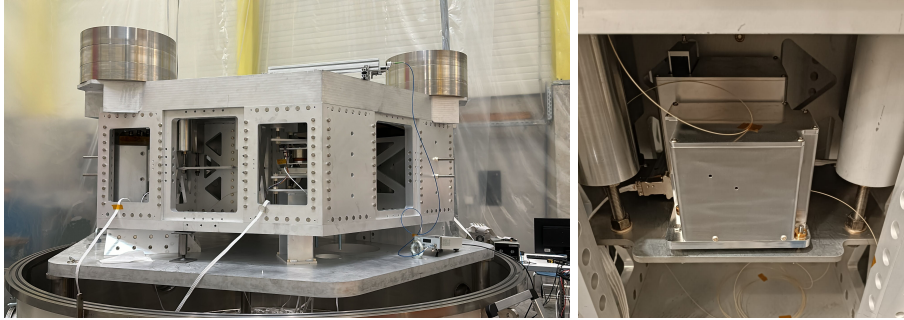


Figure 2.40: (Left) Picture of the E-TEST Active Platform (inverted pendulum not mounted). (Right) Close-up view on a E-VINS/E-HINS pair mounted on a sensor stage.

Figure 2.40 shows pictures of the integration of the E-HINS and E-VINS sensor inside the Active Platform (AP). The sensors are grouped as pairs composed of one vertical and one horizontal sensor. Three pairs are mounted in the prototype, arranged symmetrically along a radius and with the vertical axis pointed upward and the horizontal axis tangentially to the circle. Each pairs are tightly screwed on a so-called sensor stage, which can be vertically levelled to adjust the sensor's axis with the AP centre of stiffness. The sensor arrangement is shown in Figure 2.41.

MIMO inertial isolation feedback loops have been designed to control the AP in all 6 degrees of freedom. The sensors signals are recorded and demodulated on the real-time target and fed to the control system. The control system uses two intricate control loops: one inner control loop for low-authority control in a decentralized frame (from one sensor to its quasi-collocated actuator), and an outer control loop for high-authority control in the global Cartesian frame.

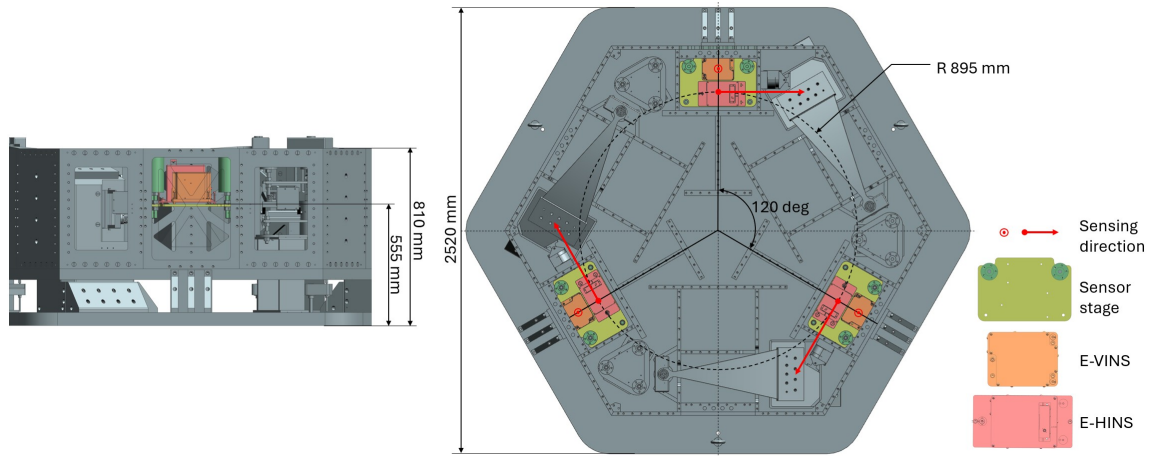


Figure 2.41: CAD view of the E-VINS/E-HINS sensor pairs integrated in the AP. (Left) Side view of a sensor pair. (Right) Top view on the AP. (Pink/orange) E-HINS/E-VINS sensor pairs oriented at 120° . (Green) Sensor stage: (light green) mounting interface, (dark green) levelling screws.

At the time of the writing of this manuscript, the control system is still under development phase, carried by fellow researchers from the PML group. Interested readers are referred to recent updates for details on the control architecture and design strategy [31–33].

2.5 Conclusion and future work

This chapter presented the design, performance and applications of the E-VINS and E-HINS optical inertial sensors designed for the active seismic isolation system of the E-TEST prototype in the 10 mHz to 10 Hz bandwidth.

Section 2.2 presented the mechanical design of both instruments. The E-VINS vertical inertial sensor uses a leafspring-suspended pendulum [56] and a clamped blade pivot joint. The suspension has been designed using nonlinear finite element modelling in METAFOR [171, 215]. The proof-mass readout is achieved using the quadrature Michelson interferometer presented in chapter 1, targeting a TIR retroreflector mounted on the proof-mass. The E-HINS horizontal inertial sensor has been designed using a Watt's Linkage suspension mechanism, inherited from [78, 79, 199–201], machined in a monolithic aluminium block using EDM. Similarly, the proof-mass motion is read by the same quadrature Michelson interferometer mounted on an alignment stage.

The characterisation of both sensors is presented in section 2.3. Tests were performed inside the HAMX-ISI isolation system [40, 220], which provides a stable, in-vacuum system instrumented with various reference sensors and isolation loops. The E-VINS and E-HINS sensors' sensitivity response functions have been measured with, respectively, a 1.453 Hz resonance frequency and 200 Q -factor, and a 0.515 Hz resonance frequency and 50 Q -factor. The sensors' resolution has also been measured from a coherent subtraction between the sensors' signals and the reference instruments, for a HAMX-ISI residual platform motion of a couple of nm. The resolution is measured at $2 \cdot 10^{-13}$ m/ $\sqrt{\text{Hz}}$ above the sensor's respective resonance, and decays as $f^{-2.5}$ below due to the mechanics and optical device self-noise. Cross-sensitivities and the nonlinear response of the sensors have also been measured by comparing the sensors' output with the ISI motion in all 6 degrees of freedom. Despite the mechanical alignment mechanism of the proof-mass and the linearisation (ellipse correction) algorithm, cross-sensitivities and nonlinearities appear to be significant limitations. However, due to the use of the sensors inside a feedback-controlled isolated stage, it is believed that the sensors will eventually operated within their linear regime when enabling the control system.

The chapter concludes with page 166, showing a brief overview of the integration of the sensors in the E-TEST active platform and preliminary control results.

This chapter presented a thorough characterisation of the sensors, but a couple of additional tests and further development could be performed for a deeper characterisation and improved performance. First, it is acknowledged that the cross-sensitivities testing might be biased by the cross-sensitivities of the HAMX-ISI system. A better characterisation would be to use a high-precision hexapod table, which would be used as a 6D shake table. Also, the sensitivity of the sensors to environmental effects has not been addressed, namely, sensitivity to magnetic field and temperature, which could have significant effects in the lower frequency range. Since cross-sensitivities has been seen significant in lower frequency, it is believed that developing a better assembly procedure, featuring a precise alignment of the proof-mass centre of mass with the pivot joint, may help reducing cross-translation sensitivity (excluding tilt-coupling). Still, the single-joint design used in E-VINS

makes the sensor inherently first order-sensitive to tilt. A reworked suspension design, including a parallel blade guidance mechanism for the proof-mass, would help in further rejection cross-axis coupling to a second order effect. Finally, an alternative design for the E-VINS leafspring suspension with lower mechanical frequency has been investigated in [171,210] and show promising potential in improving the E-VINS resolution.

Chapter 3

Compact optical accelerometer for gravimeter

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As per Newton’s law of universal gravitation, any two bodies at a distance apply a gravitational pull on each other. It results that the local value of the Earth’s gravitational acceleration is in fact a time-varying quantity, as it depends on the motion of heavy bodies in the surroundings. Notorious examples are the moon, tidal and ocean loading effects [228]. These large periodic fluctuations of gravity have peak amplitudes of typically 250 μGal ($\approx 250 \cdot 10^{-9} g$) [229], depending on the location. The measurement and study of the time-fluctuation of the local strength of the gravitational field is known as gravimetry, and the instruments used for this purpose are named gravimeters.

Gravimeters can be used for mapping out the Earth's gravitational field and quantifying gravity anomalies, or used for monitoring the underlying cause of the local gravity fluctuation, such as motion of underground bodies of water/petroleum/lava or seismic waves. Gravimeters are therefore largely used in Earth sciences as geophysics, geology and seismology [230,231]. Recent development in absolute quantum gravimeters [232–235] provided instruments capable of sensing at the μGal level [29,236], i.e. $10^{-9} g$, expanding the field of application beyond motion of massive bodies, to applications requiring ultra-high resolution. Atom gravimeters for example find applications in fundamental physics for measuring universal constants [237–243].

In gravitational-wave science, Newtonian Noise (NN), i.e. vibration noise of the detector mirrors due to local fluctuation of gravity caused by seismic waves, air pressure fluctuation, or human activity [244–247], is expected to be a limitation for the future 3rd generation of detectors in the lower frequency range [14]. Traditional method for measuring NN consists in deploying an array of seismometers to map the seismic field around the test mass of the detector and reconstruct gravity variations due to seismic waves. Seismometers are however AC instruments, suffering from drifts and blind to very-low frequency content. On the other hand, atom gravimeters are instruments capable of measuring absolute acceleration with no drift and long-term stability, therefore offering properties that are complementary to seismometer, and promising capabilities to improve the network of seismometers.

Vibration noise, i.e. ground vibration propagating to the sensing head, is a major limitation in current quantum gravimeters [41,248–250]. Several ways to reduce vibration noise include a physical (passive or active) vibration isolation of the instrument [42,251], or on (offline or online) compensation of the vibration noise using an external accelerometer [29,43,252]. The latter technique is suitable for portable instruments, as no additional mechanical components are required, but is typically limited by the vibration compensation scheme and the resolution of the accelerometer [253]. A high-resolution, broadband, optical accelerometer μVINS has been developed to be merged with the commercial Absolute Quantum Gravimeter (AQG) from EXail, ex μQuans . Similar to the EVINS sensor, the μVINS sensor uses a vertical long-period suspension of the proof mass combined with the optical interferometric readout presented in chapter 1. The μVINS is equipped with a frequency compensation scheme for broadening the sensitivity response function to comply with the bandwidth requirement of vibration compensation in gravimeters.

This chapter will present the design and performance of the μVINS sensor designed for improving the ground vibration noise compensation in an AQG. section 3.1 sets the context and provides an overview of the AQG working principle, limitations, and ground compensation scheme. The technique used for broadening the accelerometer response will also be presented. Next, section 3.2 will present the μVINS sensor mechanical design. The following section 3.3 will present two broadening techniques applied to the μVINS and the sensor characterisation. Finally, section 3.4 will show results on the integration of the accelerometer in an AQG.

3.1 Vibration noise compensation in Absolute Quantum Gravimeters

Contemporary absolute gravimeters rely on a simple, yet efficient, working principle: tracking the free-fall of a test object. The best commercially available gravimeters usually rely on the free-fall of an optical element in a sealed vacuum chamber, and an interferometer is used for reading the trajectory of the test element [234]. The FG5 gravimeter is a classic example of such an instrument. The mechanical fall of an element poses several issues: mechanical drift, wear and tear, and longer dead-time between two successive drop. More recent instrument, based on the quantum properties of atoms and wave-particle duality, eliminate all of these mechanics-related issues by dropping atoms instead of a physical object to provide measurement of g with ultra-long term stability [232,233,235]. Such instruments are named absolute quantum gravimeters.

However, as for the mechanical gravimeters, seismic vibration noise remains a challenging limitation in quantum gravimeters, and various methods have been developed to tackle the issue [254]. Using an external accelerometer to measure vibration on the gravimeter sensor head is an elegant solution for compensating the vibration noise [29, 253, 255]. However, the performance of such a method obviously depends on the resolution and bandwidth of the accelerometer. One challenge here is that the gravimeter sensitivity function to ground vibration extends over a wide spectrum of frequency, ranging from DC to hundreds of Hz [256]. This calls for the need for both high-resolution and broadband accelerometers.

This section proposes an overview of the basic working principle and vibration noise limitation in an absolute quantum gravimeter. Then, various classical techniques will be discussed for broadening the sensitivity response function of an inertial accelerometer to meet the requirements of the vibration compensation scheme.

3.1.1 Absolute Quantum Gravimeters

Absolute quantum gravimeter utilises the quantum wave-particle duality of cold atoms to produce instruments capable of sensing the absolute value of the gravitational acceleration g with a sensitivity in the range of a μGal (10^{-9} g) [29,236,255]. More precisely, they use stimulated Raman transition of atoms between two hyperfine states transition to create "splitters" and "mirror", thereby effectively creating atom-interferometers. Pioneer articles with detailed explanations include [232–235,255,256].

The basic working principle will be briefly displayed in the following, extended discussion may be found in [257–263].

Working principle

A typical configuration of a quantum gravimeter with a free-falling atom is shown in Figure 3.1 (left). A large pack of atoms, typically Rubidium or Caesium, are trapped in a Magneto-Optical

Trap (MOT) and cooled down to a few μK in an optical molasses. After few ms, atoms in the appropriate internal state, let us label $|a\rangle$, are released in free-fall in a vacuum chamber where they will be subjected to three laser pulses, forming the atom-interferometer.

The interferometer is created using stimulated Raman transition between two hyperfine levels of the falling atoms, let us call $|a\rangle$ and $|b\rangle$. The 2-photon Raman transition process is at the core of the gravimeter working principle, and is discussed in [232, 233, 235, 256]. At each pulse, the atoms are illuminated by two counter-propagating laser beams, creating a 2-photon transition. The laser frequency difference is tuned on the splitting between the two internal states $|a\rangle$ and $|b\rangle$. When an atom absorbs and emits a photon, it experiences a net momentum change $\hbar k_{\text{eff}}$, with k_{eff} the Raman pulse effective wave-vector and the Planck constant $\hbar = h/2\pi = \frac{1}{2\pi} \cdot 6.63 \cdot 10^{-34} \text{ J s}$. This thereby allows to physically split and redirect the atom cloud along different paths. As always in quantum mechanics, everything is probability. The probability of atoms to effectively undergo this absorption-emission process can be adjusted with the pulses parameters (especially their duration). For a 50% probability, the atom pack is split into two paths where half of the atoms have changed momentum and state, while the other half has remained unchanged. Such a pulse therefore acts as a "beam splitter" in optics, and is commonly named a $\pi/2$ -pulse. On the contrary, for 100% transition probability, all atoms will reverse state and the pulse acts as a "mirror", this is a π -pulse.

Quantum gravimeters typically use a $(\pi/2 - \pi - \pi/2)$ sequence, allowing to split, mirror and recombine the atom cloud in a Mach-Zehnder-like configuration. This is illustrated in Figure 3.1 (right). Each pulses are spaced by an interrogation time T during which atoms acquire velocity as they fall in the gravitational field. As per the dual properties of wave and matter, the local laser phase at each Raman transition also imprints on the atoms' phase Φ_i^A and Φ_i^B at each Raman transition. Because the atoms followed two physically different paths A and B , there will exist a net phase difference between the recombined clouds in states $|a\rangle$ and $|b\rangle$ due to the gravity-induced Doppler effect. The phase difference is

$$\Delta\Phi = k_{\text{eff}}gT^2 \quad (3.1)$$

where g is the acceleration of gravity.

At the interferometer output, the proportion of atoms in either $|a\rangle$ and $|b\rangle$ is measured by fluorescence detection. The proportion P of atom in the $|b\rangle$ state is

$$P = \frac{1}{2} (1 - C \cos \Delta\Phi) \quad (3.2)$$

where C is a contrast value. Ultimately, the detection of P combined with the properties of the Raman wave-vector k_{eff} and the interrogation time T can be used to calculate back the gravitational

acceleration value g :

$$g = \frac{1}{k_{\text{eff}} T^2} \cos^{-1} \left(\frac{1 - 2P}{C} \right) \quad (3.3)$$

In practice however, best sensitivity is obtained when operating the interferometer at mid-fringe [248]. For this reason, a frequency ramp α is applied on the Raman pulses frequency to compensate for the Doppler effect and lock the interferometer. The gravitational acceleration is then directly proportional to the frequency ramp.

$$\begin{aligned} \Delta\Phi &= (k_{\text{eff}}g - 2\pi\alpha) T^2 = 0 \\ \Leftrightarrow g &= 2\pi \frac{\alpha}{k_{\text{eff}}} \end{aligned} \quad (3.4)$$

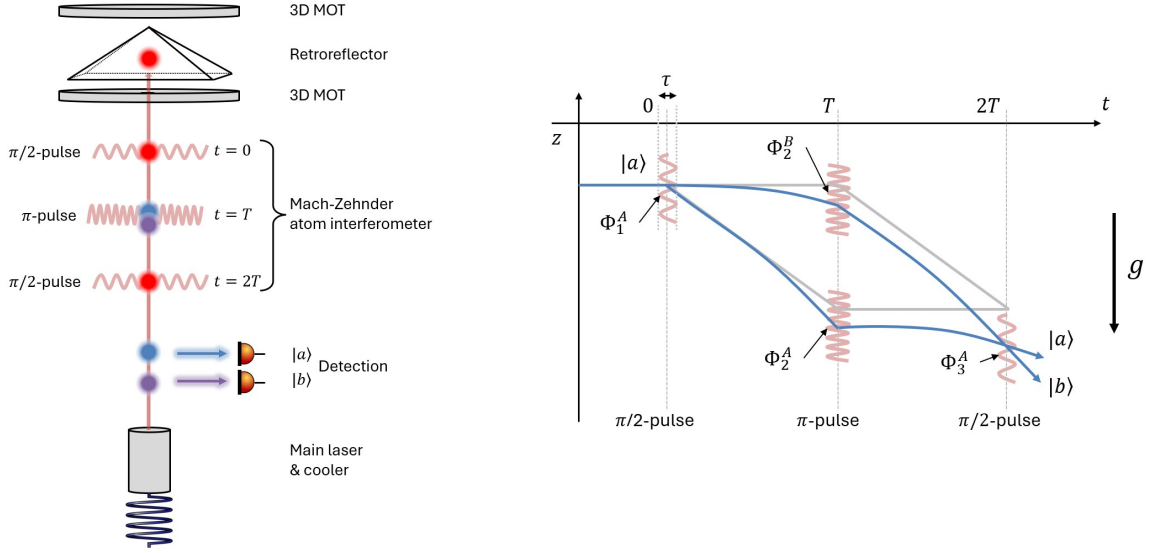


Figure 3.1: (Left) Typical configuration of a quantum gravimeter in dropped atoms configuration (Figure adapted from [29, 255]). (Right) Atom Mach-Zehnder atom interferometer (Figure adapted from [233, 263]).

For the sake of the illustration, the details of the specific interferometer used in the AQG of EXail can be found in [29, 253]. It uses Rubidium ^{87}Rb atoms and the Raman transitions are tuned on the internal states $|a\rangle = |5^2S_{1/2}, F=1, m_F=0\rangle$ and $|b\rangle = |5^2S_{1/2}, F=2, m_F=0\rangle$ of the Rubidium atoms. Approximately 10^7 atoms are trapped in the MOT and cooled down to 2 μK in an optical molasses. The pulse duration are 10 μs for the $\pi/2$ -pulses, and 20 μs for the π -pulse, and the interrogation time between two pulses is $T = 60$ ms.

Limitations

The resolution of an atomic gravimeter is set by various factors. Le Gouët *et al* proposes a thorough overview of the different noise sources that are limiting the resolution of the kind of quantum gravimeter presented above [41].

Detection noise is the accuracy with which the population of atoms in each state is measured. An ultimate limitation here is the quantum projection noise [264, 265]: consisting of population fluctuation in two-level systems according to the principle of uncertainty in quantum mechanics. Although being a hard limit due to the law of physics, the quantum projection noise can be rendered low by enlarging the number of atoms involved in the interferometer. However, quantum projection noise is rarely a limitation; electronic noise, photodiode noise, and laser frequency and intensity noise also affect the measurement and might exceed the quantum limit.

Another contribution is known as phase noise. Any noise in the optical system forming the interferometer (e.g. main/master laser, retroreflector, laser phase lock, Raman pulse synthesis, reference oscillator, etc) will imprint on the free-falling atoms and translate into noise in the atomic phase of the interferometer. The sensitivity of a quantum gravimeter to such parameters is discussed in [249] and can typically fall in the same range as detection noise.

Eventually, one last contribution is sensitivity to external factors. While the instrument can be shielded - to some degree- from some ambient loads (e.g. magnetic fields, temperature), sensitivity to ground vibration remains a large limitation, typically dominating any above-mentioned noise sources [248]. Indeed, any motion of the retroreflector, during the duration τ of a Raman pulse, translates into a spurious phase on the laser beam and thereby on the atomic phase. The transfer function $H_a(j\omega)$ from ground acceleration to the quantum gravimeter output writes [260]

$$|H_a(\omega)|^2 = \frac{16}{\omega^4} \left(\sin \left(\frac{\omega T}{2} \right) \right)^4 \quad (3.5)$$

with T the interferometer interrogation time. The sensitivity of the quantum gravimeter to seismic acceleration noise, expressed as an Allan deviation σ_g would then write

$$\sigma_g^2(\tau) = \frac{k_{\text{eff}}^2}{\tau} \sum_{k=1}^{\infty} |H_a(j\omega)|^2 S_a(2\pi k f_c) \quad (3.6)$$

where f_c is the cycling rate and S_a the acceleration noise spectrum. In the frequency domain, a typical sensitivity function of an atomic gravimeter, with an interrogation time $T = 60$ ms, to seismic acceleration noise is illustrated in Figure 3.2. It shows that seismic acceleration noise fully transmits to the gravity reading over a fairly wide range of frequencies from DC up to several multiples of the cycling frequency, typically around 10 to 20 Hz.

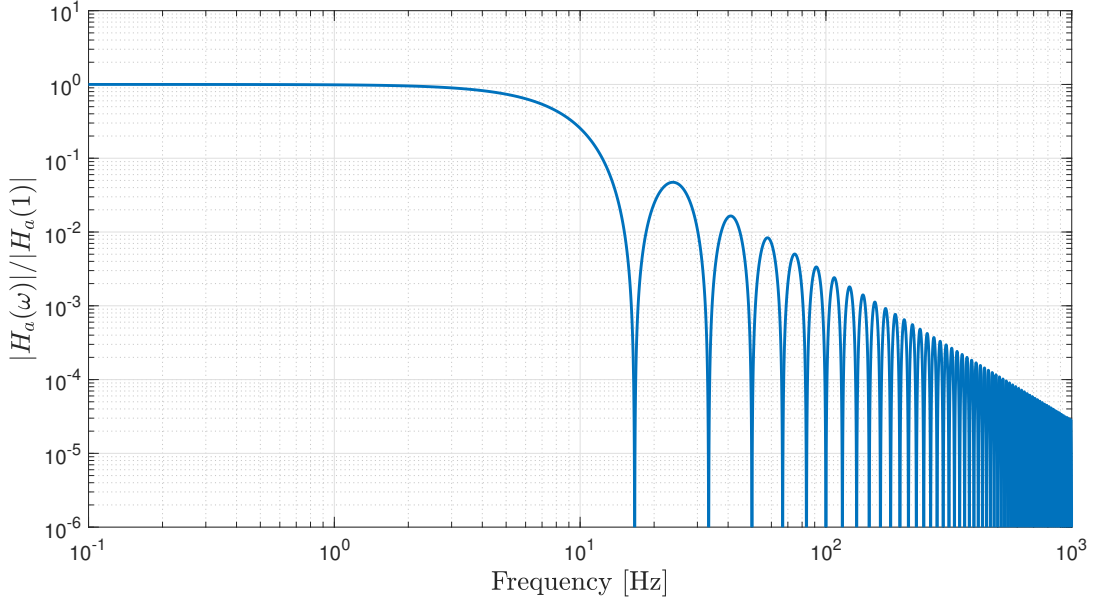


Figure 3.2: Sensitivity of a quantum gravimeter to seismic acceleration noise (Figure adapted from [260]).

Vibration noise compensation

As shown before, ground vibration noise has a significant contribution to the limitation of quantum gravimeters. Several methods have been proposed to compensate for this noise, ranging from isolating the instrument from ground motion [42, 251], compensating the vibration noise using an external accelerometer [29, 253], or hybridizing the instrument [43, 252]. A comprehensive reviews of vibration noise compensation methods can be found in [254].

Active and/or passive isolation

One straightforward and efficient way to deal with vibration noise is to isolate the quantum gravimeter from ground motion. However, because acceleration noise affects the instruments at low frequencies down to DC, the isolation system should be designed and optimised to be effective in this frequency range. For this reason, isolation system for quantum gravimeter are often large, bulky, and commonly rely on an active isolation strategy [42, 251]. Because isolation systems typically face hard limitation at sub-100 mHz frequencies, such systems are usually only capable of lowering the vibration noise in the higher frequency range. Research showed that quantum gravimeters equipped with commercial isolation systems can lower their vibration noise level down to $(2-4) \cdot 10^{-9} g$ [42, 251, 254]. However, when equipped with high-performance, research-grade, systems optimised for low-frequency vibration isolation, the vibration noise can be pushed down to the phase-noise limit [30]. The isolator was using optical inertial sensors [100, 151] and active inertial feedback con-

trol to provide isolation in the 10 mHz to 10 Hz band [221].

Compensation using an external accelerometer

This technique consists in taking advantage of the known sensitivity function of the quantum gravimeter to vibration noise [29, 253]. It uses Equation 3.6 and an external accelerometer to measure the vibration noise spectrum S_a to apply the adequate correction on the phase of the atom interferometer. Compared to the previous method, this solution comes with the advantages how being able to correct over the full range and bandwidth of the accelerometer, as opposed to actively isolated gravimeters that might be limited by saturation of the control system and other technical difficulties. It also preserves the portability of the instrument, especially convenient for field applications.

The compensation can be done either offline or online. For an offline scheme, the quantum gravimeter operates as exposed above and the atom phase is calculated from the detection of atoms in state $|b\rangle$, as per Equation 3.2. Then, a phase correction, calculated from the sensitivity of the atom interferometer phase to vibration noise and the measurement of the external accelerometer, is applied before computing the gravity acceleration value from Equation 3.1. This scheme is probably the simplest to implement. However, this does not allow the quantum gravimeter to operate in a so-called locked configuration, i.e. continuously maintaining the gravimeter at its most sensitive operating point, since the vibration noise will constantly throw the interferometer away from the mid-fringe point.

For an online compensation, the vibration noise-induced phase is fed to the laser system, and a phase correction is applied during the last (3rd) Raman pulse. Because the atom phase measured at the detection step is "free" of vibration noise, the quantum interferometer can be locked at mid-fringe (see Equation 3.4) and operates at best sensitivity. The "online" correction strategy has allowed pushing the vibration noise of the Absolute Quantum Gravimeter (AQG) equipped with the commercial Titan accelerometer down to $1 \cdot 10^{-9} g$ [29]. Of course, the performance of such techniques relies, on one hand, on how accurately the sensitivity function to ground acceleration is known and on how the correction is applied, and on the other hand, on the performance of the accelerometer. For best performance, a quantum gravimeter should be equipped with broadband accelerometers, with a bandwidth spanning from dozens of seconds to a hundred of Hz, with a sub -150 dB (ref $[(m/s^2)^2/Hz]$) resolution. In addition, the phase response of the accelerometer should be very flat over the bandwidth of the compensation for the correction to be calculated correctly.

Hybridising

Hybridising pushes the concept of online vibration correction using an external (conventional) accelerometer one step further. In this scheme, the vibration phase noise is corrected using an online compensation scheme similar to what has been explained above. In the above scheme, no actions are specifically taken to address the limitation of the external accelerometer. In particular,

classic inertial accelerometers suffer from a low-frequency drift (due to mechanical noise, temperature sensitivity, or such). This drift needs to be filtered out, which limits the bandwidth in which the vibration phase noise compensation is effective. Typically, frequencies of a few tens of mHz and below are filtered.

The hybridising scheme proposes to use an atom interferometer as a stable reference to servo-lock the DC component of the external accelerometer [43, 252]. In this way, one realises a hybrid mechanical/quantum sensor where the advantages of one sensor are used to address the limitations of the other. In terms of bandwidth, the mechanical accelerometer eliminates the dead-time issues and extends the measurement up to several hundred of Hz, and the quantum interferometer provides stability down to DC. The resolution is as high as that of a quantum gravimeter that operates as mid-fringe, i.e. of the μGal level. This method has demonstrated to lower the resolution of a quantum gravimeter close to the detection noise limit [43].

Figure 3.3 depicts schematic representations of 3 vibration compensation techniques using an external inertial sensor to measure the vibration noise. (a) Offline compensation: both the atom interferometer phase and vibration noise are simultaneously measured, and the vibration noise-induced phase is corrected based on the knowledge of the sensitivity transfer function from vibration noise to the atomic phase H_a . (b) Online compensation: the vibration noise-induced phase is fed to the laser system, and a phase correction is applied to the last Raman pulse of the atom interferometer. (c) Classical accelerometer and atom interferometer hybridising: in addition to the correction of the vibration noise-induced phase, the quantum gravimeter is used as a stable, drift-less, reference to correct the drift of the external sensor, providing a long-term and stable measurement of both the gravity signal and the vibration signal.

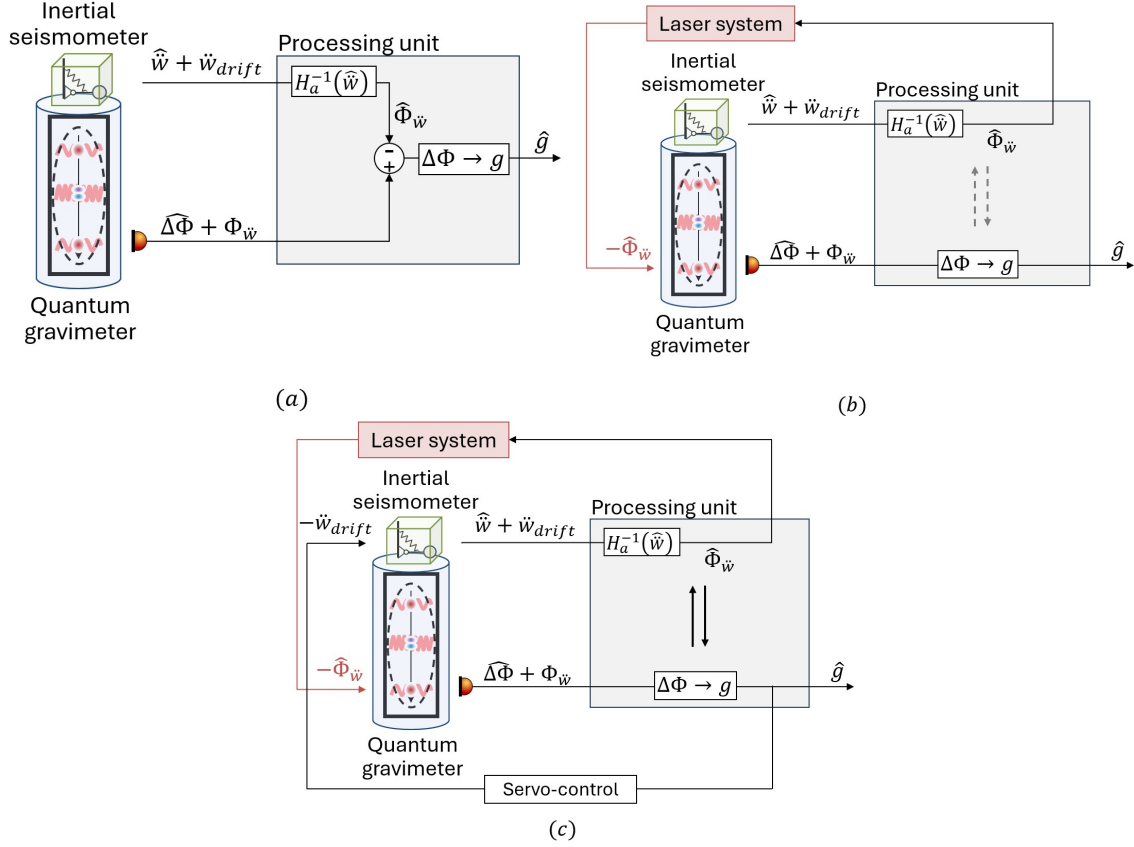


Figure 3.3: Schematic representation of the different vibration compensation techniques using an external inertial sensor. (a) Offline vibration compensation. (b) Online compensation by applying a phase correction $-\hat{\Phi}_{\ddot{w}}$ on the 3rd Raman pulse. (c) Hybridising strategy to reject both the inertial sensor drift and the quantum gravimeter vibration noise. In all figures, $\hat{w}$ is the vibration noise measured by an external (classical) inertial sensor. $\ddot{w}_{drift}$ is the drift of the inertial sensor. $\Delta\hat{\Phi}$ is the phase of the atom interferometer measured by the quantum gravimeter. $\Phi_{\ddot{w}}$ is the phase noise caused by the vibration noise. H_a is the sensitivity of the atom interferometer to vibration noise. $\hat{\Phi}_{\ddot{w}}$ is the vibration phase noise estimated from the inertial sensor. The $\Delta\Phi \rightarrow g$ is the conversion for the interferometer phase to g value. g is the acceleration of gravity. (Figures adapted from [260].)

3.1.2 Broadening an inertial sensor's sensitivity response

As exposed above, the vibration noise compensation technique is key in quantum gravimeters, and is currently the limiting noise in the EXail Absolute Quantum Gravimeter (AQG) [29, 30]. To improve the resolution of the gravimeter while preserving its portability, one needs to develop a high-resolution broadband inertial accelerometer with a bandwidth spanning from DC to 100 Hz and a flat phase response over the bandwidth, to improve the vibration compensation scheme of the AQG.

