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MODELING AND OPTIMIZATION
OF RENEWABLE ENERGY
COMMUNITIES
A LONG-TERM TO REAL-TIME APPROACH

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ENGLISH ABSTRACT

This thesis studies the internal mechanisms of renewable energy communities and proposes a series of formal approaches to support the planning, operation, and billing of these structures. Thanks to the growing penetration of residential photovoltaic production, electrical storage systems, and controllable loads, communities of prosumers can improve the synchronicity between production and consumption and pool flexibility services. However, this coordination raises technical, economic, and regulatory questions whose development requires precise operational models.

This work adopts a sequential approach: it first formalizes a general optimization framework for the distribution of flows and costs, describes a sizing method for renewable resources (PV, batteries) while integrating network constraints, then models residential demand profiles, exploits the flexibility potential of new controllable loads (Electric Vehicles, Water Boilers, Heat Pumps), and addresses real-time voltage control.

Case studies representing residential neighborhoods illustrate the methods and quantify the potential gains of the community according to the degrees of freedom allowed and the decision variables considered. This thesis concludes on the issues of equity in benefit sharing, compares allocation methods and their theoretical bases, and discusses the limits of the models while suggesting directions for extension of this work: coupling with network planning, integration of socio-behavioral aspects, and experimental validation.

FRENCH ABSTRACT

Ce manuscrit étudie les mécanismes internes aux communautés d'énergie renouvelable et propose une série d'approches formelles pour soutenir la planification, l'exploitation et la facturation de ces structures. Face à la pénétration croissante de la production photovoltaïque résidentielle, des systèmes de stockage électriques et des charges pilotables, les communautés de prosommateurs peuvent améliorer la corrélation entre production et consommation et mutualiser des services flexibles. Cependant, cette coordination soulève des questions techniques, économiques et réglementaires dont l'élaboration nécessite des modèles opérationnels précis.

Ce travail adopte une approche séquentielle : il formalise un cadre d'optimisation général pour la répartition des flux et des coûts, il propose un modèle de dimensionnement des ressources renouvelables (PV, batteries) en intégrant les contraintes réseau, il modélise ensuite des profils de demande résidentielles, utilise le potentiel de flexibilité des nouvelles charges contrôlables (Voitures Électriques, Chauffe-Eau, Pompes à Chaleur), il s'intéresse ensuite au contrôle temps-réel de la tension du réseau.

Des études de cas représentant des quartiers résidentiels illustrent les méthodes et quantifient les gains potentiels de la communauté selon les degrés de liberté permis et les variables de décisions considérées. Cette thèse conclut sur les enjeux d'équité dans le partage des bénéfices, compare des méthodes d'allocations et leurs bases théoriques, et discute les limites des modèles en suggérant des pistes d'amélioration : le couplage avec la planification réseau, l'intégration d'aspects socio-comportementaux et une validation expérimentale.

INTRODUCTION

OUTLINE

This manuscript is structured into four parts, each addressing a specific aspect of this research on renewable energy communities and their internal market structures. The chapters are designed to build upon each other, leading to a comprehensive overview of the topic.

The first part, **Background**, sets the stage by providing the necessary context and background information on the energy transition and renewable energy communities. The second part, **Planning**, outlines the planning phase of the research, including the sizing of distributed energy resources and the understanding of residential flexible load profiles. The third part, **Reaction**, focuses on the real-time aspects of the community operation, the real-time voltage control of the network. Finally, the fourth part, **Conclusion**, addresses the settlement processes within the internal market structure, including fairness considerations, and summarizes the findings of the research.

PART I Background: *Context and motivation for the research.*

- Chapter 1 provides a general definition for renewable energy communities, overviews some regulations and implementations, then motivates their existence as a partial solution to the energy transition.
- Chapter 2 defines the idealized framework that will be studied in the following parts, as well as a base mathematical model for Renewable Energy Community (REC) optimization.

PART II Planning: *Optimal decision a priori.*

- Chapter 3 discusses the optimal sizing of distributed energy resources in renewable energy communities.
- Chapter 4 presents a synthetic residential load profile generator that include a flexibility component.
- Chapter 5 proposes a method for harnessing implicit flexibility of residential loads.

PART III Reaction: *Decision changes based on the current state of the system.*

- Chapter 6 proposes a distributed real-time voltage control algorithm to ensure safe operation of the grid.

PART IV Conclusion: *Fairness considerations and concluding remarks.*

- Chapter 7 focuses on settlement processes and discusses fairness considerations.
- Chapter 8 summarizes the findings and implications of the research.

PUBLICATIONS

Most of the contributions presented in this manuscript are adaptations of my personal and collaborative peer-reviewed publications as a (co-)first author.

- Chapter 3:
 - *Sizing distributed energy resources in a renewable energy community with a grid-aware internal market structure*, T. Stegen, M. Glavic, S. Paoletti, A. Giannitrapani and B. Cornélusse. XIII Power Systems Computation Conference (PSCC), 2024 [Stegen et al., 2024b].
 - *Impact of distributed energy resources investment capacities on the carbon footprint of renewable energy communities*, T. Stegen, M. Glavic and B. Cornélusse. 14th Mediterranean Conference on Power Generation Transmission, Distribution and Energy Conversion (MEDPOWER), 2024 [Stegen et al., 2024a].
- Chapter 4:
 - *ResFlex: a residential load profile generator to model individual demand response in distribution grids*, T. Stegen, J. Castiaux, N. Diffels, M. Duchesne and B. Cornélusse. 28th Conference and Exhibition on Electricity Distribution (CIRED), 2025 [Stegen et al., 30 June 2025].
- Chapter 5:
 - *Quantifying, Activating and Rewarding Flexibility in Renewable Energy Communities*, J. Allard, T. Stegen, M. Glavic, F. Vallée, Z. De Grève and B. Cornélusse. IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe), 2024 [Allard et al., 2024].
 - *Explicit Reward Mechanisms for Local Flexibility in Renewable Energy Communities*, T. Stegen, J. Allard, N. Diffels, M. Glavic, F. Vallée, Z. De Grève and B. Cornélusse. XIV Power Systems Computation Conference (PSCC), 2026 [Stegen et al., 2026].
- Chapter 6:
 - *Fully distributed real-time voltage control in active distribution networks with large penetration of solar inverters*, A. Colot, T. Stegen, B. Cornélusse. IEEE Powertech, 2023 [Colot et al., 2023].

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GLOSSARY

ADMM	Alternating Direction Method of Multipliers.
BSS	Battery Storage System.
CEC	Citizen Energy Community.
CO	Community Operator.
DER	Distributed Energy Resource.
DSO	Distribution System Operator.
EC	Energy Community.
EV	Electric Vehicle.
HEMS	Home Energy Management System.
HP	Heat Pump.
HVAC	Heating, Ventilation and Air Conditioning.
ICA	International Cooperative Alliance.
KoR	Key of Repartition.
LV	Low-Voltage.
MILP	Mixed-Integer Linear Program.
MISOCP	Mixed-Integer Second-Order Cone Programming.
OPF	Optimal Power Flow.
PCC	Point of Common Coupling.
PV	Photovoltaic.

REC	Renewable Energy Community.
SDGs	Sustainable Development Goals.
SoC	State of Charge.
SOCP	Second Order Cone Programming.
WB	Water Boiler.
WG	White Goods.

Part I

BACKGROUND

Context and motivation for the research

CONTEXT

Outline

This chapter defines the concept of Energy Communities (ECs), describes their goals and motivates the way they can help achieving them.

Energy Communities have been proposed as a promising response to some challenges of the energy transition. They act as a bottom-up solution that aims at solving problems arising from the increased decentralization of energy systems.

They consist of a group of actors willing to cooperate for improving their energy use. This group can take multiple forms and be studied from many angles. In this manuscript, the chosen perspective is electrical engineering, and the renewable energy considered is limited to electricity. Thus, Energy Communities (ECs) will be considered only for the electricity vector.

The first goal of the ECs is direct energy sharing through an existing network. However, they can share other objectives such as co-investments, collective storage of energy, and general cooperation around the energy subject. This chapter will provide an explicit list of the potential impacts of such initiatives.

The legal framework defines two types of ECs, Renewable Energy Communities (RECs) and Citizen Energy Communities (CECs). This work will focus on local and renewable ECs.

The complete study angle of this thesis will be further described in Chapter 2 with multiple additional applications and challenges that energy communities can face.

1.1 DEFINITION OF AN ENERGY COMMUNITY

According to the European Commission [European Commission, 2018], energy communities are:

Definition 1.1. *Citizen-driven energy actions that contribute to the clean energy transition, advancing energy efficiency within local communities. They can contribu-*

te to increasing public acceptance of renewable energy projects and make it easier to attract private investments in the clean energy transition. Energy communities can be an effective means of re-structuring our energy systems, by empowering citizens to drive the energy transition locally and directly benefit from better energy efficiency, lower bills, reduced energy poverty and more local green job opportunities.

This first definition comes from the European Commission's *Clean Energy for All Europeans* package, which was adopted in 2019 and introduced the concept of energy communities into EU legislation. This directive provided both an obligation and a common base for member countries to establish a local legal framework around Renewable Energy Communities (RECs) and Citizen Energy Communities (CECs). It provides some legal constraints for the paradigm of energy communities.

Member states would then have to implement it in their own laws. The individual definitions and legal frameworks can differ, but they still need to comply with the initial European decree.

REScoop, the European federation of renewable energy cooperatives provides a more restrictive definition of energy cooperatives:

Definition 1.2. *An energy community is a way to "organise" citizens that want to cooperate together in an energy-sector related activity based on open and democratic participation and governance, so that the activity can provide services or other benefits to the members or the local community. In this sense, energy communities represent an alternative type of market actor, and a different way/philosophy to do business, which is now acknowledged by the CEP (European Commission). The primary purpose of energy communities is to create social innovation - to engage in an economic activity with non-commercial aims. REScoop.eu considers Renewable Energy Source Cooperatives (REScoops) to be a type of energy community that adheres specifically to criteria based on the seven International Cooperative Alliance principles.*

The second definition emanates from a non-profit organization whose goal is to link local energy cooperatives at European level. It provides a more down-to-earth definition with a glimpse of how this law could and should be implemented in practice. This also includes an idealized social objective for energy communities.

REScoop provides a set of constraints for energy communities to be part of their network as an energy cooperative. The set of rules is based on the 7 principles of the International Cooperative Alliance (ICA) [[Alliance, 2015](#)]:

- Voluntary and Open Membership
- Democratic Member Control
- Economic Participation through Direct Ownership
- Autonomy and Independence

- Education, Training and Information
- Cooperation among Cooperatives
- Concern for Community

According to REScoop, this is necessary for a democratic bottom-up transition using energy communities.

REScoop also provides a synthesis of the European legislation, comparing REC and CEC [REScoop.eu, 2019]:

Table 1.1: Legal Definitions of Energy Communities in EU Directives

Renewable Energy Community <i>Article 2(16) Recast Renewable Energy Directive</i>	Citizen Energy Community <i>Article 2(11) Recast Electricity Directive</i>
A legal entity: (a) which, in accordance with the applicable national law, is based on open and voluntary participation, is autonomous, and is effectively controlled by shareholders or members that are located in the proximity of the renewable energy projects that are owned and developed by that legal entity; (b) the shareholders or members of which are natural persons, SMEs or local authorities, including municipalities; (c) the primary purpose of which is to provide environmental, economic or social community benefits for its shareholders or members or for the local areas where it operates, rather than financial profits.	A legal entity that: (a) is based on voluntary and open participation and is effectively controlled by members or shareholders that are natural persons, local authorities, including municipalities, or small enterprises; (b) has for its primary purpose to provide environmental, economic or social community benefits to its members or shareholders or to the local areas where it operates rather than to generate financial profits; and (c) may engage in generation, including from renewable sources, distribution, supply, consumption, aggregation, energy storage, energy efficiency services or charging services for electric vehicles or provide other energy services to its members or shareholders;

The bolded elements showcase the main differences between the two definitions. The REC focuses on the proximity of the members and the renewable sources of energy. RECs are also more restrictive in terms of membership allowances.

Some other restrictions, while not part of this definition have been stated in recitals, and should apply:

- RECs are entitled to produce, consume, store and sell renewable energy.
- CECs could open to all categories of entities, allowing entry for medium and large enterprises.
- The companies constituting an EC should not be engaged in large-scale commercial activity primarily in the energy sector.

1.2 MOTIVATION

Modern electricity grids were first designed for centralized power production units, usually connected to large-scale transmission systems. The residential power consumers are connected to lower voltage distribution grids.

With decentralized power generation, some of the low-voltage consumers also became part-time producers, thus becoming a bidirectional point in the network (prosumers). This was not considered in the design of the radial distribution grids. A bidirectional flow of power in those can cause deleterious effects, such as over-voltages. Retailers are usually not interested in energy generated by uncertain and intermittent Distributed Energy Resources (DERs). In case of rooftop Photovoltaic (PV) systems, the profiles have important synchronicity between neighboring production units.

The current trend towards electrification of usage also increases the loading of the low-voltage networks. New electrical loads, such as Electric Vehicles (EVs) and Heating, Ventilation and Air Conditioning (HVAC), also have much higher nominal power than conventional loads. The power consumption of an EV charger can go above 10 kW and around 5 kW of electrical power can be consumed by a Heat Pump (HP). In Wallonia, one Distribution System Operator (DSO) estimated its synchronous peak to be 1.1 kVA per household connected to its network [RESA, 2025]. This is computed using their total annual peak power consumption without professional connections divided by the number of residential connection points. This number is lowered by the low synchronicity of traditional loads.

The average individual peak of a household is around 2.5 to 3 kVA and this value was used for network sizing [RESA, 2023]. Due to their high energy demand, the new loads will span over a longer timeframe, increasing their effect on the synchronicity of the network peaks.

There is a constant balance between consumption and production of electrical energy in the power grid at a system level (*i.e.*, European Network). The EC framework pushes towards a local energy balance to decrease the need for larger power transmission and distribution infrastructures.

Without control, the synchronicity between local generation and consumption is not ensured. Individually, part of the energy produced is absorbed, but as a group, with different profiles and capabilities, the potential for direct consumption can be

improved. Also, co-investment and better use of diverse resources and flexibility sources can be attained with coordinated control.

ECs have the potential to provide environmental, economic, social, and technical benefits. The following section reviews these benefits individually and then discuss how they contribute to a broader societal impact.

1.2.1 Ecological

The main goal of ECs is to boost the energy transition, going from carbon-intensive centralized power plants to greener energy production units that can be decentralized. Net zero emissions by 2050 will go through massive electrification of usage. This will require significant investments in both energy resources and infrastructure.

Through access to cheaper and more adapted renewable sources; a community could achieve faster transition to low-carbon technologies. At a low-voltage level, an increase in offsetting local energy fluxes will decrease the need for network reinforcement, thus, diminishing the impact of electrification. At a system scale, broad acceptance of EC will also reduce the need for power transmission and centralized power plants. For both investments and infrastructure, if the resources used are optimized, their negative impact is minimized.

More consumer responsibility for energy use will bring an increased accountability for the impact of consumption. The consumer-centric approach of energy communities could help people involved understand the direct impact of their behaviors.

1.2.2 Economic

The economic impact of EC can be described at different levels and from different perspectives:

- the consumers may gain access to cheaper electricity through the sharing,
- the producers could generate additional revenues, by selling their excess.
- with a longer-term ambition, the community could also provide a boost for joint DER investment, with reduced initial cost, economies of scale and additional profitability of the installation.

In addition to this, cooperatives could directly bargain with retailers to have more advantageous group tariff by making all community members sign up for a bulk purchase agreement. By taking part in the community, the power plant's profitability can be increased and ensured through long-term contracts. On the other hand, buyers can obtain fixed prices, reducing their risks of price volatility.

The community framework is also intended to provide access to new markets and service provision opportunities. Indeed, the community will create a means for aggregation (even through multi-community cooperation) that will result in more capacity for grid or other services. This could result in value stacking for the community members, first through energy arbitrage, then through explicit flexibility, balancing, or grid congestion management, among others.

1.2.3 Social

Although it is not its main objective, the EC can have a social role through multiple aspects:

- Socio-economic: as a vector of change, the EC could devote part of its benefits to reduce energy poverty for its members in need.
- Socio-educational: by promoting energy efficiency and sufficiency, encouraging self-consumption and self-sufficiency, promoting other good practices regarding energy use, and broadening this scope outside the community to reach new members.
- Socio-relational: the community aspect of the structure allows for people to exchange, whether it is energy or others, it fosters social ties. The EC is a platform where the exact rules and organization are left for the members to decide.

REScoop provides a "community of communities" platform for such initiatives. They are called energy cooperatives rather than energy communities. The energy cooperative, although more restrictive than energy community, is compatible in many overlapping parts of activities. As a voluntary and proactive entity that is often embedded in a local context, the REC can have good leverage on impactful social initiatives.

1.2.4 Technical

In addition to the benefits previously mentioned, an energy community can pose some technical challenges and bring opportunities. Technical progress could be achieved through the acceptance of REC, with direct impact on the prosumers.

When a community is created, the first focus is the internal billing. However, this billing, without any change in energy use will only provide an economic advantage to some users, if allowed by the community rules. Without adequate incentivization, members will not act on their energy use, which is necessary to create a shift. It can be done gradually through education and a change of habits, but technology can provide automatic solutions to optimize this shift.

By using optimization and forecasting, energy consumption can be matched with the local production to maximize the community impact.

1.2.4.1 Home energy management systems

A Home Energy Management System (HEMS) is a link coordinating and controlling "smart" appliances in a household. It can be composed of production units (PV systems or others), flexible consumption appliances, and Battery Storage Systems (BSSs). By learning and planning the daily production and consumption profiles, the HEMS can adapt the control to increase the overlap of the profiles. This can be achieved by using a BSS as a first buffer. The remaining energy can be exchanged with the retailer to the grid that acts as an energy slack. The usual goal of HEMS is to minimize cost through increased self-consumption and self-sufficiency.

Self-Consumption rate

$$SC = \frac{\text{Energy consumed from production}}{\text{Total energy produced}} = 1 - \frac{\text{Energy export}}{\text{Energy produced}} \quad (1.1)$$

Self-Sufficiency rate

$$SS = \frac{\text{Energy consumed from production}}{\text{Total energy consumed}} = 1 - \frac{\text{Energy import}}{\text{Energy consumed}} \quad (1.2)$$

An energy community can act as a second buffer to further improve the temporal correlation between consumption and production. This can be viewed as improving collective self-sufficiency and collective self-consumption.

As long as there is an advantage for both the producer and the consumer to exchange energy inside the community, the HEMS should promote profile complementarity.

1.2.4.2 Coordinated control

While the HEMS are currently designed for local optimization, they could be extended for coordinated control among the community. This can be seen as a secondary objective of the control strategy. The individual use is first optimized, and then collective usage can be further improved.

Modeling and controlling the assets can be done through coordinated control. With a clear view on the objectives and payoffs for both local profile concordance and collective participation, the HEMS could decide on the optimal policy for each user.

There are three main paradigms for coordinated control, as illustrated by fig. 1.1:

- An omniscient centralized controller has access to all information to take the best collective control decision. Practically, this optimal solution would face

issues for public acceptance. Outsourcing decisions on critical and personal appliances at home is not something that everybody is willing to do without complete faith in the system.

- A distributed controller allows for local exchanges without the need of a coordinator. Members only share individual information locally to achieve a global goal. This allows for a partially-observed state of the system to be shared to reach eventually the global optimum.
- A decentralized approach, which centers around individual decisions:
 - Fully decentralized: individuals control their outputs with only local information.
 - Decentralized with central broadcast: individuals control their outputs with local information and additional non-personal info from a central coordinator.

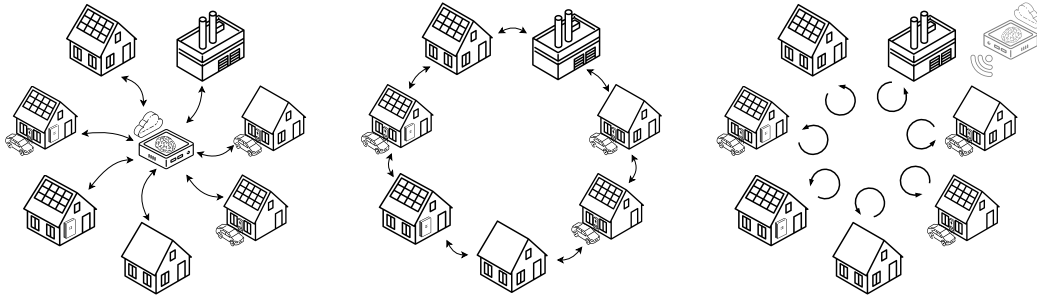


Figure 1.1: Control architecture paradigms: centralized, distributed, and decentralized approaches.

Coordinated control does not always mean perfect cooperation. Even if ECs have been presented as groups of benevolent individuals, it could happen that some are seeking to manipulate the outcome for additional revenues.

Although the theoretical goal of an EC is to cooperate towards an optimum, there could be room for adversarial behaviors to maximize individual benefits. When designing the retribution scheme, it should be ensured that, while the incentivization is fair, it encourages cooperation. Game theory can help model these incentives and ensure cooperation is maintained through careful design of reward mechanisms that discourage self-serving behaviors [Sadoine, 2025].

1.2.4.3 Grid

The energy transition is aiming towards a massive electrification of the loads, which will probably lead to massive reinforcement of existing grids. On the other hand, some networks are already facing some overvoltage issues due to the rise of DER [Smith et al., 2011]. The peak power production on distribution grid is causing

reverse power flow in weaker grids that have been initially designed for unilateral flows of consumption. This leads to overvoltages and ultimately, disconnection of power production units, resulting in a loss of production.

A first solution to adapt the usage of new loads is through the use of redesigned energy tariffs. The Walloon regulator published a report motivating updated grid tariffs for 2026 [CWaPE, 2024]. These tariffs are designed to encourage consumption at peak production time. However, with the arrival of high power demanding loads, a poorly designed tariff could lead to even higher consumption peaks, thus undervoltages, as shown in table 1.3. Indeed, the new loads will result in an important growth in the peak demand, creating the need for distribution grid expansion and redesign. This comes at a very high societal cost, especially due to the magnitude of the changes:

Table 1.3: Annual expected synchronous peak for different loads [CWaPE, 2024].

Scenario	Peak (kW)	Cumulative Peak (kW)
Base	0.60	0.60
WB	0.29	0.89
HP	2.20	3.09
EV a	7.36	10.45
EV b	3.70	6.79
EV c	2.20	5.29

EV a: Fast charger, EV b: Semi-fast charger and EV c: Slow charger

The expected average synchronous peak for 2050 is 6.6 kW per point of connection [VREG, 2023]. Considering that the network is currently designed for a 2.5 kW peak, the reinforcement would at least need to double all existing infrastructure. This increasing power burden on the grid could be compensated by flexibility. Without modification on the total energy consumption, a smoother power profile would have less impact on grid congestion. This could be achieved through:

- Variable grid and/or energy tariffs.
 - Peak tariff with a cost for the peak over a defined period. The period over which this is billed is crucial, as a too long interval would have a lesser impact on non-critical days, while a too short one would reduce the cost for the highly critical periods.
 - Time of Use with different periods depending on the potential congestion, as implemented by CWaPe in 2026 [CWaPE, 2024]. Unfortunately, the constant nature of this measure does not account for time- or space-dependant congestion, it only aims for average effects .
- Local energy exchanges at a discounted rate would truly reflect the current state of the local distribution network congestion. Local exchanges cancel external fluxes, thus reduce the need for transport infrastructure.

- Optimized fluxes through BSS usage, shifting a part of the production to when it is needed, first for individual self-consumption (1.1) or for tariff optimization (possibly through EC tariffs and collective self-consumption).

The REC can have a positive impact on the electrical grid by favoring local exchanges [Weckesser et al., 2021]. Moreover, RECs can provide good incentives and other social motivations for local consumption and the ability to invest in collectively owned and adapted DERs. An adequate local usage of existing resources or a collective redesign of the local production and storage assets will minimize the need for power supply from the external grid by maximizing self-consumed energy. This is an additional potential that could be unlocked and handled by the Community Operator (CO), while synergizing with the explicit flexibility activation in order to provide external grid services.

1.2.4.4 *Market structure*

A consumer-centric (pseudo-)market structure is needed for ECs. This market will have to be tailored to the community needs. In practice, the simplest pseudo-market implementation is a constant price and a constant retribution for internal exchange of energy. However, this approach, while being easy to implement and to be understood by users, does not reflect the true value for energy.

An intra-community market through shadow prices or regular bidding is a solution to have the economically optimal value for energy trading in the grid.

In addition to the local energy market, the energy community could be a driver for the emergence of local flexibility markets. The increased volumes obtained through communities will also open access to external explicit flexibility markets or other ancillary grid services.

1.2.5 **Societal**

The four previously mentioned parts all have a societal impact. Through the development of energy communities, or through the development of the underlying technologies, part of the energy transition can be eased. Sometimes, trade-offs are to be made, but many of these societal aspects synergize as Figure 1.3 shows.

1.2.5.1 *Sustainable development goals*

The Sustainable Development Goals (SDGs), introduced by the United Nations and Development [2015], constitute a broader and more standardized framework to assess the societal impact of a project. These consist of 17 objectives that have been defined as general directions for a fair and sustainable future. They are summarized in Figure 1.2.



Figure 1.2: The 17 SDGs defined by the United Nations and Development [2015].

On the topic of energy communities, the straightforward goal that can be identified is SDGs **7**: affordable and clean energy. Indeed, the exact goal of the energy community legislation is to boost energy transition to a cleaner mix at a lower cost. Some of the direct subsequent impacts of the rise of EC would be:

- 8**: Decent work and economic growth, through the development of the necessary technologies and knowledge.
- 9**: Industry innovation and infrastructure, through the investment in renewable energy resources.
- 11**: Sustainable cities and communities, through the creation of such communities and their implementation in the urban fabric and broader ecosystem.
- 12**: Responsible consumption and production: through the incentivization of the consumption of locally produced energy.

Additionally, [Otamendi-Irizar et al. \[2022\]](#) have identified all the positive impacts of existing ECs which have a social dimension and linked them with the SDGs. Their results are summarized in Table 1.4.

Table 1.4: Alignment between SDGs and actions promoted by LECs that activate local sustainable development [Otamendi-Irizar et al., 2022].

Initiatives promoted by LECs	1 No poverty	2 Zero hunger	3 Good health and well being	4 Quality education	5 Gender equality	6 Clean water and sanitation	7 Affordable and clean energy	8 Decent work and economic growth	9 Industry, innovation and infrastructure	10 Reduced inequalities	11 Sustainable cities and communities	12 Responsible consumption and production	13 Climate action	14 Life below water	15 Life on land	16 Peace, justice and strong institutions	17 Partnership for the goals
Social contribution	X												X				X
Research			X					X			X	X					
Job creation								X			X	X	X				X
Improvement of public space											X		X				
Public awareness/Citizen involvement	X										X		X				
Agri-food				X								X	X				
Promotion of cultural heritage				X													
Educational programs				X													
Environment conservation	X											X				X	
Electric mobility							X						X				
Other local development initiatives																	
Promotion of energy saving actions												X	X				

1.2.5.2 *Regarding human interactions*

The energy community framework implies human interactions. First, from CO to member to achieve acceptance, then between members to ensure stability in operation.

In the design of an EC, many questions can arise:

- Is energy efficiency more important than fairness?
- How much do the economic gains compete with a loss in comfort?
- How much time and energy are people willing to invest for running this community?

Many of these questions, despite having a high importance, will not be addressed in this thesis. This work lies in the field of engineering sciences and the question of how to make it happen from a human perspective will have to be tackled through a sociologist's point of view.

Nevertheless, throughout this work, the goal always has been to find a simple solution to a complex problem. Something, that could still be intuitively explained to ensure possible acceptance by non-expert peers.

1.3 OBJECTIVE OF THIS THESIS

In the context of this thesis, it was decided to step aside from current legislation. This decision was made to focus on the benefits that ECs could bring in an unconstrained implementation instead of trying to optimize their operation in a specific legal framework. The goal here is to consider ECs from a technical point of view, focusing on what they could bring to the energy transition, rather than what they currently provide. Specifically, RECs will be considered throughout this thesis. Their local and renewable aspects made this decision more relevant in the context of energy transition and to emphasize on their direct impact on the grid. In the next chapters, EC will be used as a generic term for the framework whereas REC will refer to a community constrained in energy sources and location. The details of these assumptions will be listed in Section 2.2.

Moreover, as presented, the ECs will impact the energy transition by changing their participants' behavior. A simple rewarding scheme for self-consumption with a business-as-usual approach may be economically interesting but this is not enough. The ECs must either help with the change of production means for its members or the members should change their consumption behavior to match the production profile.

To do that, a generic framework will be proposed. It models fully autonomous communities, which can decide completely freely and at all times individually:

- The retribution that they give to the producers for the produced energy;
- The cost of energy for the consumers;
- The allocation of community and retailer virtual energy exchanges;
- The investment in DERs within the community;
- The operational strategies for energy management within the community.

This should be done while ensuring that each member's behavior is correctly modeled and that it aligns with its personal optimal strategy. Additionally, considerations for the fairness, equity, and clarity of the retribution mechanisms will be taken into account.

This thesis will try to develop a model for each of these changes in behavior and then optimize them, at every step of the decision-making process, from the long-term to real-time. Figure 1.3 displays some of the positive impacts of ECs and shows their underlying themes and the interactions between them.

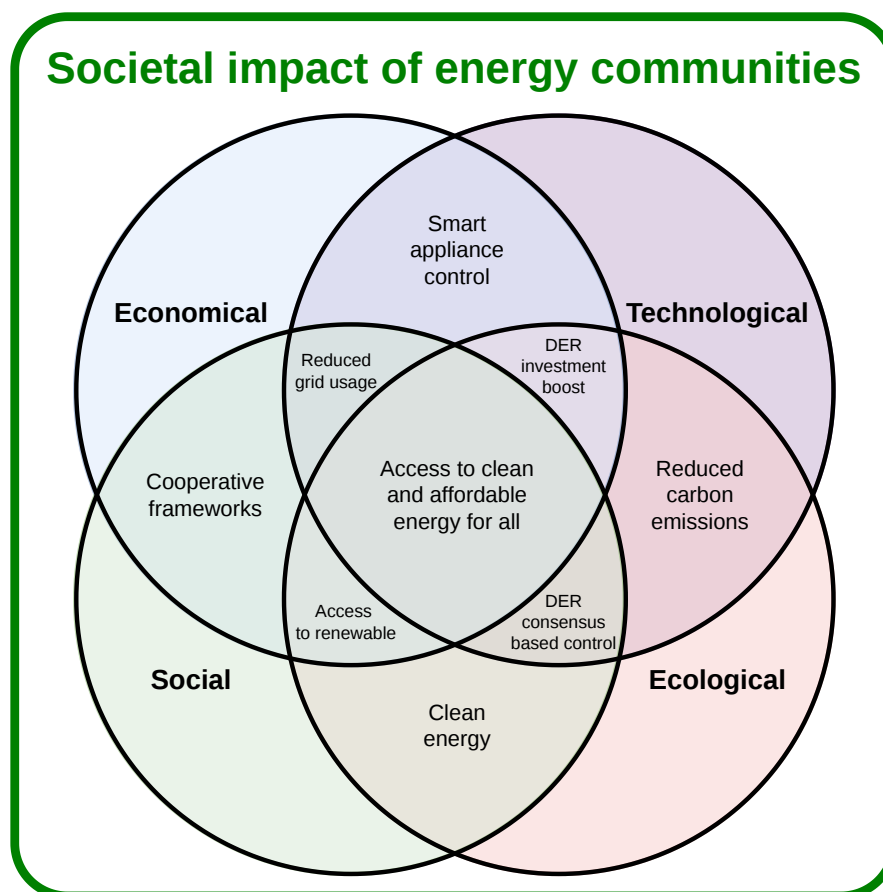


Figure 1.3: Key goals of RECs and motivations addressed in this thesis.

GENERAL PROBLEM

Outline

This chapter presents the general assumptions about the community organization, scope, and the general formulation of the mathematical model that will be particularized for the specific needs of the next chapters. It also provides an overview of state-of-the-art references on the topic.