Bandwidth and resolution are however conflicting characteristics in inertial sensors. As exposed in section 2.1, a high resolution sensor necessarily call for a low mechanical resonance frequency, as

the resolution of an inertial accelerometer with a mechanical frequency of ω_0 scales as $1/\omega_0^2$. Yet, the bandwidth and phase of an (open-loop) accelerometer is a second-order low-pass filter with a cut-off frequency at ω_0 . To illustrate with a short numerical example: for an accelerometer to have a flat phase and bandwidth from DC to 100 Hz, the mechanical resonance should be at least 100 Hz (assuming a high Q -factor). The sensitivity of such an accelerometer, even equipped with a 1 pm high-resolution readout, would be of orders of 400 nm/s^2 , far exceeding the requirements of quantum gravimeters.

Techniques exist for extending the bandwidth of sensors with a low mechanical resonance frequency. One of such methods is the force-balance principle [55, 57, 155, 157]. The technique consists in feeding back the reading of the relative motion sensor to an actuator and balancing the proof-mass motion around a null position. This method is commonly used in broadband seismometers and has been shown to effectively extend the bandwidth while keeping the detectivity unchanged (under the assumption of negligible control noise). Another method consists of using a frequency compensation filter for stretching the response of the sensor [154]. This fundamentally consists of applying filtering to the sensor output, which inverts the sensor frequency response and shapes it to the desired bandwidth, without degrading the signal-to-noise ratio since it affects signal and noise equally. This technique has been widely used for decades in microelectronics [266, 267], and has recently been proposed to be used in inertial sensors [154, 268, 269].

Force-Balanced Inertial sensor

The force-balance principle is a very well-known technique that has become a standard in virtually all commercial very broadband (VBB) seismometers and force-balance accelerometers (FBA), such as Guralp T6, Nanometrics Titan or Trillium T240 or Streckeisen STS2 [50, 51, 176]. Several recent research instruments also rely on this technique [49, 78, 79, 106, 198, 222, 270–272]. The technique consists in feeding back the reading of the relative motion sensor to an actuator and balance the proof-mass motion around a null position [55, 57, 154, 155, 157]. This way, the bandwidth can be extended beyond the mechanical response of the sensor, typically being limited by the control system, while preserving the resolution of a low ω_0 sensor (under the assumption of negligible control noise). In addition, freezing the motion of the proof-mass linearises the mechanics of the sensor and shifts the linearity requirement from the readout system to the control electronics. This is particularly valuable, especially for accelerometers that use optical readout devices, since fringe-counting interferometers suffer from strong non-linear behaviour [134].

Figure 3.4 shows a typical representation of a force-balanced inertial sensor. The mechanics is modelled the same way as exposed in section 2.1: the proof-mass is modelled as a 1 degree of freedom element m ([kg]) and suspension is modelled as a spring-dashpot system of stiffness k ([N/m]) and damping coefficient c ([Ns/m]). Ground acceleration $\ddot{w}$ ([m/s²]) transmits to the proof-mass motion x ([m]) through the suspension. The proof-mass relative motion y is measured by a relative displacement readout device g ([V/m]) to produce a readout signal r ([V]). The readout signal is fed to the control electronics with a feedback filter H ([V/V]). The controller is traditionally

a classical PID or lead-lag-type filter, but can have more sophisticated shapes. The control signal u ([V]) is then used to drive an actuator with force constant T ([N/V]) and produce the feedback force f ([N]) that balances the proof-mass. In this scheme, the sensor output is the control signal used for balancing the mass. The sensor output is tainted by various noise sources. As previously, the noise sources primarily consist of internal (readout) noise $\tilde{n}_i$ ([V]), external (e.g. thermal) noise $\tilde{n}_e$ ([N]). In addition, control noise $\tilde{n}_c$ ([V]) also adds up in the noise budget, corresponding to the noise in the control electronics and actuation system.

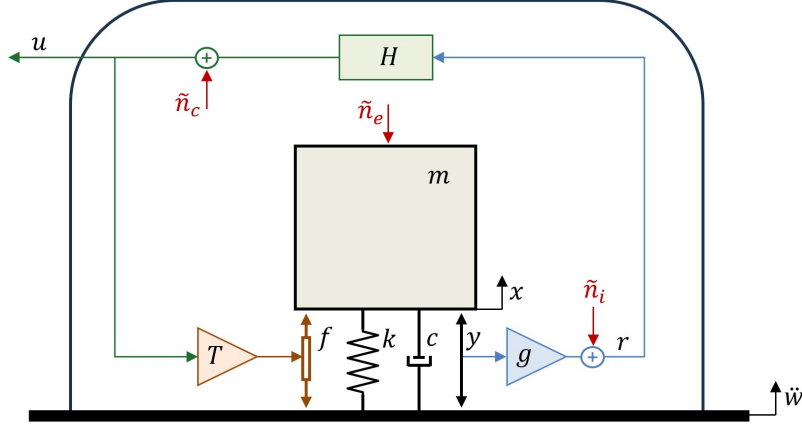


Figure 3.4: Symbolic schematic diagram of a closed-loop Force-Balanced Accelerometer. k ([N/m]), c ([Ns/m]) and m ([kg]) are the sensor's mechanical properties: suspension stiffness, damping coefficient and proof-mass, respectively. Ground acceleration $\ddot{w}$ ([m/s²]) translates into proof-mass motion x ([m]) with relative motion y ([m]). (Blue) Relative displacement sensor: readout gain g ([V/m]) and relative sensor output r ([V]). (Green) Control electronics: H feedback filter ([V/V]) and control signal u ([V]). (Orange) Actuation system: actuator gain T ([N/V]) and actuation force f ([N]). (Red) Noise sources: internal noises $\tilde{n}_i$ ([V]), control noise $\tilde{n}_c$ ([V]) and external noises $\tilde{n}_e$ ([N]).

The sensor equation is obtained by solving a set of 4 equations: the mechanics equation (Newton's Law applied to the proof-mass suspension), the readout equation, the feedback equation and the actuator equation:

$$\begin{aligned}
 \text{Mechanics equation:} \quad & m\ddot{x} + c(\dot{x} - \dot{w}) + k(x - w) = f + \tilde{n}_e \\
 \text{Readout equation:} \quad & r = g \cdot y + \tilde{n}_i \\
 \text{Feedback equation:} \quad & u = -H \cdot r + \tilde{n}_c \\
 \text{Actuator equation:} \quad & f = T \cdot u
 \end{aligned} \tag{3.7}$$

Writing the mechanics equation for the relative motion $y = x - w$ and using the Laplace transform leads to

$$(ms^2 + cs + k)Y = -ms^2W + F + N_e \tag{3.8}$$

On the other hand, combining the other 3 equations, the feedback force F writes

$$\begin{aligned} F &= T(-H \cdot R + N_c) \\ &= T(-H(g \cdot Y + N_i) + N_c) \\ &= -gHT \cdot Y - HT \cdot N_i + T \cdot N_c \end{aligned} \quad (3.9)$$

Substituting in Equation 3.8 and solving for Y yields

$$\begin{aligned} (ms^2 + cs + k + gHT)Y &= -ms^2W - HT \cdot N_i + T \cdot N_c + N_e \\ \Leftrightarrow Y &= \frac{-m}{ms^2 + cs + k + gHT}s^2W + \frac{-HT}{ms^2 + cs + k + gHT}N_i \\ &\quad + \frac{T}{ms^2 + cs + k + gHT}N_c + \frac{1}{ms^2 + cs + k + gHT}N_e \end{aligned} \quad (3.10)$$

Finally substituting again in the readout and feedback equation provides the sensor equation:

$$\begin{aligned} U &= -gH \cdot Y - H \cdot N_i + N_c \\ &= \frac{mgH}{ms^2 + cs + k + gHT}s^2W + \left(\frac{gH^2T}{ms^2 + cs + k + gHT} - H \right) N_i \\ &\quad + \left(\frac{-gHT}{ms^2 + cs + k + gHT} + 1 \right) N_c + \left(\frac{-gH}{ms^2 + cs + k + gHT} \right) N_e \\ &= \frac{mgH}{ms^2 + cs + k + gHT}s^2W + \left(\frac{H(ms^2 + cs + k) + gH^2T - gH^2T}{ms^2 + cs + k + gHT} \right) N_i \\ &\quad + \left(\frac{ms^2 + cs + k + gHT - gHT}{ms^2 + cs + k + gHT} \right) N_c + \left(\frac{-gH}{ms^2 + cs + k + gHT} \right) N_e \end{aligned} \quad (3.11)$$

Normalising with the mass m and using the definitions of the suspension resonance frequency $k/m = \omega_0^2$ and Q -factor $c/m = \omega_0/Q$, the sensor equation writes

$$\begin{aligned} U &= \frac{gH}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2 + gHT/m}s^2W + \frac{H(s^2 + \frac{\omega_0}{Q}s + \omega_0^2)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2 + gHT/m}N_i \\ &\quad + \frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2 + gHT/m}N_c + \frac{1}{m} \frac{-gH}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2 + gHT/m}N_e \end{aligned} \quad (3.12)$$

From the sensor equation, it can be seen that the sensitivity response function S_{FBA} ($[V/(m/s^2)]$) of the inertial sensor in force-balance mode to ground acceleration is now

$$S_{\text{FBA}} = \frac{U}{s^2W} = \frac{gH}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2 + gHT/m} \quad (3.13)$$

It can therefore be tuned from the design of the control filter H . Traditionally, the feedback filter

is a classical PID or lead-lag controller, but can be designed with more sophisticated approaches.

Figure 3.5 shows an example of the sensitivity response function for a force-balanced accelerometer with a 1 Hz mechanical resonance frequency, a Q -factor of 200 and equipped with a relative displacement readout with gain $g = 1$ V/m and actuator $T = 1$ N/V. The controller is a simple lead $H = 5 \cdot 10^6(s + 2\pi \cdot 70)/(s + 2\pi \cdot 500)$ designed for a unity-gain frequency at about 200 Hz (dotted orange trace). The plot shows the sensitivity response for both the passive mechanical accelerometer (dashed black) and the FB accelerometer (solid blue). It can be seen that the passive sensor shows a typical roll-off after the mechanical resonance at 1 Hz and a DC gain of $1/\omega_0^2 = 0.0253$. The FB accelerometer effectively extended for a flat response and phase from DC up to 100 Hz. In practice, it is however very common and good practice to add a DC-filter to prevent injecting low-frequency drift caused, notoriously, by temperature sensitivity and mechanical drift. Also, this model excludes spurious high-frequency modes in the sensor's mechanics, which often limits the unity-gain frequency to lower values.

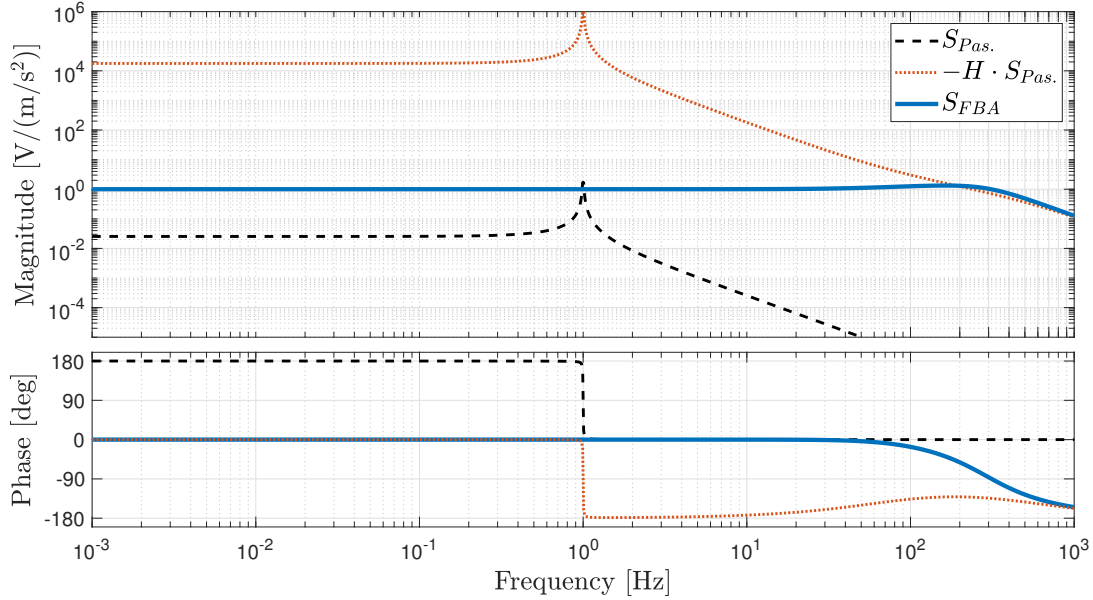


Figure 3.5: Typical sensitivity response functions of a 1 Hz force-balance accelerometer with a 200 Q -factor, a relative readout gain of 1 V/m. (Dashed black) Passive $S_{pas.} = U/s^2W$ sensitivity response of the inertial sensor. (Dotted orange) Open-loop gain: control filter $H \times$ passive sensitivity $S_{pas.}$. (Solid blue) $S_{FBA} = U/s^2W$ sensitivity response of the inertial sensor equipped with force-balanced feedback loop with control filter H and actuator $T = 1$ N/V.

Correction the sensor's equation for the FBA sensitivity response, one obtains

$$\begin{aligned} \widehat{s^2W} &= S_{FBA}^{-1} \cdot U \\ &= s^2W + \left(\frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{g} \right) N_i + \left(\frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{gH} \right) N_c + \frac{-1}{m} N_e \end{aligned} \quad (3.14)$$

where $\widehat{s^2 W}$ is the ground acceleration measured by the force-balanced sensor. Writing in the power spectral density domain Φ . $([(\cdot)^2/\text{Hz}])$, the FB accelerometer noise is expressed as

$$\Phi_{\widehat{s^2 W}} = \Phi_{s^2 W} + \underbrace{\left| \frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{g} \right|^2 \Phi_r + \left| \frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{gH} \right|^2 \Phi_c + \left| \frac{1}{m} \right|^2 \Phi_{\text{th}}}_{\text{FBA noise}} \quad (3.15)$$

where it is assumed that internal noises are primarily dominated by readout noise Φ_r ($[\text{V}^2/\text{Hz}]$) and external noises by thermal noise Φ_{th} ($[\text{N}^2/\text{Hz}]$).

Eventually, Equation 3.6 for the FB accelerometer noise shows that the readout and thermal noises only scale with the mechanical properties of the suspension, and not with feedback gain or control filter design. This equation is to be compared with that of a passive inertial accelerometer (see Equation 2.8 from section 2.1), where it can be seen that the noise scaling is identical, only one additional noise term Φ_c is present. This noise corresponds to the electrical noise in the control electronics and/or actuator noise. However, due to its $1/H$ scaling, this noise can be rendered low with a high feedback gain value. Equivalently, Figure 3.6 shows a typical noise diagram for a force-balanced accelerometer. Solving the block diagram would lead to the same conclusion.

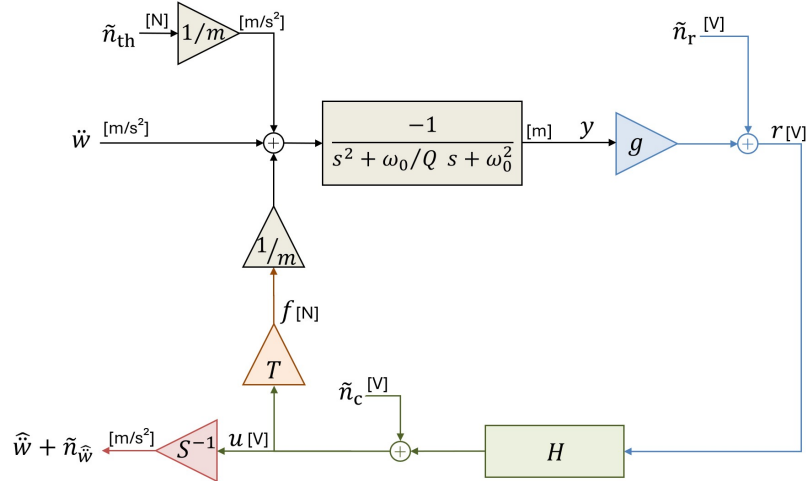


Figure 3.6: Typical noise diagram for a Force-Balanced Accelerometer. $\ddot{w}$ and $\hat{\ddot{w}}$ are respectively the ground acceleration and the sensor-measured ground acceleration. y and r are respectively the proof-mass relative motion and the readout measured motion. The brown blocks represent the sensor's mechanical response to inputs, the blue block the readout g , the green block the control electronics and the orange block the actuation system. The S^{-1} red block represents the digital inversion of the sensor's sensitivity response. $\tilde{n}_{\text{th}}$ represents thermal noise(s), $\tilde{n}_r$ the readout noise, $\tilde{n}_c$ the control noise and $\tilde{n}_{\hat{\ddot{w}}}$ ($[\text{m/s}^2]$) the global inertial sensor noise.

In fine, the force-balance scheme effectively allows to extend the bandwidth of an inertial accelerometer typically one to two orders of magnitude above its mechanical resonance frequency, while preserving the noise performance of a low ω_0 sensor since readout and thermal noise are not

scaled with the control designed. This makes the force-balance technique efficient for broadening and linearising the response of an inertial sensor. However, care should be paid in the selection of great control electronics and actuation system to minimise control noise propagation to the sensor's output.

Frequency-compensation scheme

This consists of using a filter to stretch the (passive) mechanical response of the inertial accelerometer [154]. This fundamentally consists in filtering the sensor output, inverting the sensor frequency response, and reshaping it to the desired bandwidth. Such a technique has been widely used for decades in microelectronics [266,267] for compensating the frequency response of op-amps and LDOs, and has recently been proposed to be used in inertial sensors [154,268,269].

A symbolic schematic of an inertial sensor operating with a frequency compensation scheme is shown in Figure 3.7. It uses the same notation as previously. In this scheme, the reading of the relative motion sensor r is processed by means of a filter F ([V/V]) to produce the sensor output u . The compensation filter F can be either analogue or digital. In the case of an analogue filter, the filter electronics noise could be accounted for in the modelling. The digital option allows more freedom in the design of the filter, is simpler to modify in case of drifting in the sensor's mechanical response (due to, e.g., ageing), and can be considered noiseless. Thanks to recent advances in digital electronics and programmable FPGAs, it is also possible to provide digital electronics with real-time processing power. The model assumes a digital filter with minimal noise consisting of only Analogue-to-Digital Conversion noise, which can be considered as part of the sensor's internal noise $\tilde{n}_i$.

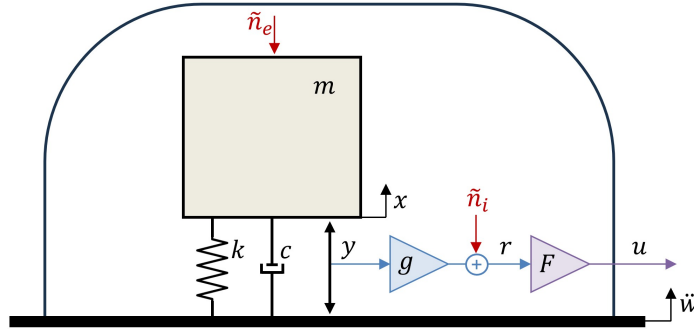


Figure 3.7: Symbolic schematic diagram of an open-loop inertial sensor with frequency compensation filter. k ([N/m]), c ([Ns/m]) and m ([kg]) are the sensor's mechanical properties: suspension stiffness, damping coefficient and proof-mass, respectively. Ground motion $\ddot{w}$ ([m]) translates into proof-mass motion x ([m]) with relative motion y ([m]). (Blue) Relative displacement sensor: readout gain g ([V/m]) and relative sensor output r ([V]). (Purple) Frequency compensation filter F ([V/V]). (Red) Noise sources: internal noises $\tilde{n}_i$ ([V]) and external noises $\tilde{n}_e$ ([N])

The inertial sensor equation is obtained combining the mechanics equation, the readout equation

and the filter equation

$$\begin{aligned}
 \text{Mechanics equation:} \quad & m\ddot{x} + c(\dot{x} - \dot{w}) + k(x - w) = \tilde{n}_e \\
 \text{Readout equation:} \quad & r = g \cdot y + \tilde{n}_i \\
 \text{Filter equation:} \quad & u = F \cdot r
 \end{aligned} \tag{3.16}$$

Solving the mechanics equation in the Laplace domain for the relative motion $y = x - w$:

$$Y = \frac{-m}{ms^2 + cs + k} s^2 W + \frac{1}{ms^2 + cs + k} N_e \tag{3.17}$$

Substituting in the readout equation and in then in the filter equation leads to

$$\begin{aligned}
 U &= F \cdot (g \cdot Y + N_i) \\
 &= \frac{-gFm}{ms^2 + cs + k} s^2 W + F \cdot N_i + \frac{gF}{ms^2 + cs + k} N_e
 \end{aligned} \tag{3.18}$$

Normalising with m leads to

$$U = \frac{-gF}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} s^2 W + F \cdot N_i + \frac{1}{m} \frac{gF}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} N_e \tag{3.19}$$

From Equation 3.19, it can be seen that the effective sensitivity response of the inertial accelerometer is simply the product of its mechanical response to the digital filter.

$$S_{\text{FCA}} = \frac{U}{s^2 W} = \frac{-gF}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \tag{3.20}$$

The sensitivity can therefore be arbitrarily modified to that of an inertial accelerometer with larger mechanical frequency $\omega_0^* > \omega_0$ and appropriate damping Q^* with a simple filter design such as

$$F = -\frac{(\omega_0^*)^2}{g} \frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0^*}{Q^*}s + (\omega_0^*)^2} \tag{3.21}$$

An example of the sensitivity response function of a frequency-compensated accelerometer is shown in Figure 3.8. A simple filter as in Equation 3.21 is used to extend the bandwidth of a 1 Hz and 200 Q -factor inertial accelerometer to a corner frequency of 300 Hz with a Q -factor of $\sqrt{2}/2$. The dashed back trace shows the mechanical response of the passive accelerometer, the dotted orange trace the frequency compensation filter and the solid blue trace the response of the filtered accelerometer. Obviously, the quality of the frequency compensation strongly depends on the characterisation of the passive mechanical response, and especially on the characterisation of ω_0 and Q . Inverting a high Q value is often not very robust, so that the frequency compensation

technique can be combined with a R-shunt technique (cf. section 2.1) to damp the resonance of a high Q -factor inertial accelerometer. The method is illustrated here for an inertial accelerometer, but also applies to any other inertial sensors (geophones, seismometers, etc).

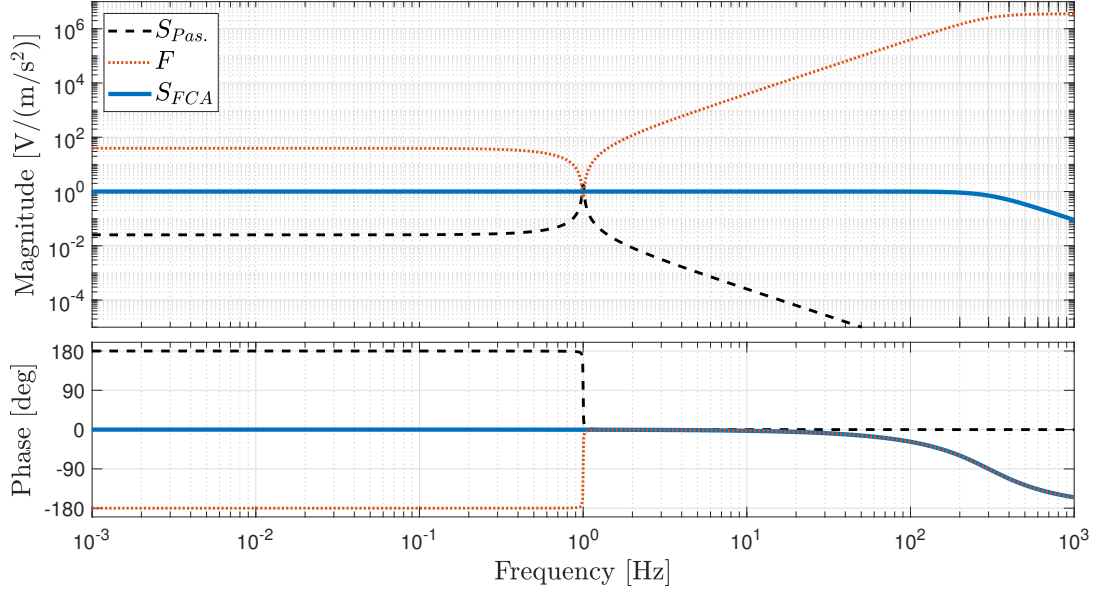


Figure 3.8: Typical sensitivity response functions of a 1 Hz inertial sensor with a 200 Q -factor and a relative readout gain of 1 V/m. (Dashed black) Passive $S_{Pas.} = U/s^2W$ sensitivity response of the inertial sensor. (Dotted orange) Frequency compensation filter F . (Solid blue) $S_{FCA} = U/s^2W$ sensitivity response after filtering.

Correcting the sensor output U for the frequency-compensated sensitivity yields

$$\begin{aligned} \widehat{s^2W} &= S_{FCA}^{-1} \cdot U \\ &= s^2W + \left(\frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{-g} \right) N_i + \left(\frac{-1}{m} \right) N_e \end{aligned} \quad (3.22)$$

So that the resolution of the frequency compensated accelerometer in power spectral densities Φ , writes

$$\Phi_{\widehat{s^2W}} = \Phi_{s^2W} + \underbrace{\left| \frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{g} \right|^2 \Phi_r + \left| \frac{1}{m} \right|^2 \Phi_{th}}_{\text{FCA noise}} \quad (3.23)$$

which is strictly identical to that of a passive inertial sensor (see Equation 2.8), and only depends on the mechanical properties of the suspension. Frequency compensation is therefore a simple and effective way to extend the bandwidth of an inertial sensor without degrading its noise performance.

Compared to the force-balance scheme, this method avoids injection of control noise from the feedback electronics, but loses the ability to linearise the instrument around an equilibrium position. A typical noise diagram for such a frequency-compensated accelerometer is shown in Figure 3.9.

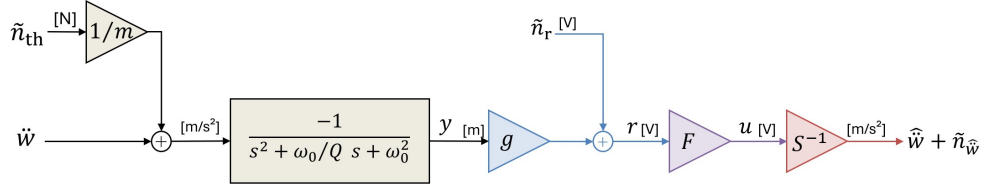


Figure 3.9: Typical noise diagram for an open-loop inertial sensor with frequency compensation filter. w and $\hat{w}$ are respectively the ground motion and the sensor-measured ground motion. y and r are respectively the proof-mass relative motion and the readout measured motion. The brown blocks represent the sensor's mechanical response to inputs, the blue block the readout g and the purple block the frequency compensation filter. The S^{-1} red block represents the digital inversion of the sensor's sensitivity response. $\tilde{n}_{th}$ represents thermal noise(s), $\tilde{n}_r$ the readout noise, $\tilde{n}_c$ the control noise and $\tilde{n}_{\hat{\ddot{w}}}$ ([m/s²]) the global inertial sensor noise.

3.2 Mechanical design

The Micro Verical INterferometric inertial Sensor (μ VINS) is a high resolution, compact, broadband inertial sensor intended to improve the vibration compensation systems in EXail (formally, μ Quans) AQQ (Absolute Quantum Gravimeter). Following the requirements of the vibration compensation technique, the μ VINS is designed for a bandwidth spanning from dozens of seconds to 100 Hz with a $(\text{nm}/\text{s}^2)/\sqrt{\text{Hz}}$ -like resolution. Inspired from the same prototypes as the E-VINS sensors [46,73,100,108,151,214], it is based on a leafspring suspension of a pendulum-style proof mass hinged by a clamped-blade. The design has been squeezed to fit a $10 \times 10 \times 10 \text{ cm}^3$ box for an integration of the accelerometer inside the AQQ, at the closest to the head of the atom interferometer. It uses the quadrature Michelson interferometer showcased in chapter 1 for reading out the proof-mass motion.

Figure 3.10 shows a picture of the μ VINS sensor (right). The 4 major sensor sub-assemblies are highlighted in colours in the CAD view (left). They are the static frame (light blue colour), proof-mass assembly (brown colour), readout assembly (yellow colour) and actuator assembly (pink colour). The mechanical design and drawing resulted from a joint study with Dr. L. Améz-Droz, who researched on developing a fused-silica pivot and guiding mechanism for the proof-mass suspension produced with femtosecond laser etching [211]. Details on the mechanical design can be found in [210]. Table 3.1 contains references to the various components in the assembly.

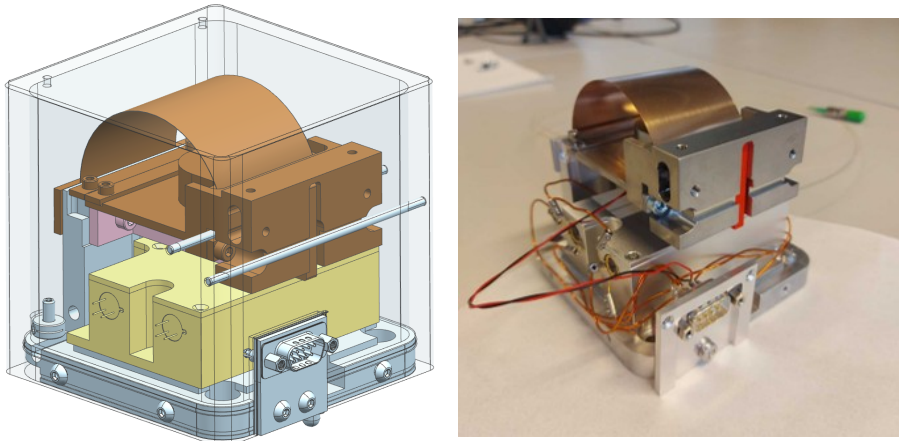


Figure 3.10: μ VINS picture (right). μ VINS CAD view (left). Sensor body (light blue), proof-mass assembly (brown), readout assembly (yellow) and actuator assembly (pink).

Sub assembly	ID	Component	Manufacturer	Ref.
Static frame	1a	Sensor body (Al)	-	L. Amez-Droz [210]
Static frame	1b	Connector interface	-	L. Amez-Droz [210]
		D-sub 9 Male connector	RS	Panel Mount D-sub 9 M Connector
		Internal cabling	RS	Block Enamelled Copper Wire 31 AWG
Static frame	1c	Levelling feet	Newport	Miniature Hex Adjustment Screw
Static Frame	1d	Slots for mounting screws	-	-
Static Frame	1e	Blocking mechanism	-	L. Amez-Droz [210]
Proof-mass	2a	Pendulum bob (SS.)	-	L. Amez-Droz [210]
Proof-mass	2b	CuBe2 suspension leafspring	Laminerie MATTHEY	CuBe2 190/C17200/CW101C
Proof-mass	2c	CuBe2 campled-blade hinge joint	Laminerie MATTHEY	CuBe2 190/C17200/CW101C
Proof-mass	2d	Retroreflector interface (SS.)	-	L. Amez-Droz [210]
Proof-mass	2e	Tuning mass (SS.)	-	Headless hex socket M8x12
Readout	3a	Quadrature Michelson interferometer	EXail	Section 1.2
Readout	3b	Alignment holes	-	L. Amez-Droz [210]
Voice-coil	4a	Large gap voice-coil actuator	Moticont	LVCN-022-013-02 (Coil only)
Voice-coil	4b	Magnet assembly	-	Appendix D
		Neodymium N45 magnets	Supermagnets	S-12-03N
Voice-coil	4c	Static frame interface (Al.)	-	L. Amez-Droz [210]
Voice-coil	4d	Proof-mass interface (Al.)	-	L. Amez-Droz [210]

Table 3.1: List of components in the μ VINS sensor assembly.

Static frame assembly

Figure 3.11 shows the μ VINS static assembly. Technical drawings can be found in [210]. It mostly consists of a rigid body of Al 1060 alloy onto which the other sub-assemblies connect (1a). An interface for a D-sub 9 Male connector (1b) is attached to the sensor body for connecting internal wiring. The pinout map for the μ VINS sensor can be found in Appendix B. The sensor is equipped with 3 feet, the two back ones are adjustable for levelling of the proof-mass centre of mass with the pivot joint. Holes (1d) are drilled in the frame for fixing the sensor to the AQG head. Two blocking rods (1e) can be inserted through the casing and slide inside slots in the proof-mass to secure it during manipulation or transportation.

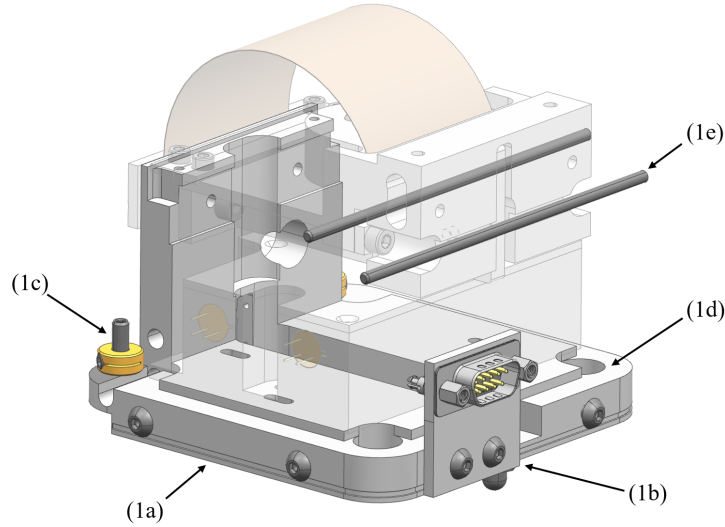


Figure 3.11: μ VINS frame assembly. Sensor body (1a), connector interface (1b), levelling feet (1c), fixing holes (1d), blocking mechanism (1e).

Proof-mass assembly

Figure 3.12 depicts the proof-mass assembly. Technical drawings for the proof-mass and interfaces can be found in [210]. The proof-mass is a leafspring suspended pendulum (2a) weighing approximately 0.4 kg, accounting for the AISI 1020 stainless steel body of the pendulum, the additional tuning masses (2e) and the actuator magnet (4b, see Figure 3.15). As for the E-VINS sensor, the suspension of the proof mass relies on a CuBe2 leafspring (2b) clamped at one end on the static frame, and at the other end on the pendulum bob. The proof-mass rotates on a horizontal axis defined by a CuBe2 clamped-blade hinge joint (2c). The interferometer shoots at a retroreflector mounted underneath the centre of mass (2d). A set of horizontal and vertical headless screws is available on the proof-mass end to allow a fine-tuning of the pendulum inertia and static position (2e), which is critical for aligning the proof-mass centre of mass in the same horizontal plane as the hinge joint, and thus minimising the sensitivity of the suspension to horizontal motion.

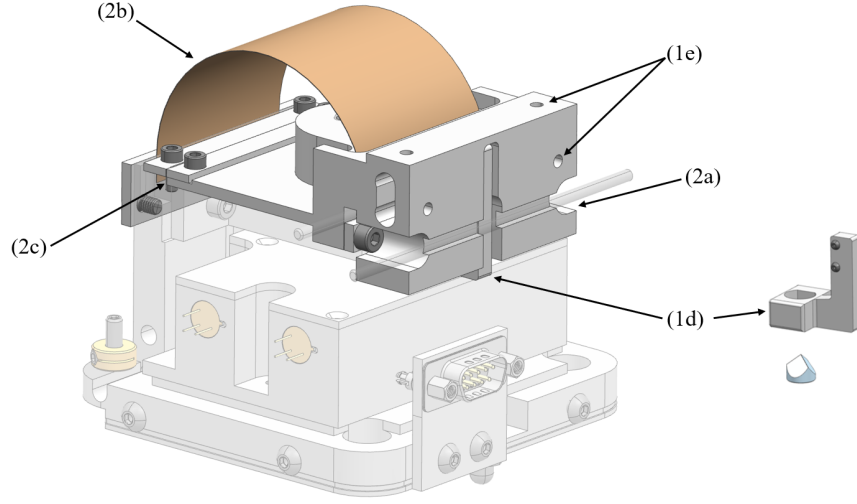


Figure 3.12: μ VINS proof-mass assembly. Proof-mass (≈ 0.4 kg) (2a), CuBe2 leafspring suspension (2b), clamped-blade hinge joint (2c), retroreflector and interface (2d), tuning mass (2e).

Similarly to the E-VINS sensor, the μ VINS uses a leafspring suspension for its proven, theoretical infinite period, performance and compact design [56]. The mechanical period of the μ VINS sensor can therefore be calculated using the same methodology as presented in subsection 2.2.1. Because the vibration compensation in the AQG requires a broadband bandwidth up to 100 Hz, the μ VINS suspension was designed for a mechanical resonance lying approximately at a central frequency, i.e. approximately 2-3 Hz. For this reason, the geometry of the suspension is simpler than for the E-VINS, and is optimised for a more compact design and larger mechanical frequency.

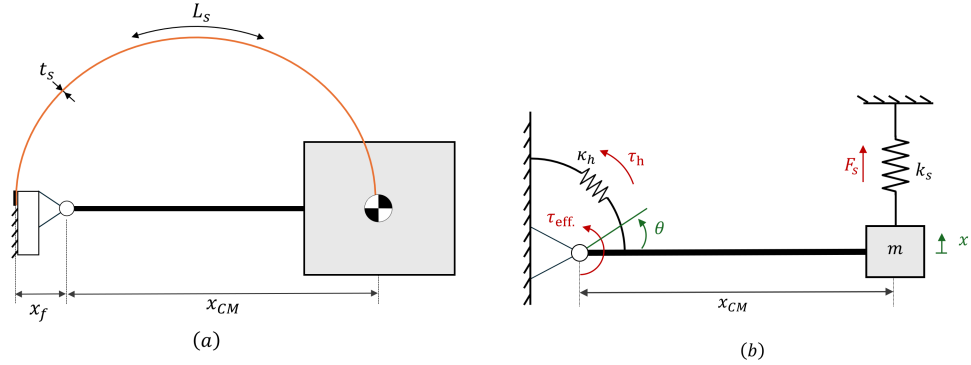


Figure 3.13: (a) Geometry of the μ VINS leafspring suspension. Blade geometry $\{L_s, w_s, t_s\} = \{105, 45, 0.24\}$ mm. Center of mass position $\{x_{CM}, y_{CM}\} = \{57.2, 0\}$ mm, frame clamping point $\{x_f, y_f\} = \{10.5, 0\}$ mm. (b) Mechanical model of the μ VINS leafspring suspension.

A schematic of the suspension geometry is shown in Figure 3.13 (a). The leafspring is a CuBe2 blade with a fairly circular shape. The blade length is $L_s = 105$ mm, the blade width is $w_s = 45$ mm and the blade thickness is $t_s = 0.24$ mm. The blade clamping points are in the same horizontal

plane as the pivot joint. The clamping point of the pendulum bob lies near the centre of mass, at a distance x_{CM} from the pivot of 57.2 mm. The clamping point of the blade on the static frame lies 10.5 mm behind the pivot. Using the analytical approach derived by M. Zeoli [171] from the energy theorem, the stiffness of such a leafspring is estimated as

$$k_s = \frac{\pi^2}{6} \frac{E w_s t_s^3}{L_s^3} = 110.49 \quad [\text{N/m}] \quad (3.24)$$

The pivot joint is similar to that used in the E-VINS sensor. It consists of a clamped thin blade of CuBe2. The blade has a width of $w_h = 45$ mm, a thickness of $t_h = 0.05$ mm and a free length L_h manually adjusted to about half a mm. The corresponding hinge angular stiffness is therefore

$$\kappa_h = \frac{1}{12} \frac{E w_h t_h^3}{L_h} = 0.117 \quad [\text{Nm}] \quad (3.25)$$

Using the kinematic model of the μVINS suspension in Figure 3.13 (b), the equivalent global stiffness is

$$\kappa_{\text{eff.}} = \kappa_h + k_s x_{\text{CM}}^2 = 0.479 \quad [\text{Nm}] \quad (3.26)$$

This results in a mechanical frequency of

$$f_{0,\mu\text{VINS}} = \frac{1}{2\pi} \sqrt{\frac{\kappa_{\text{eff.}}}{m x_{\text{CM}}^2}} = 3.044 \quad [\text{Hz}] \quad (3.27)$$

Readout assembly

Figure 3.14 pictures the readout assembly. The readout is the optical device presented in section 1.2. The size limitation does not allow a multiple degree of freedom alignment system similar to that designed for the E-VINS sensor. Instead, the optical devices are simply screwed in oblong holes drilled in the sensor frame (2b). The oblong holes allow the interferometer to slide along one degree of freedom, and are slightly oversized to allow some degree of twisting adjustment as well. Thanks to the use of a retroreflector with double-pass, this rough alignment strategy was proven sufficient to obtain decent contrast values ($> 75\%$).

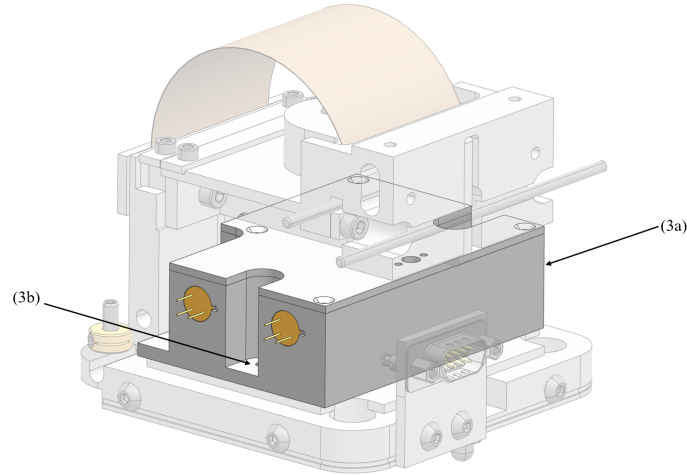


Figure 3.14: μ VINS readout assembly. Optical readout (3a), interface with the static frame (3b).

Actuator assembly

The actuator assembly is shown in Figure 3.15. The μ VINS uses the same design of a contactless voice-coil actuator with a quadrupole magnet design for minimal sensitivity to stray magnetic fields and limited eddy-current generation [216–218]. It uses a Moticont LVMC-022-013-02 coil (4a) interfaced to the static frame (4c) to minimise wiring rubbing and friction. The quadrupole magnet (4b) attaches to the proof-mass with an interface close to the centre of mass (4d). The design of the quadrupole magnet is detailed in Appendix D. The voice-coil actuator has been designed to allow for closing force-balance loops or to be used in a shunt operation to provide passive damping. This linearises the sensor reading by locking the proof-mass in a null position, therefore shifting the linearity requirement from the mechanics and the optical sensor to the actuator [55–58, 152, 157]. This can be an asset for a sensor with an optical reading since the quadrature Michelson interferometer has been measured in section 1.3 to show a nonlinear noise with a couple of $\text{nm}/\sqrt{\text{Hz}}$ level under normal seismic activity conditions.

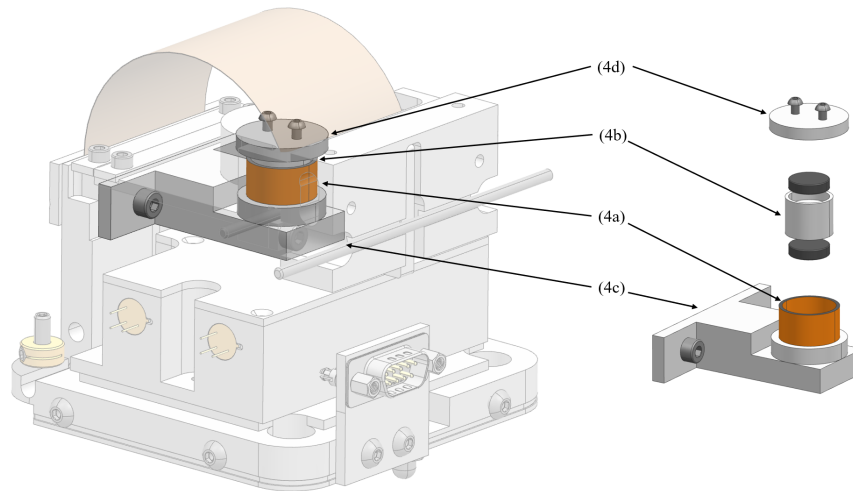


Figure 3.15: μ VINS actuator assembly. Static coil (4a), moving magnet assembly (4b), static frame interface (4c), proof-mass interface (4d).

3.3 Bandwidth broadening and characterisation

In order to have broadband acceleration measurement bandwidth spanning dozens of seconds to 100 Hz with a $(\text{nm}/\text{s}^2)/\sqrt{\text{Hz}}$ -ish resolution, the μVINS inertial accelerometer has been designed with a mid-range mechanical resonance frequency and has its bandwidth extended towards higher frequencies. The mechanical suspension of the proof-mass has been modelled in page 190 with a resonance frequency of approximately 2 Hz. Two methods are used for broadening the bandwidth to a target corner frequency of 100 Hz. They consists in the well-known force-balance principle [55, 57, 154, 155, 157] and in using a real-time frequency-compensation filter [122, 266–269]. Both methods will be compared in terms of capability in reaching the target frequency bandwidth and in terms of resolution, and the best method will be selected for integration in the AQG.

This section contains a more detailed discussion on the results published in [271].

3.3.1 Passive mechanical response

First, the passive mechanical response of the μVINS sensor is identified. A similar procedure as for the E-VINS and E-HINS sensor is used. The μVINS is placed side-by-side with another reference seismometer with a known sensitivity response. A measurement of the transfer function between both acceleration signals allows to extract of the sensitivity of the μVINS sensor. For these tests, a Guralp 6T seismometer is used as a reference, and its output is derived to measure acceleration. Both sensors are placed next to each other on solid ground, and both acceleration outputs are recorded on the MicroLab Box real-time target for 300 s. The demodulation of the relative motion by the interferometric device is implemented in the real-time target system, and the $g_{\text{IFO}} \approx 8\pi/\lambda$ rad/m optical gain is already accounted for in the demodulation.

The transfer between both sensors and corrected for the sensitivity of the Guralp CMG6T instrument is shown in Figure 3.16. The figure shows a clean measurement of the passive sensitivity response function from 0.1 Hz to 100 Hz. The coherence is great in the entire bandwidth, the coherence drop below 0.1 Hz is expected to be caused by the Guralp 6T, which is self-noise-limited below that frequency (see Figure 3.20). The first flexible mode of the mechanics appears at approximately 157 Hz. This mode is most probably the torsion mode of the suspended pendulum, as investigated in [210]. A fit to the measured transfer function estimates the sensor mechanical resonance at 2.7 Hz with a Q -factor of 200. This is slightly larger than the designed natural frequency of 2 Hz but remains in the target range.

$$S_{\mu\text{VINS}}^{\text{Mec.}} = g_{\text{IFO}} \cdot \frac{-1}{s^2 + 0.0848s + 287.80} \quad [\text{rad}/(\text{m}/\text{s}^2)] \quad (3.28)$$

The passive mechanical sensitivity of the μVINS sensor to ground acceleration is therefore flat up to 2 Hz, with a $1/\omega_0^2 = 4 \cdot 10^3$ DC gain, and then shows a $1/f^2$ roll off until 157 Hz, limited by the first flexible mode. The sensitivity bandwidth can therefore be extended up to this frequency

value.

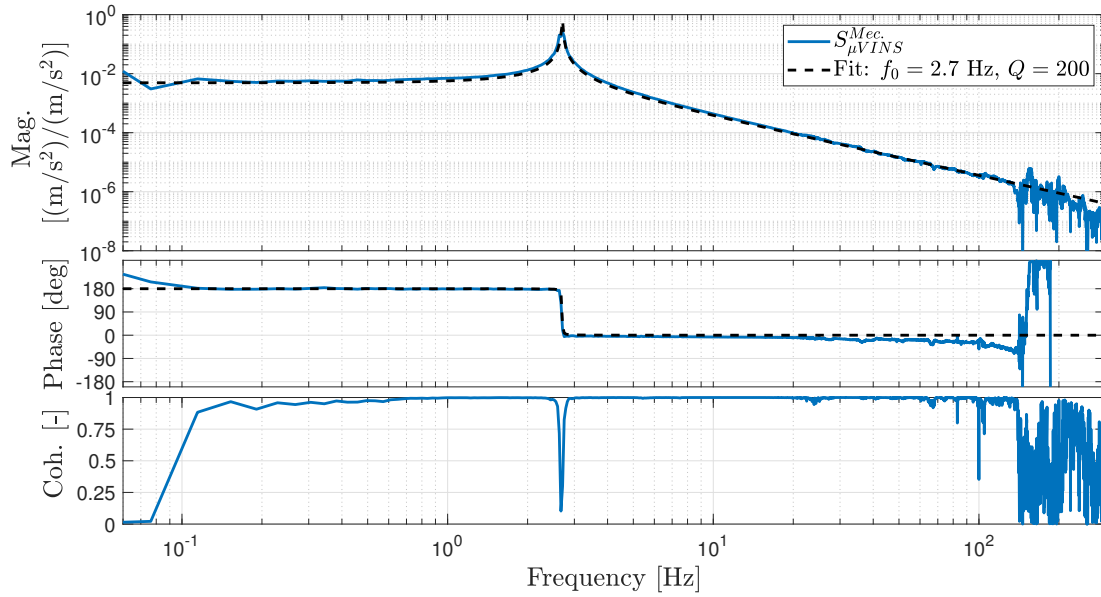


Figure 3.16: μ VINS mechanical response function $S_{\mu VINS}^{Mec.}$ measured with respect to a reference Guralp 6T seismometer. (Solid blue) Measured sensitivity function. (Dotted black) 1 DOF fit model.

3.3.2 Force-balance-accelerometer mode

The force-balance principle consists in feeding back the proof-mass relative motion read by the optical device to the quadrupole voice-coil actuator. The control loop therefore balances the proof-mass in around a null relative motion equilibrium, and the control signal becomes the inertial sensor output [55, 57, 154, 155, 157]. As a result, the effective sensitivity response function of the sensor in closed-loop is shaped by the control filter and can be extended to higher frequencies. Additionally, since the relative motion readout is locked at a null position, the dynamic range and linearity requirements are shifted to the control electronics. This can be particularly valuable for the μ VINS, as it relies on a fringe-counting type of interferometer and as these devices are notoriously known for their strong nonlinear behaviour [134]. This section presents the performance of the μ VINS in force-balanced-accelerometer (FBA) mode.

Control loop design

The first step in designing the control filter is extracting the open-loop transfer function from the quadrupole voice-coil actuator to the relative motion readout. This is done by injecting a sine-sweep excitation in the voice-coil actuator F and recording the optical relative displacement reading Y . The open-loop transfer function T_{FY} is shown in Figure 3.17 (solid blue trace). The sine excitation

sweeps from 0.1 Hz to 1 kHz with a sweep time of 60 s, for a total recording time of 600 s. The measured transfer function includes the control electronics and the actuator gains and is therefore in units of m/V. The measurement shows a clean transfer function and great coherence in the entire bandwidth.

A fit on this measurement (dashed black trace) is used for designing the feedback control filter H designed for the open-loop gain $H \cdot T_{FY}$ to have a unity-gain frequency of 60 Hz with approximately 15° of phase margin (solid orange trace). The control filter mostly consists of a large proportional gain ($9 \cdot 10^6$) combined with a lead action centred at the cross-over frequency of 60 Hz. The cross-over frequency could not be pushed higher due to flexible modes of the mechanics causing the phase drop at above 100 Hz. It also features a DC filter at 10 mHz to prevent DC current injection in the coil, and a low-pass filter at 300 Hz to prevent high-frequency noise injection.

Unfortunately, the presence of spurious poles starting at 100 Hz prevents pushing the unity gain to higher frequencies and will therefore impose a limit to the broadening of the frequency response.

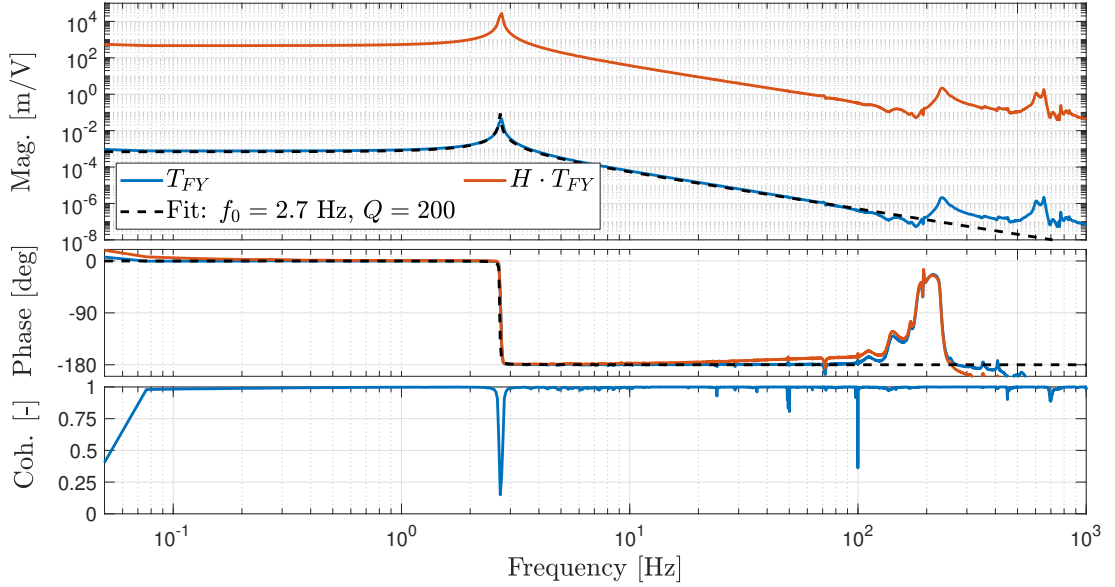


Figure 3.17: μ VINS open-loop transfer function T_{FY} and open-loop gain $H \cdot T_{FY}$.

Frequency response

After closing the force-balance loop, the sensitivity response function of the μ VINS can be measured the same way as explained above for extracting the passive mechanical response. The FBA μ VINS is placed on solid ground side-by-side with a reference Guralp 6T seismometer, and both acceleration readings are recorded on the real-time target for 300 s. After compensation for the Guralp response, the sensitivity response function of the FBA μ VINS $S_{\mu VINS}^{FBA}$ is extracted. It is shown in Figure 3.18 (blue trace). The passive mechanical response of the μ VINS is also superimposed in the figure as a

comparison (orange trace). Comparing both sensitivities, it can be seen that the bandwidth of the μ VINS is indeed extended for a flat magnitude and phase up to the cross-over frequency, i.e. 60 Hz. The measurement also shows a great match with a prediction of the closed-loop sensitivity $S_{\text{mod.}}^{\text{FBA}}$, calculated from a fit of the mechanical response and open-loop gain as

$$S_{\text{mod.}}^{\text{FBA}} = \frac{H \cdot S_{\mu\text{VINS}}^{\text{Mec.}}}{1 + H \cdot T_{\text{FY}}} \quad (3.29)$$

$$= 8.26 \cdot 10^8 \frac{s(s + 623.8)}{(s + 2584)(s + 1465)(s + 0.0001476)(s^2 + 72.78s + 1.365 \cdot 10^5)}$$

As explained above, spurious modes limit the control unity gain frequency to 60 Hz. As a result, the control gain cannot be pushed high enough for the closed-loop sensitivity to reach the targeted bandwidth of 100 Hz, and the sensitivity response and phase are only flat to 50 Hz (with approx. 50° phase lag). Also, Figure 3.18 shows that the FBA μ VINS loses coherence below 1 Hz, whereas the passive μ VINS is coherent with the reference sensor down to 0.1 Hz (limited by the Guralp 6T self noise). As it will be shown in the noise budget, this is caused by control noise propagating in the feedback loop, and limiting the resolution of the μ VINS below 1 Hz.

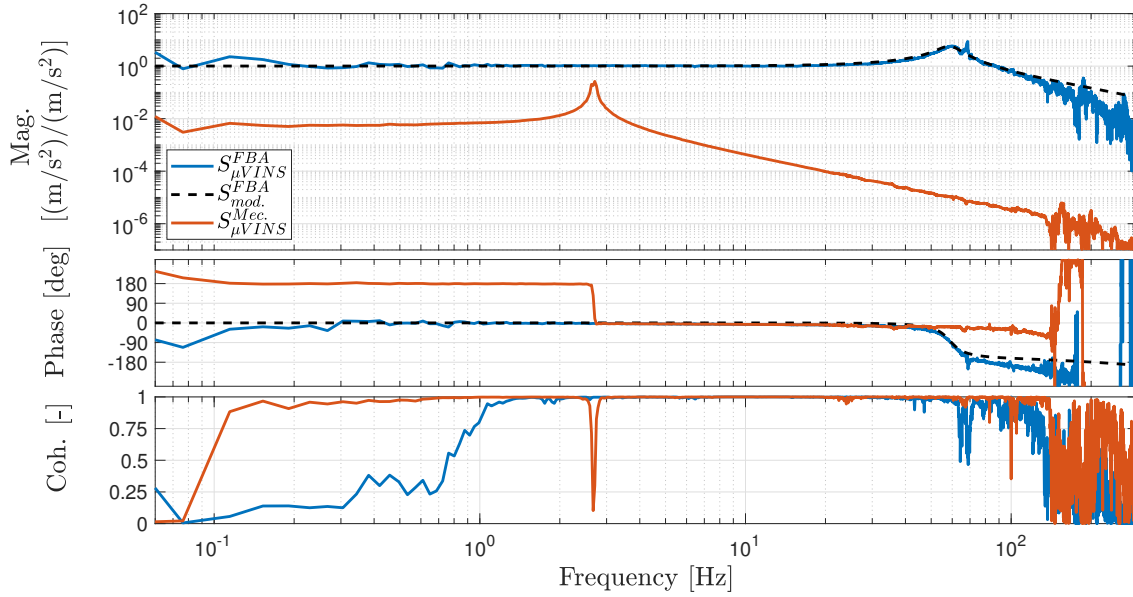


Figure 3.18: μ VINS sensitivity response function in force-balance-accelerometer mode $S_{\mu\text{VINS}}^{\text{FBA}}$ (Blue). Passive mechanical response $S_{\mu\text{VINS}}^{\text{Mec.}}$ (orange). Modelled closed-loop sensitivity response $S_{\text{mod.}}^{\text{FBA}}$ (dashed black)

Noise figure

Figure 3.19 shows a noise block-diagram for the FBA μ VINS. Ground acceleration $\ddot{w}$ transmits through the mechanical suspension to cause the proof-mass motion y . That motion is read out by the interferometer to output a digital signal $\hat{y}$, tainted by the MicroLab Box ADC noise $\tilde{n}_{\text{ADC}}$ and the quadrature interferometer self-noise $\tilde{n}_{\text{IFO}}$. This signal is fed to the control filter H , which outputs the control signal u with the MicroLab Box DAC noise $\tilde{n}_{\text{DAC}}$. The control signal is fed to a coil driver with gain $B = 0.20$ A/V that drives the quadrupole voice-coil actuator, characterised by a force constant $T = 0.65$ N/A. The coil driver is part of the signal conditioning units and is detailed in Appendix C. Likewise, details on the characterisation of the quadrupole voice-coil can be found in Appendix D.

The noise budget of the μ VINS in FBA mode is drawn by measuring individually each noise contribution and propagating them through the noise diagram. Following the same development as in section 3.1, the ground acceleration spectrum $\widehat{\Phi_{s^2W}}$ ($[(\text{m/s}^2)/\sqrt{\text{Hz}}]$) measured by the μ VINS in FBA mode writes

$$\begin{aligned}\widehat{\Phi_{s^2W}} &= (S_{\mu\text{VINS}}^{\text{FBA}})^{-2} \cdot U \\ &= \Phi_{s^2W} + \left| \frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{g_{\text{IFO}}} \right|^2 (\Phi_{\text{ADC}} + \Phi_{\text{IFO}}) + \left| \frac{B \cdot T}{m} \right|^2 \Phi_{\text{DAC}} + \left| \frac{T}{m} \right|^2 \Phi_{\text{CD}} + \left| \frac{1}{m} \right|^2 \Phi_{\text{th}}\end{aligned}\quad (3.30)$$

where $\omega_0 = 2\pi 2.7$ rad/s, $Q = 200$ and $m = 0.4$ kg are the mechanical properties of the proof-mass suspension, $g_{\text{IFO}} \approx 8\pi/\lambda$ rad/m the interferometer gain, $T = 0.65$ N/A the voice-coil actuator gain and H the control filter.

The spectral densities of each noise source have been identified. The ADC and optical readout noises Φ_{ADC} and Φ_{IFO} have been characterised in section 1.3 with a white noise level of $2 \cdot 10^{-13}$ m/ $\sqrt{\text{Hz}}$ at 10 Hz and a $1/f^{0.5}$ contribution kicking in at lower frequencies for the ADC noise, with $1/f$ excess noise below 1 Hz for the interferometer noise. The DAC noise has been measured by injecting a null (0 V) signal for a DAC port of the MicroLab Box into a 24-bit National Instruments recorder. It is estimated as $\Phi_{\text{DAC}} \approx 2 \cdot 10^{-7} \sqrt{(1 + 50f^{-1})}$ V/ $\sqrt{\text{Hz}}$, i.e. a white noise at $2 \cdot 10^{-7}$ V/ $\sqrt{\text{Hz}}$ with a $1/f^{0.5}$ contribution below 50 Hz. The coil driver noise has been measured as $\Phi_{\text{CD}} \approx 2 \cdot 10^{-8} \sqrt{(1 + 50f^{-1})}$ A/ $\sqrt{\text{Hz}}$ by feeding a null signal to the coil driver unit and recording the output current on the same 24-bit NI recorder. More details can be found in Appendix C. Thermal noise is modelled from the fluctuation-dissipation theorem [159] as detailed in section 2.1. Because the μ VINS operates at ambient pressure, the thermal noise is assumed to be Brownian-noise dominated, leading to $\Phi_{\text{th}} = \sqrt{4k_B T m \omega_0 / Q} = 2.34 \cdot 10^{-11}$ N/ $\sqrt{\text{Hz}}$.

Feeding the above measurements into Equation 3.30 leads to the noise figure shown in Figure 3.20. Superimposed in the figure is the acceleration spectrum measured by a reference instrument (Guralp 6T) placed next to the μ VINS. The figure shows that the acceleration spectrum of the μ VINS (solid

blue trace) is in really good agreement with the reference (solid red trace) in the 1 to 100 Hz bandwidth. The Guralp 6T self-noise is measured by coherent subtraction between two of those sensors placed side-by-side (dashed pink trace). It shows that the Guralp 6T is self-noise-limited below 0.1 Hz and above 100 Hz, explaining the coherence drop from previous measurements outside of this bandwidth. On the other hand, the global μ VINS noise $\Phi_{\widehat{s^2W}}$ is calculated as the quadrature sum of all contributions and is shown as the black trace. The spectrum of the individual noise sources (dashed traces) shows that the coil-driver electronic and the DAC electronic noises are far larger than other noise sources and dominate the entire bandwidth from DC to 100 Hz. In particular, it can be seen that they dominate the signal below 1 Hz, explaining the coherence drop observed in Figure 3.18 at the same frequency. Because these two noises arise from the control system, they are also often referred to as control noise. Only above 100 Hz does the interferometer noise dominate the noise budget.

In the end, the noise budgeting shows that the μ VINS in FBA mode has a resolution peaking at roughly $8 \cdot 10^{-8} \text{ (m/s}^2\text{)}/\sqrt{\text{Hz}}$ at 50 Hz, set by the control electronics. It shows a $f^{-0.5}$ increase below that frequency, dominated by control noise and increases as f^2 above that frequency, dominated by ADC and interferometer noise. Under normal seismic activities, the sensor is therefore noise-limited below 1 Hz. Because of the large noise level and limited bandwidth, the FBA operation will most probably be a limitation in the vibration compensation in the AQG.

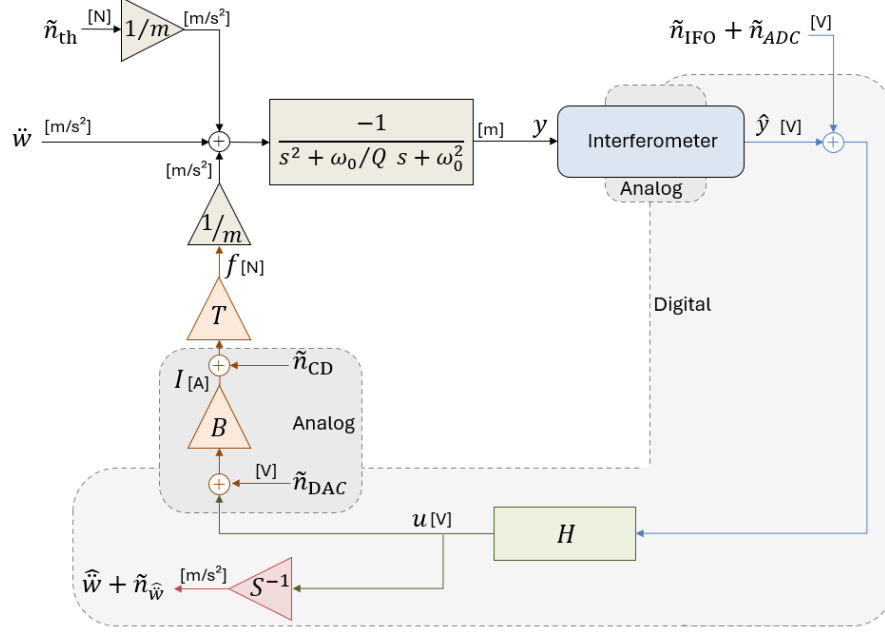


Figure 3.19: Noise diagram for the force-balance-accelerometer mode μ VINS. $\ddot{w}$ and $\hat{\ddot{w}}$ are respectively the ground acceleration and the sensor-measured ground acceleration. y and $\hat{y}$ are respectively the proof-mass relative motion and the readout measured motion. The brown blocks represent the sensor's mechanical response to inputs, the blue block the optical readout and includes the acquisition chain, the green block the control electronics and the orange block the actuation system. The $(S^{\text{FBA}})^{-1}$ red block represents the digital inversion of the sensor's sensitivity response. $\tilde{n}_{\text{th}}$ represents thermal noise(s), $\tilde{n}_{\text{IFO}}$ the interferometer noise, $\tilde{n}_{\text{ADC}}$ the MicroLab Box ADC noise, $\tilde{n}_{\text{DAC}}$ the MicroLab Box DAC noise, $\tilde{n}_{\text{CD}}$ the coil driver noise and $\tilde{n}_{\hat{w}}$ ($[\text{m/s}^2]$) the total inertial sensor noise.