In this work, the community and all of its members will be presented as mathematical models. These entities will all seek a goal that will be reached through mathematical optimization. In general, we will model the objective as the energy cost for each user or for the complete community and minimize it.

However, before modeling both the interests of the community and of its individual users, it may be useful to take a step back. This chapter will thus present the general problem that will be tackled through this work as a combination of simpler problems. Indeed, the multiple objectives and constraints can be separated by subject, namely:

- Shared billing: to individualize the quantities exchanged in the community;
- Community market: to determine optimal price for the commodity;
- Operational planning: to schedule the controllable appliances;
- Optimal power flow: to ensure adequate operation of the distribution grid;
- Sizing: to determine optimal investment sizes for new DERs.

2.1 CONSTITUTING PARTS OF THE MODEL

2.1.1 Bill sharing

The first task of the community is to share energy between its members. As previously stated, this will create a modified bill partly from the retailer and partly for the CO. All modifications in this bill come from the counterbalancing of the member

profiles (*i.e.* some producing while others are consuming). The producing members will sell their excess energy to the consumers. Usually, this is done using the individual power injection metering timeseries (P^{inj}). In case of negative injection, the user will consume a quantity of power (import i), a positive value will represent a production (export e). The sum of all individual values determines the total community injection ($P_{\text{com}}^{\text{inj}}$).

At each timestep (t), three cases can occur at the community level:

$P_{\text{COM},t}^{\text{INJ}} < 0$: Net community consumption, the community is importing. Therefore, if some members are producing, they can sell 100 % of their energy to the current consumers through the community. The consumers' import will not all be covered by the other members' export. They need to all get an individual share of their consumption that will be bought through the community and the rest from the external grid (*i.e.* retailer).

$P_{\text{COM},t}^{\text{INJ}} > 0$: Net community production, the community is exporting. Therefore, if some members are consuming, they can buy 100 % of their energy from the current producers. The producers' export will not all be covered by the other members' import. They need to all get an individual share of production to be sold through the community and the remaining go to the external grid (*i.e.* retailer).

$P_{\text{COM},t}^{\text{INJ}} = 0$: The community is perfectly balanced and members do not interact with the external network at this moment. All energy produced by the members can directly be sold to the consumers through the community.

One of the goals of energy communities could be to achieve internal balance as often as possible. This would maximise the autonomy of the energy community. However, to have full independance this needs to be achieved continuously, making the community work as an isolated microgrid. Aiming for this complete autarchy at all time is not specifically the most profitable, as the grid can provide a low-cost back-up solution. Handling the power peaks and unforeseen events can come at great marginal costs compared to what using the external network as a slack would cost.

In case of imbalance, a rule for sharing the power volumes needs to be chosen. This can be done either *ex ante* with a fixed key of repartition or *ex post* with another method which depends on members' action and system state. Here are some examples of repartition keys:

- Equal: All members have the same portion of the available power.
- Fixed: Each participant has a specific portion of the available power.
- Proportional: Each member is assigned power proportionally to its current consumption or production depending on the current collective power lack or surplus.
- Combination of previous: Some of the previous keys do not always give a complete sharing in one computation. Sometimes, energy can be left "unassigned".

It is then possible to reapply the same key or a different one. The quantity left-over is the new one to be shared, and members for whom the maximum has been reached are now excluded from the sharing. This can also be combined with another repartition below, which will share the remaining quantities between the remaining members

This list is inspired from the Walloon regulation introduced by CWaPe [CWAPE, 2024], and represents the keys currently available in Wallonia. Some more advanced allocation coefficient mechanisms are proposed in the literature:

- Equal among active members: The equal share of power is only allocated among members who are currently needing a share.
- Cascade: Equivalent to allocating an equal share multiple times, until all left-over power has been assigned.
- Shapley: Using game theory, each member gets assigned a share of the benefits proportional to its Shapley value [Shapley, 1953].
- Nucleolus: Another approach from game theory, each participants gets a profit depending on the value of their Nucleolus if it exists [Schmeidler, 1969].

In case of a simple bill sharing problem, the commodity price for energy is usually decided *ex ante*. The CO will bill the computed individual community imports to the consumers and will transfer the revenue to the producers. Another approach could be to allocate the revenue directly. This would be done by computing the full community cost, and the profit it generates compared to individual operation, and then only sharing this profit.

The result of the sharing model can be seen as a division of physical power flows in the community grid among intra-community exchanges ($e^{\text{com}}|i^{\text{com}}$) and extra-community exchanges to the retailers ($e^{\text{ret}}|i^{\text{ret}}$):

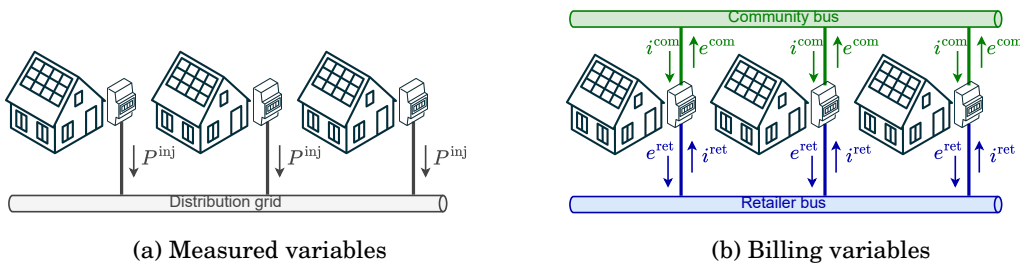


Figure 2.1: Representation of physical and virtual flows

2.1.1.1 Algorithms

This section will present four keys of repartition algorithms to share power injections among community members. These quantities, converted to energy, will be billed by the CO at community price, the remaining will be billed by the retailer.

Parameters:

- $P_{u,t}^{\text{inj}}$: Measured power injection at node u , at time t [kW];
- $P_{\text{COM},t}^{\text{inj}} = \sum_{u \in \mathcal{U}} P_{u,t}^{\text{inj}}$: Power injection for the aggregated community at time t [kW].

Variables:

- $e_{u,t}^{\text{com}}$: Power export billed to the community by the user u , at time t [kW];
- $i_{u,t}^{\text{com}}$: Power import billed by the community to the user u , at time t [kW];
- $e_{u,t}^{\text{ret}}$: Power export billed to the retailer by the user u , at time t [kW];
- $i_{u,t}^{\text{ret}}$: Power import billed by the retailer to the user u , at time t [kW].

The first two algorithms are the equal key Algorithm 2.1 and its "naive" variant Algorithm 2.2. The naive variant is the suboptimal one proposed by the Walloon regulator [Allard, 2026]. It proposes an equal share of power to everyone, even to net producers when sharing consumed power. The second one, which will be used in following parts, only considers equal sharing among those who can use the allocated power. Both of these algorithms are suboptimal as they do not always allocate 100 % of the available quantity to community exchanges. Indeed, a member can get an allocation higher than its actual capacity, resulting in unused allocation.

```

Input:  $P_{u,t}^{inj}, \forall u \in \mathcal{U}, t \in \mathcal{T}$ 
1 foreach  $t \in \mathcal{T}$  do
2    $P_{COM,t}^{inj} = \sum_{u \in \mathcal{U}} P_{u,t}^{inj}$ 
3   if  $P_{COM,t}^{inj} < 0$  % Excess of
      consumption do
4      $P_{tot,t}^{cons} = -\sum_{u \in \mathcal{U}: P_{u,t}^{inj} < 0} P_{u,t}^{inj}$ 
5     foreach  $u \in \mathcal{U} : P_{u,t}^{inj} < 0$  do
6        $i_{u,t}^{com} =$ 
           $\min\{P_{tot,t}^{cons} / |\mathcal{U}|; -P_{u,t}^{inj}\}$ 
7        $i_{u,t}^{ret} = -P_{u,t}^{inj} - i_{u,t}^{com}$ 
8     end
9   end
10  if  $P_{COM,t}^{inj} > 0$  % Excess of
      production do
11     $P_{tot,t}^{prod} = \sum_{u \in \mathcal{U}: P_{u,t}^{inj} > 0} P_{u,t}^{inj}$ 
12    foreach  $u \in \mathcal{U} : P_{u,t}^{inj} > 0$  do
13       $e_{u,t}^{com} =$ 
           $\min\{P_{tot,t}^{prod} / |\mathcal{U}|; P_{u,t}^{inj}\}$ 
14       $e_{u,t}^{ret} = P_{u,t}^{inj} - e_{u,t}^{com}$ 
15    end
16  end
17 end

```

Algorithm 2.1: Naive equal Key

```

Input:  $P_{u,t}^{inj}, \forall u \in \mathcal{U}, t \in \mathcal{T}$ 
1 foreach  $t \in \mathcal{T}$  do
2    $P_{COM,t}^{inj} = \sum_{u \in \mathcal{U}} P_{u,t}^{inj}$ 
3   if  $P_{COM,t}^{inj} < 0$  % Excess of
      consumption do
4      $P_{tot,t}^{cons} = -\sum_{u \in \mathcal{U}: P_{u,t}^{inj} < 0} P_{u,t}^{inj}$ 
5     foreach  $u \in \mathcal{U} : P_{u,t}^{inj} < 0$  do
6        $i_{u,t}^{com} = \min\{P_{tot,t}^{cons} /$ 
           $|\mathcal{U} : P_{u,t}^{inj} < 0|; -P_{u,t}^{inj}\}$ 
7        $i_{u,t}^{ret} = -P_{u,t}^{inj} - i_{u,t}^{com}$ 
8     end
9   end
10  if  $P_{COM,t}^{inj} > 0$  % Excess of
      production do
11     $P_{tot,t}^{prod} = \sum_{u \in \mathcal{U}: P_{u,t}^{inj} > 0} P_{u,t}^{inj}$ 
12    foreach  $u \in \mathcal{U} : P_{u,t}^{inj} > 0$  do
13       $e_{u,t}^{com} = \min\{P_{tot,t}^{prod} /$ 
           $|\mathcal{U} : P_{u,t}^{inj} > 0|; P_{u,t}^{inj}\}$ 
14       $e_{u,t}^{ret} = P_{u,t}^{inj} - e_{u,t}^{com}$ 
15    end
16  end
17 end
18 end
19 end

```

Algorithm 2.2: Equal Key

The two following algorithms allow complete allocation of the exchanges. The first one proposes a proportional approach, Algorithm 2.3 while the second one minimizes the maximal amount to be allocated Algorithm 2.4.

```

Input:  $P_{u,t}^{\text{inj}}, \forall u \in \mathcal{U}, t \in \mathcal{T}$ 
1 foreach  $t \in \mathcal{T}$  do
2    $P_{\text{COM},t}^{\text{inj}} = \sum_{u \in \mathcal{U}} P_{u,t}^{\text{inj}}$ 
3   if  $P_{\text{COM},t}^{\text{inj}} < 0$  % Excess of
     consumption do
4      $P_{\text{tot},t}^{\text{cons}} = -\sum_{u \in \mathcal{U}: P_{u,t}^{\text{inj}} < 0} P_{u,t}^{\text{inj}}$ 
5     foreach  $u \in \mathcal{U} : P_{u,t}^{\text{inj}} < 0$  do
6        $i_{u,t}^{\text{com}} = -k * P_{u,t}^{\text{inj}} / P_{\text{tot},t}^{\text{cons}}$ 
7        $i_{u,t}^{\text{ret}} = -P_{u,t}^{\text{inj}} - i_{u,t}^{\text{com}}$ 
8     end
9   end
10  if  $P_{\text{COM},t}^{\text{inj}} > 0$  % Excess of
     production do
11     $P_{\text{tot},t}^{\text{prod}} = \sum_{u \in \mathcal{U}: P_{u,t}^{\text{inj}} > 0} P_{u,t}^{\text{inj}}$ 
12    foreach  $u \in \mathcal{U} : P_{u,t}^{\text{inj}} > 0$  do
13       $e_{u,t}^{\text{com}} = k * P_{u,t}^{\text{inj}} / P_{\text{tot},t}^{\text{prod}}$ 
14       $e_{u,t}^{\text{ret}} = P_{u,t}^{\text{inj}} - e_{u,t}^{\text{com}}$ 
15    end
16  end
17 end

```

Algorithm 2.3: Proportional Key

```

Input:  $P_{u,t}^{\text{inj}}$ 
1 foreach  $t \in \mathcal{T}$  do
2    $P_{\text{COM},t}^{\text{inj}} = \sum_{u \in \mathcal{U}} P_{u,t}^{\text{inj}}$ 
3   if  $P_{\text{COM},t}^{\text{inj}} < 0$  % Excess of
     consumption then
4      $P_{\text{tot},t}^{\text{prod}} = \sum_{u \in \mathcal{U}: P_{u,t}^{\text{inj}} > 0} P_{u,t}^{\text{inj}}$ 
5     while  $P_{\text{tot},t}^{\text{prod}} \neq 0$  do
6       // Apply the equal key
        algorithm
7       // Compute remaining
        excess of consumption
8     end
9   end
10  if  $P_{\text{COM},t}^{\text{inj}} > 0$  % Excess of
     production then
11     $P_{\text{tot},t}^{\text{cons}} = -\sum_{u \in \mathcal{U}: P_{u,t}^{\text{inj}} < 0} P_{u,t}^{\text{inj}}$ 
12    while  $P_{\text{tot},t}^{\text{cons}} \neq 0$  do
13      // Apply the equal key
        algorithm
14      // Compute remaining
        excess of consumption
15    end
16  end
17 end

```

Algorithm 2.4: Cascade Key principle

2.1.2 Community market

The second part of this problem is choosing the price of energy exchanges within the community. This can be done through a market model, in this case, a community market.

A very simple example has been introduced by [Sousa et al., 2018]:

$$\min \quad \sum_{u \in \mathcal{U}} \sum_{t \in \mathcal{T}} \Delta_t (\pi_{u,t}^{\text{i,ret}} i_{u,t}^{\text{ret}} - \pi_{u,t}^{\text{e,ret}} e_{u,t}^{\text{ret}} + \gamma^{\text{com}} (e_{u,t}^{\text{com}} + i_{u,t}^{\text{com}})) \quad (2.1a)$$

$$\text{s.t.} \quad \Delta_t P_{u,t}^{\text{inj}} = \Delta_t (e_{u,t}^{\text{com}} + e_{u,t}^{\text{ret}} - i_{u,t}^{\text{com}} - i_{u,t}^{\text{ret}}) \quad \forall u \in \mathcal{U}, t \in \mathcal{T}, \perp \pi_{u,t}^{\text{com}} \quad (2.1b)$$

$$\Delta_t \sum_{u \in \mathcal{U}} e_{u,t}^{\text{com}} = \Delta_t \sum_{u \in \mathcal{U}} i_{u,t}^{\text{com}} \quad \forall t \in \mathcal{T}, \perp \pi_t^{\text{com}} \quad (2.1c)$$

The objective of this model is to minimize the total energy cost inside the community. This problem can output community prices as the dual of the energy balance

constraints (π_t^{com}), also called a shadow price. These prices can also be individualized, using the dual variables of local energy balance constraints ($\pi_{u,t}^{\text{com}}$). This can be used as a global price for energy usage from the grid and the community. Or, a single commodity price can be determined for the whole community as the dual of the energy balance in the community.

Additional parameters:

- Δ_t : Timestep [h];
- γ^{com} : Community exchange fees, paid by both the buyer and seller to the CO for running the community [€/kWh];
- $\pi_{u,t}^{\text{i,ret}}$: Price paid by user u to its retailer for energy bought at time t [€/kWh];
- $\pi_{u,t}^{\text{e,ret}}$: Price paid to user u by its retailer for energy sold at time t [€/kWh];

Additional variables:

- π_t^{com} : Price of energy inside of the community for all users at time t [€/kWh];
- $\pi_{u,t}^{\text{com}}$: Individualized price of energy in the community for user u at time t [€/kWh];
- $P_{u,t}^{\text{inj}}$: Power injection can be a decision variable [kW].

2.1.3 Operational planning

The last two sections presented the two main decisions to be made from the CO point of view. Until now, the user did not specifically have to change their consumption or behaviors for the community. This means that this could have until now simply been a rebilling problem, with new quantities exchanged at a new price. In order to have a real effect, the community should incentivize the local energy exchange through collective operational planning.

In this work, the day-ahead control strategy of energy assets will be referred to as operational planning. This is, for a residential prosumer, the decision regarding controllable assets such as:

- Charging and discharging profile of the BSS;
- Curtailement of the PV system (when needed);
- Starting time for programmable devices, referred as White Goods (WG);
- Charging (and discharging) profile for the EV;
- Temperature and power setpoints for the HP;
- Activation of the Water Boiler (WB).

In practice, the scheduling problem for all these appliances is not straightforward. At household-level, all installed appliances must be connected, communicate and be controlled automatically. Their energy needs for the next day should be properly chosen by the user or forecasted by an automatic tool. In an energy community, adequate control requires coordination with other members' systems while prioritizing individual user interests.

For a single home not being part of an EC, the operational planning problem can be presented as such:

$$\min \quad \Pi(P^{\text{inj}}) \quad (2.2a)$$

$$\text{s.t. Internal balance: } P_t^{\text{inj}} = P_t^{\text{PV}} + P_t^{\text{dis}} - P_t^{\text{fix}} - P_t^{\text{flex}} - P_t^{\text{cha}} \quad \forall t \in \mathcal{T} \quad (2.2b)$$

$$\text{Flexible power: } P_t^{\text{flex}} = P_t^{\text{HP}} + P_t^{\text{EV}} + P_t^{\text{WB}} \quad \forall t \in \mathcal{T} \quad (2.2c)$$

$$\text{Flex devices constraints: } P_t^{\text{flex}} \in \mathcal{S}^{\text{flex}} \quad \forall t \in \mathcal{T} \quad (2.2d)$$

$$\text{BSS constraints: } \{P^{\text{cha}}, P^{\text{dis}}\} \in \mathcal{S}^{\text{BSS}} \quad \forall t \in \mathcal{T} \quad (2.2e)$$

$$\text{PV constraints: } P^{\text{PV}} \in \mathcal{S}^{\text{PV}} \quad \forall t \in \mathcal{T} \quad (2.2f)$$

where $\Pi(P^{\text{inj}})$ represents the cost function for energy. The sets $\mathcal{S}$ represent the power injection feasibility of each type of devices. If a device is not available, its power output is simply constrained to 0.

Additional variables:

- P^{inj} : Nodal (resulting) power injection [kW];
- P^{PV} : Photovoltaic power injection [kW];
- $P^{\text{cha}}, P^{\text{dis}}$: Battery storage system charge and discharge power [kW];
- P^{flex} : Flexible power consumption [kW];
- $P^{\text{HP}}, P^{\text{EV}}, P^{\text{WB}}$: Power consumption of the HeatPump, Electric Vehicle and Water Boiler, respectively [kW].

Additional parameters:

- P^{fix} : Fixed power consumption [kW];
- $\Pi()$: Cost function [€].

2.1.4 Optimal power flow

An EC should not only care about virtual exchanges, as the power transits through the electrical network. In our case, we consider residential communities, connected through a Low-Voltage (LV) distribution grid. This grid has a fixed capacity, and its secure operation should be ensured at all times. The LV grids have a particular structure. Unlike transmission grids, they usually do not contain loops in operation. They are organized in feeders, long radial lines that each customer can connect to.

The graph constituted by the network is a tree, which brings interesting properties to simplify the power flow computation. Indeed, in a tree, there is a unique route between any node and the root node. In distribution grids, the root node is usually chosen to be the slack bus, the connection to the medium-voltage grid.

For distribution grids, the most used model is the DistFlow [Farivar and Low, 2013] and its relaxations. A directed graph is defined with an ancestor-children relationship. The slack bus will be the node 0, the only node without ancestor. Going successively, the next nodes in the lines are the successors, making a hierarchical relationship from the root to the leaves (nodes that do not have successors).

$$\min \quad \sum_{u \in \mathcal{U}} f_u(P_u^{\text{inj}}, Q_u^{\text{inj}}) \quad (2.3a)$$

$$\text{s.t.} \quad P_u^{\text{inj}} = \sum_{c_{u,n} \in \mathcal{C}_u} P_{u,c_{u,n}} - (P_{a_u,u} - r_{a_u,u} I_{a_u,u}^{\text{sqr}}) \quad \forall u \in \mathcal{U} \quad (2.3b)$$

$$Q_u^{\text{inj}} = \sum_{c_{u,n} \in \mathcal{C}_u} Q_{u,c_{u,n}} - (Q_{a_u,u} - x_{a_u,u} I_{a_u,u}^{\text{sqr}}) \quad \forall u \in \mathcal{U} \quad (2.3c)$$

$$V_u^{\text{sqr}} = V_{a_u}^{\text{sqr}} - 2(r_{a_u,u} P_{a_u,u} + x_{a_u,u} Q_{a_u,u}) + (r_{a_u,u}^2 + x_{a_u,u}^2) I_{a_u,u}^{\text{sqr}} \quad \forall u \in \mathcal{U} \quad (2.3d)$$

$$I_{a_u,u}^{\text{sqr}} = (P_{a_u,u}^2 + Q_{a_u,u}^2) / V_{a_u}^{\text{sqr}} \quad \forall u \in \mathcal{U} \quad (2.3e)$$

$$(P_u^{\text{inj}}, Q_u^{\text{inj}}) \in \mathcal{S}_u \quad \forall u \in \mathcal{U} \quad (2.3f)$$

$$\underline{V}^2 \leq V_u^{\text{sqr}} \leq \bar{V}^2 \quad \forall u \in \mathcal{U} \quad (2.3g)$$

$$V_{\text{slack}}^{\text{sqr}} = 1 \quad (2.3h)$$

This problem is non-convex due to (2.3e). A common relaxation consists in replacing the strict equality with an inequality:

$$I_{a_u,u}^{\text{sqr}} \geq (P_{a_u,u}^2 + Q_{a_u,u}^2) / V_{a_u}^{\text{sqr}} \quad \forall u \in \mathcal{U} \quad (2.4)$$

This transforms the model to a Second Order Cone Programming (SOCP) problem, and the relaxation will be exact when the inequalities are tight. This can be done by having an objective function minimizing the squared currents (I^{sqr}), pushing them toward their minimal value (*i.e.*, the right-hand side of the equation (2.4)). A natural way of doing that is by adding a loss-minimization term for active power in the objective function. In practice, it is generally relevant in terms of system costs.

$$P^{\text{loss}} = \sum_{u \in \mathcal{U}} r_{a_u,u} I_{a_u,u}^{\text{sqr}} \quad (2.5)$$

This model does not include time-coupling for the system. In a multi-period optimization horizon, it can be directly extended by adding an additional dimension $\mathcal{T}$.

Additional variables:

- V_u^{sqr} : Squared voltage of node u [pu];

- I_u^{sqr} : Squared current in line u (going from its ancestor to node u) [pu];
- $P_{a_u,u}$: Active power flowing on the line u (from its ancestor to node u), on the ancestor's end [kW];
- $Q_{a_u,u}$: Reactive power flowing on the line u , on the ancestor's end [kvar];
- $P_{u,c_{u,n}}$: Active power flowing on the line $c_{u,n}$ (from u to its n^{th} children), on the end of u [kW];
- $Q_{u,c_{u,n}}$: Reactive power flowing on the line $c_{u,n}$ (from u to its n^{th} children), on the end of u [kvar];

Additional parameters:

- $f_u()$: Objective function of node u [€];
- P_u^{inj} : Injection active power [kW];
- Q_u^{inj} : Injection reactive power [kW];
- $V_{\text{slack}}^{\text{sqr}}$: Square of the slack bus, or Point of Common Coupling (PCC) [pu];
- $\underline{V}^2, \bar{V}^2$: Voltage boundaries [pu];
- $r_{a_u,u}, x_{a_u,u}$: Resistance and reactance of the line between node u and its ancestor node [Ω].

2.1.5 Sizing

A longer-term approach could also consider the new investments in DER. This problem can be modeled using a simple CAPEX- and OPEX-weighted objective. Here is a simple example for PV and BSS sizing, but other assets could also be considered.

$$\min \quad \Pi^{\text{op}}(P^{\text{inj}}) + \Pi^{\text{cap}}(C^{\text{pv}}, C^{\text{BSS}}) / I^{\text{hor}} \quad (2.6a)$$

$$\text{s.t.} \quad P_{u,t}^{\text{inj}} = P_t^{\text{PV}} + P_t^{\text{dis}} - P_t^{\text{fix}} - P_t^{\text{cha}} \quad \forall t \in \mathcal{T} \quad (2.6b)$$

$$P_t^{\text{PV}} \leq C^{\text{PV}} \cdot \bar{p}_t^{\text{PV}} \quad \forall t \in \mathcal{T} \quad (2.6c)$$

$$s_t = s_{t-1} + \Delta_T \left(P_t^{\text{cha}} \eta - P_t^{\text{dis}} / \eta \right) \quad \forall t \in \mathcal{T} \quad (2.6d)$$

$$s_t \in [\underline{s}C^{\text{BSS}}, \bar{s}C^{\text{BSS}}] \quad \forall t \in \mathcal{T} \quad (2.6e)$$

The community can have two impacts here:

- Optimal sizing for the collective;
- Reduced costs due to economies of scale;
- Unlocked access to new technologies or bigger capacities with co-investment.

Additional variables:

- C^{PV} : PV system capacity.
- C^{BSS} : Battery capacity.

Additional parameters:

- Π^{op} : Opex function;
- Π^{cap} : Capex function;
- I^{hor} : Investment horizon.

2.2 GENERAL PRELIMINARY ASSUMPTIONS

In this thesis, many preliminary hypotheses have been made for tractable mathematical modelling that allow benefits for the community.

COLLABORATIVE ASPECT: The models in this thesis present a collaborative framework. The results presented here try to assess a maximal benefit of a well-behaved community.

REDUCED GRID TARIFFS (γ^{com}): To propose a profitable business model and incentivize exchanges, the intra-community exchanges are not subject to all regular grid tariffs. This hypothesis makes sense in case of a local community, as a reduced portion of the grid is used and other fees like taxes could be exonerated. Part of this reduced fee can still be paid to the community operator to cover its costs.

PRIVATE LOCAL GRID: Some of the proposed models present a specific case where all members of the grid beyond a point constitute the EC. In these cases, the CO could be responsible for grid operation before this point, which we will call PCC. This could be done through privatization or rental of the infrastructure.

ELECTRIFIED HOUSEHOLD: Most households are assumed to be equipped with DER (PV, BSS) and other controllable appliances, representing a post energy transition residential area.

RATIONAL BEHAVIORS: All EC members are assumed to act rationally according to economic incentives and community membership.

AUTOMATIC FORECASTING AND CONTROL CAPACITY: The community and its members have access to perfect forecasts and automatic control systems for all flexible devices without user intervention.

BENEVOLENT AND VOLUNTARY COMMUNITY OPERATOR: The CO acts in the best interest of the community and its members, without profit maximization for self-interest.

PERFECT FORECAST: To quantify maximum potential of the EC, idealized operational conditions are considered. In a practical setup, uncertainties should

be considered with robust or stochastic programming. This also includes full availability of load and production data.

2.3 ORGANIZATION OF THE SUBSEQUENT CHAPTERS

All the subproblems presented in Section 2.1 will be implemented together to assess the full impact of an optimal REC. This will be done gradually following a time-decision logic as illustrated by Figure 2.2. The contents will start by tackling long-term planning through the decision on investments and end with the settlement on *ex ante* sharing of benefits.

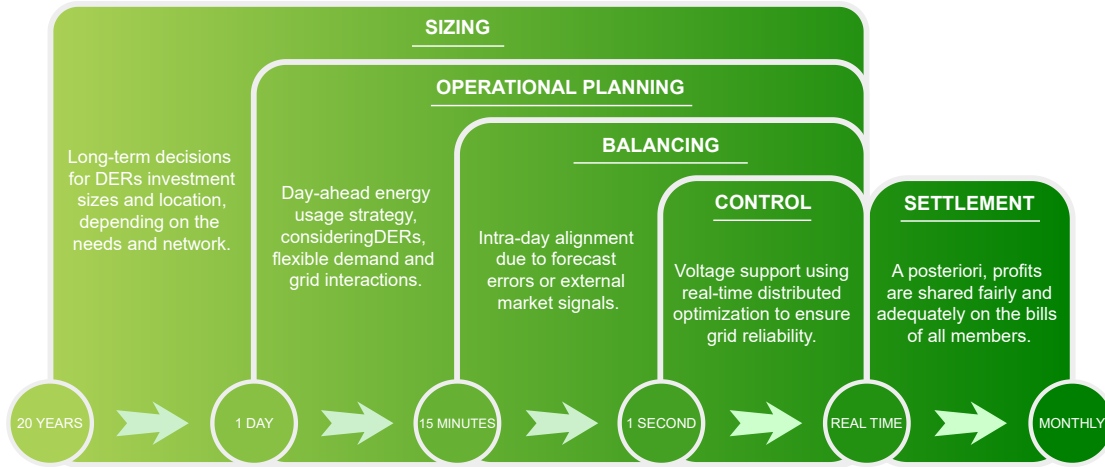


Figure 2.2: Summary of the problem structure and sequentiality between decisions.

2.4 BASE MODEL DESCRIPTION

This thesis models the EC as an optimality-seeking group that jointly tries to minimize its energy cost. As such, mathematical modeling is proposed as a way to assess the efficiency of such a framework in an idealized case.

$$\begin{aligned}
 & \min_{\Omega} && \text{Objective function (2.8a)} \\
 & \text{subject to} && \text{Cost constraints (2.8b),} \\
 & && \text{Energy exchange constraints (2.8c) – (2.8e),} \\
 & && \text{Devices constraints (2.8f) – (2.8h),} \\
 & && \text{Power flow equations (2.8j).}
 \end{aligned}$$

To present most of the variables and constraints, (2.8) provides a general approach that will be specified further for the needs of next chapters.

$$\begin{aligned}
\min_{\Omega} \quad & \sum_{u \in \mathcal{U}} f(\omega_u) & (2.8a) \\
\text{s.t.} \quad & \Pi_{u,t} = (\pi_{u,t}^{\text{i,ret}} i_{u,t}^{\text{ret}} - \pi_{u,t}^{\text{e,ret}} e_{u,t}^{\text{ret}} + \gamma^{\text{com}}(i_{u,t}^{\text{com}} + e_{u,t}^{\text{com}})) & \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (2.8b) \\
& P_{u,t}^{\text{inj}} = P_{u,t}^{\text{PV}} + P_{u,t}^{\text{dis}} - P_{u,t}^{\text{fix}} - P_{u,t}^{\text{flex}} - P_{u,t}^{\text{cha}} & \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (2.8c) \\
& P_{u,t}^{\text{inj}} = e_{u,t}^{\text{ret}} + e_{u,t}^{\text{com}} - i_{u,t}^{\text{ret}} - i_{u,t}^{\text{com}} & \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (2.8d) \\
& \sum_{u \in \mathcal{U}} e_{u,t}^{\text{com}} = \sum_{u \in \mathcal{U}} i_{u,t}^{\text{com}} & \forall t \in \mathcal{T} \quad (2.8e) \\
& P_{u,t}^{\text{flex}} = P_{u,t}^{\text{EV}} + P_{u,t}^{\text{WB}} + P_{u,t}^{\text{HP}} & \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (2.8f) \\
& P_{u,t}^{\text{PV}} \leq C_u^{\text{PV}} \bar{P}_{u,t}^{\text{PV}} & \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (2.8g) \\
& s_{u,t}^{\text{BSS}} = s_{u,t}^{\text{prev, BSS}} + \Delta_T (\eta_u^{\text{BSS}} P_{u,t}^{\text{cha}} - P_{u,t}^{\text{dis}} / \eta_u^{\text{BSS}}) / C^{\text{BSS}} & \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (2.8h) \\
& (P_u^{\text{inj}}, Q_u^{\text{inj}}) \in \mathcal{S}_u & \forall u \in \mathcal{U} \quad (2.8i) \\
& (V_t^{\text{sqr}}, I_t^{\text{sqr}}) = \mathcal{H}(P_t^{\text{inj}}, Q_t^{\text{inj}}) & \forall t \in \mathcal{T} \quad (2.8j)
\end{aligned}$$

Equation (2.8b) computes the community members' bills based on grid tariffs and community fees, (2.8c) expresses the physical or internal power balance, (2.8d) separates power exchanges into virtual community and grid/retailer flows, (2.8e) ensures internal energy balance within the community, (2.8f) represents the aggregation of flexible loads, (2.8g) limits PV production to available resources, (2.8h) tracks battery State of Charge (SoC) dynamics with round-trip efficiency, (2.8i) constrains the injection to feasible operating regions depending on the installed assets, and (2.8j) represents power flow equations in the distribution network.