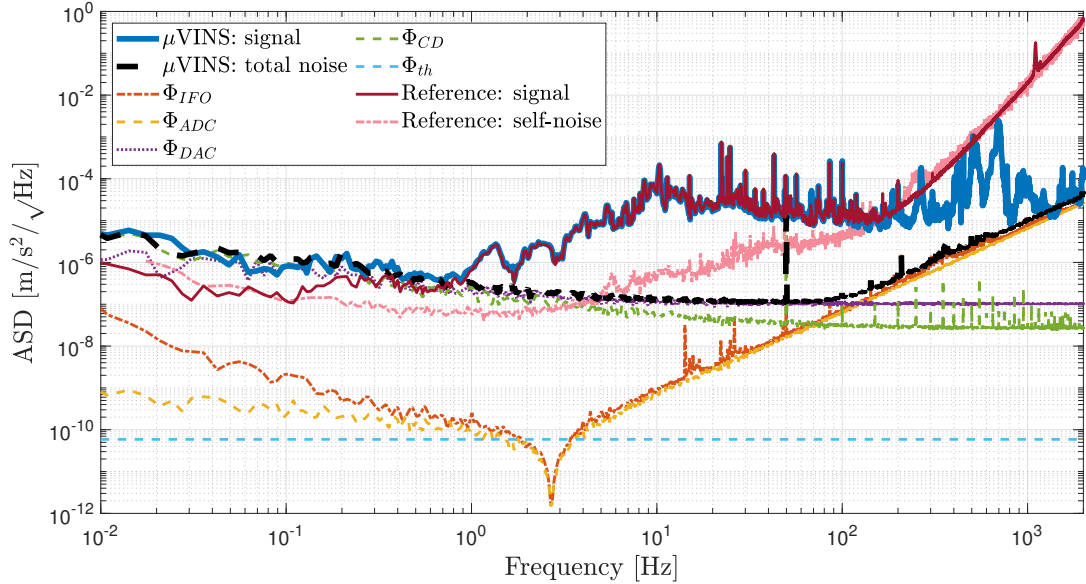


Figure 3.20: μ VINS noise budget in force-balance-accelerometer mode. (Solid blue) μ VINS Acceleration signal. (Dashed orange) Optical readout noise Φ_{IFO} . (Dashed yellow) MicroLab Box ADC noise Φ_{ADC} . (Dashed purple) MicroLab Box DAC noise Φ_{DAC} . (Dashed green) Coil driver noise Φ_{CD} . (Dashed light blue) Brownian thermal noise Φ_{th} . (Dashed black) Total noise (quadrature sum). (Solid red and dashed pink) Reference instrument (Guralp 6T) signal and self noise.

3.3.3 Frequency-compensation-accelerometer mode

The frequency-compensation-accelerometer (FCA) mode for the μ VINS suggests filtering the sensor output in a way to reshape its sensitivity response to the desired bandwidth. Such a technique has been widely used for decades in microelectronics [266, 267] for compensating the frequency response of op-amps and LDOs, and has recently been proposed to be used in inertial sensing [154, 268, 269]. While such filters were traditionally designed with analogue circuitry, the recent advances in microprocessor and programmable FPGA now allow "arbitrarily complex" digital filters to be designed and run in real-time processing units.

Filter design

The filter design fundamentally involves the inversion of the μ VINS mechanical response. Even for a good characterisation of the mechanical sensitivity, the inversion of a sharp, undamped pair of poles is often not robust and therefore not desired. For this reason, the μ VINS resonance is passively damped by shunting the quadrupole voice-coil actuator, as explained in section 2.1. In order to maximise shunt-damping $\frac{c + \frac{R^2}{m}}{R_{\text{coil}}}$, the coil wires are shorted so as to obtain the lowest shunt resistance $R_{\text{coil}} = 4.9 \, \Omega$ corresponding to the coil wire resistance. To further increase damping, a small soft rubber pad is placed close to the pivot of the pendulum-like proof mass, in between the base of the pendulum and the coil interface, as shown in Figure 3.21. Various locations for the damping pad have been considered, having the pad near the pivot allowed to bring sufficient additional damping with minimal additional rotational stiffness, thus a minimal increase of the resonance frequency.

The newly damped mechanical response of the μ VINS is characterised as previously, recording ground acceleration with a reference sensor (Guralp 6T) placed side-by-side. The damped mechanical response $S_{\mu\text{VINS}}^{\text{Mec.}'}$ is shown in Figure 3.22. A fit to the transfer function leads to a suspension mode of $f_0 = 2.85 \, \text{Hz}$, which is an increase of 5% compared to the suspension without damping of 2.7 Hz, and a Q -factor of 25. This is a significant reduction by almost a tenfold, making the filter design a lot simpler and more robust.

$$S_{\mu\text{VINS}}^{\text{Mec.}'} = g_{\text{IFO}} \cdot \frac{-1}{s^2 + 0.7163s + 320.7} \quad [\text{rad}/(\text{m}/\text{s}^2)] \quad (3.31)$$

A filter F is designed based on this new model. With a target sensitivity $S_{\text{mod.}}^{\text{FCA}}$ with a corner frequency at 500 Hz and critical damping, the filter F is

$$F = \frac{S_{\text{mod.}}^{\text{FCA}}}{S_{\mu\text{VINS}}^{\text{Mec.}'}} = 4.45 \cdot 10^{10} \frac{s(s^2 + 0.7162s + 320.7)}{(s + 0.06283)(s + 6283)(s^2 + 3927s + 9.87 \cdot 10^6)} \quad (3.32)$$

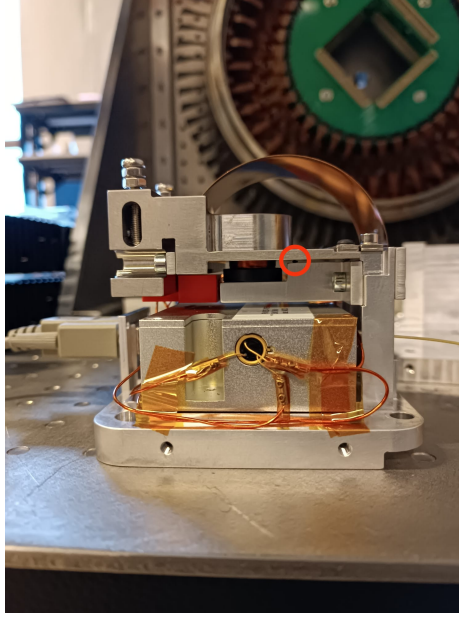


Figure 3.21: Dampening of the μ VINS mechanical resonance using of small pad of rubber (circled in red) underneath the proof-mass.

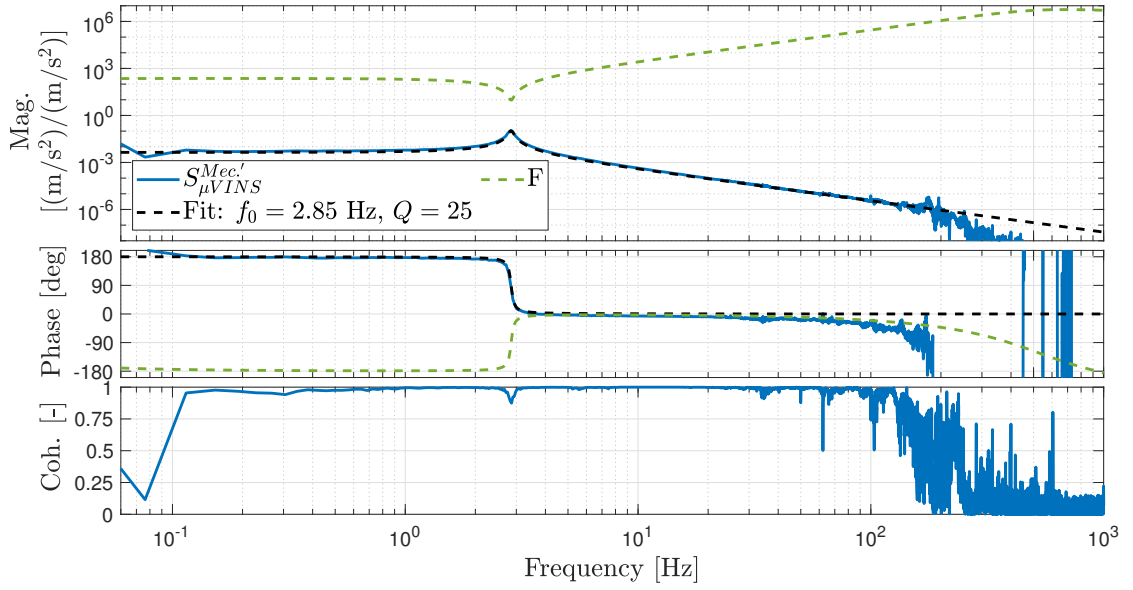


Figure 3.22: μ VINS damped mechanical response $S_{\mu VINS}^{Mec.}$ and frequency compensation filter F .

Frequency response

The above filter F is implemented in the MicroLab Box real-time target and applied to the μ VINS signal after demodulation of the interferometer reading. The sensitivity response function of the sensor in FCA mode $S_{\mu VINS}^{FCA}$ is measured using again a Guralp 6T as a reference instrument. Results are shown in Figure 3.23 (blue trace). The passive (damped) mechanical response $S_{\mu VINS}^{Mec.}$ is also shown as a comparison (orange trace). It can be seen that the magnitude and phase of the FCA μ VINS are rendered flat over a large bandwidth up to almost 100 Hz, with no degradation in the coherence (which is presumably limited by the reference instrument). The FCA sensitivity is also in very good agreement with the modelled sensitivity (dashed black trace). The matching however degrades at above 200 Hz due to the flexible mode of the suspension mechanism. If those modes were to be included in the filtering, the bandwidth could be extended further. This results in an excess of phase loss at 100 Hz: the filter was designed with a corner frequency at 500 Hz so that the phase lag at 100 Hz is less than 20° , set by the requirement of the vibration compensation in the AQG. Because of flexible modes, the actual phase lag at 100 Hz is approximately 50° .

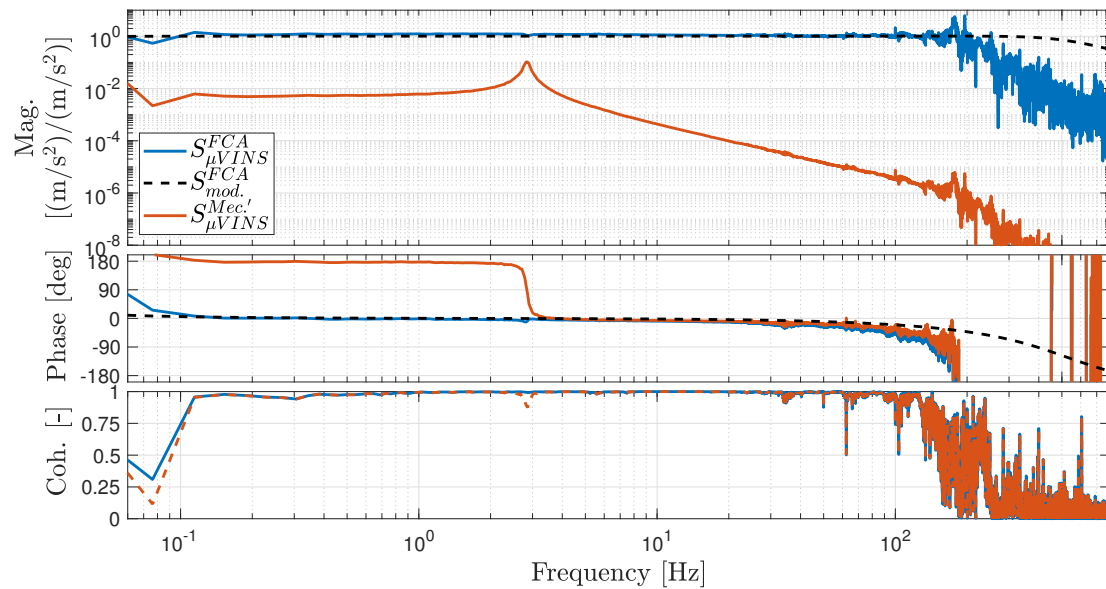


Figure 3.23: μ VINS sensitivity response function in frequency-compensation-accelerometer mode $S_{\mu VINS}^{FCA}$ (Blue). Passive damped mechanical response $S_{\mu VINS}^{Mec.}$ (orange). Modelled filtered sensitivity response $S_{mod.}^{FCA}$ (dashed black)

Noise figure

A block-diagram representation of the μ VINS in FCA mode is shown in Figure 3.24. The block diagram is a lot simpler than for the FBA mode, as there is no feedback involved, and therefore avoids control noise (DAC and coil driver). Ground acceleration $\ddot{w}$ transmits to the proof-mass, and

the relative motion y is read out by the quadrature Michelson interferometer. The MicroLab Box real-time target is used for gathering the photodiode signals and adds ADC noise $\tilde{n}_{\text{ADC}}$, and the signal is demodulated online. It also includes the readout noise $\tilde{n}_{\text{IFO}}$. The signal is then filtered by F , and because the frequency-compensation is implemented digitally in the MicroLab Box it can be assumed noiseless.

Proceeding as for the FBA mode, each noise contribution is individually measured and propagated through the noise diagram to draw the noise figure. The ground acceleration spectrum $\Phi_{\widehat{s^2 W}}$ ($[(\text{m/s}^2)/\sqrt{\text{Hz}}]$) measured by the μVINS in FCA mode writes

$$\begin{aligned}\Phi_{\widehat{s^2 W}} &= (S_{\mu\text{VINS}}^{\text{FCA}})^{-2} \cdot U \\ &= \Phi_{s^2 W} + \left| \frac{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}{g_{\text{IFO}}} \right|^2 (\Phi_{\text{ADC}} + \Phi_{\text{IFO}}) + \left| \frac{1}{m} \right|^2 \Phi_{\text{th}}\end{aligned}\quad (3.33)$$

with $\omega_0 = 2\pi \cdot 85 \text{ rad/s}$, $Q = 25$ and $m = 0.4 \text{ kg}$ are the mechanical properties of the damped suspension and $g_{\text{IFO}} \approx 8\pi/\lambda \text{ rad/m}$ the interferometer gain.

Because the μVINS in FCA mode utilises the same real-time target and optical readout as in the FBA mode, the readout Φ_{IFO} and ADC Φ_{ADC} noise spectrum are identical to the traces in Figure 3.20. The thermal noise is however increased by a factor of approximately 3 ($\sqrt{10}$) due to the tenfold decrease in the Q -factor from the padding and shunting. Using the fluctuation-dissipation theorem [159] and assuming Brownian damping, it writes $\Phi_{\text{th}} = \sqrt{4k_B T m \omega_0 / Q} = 6.85 \cdot 10^{-11} \text{ N}/\sqrt{\text{Hz}}$.

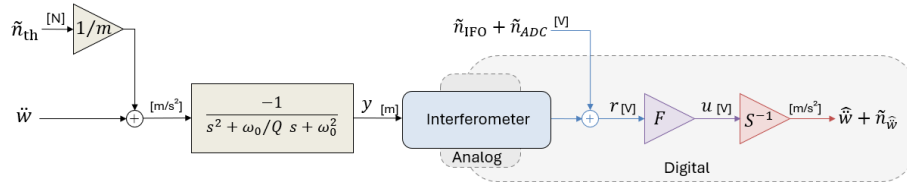


Figure 3.24: Noise diagram for the frequency-compensation-accelerometer mode μVINS . $\ddot{w}$ and $\hat{w}$ are respectively the ground acceleration and the sensor-measured ground acceleration. y and $\hat{y}$ are respectively the proof-mass relative motion and the readout measured motion. The brown blocks represent the sensor's mechanical response to inputs, the blue block the optical readout and includes the acquisition chain and the purple block represent the frequency compensation filter. The $(S^{\text{FCA}})^{-1}$ red block represents the digital inversion of the sensor's sensitivity response. $\tilde{n}_{\text{th}}$ represents thermal noise(s), $\tilde{n}_{\text{IFO}}$ the interferometer noise, $\tilde{n}_{\text{ADC}}$ the MicroLab Box ADC noise and $\tilde{n}_{\hat{w}}$ ($[\text{m/s}^2]$) the total inertial sensor noise.

Plugging these measurements in Equation 3.33 leads to the noise budget shown in Figure 3.25. Superimposed in the figure is the acceleration spectrum measured by a reference instrument (Guralp 6T) placed next to the μVINS . The acceleration signal spectrum measured by the FCA μVINS is in very good agreement with the reference in the entire range from 0.1 Hz to 100 Hz. Below 0.1 Hz

and above 100 Hz, the reference Guralp 6T is self-noise limited, as shown by the dashed pink trace measured by coherent subtraction of the signal of two Guralp 6T. The FCA μ VINS resolution is shown as the dashed black trace with a peak sensitivity of $2 \cdot 10^{-10} \text{ (m/s}^2\text{)}/\sqrt{\text{Hz}}$ at 2.85 Hz, and is fully dominated by the ADC noise above 3 Hz, and by the excess readout noise below 1 Hz.

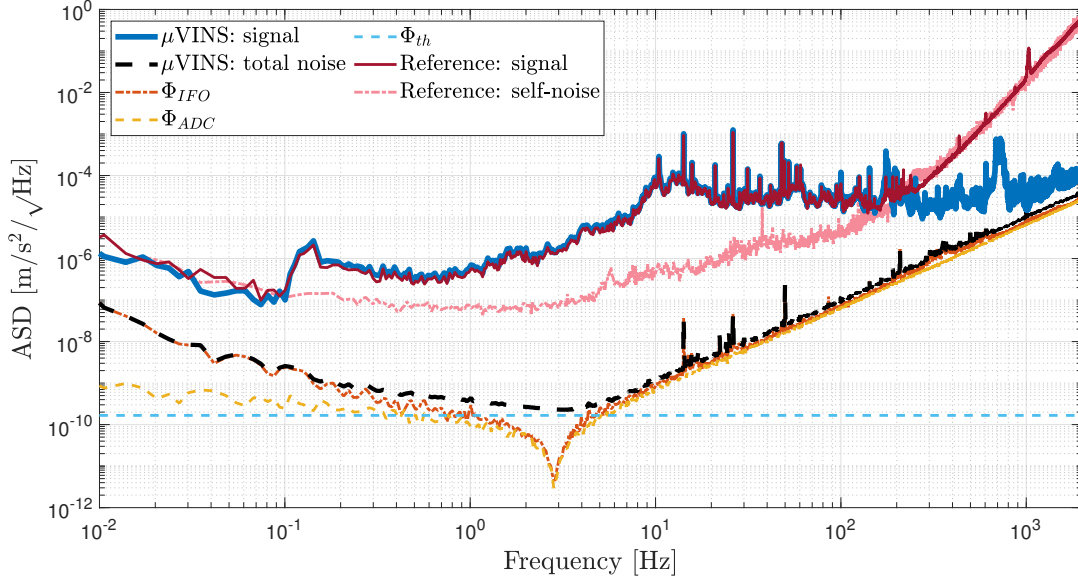


Figure 3.25: μ VINS noise budget in frequency-compensation-accelerometer mode. (Solid blue) μ VINS Acceleration signal. (Dashed orange) Optical readout noise Φ_{IFO} . (Dashed yellow) MicroLab Box ADC noise Φ_{ADC} . (Dashed light blue) Brownian thermal noise Φ_{th} . (Dashed black) Total noise (quadrature sum). (Solid red and dashed pink) Reference instrument (Guralp 6T) signal and self noise.

3.3.4 Comparison

A comparison of the performance of the μ VINS in both operation modes is shown in Figure 3.26. For each mode, the resolution (solid traces), calculated from the noise budget, and an estimation of the dynamic range (dashed traces) are plotted. The area bounded by the resolution, i.e., the smallest measurable motion, and the dynamic range, i.e., the largest measurable motion, represent the operating range of the sensor. The various limitations to the resolution and dynamic range are indicated in the plot.

From the above noise budgeting, it has been measured that the FBA μ VINS resolution is limited by control noise, i.e. ADC and coil driver electronics, propagating in the feedback loop. This is a severe limitation, dominating the resolution at approximately $80 \text{ (nm/s}^2\text{)}/\sqrt{\text{Hz}}$ at 50 Hz. The ADC noise dominates the higher range of the spectrum with a f^2 contribution and control noise dominates the lower frequency bandwidth with a $f^{-0.5}$ contribution. Because the proof mass is locked in null relative motion by the force-feedback loop, the dynamic range is set by the saturation

of the control electronics. Knowing the saturation limits of the MicroLab Box DAC (10 V), of the coil driver electronics (2 A), and the feedback system control effort (i.e., transfer function from input acceleration to output control signal), the dynamic range of the FBA μ VINS can be calculated at roughly 1 mm/s^2 . Because of the suspension's flexible modes, the control bandwidth was limited to a cross-over frequency at 60 Hz, so that the sensitivity response has a flat magnitude and phase up to 50 Hz (with approx. 30° phase lag). The phase lag at 100 Hz far exceeds the requirement of the vibration noise compensation in the AQG ($> 180^\circ$).

On the other hand, the FCA μ VINS can fully benefit from the high resolution provided by the quadrature interferometer since there is no control involved. The resolution has been estimated in the noise budget to reach a peak value of $0.2 \text{ (nm/s}^2\text{)}/\sqrt{\text{Hz}}$ at 2.85 Hz, limited by thermal noise. At higher frequencies, the resolution is dominated by the ADC noise with a f^2 contribution. At lower frequencies, it is being dominated by the interferometer noise with a f^{-1} contribution. Because the proof mass is free to move, the dynamic range is limited by the range of the quadrature Michelson interferometer. Even though the optical readout is a fringe-counting type of interferometer capable of demodulating multiple wavelengths ($\lambda = 1550 \text{ nm}$), nonlinearities can be assumed to limit the range to typically a dozen of wavelengths. The dynamic range can be estimated to $100 \text{ }\mu\text{m/s}^2$. The frequency compensation filter has been shown to be successful in extending the sensor bandwidth up to 100 Hz. Due to spurious mechanical resonances not accounted for in the filter design, a slight excess of phase lag (50°) remains at 100 Hz.

Comparing both operating modes, the resolution of the FCA μ VINS is two to three orders of magnitude better than in the FBA mode in almost the entire bandwidth from DC to 100 Hz. As a trade-off, the linearisation of the sensors in FBA mode provides a dynamic range a factor of 100 larger. Regarding sensitivity response bandwidth, only the FCA mode is capable of extending the bandwidth up to 100 Hz, with minimal phase lag of 50° , which is slightly above the AQG requirements.

For these reasons, the FCA mode is assumed better suited for the application in vibration compensation noise and will be used for the integration in the AQG, although a better designed control electronics for the FBA mode can certainly compete with the FCA mode.

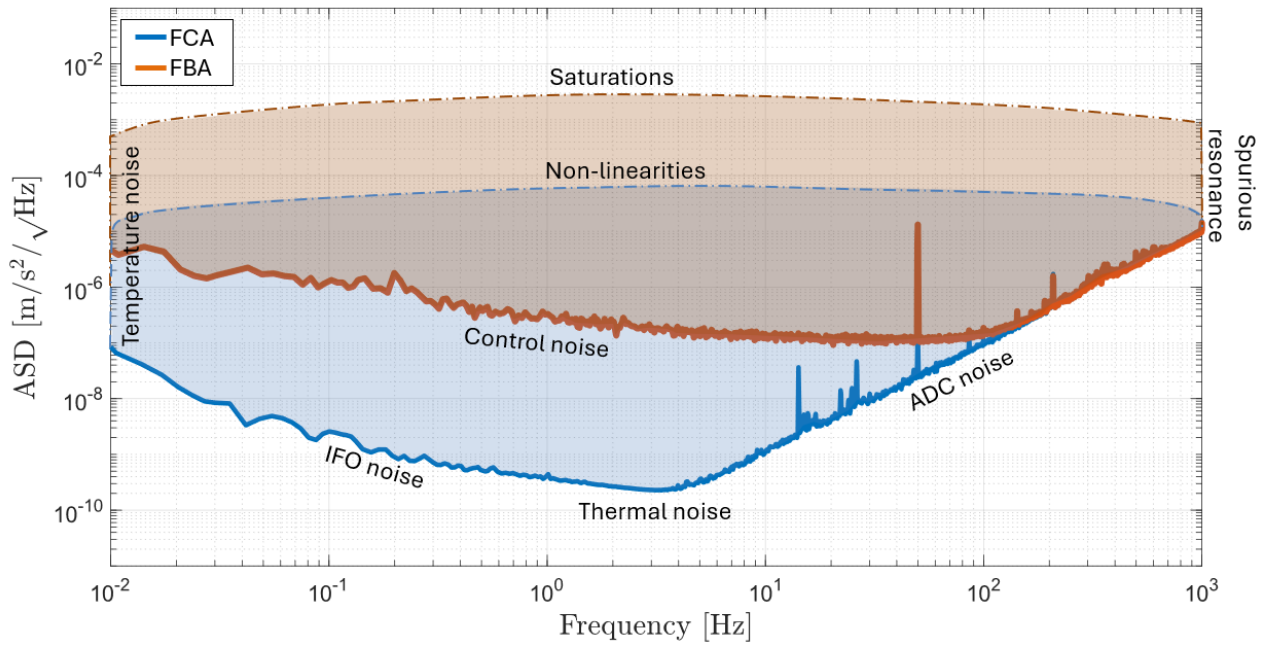


Figure 3.26: Qualitative comparison of the sensitivity and dynamic range of the μVINS working in frequency-compensation-accelerometer mode (blue) and force balanced-accelerometer (orange).

3.4 Integration in the quantum gravimeter

The ultimate goal of the μ VINS sensor is to enhance the performance of a quantum gravimeter, so as to meet the requirements of the gravitational wave observation, by improving its seismic noise compensation system. As briefly discussed in section 3.1, the vibration compensation performance is expected to improve with the resolution of the accelerometer and its bandwidth. The μ VINS has therefore been specifically tailored to an Absolute Quantum Gravimeter (AQG) from the EXail company with the objectives of a better resolution and bandwidth than the commercial accelerometer currently used in the AQG, a Titan accelerometer from the Nanometrics company.

This section first validate the frequency-compensation filter strategy, used for broadening the optical accelerometer bandwidth, by applying a similar filter on the native accelerometer used in the AQG. Then, the sensitivities of the Titan and μ VINS frequency-compensated accelerometers (FCA) in terms of resolution and bandwidth. Finally, the merging process of the μ VINS optical accelerometer with an AQG from the EXail company is presented. Preliminary results of the merging show a successful integration of the accelerometer in the AQG and promising performance improvements. The author acknowledges fellow PhD researchers from the PML group for their contribution to the following work - mainly Mayana Teloi and Briex Thibaut - as well as the gravimeter engineering department from the EXail company.

3.4.1 Validation of the frequency compensation filter approach

From discussion with the EXail gravimetry team, improvement in the ground vibration compensation using an eternal accelerometer can be made using a broadband sensor displaying flat sensitivity, and mostly a very limited phase lag, in a large bandwidth spanning several dozens of second up to hundred of Hz. Whereas this is commonly achieved in a force-balanced accelerometer using a servo-control system, the μ VINS has been designed with a frequency-compensation filter. As discussed thoroughly in section 3.3, the frequency-compensation filter allows for broadening the bandwidth of the sensor, without adding control noise, inherent to the feedback loop.

In order to validate this approach, and that a larger bandwidth is indeed key for better vibration compensation and thus better performance of an AQG, a frequency-compensation filter is designed for the accelerometer currently used in the AQG. The design process is very similar to that explained in a previous chapter: The filter first consists of a plant inversion that corrects for the magnitude and phase drop of the Titan accelerometer, and add new cut-off frequencies at arbitrarily lower/higher frequencies. The filter has been designed in MATLAB, and converted to C++ language so that it could be implemented in the digital micro-processor embedded in the AQG.

Various designs have been produced. The retained one, yielding the best performances, consists of a filter that extends the bandwidth up to 500 Hz, with a high-pass filter at 10 mHz added for stability and noise reduction. Figure 3.27 shows the frequency bandwidth and phase lag of the Titan Nanometrics. The bandwidth also includes magnitude and phase drops from the analogue

acquisition of the AQG processing unit, as communicated by the EXail gravimetry team. From this response, a filter has been designed (dashed yellow) in order to flatten the phase drop at higher and lower frequencies. The filtered response effectively shows a phase lag that has lowered from -43° to -26° at 100 Hz, and with a significantly flatter magnitude and phase response at lower frequencies down to 100 mHz.

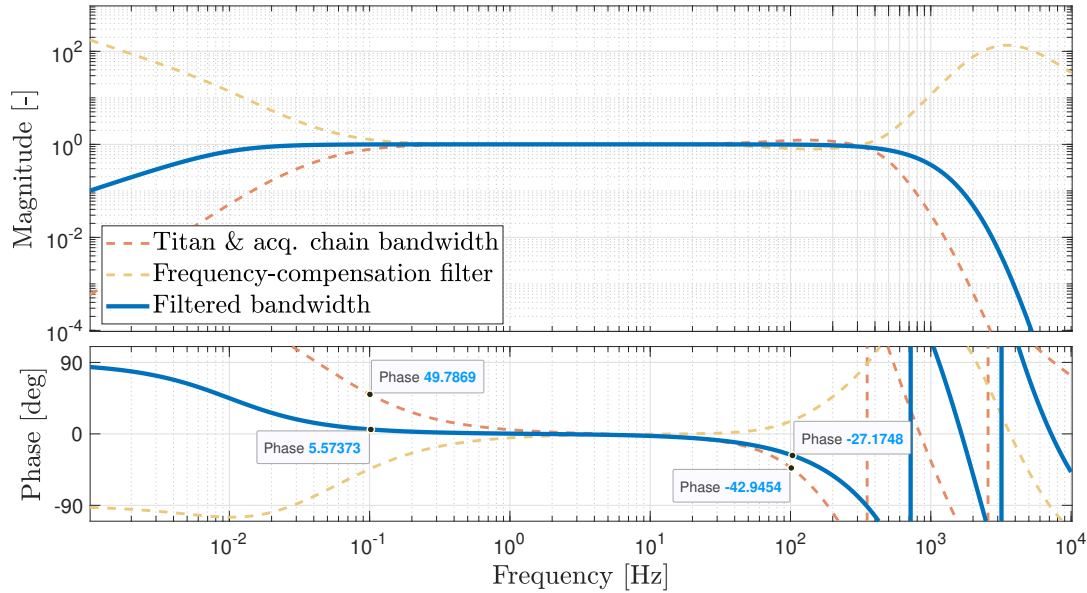


Figure 3.27: Frequency compensation filter applied on the Nanometrics Titan accelerometer. Initial sensitivity and phase drop (dashed red), frequency-compensation filter (dashed yellow), and filtered response (solid blue).

The filter has been implemented in the digital controller of the AQG by the EXail gravimeter engineer. For estimating the impact of the filter on the AQG performance, the EXail engineer calculated the Allan deviation of the measured gravity signal of the AQG with the frequency compensation filter enabled (Stretcher ON) and disabled (Stretcher OFF). The Allan deviation is a time-domain tool that measures the stability of a signal. It indicates how stable a device's signal is over different averaging times (τ), by analysing how noise averages out. It is commonly used in oscillators, clocks, and other DC sensors where long-term stability is desired (such as gyroscopes, gravimeters, MEMS accelerometers). Lower Allan deviation indicates better stability and lower noise overall. The Allan deviation will naturally be decreasing with the averaging time, as noises average themselves out, but the averaging time at which the curve eventually start increasing again shows the limit of the long-term stability of the instrument. The slope of the Allan deviation is related to the broadband white noise of the instrument.

Figure 3.28 shows the Allan deviation of the same AQG with and without the frequency compensation filter enabled. The Allan deviation with the filter engaged (Stretcher ON) shows an obvious

improvement. This is a solid indication that broadening the sensitivity response of the accelerometer in magnitude and phase is improving the performance of the AQG, as the same sensor, with the same noise characteristics, yields better vibration noise rejection when extended over a broader bandwidth.

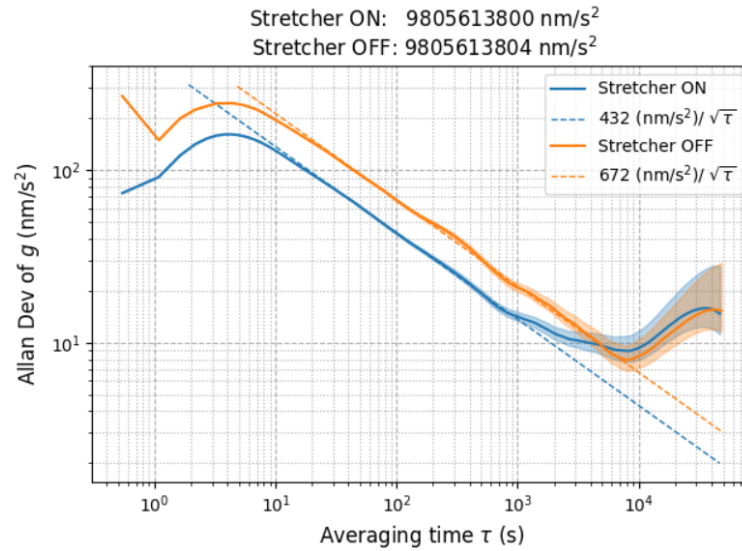


Figure 3.28: Comparison of an AQG resolution equipped with a Titan accelerometer before applying a frequency compensation filter (orange) and after filtering (blue). **Figure obtained with the courtesy of the EXail gravimetry staff.**

Obviously, and as discussed above, the filter applies on both the signal and the noise, and therefore has no impact on the resolution nor on the temperature sensitivity. The latter being particularly related to the phase drop at low frequency, as the AQG processing unit includes a DC-block filter to avoid temperature drift being injecting to the control system.

3.4.2 Sensitivity comparison between μ VINS and Titan accelerometers

As per its lower mechanical resonance frequency and high-resolution optical readout, the resolution of the μ VINS accelerometer is expected to be significantly lower than that of the Titan.