Future chapters will provide variations for this model by adding other decision variables, constraints or decomposition techniques. They will focus on a specific part of this model, while this base will be used for the remaining of the thesis.

Table 2.1 summarizes all the parameters and variables used in this general problem.

2.5 LITERATURE REVIEW

This section will give an overview of other relevant research for the studied problem. Divided by the chapters they refer to, these references have been studied at the time of the initial publication of these works. This constitutes the literature review for this complete thesis.

2.5.1 Sizing distributed energy resources in renewable energy communities

To be a truly viable solution, the energy communities have to be properly sized and managed while taking into account the interaction with the grid to which they

Table 2.1: Variables and parameters of the model

	Dim	Dom	Unit	Description
P^{fix}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Fixed power consumption
$\overline{P}^{\text{PV}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Max PV power production
C^{PV}	$\mathcal{U}$	$\mathbb{R}_+$	kWp	PV system size
$s^{\text{init,BSS}}$	$\mathcal{U}$	$[0, 1]$	-	Initial battery SoC
C^{BSS}	$\mathcal{U}$	$\mathbb{R}_+$	kWh	Battery capacity
η^{BSS}	$\mathcal{U}$	$[0, 1]$	-	Battery efficiency
$\pi^{\text{i,ret}}$	$\mathcal{T}$	$\mathbb{R}_+$	€/kWh	Retailer import price
$\pi^{\text{e,ret}}$	$\mathcal{T}$	$\mathbb{R}_+$	€/kWh	Retailer export revenue
γ^{com}	$\mathcal{T}$	$\mathbb{R}_+$	€/kWh	Community fee
e^{com}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Community export power
i^{com}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Community import power
e^{ret}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Retailer export power
i^{ret}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Retailer import power
P^{PV}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	PV production power
P^{cha}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	BSS charging power
P^{dis}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	BSS discharging power
P^{inj}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Node injection power
P^{flex}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Flexible load power

are connected. Chapter 3 proposes a model to collectively size the assets and to retribute the community investors, then a carbon emission impact analysis is conducted. Prior works had already tackled parts of this problem separately.

Several works have dealt with the sizing of DERs in ECs, and notable ones include [Aviles et al., 2019; Fioriti et al., 2021] for ECs in general and [Weckesser et al., 2021],[Cosic et al., 2021],[Hupez et al., 2021] for REC in particular. In [Aviles et al., 2019], bilevel programming is proposed as the solution approach for an EC investment and operation problem with a business model incorporation.

Game theory reward distribution schemes have been used by Fioriti et al. [2021] for the sizing and fair revenue sharing among community members. This work has suggested Shapley/Core, Variance/Core, and Nucleolus reward distribution schemes as the solution to the problem. Weckesser et al. [2021] compared three strategies for DERs sizing in RECs: economic optimization, peak shaving, and self-sufficiency. These models are tested on multiple types of EC networks (urban, suburban, and rural), and their impact on the grid is quantified. The work shows that the best strategy for grid operation changes depending on the grid type. Cosic et al. [2021] proposed to use a Mixed-Integer Linear Program (MILP) to show that a REC can

reduce the CO₂ emissions related to energy usage. This reduction is amplified when the participants invest in a central DER rather than multiple personal DERs.

Two cooperative frameworks using game theory were developed by [Hupez et al. \[2021\]](#). The first one uses Shapley values to determine distribution keys for profit. This method is computationally intensive, but adequately rewards the mobilized flexibility. The second one finds a natural Nash equilibrium, which tends to reward the potential flexibility of end-users.

As noted by [Fioriti et al. \[2021\]](#), ECs are designed through non-cooperative or cooperative approaches. In non-cooperative approaches, each EC member acts on its own interest in the local energy market [[Cornélusse et al., 2019](#)], while in cooperative ones the EC operator takes the task of trading on behalf of EC members. The cooperative approaches align with the European context [[Cappellaro et al., 2022](#)], [[European Commission, 2018](#)], [[Berka and Creamer, 2016](#)] and require a fair scheme for sharing the benefits to all EC members.

A life cycle assessment approach [[National Renewable Laboratory \(NREL\), 2021](#); [Gao et al., 2021](#)] is usually employed to quantify the environmental impact (carbon footprint) of a product. It covers the whole lifespan of a product (starting from the extraction of raw materials up to disposal and recycling).

The work presented by [Elomari et al. \[2024\]](#) suggests a data-driven framework to design a REC considering technical objectives and environmental impact. Formulated multi-criteria decision-making problem is solved by integrating machine learning and life cycle assessment.

A REC multi-objective planning method considering economic (total annualized costs), environmental (carbon footprint) and social (user comfort) objectives is presented by [Mariuzzo et al. \[2023\]](#). The results of this work shows improvements in economic benefits and self-sufficiency of a REC when all community members modify their behavior towards an objective.

2.5.2 Load profiles with a flexibility component

Chapter 4 proposes a tool to generate load profiles with a flexibility component. This tool was created due to the incomplete or closed solutions that were available.

A few projects measured real-world data and included some separation of the households' appliances [[Open Power System Data, 2018](#); [Pecan Street Inc., 2024](#)]. However, these are limited in the number of homes and do not allow the modeling of extensive populations.

Other projects for district energy demand modeling [[Fonseca et al., 2016](#); [Pflugradt et al., 2022](#)] exist. However, despite providing efficient solutions to generate load profiles, those tools are opaque solutions. They include district heating needs and other types of customers to model complete cities.

The work presented here is a fully customizable tool where appliances can include a flexibility component. Open source libraries such as [oemof Developer Group, 2016; Baetens and Saelens, 2016] are more relevant in this work due to their accessibility and configurability, and this work is built upon them.

Another approach involves a machine learning technique to detect and cluster the appliances activation in measured household load profiles [Ruzzelli et al., 2010; Liao et al., 2014; Dongre et al., 2021]. This method has given decent results but still requires real data, which induces customer concerns about privacy issues for data coming from smart meters.

2.5.3 Flexibility on the demand in Renewable Energy Communities

Chapter 5 motivates the need for activating the flexibility of the residential loads in a community grid. It also quantifies its potential and proposes methods to activate and retribute it. Here is the prior work that was reviewed when this work was conducted.

The potential of home appliance flexibility was studied by Limmer et al. [2021]. A mixed integer linear programming optimization problem is proposed to coordinate home appliances for demand response provision (load control and time-shifting of the appliances). The focus is on efficient reformulation to solve problems involving up to 200 appliances.

REC flexibility provision through load shifting, demand response and energy storage was reviewed by Ponnaganti et al. [2023]. The identified outcomes from the provision include peak shaving and shifting, energy cost minimization, being a part of demand-side management, and self-sufficiency in islanded mode of operation. Load flexibilization within REC was also considered by Dorahaki et al. [2023] and [Barnes et al., 2022]. The later revealed the potential for REC to provide demand-side solutions by creating new business models.

End-users' discomfort modeling issues were addressed in [Javadi et al., 2021; Dorahaki et al., 2023]. The thermal end-user comfort was modeled in [Dorahaki et al., 2023] by the deviation of the indoor temperature from the setpoints. Javadi et al. [2021] considered this through a discomfort index of the consumer. This index was defined by a linear penalization for shifted time slots compared to ideal end-user preference.

An incentive-based mechanism and a price-based mechanism (based on the financial compensation of the participants) for optimization-based demand response were considered in [Asadinejad and Tomsovic, 2017]. They calculated the load change for the incentive and the best times to realize the demand response while accounting for the comfort level. Mustika et al. [2023] proposed a sharing mechanism based on Key of Repartition (KoR) for the benefits linked to the frequency reserve provi-

sion by REC storage systems. The benefits for the local arbitrage are shared using another standard KoR.

To unlock the maximum potential of a residential REC, flexible consumption of end-users must be incentivized [Pannaganti et al., 2023; Askeland et al., 2025]. As highlighted by some research work, the REC framework increases the willingness of consumers to be flexible with respect to individual consumers [Luzzati et al., 2024; Dudka, 2025]. Additionally, the growing penetration of low-carbon technologies in residential loads (electric vehicles, water boilers and heat pumps) increases the needs and possibilities for response to residential demand [Mathieu et al., 2025] and their potential to maximize utility in the residential sector has already been demonstrated Lopez et al. [2024].

The work presented in [Dukovska et al., 2021] formulated the coordination of REC members for collective energy consumption and production as a mixed-integer linear problem with coupling constraints. The optimization problem is decentralized and solved using a decomposition method.

Alternating Direction Method of Multipliers (ADMM), for distributed optimisation, has been considered in [Lilla et al., 2020; Stephant et al., 2021]. Lilla et al. [2020] focused on a day-ahead operational planning, and privacy-preserving computation is achieved through ADMM, while Stephant et al. [2021] dealt with the improvement of REC self-consumption. In the latter work, the equilibrium between the community operator and community members was achieved when the grid imported power was below a predefined threshold, instead of focusing on users preferences satisfaction.

The collective self-consumption is the focus of Mustika et al. [2022], which proposes a two-stage approach, decoupling the operational phase from the settlement of energy allocation. A bilevel model is built by Li et al. [2021] to solve the scheduling problem considering renewable uncertainty and demand response with an emphasis on the flexibility potential of electric vehicles.

2.5.4 Distributed control for optimal power flow in distribution grids

Chapter 6 proposes a method to control the voltage level in distribution grids with an ADMM algorithm. This work extended existing literature on distributed method by developing an ADMM algorithm that features nodal decomposition and iterative linearization of the non-convexity of the OPF.

Li et al. [2019] and Bolognani et al. [2014] used a dual-ascent method to solve an optimization problem that is decomposed over each electrical node. After updating the Lagrangian multipliers, they are exchanged among neighbors through a communication network. Zhang et al. [2014] also employed a dual-ascent method, but they exchange Lagrangian multipliers to equalize the angle differences across a line from both ends.

Šulc et al. [2014] showed that the ADMM has superior convergence properties to dual-ascent methods, and consider a consensus-based version of ADMM. The convergence properties of distributed algorithms are actually important when it comes to real-time control of distribution networks, since long convergence times lead to suboptimal operation of the system. In [Xu and Wu, 2020], the problem is split into several zones consisting of several electrical nodes, and a consensus-based ADMM is also implemented. They use a quasi-Newton approach for the dual update to further enhance convergence properties. A convex surrogate of a distributed AC-OPF problem is solved using ADMM in [Zhang et al., 2017], although they use a centralizer to compute voltage-related vectors and to broadcast them to the different controllable units.

Part II

PLANNING

Optimal decision a priori

SIZING

Outline

This chapter covers long term design of the community through distributed energy resources investments, and propose a way to compare financial and ecological optimums.

The goal of this chapter is to develop an optimization model that is able to evaluate the benefits of ECs on three important aspects at once:

- The direct benefits of the community members;
- The improvement on grid operation due to the community;
- The incentive bonus of EC in new DER investments.

Then this financial assessment will be extended to CO₂ emissions in the economic optimum. An ecological optimum will be defined to compare the solutions of both objectives.

CONTEXT AND OBJECTIVES OF THIS CHAPTER

The work presented in this chapter proposes four main contributions identified from the research gaps of Section 2.5.1:

- A REC framework that includes a sizing, a planning, and a grid model.
- A co-investment model with ensured return for community members.
- A carbon footprint optimization that directly related to the installed capacity embedded emission instead of the energy outputs.

Some hypotheses have been taken in this work to ensure the relevance and the trackability of the model. The REC is composed of the complete downstream LV network, which is necessary to isolate the direct impact of the community on the grid. The CO is located at the PCC and handles the collective bill for energy measured at this point. He is then responsible for bill sharing among the members.

One of the key assumptions of this model is the deterministic and optimal operation of the community. This setup is similar to a sizing problem with historical

data. Here the considered data spans over one year and has been reduced to 12 representative days (one per month). These days have been chosen to represent the magnitude of the total produced and consumed energy for each member.

This model reduce the yearly scenario to twelve days to keep trackability of the solution. It is a Mixed-Integer SOCP problem, with around 13,000 second-order constraints, 180 binary variables and 225,000 variables. This model does not pretend to give an exact solution for this specific problem. The actual solution will highly depend on the hypothesis and scenario modelling for the long term operation of the REC.

The goal of this model is to provide a proof of concept which shows the potential impact of new investments on the community members and the grids. With the given assumptions, three cases can be compared, investment to act alone, individual investment inside a community, and co-investment in the community. While these results are not meaningful themselves as they are heavily case-dependant, the comparison among them is the main outcome of this model.

Table 3.1 presents the variables and parameters used in this chapter.

	Dim	Dom	Unit	Description
Main decision variables (Sizing)				
C^{PV}	$\mathcal{U}$	$\mathbb{R}_+$	[kW]	Invested PV capacity
C^{BSS}	$\mathcal{U}$	$\mathbb{R}_+$	[kWh]	Invested BSS capacity
b^{PV}	$\mathcal{U}$	[0;1]	[-]	PV investment flag
d^{PV}	$\mathcal{U}$	[0;1]	[-]	PV discount flag
b^{BSS}	$\mathcal{U}$	[0;1]	[-]	BSS investment flag
d^{BSS}	$\mathcal{U}$	[0;1]	[-]	BSS discount flag
$\bar{s}_u, \underline{s}_u$	$\mathcal{U}$	$\mathbb{R}_+$	[kW]	BSS maximum charging and discharging rate
Secondary decision variables (Planning)				
P^{PV}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kWp]	PV investment
P^{dis}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	BSS discharge power
P^{cha}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	BSS charge power
e^{ret}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	Power export billed to the retailer
i^{ret}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	Power import billed by the retailer
e^{com}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	Power export billed to the retailer
i^{com}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	Power import billed by the retailer

Q^{PV}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kvar]	Reactive power injection of the PV inverter
Q^{BSS}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kvar]	Reactive power injection of the BSS inverter
Auxiliary and resulting variables				
$D^{\text{PV}}, D^{\text{BSS}}$	$\mathcal{U}$	[€]	$\mathbb{R}_+$	PV and BSS discount
P^{inj}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	Active power injection
P^{loss}	$\mathcal{U}$	$\mathbb{R}_+$	pu	Squared voltage magnitude
$\bar{p}$	/	$\mathbb{R}_+$	[kW]	Peak power
Q^{inj}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kvar]	Reactive power injection
V^{sqr}	$\mathcal{U}$	$\mathbb{R}_+$	[V ²]	Squared voltage magnitude
I^{sqr}	$\mathcal{U}$	$\mathbb{R}_+$	[A ²]	Squared current magnitude
P_u^{line}	$\mathcal{U}$	$\mathbb{R}$	[kW]	Active power in line u
Q_u^{line}	$\mathcal{U}$	$\mathbb{R}$	[kW]	Reactive power in line u
Parameters				
$\pi^{\text{i,ret}}$	/	0.40	[€/kWh]	Retailer energy tariff
$\pi^{\text{e,ret}}$	/	0.10	[€/kWh]	Retailer feed-in tariff
γ^{com}	/	0.01	[€/kWh]	Community fees
γ^{bss}	/	0.01	[€/kWh]	BSS fees
π^{peak}	/	10	[€/(kW.y)]	Peak power tariff
w	$\mathcal{D}$	$\mathbb{R}_+$	[-]	Weight factor for representative days
Δ_T	/	0.25	[h]	Timestep duration
η^{cha}	$\mathcal{U}$	0.95	[-]	BSS charge efficiency
η^{dis}	$\mathcal{U}$	0.95	[-]	BSS discharge efficiency
P^{fix}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	Fixed consumption
$\bar{P}^{\text{PV}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	PV maximum power output
$\bar{P}^{\text{cha}}$	$\mathcal{U}$	$\mathbb{R}_+$	[kW]	Maximum battery charging power
$\bar{P}^{\text{dis}}$	$\mathcal{U}$	$\mathbb{R}_+$	[kW]	Maximum battery discharging power
$\underline{C}^{\text{PV}}, \bar{C}^{\text{PV}}$	/	{2; 20}	[kWp]	Minumum and maximum PV investment capacities
$\underline{C}^{\text{BSS}}, \bar{C}^{\text{BSS}}$	/	{4; 40}	[kWh]	Minumum and maximum BSS investment capacities
$\pi_f^{\text{PV}}, \pi_f^{\text{BSS}}$	/	{4; 2}	[k€]	Fixed prices for PV and BSS investments
$\pi_v^{\text{PV}}, \pi_d^{\text{PV}}$	/	{1.6; 0.5}	[k€/kWp]	Variable and discount prices for PV investment

$\pi_v^{\text{BSS}}, \pi_d^{\text{BSS}}$	/	$\{0.6; 0.2\}$	[€/kWh]	Variable and discount prices for BSS investment
$\mathcal{B}$	$\mathcal{U}$	$\mathbb{R}_+$	[€]	Maximum investment budget
$\bar{P}_{u,t}^{\text{PV},i}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	[kW]	Initial PV system maximum power output
$\underline{s}_u^i, \bar{s}_u^i$	$\mathcal{U}$	$\mathbb{R}_+$	[kWh]	Initial BSS minimum and maximum SoCs
$\bar{P}_u^{\text{cha},i}, \bar{P}_u^{\text{dis},i}$	$\mathcal{U}$	$\mathbb{R}_+$	[kW]	Initial BSS maximum charging and discharging powers
Q^{fix}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}$	[kvar]	Load reactive power
r	$\mathcal{U}$	$\mathbb{R}_+$	[Ω]	Line resistance
x	$\mathcal{U}$	$\mathbb{R}_+$	[Ω]	Line reactance
$\underline{V}^{\text{sqr}}$	/	0.95^2	[V ²]	Minimum nodal squared voltage
$\bar{V}^{\text{sqr}}$	/	1.05^2	[V ²]	Maximum nodal squared voltage
$\bar{I}^{\text{sqr}}$	/	$\mathbb{R}_+$	[A ²]	Maximum line squared current

The SoC variables of this problem are expressed in kWh to allow for a linear

3.1 OPTIMAL INVESTMENTS IN DERS

The contribution is twofold. First, an Optimal Power Flow (OPF) problem is integrated into the model presented by [Cornélusse et al. \[2019\]](#). In particular, a branch flow OPF [[Farivar and Low, 2013](#)] is adopted to account for network losses and ensure proper grid operation. Second, a technique to optimize the investment in DER of EC members is proposed to determine the optimal allocation of new DER within the EC. Three scenarios are compared:

- for each user independently without a community;
- for each community member when they invest separately;
- for the community as a whole.

The proposed two-step approach is inspired by [Cornélusse et al. \[2019\]](#), which merges the two steps in a bilevel formulation but does not include a grid model or new investments sizing. Although the problem addressed here could also be modeled as a bilevel program, it would result in a mixed-integer non-convex program. The proposed two-step approach approximates this bilevel problem by soft-linking the sizing problem, a mixed-integer second-order cone problem, and the profit-sharing problem, a quadratic program. The first optimization step still ensures economic optimality through collective bill minimization.

3.1.1 Minimizing community costs considering network and investments

The first optimization problem minimizes the community costs, considering all its users' investment and operation costs. The CO also ensures reliable community grid operation. This problem has a double role, finding optimal sizes and locations for new investments while establishing the operational behavior for the EC. The overall problem can be formulated as,

$$\begin{aligned}
 & \min && \text{Total cost for the community (3.2)} && (3.1) \\
 & \text{subject to} && \text{Price and exchange constraints (3.3) – (3.5),} \\
 & && \text{Devices constraints (3.6) – (3.7g),} \\
 & && \text{Sizing constraints (3.8) – (3.10d),} \\
 & && \text{Power flow equations (3.14a) – (3.20).}
 \end{aligned}$$

The total cost for the community is defined as,

$$\sum_{u \in \mathcal{U}} (\Pi_u^{\text{com}} + \Pi_u^{\text{inv}} / I^{\text{hor}}) + \pi^{\text{peak}} \bar{p} + \pi_t^{\text{i,ret}} \sum_{t \in \mathcal{T}} w_{dt} \Delta_T P_t^{\text{loss}}, \quad (3.2)$$

where $\mathcal{U}$ is the set of users of the LV grid, and $\mathcal{T}$ is the set of time periods of the simulation. This set can also be divided in several representative days d which constitute the set $\mathcal{D}$. Π_u^{com} is the total yearly operation cost for user u considering its energy exchanges over the whole time horizon, Π_u^{inv} is its investment in new DER, and I^{hor} is the investment horizon in years. $\bar{p}$ is the yearly peak power injection at the PCC of the community, which is charged at price π^{peak} . The last term penalizes the energy lost (P_t^{loss}) in the electrical lines. The time is discretized in time steps of duration Δ_T .

In this model, the CO is in charge of grid operation behind the PCC. The hypothesis of this model is that the CO is virtually connected to the slack bus of the network and is billed depending on the energy metered at this point. It then passes the energy costs on the individual members. The cost of downstream network losses should be covered by the community network fees (γ^{com}).

The operational community cost for each user (Π_u^{com}) is defined by

$$\begin{aligned}
 \Pi_u^{\text{com}} = & \Delta_T \sum_{t \in \mathcal{T}} w_{dt} (\pi_t^{\text{i,ret}} i_{u,t}^{\text{ret}} - \pi_t^{\text{e,ret}} e_{u,t}^{\text{ret}} + \gamma^{\text{com}} (e_{u,t}^{\text{com}} + i_{u,t}^{\text{com}}) \\
 & + \gamma^{\text{bss}} (\eta_u^{\text{cha}} P_{u,t}^{\text{cha}} + P_{u,t}^{\text{dis}} / \eta_u^{\text{dis}})) \quad \forall u \in \mathcal{U}. \quad (3.3)
 \end{aligned}$$

Variables $e_{u,t}^{\text{ret}}$ and $e_{u,t}^{\text{com}}$ (resp. $i_{u,t}^{\text{ret}}$ and $i_{u,t}^{\text{com}}$) represent the power sold (resp. bought) to the retailer and the community by user u at time t . Simultaneous virtual power import and export is prevented by a grid import price ($\pi_t^{\text{i,ret}}$) greater than the grid selling price ($\pi_t^{\text{e,ret}}$) at all time. The CO charges a fee for the intra-community exchanges valued at γ^{com} and the storage usage is valued at γ^{bss} for battery wear. The benefits are weighted with w_{dt} which depends on the day d of the current timestep

t . The weights are designed to represent a complete year with a limited number of days.

Equation (3.4a) ensures that internal community power exchanges are balanced at all times, and equation (3.4b) determines the peak ($\bar{p}$) for the whole community.

$$\sum_{u \in \mathcal{U}} (i_{u,t}^{\text{com}} - e_{u,t}^{\text{com}}) = 0 \quad \forall t \in \mathcal{T}, \quad (3.4a)$$

$$\sum_{u \in \mathcal{U}} (i_{u,t}^{\text{ret}} - e_{u,t}^{\text{ret}}) + P_t^{\text{loss}} \leq \bar{p} \quad \forall t \in \mathcal{T}. \quad (3.4b)$$

The power balance for each community member defines its power exchanges depending on its internal devices and usage,

$$e_{u,t}^{\text{ret}} - i_{u,t}^{\text{ret}} + e_{u,t}^{\text{com}} - i_{u,t}^{\text{com}} = P_{u,t}^{\text{PV}} - P_{u,t}^{\text{fix}} + P_{u,t}^{\text{dis}} - P_{u,t}^{\text{cha}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.5)$$

where $P_{u,t}^{\text{PV}}$ and $x_{u,t}$ are the power produced by the PV system and the fixed power consumption of user u at time t . Variables $P_{u,t}^{\text{cha}}$, and $P_{u,t}^{\text{dis}}$ are the active powers going in and out of the BSS.

3.1.1.1 Photovoltaic installations

Photovoltaic power injection ($P_{u,t}^{\text{PV}}$) is bounded for every time period t . The upper bound value is defined as the PV profile $\bar{P}_{u,t}^{\text{PV}}$ depending on both weather conditions and PV installation size,

$$P_{u,t}^{\text{PV}} \in [0, \bar{P}_{u,t}^{\text{PV}}] \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (3.6)$$

3.1.1.2 Battery storage systems

The following equations model the BSS state of charge evolution. Constraint (3.7c) enforces an identical state of charge ($s_{d,u}^{\text{init}}$) at the beginning ($t_{d,0}$) and end ($t_{d,T}$) of each representative day d . This value is defined as a variable to be optimally determined.

$$s_{u,t_{d,1}} = s_{u,d}^{\text{init}} + \Delta_T (\eta_u^{\text{cha}} P_{u,t_{d,1}}^{\text{cha}} - P_{u,t_{d,1}}^{\text{dis}} / \eta_u^{\text{dis}}) \quad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}, \quad (3.7a)$$

$$s_{u,t} = s_{u,t-1} + \Delta_T (\eta_u^{\text{cha}} P_{u,t}^{\text{cha}} - P_{u,t}^{\text{dis}} / \eta_u^{\text{dis}}), \quad \forall u \in \mathcal{U}, t \in \{t_{d,2} : t_{d,T}\}, \forall d \in \mathcal{D}, \quad (3.7b)$$

$$s_{u,t_{d,T}} = s_{u,d}^{\text{init}} \quad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}, \quad (3.7c)$$

$$s_{u,t} \in [\underline{s}_u; \bar{s}_u] \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.7d)$$

$$s_{u,d}^{\text{init}} \in [\underline{s}_u; \bar{s}_u] \quad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}, \quad (3.7e)$$

$$P_{u,t}^{\text{dis}} \in [0; \bar{P}_u^{\text{dis}}] \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.7f)$$

$$P_{u,t}^{\text{cha}} \in [0; \bar{P}_u^{\text{cha}}] \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.7g)$$

with $s_{u,t}$ the state of charge of the storage of user u at the end of period t . These are bounded by variables depending on the sizing results introduced in the next section.

3.1.1.3 Sizing

Some of the optimization variables of this first problem are the sizes of the new DERs. Each member can invest in new PV (C_u^{PV}) or BSS (C_u^{BSS}) capacities at its location.

The PV and BSS sizes are optimized for each user considering its location and the investment costs with a function modelling economies of scale and fixed prices,

$$\Pi_u^{\text{inv}} = \Pi^{\text{PV}}(C_u^{\text{PV}}) + \Pi^{\text{BSS}}(C_u^{\text{BSS}}) \quad (3.8)$$

where $\Pi^y(x)$ is the cost of installation for a system y of size x , which translates to Figure 3.1 using the chosen parameters:

$$\Pi^{\text{BSS}} = b^{\text{BSS}}\pi_f^{\text{BSS}} + C^{\text{BSS}}\pi_v^{\text{BSS}} - D^{\text{BSS}} \quad (3.9a) \quad \Pi^{\text{PV}} = b^{\text{PV}}\pi_f^{\text{PV}} + C^{\text{PV}}\pi_v^{\text{PV}} - D^{\text{PV}} \quad (3.10a)$$

$$0 \leq D^{\text{BSS}} \quad (3.9b) \quad 0 \leq D^{\text{PV}} \quad (3.10b)$$

$$D^{\text{BSS}} \leq d^{\text{BSS}}(C^{\text{BSS}} - \bar{C}^{\text{BSS}}/2)\pi_d^{\text{BSS}} \quad (3.9c) \quad D^{\text{PV}} \leq d^{\text{PV}}(C^{\text{PV}} - \bar{C}^{\text{PV}}/2)\pi_d^{\text{PV}} \quad (3.10c)$$

$$b^{\text{BSS}}\underline{C}^{\text{BSS}} \leq C^{\text{BSS}} \leq b^{\text{BSS}}\bar{C}^{\text{BSS}} \quad (3.9d) \quad b^{\text{PV}}\underline{C}^{\text{PV}} \leq C^{\text{PV}} \leq b^{\text{PV}}\bar{C}^{\text{PV}} \quad (3.10d)$$

with π_f^y , π_v^y and π_d^y respectively the fixed, variable and discounted cost coefficients. The variable D^y is positive and gives a discount of π_d^y per unit of capacity installed above a system size threshold (*i.e.*, half of the maximum). The fixed cost and discount terms allow to represent economies of scale in new investments as shown in Fig. 3.1. Equations (3.10d),(3.9d) bound the system sizes and fixes the binary variable indicating the user's investment in technology y (b_u^y). The binary variable d_u^y indicates that the size the investment allows discounted prices.

When tight, which is ensured by the cost minimization objective, the cost function is:

$$\Pi(C^y) = \begin{cases} 0 & \text{if } C^y = 0 \\ \pi_f^y + C^y\pi_v^y, & \text{if } C^y \in [\underline{C}^y, \bar{C}^y/2] \\ \pi_f^y + C^y\pi_v^y - (C^{\text{PV}} - \bar{C}^{\text{PV}}/2)\pi_d^y, & \text{if } C^y \in [\bar{C}^y/2, \bar{C}^y] \end{cases} \quad (3.11)$$

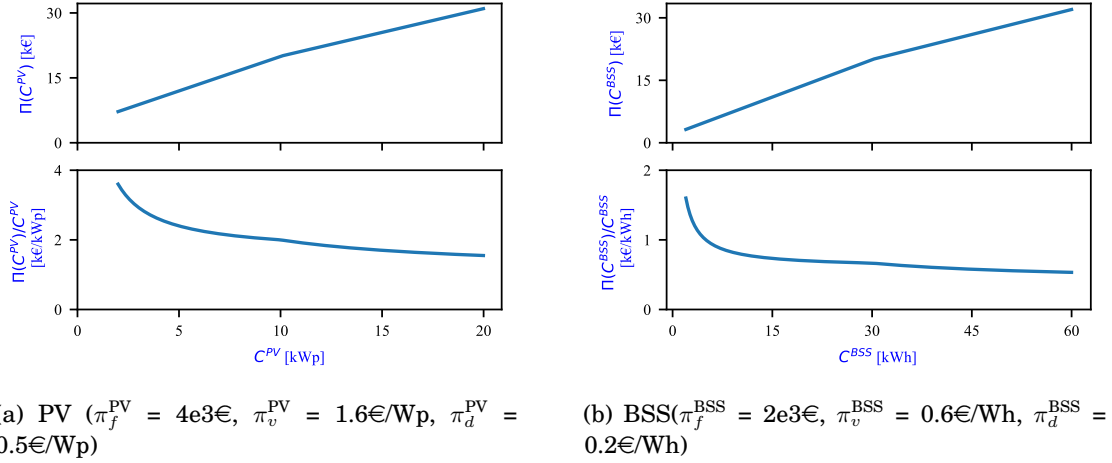


Figure 3.1: Investment costs and capacity costs for PV and BSS installations.

The investment capacities are also limited by the budget of the community participants ($\bar{\mathcal{B}}_u$). The total investment costs for the whole community is limited by the budget of all participants pooled together:

$$\sum_{\forall u \in \mathcal{U}} \Pi_u^{\text{inv}} \in [0; \sum_{\forall u \in \mathcal{U}} \bar{\mathcal{B}}_u]. \quad (3.12)$$

The decision variables for investment capacities C_u^{PV} and C_u^{BSS} are used to define the bounds for the power injections in the operational planning,

$$\bar{P}_{u,t}^{\text{PV}} = \bar{P}_{u,t}^{\text{PV,i}} + C_u^{\text{PV}} \bar{p}_{u,t}^{\text{PV}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.13a)$$

$$\underline{s}_u = \underline{s}_u^i + 0.1 C_u^{\text{BSS}} \quad \forall u \in \mathcal{U}, \quad (3.13b)$$

$$\bar{s}_u = \bar{s}_u^i + 0.9 C_u^{\text{BSS}} \quad \forall u \in \mathcal{U}, \quad (3.13c)$$

$$\bar{P}_u^{\text{cha}} = \bar{P}_u^{\text{cha}_i} + \bar{P}_u^{\text{cha}} C_u^{\text{BSS}} \quad \forall u \in \mathcal{U}, \quad (3.13d)$$

$$\bar{P}_u^{\text{dis}} = \bar{P}_u^{\text{dis}_i} + \bar{P}_u^{\text{dis}} C_u^{\text{BSS}} \quad \forall u \in \mathcal{U}, \quad (3.13e)$$

where $\bar{P}_{u,t}^{\text{PV,i}}$ is the maximum PV production of user u at time t before investment and $\bar{p}_u^{\text{PV}}$ is the scaled PV profile associated with user u . Variables $\underline{s}_u^i$ and $\bar{s}_u^i$ represent the initial minimum and maximum values for the state of charge of user u . The newly invested capacity C_u^{BSS} can only be used ranging from 10 to 90% of its maximum capacity. Variables $\bar{P}_u^{\text{cha}_i}$ and $\bar{P}_u^{\text{dis}_i}$ denote the charging and discharging rates, respectively, for user u before investment and variables $\bar{P}_u^{\text{cha}}$ and $\bar{P}_u^{\text{dis}}$ represents the maximal charging and discharging power of the new BSS per unit of capacity installed.

3.1.1.4 Power flow model

Power flow constraints are added to this optimization problem to ensure a reliable operation of the distribution network. The radial configuration of the distribution grid allows us to use a relaxed DistFlow model [Farivar and Low, 2013].

First, physical network limits are represented by limits on the squared line currents (I^{sqr}) and squared node voltages (V^{sqr}),

$$\underline{V}^{\text{sqr}} \leq V_{u,t}^{\text{sqr}} \leq \overline{V}^{\text{sqr}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.14a)$$

$$I_{u,t}^{\text{sqr}} \leq \overline{I}_u^{\text{sqr}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (3.14b)$$

The set of network lines is actually identical to set $\mathcal{U}$. There are $|\mathcal{U}|$ lines in the radial distribution network due to the existence of $|\mathcal{U}| + 1$ electrical nodes. The additional node is the slack bus and does not host any user.

$\mathcal{C}_u$ is the set of children of user u in the tree graph of the electrical network. Similarly, $\mathcal{A}_u$ is the set of ancestors of user u . Due to the tree structure of the graph, there is always a single ancestor to u (a_u).

Line u is the line reaching user u from its ancestor with impedance $Z_u = R_u + jX_u$. The DistFlow model is implemented as:

$$P_{u,t}^{\text{inj}} + P_{u,t}^{\text{line}} - R_u I_{u,t}^{\text{sqr}} = \sum_{c \in \mathcal{C}_u} P_{c,t}^{\text{line}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.15a)$$

$$Q_{u,t}^{\text{inj}} + Q_{u,t}^{\text{line}} - X_u I_{u,t}^{\text{sqr}} = \sum_{c \in \mathcal{C}_u} Q_{c,t}^{\text{line}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.15b)$$

$$V_{u,t}^{\text{sqr}} = V_{a_u,t}^{\text{sqr}} - 2(R_u P_{u,t}^{\text{line}} + X_u Q_{u,t}^{\text{line}}) + (R_u^2 + X_u^2) I_{u,t}^{\text{sqr}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.15c)$$

$$I_{u,t}^{\text{sqr}} V_{a_u,t}^{\text{sqr}} \geq P_{u,t}^{\text{line}^2} + Q_{u,t}^{\text{line}^2} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.15d)$$

where equations (3.15a) and (3.15b) represent power balance at each node u , with P_u^{line} and Q_u^{line} the active and reactive powers flowing in line u . Equation (3.15c) and (3.15d) determine respectively the voltage drop and the squared current in each line u .

The variables for the slack bus are defined as,

$$V_{\text{slack},t}^{\text{sqr}} = 1 \quad \forall t \in \mathcal{T}, \quad (3.16a)$$

$$P_{\text{slack},t}^{\text{inj}} = \sum_{c_s \in \mathcal{C}_{\text{slack}}} P_{c_s,t}^{\text{line}} \quad \forall t \in \mathcal{T}, \quad (3.16b)$$

$$Q_{\text{slack},t}^{\text{inj}} = \sum_{c_s \in \mathcal{C}_{\text{slack}}} Q_{c_s,t}^{\text{line}} \quad \forall t \in \mathcal{T}. \quad (3.16c)$$

The maximum of $P_{\text{slack},t}^{\text{inj}}$ corresponds to the peak power of the community.

These power flow variables and constraints are linked to those of the community problem with:

$$P_{u,t}^{\text{inj}} = e_{u,t}^{\text{ret}} + e_{u,t}^{\text{com}} - i_{u,t}^{\text{ret}} - i_{u,t}^{\text{com}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.17)$$

$$Q_{u,t}^{\text{inj}} = Q_{u,t}^{\text{PV}} + Q_{u,t}^{\text{BSS}} - Q_{u,t}^{\text{fix}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.18)$$

where $Q_{u,t}^{\text{fix}}$ is the fixed reactive part of the load profile of u . $Q_{u,t}^{\text{PV}}$ and $Q_{u,t}^{\text{BSS}}$ are the reactive power injection or withdrawal of the PV and battery inverters. To model technical and regulatory limitations, the magnitudes of these variables are bounded to 30% of the current power flowing through the inverter:

$$-0.3P_{u,t}^{\text{PV}} \leq Q_{u,t}^{\text{PV}} \leq 0.3P_{u,t}^{\text{PV}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (3.19a)$$

$$-0.3(P_{u,t}^{\text{cha}} + P_{u,t}^{\text{dis}}) \leq Q_{u,t}^{\text{BSS}} \leq 0.3(P_{u,t}^{\text{cha}} + P_{u,t}^{\text{dis}}) \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (3.19b)$$

The total active power losses in the system (P_t^{loss}) for each time step are:

$$P_t^{\text{loss}} = \sum_{\forall u \in \mathcal{U}} R_u I_{u,t}^{\text{sqr}} \quad \forall t \in \mathcal{T}. \quad (3.20)$$

Indeed, with equations (3.15a), (3.17), and (3.4a), equation (3.16b) can be rewritten as,

$$P_{\text{slack},t}^{\text{inj}} = \sum_{\forall u \in \mathcal{U}} (i_{u,t}^{\text{ret}} - e_{u,t}^{\text{ret}}) + P_t^{\text{loss}} \quad \forall t \in \mathcal{T}.$$

3.1.1.5 Output of the first problem

After computing the maximum profit of the community as a whole, the solution to this problem is used as parameters for the second step of optimization to ensure a fair sharing of the costs and benefits generated by the community. Both investments and energy usage are considered, so every community member gets billed fairly. The power flows in the network are, at this point, totally fixed, as well as the peak of the community whose bill will have to be shared among participants.

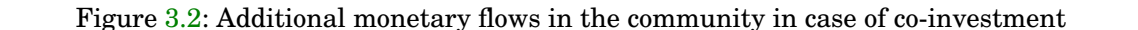
To compute the energy bills of the members, the CO has to consider all the power exchanges with the grid and the community during the simulation horizon as well as the battery charging and discharging, which is also valued.

One of the goals of the second optimization problem (3.21) is to account for the distribution of the co-invested benefits. The distribution key for this is defined by a simple scheme and is computed after solving (3.1) to be used as parameters in (3.21):

- If a participant has invested and there is an investment at its location, he will have priority over it;
- If a participant has more to invest than what is needed at its location, the excess goes to a community investment pool;
- If a participant has less to invest than needed at its location, the community investment pool pays the remaining.

Two parameters are computed for each participant, with a procedure illustrated in Figure 3.2:

- i_u the share of u in the community budget pool;
- k_u the portion of DER investment at node u that has been paid for by the community pool.



Algorithm 3.1 shows how the profit from energy exported by co-invested assets to the grid ($B^{\text{gr}, \text{pool}}$) and the power exported from the community through the pool ($e^{\text{com}, \text{pool}}$, which is negative for imports) are computed. The grid export will then be valued at a known price ($\pi^{\text{i}, \text{ret}}$ or $\pi^{\text{e}, \text{ret}}$) and the power exported to the community ($e_{u,t}^{\text{com}, \text{pool}}$) will then be valued in the profit sharing problem. The total profit will then be shared proportionally to each user's share in the community budget pool (i_u).

```

1 for  $u$  in  $\mathcal{U}$  do
2   for  $t$  in  $\mathcal{T}$  do
3     if  $k_u \neq 0$  then
4       // Net production of co-invested asset:
5        $e_{u,t}^{inv} = C_u^{PV} \bar{p}_{u,t}^{PV} + P_{u,t}^{dis}(C_u^{BSS}) - P_{u,t}^{cha}(C_u^{BSS})$ ;
6       // Net export of user  $u$ :
7        $e_{u,t} = e_{u,t}^{ret} + e_{u,t}^{com} - i_{u,t}^{ret} - i_{u,t}^{com}$ ;
8
9       // 1. User self-consumes or self-produces:
10      if  $(e_{u,t}^{inv} > 0 \& e_{u,t} < 0)$  or  $(e_{u,t}^{inv} < 0 \& e_{u,t} > 0)$  then
11         $e_{u,t}^{com,pool} = k_u e_{u,t}^{inv}$ ;
12        // Valued at community price
13      end
14      // 2. Net consumption and user imports:
15      else if  $(e_{u,t}^{inv} < 0 \& e_{u,t} < 0)$  then
16         $e_{u,t}^{com,pool} = k_u e_{u,t}^{inv} i_{u,t}^{com} / (i_{u,t}^{ret} + i_{u,t}^{com})$ ;
17         $B_{u,t}^{gr,pool} = \pi_t^{i,ret} k_u \Delta T e_{u,t}^{inv} i_{u,t}^{ret} / (i_{u,t}^{ret} + i_{u,t}^{com})$ ;
18      end
19      // 3. Net production and user exports:
20      else if  $(e_{u,t}^{inv} > 0 \& e_{u,t} > 0)$  then
21         $e_{u,t}^{com,pool} = k_u e_{u,t}^{inv} e_{u,t}^{com} / (e_{u,t}^{ret} + e_{u,t}^{com})$ ;
22         $B_{u,t}^{gr,pool} = \pi_t^{e,ret} k_u \Delta T e_{u,t}^{inv} e_{u,t}^{ret} / (e_{u,t}^{ret} + e_{u,t}^{com})$ ;
23      end
24    end
25  end
26 end
27 return  $B^{gr,pool}, e^{com,pool}$ ;

```

Algorithm 3.1: A *posteriori* computation of the pool profit and coinvested assets exports ($B^{gr,pool}$ and $e_{u,t}^{com,pool}$).

This algorithm computes for each time step the net production of the jointly invested asset ($e_{u,t}^{inv}$) and the net export ($e_{u,t}$) at the node where the investment is located. Then, three cases can occur:

- (a) If the asset produces and the user consumes (resp. the asset consumes and the user produces), the part (k_u) of energy produced (resp. consumed) by the pool investment will be valued at community price in the next problem. Indeed this energy is beneficial for the user due to the co-investment and the community exchanges.
- (b) If the asset and the users both have a negative net injection, both are consuming energy. This will be valued at grid tariff for a portion equal to $i_{u,t}^{ret} / (-e_{u,t})$

and the rest is considered as community exchanged and will be valued in the next problem.

- (c) In the last case, both have a positive net injection. Profits are computed similarly to case b with export prices.

3.1.2 Benefits distribution among participants

Using the results of (3.1), a second optimization problem is solved *a posteriori* to share the benefits of the community between its participants. At this point, all the exchanges and investments have been computed in the first optimization model and are fixed. This results in a fixed community benefit to be shared.

In practice, this would be done during operation. To share all the additional benefits generated by the community, fair internal exchange prices need to be determined for the community. These prices are the main decision variables of this problem and will be used for both direct exchanges between members and return on investment from a co-invested asset to the participants of the community budget pool.

The results of this problem are highly dependent on the distribution key used by the community operator. The discussion of different sharing mechanisms and their fairness is outside the scope of this work. Here, only a simple scheme will be proposed: maximizing the minimum improvement in the user's benefit compared to a case without community involvement.

$$\begin{aligned}
 & \max && \text{Minimum additional member profit (3.22)} && (3.21) \\
 & \text{subject to} && \text{Minimum profit (3.23),} \\
 & && \text{Profit sharing constraints (3.24) – (3.27),} \\
 & && \text{Price constraints (3.28) – (3.29).}
 \end{aligned}$$

The objective function of this problem is defined as,

$$F = \alpha - N(\pi^{com}), \quad (3.22)$$

where $N(\pi^{com})$ the price regularization term. The main term α represents the minimum additional benefit of a community member, compared to its standalone situation,

$$B_u \geq B_u^{\text{SU}} + \alpha, \quad \forall u \in \mathcal{U}. \quad (3.23)$$

The parameter B_u^{SU} is the optimal equivalent yearly profit of user u computed with a problem similar to (3.1) where the community does not exist. B_u defines

the optimal yearly profit of user u in the community, computed from the yearly operational cost of electricity (B_u^{energy}), negative peak profit (B_u^{peak}), negative profit to return to co-investors (B_u^{return}), and profit from co-investing (B_u^{coinvest}):

$$B_u = B_u^{\text{energy}} + B_u^{\text{peak}} + B_u^{\text{return}} + B_u^{\text{coinvest}}. \quad (3.24)$$

$$\begin{aligned} B_u^{\text{energy}} = & -\Delta_T \sum_{t \in \mathcal{T}} (\pi_t^{\text{i,ret}} i_{u,t}^{\text{ret}} - \pi_t^{\text{e,ret}} e_{u,t}^{\text{ret}} \\ & + \gamma^{\text{com}} (e_{u,t}^{\text{com}} + i_{u,t}^{\text{com}}) - \pi_t^{\text{com}} (e_{u,t}^{\text{com}} - i_{u,t}^{\text{com}}) \\ & + \gamma_u^{\text{bss}} (\eta_u^{\text{cha}} P_{u,t}^{\text{cha}} + P_{u,t}^{\text{dis}} / \eta_u^{\text{dis}})). \end{aligned} \quad (3.25)$$

This equation differs from (3.3) by the term which represents the monetary exchanges between community members. Variable π_t^{com} represents the price at which the energy will be sold within the community at time period t of the simulation. The peak of the community ($\bar{p}$) is shared between members using equation (3.27), each member is then accountable to pay an amount $-B_u^{\text{peak}}$ defined by its positive share of the peak ($\bar{p}_u$) and the peak price (π^{peak}).

$$B_u^{\text{peak}} = -\pi^{\text{peak}} \bar{p}_u, \quad (3.26)$$

$$\bar{p} = \sum_{u \in \mathcal{U}} \bar{p}_u. \quad (3.27)$$

This problem aims to optimize the personal share of each user by adapting the community energy prices π_t^{com} . To have sensible results, additional constraints are added to the prices to make them coherent,

$$\pi_t^{\text{com}} \in [\pi_t^{\text{e,ret}} + \gamma^{\text{com}}; \pi_t^{\text{i,ret}} - \gamma^{\text{com}}] \quad \forall t \in \mathcal{T}, \quad (3.28)$$

$$N(\pi^{\text{com}}) = \omega_N \sum_{t \in \mathcal{T}} (\pi_t^{\text{com}} - (\pi_t^{\text{i,ret}} + \pi_t^{\text{e,ret}})/2)^2, \quad (3.29)$$

where (3.28) ensures that there is no time period where the community prices are less interesting than the grid fees, be it for the buyer or seller. Equation (3.29) defines the regularization term, where ω_N is a weight factor such that $\alpha \gg N(\cdot)$. The role of this term is to reduce the variability in the price by driving it closer to a constant value.

The additional profit of u that needs to be returned to the pool (B_u^{return}) and the additional profit of a pool participant (B_u^{coinvest}) can be computed using the profit made by the asset from grid exchanges with fixed price ($B_{u,t}^{\text{gr,pool}}$) and the quantities exchanged inside the community ($e_{u,t}^{\text{com,pool}}$) with a price to be defined (π_t^{com}).

$$B_u^{\text{return}} = -\sum_{t \in \mathcal{T}} (\Delta_T \pi_t^{\text{com}} e_{u,t}^{\text{com,pool}} + B_{u,t}^{\text{gr,pool}}) \quad \forall u \in \mathcal{U}, \quad (3.30)$$

$$B_u^{\text{coinvest}} = -i_u \sum_{u \in \mathcal{U}} B_u^{\text{return}} \quad \forall u \in \mathcal{U}. \quad (3.31)$$

3.1.3 Case study

The problem is run on a benchmark Dickert-LV network [Dickert et al., 2013] consisting of three feeders for a total of 45 buses. The community spans the whole network and involves 39 residential users, five industrial consumers, and 1 PCC (CO), which is the slack bus in the OPF model. The role of this CO is to provide for the net imbalance in the community and to compensate for the power losses in the network.

Range	Linetype	Customer	Case	Voltage level	Line length
Middle	Cable	Multiple	Good	230 V	40 m

Table 3.2: Parameters of the dickert LV-Network.

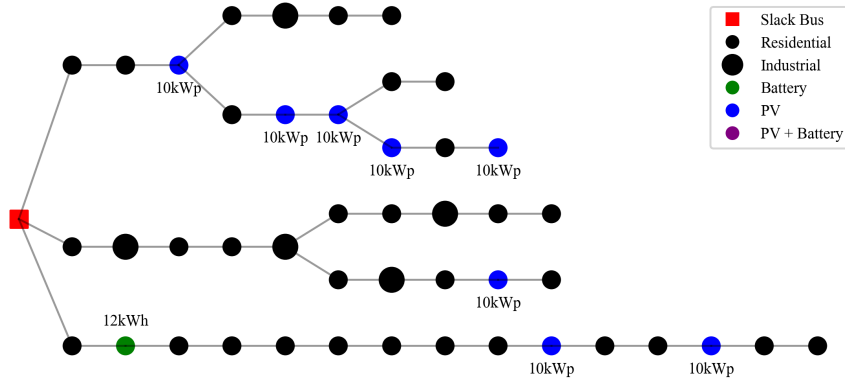


Figure 3.3: Initial network of the EC.

Before new investments, the network already includes eight PV installations and one BSS. The locations are shown in Fig. 3.3. In this case study, all users have an equal price for energy sold to and bought from the retailer, which is constant for the whole simulation, $\pi^{e,ret} = 0.05\text{€/kWh}$ and $\pi^{i,ret} = 0.40\text{€/kWh}$. The fee imposed by the CO is $\gamma^{com} = 0.01\text{€/kWh}$.

3.1.4 Simulation

The simulation is run with a one-hour resolution using 12 representative days each divided in 24 periods. This models a full year with one day per month weighted with w_{d_t} . Data for hourly consumption and PV production profiles are extracted from Open Power System Data [2018]. The operational planning costs of each day is weighted to have a total amount corresponding to the annual OPEX of the system. These days are all modeled and solved by a single optimization model to have common variable DER sizes determining the yearly CAPEX of each user. The problem

is implemented using the Julia JuMP package [Lubin et al., 2023] and solved with the Gurobi solver [Gurobi Optimization, LLC, 2023]. The results of the model are then run through the Pandapower python library [Thurner et al., 2018] to verify the grid state at every timestep.

3.1.5 Scenarios

Three investment scenarios are compared to assess the impact of the EC on its members and the grid.

3.1.5.1 Individual model

In this model, each user can only invest at his electrical node with an amount limited by his budget,

$$\Pi_u^{\text{inv}} \in [0; \bar{B}_u] \quad \forall u \in \mathcal{U}, \quad (3.32)$$

and community exchanges are not allowed,

$$i_{u,t}^{\text{com}} = e_{u,t}^{\text{com}} = 0 \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (3.33)$$

The second optimization problem is thus irrelevant since each user can only exchange energy with the grid (*i.e.* its retailer). The value of B_u^{energy} obtained in this case defines, for each user u , the parameter B_u^{SU} used for the benefit distribution in the next scenarios.

3.1.5.2 REC with self-investments

In this scenario, each member's budget can only be used for personal DERs, ensured by constraint (3.32). A community is created, and constraint (3.33) is thus relaxed. This represents a community where each participant can invest in devices located behind its own energy meter. The benefits of the community mostly come from the gap between the import ($\pi^{\text{i,ret}}$) and export ($\pi^{\text{e,ret}}$) prices. When the community price is set inside this range, the community creates value for both the buyer and the seller. Equation (3.24) is reduced to

$$B_u = B_u^{\text{energy}} + B_u^{\text{peak}} \quad \forall u \in \mathcal{U}, \quad (3.34)$$

which does not account for DER co-investments.

3.1.5.3 REC with co-investments

All the community members can pool their capital together, the CO can optimally size and place the new DERs. This scenario uses the full model developed in Sections 3.1.1 and 3.1.2.

3.1.6 Results

The results show that the community can create benefits for all users. The optimal investments considering the three scenarios are compared in Fig. 3.4, which shows that the individual and community with self-investment models have similar results.

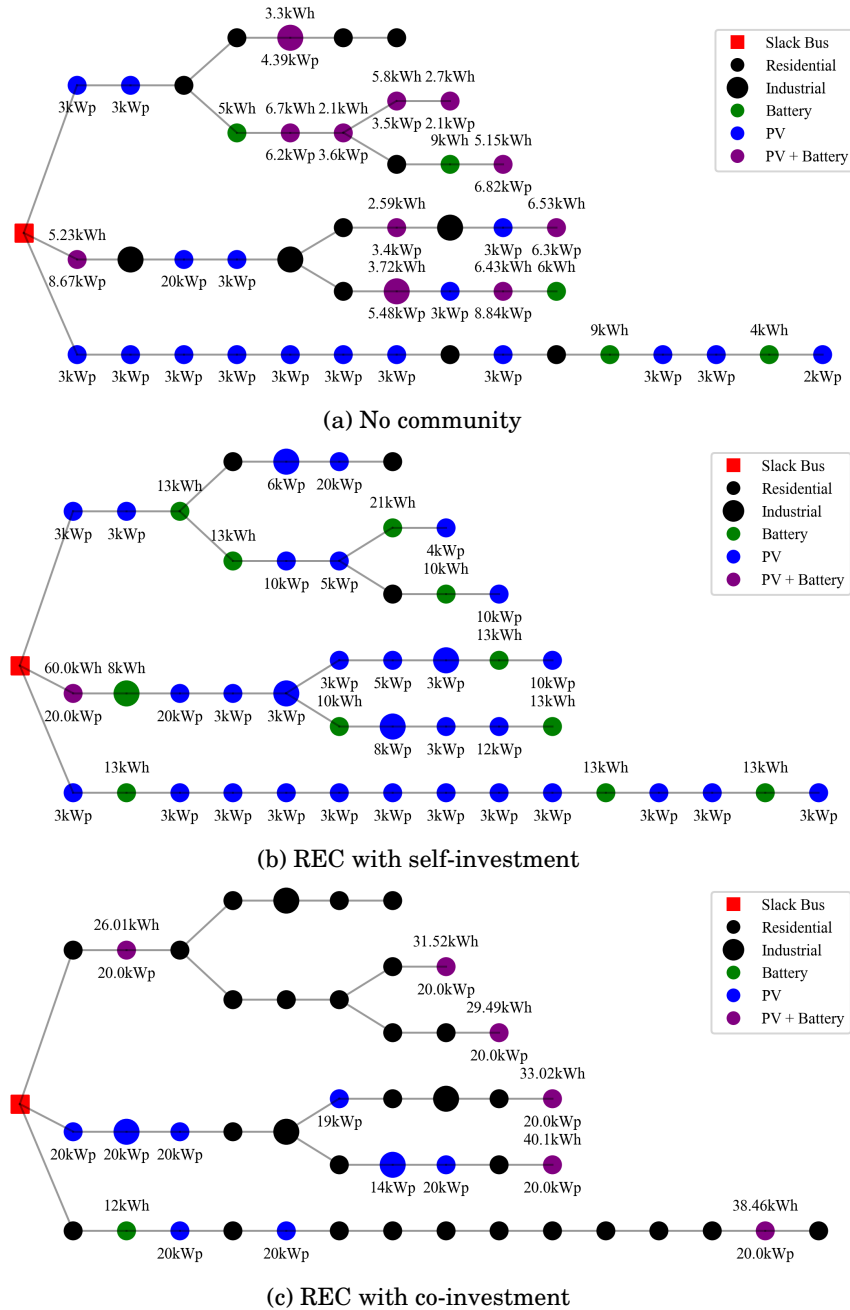


Figure 3.4: Optimal new investment locations and sizes for the three scenarios.

This is because the personal budget is highly limiting in these two first cases. The optimum of each member primarily depends on its own energy usage. In scenario 2, we can see an increase in BSS investment. This can be explained by the added profitability of maximized collective self-consumption for the community. These results are defined by the first step, which considers the community as a whole. The results for the second scenario show a totally different approach. Co-investment allows users to invest in fewer assets with a bigger capacity to take advantage of the economies of scale.

One hypothesis made in problem (3.1) was that the community would be accountable for loss compensation. For this assumption to be realistic, the CO would need enough income to pay for this energy. In the basic simulation design with $\gamma^{\text{com}} = 0.01$ €/kWh, this hypothesis is already validated with 3246 € of CO revenue per year for 1115 € of costs for losses in scenario 2 and 3160 € of revenue for 918 € of costs for scenario 3.

Figure 3.5 shows the intra-community energy prices for the 12 representative days. With the proposed profit-sharing strategy, the prices tend to be equal to their upper or lower bound, depending on the excess or deficit of power. Some prices are equal to the mean value due to the regularization term and the absence of energy to be sold in the community. In the scenario with co-investment, the prices are higher due to the necessity for co-investors to have a good return on investment.

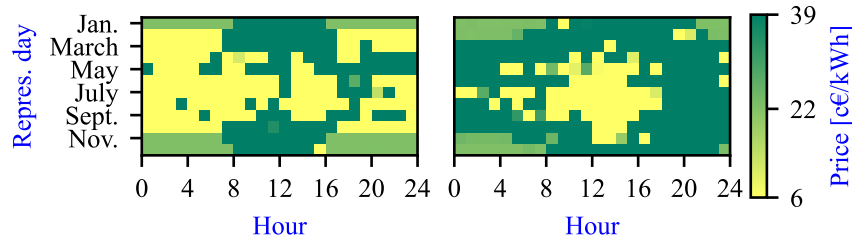


Figure 3.5: Price in the community for energy exchange during 12 representative days, REC without co-investment on the left, REC with co-investment on the right.

The proposed model ensures profitability for all community members compared to their optimal behavior in the same conditions without the community's existence. Figure 3.6 shows how much the profit can be increased with the two community scenarios.

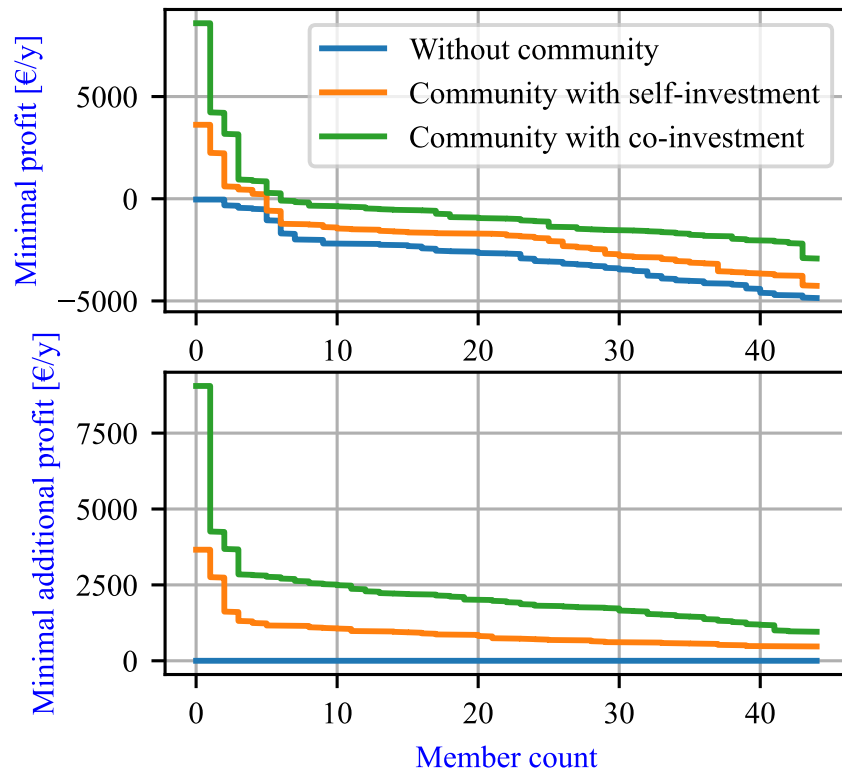


Figure 3.6: Community members' increase in benefits.

Table 3.3: Summary of network usage for the three scenarios.

Case	Ind. model	EC self-inv.	EC co-inv.
Total losses [MWh]	3.04	2.79	2.29
Max voltage [pu]	1.03	1.04	1.05
Min voltage [pu]	0.95	0.95	0.95
Max line loading [%]	83.54	84.83	71.21
Self-consumption [%]	37.49	88.23	81.38
Self-sufficiency [%]	70.95	73.87	79.48
PCC peak [kW]	160.56	56.58	69.50

Table 3.3 summarizes the impact of the EC on the distribution grid. For the self-investment models, the results are comparable as the topology of new investments is very similar.

The third scenario's improvement in most aspects is achieved through increased flexibility in DER placement and sizes. This is because the CO can place the assets at the optimal location, with decreased limitation on system size. However, this larger overall DER capacity slightly increases maximum voltage and decreases self-consumption.

Case	Ind. model	EC self-inv.	EC co-inv.
Total cost [k€/y]	147	108	97
Total OPEX [k€/y]	123	77	66
Total CAPEX [k€]	486	607	636
PV inv. [kWp]	157	217	260
BSS inv. [kWh]	119	172	253
Min additional profit [€/y]	0	483	938

Table 3.4: Summary of financial results for the three scenarios.

From these results, we can see that the community can improve the users' benefits in both cases. The co-investment scenario allows the minimal additional benefit (α) to be nearly doubled compared to the self-investment.

The total investment of the community is also increased in the co-investment community. This is mainly due to specific users for which the co-investment can bring new possibilities. Indeed, members' budgets can be greater than the maximum investment at one location, which is limited by maximum size for BSS and PV installation. Bigger and more profitable investments can be made from this budget. Some other members which did not have enough budget to realize new investments on their own, due to fixed costs and minimal capacities, now can use co-investment to generate profit.

3.2 CARBON EMISSIONS OF RENEWABLE ENERGY COMMUNITIES

This second section proposes a way to assess the carbon footprint of such a community. It will reuse a similar model with modified objectives to challenge some of the numerical hypothesis and assess the sensibility of the results to:

- The objective, price or CO₂ minimization;
- The PV potential and the carbon intensity at the location;
- The members' investment budgets;
- The intra-community exchange fees imposed by the CO.

3.2.1 Problem formulation

The models are very similar to the previous ones. CO₂ constraints were added for the sake of completeness. The optimization problem, in a descriptive form, is as follows,

$$\begin{aligned}
& \min && \text{Cost/CO}_2 \text{ footprint of the REC (3.36)/(3.37)} && (3.35) \\
& \text{subject to} && \text{Cost and CO}_2 \text{ constraints (3.38) – (3.40),} \\
& && \text{Price and exchange constraints (3.3) – (3.5),} \\
& && \text{Devices constraints (3.6) – (3.7g),} \\
& && \text{Sizing constraints (3.8) – (3.10d),} \\
& && \text{Power flow equations (3.14a) – (3.20).}
\end{aligned}$$

3.2.1.1 Objectives

We consider two options for the objective function, to minimize either the total REC's cost or the carbon footprint, considering impacts from both investments and operation,

$$\text{Total cost} = \sum_{u \in \mathcal{U}} \left(\Pi_u^{\text{com}} + \frac{\Pi_u^{\text{inv}}}{I^{\text{hor}}} \right) + \pi^{\text{peak}} \bar{p} + \pi_t^{\text{i,ret}} \sum_{t \in \mathcal{T}} w_{dt} \Delta_T P_t^{\text{loss}}, \quad (3.36)$$

$$\text{Total CO}_2 = \sum_{u \in \mathcal{U}} \left(K_u^{\text{op}} + \frac{K_u^{\text{inv}}}{I^{\text{hor}}} \right) + \kappa_t^{\text{gri}} \sum_{t \in \mathcal{T}} w_{dt} \Delta_T P_t^{\text{loss}}. \quad (3.37)$$

Equation (3.36) models the total users' operational and weighted investment costs, peak price for the community and network losses, valued at grid import price. Equation (3.37), models the CO₂ emissions of the community.