The Titan accelerometer indeed has a natural frequency of around 10 Hz and is equipped with a capacitive readout system, typically reaching resolution in the 0.1 nm-range. On the other hand, the optical readout used in the μ VINS has been shown in section 1.3 to reach sub-pm resolution at 1 Hz, and the sensor has a mechanical resonance of 2.8 Hz, a factor of 3 lower than the Titan. All in all, this yields a resolution that is expected to be a factor 300 better than the Titan accelerometer.

The above rough estimation is further supported by a measurement of the signal of both ac-

celerometers. The two accelerometers are placed side-by-side on solid ground, and their signal is recorded with the Nanometrics Centaur 24-bit recorder set to the 20 V_{pp} range. Measurement is taken at night for lower background noise. Figure 3.29 shows acceleration spectra of both sensors in the 10 mHz to 100 Hz range. It can be seen that both sensors are in excellent agreement in the 100 mHz to 100 Hz. In the figure are superimposed resolutions (self-noise) of both sensors. The self-noise of the Titan accelerometer can be found from its datasheet and is adapted to Titan clipping and Centaur gain settings. The resolution of the μ VINS is calculated from the resolution of the optical sensor when recording with the Centaur 24-bit acquisition system (see subsection 1.3.3) and is processed as discussed in section 3.3. The self-noise curves show that the resolution of the μ VINS is indeed a factor of at least 100 to 300 better than that of the Titan, as a function of frequency, in the entire target bandwidth. This allows the μ VINS sensor to have an accurate measurement of ground vibrations in the 10 mHz to above 100 Hz bandwidth, whereas the Titan sensor is clearly noise-limited below 100 mHz and above 100 Hz, and its noise figure is only slightly below background vibration noise in the 1-2 Hz range.

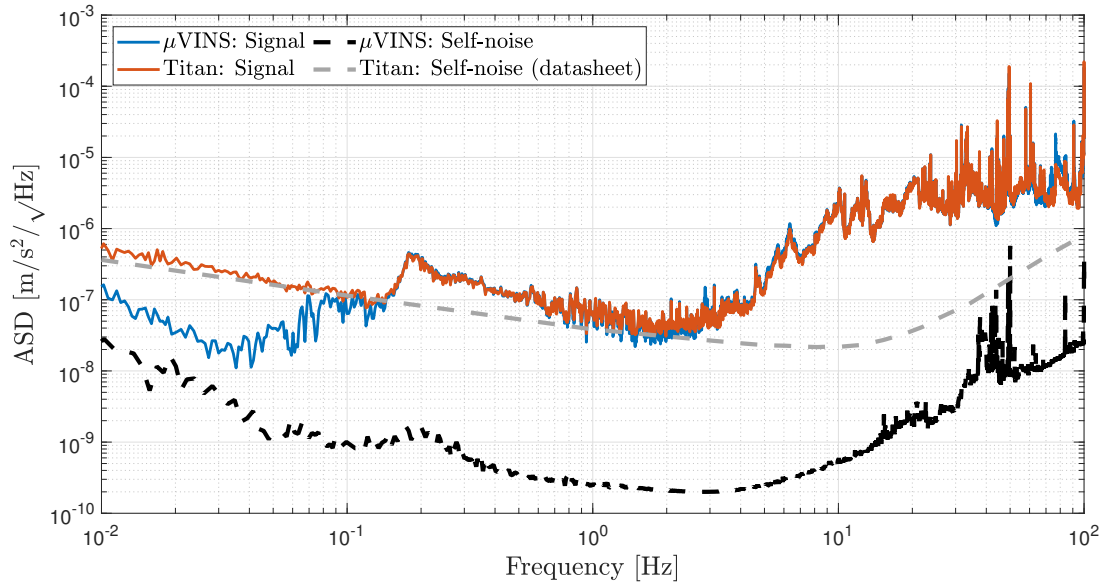


Figure 3.29: Comparison of the Nanometrics Titan accelerometer and of the μ VINS optical accelerometer resolutions, upon recording on a 24-bit acquisition system (Nanometrics Centaur). The μ VINS sensor shows a resolution a factor of at least 100 (40 dB) better than the Titan in a wide frequency range.

Figure 3.30 compares both accelerometers in terms of gain and phase bandwidth. Note that phase lag is especially relevant to the AQG application and should be kept to minimal values in the entire target bandwidth. As per its datasheet, the Titan is a force-balance accelerometer (FBA) which uses a force-feedback loop to extend its bandwidth from its mechanical resonance (≈ 10 Hz) to "DC"-430 Hz. However, the relatively "high" noise figure at low-frequency, coupled with

temperature sensitivity, typically prevent DC measurement and so the AQG uses a high-pass filter to cut drift, at the cost of decreased bandwidth for vibration noise compensation. The μ VINS sensor uses a frequency-compensation filter to extend its bandwidth from mechanical resonance (2.8 Hz) to above 100 Hz. Its sensitivity bandwidth has been measured using a Guralp seismometer as a reference sensor, as discussed in section 3.3. Figure 3.30 shows that the μ VINS sensor reaches a sensitivity very similar to that of the Titan sensor, despite the lower mechanical resonance, and without the need of a high-gain feedback loop, which would add control noise to the system. A key value to note is the phase lag at 100 Hz, being approximately 30 deg for both sensors.

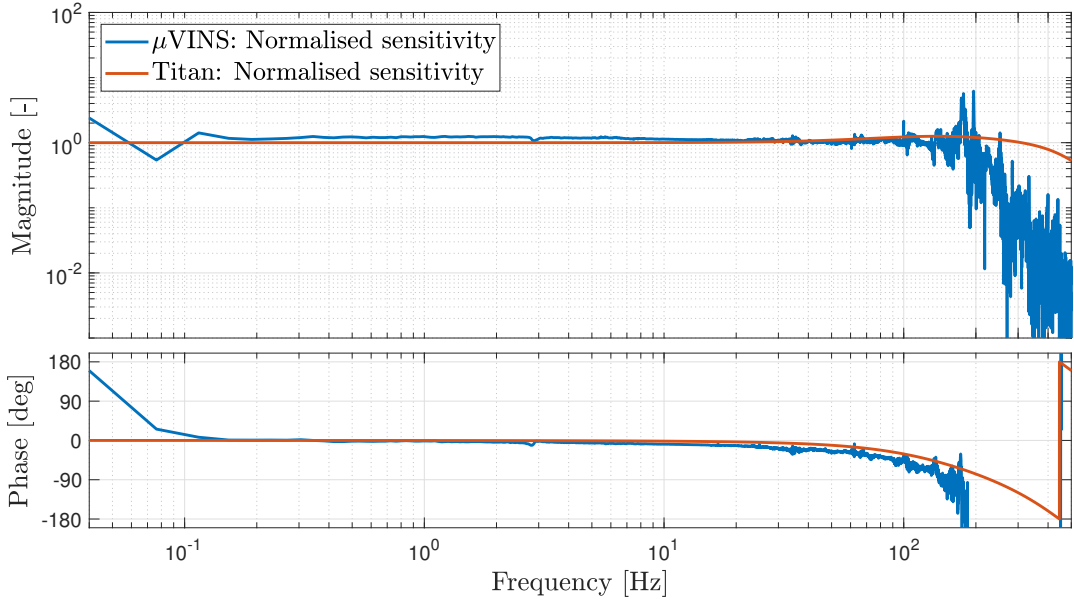


Figure 3.30: Comparison of the Nanometrics Titan accelerometer and of the μ VINS optical accelerometer sensitivities. The μ VINS sensor shows a sensitivity similar to the Titan despite the significantly lower mechanical resonance.

3.4.3 μ VINS integration and enhanced AQG performance

A first integration of the μ VINS in an AQG as a substitute for the Titan accelerometer has been performed at EXail premises in Gradignan, France, with the assistance of the gravimeter engineering team. The integration process is first briefly described. Then, preliminary results of the performance of the AQG are presented. These results are issued from a test campaign performed at EXail facilities in Gradignan, carried out by Mayana Teloi and Briex Thibaut from the PML group. The author acknowledges his fellow PhD colleagues as well as the EXail staff for their assistance and contribution. This section only contains preliminary results from this test campaign and only aims in showcasing the correct operation of the AQG merged with the μ VINS and the potential improvement of the vibration noise compensation technique. A more detailed discussion will be presented in another

publication by Mayana Teloi, currently under preparation at the time of this writing [35,36].

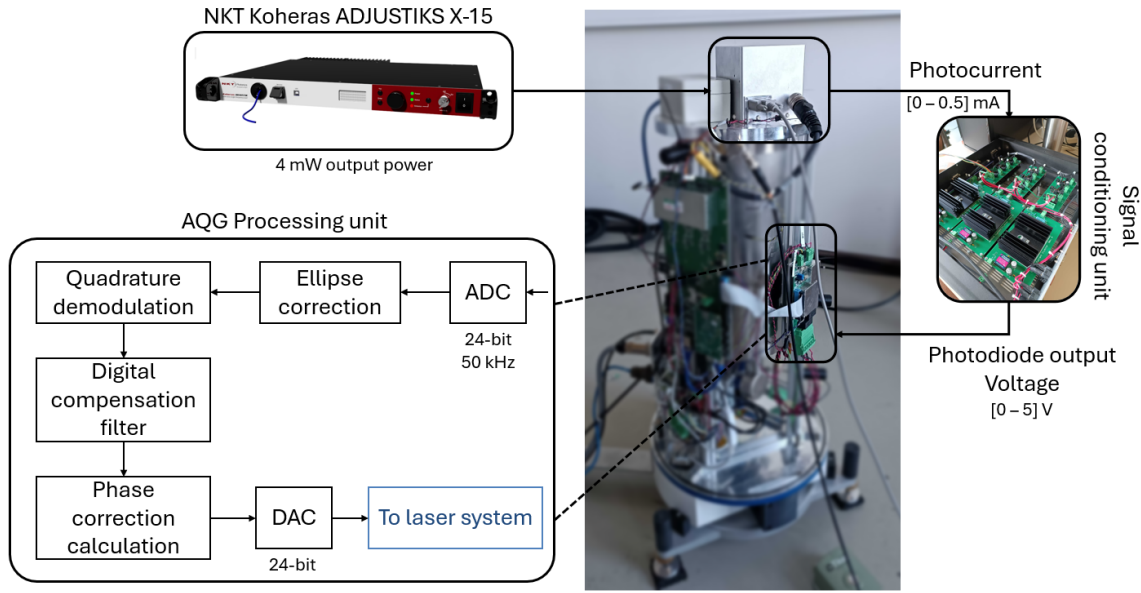


Figure 3.31: Integration of the μ VINS accelerometer in the AQG and schematic working principle. Picture blurred intentionally.

A schematic of the integration is shown in Figure 3.31. The μ VINS sensor is placed inside the AQG casing, on top of the vacuum vessel containing the dropping atom and interferometry system. The sensor is centred as best as possible with the axis of the retroreflecting mirror, as this is the element that transmits vibration noise as phase noise in the interferometer, and that needs to be cancelled. Using the same hardware as exposed previously, the μ VINS is powered by its own laser system using a Koheras ADJUSTIKS X15 laser with 4 mW input power, and the optical readout output is processed into voltage readings with a 5 V range by the custom signal conditioner unit (Appendix C). The output voltage is then demodulated and processed into an acceleration signal in the AQG microprocessor unit. The demodulation algorithm, including conditioning of the interferometer signals, ellipse correction and frequency-compensation filter, has been transposed in C++ and communicated to EXail software engineers for implementation in the AQG digital processing unit. The acceleration signal is eventually fed to the vibration noise compensation system, which calculates the appropriate phase correction and drives the laser system.

The performance of the AQG merged with the μ VINS sensor is assessed by comparison with another AQG using a traditional Titan accelerometer. In the figures below, the A04 AQG refers to the AQG equipped with the μ VINS optical accelerometer, whereas the A09 AQG refers to another AQG equipped with a Titan accelerometer.

Figure 3.32 shows the *residual* gravity fluctuation measured by the A04 and A09 AQGs over the course of a 12 hours. The residual gravity is calculated for the raw gravity measurement, and is obtained by subtracting the average "DC" gravity constant, mentioned in the figure, and

other systematic or periodic effects such as the moon or tide-induced gravity fluctuations. During nighttime (highlighted in yellow/orange in the graphs), when vibration noise is inherently low, it can be seen that both AQGs measurements are in good agreement. Using the measurement of A09 as a reference value, the A04 AQG measures g_0 with an accuracy of 2100 nm/s^2 . The 3rd plot, showing standard variation statistical value calculated over a timelaps of 10 min, confirms that both instruments are equally stable during this time window. However, the A04 AQG still shows larger fluctuation during later afternoon and early-morning times, presumably caused by excess vibration noise in the residual gravity signal. This implies that the merging process can still be optimised.

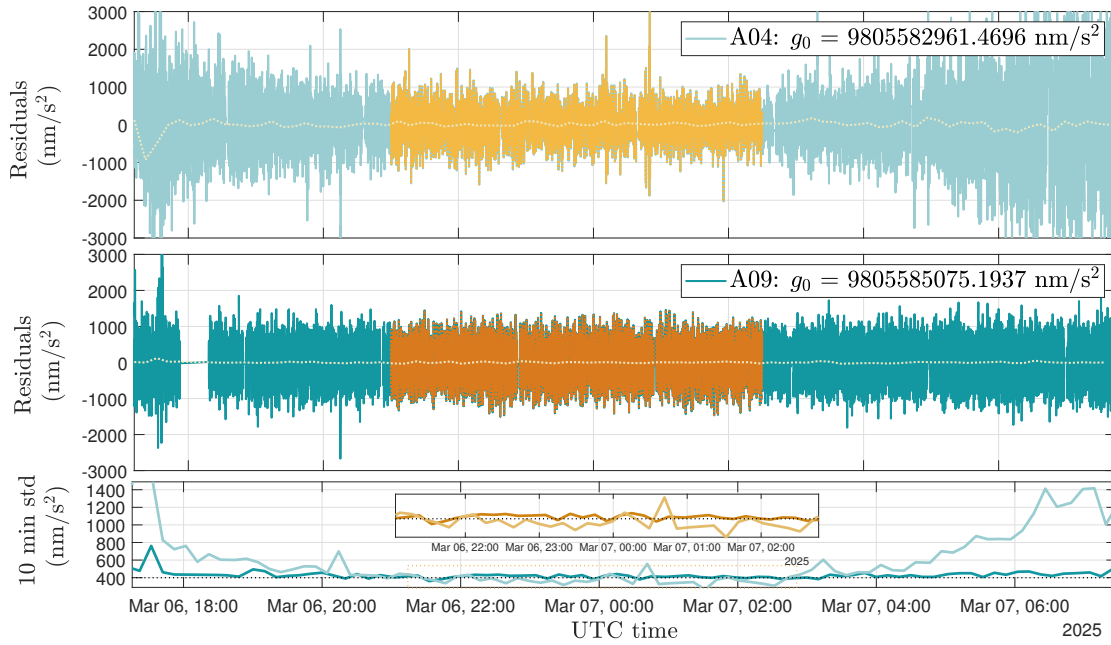


Figure 3.32: Comparison of the A04 and A09 AQGs performances. Top two graphs: residual gravity variations. Bottom graph: 10-minutes standard deviation value.

The above data can be processed with Allan deviation to study the long-term stability of the AQGs. This is shown in Figure 3.33. Superimposed to the figure is the ultimate, theoretical, AQG noise floor, which primarily consists of the quantum projection and laser phase noise. This plot confirms the above discussion: the A04 AQGs provides a stable signal with no long-term drift, which is a sign of a successful integration of the optical accelerometer. The Allan deviation, however, still sits above the A09 AQG, meaning that vibration noise compensation is currently suboptimal.

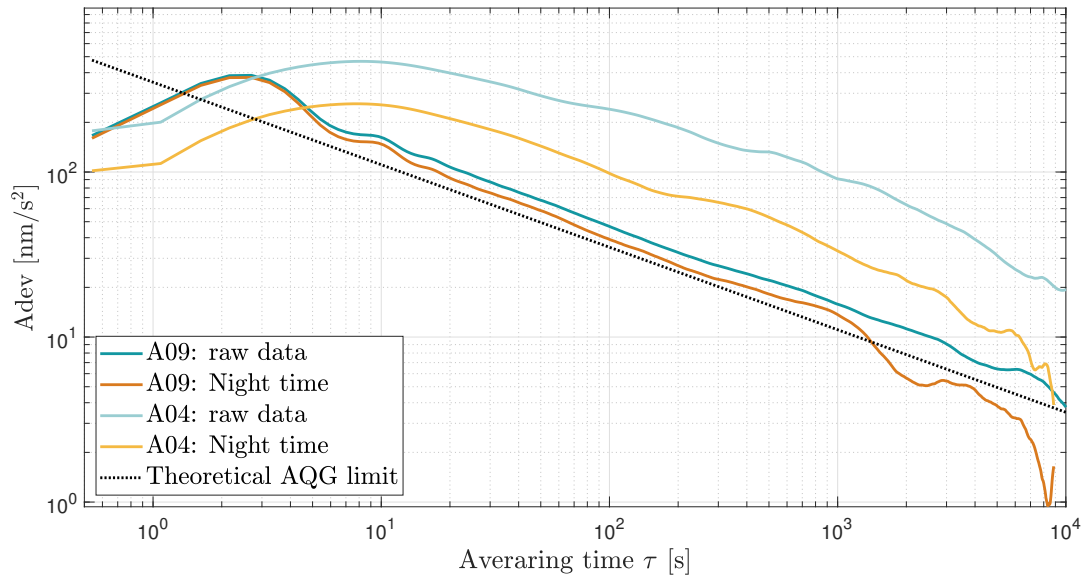


Figure 3.33: Comparison of the A04 AQG and A09 AQG resolutions.

Since it has been shown in Figure 3.29 and Figure 3.30 that the μ VINS optical accelerometer has been measured to have a significantly better noise figure, and a very similar frequency response function, it is believed that the current performances are limited by the merging process rather than the accelerometer itself. First, the alignment of the μ VINS with the AQG retroreflecting mirror is currently approximate and could be improved. Second, the vibration noise compensation algorithm of the AQG currently re-uses the same filtering process as with the Titan accelerometer previously used. According to the expertise of the EXail engineers, these filters should be optimised for the current accelerometer, and this could have a significant impact on the results. This optimisation process fundamentally uses a trial-and-error approach and is fairly time consuming, and has not been performed for this first integration run.

3.5 Conclusion and future work

This chapter presented the design and performance characterisation of the μ VINS optical accelerometer designed for active vibration noise compensation in an Absolute Quantum Gravimeter from the EXail company [29, 253]. For better vibration compensation performance, a technique is proposed for broadening the bandwidth of the accelerometer beyond its mechanical mode, with an emphasis on a flat phase response up to several Hz [41, 43].

First, section 3.2 presented the mechanical design of the μ VINS accelerometer. The proof-mass is suspended with a CuBe_2 leafspring [56] and uses a clamped blade pivot joint to form a 1 dof mechanism. The mechanical design achieve a 2.8 Hz resonance frequency in a fairly compact design fitting a 10 x 10 x 10 cm box. The reading of the proof-mass is achieved using the quadrature Michelson interferometer presented in chapter 1, targeting a TIR retroreflector mounted on the proof-mass.

Then, the mechanical sensitivity and noise performance are measured using an external reference seismometer in section 3.3. Two methods are presented and compared for extending the sensor bandwidth. One method is the industry-standard force-balanced method [55, 57, 154, 155, 157] that uses a custom self-shielded quadrupole voice-coil actuator integrated in the design to close a feedback control loop. The other method uses a real-time frequency-compensation filter [122, 266–269] to digitally compensate for the magnitude drop and phase lag of the accelerometer response. After implementation of both methods in the μ VINS and comparison of the performances, mostly in terms of resolution, it was found that the frequency compensation technique allows a lower noise floor. This is because it does not use a feedback loop, which causes control noise propagation in the case of the force-feedback accelerometer. However, it is worth noting that the frequency compensation filters do not affect the dynamics of the proof-mass, unlike the other method, and therefore does not help in linearising the sensor reading or dynamic range, which might be valuable in other applications.

Eventually, section 3.4 presents preliminary results of the integration of the optical accelerometer in an AQG for vibration noise compensation. A comparison of the reading of the AQG merged with the μ VINS with another standard AQG shows a successful integration of the optical accelerometer.

Despite the promising results, the comparison campaign between the AQG enhanced with μ VINS and the reference AQG showed that there is still margins for improvement. The merging currently re-uses the same filtering process as with the Titan accelerometer, traditionally used in AQGs. According to the expertise of the EXail engineers, these filters should be optimised for the current accelerometer, and this could have a significant impact on the results. As for the optical accelerometer itself, there are concerns about the sensitivity of the sensor to environmental effects. First, the sensitivity of the accelerometer to magnetic fields has not been properly characterised. This might be relevant since the sensor operates close to the MOT (Magneto-Optical Trap) of the AQG, which might leak a significant amount of stray field. Also, one limitation of the performance of the vibration compensation at low frequencies is the accelerometer's temperature sensitivity. Preliminary measurements indicated that the sensitivity of the μ VINS to temperature, driven by the temperature

sensitivity of the suspension leafspring, is similar to that of the Titan accelerometer. Improvements for lowering the sensitivity are under investigation: using an alternative material for the suspension with lower temperature sensitivity, or coupling different blades with different thermal behaviour, forming a "self-compensated" pair, or using a servo-control alignment of the proof-mass for compensating thermal drifts. The active alignment mechanism could act directly on the proof-mass itself, for instance using a voice coil-like actuator, or as a motorised levelling foot that control the inclination of the whole device. Related to thermal drift is sensitivity to cross directions. The latter technique, based on servo-controlled alignment, might have the additional benefit of re-aligning the proof-mass with the pivot joint, thereby minimising sensitivity to cross-directions. Eventually, since the frequency-compensation filter only digitally increases the bandwidth of the sensor, it allows the proof-mass to move freely and therefore leaves the sensor sensitive to the nonlinear response of the interferometer. A dampening system of the proof-mass could help in improving the overall sensor nonlinearity level.

Chapter 4

Conclusion

This thesis has focused on the development of multiple novel high-resolution sensing devices designed for active isolation in gravitational wave detectors. This thesis falls in the framework of the E-TEST project [21, 22]. One of its major objective is to develop a novel active seismic isolation system for the future ET gravitational wave detector that consists in an inverted pendulum, from which hangs the penultimate mirror, sitting on a 6D actively isolated platform.

A core technology that has been developed is a high-resolution optical readout based on homodyne quadrature Michelson interferometry [46, 72, 74–76, 83, 108]. Multiple devices have been produced and are used in the E-TEST prototype for alignment and active damping of the inverted pendulum. Their optical layout, demodulation process and characterisation have been presented in chapter 1. The device’s optical layout is also uses a double-pass reflection in the measuring arm to accommodate rotational motion of the target with minimal fringe visibility loss. A pre-processing and ellipse correction algorithm [46, 76, 113, 114] are used for mitigating most of the nonlinearities issued from the optical systems, such as misalignment, components imperfections or electronics imbalances have been presented to minimise nonlinear effects and compensate for sub-optimal alignment of the interferometer. The entire ellipse correction and demodulation have been implemented in MATLAB SIMULINK environment and are run with DSpace MicroLab Box real-time targets for integration of the sensors in real-time feedback control loops. A thorough characterisation of the optical sensor, mostly in terms of noise performance, has been performed. A measurement of the resolution by mechanically blocking the measuring arm of the device showed a resolution of $2 \cdot 10^{-13}$ m/ $\sqrt{\text{Hz}}$ at frequencies above 10 Hz when connected to a real-time target, or $8 \cdot 10^{-14}$ m/ $\sqrt{\text{Hz}}$ using a high-resolution (24-bit) Centaur recorder. A noise budget of the device points out the ADC noise as the major contribution at higher frequencies (> 10 Hz). The lower frequency range (sub 1 Hz), are suspected to be limited by the laser frequency noise. A deeper, in-depth, noise budget has been laid out in subsection 1.3.3.

Building up upon the interferometer technology, two vacuum-compatible optical inertial sensor designs have been produced and presented in section 2.4. One design is an optical inertial designed

for measuring vertical inertial motion (E-VINS), and the other sensor design measures the horizontal motion (E-HINS). 3 of each units have been produced and are used as high-resolution seismometers for inertial feedback control of the E-TEST active platform. The E-VINS vertical inertial sensor uses a leafspring-suspended pendulum [56] and a clamped blade pivot joint. The E-HINS horizontal inertial sensor has been designed using a Watt's Linkage suspension mechanism, inherited from [78, 79, 199–201]. Both sensors uses the above quadrature interferometer, shooting a corner-cube mounted on the proof-mass, as a readout device. The sensors have been characterised at the LIGO-MIT lab, in the HAMX-ISI system [40], which is a replica of the LIGO active seismic isolation system. The sensors' sensitivity response functions and resolution have been measured with, respectively, a 1.453 Hz resonance frequency and 200 Q -factor, and a 0.515 Hz resonance frequency and 50 Q -factor. The resolution is measured at $2 \cdot 10^{-13}$ m/ $\sqrt{\text{Hz}}$ above the sensor's respective resonance, and decays as $f^{-2.5}$ below due to the mechanics and optical device self-noise. Cross-sensitivities and the nonlinear response of the sensors have also been measured by comparing the sensors' output with the ISI motion in all 6 degrees of freedom. Despite the mechanical alignment mechanism of the proof-mass and the linearisation (ellipse correction) algorithm, cross-sensitivities and nonlinearities appear to be significant limitations. An overview of the integration of the sensors in the E-TEST active platform is also presented at the end of the chapter.

Lastly, chapter 3 presented the design of another optical vertical inertial sensor (accelerometer), μ VINS. Its application is active vibration noise compensation in an Absolute Quantum Gravimeter (AQG) from the EXail company [29, 253]. The μ VINS design is essentially similar to the E-VINS sensor, using proof-mass suspended with a CuBe₂ leaf-spring [56] and a clamped blade pivot, but twice as compact to fit the head of the AQG. The novelty here lies in the strategy used for broadening the bandwidth of the accelerometer to higher frequencies, as this is one key parameter for a successful integration in the AQG. Two methods are presented and compared for extending the sensor bandwidth. One method is the industry-standard force-balanced method [55, 57, 154, 155, 157] that uses a custom self-shielded quadrupole voice-coil actuator integrated in the design to close a feedback control loop. The other method uses a real-time frequency-compensation filter [122, 266–269] to digitally compensate for the magnitude drop and phase lag of the accelerometer response. After implementation of both methods in the μ VINS and comparison of the performances, mostly in terms of resolution, it was found that the frequency compensation technique allows a lower noise floor. This is because it does not use a feedback loop, which causes control noise propagation in the case of the force-feedback accelerometer. The μ VINS accelerometer has been integrated in the AQG at EXail facilities, and promising preliminary results have been obtained.

Perspectives for improvements

The thorough characterisation of the above devices has revealed several areas with margins for improvements, or alternative design concepts that may be beneficial for the device performance:

- In subsection 1.3.3, despite the massive investigation effort, the origin of the excess noise at

sub-Hz frequencies, with a noise spectrum growing with a slope steeper than the expected $f^{-0.5}$, is still unclear. Whereas several measurement may point out the laser source frequency noise as the potential limitation, no direct measurement could be performed. Temperature noise [122, 128–130] was also suspected to be a potential major contribution. It is believed that further characterisation work, for example, using a temperature-controlled enclosure, equipped with a high-resolution and low-frequency temperature sensor, should help in measuring the temperature sensitivity of the interferometer.

- Then, initial measurements of the E-VINS and E-HINS sensors' cross-sensitivities have indicated that they can be a significant limitation, and especially complicating the control system design. It is acknowledged that the measurement of the cross-sensitivities might have been biased by the cross-sensitivities of the HAMX-ISI system. A better characterisation would be obtained by using a high-precision hexapod table, which would be used as a 6D shake table. Also, the sensitivity of the sensors to environmental effects has not been addressed, namely, sensitivity to the magnetic field and temperature, which could have significant effects in the lower frequency range. A first solution for mitigating cross-axis sensitivities would be to develop a better assembly procedure, featuring a precise alignment of the proof-mass centre of mass with the pivot joint, may help reduce cross-translation sensitivity (excluding tilt-coupling). However, the single-joint design used in E-VINS makes the sensor inherently first order-sensitive to tilt. A reworked suspension design, including a parallel blade guidance mechanism for the proof-mass, would help in further rejection cross-axis coupling to a second order effect. Finally, an alternative design for the E-VINS leafspring suspension with lower mechanical frequency has been investigated in [171, 210] and shows promising potential in improving the E-VINS resolution.
- Finally, despite the promising results, the comparison campaign between the AQG enhanced with μ VINS and the reference AQG showed that there is still a clear margin for improvement. The merging currently re-uses the same filtering process as with the Titan accelerometer, traditionally used in AQGs. According to the expertise of the EXail engineers, these filters should be optimised for the current accelerometer, and this could have a significant impact on the results. As to the optical accelerometer itself, there are concerns about the sensitivity of the sensor to environmental effects. The sensitivity of the accelerometer to magnetic fields has not been properly characterised. This might be relevant since the sensor operates close to the MOT (Magneto-Optical Trap) of the AQG, which might leak a significant amount of stray field. Also, the EXail engineers report that temperature sensitivity is currently a limitation to the vibration noise compensation using a Titan accelerometer at low frequency. Preliminary measurements indicated that the sensitivity of the μ VINS to temperature, driven by the temperature sensitivity of the suspension leafspring, is similar to that of the Titan accelerometer. Improvements for lowering the sensitivity are under investigation: using an alternative material for the suspension with lower temperature sensitivity, or coupling different blades with different thermal behaviour, forming a "self-compensated" pair, or using a servo-control alignment of the proof-mass for compensating thermal drifts. Related to thermal

drift is sensitivity to cross directions. The active alignment mechanism could act directly on the proof-mass itself, for instance using a voice coil-like actuator, or as a motorised levelling foot that control the inclination of the hole device. The latter technique, based on servo-controlled alignment, might have the additional benefit of re-aligning the proof-mass with the pivot joint, thereby minimising sensitivity to cross-directions. Eventually, since the frequency-compensation filter only digitally increases the bandwidth of the sensor, it allows the proof-mass to move freely and therefore leaves the sensor sensitive to the nonlinear response of the interferometer. A dampening system of the proof-mass could help in improving the overall sensor nonlinearity level.

Appendix A

eHINS drawings

Figure A.1 lays out the E-HINS optical inertial sensor major components and sub-assemblies. They are:

1. Sensor body assembly.
 - 1.1. Watt's Linkage. Ref: VU Amsterdam, Nickef.
 - 1.2. Retroreflector interface.
 - 1.3. TIR corner-cube retroreflector with flat side. Ref: EXail.
2. Base plate.
3. Readout stage.
 - 3.1. Optical readout. Ref: section 1.2.
 - 3.2. Readout interface.
 - 3.3. Alignment brackets
4. Actuator stage.
 - 4.1. Vacuum-compatible coil. Ref: H2W NCC03-07-001-1V (Coil only)
 - 4.2. N45 Neodymium magnet. Ref: Supermagnet S-12-03N
 - 4.3. Magnet assembly.
 - 4.4. Magnet assembly interface.
 - 4.5. Moving magnet interface.
 - 4.6. Static coil interface.
5. Connector assembly.
 - 5.1. Connector front interface.
 - 5.2. Connector back interface.
 - 5.3. Connector Sub-D9 M interface. Ref: Arun MicroElectronics VC9DM
6. Sensor feet.
 - 6.1. Thorlabs piezo inertia actuator. Ref: Ref: Thorlabs PIAK10VF
 - 6.2. Motor interface.
7. Manual feet. Ref: VU Amsterdam, Nickef.
8. Spring-loaded screw. Ref: LeeSpring LHP038B01S

9. Cover assembly.

9.1. Side walls.

9.2. Top cover.

The technical drawings of items that have been designed in-house will be shared in the following.

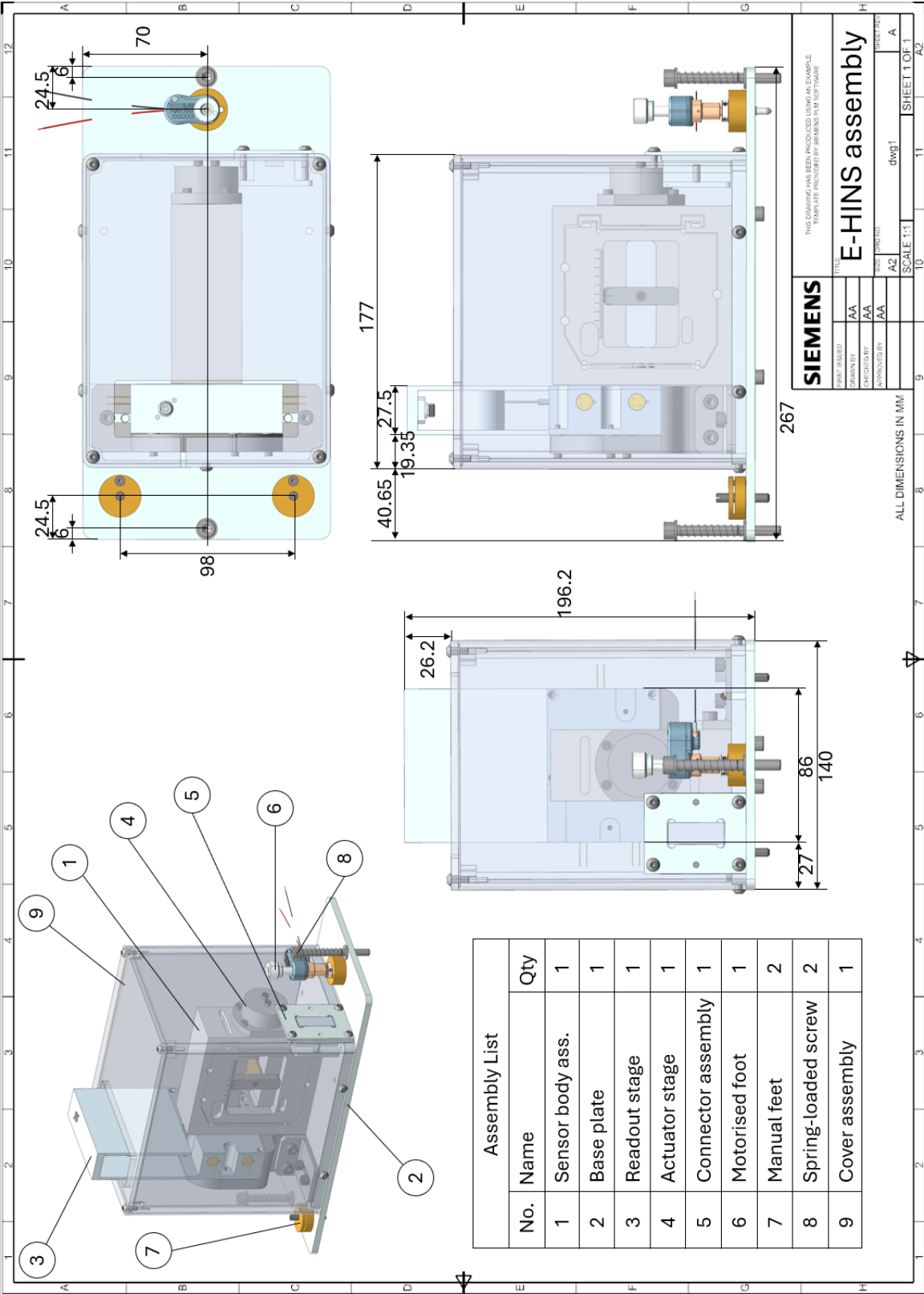


Figure A.1: E-HINS assembly drawing and global dimensions.

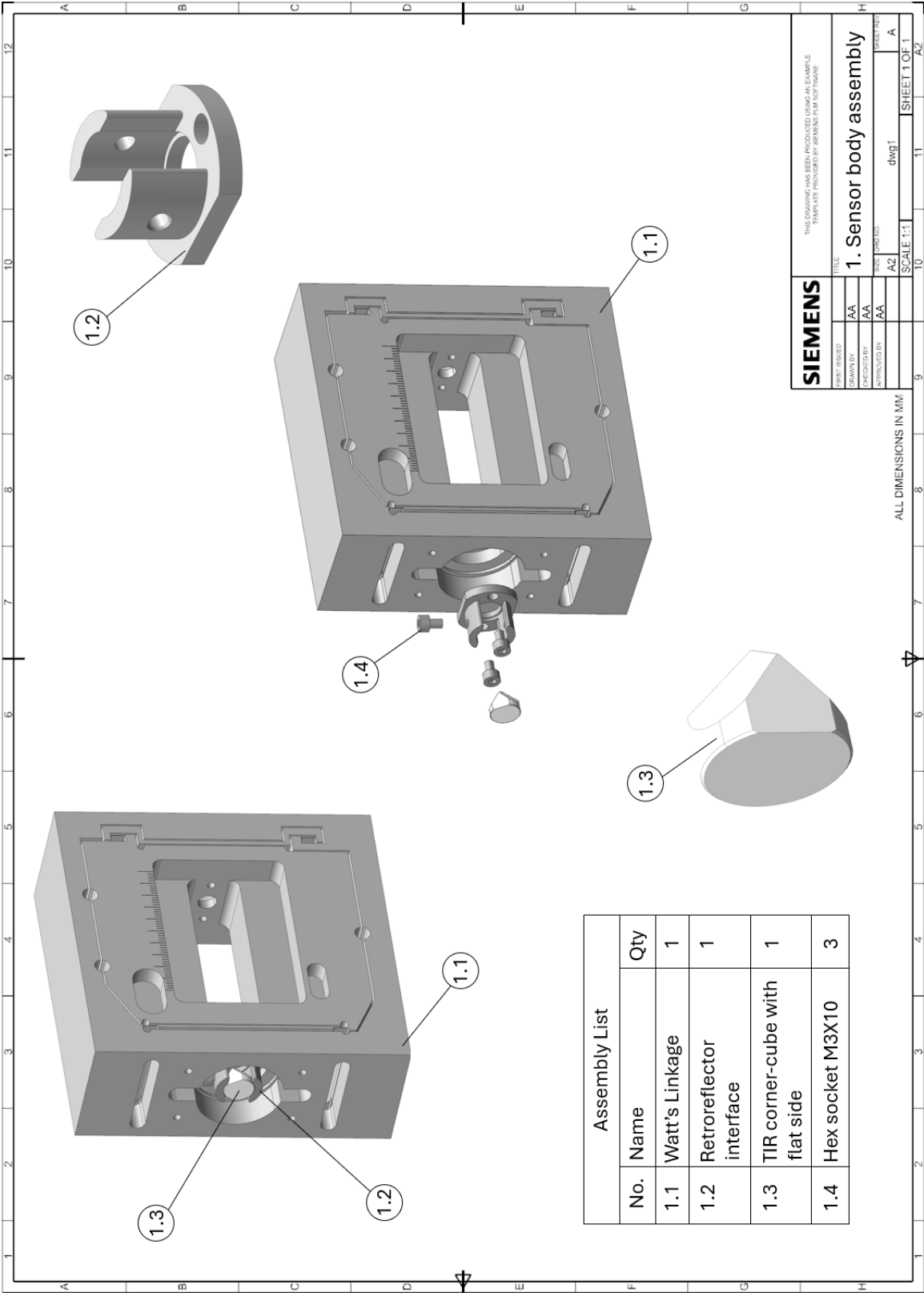


Figure A.3: E-HINS sensor body assembly drawing.



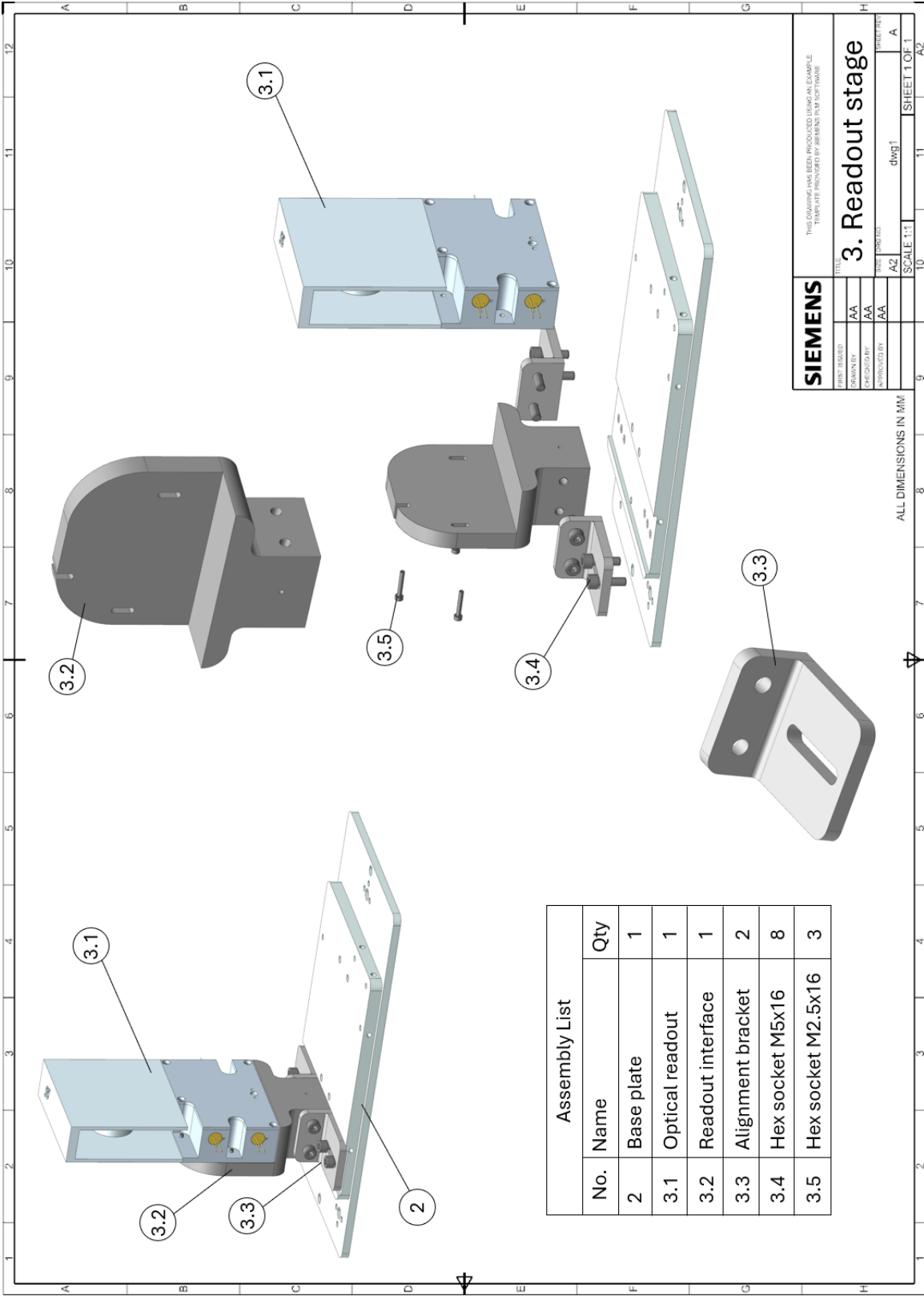


Figure A.5: E-HINS readout stage assembly drawing.

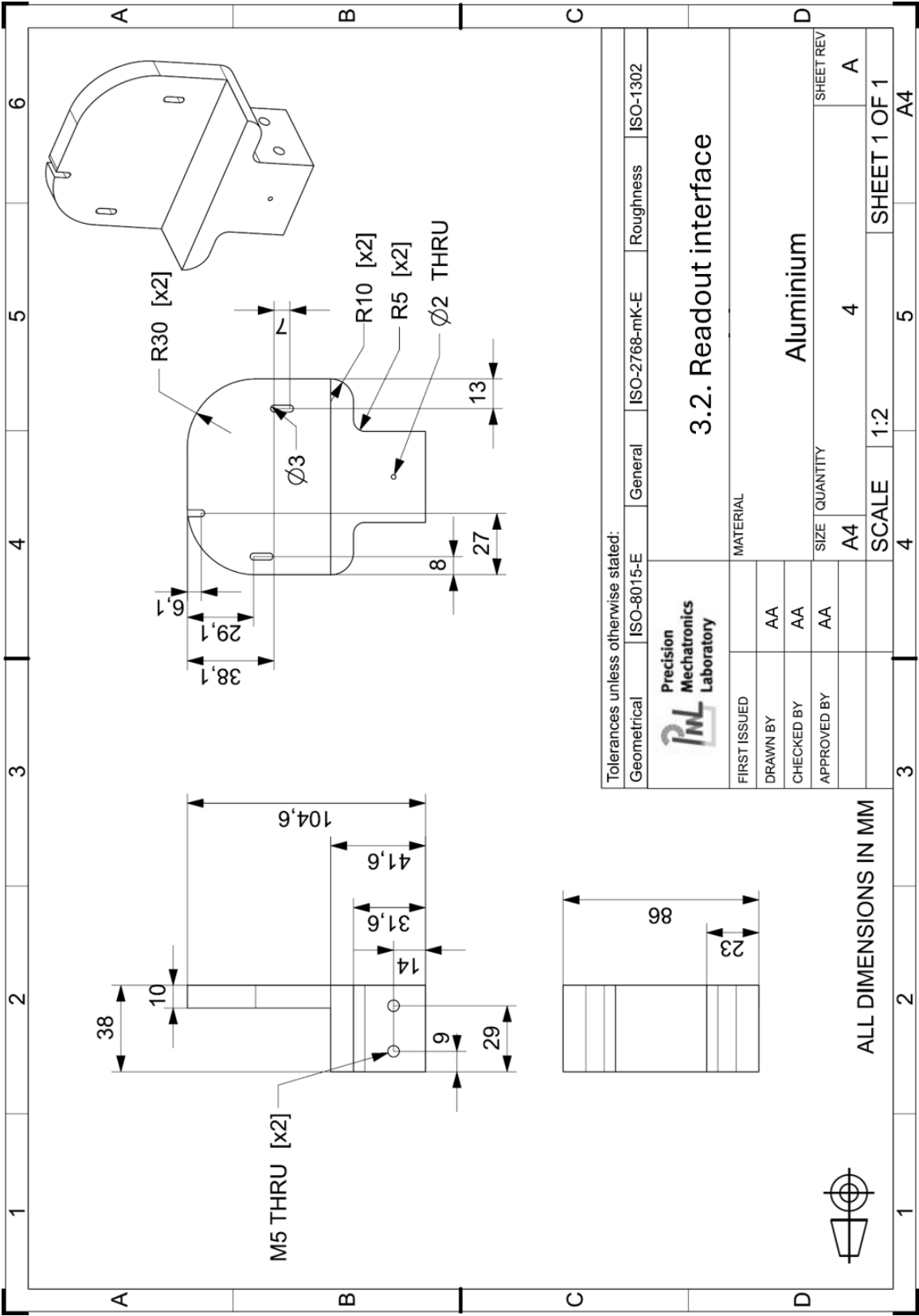


Figure A.6: E-HINS readout interface drawing.

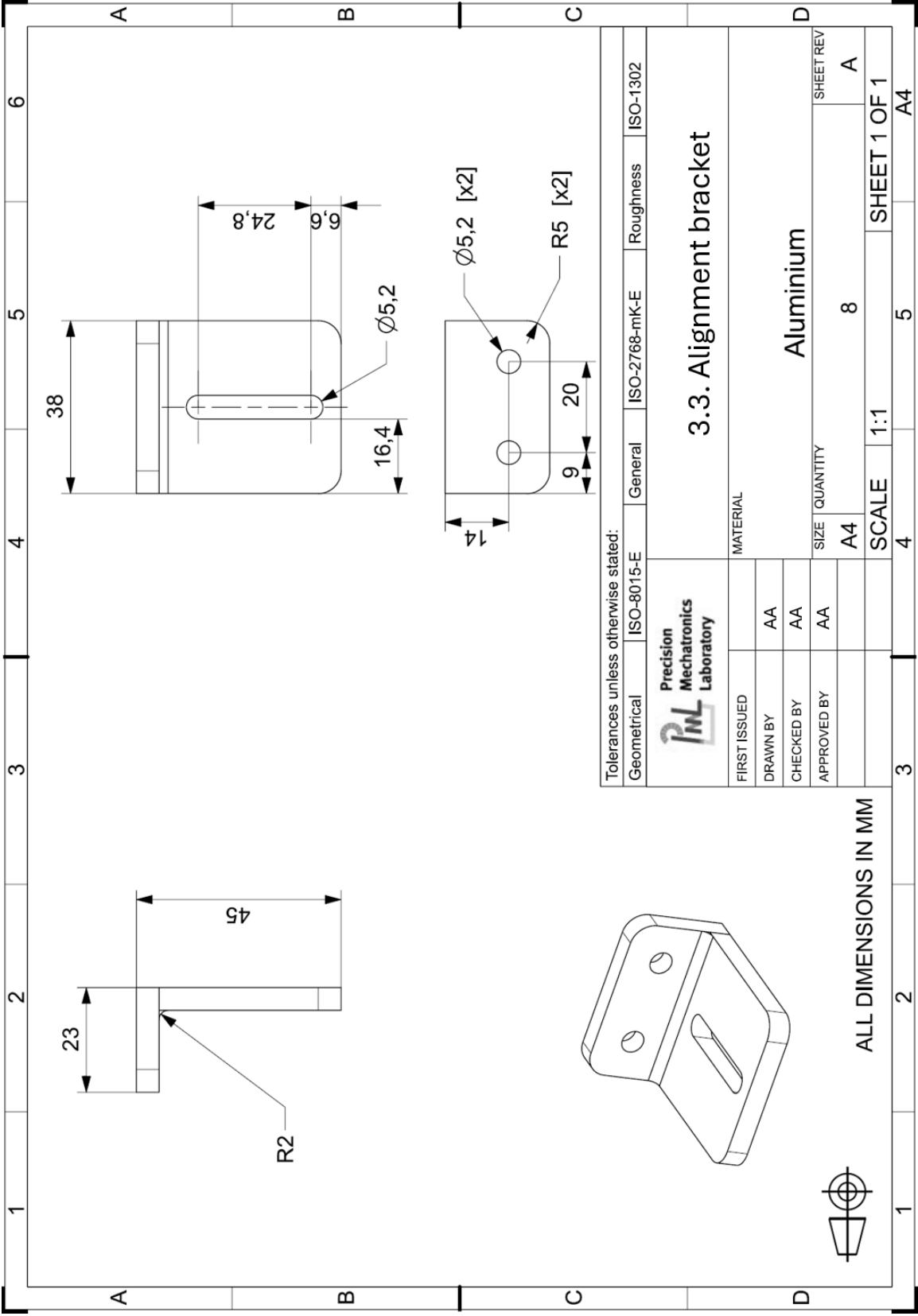


Figure A.7: E-HINS alignment bracket drawing.

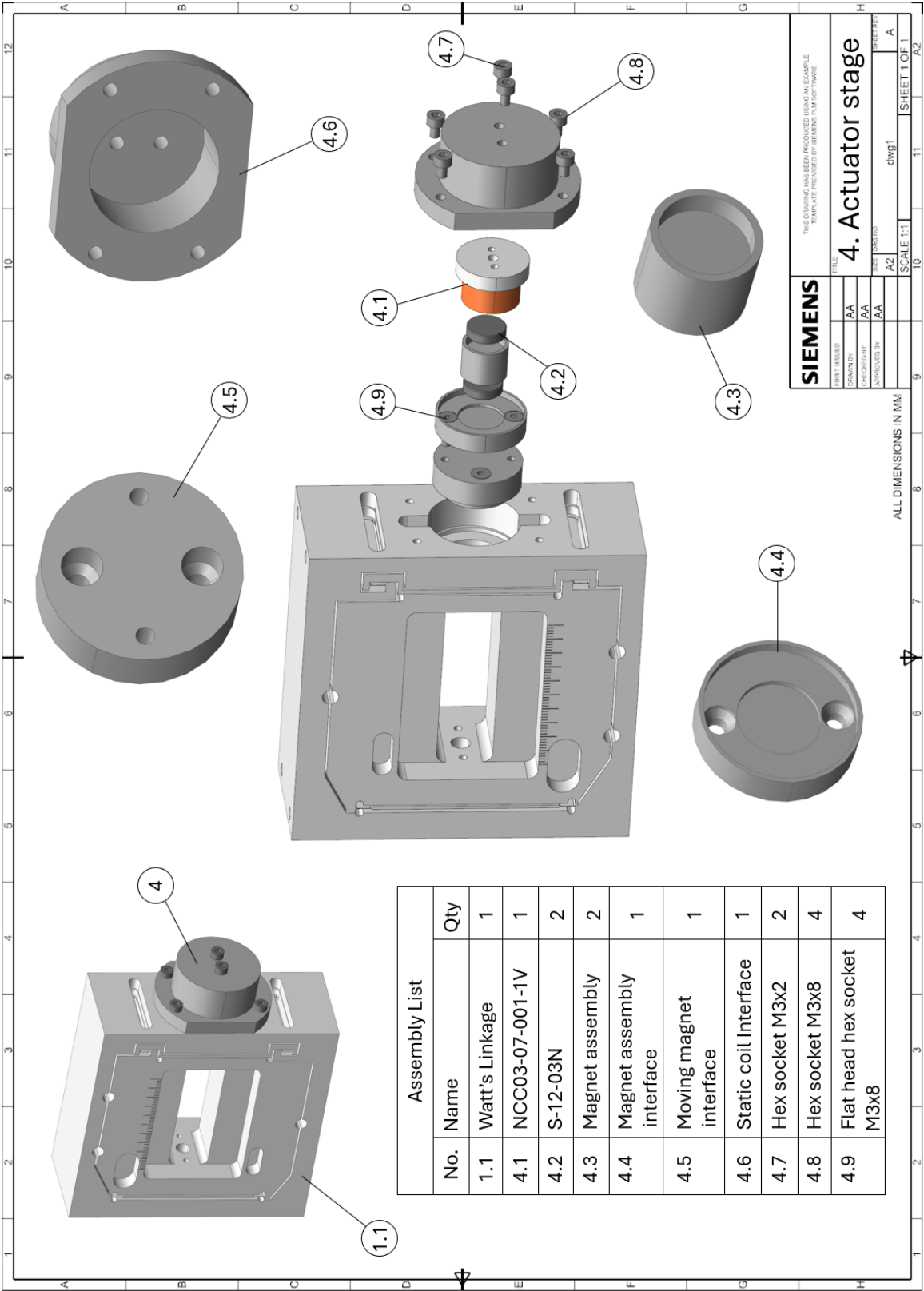


Figure A.8: E-HINS actuator stage assembly drawing.

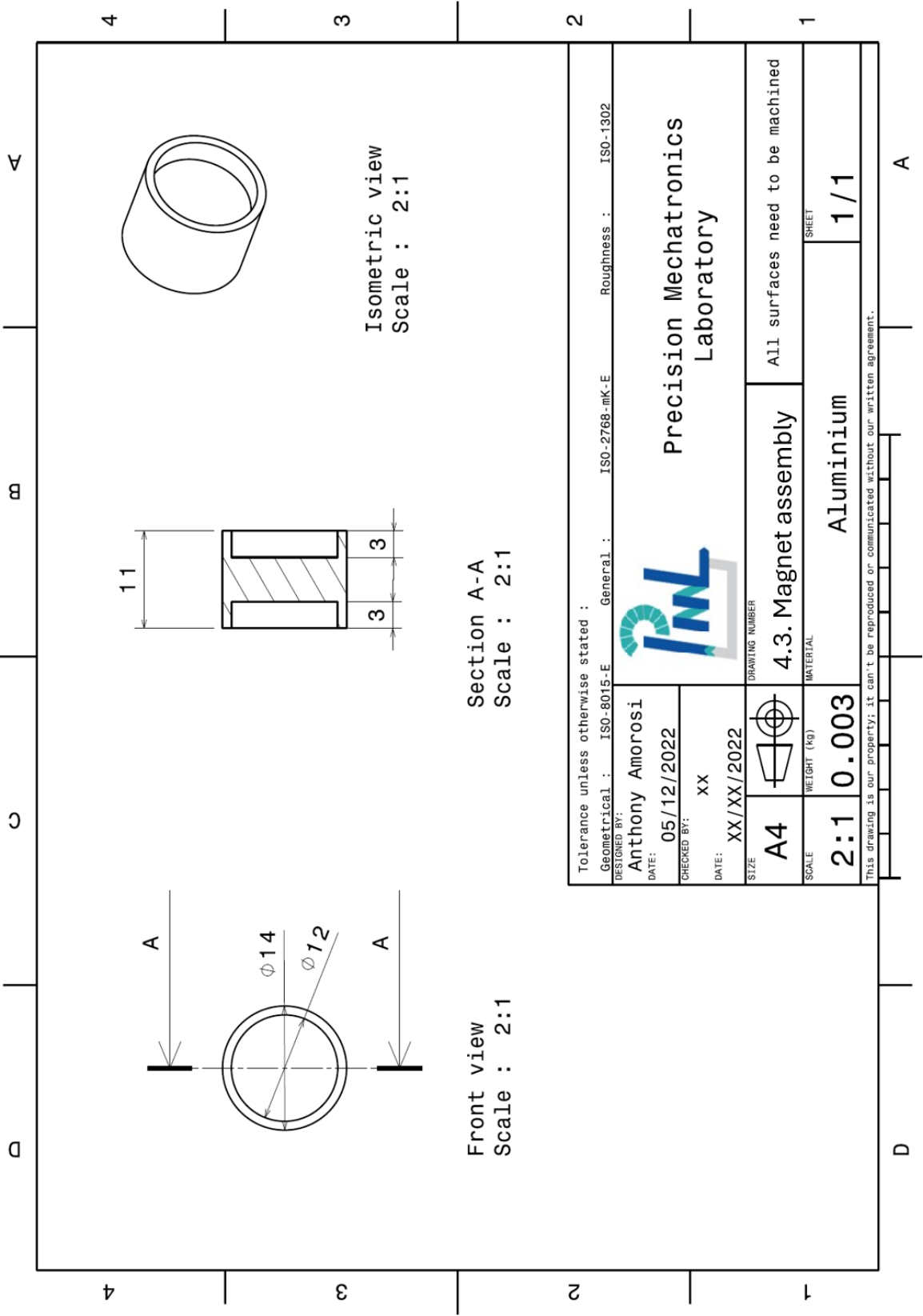


Figure A.9: E-HINS magnet assembly drawing.



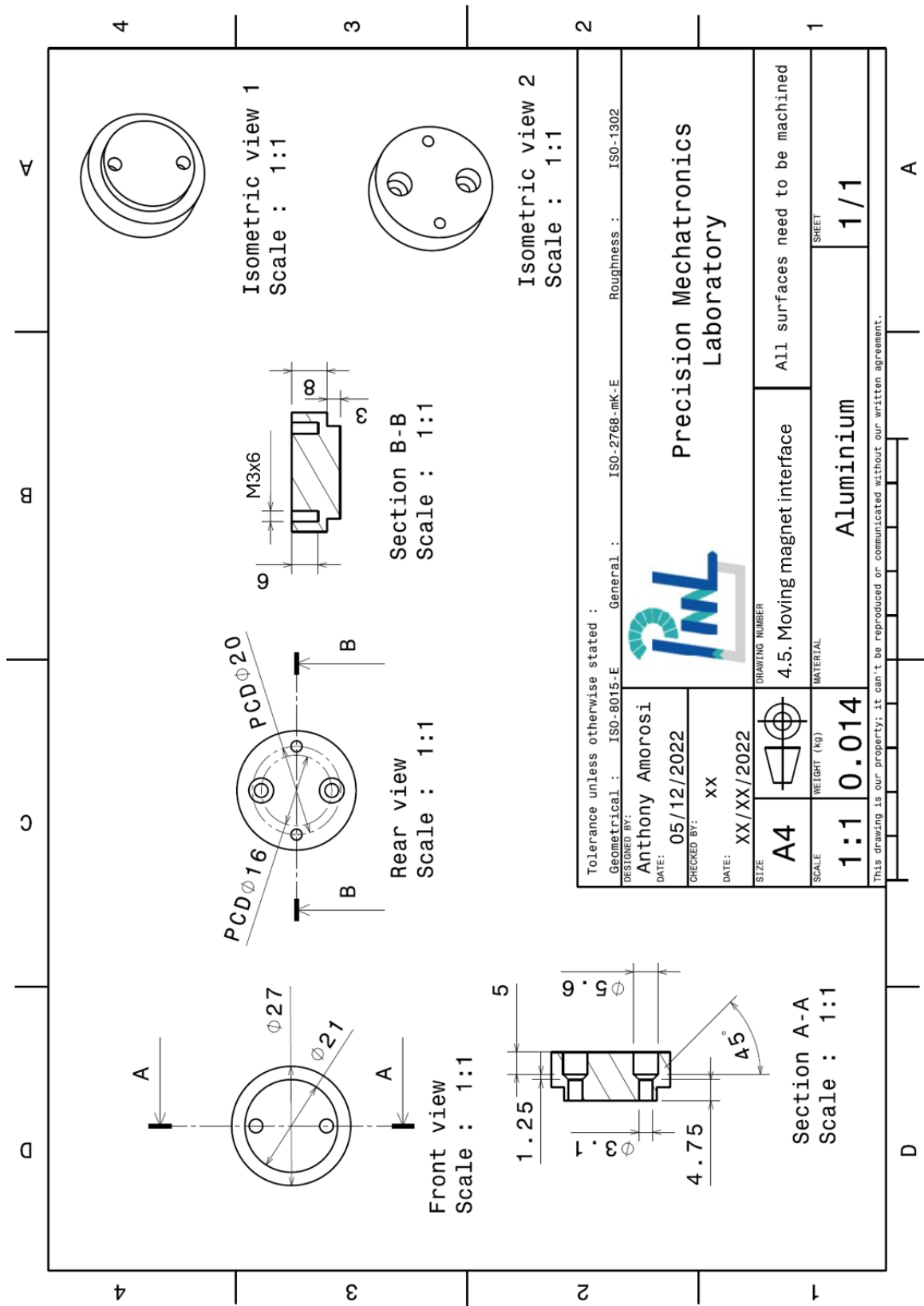


Figure A.11: E-HINS moving magnet interface drawing.

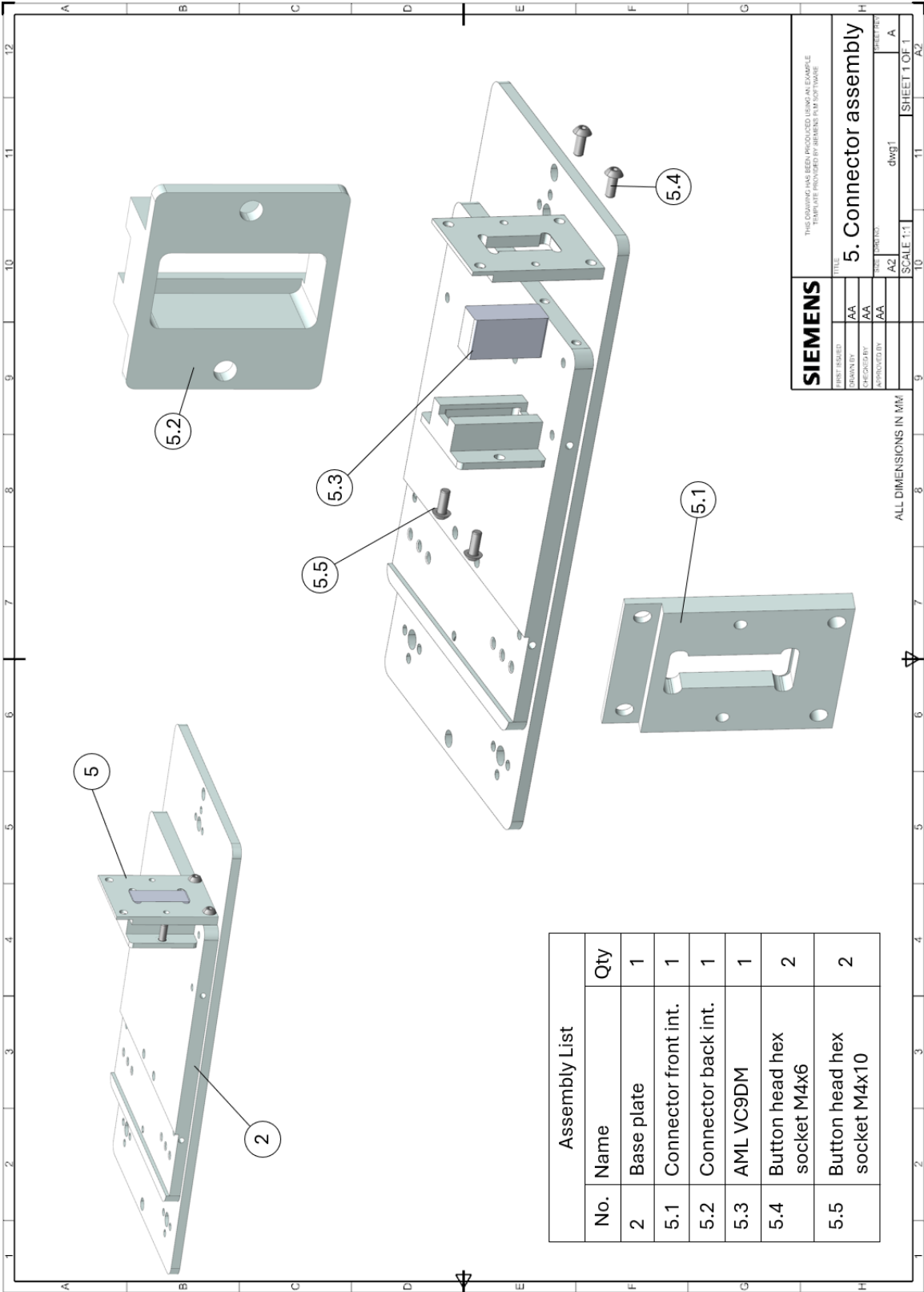


Figure A.13: E-HINS connector assembly drawing.