Each user's operational cost for the community still writes as:

$$\begin{aligned}
\Pi_u^{\text{com}} = & \Delta_T \sum_{t \in \mathcal{T}} w_{dt} \left(\pi_t^{\text{i,ret}} i_{u,t}^{\text{ret}} - \pi_t^{\text{e,ret}} e_{u,t}^{\text{ret}} + \gamma^{\text{com}} (e_{u,t}^{\text{com}} + i_{u,t}^{\text{com}}) \right. \\
& \left. + \gamma^{\text{bss}} \left(\eta_u^{\text{cha}} P_{u,t}^{\text{cha}} + P_{d,t}^{\text{dis}} / \eta_u^{\text{dis}} \right) \right) \quad \forall u \in \mathcal{U}.
\end{aligned} \quad (3.38)$$

To compute the CO₂ intensity of the community energy use, several sources of emissions are considered:

- CO₂ intensity of energy taken from the grid (κ^{gri}) defines the operational emissions (K^{op}):

$$K_u^{\text{op}} = \Delta_T \sum_{t \in \mathcal{T}} w_{dt} \kappa_t^{\text{gri}} e_{u,t}^{\text{ret}} \quad (3.39)$$

- CO₂ emissions due to the new investments of the community (K^{inv}) depend on the life cycle assessment for PV (κ^{PV}) and battery storage system (BSS) (κ^{BSS}) installations and their sizes:

$$K_u^{\text{inv}} = \kappa^{\text{PV}} C_u^{\text{PV}} + \kappa^{\text{BSS}} C_u^{\text{BSS}} \quad (3.40)$$

- CO₂ cost for energy losses in the LV network, with a CO₂ intensity equal to grid emissions, because these losses will be compensated by external energy.

3.2.1.2 Constraints

The constraints used in the optimization model have been detailed in the previous section and are simply reused with a new objective. Problem (3.35) lists all the reused equations.

3.2.2 Experimental setup

3.2.2.1 Case study

The problem is run on a benchmark Dickert-LV network [Dickert et al., 2013] consisting of three feeders for a total of 11 buses. The community spans the whole network and involves nine residential users, one industrial consumer, and PCC, which is the slack bus in the OPF model. The role of this slack bus is to provide for the net imbalance in the community and to compensate for the power losses in the network.

3.2.2.2 Simulation

The simulation is run with a one-hour resolution using 12 representative days each divided into 24 periods of one hour. This models a full one-year period with one day per month weighted with values of w_{dt} . Data for yearly consumption and PV production come from Open Power System Data [2018]. The operational planning costs of each day are weighted to represent the annual OPEX of the system. Yearly results are obtained from a single optimization model to determine unique DER sizes and the annual CAPEX of each user.

This Mixed-Integer Second-Order Cone Problem (MISOCP) is implemented using Julia JuMP package [Lubin et al., 2023] and solved by Gurobi [Gurobi Optimization, LLC, 2023]. The exact relaxation of the power flow constraints is checked *ex post*.

3.2.2.3 Scenarios

The three investment scenarios presented in the previous section are compared to assess the cross impact of the EC on members' energy bills and carbon footprints.

IND. *Individual model* described in section 3.1.5.1.

EC *REC with self-investment* described in section 3.1.5.2.

CO. *REC with co-investment* described in section 3.1.5.3.

3.2.2.4 Simulation parameters

This study is initially based on a Belgian case study, where the annual CO₂ emissions for the electricity taken from the grid is 175 g/kWh [Electricity Maps, 2024]. The new investments have a fixed CO₂ intensity per capacity: 1590 kg/kWp for PV [de Wild-Scholten, 2013], 106 kg/kWh for battery capacity [Emilsson and Dahllöf, 2019] and 70 kg/kW for each of the inverters [Tschümperlin L, 2016]. All these values come from life-cycle assessments for the device and depend on its capacity.

The price of imports from the grid (retailer) is valued at 40 c€/kWh and exports are sold at 10 c€/kWh, the base community fee (γ^{com}) imposed by the CO is 1 c€/kWh for both import and exports, battery fees (γ^{bss}) are valued at the same amount. Most community members willing to create the community also have an important investment budget for renewable energy resources summing up to 156 k€, including an important capital from the industrial member.

3.2.3 Results

Table 3.5 summarizes all six simulation results. The price-optimal results align with those from the previous section, with a REC model delivering savings and better grid performance metrics that could be further improved through the co-investment mechanism within the community. Similarly, the CO₂-optimal results are enhanced by the community and further improved through the co-investment mechanism.

Table 3.5: Summary of optimal results for the two objectives and three frameworks.

Optimum	Price			CO ₂		
Case	Ind.	EC	Co.	Ind.	EC	Co.
Cost [k€/y]	22.9	19.4	18.0	25.6	20.9	20.5
π^{com} [k€/y]	15.6	11.4	10.1	18.7	13.1	11.8
Π^{inv} [k€]	145.0	156.0	156.0	117.8	156.0	154.2
CO ₂ [t/y]	12.3	10.7	10.6	11.5	10.15	9.9
K^{op} [t/y]	7.55	5.1	4.6	8.2	5.2	4.5
K^{inv} [t]	95.1	112.8	119.0	65.6	98.4	107.9
PV [kWp]	55.2	65.0	68.4	36.6	54.7	60.0
BSS [kWh]	27.7	40.0	44.3	38.7	60.7	67.0
Self suff [%]	52.4	70.6	73.8	49.1	68.9	76.2
Self cons [%]	71.1	83.2	83.6	92.5	99.9	99.9
Peak [kW]	32.7	15.2	14.8	46.8	23.82	21.43
Losses [kWh]	296.0	237.0	224.9	252.1	211.3	167.6

Comparing the two objectives, we can see that the results are not drastically different. In the optimal case (co-investment), minimizing the CO₂ instead of the energy bill provides an emission decrease of 700 kg/y for the whole community at an additional cost of 2500 €/y. More generally, going from objective (3.36) to (3.37) provides results around 5% better in terms of CO₂ emissions yet requires 10% higher costs.

For CO₂ minimization, the BSSs are more favorable compared to price minimisation, enhancing self-sufficiency with a lower investment penalty. Self-consumption significantly increases under the second objective because the excess energy, which is not valued, is curtailed to prevent increased grid losses.

The REC results for the following sections are always carried out with the co-investment scenario, as it provides the best results overall.

3.2.3.1 *Impact of the location of the REC*

In these simulations, two key parameters are analyzed: the CO₂ intensity of the energy mix from the grid and the photovoltaic capacity of the location. The benefits of the REC when minimizing CO₂ emissions highly depend on both the efficiency of new investments and how much emissions the DER can avoid.

Here, three new models are solved for each pair of parameters with objective (3.37).

- Scenario **A**, is the individual model with no investment budget, to show what the starting point of the member would be with grid import only.
- Scenario **B** restores the available budgets, thus corresponding to the individual model from section 3.1.5.1.
- Scenario **C** is the full problem with REC and co-investment.

Figure 3.7 shows the optimal CO₂ emissions of the community for all considered energy mixes and PV potentials. As expected, the best results are obtained with higher values for the PV potential, because the same investments enable a larger energy output. The main factor for the carbon efficiency of the community's energy usage is the CO₂ emissions for the grid imports. This value directly defines all the model's operational emissions.

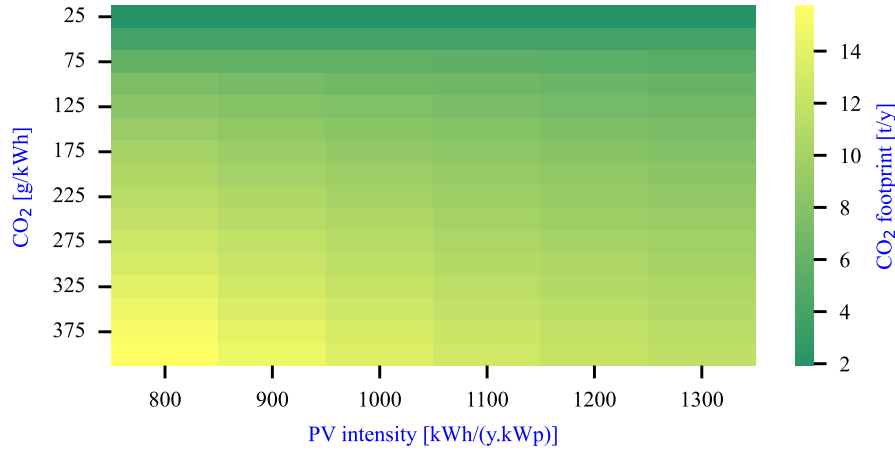


Figure 3.7: Optimal CO₂ emissions of the REC framework (scenario C).

Figure 3.8 shows the impact of the community framework. It displays the difference between the optimal footprints in scenario B and C. Results show a positive impact of the REC when the grid CO₂ intensity and the PV potential increase. Indeed, in these cases, local exchanges avoid larger amounts of emissions. At very low carbon energy mixes (25-75 g/kWh), the REC cannot further reduce the CO₂ emissions. This is because the energy imported is as clean as the energy produced by the decentralized PV resources.

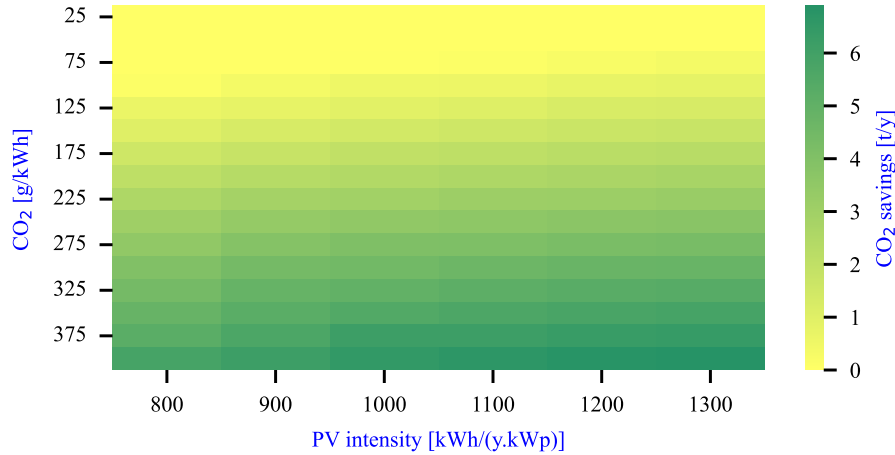


Figure 3.8: CO₂ savings of the REC framework over the individual (B-C).

Table 3.6 depicts CO₂ emission savings for chosen countries in the EU with defined parameters for mean solar potentials from Suri et al. [2007] and average values for energy mixes in 2024 from Electricity Maps [2024]. As expected, a country with high solar and CO₂ intensity can reach the highest emissions savings, with the impact of grid import emissions prevailing over PV potential.

Country	CO ₂ [*]	PV pot. [g/kWh]	A-B [t/y]	B-C [t/y]	A-C [t/y]
Sweden	25	800	0.0	0.0	0.0
France	50	1100	0.0	0.0	0.0
Spain	150	1300	2.5	1.77	4.27
Belgium	175	800	1.99	1.57	3.56
Italy	350	1200	9.88	5.79	15.67
Germany	400	900	10.12	6.15	16.27

Table 3.6: Rounded parameters and CO₂ savings for selected European countries.

[*]: [kWh / (y.kWp)]

A-B: CO₂ savings from investments only.

B-C: CO₂ savings from REC creation.

A-C: CO₂ savings overall.

3.2.3.2 Sensitivity to available budgets

The magnitude of the emissions and financial savings highly depend on the investment budgets of the members. Previous results were obtained using an average budget of 15.6 k€ per member. In this section, financial and emission optimization results considering increasing member investment capacity are shown. The case study is in Belgium and allows co-investment in the REC. Figure 3.9 shows how the minimal costs and emissions evolve with varying budgets.

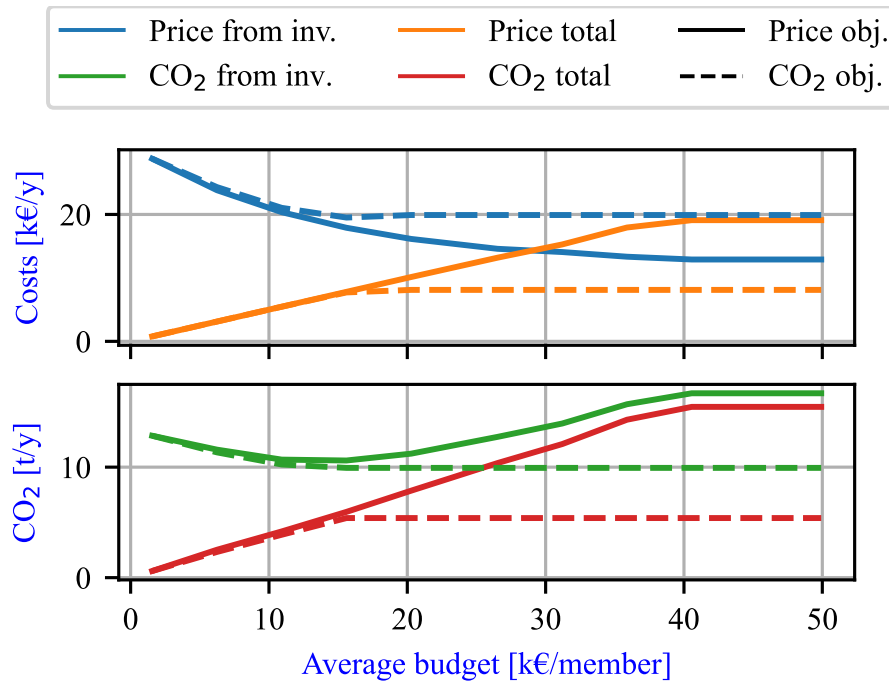


Figure 3.9: Results with varying investment budgets.

These results show that with a low budget, the price and CO₂ minimization provide the same results. All members' budget is invested in renewable to provide maximized self-sufficiency. The minimum for CO₂ emissions is reached with a budget of approximately 15 k€/member, over this value, marginal investments do not save CO₂ and the dotted curves flatten. Minimal energy costs are still decreased until a budget of 40 k€/member is reached, which results in increased CO₂ emissions due to over-investment.

The maximal useful budget value mainly depends on the ratio between capital and operational weight factors in the objective. Indeed, the ratio between CO₂ costs for grid imports and investments ($\kappa^{\text{gri}}/\kappa^{\text{PV or BSS}}$) is lower than for OPEX over CAPEX. Thus, with equal energy output, investing stops being profitable in terms of CO₂ emissions before it stops being profitable for costs.

Members' ability to sell the excess energy to the grid amplifies this difference in capital versus operational trade-offs between both objectives and encourages over-investment by valuing energy surplus.

3.2.3.3 *Impact of community operator fees*

Community operator fees are an impacting parameter of the simulation. They define what value is lost for the community members to the profit of the CO and provide a lever for incentivizing local exchanges. These fees should be well designed to ensure that the CO can pay for all grid-related costs: energy losses, grid tariffs and/or grid rent.

Figure 3.10 shows the impact of γ^{com} for cost minimization in the base Belgian case with co-investment. There is a small dependence of the emissions of CO₂ on the CO fee, they increase slightly until around 13 c€/kWh where internal exchanges stop being profitable due to the internal fees. Additionally, this figure shows that incentivized exchanges provide far better financial results and would promote participation in the REC and the energy transition.

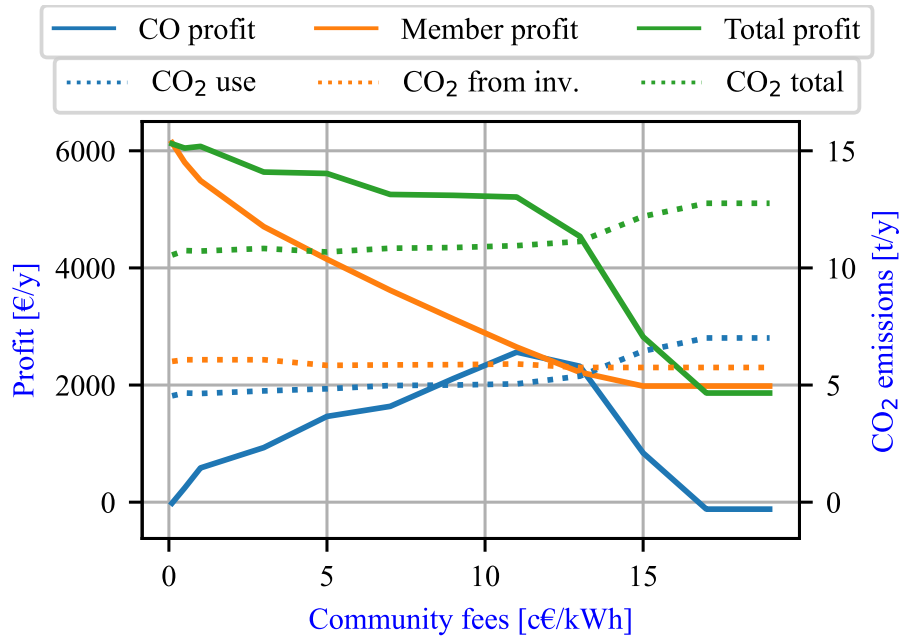


Figure 3.10: Results with varying community exchange fees (γ^{com}).

3.3 CONCLUSION

The two presented works provided motivation for DER investments in REC.

First, considering price-optimality and benefits sharing, section 3.1 showed:

- That the community can provide an incentive for new investments that could provide benefits to all members.
- That a co-investment framework could increase the overall profits and that they could be efficiently shared.
- That the grid can have a lower to same usage with increased distributed generation and adapted control of the assets through the CO.

The second work, presentend in section 3.2, offers some insights into the impact of distributed generation investment capacities on the carbon footprint in renewable energy communities. Based on the results of the case studies, the following conclusions can be drawn:

- Energy communities can provide a beneficial framework for investing in distributed energy resources which is profitable for the transition to low-carbon energy.
- The magnitude of this positive impact highly depends on the solar power generation potential and energy mix of the country where the EC is located.

- Price minimization can provide an adequate proxy and direct motivation for CO₂ efficiency. However, with large investment budgets and important monetary value for selling energy excess, the objectives diverge. The investments for optimal prices are oversized compared to those that minimize CO₂. This can partly be explained by the lack of economies of scale in the CO₂ cost compared to investment. Other main factors include lack of CO₂-valuation of the excess energy and more generally numerical hypothesis on investment costs and emissions.
- Encouraging the internal community exchanges with reduced grid tariffs benefits the community outcome. This value still needs to be chosen adequately. Ensuring correct allocation of the benefits between members and CO, which needs to be incentivized while ensuring correct operation and payment for the grid, is crucial.

Outline

This chapter presents a synthetic residential load profile generator that includes a flexibility component. This tool named *ResFlex* is designed to generate realistic data that will be used in the next chapters.

The long-term planning impact for the community only depends on the production and storage capacities of the member to model a change in power injection. For day-ahead scheduling of the community, flexible load profiles are needed to assess the impact of the community. Indeed, with fixed load profiles in daily operation, the community would simply conduct business as usual with a differentiated tariff for intra-community exchanges.

To model adequately the flexibility potential of residential area, data and models are needed. However, in already available data tools, complete model availability has not been found. In existing datasets, flexibility models were also not present. Some works quantified flexibility potential as a portion of the total power [Allard, 2024]. A more precise model is needed in the subsequent chapters for a two-step model, that first optimizes flexibility for the exchanges and then for external market signals.

This chapter presents ResFlex, a tool to generate residential load profiles with flexible appliances. The output includes a reference load profile and data to remodel all flexible appliances in each house (SoC for EV and temperature dynamics for thermal load).

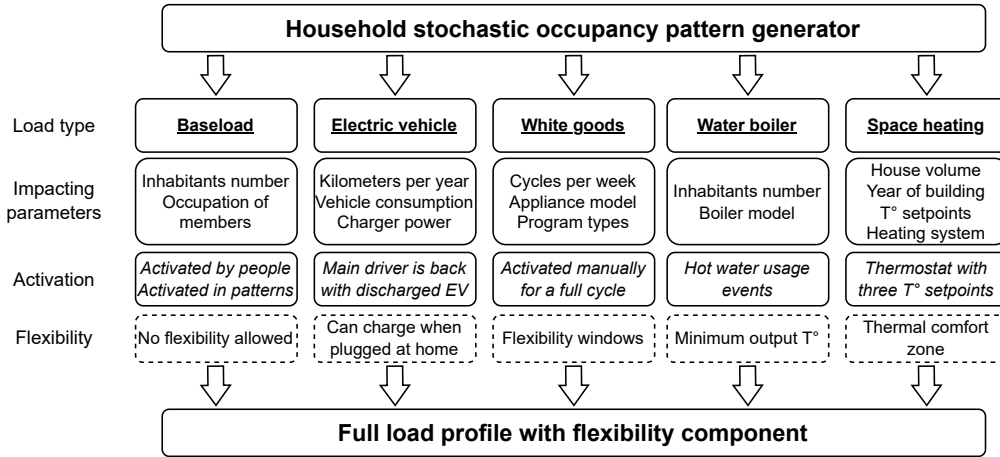


Figure 4.1: Load profile generator organization overview.

The load profile generator is based on a stochastic household occupancy pattern generator [Baetens and Saelens, 2016]. This pattern is used in all parts of the load profile and depend on the input parameters representing the installed appliances.

The goal of this tool is to provide individually coherent profiles that can match a given population.

4.1 FLEXIBLE LOAD PROFILE GENERATOR

As shown in Figure 4.1, the output load is composed of five terms that represent each type of appliance, with a corresponding flexibility component. These components depend on the input parameters, the occupancy pattern generated by Baetens and Saelens [2016], and some external parameters detailed below.

4.1.1 Stochastic occupancy

The weekly occupancy profile depends on the type of resident (Working hours, student, unemployed) and is repeated over the full year. This will be the base data for generating all the different appliances' energy needs.

4.1.2 Baseload

The baseload includes all non-programmable appliances, the lights and other loads plugged through a regular outlet. This load component can be considered as the fixed demand. It would not be realistic to consider consumers who adapt their

baseload to external signals. This category includes most plugged-in appliances: fridges, kitchen appliances, computers, televisions, lights, etc.

All data for the activation of the appliances and power profiles come from [Baetens and Saelens \[2016\]](#), which provides a realistic load profile. When specified, the yearly baseload input parameter is converted to scale this part of the profile.

4.1.3 White goods

White goods, also called wet appliances (washing machines, driers, and dishwashers), can act as time-shiftable controllable loads. Depending on the family size, a number of weekly cycles is assigned to each appliance; this number can be overwritten by an input. Cycles can vary in profile and length according to the appliance model and program. The type of program is randomly selected from predefined data [[Mazidi et al., 2023](#)].

4.1.4 Electric Vehicle

To generate car usage patterns, a main driver attribute is first assigned to one of the household's members. Then, the owner of the car leaving the home for more than 15 minutes is detected in the occupancy pattern, creating a "driving event". This driving event will last until the driver is back.

Depending on the duration of the trip and the annual mileage input parameter, a stochastic length in kilometers is assigned to each driving event. Then, this length is converted to an energy need according to the vehicle's size and then to a return SoC.

The possibility of charging outside the house is also modeled, and this energy usage is removed from the load profile. The probability of a charging event outside the home $P(\text{ch,out})$ increases when the ratio between the required and available energy decreases. The probability $P(\text{ch,out})$ is mathematically defined as:

$$r = \frac{E_j}{E_l} \tag{4.1}$$

$$P(\text{ch,out}) = \begin{cases} 1, & \text{if } r > 1.0 \\ 0, & \text{if } r < 0.05 \\ r, & \text{otherwise.} \end{cases} \tag{4.2}$$

Where the dimensionless ratio r is defined by the energy required for the considered journey E_j in kWh, divided by the available energy in the EV's battery E_l in kWh.

When this happens, the initial vehicle discharge is divided by two. This represents a charging event happening halfway through the trip (*i.e.* at the destination).

When the EV returns home, it is directly plugged in and charged at full power until the maximum SoC is reached; this represents the initial reference load, which mimics the standard use of an EV without smart charging.

4.1.5 Space Heating

A complete house thermal model is considered. Based on the occupancy profile, three temperature setpoints are defined:

- High when someone is present;
- Medium when everybody present is sleeping;
- Low when no-one is home.

The exact temperature level for each setpoint is one of the input parameters. Based on previous work from [Georges et al. \[2013\]](#), each house is classified into one of the five construction year ranges (from before 1945 to after 2008). The corresponding thermal and sizing properties are then determined with some stochasticity: ground surface and air volume, windows surface and wall insulation.

The space heating model can be summarized in four equations:

$$T_{t+1}^{in} = T_t^{in} + \Delta_T k^{th} \cdot (P_t^{hp,th} - P_t^{hp,loss}) \quad (4.3)$$

$$P_t^{hp,th} = COP * P_t^{hp,el} \quad (4.4)$$

$$k^{th} = k_{air}^{th} + k_{env}^{th} \quad (4.5)$$

$$P_t^{loss} = P_t^{exf} + P_t^{cond} - P_t^{irr} \quad (4.6)$$

where t is the considered time step. The subscripts *air* and *env* refer to the modeling of the house, which is considered as 4 walls (with windows), a floor, and a ceiling constituting the envelope of a parallelepiped filled with air. Furthermore,

- T^{in} is the current indoor temperatures,
- k^{th} is the thermal coefficient expressed in K/kWh,
- COP is the coefficient of performance of the HP, $P^{hp,el}$ is its electric power input and $P^{hp,th}$ is its thermal power output,
- $P^{hp,loss}$ are the total thermal losses composed of P^{exf} (air exfiltration), P^{cond} (conduction of the envelope), and P^{irr} (irradiation).

A simple control strategy with temperature dead bands around the setpoint (T^{set}) is then used to convert the temperature profile to a reference power profile. $P^{hp,nom}$ is the nominal power of the HP and $P^{hp,el}$ its instantaneous setpoint.

The model simulates indoor air temperature dynamics based on a simplified thermal representation of a dwelling, where the building is treated as a cube of air and an envelope (walls, floor, ceiling). Considered heat gains and losses are walls conduction, air exfiltration, and solar irradiation. The energy demand associated to the space heating is then derived from this control behavior.

4.1.6 Water Boiler

Resflex includes a complete thermal model of the water tank and boiler [Leclercq et al., 30 June 2024].

$$T_{t+1}^{wb} = T_t^{wb} + \Delta T k^{wb} (P_t^{wb} - P_t^{wb,loss} - P_t^{wb,use})$$

The internal temperature evolution is computed using this thermal capacity of the water volume k^{wb} . The stochastic water usage events depend on the occupancy, they cause power losses due to the lower temperature of the water inlet $P_t^{wb,use}$. The losses through the tank envelope $P_t^{wb,loss}$ are proportional to the difference between the water temperature and the room temperature, which is assumed constant.

The water boiler is either activated at nominal power or deactivated. When it is below a temperature threshold, it activates until the setpoint is reached again.

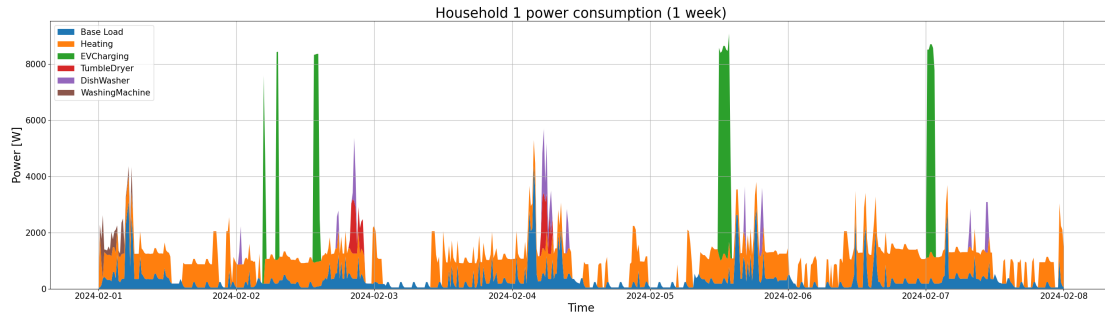


Figure 4.2: Example of a weekly load profile

4.1.7 Flexibility output

The described consumptions are considered as the initial fixed profile of the household. Some plausible flexibility boundaries must be provided along this profile to model flexibility, capacity and reactions. This flexibility depends on the appliance type:

- *Baseload*: No flexibility is possible, this is the basic usage of non-controllable appliances;

- *White goods*: A maximum (in hours) can be set to define the maximum shift in delay or advance for each cycle of each wet appliance. Once launched, the program profile will have to stay identical;
- *EV*: The starting time can be delayed and the power magnitude decreased. By knowing the necessary energy and the power rating of the charger, the flexibility window can be computed, with the a full-charge constraint at EV departure. Vehicle-to-grid or vehicle-to-home could also be considered;
- *Water boiler and Space heating*: The thermal appliances can be modeled using a larger temperature deadband on the thermostatic control, or a discomfort measure can be defined. The thermal model of the appliance is still needed for this flexibility component.

4.1.8 Inputs description

Table 4.1 summarizes all generator inputs for a single household.

Table 4.1: Inputs of the generator for a single household.

	Name	Description
General	Timestep	Granularity of the output, minimum 1 minute (min)
	Duration	Number of days to be generated (day)
	Starting date	Time of the year at which simulation is starting (day)
Base	Family size	Number of people in the house (pers.)
	Occupation	Unemployed, Full/Part-time job, Student or Child, for each person
	Baseload	Yearly baseload can be rescaled (kWh/y)
EV	Annual use	Average yearly use of the EV (km/y)
	EV type	Small, medium or big: converted to a consumption (kWh/100km) and battery size (kWh)
	Charger	Maximal power output of the car charger (kW)
WG	Washing	Number of cycles per week (optional)
	Drier	Number of cycles per week (optional)
	Dishwash	Number of cycles per week (optional)
WB	Boiler type	None, Electrical or Thermodynamic boiler
	Boiler size	Water tank volume (L)
	T° setpoint	Output temperature setpoint (°C)

Continued on next page

Table 4.1: Inputs of the generator for a single household (continued).

	Name	Description
HP	T° setpoints	Low, medium and high temperature setpoints (°C)
	Surface	Ground surface of the house (m ²)
	Height	Height of the house (m)
	Construction	Year of construction, converted to different insulation layers affecting heating dynamics (years)
	Outside T°	Yearly temperature profile to compute heat exchanges (°C)
	Power	Nominal input power of the heating system (kW)

4.1.9 Aggregation

The tool is also designed to output multiple load profiles, to model a whole population, and to see the impact on the grid infrastructure. This way, a neighborhood, a city, or even a representation of an entire country can be modeled quickly. To this end, the input values in Table 4.1 have to be replaced by lists with associated proportions, which the generator will interpret as probabilities.

The additional parameters for population modeling are presented in Table 4.2. All parameters in Table 4.1 should also be provided, converted to lists of values with corresponding probabilities except for the "General" inputs, common for all profiles.

	Name	Description
	Number	Amount of profiles to be generated
Penetration	EV	Portion of household profiles which include an EV
	Direct WB	Portion of households equipped with an electrical boiler
	Therm. WB	Portion of households equipped with a thermodynamic boiler
	HP	Portion of profiles equipped with a HP
	Direct heating	Portion of profiles with direct heating

Table 4.2: Additional inputs of the generator for a population.

The penetration parameters are optional, as this can be achieved by selecting a portion of appliances with zero nominal power. However, they can become relevant for multi-year studies by choosing a given proportion for each type of appliance and then slowly increasing their penetration.

4.2 CONCLUSION

This tool allows to generate an appliance-dependent load profile for households to model the possibilities for demand-side management at the residential low-voltage level.

It proposes a fully functional and customizable tool that avoids taking intrusive and long-lasting measurements. By aggregating multiple profiles, modeling larger-scale systems is made possible with simple input parameters' adaptation. These results can also be used for longer-term planning by generating the same population over multiple years with scenarios for evolving penetrations of low-carbon appliances, PV, and battery systems that have to be considered and included in the residential energy management system.

The result of this generator is realistic in a bottom-up approach using appliance-based individual load profiles and can be aggregated to be statistically representative for populations with defined load magnitude and patterns as well as low-carbon technologies penetration features.

The aggregated output results have been validated on Belgian data [Fluvius, 2024].

Outline

This chapter propose a way to use the load flexibility available from controllable appliances to maximize the internal exchanges through collective self-consumption and self-sufficiency. First, a centralized approach is presented to quantify, activate and retribute load flexibility. Then, a more intuitive decentralized approach is proposed to model more precisely the loads, the members' reactions and the effect on their discomfort.

As expressed in Section ??, the energy community will have a positive impact on the environment if it allows for increased self-consumption and/or self-sufficiency. Without behavior changes, the framework can still create profit through collective billing. This chapter models demand response for local arbitrage in the REC.

CONTEXT AND OBJECTIVES

The models presented in this chapter propose a way to model demand shifting. The first objective is to propose a realistic load modelling to assess the potential. The second is to ensure that the response is realistic and interpretable by the members. The realistic response is assumed through the use of automatically controllable loads. The interpretability comes from the iterative procedure with embedded KoR activation.

From the litterature review in Section 2.5.3, several gaps and contributions have been identified:

- Scenario-based isolation of the load-shifting impact specifically for EC bill optimization through implicit flexibility.
- Physical load models, with realistic flexibility bounds and discomfort functions.
- Integrated grid constraint to show the physical effect of the activation on the network.
- Interpretable process with embedded KoR.

Some of the key preliminary assumptions of this work are the constant total demand of the community, perfect forecast of day-ahead load and production capacities, and indistinguished cooperation of the members.

Preliminary notes on the model and decision variables

The model is a day-ahead linear program, with a limited number of variables, the dimension and tractability of the solution is not problematic. The decentralized algorithm is proposed to solve the privacy and hierarchical issues of the centralized method.

Table 5.1 presents the main decision variables in this process, while Table 5.2 lists all parameters that allowed flexibility modelling.

Table 5.1: Variables of the model

	Dim	Dom	Unit	Description
e^{com}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Community export power
i^{com}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Community import power
e^{ret}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Retailer export power
i^{ret}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Retailer import power
P^{PV}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	PV production power
P^{cha}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	BSS charging power
P^{dis}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	BSS discharging power
P^{inj}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Node injection power
P^{flex}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Flexible load power
P^{EV}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	EV charging power
P^{WB}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	WB power setpoint
P^{HP}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	HP power setpoint
s^{EV}	$\mathcal{U} \times \mathcal{T}$	$[0, 1]$	-	EV state of charge
T^{WB}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	°C	WB tank temperature
T^{HP}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	°C	HP indoor temperature
J^{EV}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	€	Discomfort from EV flex
J^{WB}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	€	Discomfort from WB flex
J^{HP}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	€	Discomfort from HP flex

Table 5.2: Inputs or parameters of the model

	Dim	Dom	Unit	Description
P^{fix}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Fixed power consumption
$\bar{P}^{\text{PV}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Max PV power production
C^{PV}	$\mathcal{U}$	$\mathbb{R}_+$	kWp	PV nominal power
$s^{\text{init,BSS}}$	$\mathcal{U}$	$[0, 1]$	-	Initial battery state of charge
C^{BSS}	$\mathcal{U}$	$\mathbb{R}_+$	kWh	Battery capacity
η^{BSS}	$\mathcal{U}$	$[0, 1]$	-	Battery efficiency
$\pi^{\text{i,ret}}$	$\mathcal{T}$	$\mathbb{R}_+$	€/kWh	Retailer import price
$\pi^{\text{e,ret}}$	$\mathcal{T}$	$\mathbb{R}_+$	€/kWh	Retailer export revenue
γ^{com}	$\mathcal{T}$	$\mathbb{R}_+$	€/kWh	Community fee
P^{ref}	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Reference flexible profile
E^{Cap}	$\mathcal{U}$	$\mathbb{R}_+$	kWh	Total flexible load
Electric Vehicle				
a^{EV}	$\mathcal{U} \times \mathcal{T}$	$\{0, 1\}$	-	EV plugged
$t^{\text{arr,EV}}$	$\mathcal{U} \times \mathcal{T}$	$\{0, 1\}$	-	EV arriving
$t^{\text{dep,EV}}$	$\mathcal{U} \times \mathcal{T}$	$\{0, 1\}$	-	EV leaving
$s^{\text{arr,EV}}$	$\mathcal{U} \times \mathcal{T}$	$[0, 1]$	-	EV state of charge at arrival
$s^{\text{ref,EV}}$	$\mathcal{U} \times \mathcal{T}$	$[0, 1]$	-	EV reference state of charge
$P^{\text{ref,EV}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Power from reference profile
$\bar{s}^{\text{EV}}$	$\mathcal{U}$	$[0, 1]$	-	Target state of charge
$s^{\text{init,EV}}$	$\mathcal{U}$	$[0, 1]$	-	Initial state of charge
η^{EV}	$\mathcal{U}$	$[0, 1]$	-	Battery efficiency
C^{EV}	$\mathcal{U}$	$\mathbb{R}_+$	kWh	Battery capacity
$\bar{P}^{\text{EV}}$	$\mathcal{U}$	$\mathbb{R}_+$	kW	Charger maximum power
α^{EV}	$\mathcal{U}$	$\mathbb{R}_+$	€/kWh ²	Discomfort reluctance
$E^{\text{Cap,EV}}$	$\mathcal{U}$	$\mathbb{R}_+$	kWh	Total energy consumption
Water Boiler				
$T^{\text{ref,WB}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	°C	Reference temperature
$T^{\text{set,WB}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	°C	Setpoint temperature
$P^{\text{l,WB}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Losses through envelope
$P^{\text{u,WB}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Losses from water use
$P^{\text{ref,WB}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Power reference
$t^{\text{u,WB}}$	$\mathcal{U} \times \mathcal{T}$	$\{0, 1\}$	-	Water usage event
$T^{\text{init,WB}}$	$\mathcal{U}$	$[0, 1]$	-	Initial temperature

Continued on next page

Table 5.2: Inputs or parameters of the model (continued)

	Dim	Dom	Unit	Description
$C^{\text{th,WB}}$	$\mathcal{U}$	$\mathbb{R}_+$	$^{\circ}\text{kWh}$	Thermal coefficient
$\bar{P}^{\text{WB}}$	$\mathcal{U}$	$\mathbb{R}_+$	kW	Water usage event
α^{WB}	$\mathcal{U}$	$\mathbb{R}_+$	$\text{€}/^{\circ}\text{h}$	Discomfort reluctance
$E^{\text{Cap,WB}}$	$\mathcal{U}$	$\mathbb{R}_+$	kWh	Total energy consumption
Heat Pump				
$T^{\text{ref,HP}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	$^{\circ}\text{C}$	Reference temperature
$T^{\text{set,HP}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	$^{\circ}\text{C}$	Setpoint temperature
$P^{\text{l,HP}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Power losses through walls
$P^{\text{ref,HP}}$	$\mathcal{U} \times \mathcal{T}$	$\mathbb{R}_+$	kW	Power from reference profile
$T^{\text{init,HO}}$	$\mathcal{U}$	$[0, 1]$	-	Initial temperature
$C^{\text{th,HP}}$	$\mathcal{U}$	$\mathbb{R}_+$	$^{\circ}\text{kWh}$	Thermal coefficient
$\bar{P}^{\text{HP}}$	$\mathcal{U}$	$\mathbb{R}_+$	kW	Nominal thermic power
COP^{HP}	$\mathcal{U}$	$\mathbb{R}_+$	-	Coefficient of performance
α^{HP}	$\mathcal{U}$	$\mathbb{R}_+$	$\text{€}/^{\circ}\text{h}$	Discomfort reluctance
$E^{\text{Cap,HP}}$	$\mathcal{U}$	$\mathbb{R}_+$	kWh	Total energy consumption

5.1 CENTRALIZED FLEXIBILITY ACTIVATION

5.1.1 Optimization model

The first step of the proposed procedure aims to optimally schedule (*i.e.*, day-ahead) the use of flexible devices, battery storage systems and economic exchanges in the REC. The problem is a Mixed-Integer Second-Order Cone Programming (MISOCP) problem and can be written in general terms as:

$$\begin{aligned}
 & \min_{\Omega} \quad \text{Objective function (5.2)} & (5.1) \\
 & \text{subject to} \quad \text{Cost constraints (5.3),} \\
 & \quad \text{Energy exchange constraints (5.4a) – (5.5),} \\
 & \quad \text{Battery storage constraints (5.6a) – (5.6e),} \\
 & \quad \text{Flexible devices constraints (5.7a) – (5.10),} \\
 & \quad \text{Power flow equations (2.3b) – (2.5),} \\
 & \quad \text{Discomfort constraints (5.11) – (5.12).}
 \end{aligned}$$

5.1.1.1 Objective function

The EC objective may be of different nature (economic, ecological, societal). In this work, the EC aims to minimize its costs (*i.e.*, maximize its benefits) while considering the REC members' discomfort function related to the flexible loads usage and the REC power loss costs. The global objective therefore writes as:

$$F(\Omega) = \Pi^{REC} + \sum_{u \in \mathcal{U}} J_u^{tot} + \pi^{loss} \sum_{t \in \mathcal{T}} P_t^{loss}, \quad (5.2)$$

with $\mathcal{U}$ the set of REC users, $\mathcal{T}$ the set of optimization periods of length Δ_T , Π^{REC} the collective electricity bill in the community framework (5.3), J_u^{tot} the total discomfort function of member u (5.12) and P_t^{loss} the total power losses in the REC distribution network (2.5) compensated at price π^{loss} .

5.1.1.2 Cost structure

The total community bill is defined by:

$$\Pi^{REC} = \sum_{t \in \mathcal{T}} \sum_{u \in \mathcal{U}} [\pi^{i,ret} i_{u,t}^{ret} - \pi^{e,ret} e_{u,t}^{ret} + \gamma^{com} (i_{u,t}^{com} + e_{u,t}^{com})] \Delta_T \quad (5.3)$$

where $i_{u,t}^{ret}$ and $i_{u,t}^{com}$ (resp. $e_{u,t}^{ret}$ and $e_{u,t}^{com}$) represent the power imported from (resp. exported to) the retailer and the REC by user u at time t . $\pi^{i,ret}$ and $\pi^{e,ret}$ are retailer import and export prices accounting for commodity and upstream grid usage charges. At this point, the commodity costs for the local energy exchanges are considered zero, the distribution of the benefits is done in the second step. However, a fee is charged for the intra-community exchanges, γ^{com} , accounting for the internal grid usage costs and the community operator remuneration.

5.1.1.3 Energy exchanges

The power injected by member u at time t ($P_{u,t}^{inj}$) can be expressed as a function of physical (5.4a) or virtual (5.4b) flows:

$$P_{u,t}^{inj} = P_{u,t}^{PV} + P_{u,t}^{dis} - P_{u,t}^{fix} - P_{u,t}^{flex} - P_{u,t}^{cha} \quad \forall u \in \mathcal{U}, t \in \mathcal{T}, \quad (5.4a)$$

$$P_{u,t}^{inj} = e_{u,t}^{ret} + e_{u,t}^{com} - i_{u,t}^{ret} - i_{u,t}^{com} \quad \forall u \in \mathcal{U}, t \in \mathcal{T}, \quad (5.4b)$$

with $P_{u,t}^{PV}$ the photovoltaic power injection, $P_{u,t}^{dis}$ and $P_{u,t}^{cha}$ the active powers going in and out of the BSS, $P_{u,t}^{fix}$ the non-flexible consumption and $P_{u,t}^{flex}$ the total flexible load. Constraint (5.5) also ensures the export/import power balance in the REC.

$$\sum_{u \in \mathcal{U}} e_{u,t}^{com} = \sum_{u \in \mathcal{U}} i_{u,t}^{com} \quad \forall t \in \mathcal{T}. \quad (5.5)$$

5.1.1.4 Battery storage system

All community members may use an individual battery storage system modeled as follows:

$$s_{u,1}^{BSS} = s_{u,0}^{BSS} + \Delta_T(\eta_u^{cha} P_{u,1}^{cha} - P_{u,1}^{dis}/\eta_u^{dis}) \quad \forall u \in \mathcal{U}, \quad (5.6a)$$

$$s_{u,t}^{BSS} = s_{u,t-1}^{BSS} + \Delta_T(\eta_u^{cha} P_{u,t}^{cha} - P_{u,t}^{dis}/\eta_u^{dis}) \quad \forall u \in \mathcal{U}, t \in \mathcal{T}_{\setminus\{1\}}, \quad (5.6b)$$

$$s_{u,T}^{BSS} = s_{u,0}^{BSS} \quad \forall u \in \mathcal{U}, \quad (5.6c)$$

$$0 \leq P_{u,t}^{cha}, P_{u,t}^{dis} \leq \bar{P}_u^{BSS} \quad \forall u \in \mathcal{U}, t \in \mathcal{T}, \quad (5.6d)$$

$$\underline{s}_u^{BSS} \leq s_{u,t}^{BSS} \leq \bar{s}_u^{BSS} \quad \forall u \in \mathcal{U}, t \in \mathcal{T}, \quad (5.6e)$$

with $s_{u,t}^{BSS}$ the state of charge of the battery storage system affected by its efficiencies $\eta_u^{cha}, \eta_u^{dis}$ and $s_{u,0}^{BSS}$ the initial state of charge which is imposed to be recovered at the end of the period through constraint (5.6c). Notations $\bar{[.]}$ and $\underline{[.]}$ are used to represent respectively upper and lower bounds of decision variables.

5.1.1.5 Flexible devices

REC members may consider shifting the consumption of some appliances. This represents their flexibility potential. This work considers three types of flexible devices: white goods, electric vehicle, and thermal loads.

White goods are loads with a fixed consumption cycle but whose starting time can be modified (e.g., dishwasher). They are modeled using a binary variable ($z_{w,u,t}$) representing the starting time of the device.

$$z_{u,t} = 0 \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T} \setminus \mathcal{T}_u^{flex}, \quad (5.7a)$$

$$\sum_{t \in \mathcal{T}_u^{flex}} z_{u,t} = 1 \quad \forall u \in \mathcal{U}, \quad (5.7b)$$

$$P_{u,t}^{WG} = \sum_{\tau} z_{u,\tau} P_{u,t-\tau+1} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \tau \in [t - D + 1, t]. \quad (5.7c)$$

Constraints (5.7a) and (5.7b) respectively impose that only one cycle is performed during the day and that the starting time falls within the flexible window, $\mathcal{T}_u^{flex}$, granted by user u . D represents the cycle duration in number of periods, P is the power consumption profile during a cycle and $P_{u,t}^{WG}$ is the actual white goods consumption.

Electric vehicles are modeled as one-way batteries for which the charging power can be adjusted. The charging operation is characterized by the arrival and depar-

ture time to/from the charging station, t_u^{arr} and t_u^{dep} , and the arrival and departure state of charge, $s_u^{arr, EV}$ and $s_u^{dep, EV}$.

$$s_{u, t_u^{arr}}^{EV} = s_u^{arr, EV} + \Delta_T P_{u, t_u^{arr}}^{EV} \quad \forall u \in \mathcal{U}, \quad (5.8a)$$

$$s_{u, t}^{EV} = s_{u, t-1}^{EV} + \Delta_T P_{u, t}^{EV} \quad \forall u \in \mathcal{U}, \forall t \in [t_u^{arr}, t_u^{dep}], \quad (5.8b)$$

$$s_{u, t_u^{dep}-1}^{EV} = s_u^{dep, EV} \quad \forall u \in \mathcal{U}, \quad (5.8c)$$

$$0 \leq P_{u, t}^{EV} \leq \bar{P}_u^{EV} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (5.8d)$$

$$\underline{s}_u^{EV} \leq s_{u, t}^{EV} \leq \bar{s}_u^{EV} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (5.8e)$$

with $s_{u, t}^{EV}$ the electric vehicle state-of-charge and P^{EV} the charging power of the electric vehicle.

Thermal loads (e.g., electrical water heater) are considered to be fully power-flexible loads, represented by the following two constraints:

$$0 \leq P_{u, t}^H \leq \bar{P}_u^H \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}, \quad (5.9a)$$

$$\sum_{t \in \mathcal{T}} P_{u, t}^H = \sum_{t \in \mathcal{T}} P_{u, t}^{ref, H} \quad \forall u \in \mathcal{U}, \quad (5.9b)$$

with $P_{u, t}^H$ the power consumption of thermal loads and $P_{u, t}^{ref, H}$ the base consumption profile of those loads.

The constant total flexible load consumption is expressed as:

$$P_{u, t}^{flex} = P_{u, t}^{WG} + P_{u, t}^{EV} + P_{u, t}^H \quad \forall u \in \mathcal{U}, t \in \mathcal{T}. \quad (5.10)$$

5.1.1.6 Power flow model

Power flow constraints are added to the problem to ensure a reliable operation of the distribution network. The radial configuration of the distribution grid allows usage of a relaxed DistFlow model [Farivar and Low, 2013]. The power flow is linked to the power injection decision variables with the previously-defined formulation (2.3b)-(2.5).

The maximum of $P_{0, t}^{inj}$ at the substation (slack bus) corresponds to the peak power of the community.

5.1.1.7 Discomfort functions

Flexible devices' usage is associated with a discomfort function which is defined for each type of flexible load by:

$$J^k = \alpha_u^k \sum_{t \in \mathcal{T}} (P_{u, t}^{ref, k} - P_{u, t}^k)^2 \quad \forall k \in \mathcal{K}, \forall u \in \mathcal{U}, \quad (5.11)$$

where $\mathcal{K} := \{W, EV, H\}$ and α_u^k represents the discomfort price of each flexible load, which can be considered as the user's reluctance to load flexibilization. $P_{u, t}^{ref, K}$, the

reference consumption profile of flexible loads, represents the preferred behavior of member u which corresponds to a cycle starting at the preferred time for white goods, an electric vehicle charging at full rate upon arrival until full charge or departure, and a typical thermal load profile to meet the desired temperature. The discomfort functions of flexible loads are summed to form the total discomfort of member u .

$$J_u^{tot} = J_u^{WG} + J_u^{EV} + J_u^H \quad \forall u \in \mathcal{U}. \quad (5.12)$$

5.1.1.8 Particular Cases

Different variants (*i.e.*, benchmark cases) can be derived from the original problem. The collective framework will be compared against them to evaluate performance. In this work, three particular cases can be created by constraining two major features: the flexibility potential and the community exchanges.

The first alternative is to not consider any flexibility potential by adding the following constraint for each type of flexible load which restricts the usage of flexible loads to their reference one:

$$P_{u,t}^k = P_{u,t}^{ref,k} \quad \forall k \in \mathcal{K}. \quad (5.13)$$

The second alternative is to consider an individual framework in which the following constraint imposes no internal exchanges:

$$i_{u,t}^{com} = e_{u,t}^{com} = 0 \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (5.14)$$

5.1.2 Flexibility Reward Methods

The daily total benefits to be shared among members are split between the direct benefits from the consumption and production profiles complementarity of REC members, and benefits that are related to flexibility expressed as follows:

$$B_d^{flex} = B_d^{REC, fixed} - B_d^{REC, flexible} \quad \forall d \in \mathcal{D}, \quad (5.15)$$

with $\mathcal{D}$ the set of days in a month, $B_d^{REC, flexible}$ and $B_d^{REC, fixed}$ the daily community bills with and without flexibility potential.

The second step of the model dispatches the economic benefits created by the activation of the flexibility potential of REC members. To ensure fairness, the dispatch scheme considers both activated and available flexibility. Two mechanisms are proposed, the first one is based on KoR and the second is based on flexibility prices.

Before sharing the benefits, daily activated and offered flexibility volumes are computed for each member. The activated volume is directly computed as the deviation from the reference profile:

$$E_{u,d}^{act, flex} = \sum_{t \in \mathcal{T}} |P_{u,t}^{flex} - P_{u,t}^{ref, flex}| \Delta_T / 2 \quad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}. \quad (5.16)$$

On the other hand, the available volume is computed by solving a problem to find the maximum activated volume under the flexible device constraints:

$$E_{u,d}^{\text{cap,flex}} = \arg \max_{p^{\text{flex}}} E_{u,d}^{\text{act,flex}} \text{ s.t. (5.7a)-(5.10).} \quad (5.17)$$

5.1.2.1 Rule-based Distribution of Flexibility Benefits

The first method proposed to distribute the benefits fairly among the flexibility providers is the application of the following KoR, $k_{u,d}$, based on the pro-rata of the weighted sum of activated and offered flexibility.

$$k_{u,d} = \frac{E_{u,d}^{\text{act,flex}} + \omega E_{u,d}^{\text{cap,flex}}}{\sum_{u \in \mathcal{U}} (E_u^{\text{act,flex}} + \omega E_{u,d}^{\text{cap,flex}})} \quad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}, \quad (5.18a)$$

$$B_{u,d}^{\text{flex}} = k_{u,d} B_d^{\text{flex}} \quad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}. \quad (5.18b)$$

with $B_{u,d}^{\text{flex}}$ the daily individual share of benefits. Different values of parameter ω are compared in the results section.

5.1.3 Optimization-based Remuneration of Flexible Behavior

The second method is based on defining an optimal flexibility price by solving another optimization problem which minimizes the variance of individual profits among members:

$$\min \sum_{d \in \mathcal{D}} \sum_{u \in \mathcal{U}} [(B_{u,d}^{\text{flex}} - J_{u,d}^{\text{tot}}) - (B_d^{\text{flex}} - \sum_{u \in \mathcal{U}} J_{u,d}^{\text{tot}})/|\mathcal{U}|]^2 \quad (5.19a)$$

$$\text{s.t. } B_{u,d}^{\text{flex}} = \pi_d^{\text{act}} E_{u,d}^{\text{act,flex}} + \pi_d^{\text{cap}} E_{u,d}^{\text{cap,flex}} \quad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}, \quad (5.19b)$$

$$\pi_d^{\text{act}} E_{u,d}^{\text{act,flex}} \geq J_{u,d}^{\text{tot}} \quad \forall u \in \mathcal{U}, \forall d \in \mathcal{D}, \quad (5.19c)$$

$$\sum_{u \in \mathcal{U}} B_{u,d}^{\text{flex}} = B_d^{\text{flex}} \quad \forall d \in \mathcal{D}, \quad (5.19d)$$

with π_d^{act} and π_d^{cap} respectively the reward price for the activated and offered flexibility volumes (*i.e.*, decision variable of the problem). Equation (5.19c) ensures the total discomfort of each member is at least compensated by the benefits of their flexibility activation.

5.1.4 Case Study and Results

5.1.4.1 Case Study Parameters

The REC considered in this work is composed of 20 domestic end-users connected to a radial distribution grid [Dickert et al., 2013] as depicted in Fig.5.1. Their non-flexible consumption as well as their flexible loads usage parameters are based on

the Pecan Street project open data [Pecan Street Inc., 2024]. The first model is solved daily with a one-hour resolution for a complete year. The flexibility rewards problems are run monthly. Perfect forecasts are considered for the non-flexible and production profiles in day-ahead (*i.e.*, no deviations between the first and second steps). A constant $\alpha_{k_u} = 0.05 \text{ €/kWh}^2$ was chosen to represent a community where each participant's aversion to demand flexibilization is equivalent. In this case study, users are considered to have equal tariffs for retailer imports ($\pi^{i,\text{ret}} = 0.4 \text{ €/kWh}$) and exports ($\pi^{e,\text{ret}} = 0.1 \text{ €/kWh}$). The internal community fees are set to 0.01 €/kWh . All input data is available in [Online Companion \[2024\]](#).

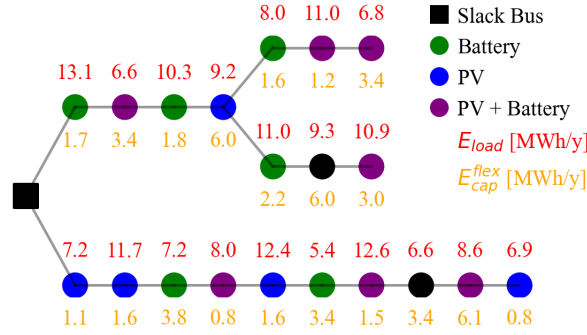


Figure 5.1: Case study considered for the simulation.

5.1.4.2 Collective Economic Surplus

Figure 5.2 shows the monthly total cost of energy for the four variants of the problem where “Ind.” states for *Individual* and relates to the case without local exchanges (5.14) and “fix” relates to the case without flexibility potential (5.13). Table 5.3 displays yearly financial and flexibility usage results. As expected from previous works, the community creation significantly reduces the total energy bill. Unlocking end-users’ flexibility further improves the benefits of the community, especially during spring and summer when the solar potential is the highest (*i.e.*, more local production). Those additional benefits related to flexibility are more important in the EC framework thanks to the internal exchange possibility but must also be put in perspective to the discomfort costs to be compensated which represent a higher share of the benefits in the EC framework, 14.2% versus 8.8%.

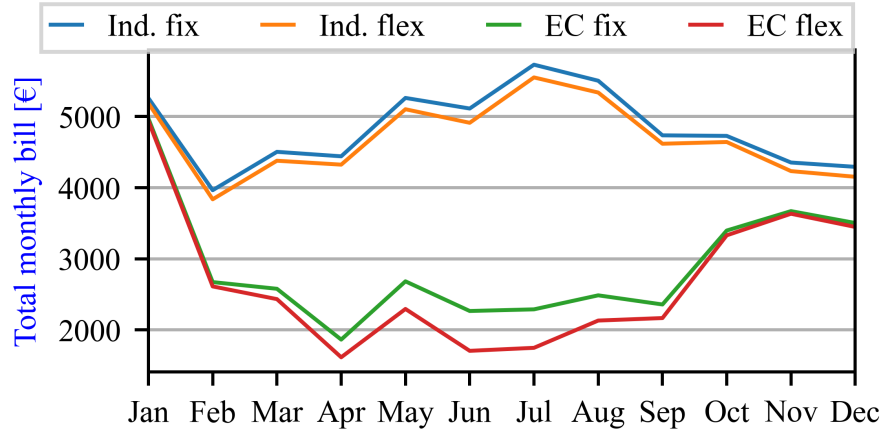


Figure 5.2: Monthly bills of the whole community for the four problems.

Table 5.3: Financial and flexibility results for the four problems.

	Indiv.		REC	
	Fixed	Flex.	Fixed	Flex.
Total costs [€]	57,900	56,300	34,700	32,000
Total discomfort [€]	0	142	0	382
Activated flex. [MWh]	0	5.3	0	9.2

5.1.5 Flexible Loads Usage and Discomfort level

Table 5.4 displays flexible load capacity, usage and discomfort for each type of load. It shows that EVs represent the biggest share of available flexibility as they are more energy-intensive assets. They are also the most activated flexibility as the power level can be adjusted during each period. On the contrary, white goods usage is less modulated as it implies the shift of the full consumption window. Discomfort levels are directly linked to flexibility activation.

Table 5.4: Flexibility usage and associated discomfort.

	Individual				REC			
	WG	EV	H	Tot.	WG	EV	H	Tot.
E^{cap} [MWh]	9.1	40.4	4.8	54.3	9.1	40.4	4.8	54.3
E^{act} [MWh]	0.7	2.5	2.0	5.3	1.0	5.1	3.1	9.2
J [€]	37	79	26	142	51	269	61	382

5.1.5.1 Individual Economic Surplus

The first part of Figure 5.3 shows the individual activated and offered flexibility volumes ordered by ascending activated flexibility and the second part compares the individual benefits obtained from the different settlement methods. It clearly shows that for all methods, the individual discomforts are compensated. Compared to an equal distribution of the benefits (solid blue line), the different proposed methods (green, red and purple bars) follow the individual flexibility level. Green bars (*i.e.*, activated flexibility only KoR) do not exactly follow the activated flexibility volume as the benefits are aggregated over one year with a daily settlement frequency.

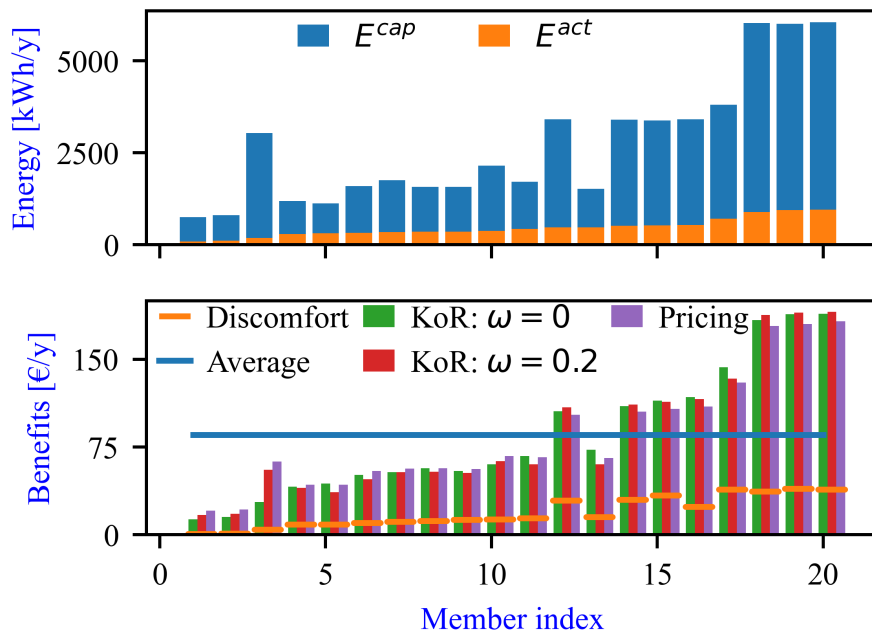


Figure 5.3: Individual flexibility and benefits for users depending on the settlement method.

5.1.5.2 Grid Usage

Regarding the stress on the distribution network, Table 5.5 shows that if several benefits arise by aggregating end-users in a community, as already demonstrated in Allard et al. [2023], the activation of flexibility has a relatively low impact. It mainly improves the losses, the self-consumption and self-sufficiency rate in both individual and EC frameworks.

Table 5.5: Summary of network usage for the four problems.

Framework	Individual		REC	
Demand	Fixed	Flex.	Fixed	Flex.
Total losses [MWh]	143.8	134.6	94.7	86.9
Max voltage [pu]	1.03	1.03	1.02	1.03
Max line loading [%]	74	74	69	70
Self-consumption [%]	51	52	84	87
Self-sufficiency [%]	40	42	67	69

5.1.6 Finding and limitations of the centralized model

This first model, despite its intrinsic limitations, showed the interest of load management for community bill minimization through self-consumption.

Three main weaknesses have also been identified in this preliminary work:

UNREALISTIC FLEXIBLE LOADS MODELLING: This model was developed before the tool presented in Chapter 4. The use of new load profiles will improve this point.

BASIC DISCOMFORT MODELLING: The discomfort function (5.11), (5.12) are basic and do not relate to true member discomfort. They are proportional to energy displaced.

CENTRALIZED SOLVING WITH UNKNOWN RETRIBUTION: This approach proposed a central approach that does not ensure adequate distribution. A decentralized approach with included allocation mechanism would ensure fairer retribution while allowing members to respond individually to the actual flexibility demand.

This is tackled by the second REC flexibility model presented in the next section.

5.2 EXPLICIT REWARD FOR DECENTRALIZED FLEXIBILITY ACTIVATION

5.2.1 Introduction

Building upon the model of the previous section, the objective is to quantify the gap in collective cost savings between centralized and decentralized coordination of end-user flexible resources. Compared to the aforementioned works, this one proposes an intuitive privacy-preserving approach to solve a decentralized problem that allows for embedding more precise (and potentially non-convex) models of flexible appliances, which would noticeably increase the computational complexity of state-of-the-art bilevel and ADMMs methods.