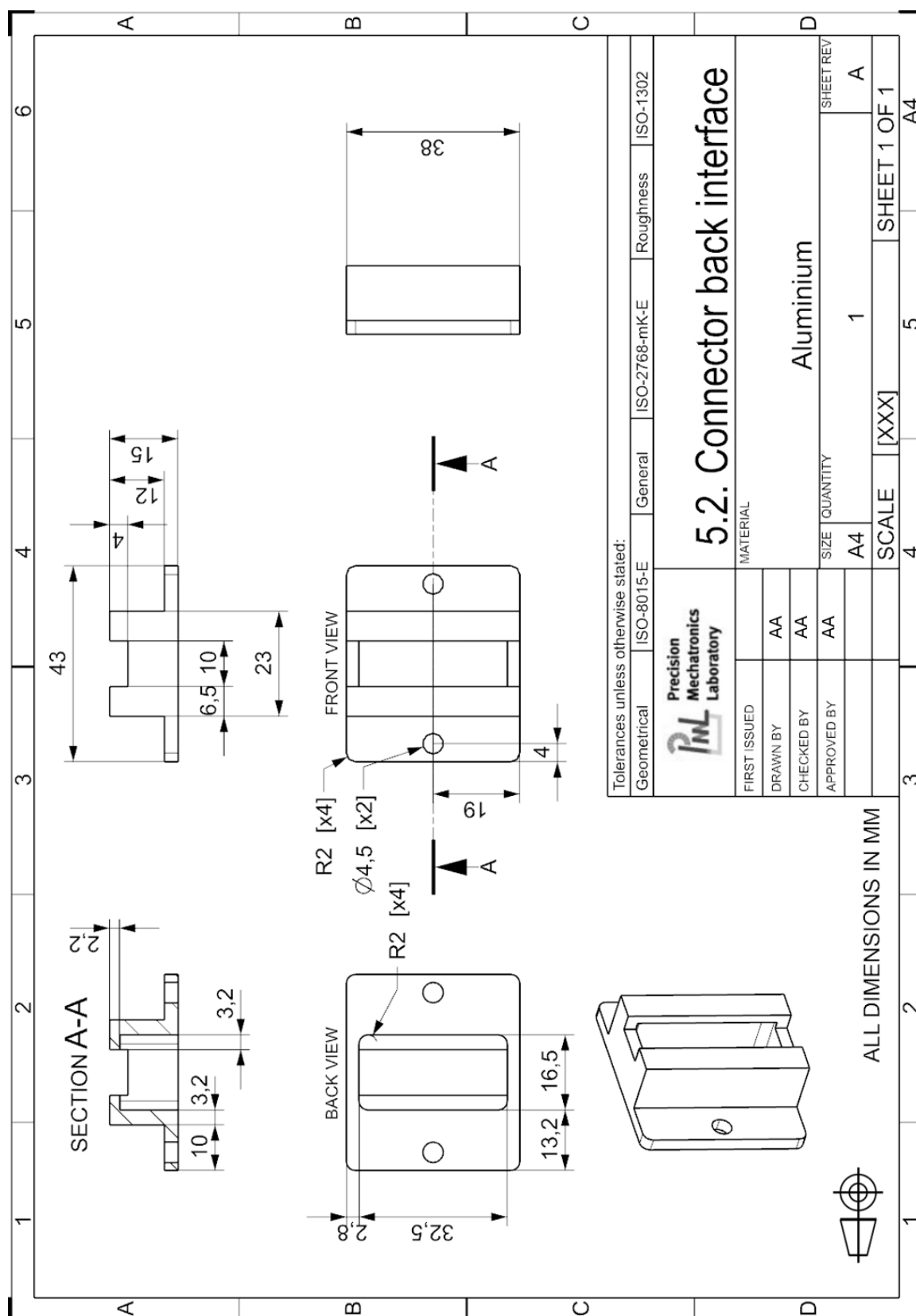


Figure A.15: E-HINS connector back interface drawing.

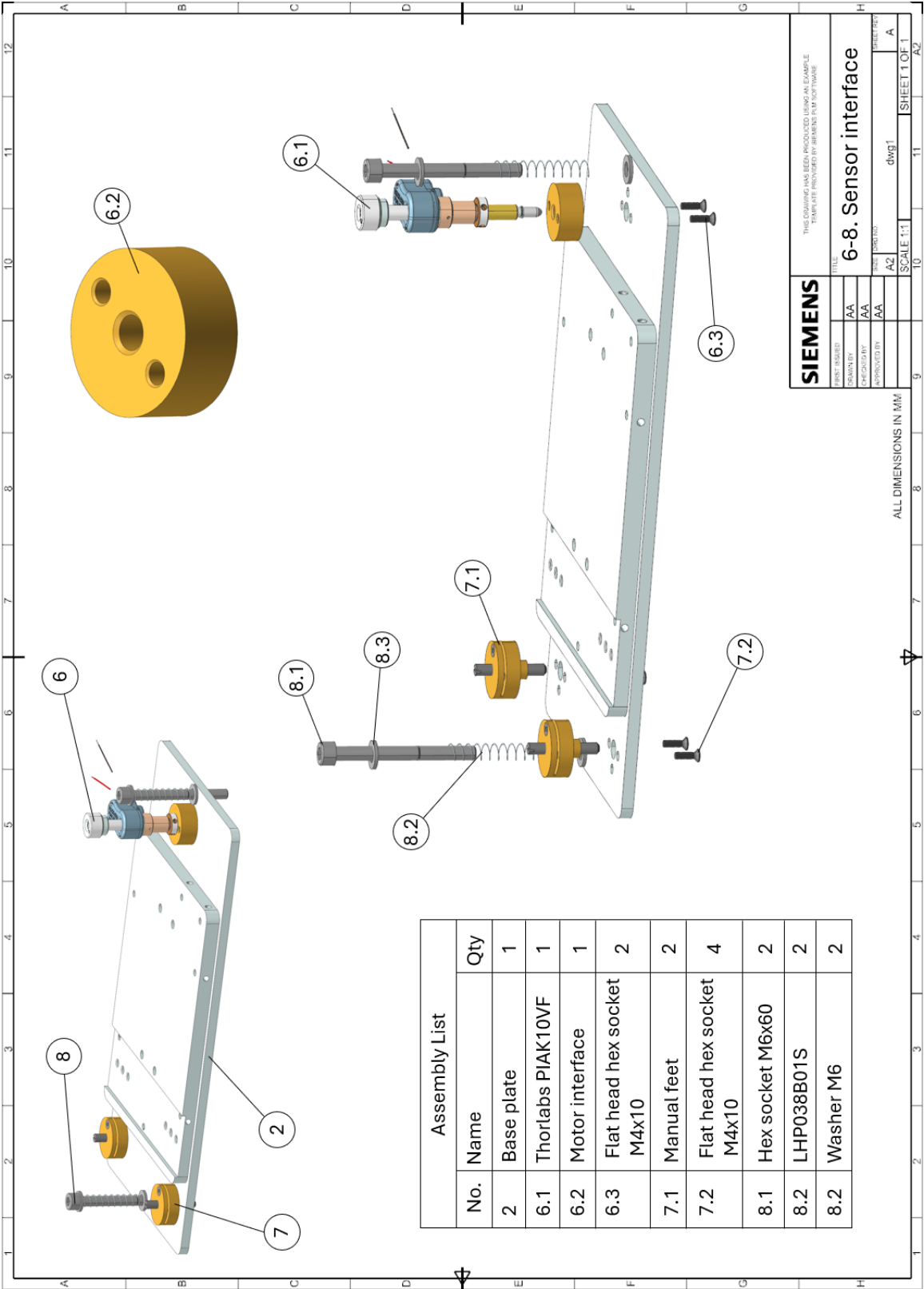


Figure A.16: E-HINS baseplate interface assembly drawing.

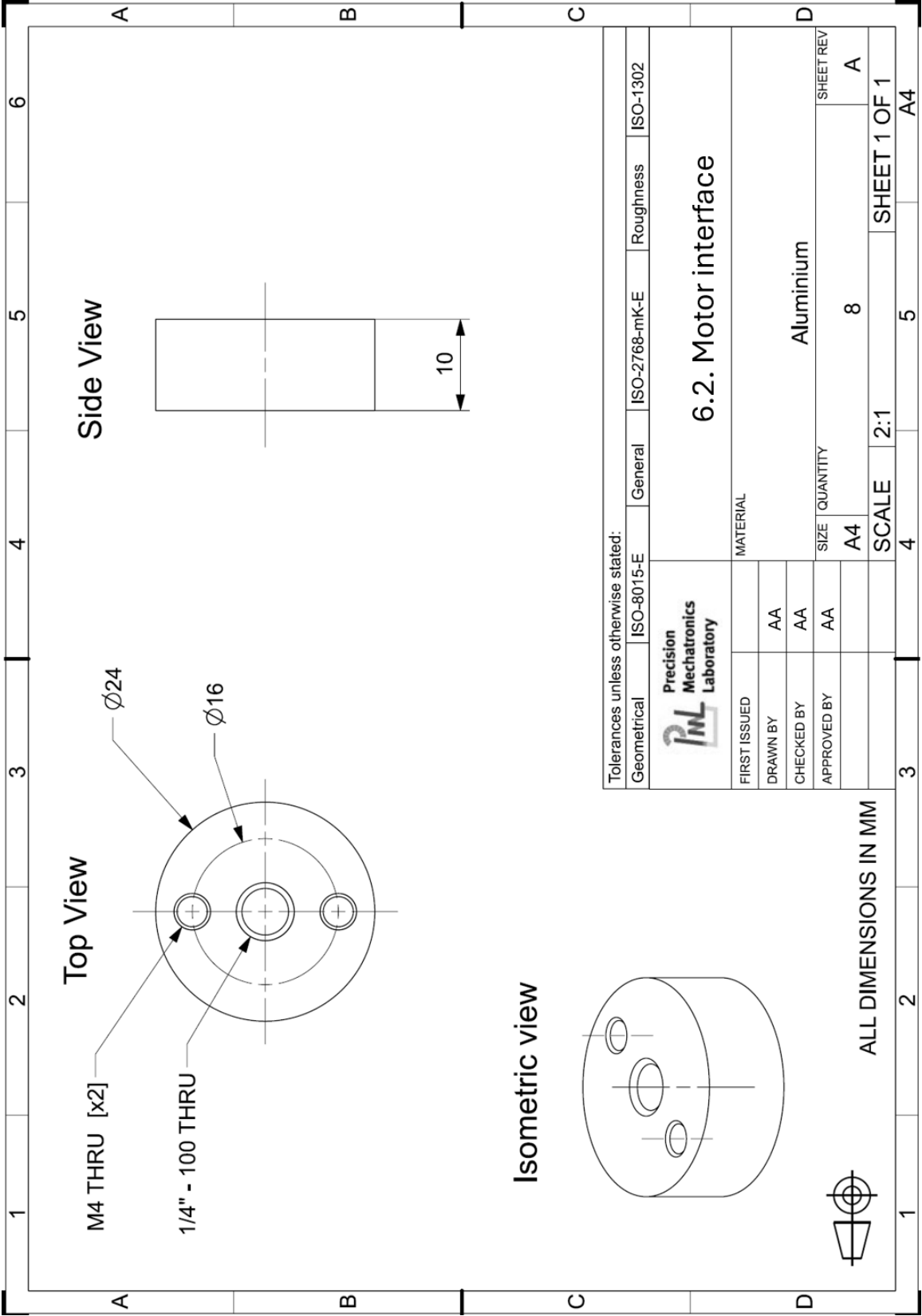


Figure A.17: E-HINS motor interface drawing.

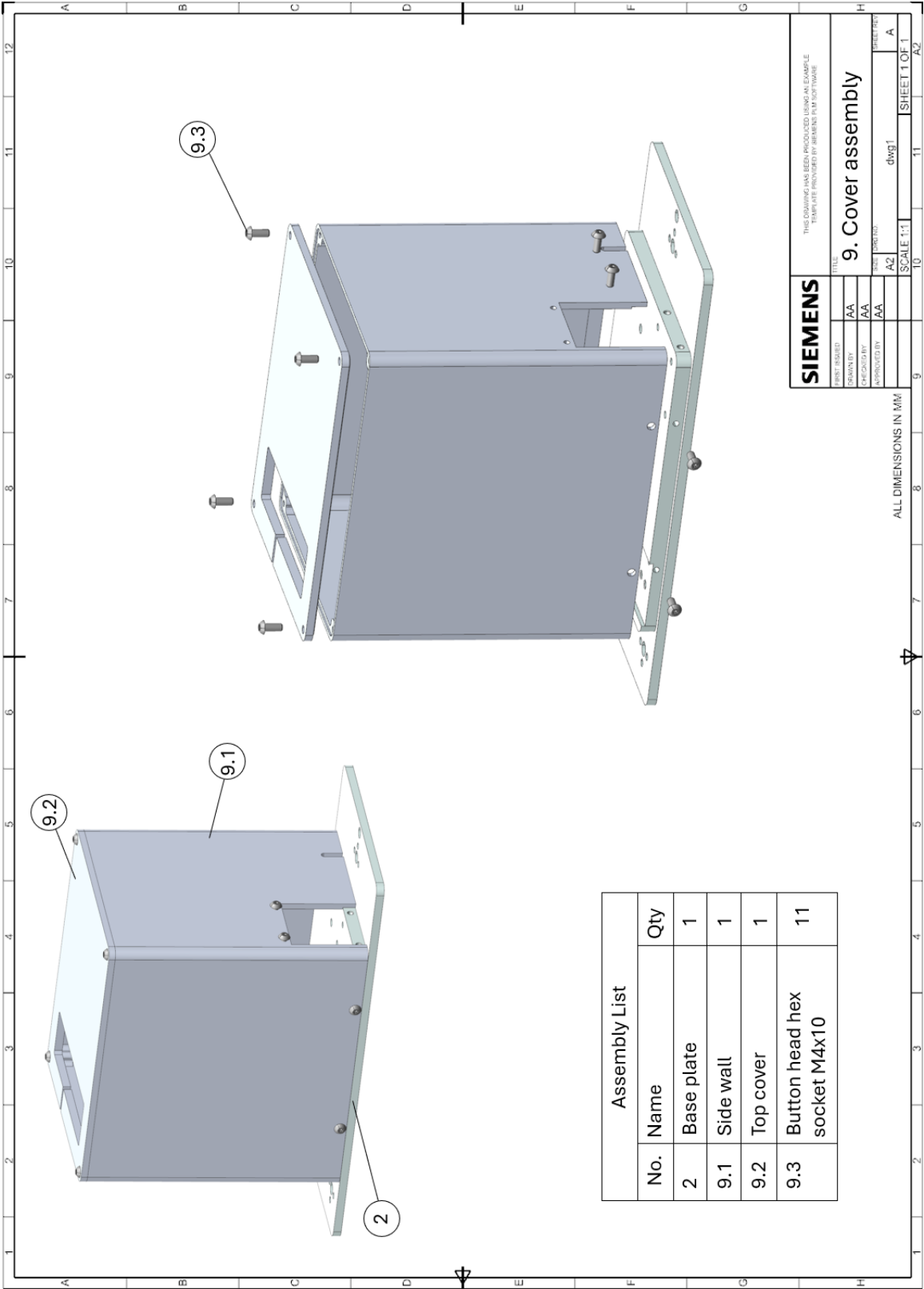


Figure A.18: E-HINS cover assembly drawing.

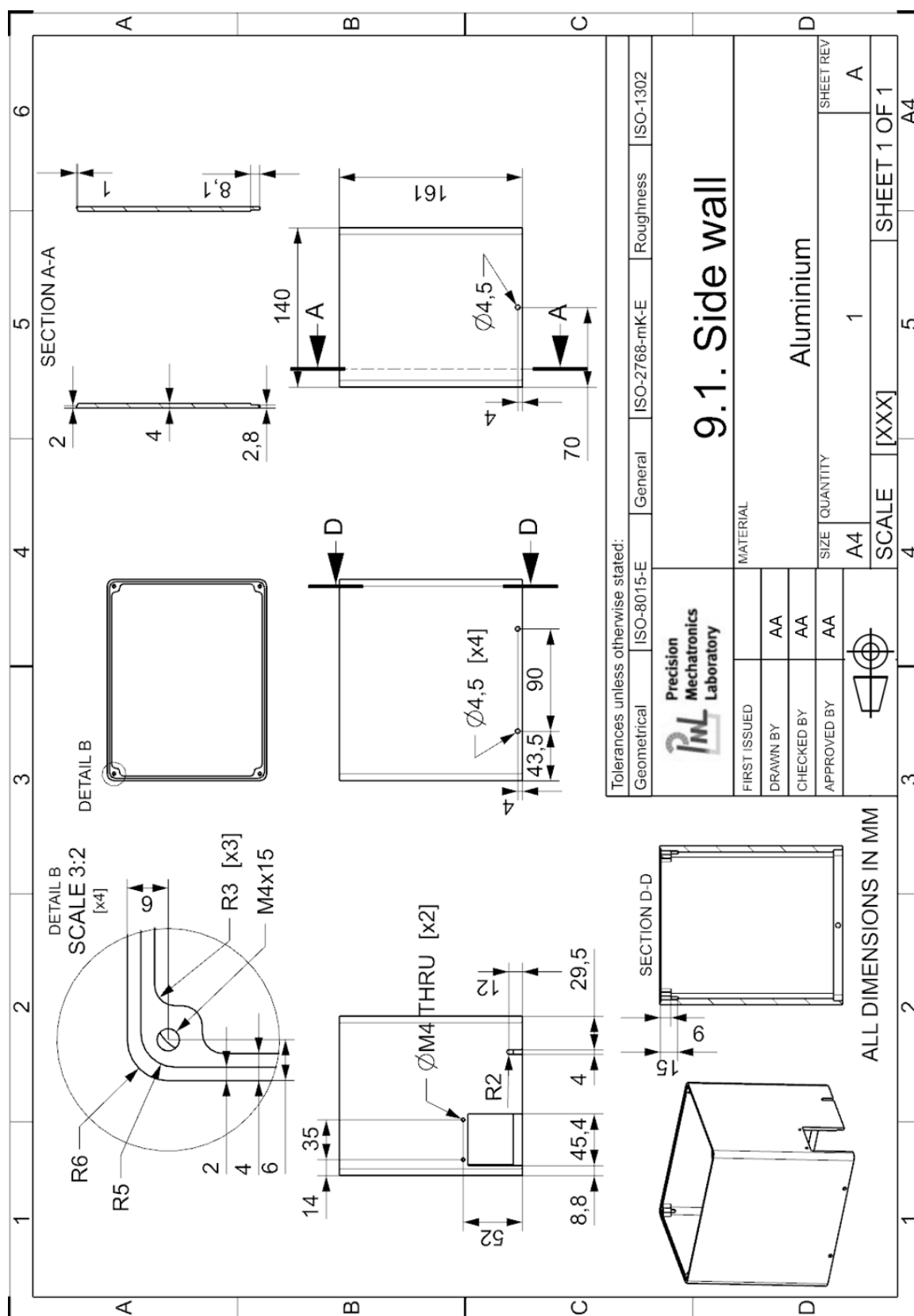


Figure A.19: E-HINS side wall drawing.

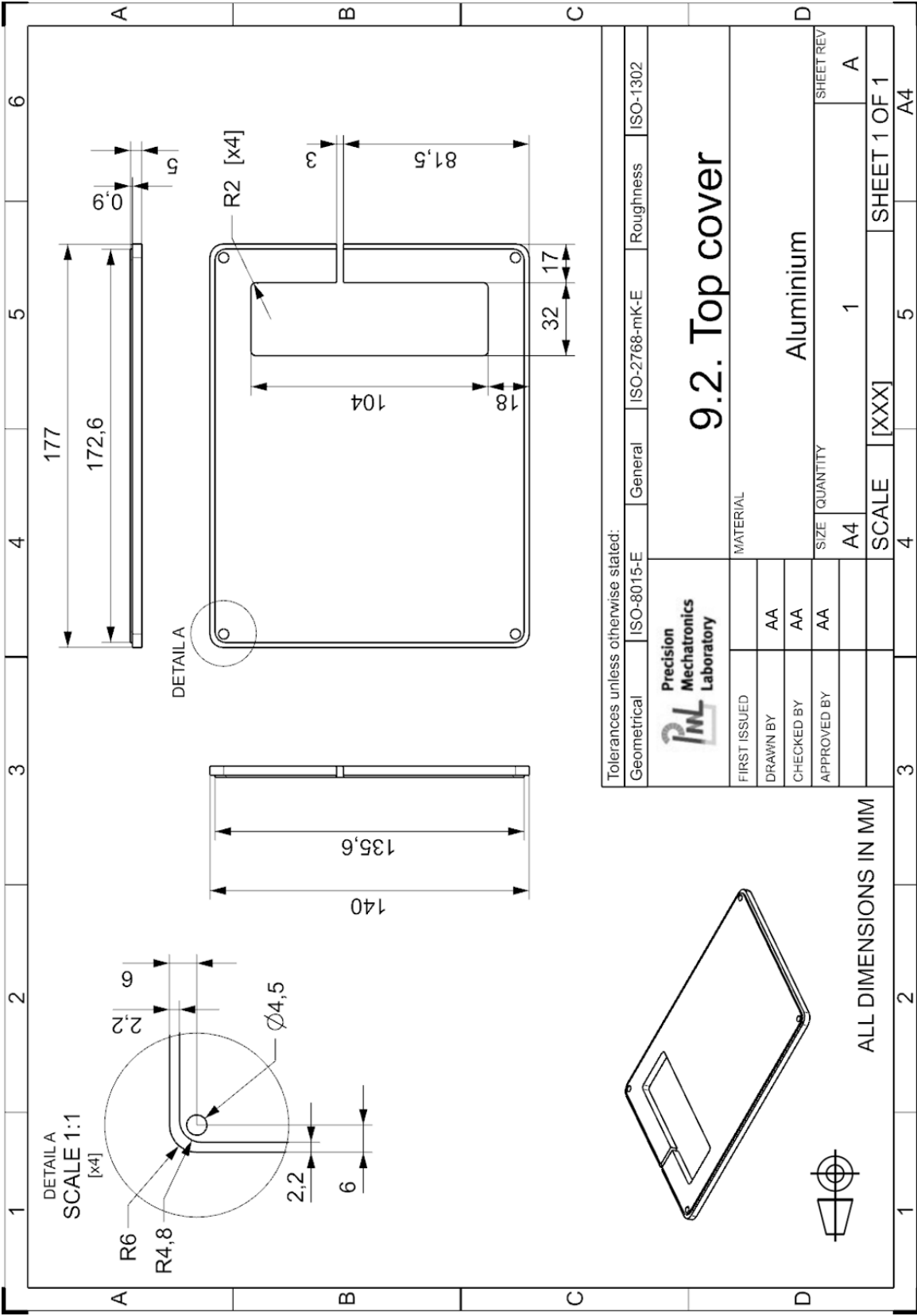


Figure A.20: E-HINS top cover drawing.

Appendix B

Pinout mappings

This appendix contains the pinout maps for the various sensors and electronics.

B.1 E-VINS pinout map

The E-VINS vertical optical inertial sensor is equipped with a D-sub 9 male connector. The pins of the 3 FGA21 photodiodes (PDx) optical readout (IFO) are wired to 7 pins of the D-sub connector, photodiode ground (GND) are common. The last two pins are unused/reserved for the voice-coil actuator (VCA).

PIN	Signal	Purpose
1	PD1-S	Photodiode 1, Signal
6	PD1-B	Photodiode 1, Bias voltage
2	PD2-S	Photodiode 2, Signal
7	PD2-B	Photodiode 2, Bias voltage
3	PD3-S	Photodiode 3, Signal
8	PD3-B	Photodiode 3, Bias voltage
4	N/C	Not connected/reserved for actuator
9	N/C	Not connected/reserved for actuator
5	GND	Photodiode 1,2,3 ground

Table B.1: E-VINS D-sub 9 pinout map.

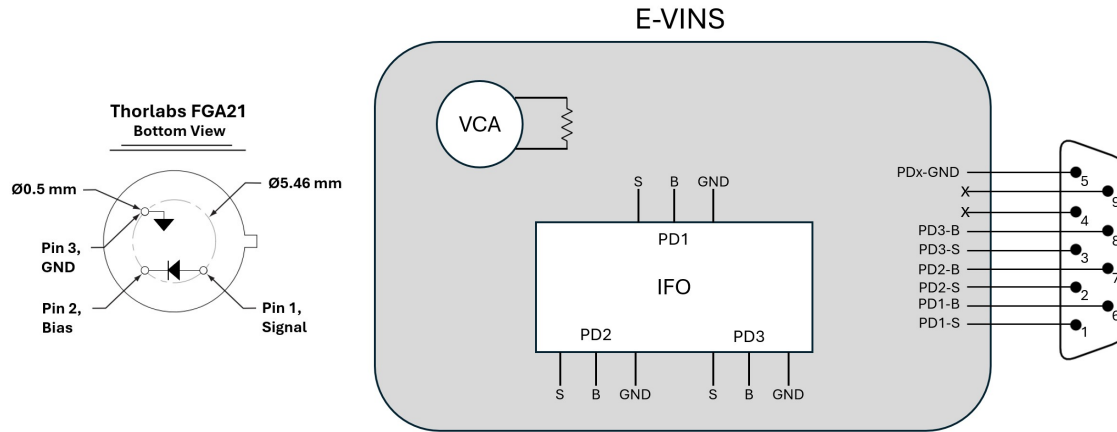


Figure B.1: Thorlabs FGA21 pinout diagram (left). E-VINS pinout diagram (right)

B.2 E-HINS pinout map

The E-HINS horizontal optical inertial sensor is equipped with a D-sub 9 male connector. The pins of the 3 FGA21 photodiodes (PDx) optical readout (IFO) are wired to 7 pins of the D-sub connector, photodiode ground (GND) are common. The last two pins are connected to the PIAK10VF motorised piezoelectric inertia actuator (MA).

PIN	Signal	Purpose
1	PD1-S	Photodiode 1, Signal
6	PD1-B	Photodiode 1, Bias voltage
2	PD2-S	Photodiode 2, Signal
7	PD2-B	Photodiode 2, Bias voltage
3	PD3-S	Photodiode 3, Signal
8	PD3-B	Photodiode 3, Bias voltage
4	MA-P	Motorised actuator, P
9	MA-N	Motorised actuator, N
5	GND	Photodiode 1,2,3 ground

Table B.2: E-HINS D-sub 9 pinout map.

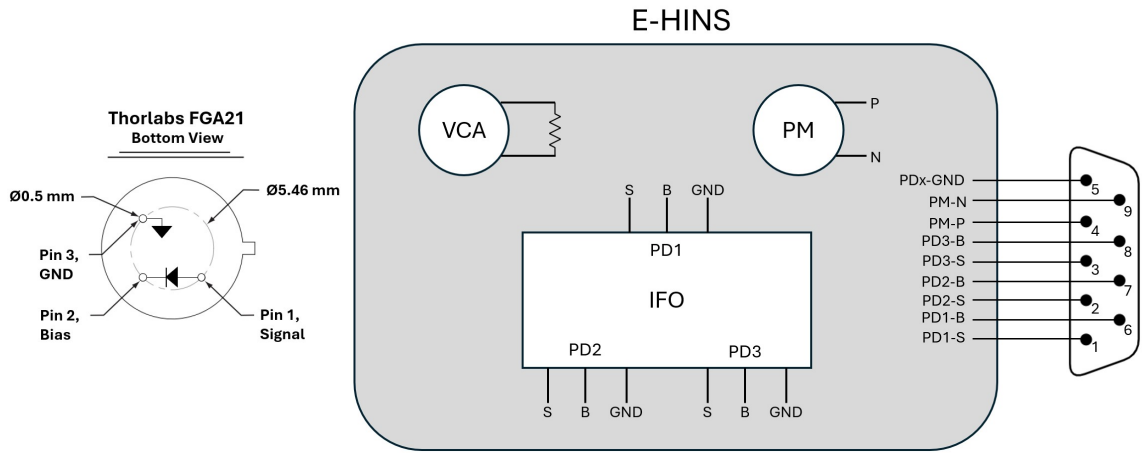


Figure B.2: Thorlabs FGA21 pinout diagram (left). E-HINS pinout diagram (right)

B.3 μ VINS pinout map

The μ VINS vertical optical inertial sensor is equipped with a D-sub 9 male connector. The pins of the 3 FGA21 photodiodes (PDx) optical readout (IFO) are wired to 7 pins of the D-sub connector, photodiode ground (GND) are common. The last two pins are connected to the coil of voice-coil actuator (VCA).

PIN	Signal	Purpose
1	PD1-S	Photodiode 1, Signal
6	PD1-B	Photodiode 1, Bias voltage
2	PD2-S	Photodiode 2, Signal
7	PD2-B	Photodiode 2, Bias voltage
3	PD3-S	Photodiode 3, Signal
8	PD3-B	Photodiode 3, Bias voltage
4	VCA-P	Voice-coil drive
9	VCA-P	Voice-coil drive - GND
5	GND	Photodiode 1,2,3 ground

Table B.3: μ VINS D-sub 9 pinout map.

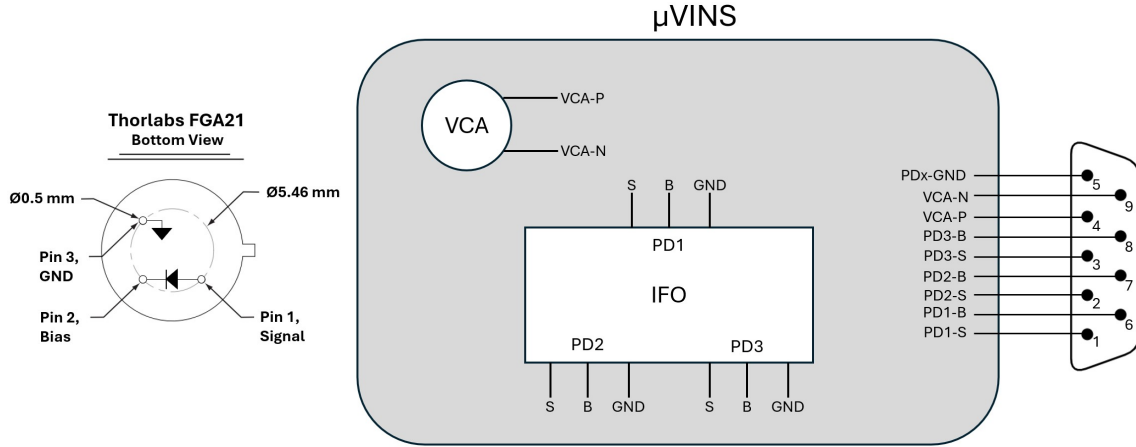


Figure B.3: Thorlabs FGA21 pinout diagram (left). μ VINS pinout diagram (right)

B.4 Signal conditioner pinout map

Figure B.4 shows the electrical layout of one signal conditioner unit. Each unit contains electronics for processing 3 optical inertial sensors (Sens I-III), i.e. 3 pre-amp boards (**Pre-Amp**), and 3 coil driver electronics (**Coil Driver**) to power 3 voice-coil actuators (**Act I-III**). The unit is powered with a ± 15 V symmetric voltage supply. The power supply connects to two switches: the first switch powers the pre-amp boards only, while the second switch also powers the coil drivers. This separation in the power supply is because it has been noticed that the high-load circuitry of the coil driver produces stray signals that interfere with the pre-amp boards, so that it is preferred to keep them off when they are not in use.

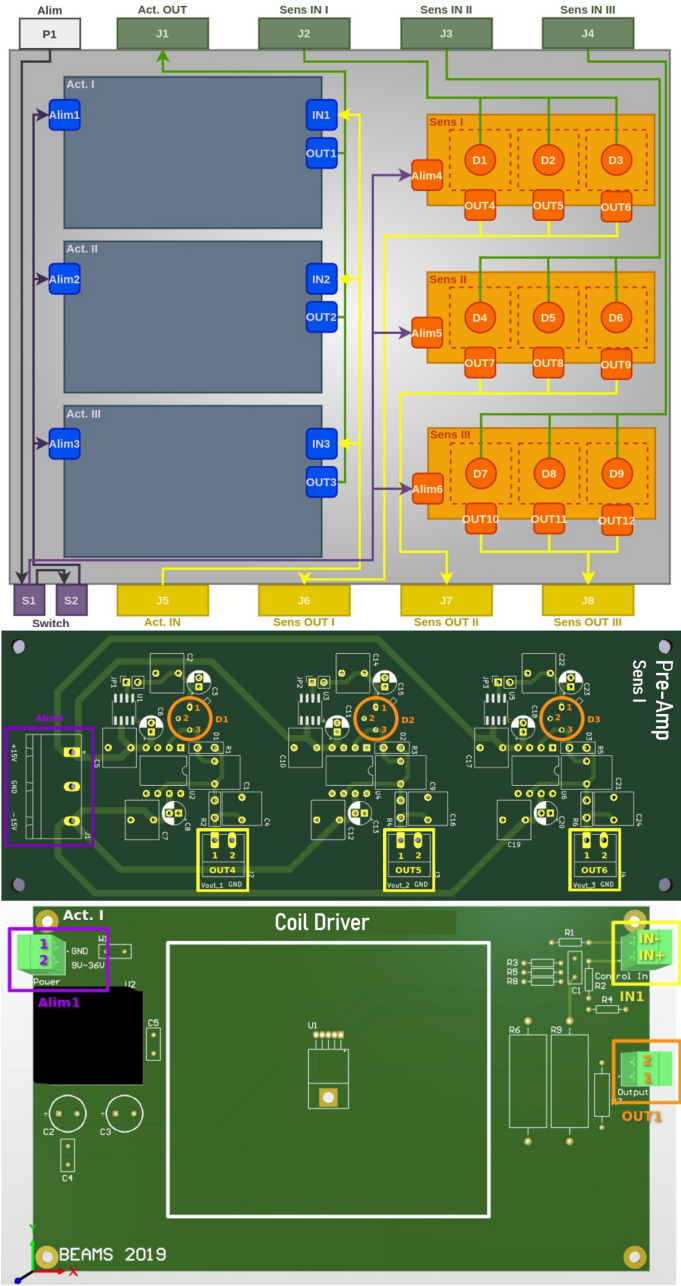


Figure B.4: Layout of 1 signal conditioner unit (top). 1 photodiode pre-amp board (middle). 1 coil driver board (bottom)

The below table shows the pinout mapping for the D-sub 9 female connector of the signal conditioner unit.