Therefore, the main contributions are:

- (a) a privacy-preserving intuitive sequential approach to solve the decentralized coordination problem of flexible resources in energy communities;
- (b) the implementation and comparison of explicit rule-based flexibility activation mechanisms which take the form of regulatory-compliant Keys of Repartition in the decentralized coordination scheme;

To model flexible resources, an embedded equivalent storage formulation of four relevant flexible domestic appliances (batteries, electric vehicles, heat pumps, water boilers) is used in the operational load scheduling problem.

5.2.2 Framework and key assumptions

This work considers a REC entity that is constituted by a group of domestic consumers and prosumers that can either exchange electricity with each other or with their traditional retailers. The organization of the local electricity sharing between participants is handled by a CO. Besides ensuring local balance and billing of members baseline exchanges within the community, the CO is also responsible for coordinated flexibility scheduling which is the main focus of this paper.

To run this task, assuming a perfect forecast of generation and baseline consumption, the CO identifies the flexibility needs of the day (in both upward and downward directions) of the community to maximize the value of local arbitrage. Then, it passes this information to members along with a reward price. Based on this volume-price pair, EC members offer flexibility capacity (based on load-shifting) to the operator. Finally, the CO activates the offered capacity explicitly by applying Keys of Repartition.

Hereafter, the different flexible loads considered in this study are presented in detail, as well as the Keys of Repartition compared for the fair activation of flexibility within the REC.

5.2.2.1 Flexible load models

With the growing electrification of energy demand, new opportunities to unlock flexibility in the consumption profiles of end-users emerge. At the residential level, three main devices have been identified and considered in this work, namely, EV, WB and HP. The first one, once plugged, allows adaptation of the charging strategy to the needs of the owner, while for the other two, the flexibility comes from the elasticity of the thermal demand as some end-users may have a comfort temperature range and not a specific desired value. In this work, all three flexible load types (or the related demand) are modeled similarly to a BSS as depicted in Fig. 5.4. It means that each one has a state variable (*i.e.*, EV's battery state-of-charge, water temperature of the boiler and indoor air temperature), incoming/outgoing electrical power flows, and a capacity that converts the power exchanges to a change of state (in kWh or in °C).

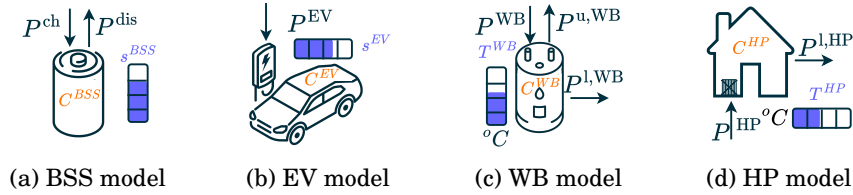


Figure 5.4: Equivalent battery models of the four domestic flexible equipments considered.

5.2.2.2 Repartition keys

Regarding the activation of flexibility, a rule-based approach is adopted by the community operator. These rules are represented by Keys of Repartition that are usually applied to share the local energy available among members in some countries (*e.g.*, Wallonia (BE) or France). The general KoR mechanism for flexibility activation can be written as:

$$P_{u,t}^{\text{act}} = \min(k_{u,t} P_{0,t}^{\text{cap}}, P_{u,t}^{\text{cap}}) \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.20)$$

where $P_{u,t}^{\text{cap}}$ is the flexible power capacity offered by each member, $P_{0,t}^{\text{cap}}$ is the requested capacity by the CO, $P_{u,t}^{\text{act}}$ is the activated flexible power of each member and $k_{u,t}$ is the value of the key of repartition for each individual.

Among the existing KoR [CWAPE \[2024\]](#); [Mustika et al. \[2022\]](#), three specific ones (*i.e.*, equal, proportional, and cascade) are considered and compared in terms of benefits distribution between flexible members. Their principles are defined as follows:

Equal Key:

This key is the simplest one as it splits the requested flexible volume equally between the participants who offer individual flexible capacity.

$$k_{u,t} = \begin{cases} \frac{1}{|\mathcal{U}_t^{flex}|} & \forall u \in \mathcal{U}_t^{flex}, \forall t \in \mathcal{T}, \\ 0, & \text{otherwise.} \end{cases} \quad (5.21)$$

with $\mathcal{U}_t^{flex} = \{u \in \mathcal{U} | P_{u,t}^{cap} > 0\}$ the set of REC participants who offered flexible capacity during period t .

Prorate Key:

This key activates flexibility volumes of each member based on its offered capacity, $P_{u,t}^{cap}$, with respect to the overall capacity available within the REC.

$$k_{u,t} = \frac{P_{u,t}^{cap}}{\sum_{u \in \mathcal{U}} P_{u,t}^{cap}} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T}. \quad (5.22)$$

Cascade Key:

This key consists of an iterative application of the *Equal Key* in order to maximize the activated volume and match the requested one. At each iteration, the set of flexibility providers $\mathcal{U}_t^{flex}$ is recomputed based on the remaining non-activated capacity of each member after the previous activation rounds. The process stops when there remains no offered capacity or the community operator's request is met. The practical application at each time period, $t \in \mathcal{T}$, of the *Cascade Key* follows Algorithm 5.1.

Input: $P_{u,t}^{cap}, P_{0,t}^{cap}$

- 1 Initialize remaining individual and requested flexible capacities:
- 2 $P_{u,t}^{cap,rem} = P_{u,t}^{cap} \quad \forall u \in \mathcal{U};$
- 3 $P_{0,t}^{cap,rem} = P_{0,t}^{cap};$
- 4 Initialize individual activated volumes:
- 5 $P_{u,t}^{act} = 0 \quad \forall u \in \mathcal{U};$
- 6 **while** $\sum_{u \in \mathcal{U}} P_{u,t}^{cap,rem} > 0$ **and** $P_{0,t}^{cap,rem} > 0$ **do**
- 7 Compute the number of remaining flexibility providers:
- 8 $\mathcal{U}_t^{flex} = \{u \in \mathcal{U} | P_{u,t}^{cap,rem} > 0\};$
- 9 Apply the Equal Key to activate flexibility:
- 10 $P_{u,t}^{act} = \max(P_{u,t}^{cap}, P_{u,t}^{act} + P_{0,t}^{cap,rem} / |\mathcal{U}_t^{flex}|) \quad \forall u \in \mathcal{U};$
- 11 Update remaining individual and requested flexible capacities:
- 12 $P_{u,t}^{cap,rem} = P_{u,t}^{cap} - P_{u,t}^{act} \quad \forall u \in \mathcal{U};$
- 13 $P_{0,t}^{cap,rem} = P_{0,t}^{cap} - \sum_{u \in \mathcal{U}} P_{u,t}^{act};$
- 13 **end**

Algorithm 5.1: Cascade Key principle

5.2.3 Centralized Flexibility Activation Problem

Building upon previous work [Allard et al. \[2024\]](#), this section describes the mathematical formulation of the day-ahead coordinated and centralized EC optimal operational planning problem with state-space modeling of the three flexible devices considered (*i.e.*, EVs, HPs and WBs). The objective is to minimize the REC electricity bill by scheduling the use of controllable loads to take advantage of local PV production while accounting for the discomfort caused by load-shifting. The complete problem, denoted as *ECFlex*, is written as:

$$\min_{\Omega} \quad \sum_{u \in \mathcal{U}} (\Pi_u + J_u^{\text{flex}}) \quad (5.23a)$$

subject to :

$$\Pi_u = \Delta_T \sum_{t \in \mathcal{T}} (\pi_{u,t}^{i,\text{ret}} i_{u,t}^{\text{ret}} - \pi_{u,t}^{e,\text{ret}} e_{u,t}^{\text{ret}} + \gamma^{\text{com}} (i_{u,t}^{\text{com}} + e_{u,t}^{\text{com}})), \quad \forall u \in \mathcal{U} \quad (5.23b)$$

$$J_u^{\text{flex}} = \sum_{t \in \mathcal{T}} (J_{u,t}^{\text{EV}} + J_{u,t}^{\text{WB}} + J_{u,t}^{\text{HP}}), \quad \forall u \in \mathcal{U} \quad (5.23c)$$

$$P_{u,t}^{\text{inj}} = P_{u,t}^{\text{PV}} + P_{u,t}^{\text{dis}} - P_{u,t}^{\text{fix}} - P_{u,t}^{\text{flex}} - P_{u,t}^{\text{cha}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23d)$$

$$P_{u,t}^{\text{inj}} = e_{u,t}^{\text{ret}} + e_{u,t}^{\text{com}} - i_{u,t}^{\text{ret}} - i_{u,t}^{\text{com}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23e)$$

$$\sum_{u \in \mathcal{U}} e_{u,t}^{\text{com}} = \sum_{u \in \mathcal{U}} i_{u,t}^{\text{com}}, \quad \forall t \in \mathcal{T} \quad (5.23f)$$

$$P_{u,t}^{\text{cha}} \leq \bar{P}_u^{\text{BSS}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23g)$$

$$P_{u,t}^{\text{dis}} \leq \bar{P}_u^{\text{BSS}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23h)$$

$$\underline{s}_u^{\text{BSS}} \leq s_{u,t}^{\text{BSS}} \leq \bar{s}_u^{\text{BSS}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23i)$$

$$s_{u,t}^{\text{BSS}} = s_{u,t}^{\text{prev,BSS}} + \Delta_T (\eta_u^{\text{BSS}} P_{u,t}^{\text{cha}} - P_{u,t}^{\text{dis}} / \eta_u^{\text{BSS}}) / C_u^{\text{BSS}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23j)$$

$$s_{u,T}^{\text{BSS}} = s_u^{\text{init,BSS}}, \quad \forall u \in \mathcal{U} \quad (5.23k)$$

$$P_{u,t}^{\text{flex}} = P_{u,t}^{\text{EV}} + P_{u,t}^{\text{WB}} + P_{u,t}^{\text{HP}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23l)$$

$$P_{u,t}^{\text{EV}} \leq a_{u,t}^{\text{EV}} \bar{P}_u^{\text{EV}}, \quad \forall u \in \mathcal{U} \quad (5.23m)$$

$$s_{u,t}^{\text{EV}} = t_{u,t}^{\text{arr,EV}} s_{u,t}^{\text{arr,EV}} + (1 - t_{u,t}^{\text{arr,EV}}) s_{u,t}^{\text{EV,prev}} + \Delta_T \eta_u^{\text{EV}} P_{u,t}^{\text{EV}} / C_u^{\text{EV}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23n)$$

$$t_{u,t}^{\text{dep,EV}} s_{u,t}^{\text{ref,EV}} \leq s_{u,t}^{\text{EV}} \leq 1, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23o)$$

$$\sum_{t \in \mathcal{T}} P_{u,t}^{\text{EV}} = \sum_{t \in \mathcal{T}} P_{u,t}^{\text{ref,EV}}, \quad \forall u \in \mathcal{U} \quad (5.23p)$$

$$J_{u,t}^{\text{EV}} \geq \alpha_u^{\text{EV}} (s_{u,t}^{\text{ref,EV}} - s_{u,t}^{\text{EV}}), \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23q)$$

$$P_{u,t}^{\text{WB}} \leq \bar{P}_u^{\text{WB}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23r)$$

$$T_{u,t}^{\text{WB}} = T_{u,t}^{\text{prev,WB}} + \Delta_T (P_{u,t}^{\text{WB}} - P_{u,t}^{\text{u,WB}} - P_{u,t}^{\text{l,WB}}) / C_u^{\text{th,WB}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23s)$$

$$\bar{T}_{u,t}^{\text{WB}} \geq T_{u,t}^{\text{WB}} \geq t_{u,t}^{\text{u,WB}} T_{u,t}^{\text{lim,WB}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23t)$$

$$\sum_{t \in \mathcal{T}} P_{u,t}^{\text{WB}} = \sum_{t \in \mathcal{T}} P_{u,t}^{\text{ref,WB}}, \quad \forall u \in \mathcal{U} \quad (5.23u)$$

$$J_{u,t}^{\text{WB}} \geq \alpha_u^{\text{WB}} (T_{u,t}^{\text{lim,WB}} - T_{u,t}^{\text{WB}}), \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23v)$$

$$P_{u,t}^{\text{HP}} \leq \bar{P}_u^{\text{HP}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23w)$$

$$T_{u,t}^{\text{HP}} = T_{u,t}^{\text{prev,HP}} + \Delta_T (COP_u^{\text{HP}} P_{u,t}^{\text{HP}} - P_{u,t}^{\text{l,HP}}) / C_u^{\text{th,HP}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23x)$$

$$\sum_{t \in \mathcal{T}} P_{u,t}^{\text{HP}} = \sum_{t \in \mathcal{T}} P_{u,t}^{\text{ref,HP}}, \quad \forall u \in \mathcal{U} \quad (5.23y)$$

$$J_{u,t}^{\text{HP}} \geq \alpha_u^{\text{HP}} (T_{u,t}^{\text{lim,HP}} - T_{u,t}^{\text{HP}}). \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.23z)$$

5.2.3.1 Additional benchmark models

In order to assess the local arbitrage benefits arising from optimal flexible devices coordination, benchmark frameworks must be defined. In this work, three additional models are investigated that derive from the problem (2.8) by adding specific additional constraints.

The complete model, *ECFlex* (5.23), is then compared to the proposed iterative heuristic (Algorithm 5.2).

Baseline problem (SoloFix):

This constitutes the reference case and is associated to the idea of solitary end-user models (*i.e.*, no local sharing activities) without flexibility concerns (*i.e.*, fixed power consumption profile). This typical scenario serves as the baseline for the subsequent models. It is obtained by adding the following constraints to problem (2.8):

$$e_{u,t}^{\text{com}} = i_{u,t}^{\text{com}} = 0 \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.24a)$$

$$P_{u,t}^{\text{flex}} = P_{u,t}^{\text{ref,EV}} + P_{u,t}^{\text{ref,WB}} + P_{u,t}^{\text{ref,HP}} \quad \forall u \in \mathcal{U}, t \in \mathcal{T} \quad (5.24b)$$

Individual self-consumption problem (SoloFlex):

The second benchmark introduces participants' flexibility potential in order to take advantage of individual local renewable generation and maximize its self-consumption. Compared to the baseline problem, in this case only the constraint (5.24a) is added to problem (2.8).

Non-flexible community problem (ECFix):

A collective problem can be formulated without addressing the management of flexible resources except for the batteries. From a mathematical point of view, this case adapts (2.8) by adding the non-flexibility constraint (5.24b) but keeping local electricity exchanges allowed.

Prioritization of individual self-consumption (ECFix'):

Usually used as a comparison benchmark, like in the previous section, the *SoloFlex* model may also be seen as preliminary actions before coordinated flexibility planning. This aspect is studied in this work by changing the reference power consumption profile of controllable appliances by the output values of P^{EV} , P^{WB} & P^{HP} from the individual self-consumption problem. In this new framework, individual self-consumption is prioritized by design before the EC coordinated flexibility actions

are taken. Note that state variables reference values are not adapted in the discomfort functions of problem (2.8), meaning that the discomfort created by individual flexibility actions is passed to the coordinated problem. The use of these new reference powers in further models will be indicated by a quotation mark ($P_{u,t}^{\text{ref,EV}}$, $P_{u,t}^{\text{ref,WB}}$, $P_{u,t}^{\text{ref,HP}}$).

5.2.4 Proposed decentralized resolution approach

If the centralized approach offers collective optimality guarantees, as presented in the previous section, it raises privacy concern and limits the active participation of end-users in the flexibility provision. The latter may also negatively affect the fairness of benefits distribution, as it may prioritize some individuals based on the resources they have and not on their willingness to participate. To alleviate this issue, the authors describe in the following the decentralized resolution approach of the flexibility activation problem presented in Section 5.2.3.

First, let us split the set of decision variables with $\Omega_u = [P^{\text{cha}}, P^{\text{dis}}, P^{\text{EV}}, P^{\text{WB}}, P^{\text{HP}}, P^{\text{cap,+}}, P^{\text{cap,-}}]$ being the individual decision variables of REC members while $\Omega_{\text{CO}} = [i^{\text{com}}, e^{\text{com}}, \bar{P}_{u,t}^{\text{act,+}}, \bar{P}_{u,t}^{\text{act,-}}]$ are the decision variables of the community operator. Then, let us define $i_{0,t}^{\text{ret}}$, $e_{0,t}^{\text{ret}}$ as the net import and net export, respectively, at the PCC of the REC, which represent the collective external exchanges obtained after resolution of the *Non-flexible community problem*. They serve as initial downward and upward flexibility requests (i.e., capacity limits) of the CO at each timestep as presented in Algorithm 5.2. The third input is the total flexible device reference power $P_{u,t}^{\text{ref}} = P_{u,t}^{\text{flex}} + P_{u,t}^{\text{cha}} - P_{u,t}^{\text{dis}}$.

Once the CO has identified the initial flexibility needs of the REC, flexible capacity will be offered by end-users and validated (or not) by the operator iteratively until there is no more flexibility request at the collective scale (i.e., no more local generation to take advantage of) or no member is offering additional flexible capacity.

At each iteration, there is a two-way communication between REC members and the CO for the activation of flexibility that follows these three steps:

- (a) Based on the community level flexibility request, each member tries to maximize its revenues from the provision of flexibility by submitting capacity offers to the CO, knowing they can earn $\pi^{\text{act}} = \pi^{\text{i,ret}} - \pi^{\text{e,ret}} - 2\gamma^{\text{com}}$ for each displaced kWh. These individual problems are constrained by the set of operating constraints of flexible resources and the following additional ones:

$$P_{u,t}^{\text{cap,+/-}} \leq \bar{P}_{0,t}^{\text{cap,+/-}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T}, \quad (5.25a)$$

$$\sum_{t \in \mathcal{T}} P_{u,t}^{\text{cap,+}} = \sum_{t \in \mathcal{T}} P_{u,t}^{\text{cap,-}}, \quad \forall u \in \mathcal{U}, \quad (5.25b)$$

$$P_{u,t}^{\text{flex}} = P_{u,t}^{\text{ref}} + P_{u,t}^{\text{cap,+}} - P_{u,t}^{\text{cap,-}}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T}. \quad (5.25c)$$

- (b) The resulting capacity offers obtained by the CO are not aware of other members' decisions. They are provided as if each end-user were the sole member of the REC. Therefore, to ensure a fair contribution to the flexibility activation for everyone, the operator refines individual activation bounds, $\bar{P}_{u,t}^{\text{act},+/-}$, by distributing the flexibility request among the members through the use of a Keys of Repartition mechanism as defined in section 5.2.2.2.
- (c) Finally, as the newly computed individual bounds for each timestep may not necessarily ensure constraint (5.25b), consumers solve again the maximization of flexibility revenues problem with the updated bounds as follows:

$$P_{u,t}^{\text{cap},+/-} \leq \bar{P}_{u,t}^{\text{act},+/-}, \quad \forall u \in \mathcal{U}, t \in \mathcal{T}. \quad (5.26a)$$

The resulting flexible resource usage decisions are effectively activated and set the new reference consumption for the next iteration. From the CO perspective, the EC level request is updated to account for the already activated flexibility.

Input: $i_0^{\text{ret}}, e_0^{\text{ret}}, P^{\text{ref}}$

```

1 Set the initial CO flexibility requests as upper bounds for EC members offers:
2  $\bar{P}_{0,t}^{\text{cap},+}, \Delta \bar{P}_{0,t}^{\text{cap},+} \leftarrow e_0^{\text{ret}} \quad \forall u \in \mathcal{U}, t \in \mathcal{T};$ 
3  $\bar{P}_{0,t}^{\text{cap},-}, \Delta \bar{P}_{0,t}^{\text{cap},-} \leftarrow i_0^{\text{ret}} \quad \forall u \in \mathcal{U}, t \in \mathcal{T};$ 
4 while  $\bar{P}_{0,t}^{\text{cap},+/-} > 0$  and  $\Delta \bar{P}_{0,t}^{\text{cap},+/-} > 0$  do
5   foreach  $u \in \mathcal{U}$  do
6     Individuals solve flexibility capacity problem:
7      $P_{u,t}^{\text{cap},+/-} \leftarrow \{\arg \max_{\Omega_u} \sum_{t \in \mathcal{T}} P_{u,t}^{+, \text{cap}} \pi^{\text{act}} \Delta_T - J_u^{\text{flex}},$ 
8       s.t. (5.23g) – (5.23z), (5.25a) – (5.25c);
9   end
10  The CO applies a key of repartition to define individual flexibility activation
    bounds:
11   $\bar{P}_{u,t}^{\text{act},+/-} \leftarrow \text{KoR}(P^{\text{cap},+/-}) \quad \forall u \in \mathcal{U}, t \in \mathcal{T};$ 
12  foreach  $u \in \mathcal{U}$  do
13    Each member activates its flexibility based on the new individual limits:
14     $P_{u,t}^{\text{act},+/-} = P_{u,t}^{\text{cap},+/-} \leftarrow \{\arg \max_{\Omega_u} \sum_{t \in \mathcal{T}} P_{u,t}^{+, \text{cap}} \pi^{\text{act}} \Delta_T - J_u^{\text{flex}},$ 
15      s.t. (5.23g) – (5.23z), (5.25b) – (5.25c), (5.26a);
16    The activated flexibility is taken into account for the next iteration by
      updating the reference load profiles:  $P_{u,t}^{\text{ref}} \leftarrow P_{u,t}^{\text{ref}} + P_{u,t}^{\text{act},+} - P_{u,t}^{\text{act},-};$ 
17  end
18  The CO updates its EC level flexibility requests and keeps track of the
    activated volume:
19   $\bar{P}_{0,t}^{\text{cap},+/-} \leftarrow \bar{P}_{0,t}^{\text{cap},+/-} - \sum_{u \in \mathcal{U}} P_{u,t}^{\text{act},+/-} \quad \forall u \in \mathcal{U}, \forall t \in \mathcal{T};$ 
20   $\Delta \bar{P}_{0,t}^{\text{cap},+/-} \leftarrow \sum_{u \in \mathcal{U}} \sum_{t \in \mathcal{T}} P_{u,t}^{\text{act},+/-};$ 
21 end
```

Algorithm 5.2: Proposed decentralized resolution algorithm *ECFlexIt*

5.2.5 Case Study and Results

This section provides detailed results for a full year of simulation and compares the economic and energy usage performances of the proposed decentralized resolution approach to the benchmark models presented in Section 5.2.3 and the centralized approach developed in 5.1.

The considered EC is composed of 20 domestic end-users whose non-flexible and flexible components are generated using a stochastic residential load profile generator *Resflex* Stegen et al. [30 June 2025]. The penetration levels of the four flexible assets considered in the 20 households are 70% for WBs, 60% for EVs, 50% for HPs, and 25% for BSS. Among the members, 15 also have PV installations ranging from 2 to 20 kW^{peak}, for a total of 147 kW^{peak} at the community level. Regarding the electricity tariffs, only flat prices are considered with an import price (including retail and grid fees) of 0.4 €/kWh, an export price of 0.1 €/kWh and a community fee of 0.01 €/kWh charged by the CO. Hence, the reward value for each kWh of flexibility activated rises to 0.28 €/kWh.

The community members have a day-ahead planning for the usage of their flexible appliances (EV charging and HP or WB temperature setpoints and predicted losses) and the model is simulated for one full year with sequential solving of daily problems. Finally, *ex-post* power flow analyses are run for the different scenarios studied, assuming that REC participants are connected to a benchmark low voltage distribution grid Dickert et al. [2013].

Table 5.6: Summary of results for selected scenarios.

	SoloFlex	ECFix	ECFlex	ECFlexIt	ECFlexIt'
[€]					
Π	6,549	6,831	5,243	5,576	5,412
J^{EV}	79	0	188	36	802
J^{WB}	0	0	0	0	0
J^{HP}	0	0	0	0	0
[MWh]					
E^{act}	40.6	0	44.9	23.7	12.7
$E^{act,EV}$	2.3	0	4.1	0.8	0.6
$E^{act,WB}$	27.4	0	30.7	17.1	8.5
$E^{act,HP}$	10.9	0	10.2	5.9	3.6
$E^{dis,BSS}$	7.3	10.2	7.4	10.1	8.6

5.2.5.1 Preserving privacy does not imply high benefits loss

One of the key findings of this study is that the coordinated decentralized iterative approach described in Section 5.2.4 converges towards the centralized methodology presented in 5.1. As illustrated in Figure 5.5, the proposed *ECFlex* and *ECFlexIt'* (*i.e.*, with and without individual prioritization and with an Equal Key) models provides a close, though suboptimal approximation of the case in which the decision-making process is fully coordinated by the CO (*i.e.*, *ECFlex*). Indeed, a residual

billing inefficiency remains, as reported in Table 5.6, with 6.35% (*ECFlexIt*) and 3.22% (*ECFlexIt'*) deviation observed in the overall community cost compared to the centralized benchmark. Nonetheless, this deviation represents the trade-off required to enhance user privacy and decentralize the decision-making process to community members, thereby ensuring individual preferences are more explicitly considered compared to the fully centralized case where the operator optimizes the overall REC outcome.

In addition, one should take a look at the other component of the objective function, that is, the discomfort generated by the activation of flexibility resources. In this perspective, the first interesting outcome observed in Table 5.6 is that water boilers and heat pumps have the potential to activate a significant amount of flexibility (up to 30.7% for WBs and 10.2% for HPs) without causing any discomfort, despite α^{HP} and α^{WB} equal to 1. This is made possible as only lower temperatures than the reference ones are penalized. This allows the thermal assets to shift the evening consumption to solar hours to "overheat" the water and air and let it cool down slowly over the end of the day without going below the limit. This emphasizes the flexibility potential at low or negligible costs from other assets than typical BSS, often unique flexible resources considered in this type of study.

On the other hand, shifting EV power only comes with discomfort costs as it delays full charge and limits anticipated departure potential. They are therefore used in cases where other assets do not have enough potential to take advantage of the overall local production available.

Comparing the different scenarios together, one can also notice that in the decentralized framework, without prior self-consumption, the initial use of storage systems before the activation of other flexibility means limits the action of the latter. The centralized approach, on the contrary, can make the direct arbitrage between the free WBs and HPs flexibility and the imperfect battery systems (*i.e.*, 95 % efficiency). The prioritization of individual self-consumption allows for applying similar arbitrage at the individual level before coordinating the responses to remaining flexibility requests of the CO.

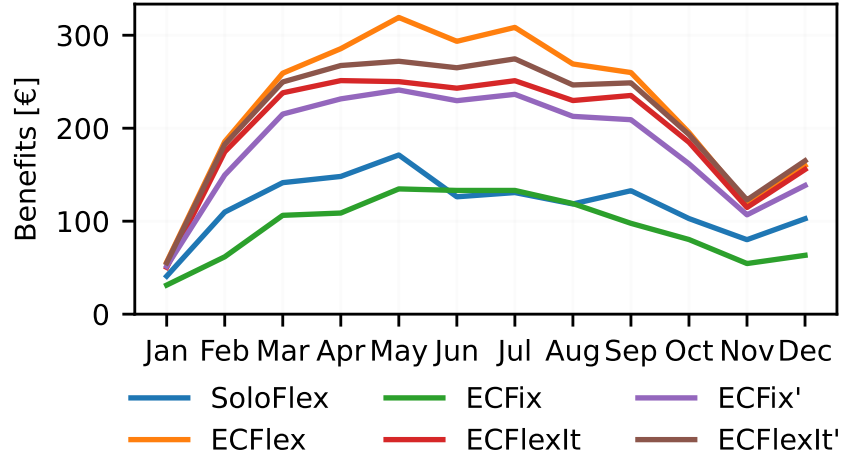


Figure 5.5: Total benefits over the SoloFix model

5.2.5.2 KoRs have a limited influence on fair benefits repartition

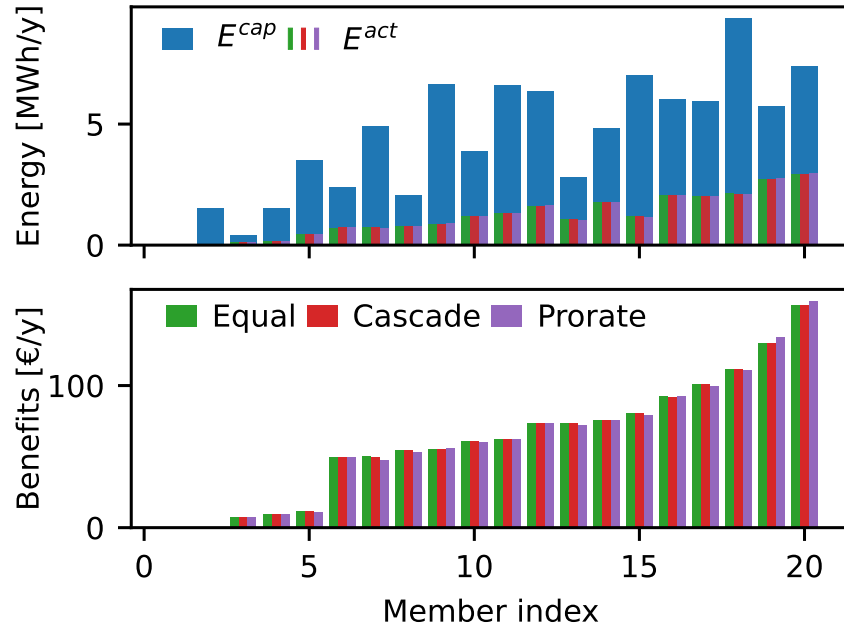


Figure 5.6: Individualization of the benefits compared to the community with fixed consumption

The proposed decentralized resolution approach has been applied without prioritization for the different Keys of Repartition presented in section 5.2.2.2 to assess their impact on the individual benefits distribution.

Nevertheless, results displayed in Figure 5.6 show that there are very small differences in terms of flexible energy activation and consequent benefits for the three different key mechanisms. Indeed, as discussed in the previous results section, the main driving factor for the participation of individuals to the flexibility coordination procedure is the flexible assets ownership. Hence, users with EVs only (User 2 in Fig. 5.6) may have an interesting flexible capacity (*i.e.*, the overall consumption of owned flexible assets), E^{cap} , but they will tend to offer limited capacity to the operator due to the discomfort that it creates. Meanwhile, REC members combining the two thermal assets (WBs and HPs) (*e.g.*, User 20 in Fig. 5.6) can offer a large part of their flexible capacity without a negative counterpart, raising their economic benefits. Regarding heat pumps, it must be pointed out that when they are only considered for heating households, their action is limited during summer periods, while those with WB exhibit the highest potential for local exchanges.

Nevertheless, the chosen Keys of Repartition have an impact on the convergence of the iterative procedure, as the cascading and proportional activation will tend to reduce the number of required iterations by dispatching all the flexibility offered at each step.

Table 5.7: Priorization impact

	With self-priorization				Without priorization			
	B [k€]	ΔB	J [€]	E^{act}	B [k€]	ΔB	J [€]	E^{act}
SoloFix	7.9				7.9		0	0
SoloFlex	6.5	-1.4	79	40				
ECFix	5.7	-0.8	79	40	6.8	-1.1	0	0
ECFlexIt	5.4	-0.3	803	53	5.6	-1.2	37	24

5.2.5.3 Flexibility impacts on the grid require explicit incentives

Regarding grid performances, it is observed that the yearly average indicators improve in scenarios where flexibility activation is enabled. Specifically, the total power losses are reduced from 840 kWh (*ECFix*) to 690 kWh (*ECFlex*) in the considered distribution network, improving the local grid efficiency. On top, the activation of flexibility mechanisms results in approximately 12 MWh less energy exchanged with the upstream grid in both directions (both imported and exported energy). This enhanced self-consumption and self-sufficiency of the community also benefits to the global grid by reducing the power flows through the upstream lines.

5.2.6 Conclusion

This work aimed to propose an intuitive resolution approach for the decentralized coordination of flexible resources within a REC to address the privacy concern

of a perfect centralized method without increasing the computational burden induces by state-of-the-art bilevel or ADMMs techniques. On a 20-household case study, results show that one can reach collective benefits close to the one obtained with centralized coordination with only 3.22% deviation in the best case. Authors also identified thermal loads such as water boilers and heat pumps as promising flexible resources as their smart management can improve the usage without generating any discomfort for the members. Finally, the investigation of several Keys of Repartition mechanism for the activation of offered flexible capacity did not lead to key differences as the individual benefits distribution is mainly driven by the flexible assets ownership.