PIN	Signal	Purpose	PIN	Signal	Purpose
Sens IN I			Sens OUT I		
1	PD1-S	Photodiode 1, Signal	1	VCos	Cosine signal
6	PD1-B	Photodiode 1, Bias voltage	6	GND	Ground
2	PD2-S	Photodiode 2, Signal	2	VSin	Sine signal
7	PD2-B	Photodiode 2, Bias voltage	7	GND	Ground
3	PD3-S	Photodiode 3, Signal	3	VmSin	-Sine signal
8	PD3-B	Photodiode 3, Bias voltage	8	GND	Ground
4	N/C		4	N/C	
9	N/C		9	N/C	
5	GND	Photodiode 1,2,3 ground	5	N/C	Not connected
Sens IN II			Sens OUT II		
1	PD1-S	Photodiode 1, Signal	1	VCos	Cosine signal
6	PD1-B	Photodiode 1, Bias voltage	6	GND	Ground
2	PD2-S	Photodiode 2, Signal	2	VSin	Sine signal
7	PD2-B	Photodiode 2, Bias voltage	7	GND	Ground
3	PD3-S	Photodiode 3, Signal	3	VmSin	-Sine signal
8	PD3-B	Photodiode 3, Bias voltage	8	GND	Ground
4	N/C		4	N/C	
9	N/C		9	N/C	
5	GND	Photodiode 1,2,3 ground	5	N/C	
Sens IN III			Sens OUT III		
1	PD1-S	Photodiode 1, Signal	1	VCos	Cosine signal
6	PD1-B	Photodiode 1, Bias voltage	6	GND	Ground
2	PD2-S	Photodiode 2, Signal	2	VSin	Sine signal
7	PD2-B	Photodiode 2, Bias voltage	7	GND	Ground
3	PD3-S	Photodiode 3, Signal	3	VmSin	-Sine signal
8	PD3-B	Photodiode 3, Bias voltage	8	GND	Ground
4	N/C		4	N/C	
9	N/C		9	N/C	
5	GND	Photodiode 1,2,3 ground	5	N/C	
Act OUT			Act IN		
1	Drive1	Drive Current 1	1	Cont1	Diff. Control Input 1 (+)
6	Drive1-GND	Drive 1 Ground	6	Cont1-GND	Diff. Control Input 1 (-)
2	Drive2	Drive Current 2	2	Cont2	Diff. Control Input 2 (+)
7	Drive2-GND	Drive 2 Ground	7	Cont2-GND	Diff. Control Input 2 (-)
3	Drive3	Drive Current 3	3	Cont3	Diff. Control Input 3 (+)
8	Drive3-GND	Drive 3 Ground	8	Cont3-GND	Diff. Control Input 3 (-)
4	N/C		4	N/C	
9	N/C		9	N/C	
5	N/C		5	N/C	

Table B.4: Signal conditioner unit pinout map.

The below table shows the wiring map from each connectors to the different coil driver and pre-amp boards.

FROM	TO	Signal	FROM	TO	Signal
Sens IN I	Sens I		SENS OUT I	Sens I	
1	D1,3	PD1-S	1	OUT4,1	VCos
6	D1,1	PD1-B	6	OUT4,2	GND
2	D2,3	PD2-S	2	OUT5,1	VSin
7	D2,1	PD2-B	7	OUT5,2	GND
3	D3,3	PD3-S	3	OUT6,1	VmSin
8	D3,1	PD3-B	8	OUT6,2	GND
4	N/C		4	N/C	
9	N/C		9	N/C	
5	D1,2 & D2,2 & D3,2	GND	5	N/C	
Sens IN II	Sens II		SENS OUT II	Sens II	
1	D4,3	PD1-S	1	OUT7,1	VCos
6	D4,1	PD1-B	6	OUT7,2	GND
2	D5,3	PD2-S	2	OUT8,1	VSin
7	D5,1	PD2-B	7	OUT8,2	GND
3	D6,3	PD3-S	3	OUT9,1	VmSin
8	D6,1	PD3-B	8	OUT9,2	GND
4	N/C		4	N/C	
9	N/C		9	N/C	
5	D4,2 & D5,2 & D6,2	GND	5	N/C	
Sens IN III	Sens III		SENS OUT III	Sens III	
1	D7,3	PD1-S	1	OUT10,1	VCos
6	D7,1	PD1-B	6	OUT10,2	GND
2	D8,3	PD2-S	2	OUT11,1	VSin
7	D8,1	PD2-B	7	OUT11,2	GND
3	D9,3	PD3-S	3	OUT12,1	VmSin
8	D9,1	PD3-B	8	OUT12,2	GND
4	N/C		4	N/C	
9	N/C		9	N/C	
5	D7,2 & D8,2 & D9,2	GND	5	N/C	
Act OUT	ACT I-III		Act IN	ACT I-III	
1	OUT1,1	Drive1	1	IN1,IN+	Cont1
6	OUT1,2	Drive1-GND	6	IN1,IN-	Cont1-GND
2	OUT2,1	Drive2	2	IN2,IN+	Cont2
7	OUT2,2	Drive2-GND	7	IN2,IN-	Cont2-GND
3	OUT3,1	Drive3	3	IN3,IN+	Cont3
8	OUT3,2	Drive3-GND	8	IN3,IN-	Cont3-GND
4	N/C		4	N/C	
9	N/C		9	N/C	
5	N/C		5	N/C	

Table B.5: Signal conditioner unit wiring map.

Appendix C

Signal conditioning unit characterisation

The quadrature Michelson interferometer (section 1.2), the E-VINS/E-HINS inertial sensors (chapter 2) and the μ VINS inertial accelerometer (chapter 3) are all operated on the same electronics, named signal conditioning unit. The architecture, circuitry and pinout map can be found in Appendix B.

One signal conditioning unit contains 3 pre-amp boards. Each pre-amp board consists of 3 low-noise transimpedance circuits for conditioning the 3 photodiode signals on one optical readout. One signal conditioning unit can therefore process up to 3 devices simultaneously. Figure C.1 (top) shows the schematics of a pre-amp board. The core of the design is an operational amplifier (Op-amp) placed in an inverting configuration with a 10 k Ω feedback resistor to produce the desired gain. The photodiode is also powered with a 2.5 V bias voltage to maximise the response and dynamic range. Noise from such an electronic circuit primarily arises from the active component, i.e. the Op-amp. The LT1792 low-noise, precision, JFET Op-amp was chosen for its high performance at lower frequencies, typically 8.3 nV/ $\sqrt{\text{Hz}}$ and 10 fA/ $\sqrt{\text{Hz}}$ at 10 Hz when supplied with ± 15 V (online datasheet). In addition, the $1/f$ Flicker-noise can become a severe limitation at sub-Hz frequencies. To limit the contribution from resistive components, especially from the feedback resistor, a Vishay Z201 ultra-high precision Z-Foil resistor with a -40 dB Noise Index (NI) is used (online datasheet). A 1.6 kHz low-pass filter is also used to avoid high-frequency noise propagation, and decoupling capacitors at the Op-amp and photodiode levels are also used to isolate from power supply noise.

One signal conditioning unit also contains 3 coil driver boards. Each board consists mostly of one instrumentation amplifier used for powering the coils of the E-VINS, E-HINS or μ VINS sensors. One signal conditioning unit can drive up to 3 coils. Figure C.1 (bottom) shows the schematics of a coil driver board. It uses a high-voltage/high-current OPA544 Op-amp in an instrumentation amplifier configuration to provide stable drive current with nearly-DC capabilities. It has a 103 dB

open-loop gain and $36 \text{ nV}/\sqrt{\text{Hz}}$ input voltage noise and $3 \text{ fA}/\sqrt{\text{Hz}}$ input current noise at 1 kHz. The circuit design is the work of the electronic workshop of the BEAMS (Service Systèmes bio et électromécaniques) department of ULB.

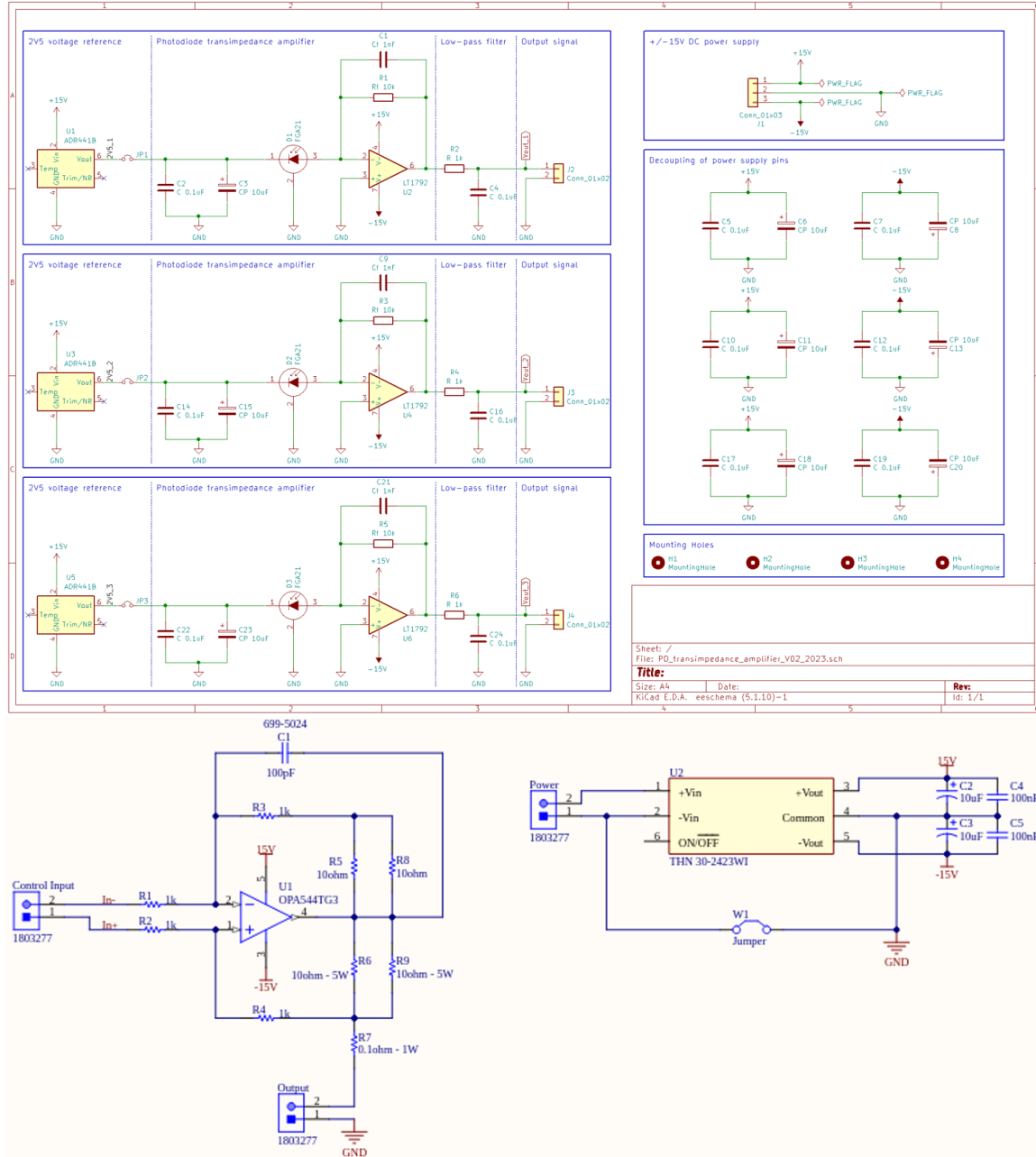


Figure C.1: (Top) Pre-amps circuit drawing. (Bottom) Coil driver circuit drawing.

The pre-amp boards have been tested by recording the voltage output of the signal conditioning unit when all input channels are shorted with an impedance matching the photodiode impedance.

The voltage outputs are recorded on the MicroLab Box (16-bit) ADCs at 10 kHz for 300 s. The voltage noise spectrums of the 3 voltage outputs (VCos, VSin and VmSin) for all 3 pre-amp boards (Sens OUT I-III) are shown in Figure C.2. They show noise levels of $5 \mu\text{V}/\sqrt{\text{Hz}}$ at 5 Hz and above, and a $1/f^{0.5}$ contribution below 5 Hz, sitting just above acquisition noise (ADC).

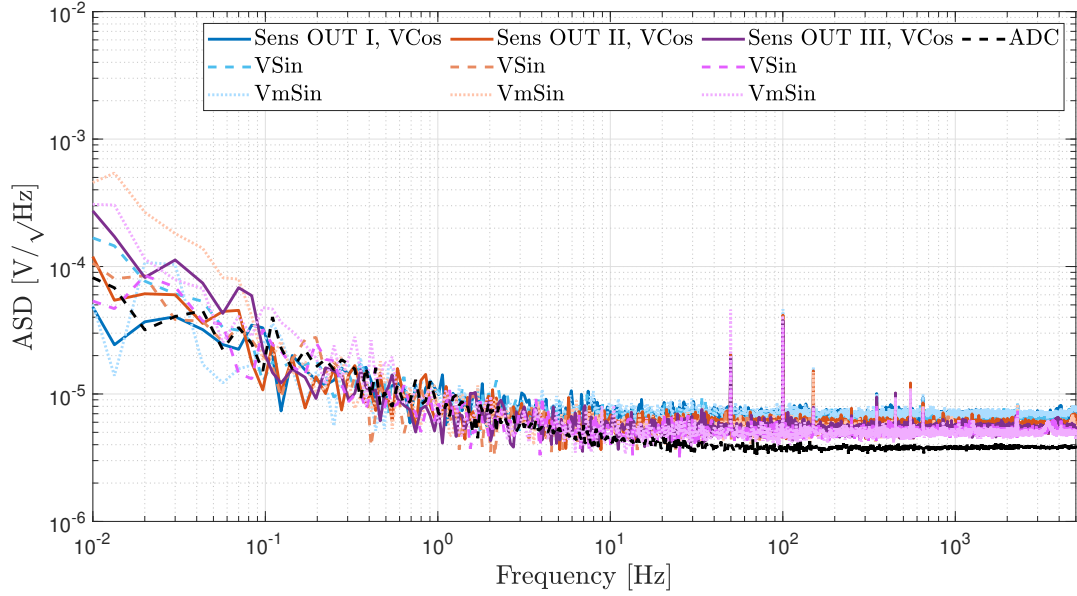


Figure C.2: Signal conditioner unit pre-amps noise spectrum.

The coil driver boards' noise level is measured by feeding a null (0 V) voltage input and recording the output drive current. The MicroLab Box is used for feeding the null signal since this is the real-time target unit that will be used in the control loops. The MicroLab Box 16-bit DAC noise therefore also propagates through the coil driver, in addition to the coil driver electronic self-noise. The output current is fed to a resistor element to produce a voltage, which is then recorded and converted back to current with Ohm's law. Because the noise level is expected to be lower than the MicroLab Box 16-bit ADC noise, a 24-bit NI acquisition system is used instead. The current noise spectrums are shown in Figure C.3. They show noise levels of $20 \text{ nA}/\sqrt{\text{Hz}}$ at 100 Hz and above, with a $1/f^{0.5}$ contribution kicking in at 50 Hz below.

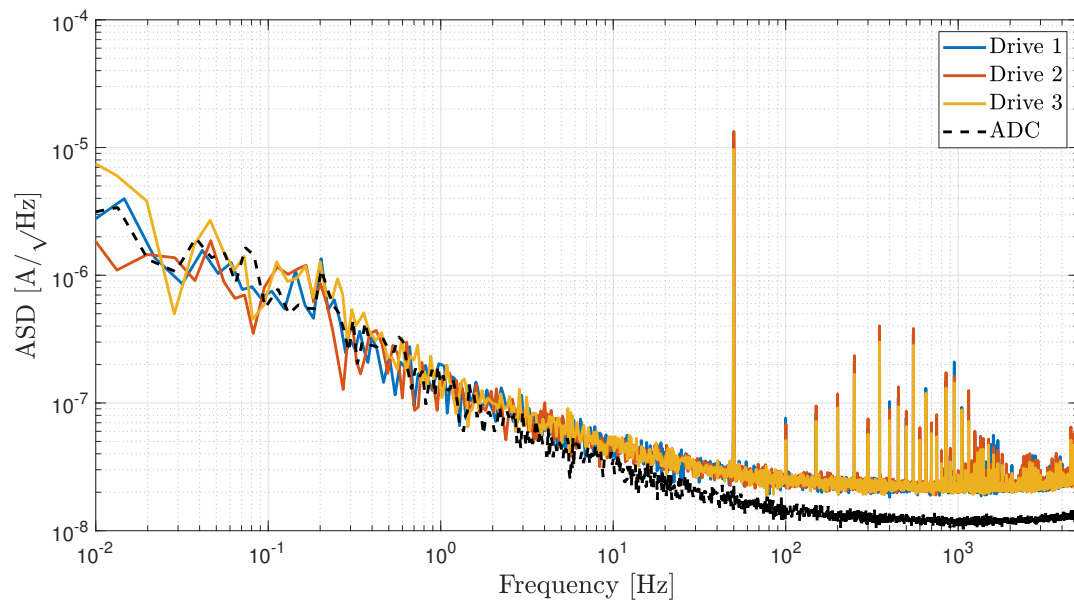


Figure C.3: Signal conditioner unit coil drivers noise spectrum.

Appendix D

Quadrupole voice-coil

Actuators have been designed to close passive (shunt) damping loops on the E-HINS/E-VINS inertial sensors (chapter 2) and μ VINS inertial accelerometer with frequency compensation (subsection 3.3.3), or close an active feedback loop on the μ VINS force-balanced accelerometer (subsection 3.3.2). The actuator device needs to be capable of applying a force on the sensor's proof mass, preferably at the closest to its centre of mass for the best value, and close to the relative motion readout for collocation. It should also be contactless so that it does not affect the inertial sensor's suspension dynamics by bringing additional stiffness.

Voice-coil actuators are great candidates for such applications because they are compact, contactless, and have a fairly large dynamic range. A voice-coil actuator primarily consists of two elements: a coil and a magnet assembly. Feeding current to the coil wiring generates a magnetic field that interacts with the magnet assembly, resulting in a contactless force as per the Lorentz law. The force strength and direction can be controlled with the current value and direction of propagation in the coil. Voice-coil actuators produce a linear force; hence, they are often referred to as linear motors, and can be guided on a linear axis or not. For the purpose of this application, the voice-coil will not be guided to minimise friction, and it will be designed with a large clearance gap (spacing between the coil inner diameter and the magnet) so that it can tolerate rotation of the proof mass to some degree. It will also be mounted in the sensor in a moving-magnet configuration: i.e. with the magnet mounted on the proof mass and the coil fixed on the static frame to avoid coil wire rubbing and friction.

However, the voice-coil moving magnets and varying magnetic field have been shown to induce Eddy-currents in the nearby conducting surfaces [218]. These in turn cause of velocity-dependent damping, which ultimately translates into thermal noise as per the fluctuation-dissipation theorem [159]. A carefully designed magnet assembly, consisting of two magnets in a quadrupole configuration, allows for a greatly reduced Eddy-current generation and also provides shielding from stray magnetic fields. Such a design has been investigated for use in gravitational-wave detectors and high-resolution inertial sensors [216–218]. The quadrupole design consists of a pair of two like-pole

facing magnets. This configuration leads to a resulting magnetic field whose intensity decays as $1/R^4$, R being the radial distance from the magnet, whereas a single traditional magnet shows a power of 3 decay. However, it comes with the trade-off of a lower net strength (force produced by unit current), due to the overall lower assembly magnetic field. Note that Eddy-current generation can also be minimised using non-conductive material for the sensor's proof mass and static frame. This section presents the optimal design of the quadrupole magnet and a brief experimental campaign validating its performance. Additional details on the quadrupole magnet design can be found in [271].

D.1 Quadrupole magnet modelling

The magnet design has been optimised to generate the required Lorentz force under minimal excitation current. The optimisation process is based on the fundamental equations of electromagnetism and the Biot-Savart law, and is extensively described in [218]. A succinct description of the methodology is presented below.

The Lorentz force from the coil acting on a magnet is

$$F_z(z) = B_r(z, R_m) \cdot I_m \cdot (2\pi R_m) \quad (\text{D.1})$$

This equation expresses the axial force $F_z(z)$ as a function of the distance between the coil and the magnet z , the coil radial magnetic field B_r , the magnet 'equivalent current-loop' I_m , and the physical properties of the magnet assembly (magnet diameter R_m).

The radial magnetic field of the coil $B_r(z, r)$ is calculated by defining the magnetic field of a single loop of the coil wire, for a given coil geometry (i.e. diameter, coil resistance), and is then integrated over the coil length L_c (height). Following the Biot-savart law, the magnetic field of one loop of wire l write

$$B(\mathbf{r}) = \int_l \frac{\mu_0 I}{4\pi |\mathbf{r} - \mathbf{R}|^3} d\mathbf{l} \times (\mathbf{r} - \mathbf{R}) \quad (\text{D.2})$$

Writing the radial component in 'solenoidal' coordinates (r, z, ϕ) leads to

$$B_r(z, r) = \frac{\mu_0 I (z - z_0) R_c}{4\pi} \int_0^{2\pi} \frac{\cos \phi}{[R_c^2 + r^2 + (z - z_0)^2 - 2rR_c \cos \phi]} d\phi \quad (\text{D.3})$$

for a loop centred at the z_0 plane.

The expression is further simplified using complete elliptic integrals of the first and second kind $K(x)$ and $E(x)$ for ease of numerical implementation, using the PYTHON `ELLIPE` and `ELLIPK` func-

tions of the SPICY.SPECIAL package:

$$B_r(z, r) = \frac{-\mu_0 I(z - z_0)}{2\pi\sqrt{C}} \left[K\left(\frac{\pi r R_c}{C}\right) - \frac{B}{A} E\left(\frac{\pi r R_c}{C}\right) \right] \quad (\text{D.4})$$

with $A = (R_c - r)^2 + (z - z_0)^2$, $B = R_c^2 + r^2 + (z - z_0)^2$ and $C = (R_c + r)^2 + (z - z_0)^2$, $K(x) = \int_0^{\pi/2} \frac{1}{\sqrt{1-x^2 \sin^2 \psi}} d\psi$ and $E(x) = \int_0^{\pi/2} \sqrt{1-x^2 \sin^2 \psi} d\psi$.

The magnetic field is then integrated over the length of the coil, leading to

$$B_r(z, r) = \frac{\mu_0 N I_c}{2\pi L_c \Delta R_c} \int_{-L_c/2}^{L_c/2} \int_{R_i}^{R_o} \frac{(z - z_c)}{\sqrt{C}} \left[K\left(\frac{\pi r R_c}{C}\right) - \frac{B}{A} E\left(\frac{\pi r R_c}{C}\right) \right] dR_c dz_c \quad (\text{D.5})$$

The second half of Equation D.1 is I_m . Here, the magnet is represented as a theoretical current loop using the equivalence between a permanent magnet with a magnetic dipole moment $\mathbf{m} = 1/\mu_0 \mathbf{B}_{\text{rem}} \mathbf{V}$ and a current loop with dipole moment $\mathbf{m} = I \mathbf{A}$. Solving for I yields the equivalent magnet current I_m

$$I_m = \frac{B_{\text{rem}} L_m}{\mu_0} \quad (\text{D.6})$$

with B_{rem} the magnet magnetic strength (remanence), L_m the magnet length and μ_0 the magnetic permeability.

Eventually, the net force is obtained by integrating the product of B_m and a unit magnet current dI_m on the full magnet length L_m .

$$F_z(z) = \frac{B_{\text{rem}}}{\mu_0} (2\pi R_m) \int_{z-L_m/2}^{z+L_m/2} B_r(z, R) dz \quad (\text{D.7})$$

This expression shows the force of the coil on 1 magnet as a function of the coil and magnet physical properties. For a quadrupole design, the force applied on each magnet are calculated individually and the total resulting force is the summation of both electromagnetic force.

These maths have been coded in PYTHON in order to determine the force constant of a 2-magnets voice-coil, based on the specifications of the coil (size, number of turns, etc), magnets (size, magnetic remanence), distance between the magnet and the centre of coil. Typically, for one given coil, the magnet sizes are first chosen to fit the coil inner diameter with acceptable clearance. One then iterate on the above calculation with various magnet material, thickness and inter-magnet distance to maximise the force/current strength of the assembly.

D.2 Voice-coil design

A quadrupole magnet assembly has been designed according to the coil characteristics. The quadrupole voice-coil actuators use off-the-shelf coils from either the moticont or H2W companies. They are the moticont LVMC-022-013-02 coil or the H2W NCC03-07-001-1V vacuum-compatible coil. Luckily, both coils have fairly similar characteristics, so that the magnet assembly has been optimised for the moticont LVMC-022-013-02 coil and then re-used without modification on the H2W NCC03-07-001-1V coil with little impact on the voice-coil constant.

The parameters of the quadrupole magnet have been optimised to maximise the actuator constant with a design that fits the coil inner diameter while allowing an acceptable radial clearance of at least 1 mm to tolerate the angular motion of the magnet. The relevant parameters are the size of the magnets, the distance of the magnet assembly to the coil centre, and the inter-magnet distance. Figure D.1 shows a schematic drawing of the custom quadrature magnet design. N45 Neodymium disc magnets are used for their large magnetic strength. The magnets are 12 mm in diameter and 3 mm thick to fit the mechanics with an acceptable clearance. A 5 mm inter-magnet distance has been optimised to allow a good far-field cancellation of the magnetic flux lines while still allowing an acceptable force per unit current to be generated by the coil. All parameters can be found in Table D.1.

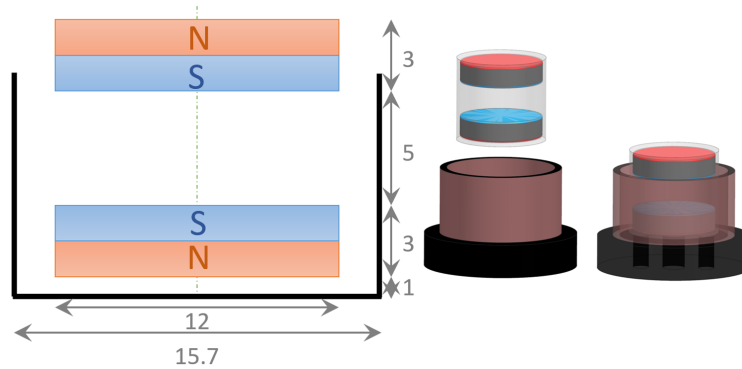


Figure D.1: (Top) Pre-amps circuit drawing. (Bottom) Coil driver circuit drawing.

Coil parameters			Magnet parameters	
Wire material		Copper	Material	N45 Neodymium
Inner diameter	[mm]	15.7	Magnet diameter	[mm] 12
Height	[mm]	9.5	Height	[mm] 3
Number of turns		165	Distance to coil centre	[mm] 2.5
Coil resistance	[Ω]	4.9	Inter-magnet distance	[mm] 5
Coil inductance @ 1 kHz	[mH]	0.5		

Table D.1: Quadrupole voice-coil actuator design parameters.

Plugging these parameters in the calculation presented above leads to a quadrupole voice-coil

actuator with a strength of approximately 0.6 N/A, where the inter-magnet distance and magnet thickness have been chosen to optimise the F/I constant for the given coil (LVMC-022-013-02) and magnets diameters, as shown in Figure D.2.

The calculations has been cross-check on FEMM, which is an open-source finite element solver for magnetic, electrostatic, heat flow and current flow problems. The simulation confirm a theoretical force constant for the quadrupole voice-coil assembly of $F/I = 0.71$ N/A, and the R^{-4} decay of the magnetic field and the far-field reduction can be seen in Figure D.3.

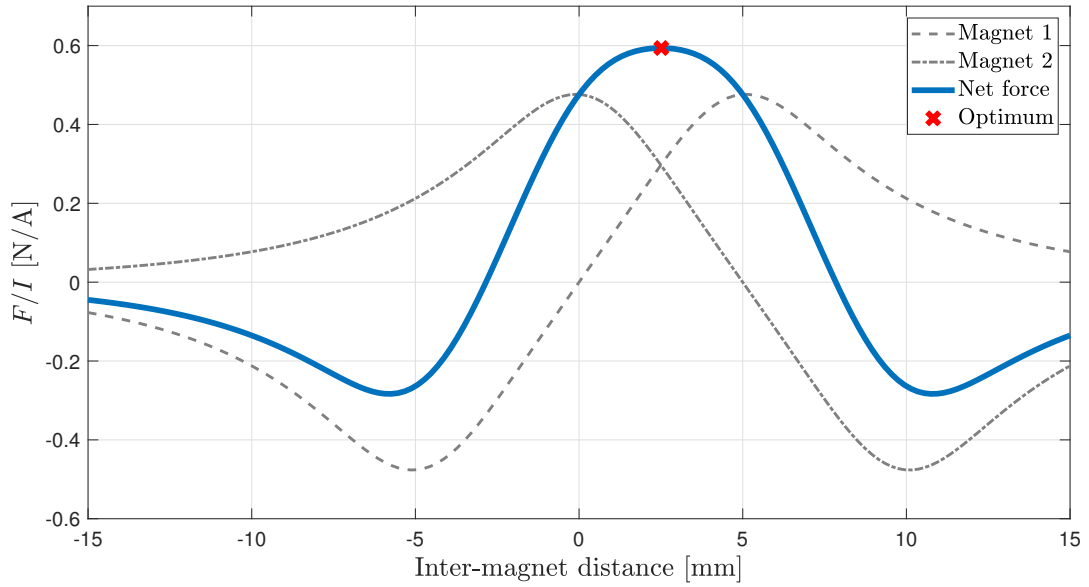


Figure D.2: Python model of the quadrupole voice-coil strength F/I in N/A as a function of the inter-magnet distance. (Dashed and solid grey) Strength of the coil to each individual magnet. (Solid blue) Strength of the quadrupole magnet assembly. (Red) Optimum distance leading to the highest strength of about 0.60 N/A

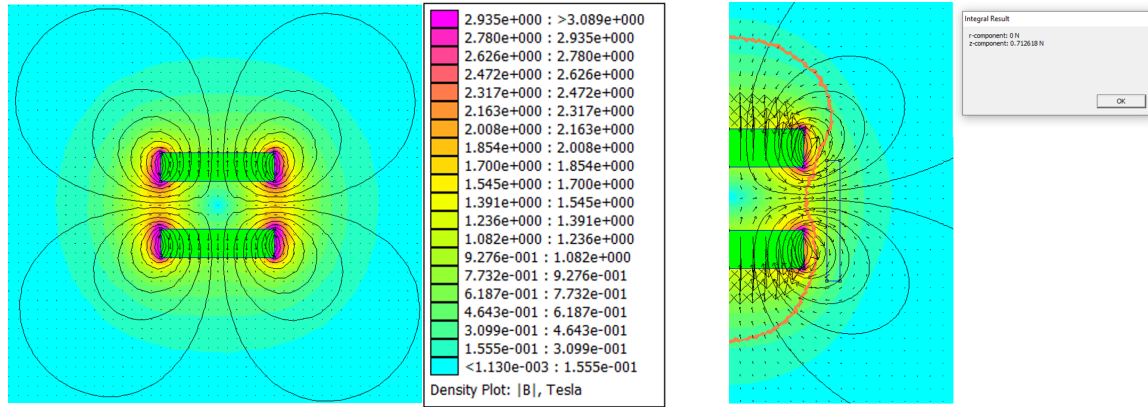


Figure D.3: Finite element modelling of the quadrupole magnet voice-coil strength in FEMM. (Left) Quadrupole magnet magnetic field. (Right) Quadrupole magnet resulting force for a 1 A current fed to the coil.

D.3 Experimental characterisation

A prototype of the quadrupole voice-coil actuator has been produced and is shown in Figure D.4 (left). The force constant has been measured by mounting the voice coil on a scale, and measuring the pulling/pushing force of the actuator when injecting constant current into the coil. The weight reading of the scale can be up-converted to a force by multiplying the mass reading by the gravitational acceleration, with a reading accuracy of 0.01 g. Results are displayed in Figure D.4 (right). The voice coil demonstrated a force constant of 0.6465 N/A, and good linearity even under input currents up to 600 mA. Assuming an RMS value for the ground motion of typically a few micrometres, it can be computed from the pendulum mechanical properties (e.g. $m = 0.4$ kg, $f_0 = 2.8$ Hz for the μ VINS sensor) and the location of the force actuator (e.g. 35 mm lever-arm distance from the joint for the μ VINS sensor) that a peak force of 5 mN would be required to balance the pendulum motion under normal seismic conditions. This leads to an average drive current of 8.5 mA.

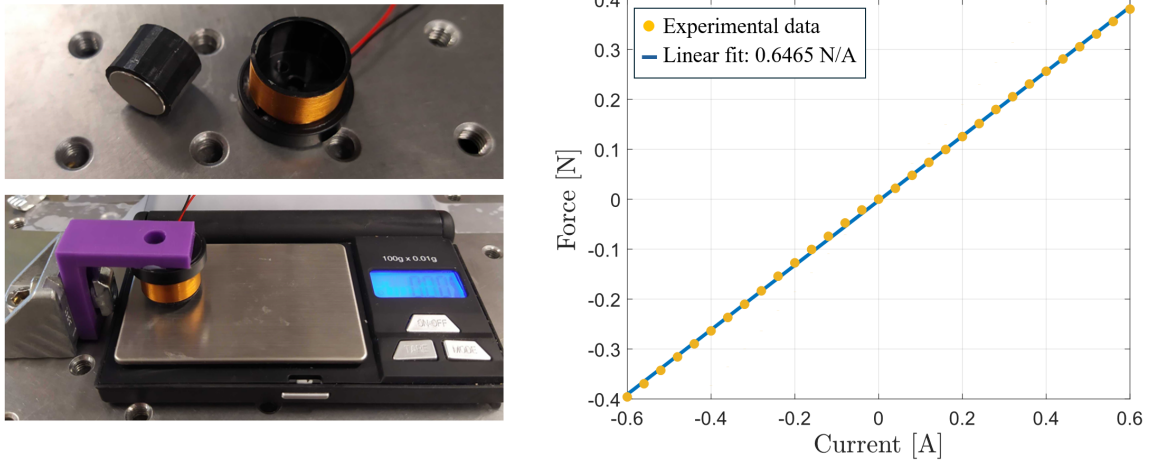


Figure D.4: (Left) Prototype of the quadrupole voice-coil actuator and experimental testbed. (Right) Experimental measurement of the quadrupole voice-coil strength F/I . (Yellow dots) Measurement data. (Solid blue) Linear fit estimating F/I as 0.6465 N/A.

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