Part III

REACTION

Decision changes based on the current state of the system

Outline

After the planning phase, there could remain some needs for voltage support in the local grids. In this chapter, a distributed algorithm is proposed to solve the AC OPF problem in distribution grids. This algorithm is presented as a cooperative, yet private way for avoiding overvoltages. REC could benefit for such algorithm to maximize power exports when it is profitable. They would also provide a perfect environment for this coordinated control and could boost adoption of this kind of initiative.

If the deviations from the forecasts are too big in intra-day, local congestion or overvoltages can exist in the distribution network.

A way to mitigate this negative impact will be proposed in this chapter after modelling an area overpopulated with PV systems, causing local overvoltages. This strategy is a proof of concept for distributed voltage control with nodal private decision. It could be facilitated by the EC paradigm, as it allows for maximization of the injected energy in a congested grid.

The CO could act as a coordinating point which ensures that every member is participating in this distributed control. Indeed, as it is presented here, optimality requires voluntary and cooperative participation of all members equipped with a controllable power-injecting device.

CONTEXT AND OBJECTIVES

Distributed control strategies are a trade-off between centralized and decentralized control algorithms. They only need reduced communication to take decisions locally, but can reach similar performance as a central controller. This makes them particularly well-suited for the voltage regulation problem in distribution networks. Additionally, they can ensure data privacy and improve robustness against cyber-attacks or communication failure. Compared to a central controller, there is no aggregator in distributed control strategies, so that there is no single-point of failure.

The contributions of this work are the following:

- No need for a central coordinator. Nodal optimization problems are solved using local estimations for power and voltage magnitude measurements.
- A linear version of the branch flow model is implemented based on estimation of the branch currents. Unlike common linearization methods, the losses are not neglected; they are evaluated for each line and each iteration. This method allows the use of a convex model which does not need the usual loss minimization term for the convex relaxation to be exact, as discussed by Farivar and Low [2013].

Preliminary notes on the model and decision variables

The optimization problem presented in this chapter is a SOCP problem. It is solved for a single timestep of 1 second, the simulation is run for one day. Thus 86,400 problems are solved sequentially, then rerun through pandapower to ensure exactness of the solution. The network includes 122 nodes, each model consists of around 750 variables, 1000 linear constraints and 122 second-order constraints.

The iterative algorithm is run at a frequency of 0.1 Hz, which means that it iterates 10 times for each problem. The last state of the ADMM algorithm is used as a warm start for the next second. Table 6.1 presents the variables and parameters used in this chapter.

Table 6.1: Variables and parameters of the model

Name	Dim	Dom	Unit	Description
Main decision variables				
p	$\mathcal{U}$	$\mathbb{R}$	kW	Active power production
q	$\mathcal{U}$	$\mathbb{R}$	kW	Reactive power production
Auxiliary and resulting variables				
V^{sqr}	$\mathcal{U}$	$\mathbb{R}_+$	pu	Squared voltage magnitude
I^{sqr}	$\mathcal{U}$	$\mathbb{R}_+$	pu	Squared current magnitude
P_{au}	$\mathcal{U}$	$\mathbb{R}$	pu	Active power in line u
Q_{au}	$\mathcal{U}$	$\mathbb{R}$	pu	Reactive power in line u
Parameters				
r	$\mathcal{U}$	$\mathbb{R}_+$	pu	Line resistance
x	$\mathcal{U}$	$\mathbb{R}_+$	pu	Line reactance
p_l	$\mathcal{U}$	$\mathbb{R}_+$	pu	Consumption active power
q_l	$\mathcal{U}$	$\mathbb{R}_+$	pu	Consumption reactive power
$\underline{V}$	/	0.95	pu	Minimum nodal voltage
$\overline{V}$	/	1.05	pu	Maximum nodal voltage

The power injection variables used in this model can be linked to the previous chapters through:

$$P_{u,t}^{\text{inj}} = S_{\text{nom}}(p_u - p_{l,u}),$$

the per unit system is used in this model to avoid numerical issues and improve convergence.

In this case study, PV system is considered as the only flexible appliance, all time-coupling state modelled energy storage appliances have been removed to solve the problem for a single timestep. The power injection is thus reduced to the internal power balance relation $P^{\text{inj}} = P^{\text{PV}} - P^{\text{fix}}$.

6.1 PROPOSED CONSENSUS-BASED ADMM APPROACH

A modified version of the branch flow model is considered:

$$\begin{aligned} \min \quad & \sum_{u \in \mathcal{U}} f_u(p_u, q_u) & (6.1a) \\ \text{s.t.} \quad & p_u - p_{l,u} = \sum_{c \in \mathcal{C}_u} P_{uc} - (P_{au} - r_{au} I_{au}^{\text{sqr}}) & \forall u \in \mathcal{U} \quad (6.1b) \\ & q_u - q_{l,u} = \sum_{c \in \mathcal{C}_u} Q_{uc} - (Q_{au} - x_{au} I_{au}^{\text{sqr}}) & \forall u \in \mathcal{U} \quad (6.1c) \\ & V_u^{\text{sqr}} = V_a^{\text{sqr}} - 2(r_{au} P_{au} + x_{au} Q_{au}) + (r_{au}^2 + x_{au}^2) I_{au}^{\text{sqr}} & \forall u \in \mathcal{U} \quad (6.1d) \\ & I_{au}^{\text{sqr}} = (P_{au}^2 + Q_{au}^2) / V_a^{\text{sqr}} & \forall u \in \mathcal{U} \quad (6.1e) \\ & (p_u, q_u) \in \mathcal{S}_u & \forall u \in \mathcal{U} \quad (6.1f) \\ & \underline{V}^2 \leq V_u^{\text{sqr}} \leq \bar{V}^2 & \forall u \in \mathcal{U} \quad (6.1g) \end{aligned}$$

where, compared to the usual formulation, a decomposable objective function is considered. It corresponds to the sum of cost functions defined at each node. Furthermore, the following change of variable was applied: $V_u^{\text{sqr}} = V_u^2$ and $I_{ij}^{\text{sqr}} = I_{ij}^2$. The decision variables p_u, q_u and the objective function are specific to each node, (*i.e.*, they depend only on quantities defined at node u). However, there are coupling constraints for quantities that are shared among different nodes. For instance, consider a node a connected to node u through a line of impedance $r_{au} + jx_{au}$, one can write $V_a e^{j\delta_a} - V_u e^{j\delta_u} = (r_{au} + jx_{au}) I_{au} e^{j\phi_{au}}$, that is Ohm's law in phasor form. Voltage magnitudes at nodes a and u are linked via the current through the line connecting a to u , and its impedance. Therefore, one cannot directly decompose the optimal power flow problem because of the power flow constraints. In this work, the consensus-based ADMM [Boyd et al., 2011] is employed to distribute the problem because of its enhanced convergence properties compared to other distributed methods.

The following conventions are considered. The network is radial, it is represented using a directed graph rooted at the substation node that is node 0. Each of the other nodes ($u \neq 0$) has a unique ancestor a and any number of children that belongs to the set $\mathcal{C}_u$. A positive power flows from the ancestor to the children and all branch powers are considered on the side of the ancestor node (see Figure 6.1 for the conventions used).

In the consensus-based ADMM, coupled variables are turned into local copies, and an additional consensus constraint imposes the equality of local copies to a shared consensus variable. The consensus constraint is then dualized. The consensus variables are computed as the mean of the local copies. As a result, a communication network matching the electrical network must be created to share corresponding local variables between neighboring nodes.

The ADMM iterative procedure is implemented as follows:

Step 1: Solve the subproblem at each node u ,

Step 2: Exchange local copies with the neighbors of u ,

Step 3: Update the consensus variables.

Step 4: Update scaled dual variables corresponding to the consensus variables using dual ascent.

These four steps are repeated until convergence of primal and dual variables.

6.1.1 Iterative procedure

Step 1: The subproblem solved at each node u is:

$$\min_{p_u, q_u} f_u(p_u, q_u) + n_u(\varepsilon^u) + m_u(V_u^{\text{sqr},u}, V_a^{\text{sqr},u}) \quad (6.2a)$$

$$\text{s.t. } p_u - p_{l,u} = \sum_{c \in \mathcal{C}_u} P_{uc}^u - (P_{au}^u - r_{au} I_{au}^{\text{sqr},u}), \quad (6.2b)$$

$$q_u - q_{l,u} = \sum_{c \in \mathcal{C}_u} Q_{uc}^u - (Q_{au}^u - x_{au} I_{au}^{\text{sqr},u}), \quad (6.2c)$$

$$V_u^{\text{sqr},u} = V_a^{\text{sqr},u} - 2(r_{au} P_{au}^u + x_{au} Q_{au}^u) + (r_{au}^2 + x_{au}^2) I_{au}^{\text{sqr},u}, \quad (6.2d)$$

$$(p_u, q_u) \in \mathcal{S}_u, \quad (6.2e)$$

$$\underline{V}^2 \leq V_u^{\text{sqr},u} \leq \bar{V}, \quad (6.2f)$$

where ε^u is a vector containing the coupling variables related to node u , (i.e., $V_u^{\text{sqr},u}$, $V_a^{\text{sqr},u}$, P_{au}^u , Q_{au}^u and $\{P_{uc}^u, Q_{uc}^u\}_{c \in \mathcal{C}_u}$).

The penalty term n_u is the augmented Lagrangian in its scaled form [Boyd et al., 2011]:

$$\begin{aligned} n_u(\varepsilon^u) = & \frac{\rho}{2} (\|P_{au}^u - \bar{P}_{au}^u + \gamma_{au}^u\|_2^2 + \|Q_{au}^u - \bar{Q}_{au}^u + \delta_{au}^u\|_2^2 \\ & + \|V_a^{\text{sqr},u} - \bar{V}_a^{\text{sqr},u} + \lambda_a^u\|_2^2 + \|V_u^{\text{sqr},u} - \bar{V}_u^{\text{sqr},u} + \lambda_u^u\|_2^2 \\ & + \sum_{c \in \mathcal{C}_u} (\|P_{uc}^u - \bar{P}_{uc}^u + \gamma_{uc}^u\|_2^2 + \|Q_{uc}^u - \bar{Q}_{uc}^u + \delta_{uc}^u\|_2^2)), \end{aligned} \quad (6.3a)$$

with the penalty parameter or update step ρ , initialized at 1 and updated using a heuristic described by Boyd et al. [2011]. $\gamma_{au}^u, \gamma_{uc}^u, \lambda_u^u, \lambda_a^u, \delta_{au}^u, \delta_{uc}^u$ are dual variables associated with consensus constraints, and quantities defined with $(\bar{\cdot})$ represent consensus variables. **Step 3** explains how those quantities are computed. Note that the constraint (6.1e) does not appear in the local subproblems, although the branch

currents I_{au}^{sqr} appear in the other constraints. This constraint makes the problem non-convex. To avoid this, the iterative procedure of the ADMM is used to consider branch currents as parameters that are updated at each iteration of the procedure using:

$$I_{au}^{\text{sqr},u} = \frac{(\bar{P}_{au}^u)^2 + (\bar{Q}_{au}^u)^2}{\bar{V}_a^{\text{sqr},u}} \quad (6.4)$$

The last term m_u is a second penalty term for voltage deviation compared to the measured value:

$$m_u(V_u^{\text{sqr},u}, V_a^{\text{sqr},u}) = c_v((V_u^{\text{sqr},u} - V_{\text{meas},u}^2)^2 + (V_a^{\text{sqr},u} - V_{\text{meas},a}^2)^2). \quad (6.5)$$

This aims to keep the local copies of the squared voltage magnitude variables close to the measured values. In distribution network, the substation node sets the reference voltage for the system, and the voltage values at other nodes are derived from this reference and the power flow in the lines. The outer nodes thus rely on information from ancestors to determine their own voltage in the ADMM algorithm.

Due to the distributed and iterative nature of ADMM, this information takes some time to propagate through the network, resulting in underestimation of voltage values and lasting overvoltages. This is actually one of the major drawback of distributed control strategies, as they take some time to converge to the optimal solution. Adding this term with a weight factor $c_v > c_q$ speeds up the convergence to solutions that satisfy Kirchhoff's laws. This term ensures faster voltage regulation while optimal operation is still reached when the algorithm has converged.

If one considers a perfect *voltage model*, and no measurement errors, the penalty term defined in (6.5) becomes zero when the ADMM procedure has converged toward a solution of problem (6.1). However, in practice, there exist some modelling uncertainties and measurements are noisy such that the penalty term is never equal to zero.

With respect to (6.1), a superscript u represents a local copy of a variable considered at node u and a is associated to the ancestor of node u (i.e., $V_a^{\text{sqr},u}$ denotes the squared voltage magnitude of the ancestor node a of node u seen from node u , while $V_u^{\text{sqr},u}$ denotes the squared voltage magnitude of node u seen from node u).

Step 2: The communication graph described in Figure 6.1 shows the variables to be exchanged after each node has solved its subproblem.

Step 3: The consensus variables are updated at each node u by computing the average value of all the local copies:

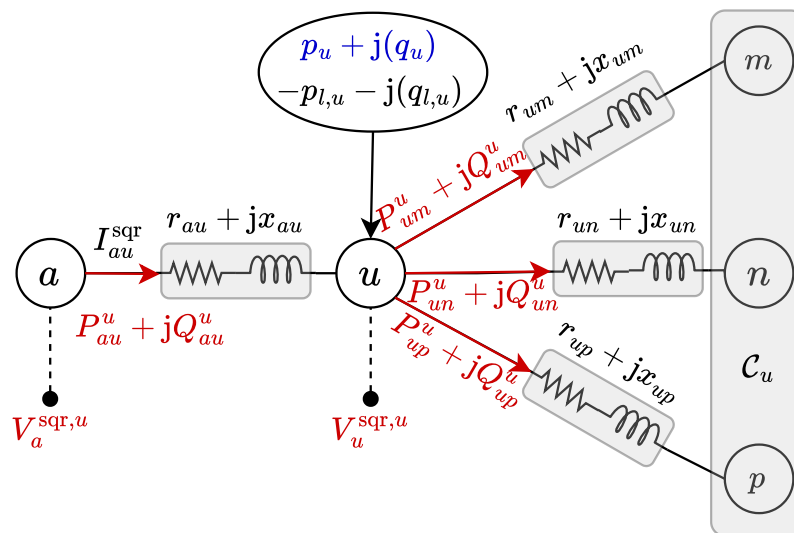
$$\bar{V}_u^{\text{sqr},u} = (V_u^{\text{sqr},u} + \sum_{c \in \mathcal{C}_u} V_u^{\text{sqr},c}) / (|\mathcal{C}_u| + 1), \quad (6.6a)$$

$$\bar{P}_{au}^u = (P_{au}^u + P_{au}^a) / 2 \quad (6.6b)$$

$$\bar{Q}_{au}^u = (Q_{au}^u + Q_{au}^a) / 2 \quad (6.6c)$$

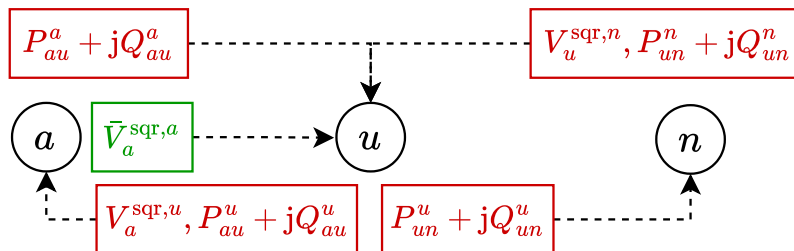
$$\bar{P}_{uc}^u = (P_{uc}^u + P_{uc}^c) / 2 \quad \forall c \in \mathcal{C}_u \quad (6.6d)$$

$$\bar{Q}_{uc}^u = (Q_{uc}^u + Q_{uc}^c) / 2 \quad \forall c \in \mathcal{C}_u \quad (6.6e)$$



Decision variables: p_u, q_u

Parameters: $I_{au}^{\text{sqr}}, r_{au} + \text{j}x_{au}, r_{un} + \text{j}x_{un}, p_{l,u}, q_{l,u}$



Consensus update for node u

$$\begin{aligned}\bar{V}_u^{\text{sqr},u} &= 0.5(V_u^{\text{sqr},u} + V_u^{\text{sqr},n}) & \bar{V}_a^{\text{sqr},u} &= \bar{V}_a^{\text{sqr},a} \\ \bar{P}_{au}^u + \mathbf{j}\bar{Q}_{au}^u &= 0.5(P_{au}^u + P_{au}^a + \mathbf{j}(Q_{au}^u + Q_{au}^a)) \\ \bar{P}_{un}^u + \mathbf{j}\bar{Q}_{un}^u &= 0.5(P_{un}^u + P_{un}^n + \mathbf{j}(Q_{un}^u + Q_{un}^n))\end{aligned}$$

Figure 6.1: Communication graph for node u .

Only communication with adjacent buses is used in the algorithm. In order to update $\bar{V}_a^{\text{sqr},u}$, information from the children nodes $n \in \mathcal{C}_a$ of the ancestor a is needed, since all the children nodes have a local copy of the ancestor voltage. This consensus variable is thus only updated in node a with data from all its children, then transmitted to the children node, including node u .

Step 4: The scaled dual variables are updated with a gradient ascend method:

$$\lambda_u^u = \lambda_u^u + V_u^{\text{sqr},u} - \bar{V}_u^{\text{sqr},u}, \quad (6.7a)$$

$$\lambda_a^u = \lambda_a^u + V_a^{\text{sqr},c} - \bar{V}_a^{\text{sqr},u}, \quad (6.7b)$$

$$\gamma_{uc}^u = \gamma_{uc}^u + P_{uc}^u - \bar{P}_{uc}^u, \quad \forall c \in \mathcal{C}_u \quad (6.7c)$$

$$\gamma_a^u = \gamma_a^u + P_a^u - \bar{P}_a^u, \quad (6.7d)$$

$$\delta_{uc}^u = \delta_{uc}^u + Q_{uc}^u - \bar{Q}_{uc}^u, \quad \forall c \in \mathcal{C}_u \quad (6.7e)$$

$$\delta_a^u = \delta_a^u + Q_a^u - \bar{Q}_a^u. \quad (6.7f)$$

6.1.2 Experimental results

6.1.2.1 Simulation setup

A Dickert benchmark LV network is used [Dickert et al., 2013], and available in Pandapower [Thurner et al., 2018]. The parameters for the Dickert network are *long range feeders, multiple customers, cables* as line type, and the case type is *average* with a total of 122 nodes. The same initial electrical loads are kept, and PV systems are added. 109 PV systems are distributed randomly in the network, corresponding to a penetration rate of 90%. The rated size of the inverters is selected among three different sizes $\{8, 9, 10\}$ kVA. Values for load consumption and PV production with a minute resolution come from [Open Power System Data \[2018\]](#). We derived 40 load profiles and 40 PV production profiles collected over several days to simulate active and reactive load consumption patterns and PV system orientations. The data are linearly interpolated to create profiles with second resolution.

The simulations are run for one day, from sunrise to sunset. Considering the very low computation time and the direct communication between nodes, the frequency of the ADMM algorithm is estimated to be 10 Hz. Every 0.1 second, the inverters receive new setpoints. The inverters can directly implement the power setpoint received. Then, voltage magnitudes measurements are made and fed back in equation (6.2a). Considering the one-second resolution for PV and load data, ADMM has 10 iterations to approach convergence before the state of the system is modified. Each second, after system modification, the algorithm benefits from previous operating points as a warm start. Operating points stay similar and dual variables for the problem are already close to convergence. Perfect communication and synchronicity between neighboring inverters is considered for these simulations.

6.1.2.2 Results

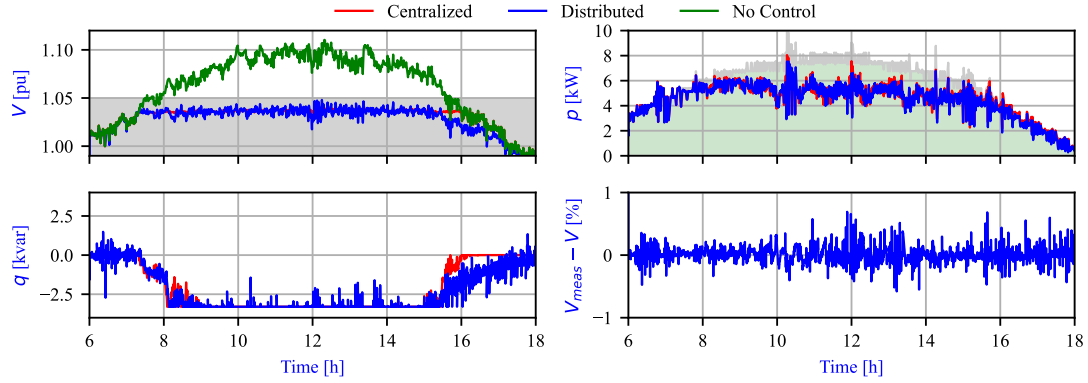


Figure 6.2: Results for one specific bus equipped with a PV inverter. V denotes the local copies of the voltage magnitude for one specific node, while V_{meas} is the actual voltage magnitude that has been measured. q and p denote the reactive power and active power injected by the DER, respectively.

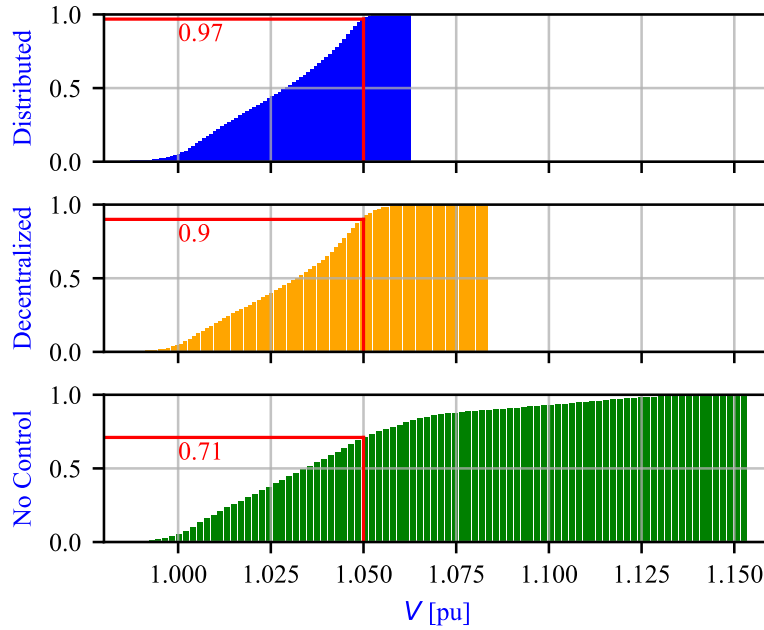


Figure 6.3: Cumulative distribution functions of voltages for all nodes in three simulations, where V denote the voltage magnitude measured throughout the network.

Figure 6.2 compares the performance of the centralized model and the distributed model. The voltage magnitude at one problematic node is shown in the top-left figure. Three simulations are run. The first two use centralized and distributed models to compute power setpoints. In the third one, no control action is taken (reactive power set to zero and no active power curtailment). The voltage level in the latter case exceeds the 1.05 pu threshold, whereas it does not in the other two cases. Even if the local voltage measurements are not outside voltage limits, the distributed

algorithm curtails the active power to avoid overvoltages in other nodes. It can achieve the same results as a centralized model without needing a coordinator with complete observability of the system state. However, the magnitude of the voltage in our model may differ from the network's actual voltage (resulting from the solution of a power flow using the power setpoints computed with the distributed algorithm), as explained before.

The bottom right plot displays the differences between the actual voltage and the voltage perceived by the distributed agents. Finally, the computed power setpoints are compared between the centralized and distributed algorithms. It is noticeable that the distributed model accurately follows the optimal power setpoints computed with the centralized model.

The control algorithms are compared in terms of overvoltages in Figure 6.3 (the centralized method is not presented as it avoids all overvoltages). The distributed algorithm limits the number of voltage violations and the maximum overvoltage, compared to the decentralized and no control scenarios.

In Figure 6.4, the energy curtailed (integral of the total active power curtailed over a specified period) is shown for three control algorithms, the centralized version of the branch flow model, the distributed approach and the a decentralized automatic control algorithm inspired by CENELEC EN50549-2 [2019] standard.

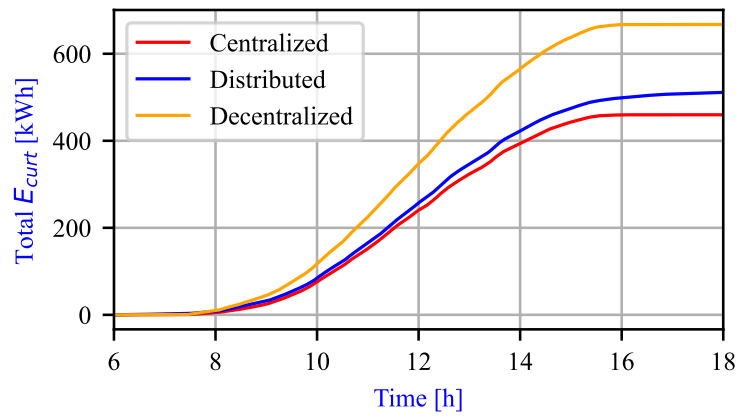


Figure 6.4: Energy curtailed for the three control algorithms.

Table 6.2 shows the comparative results for the four simulations. The centralized optimizer shows the ideal results, with minimal energy curtailment and no voltage violations. The results for the proposed distributed method are relatively close to ideal, with reduced communication needs. The decentralized controller show relatively good voltage regulation, at the expense of very large power curtailment.

Table 6.2: Results for the full simulations.

	No Ctrl	Decentr.	Distr.	Centr.
Energy produced [kWh]	5009	4342	4498	4549
Energy curtailed [kWh]	0	697	511	460
Energy lost [kWh]	212	123	186	191
Production peak [kW]	697	579	591	601
Consumption [kWh]	679			
Reactive control [kWh]	0	0	1129	1031
Max. voltage [pu]	1.15	1.08	1.06	1.05
Overvoltages [%]	29	10	3	0

6.2 SUMMARY

In this chapter, the AC-OPF problem was solved using a consensus-based ADMM, that only requires neighbor-to-neighbor communication. One subproblem per electrical node is defined, and each node communicates with its neighboring nodes. The proposed method can reach similar results as an optimal centralized controller. We used a branch flow model to model the power flow equations. A convex formulation for the branch flow model is obtained by appropriately estimating the branch currents, using the iterative procedure of ADMM. The global optimum can be reached without minimizing the network losses in the model objective thanks to the iterative evaluation of the branch currents. Numerical results on a 122-node single-phase distribution networks show performance comparable to a centralized controller, with less communication requirements.

In practice, implementing this method in a distribution would face problem of social acceptance. Indeed, this partial curtailment of solar power, while being overall more profitable, would not always be individually profitable. To allow for nodes at feeders' end to produce more, upstream producer should inject a little bit less than what they did without this algorithm. A cooperative algorithm like this would only be approved with proper incentive for everyone, this could be done through the EC. The communication burden that would also constitute a barrier for implementation would also be eased by the EC structure.

This method could also be extended to multi-timestep energy management by reducing the dimension of the future costs. These costs can be estimated with the expected value of the stored energy [Dumas et al., 2021]. However, this case study is a proof of concept. Convergence is not constrained to one specific case of separable objective functions and internal flexibility charts.

Part IV

CONCLUSION

Fairness considerations and concluding remarks

Outline

This chapter discusses the need for fairness in the incentives of the EC. To foster cooperation, the goals for all parties should be aligned. We describe a few ways of ensuring fairness, stability and efficiency of the solution for profit sharing inside the community.

One critical aspect of REC is the problem of social acceptance they face. Community members may be deterred by the limited transparency and interpretability of the underlying mathematical models. This chapter describe what an optimal theoretical framework could look like. Then it will motivate a more practical approach, adapted for implementation.

Fairness is a central component of justice, referring to impartiality, equity, and the absence of bias in decision-making and resource allocation. Fairness can either be interpreted as equality (equal shares for all) or equity (distribution according to needs, merits, or contributions). It does not only depend on the outcome but also on the process. Fairness considerations stretch beyond the expertise of applied sciences as they are originating from philosophy and social sciences. Here, we propose to highlight the relevance of fairness considerations in the community framework without the pretention of finding a solution.

7.1 FAIRNESS IN A RENEWABLE ENERGY COMMUNITY

The EC bill sharing problem is intrinsically dependent on the fairness definition within the community. In the previous chapters, different allocations were considered. This section will try to give a complete overview of how to apply the concept of fairness inside of the community.

Fairness relates not only to the individual outcome for each member, but also to the procedure and framework. A fair community needs to follow four different axes:

- Distributive fairness: how costs and benefits are shared;
- Procedural fairness: how decisions are made and how transparent the process is;

- Individual contribution fairness: accounting for assets, consumption, flexibility;
- Perceived fairness: acceptance by participants.

While being fair is important, the CO should ensure that the decision is stable, both in the short-term (*i.e.*, the result of a Nash equilibrium) and in the long-term (*i.e.*, the core is not empty). Utilitarian participants will also require the community to be efficient, the allocation procedure should in this case require global optimality of the solution [Cornélusse et al., 2018].

Three main levers (decision variables) are available to the CO:

- **Allocation:** Repartition of the energy quantities. Each community member gets a share of the available energy at each time step. KoR are a simple materialization of this.
- **Pricing:** Internal price for energy sold or bought. It could be driven by a local energy market with bids or shadow pricing. This is similar to what was proposed by Cornélusse et al. [2019].
- **Direct benefits allocation:** The profit is directly shared and the fairness of the solution depends on the direct savings of each member.

These methods are not specifically mutually exclusive, keys of repartition could be used with constant pricing or with a variable price based on a market. The direct benefits allocation could be done *a posteriori* with bounded prices and predefined energy exchanges, as presented in Section 3.1.2.

In energy communities, fairness is not limited to the retribution of the participants, it should also encompass the full community organization. As stated by [European Commission \[2018\]](#): "Energy communities should ensure non-discriminatory access and fair treatment of all members".

7.2 FAIRNESS METRICS AND OBJECTIVES

Modelling fairness is a challenge as it is not purely mathematical. Fairness encompasses normative values and social preferences that often conflict with purely mathematical optimization criteria.

The previous part of this thesis was built on optimization, focusing on minimizing costs and maximizing efficiency. Some members may prefer a solution that is fair but slightly less efficient than one that maximizes profit but creates inequity, some may not.

Fortunately, multiple works have already tackled the fairness measurement from different sides. Various frameworks exist to quantify fairness, each capturing different aspects of what constitutes an equitable outcome (*e.g.*, according to needs, merits, or contributions), both in a generic context [Xinying Chen and Hooker, 2023]

and specifically for EC [Fornier et al., 2024], [Hupez et al., 2021]. Many fairness metrics are presented and summarized by Xinying Chen and Hooker [2023].

7.2.1 Heuristics

The most straightforward way to allocate the benefits is with a direct rule.

- **Equal:** Individuals get the same share of profit;
- **Predefined share:** Individuals get a specific share of the profit;
- **Proportional:** Individuals get a share of the profit relative to their magnitude.

In the community paradigm, keys of repartition mechanisms can be seen as a simple rule-based option to share exchanged quantities.

7.2.2 Variance-based

This second way to measure fairness is through the broadness of the interval of results. It can be related to the computation of the variance in statistics.

- **Relative range:** The spread between minimum and maximum profit, normalized by the mean.
- **Relative mean deviation:** The normalized average deviation from the mean. It takes into account all utilities, rather than only the two extremes.
- **Hoover index:** The fraction of total utility that must be redistributed to achieve perfect equality. Also related to the Lorenz curve, and proportional to the relative mean deviation.
- **Gini coefficient:** Perhaps the best known measure of inequality. Proportional to the area between the Lorenz curve of the solution and a diagonal line representing perfect equality. Ranges from 0 to 1, with 0 indicating perfect equality.

7.2.3 Game theory

When considering fairness as a mean to retribute according to the utility of each individual, the field of game theory can provide an adequate solution. It allocates benefits based on merits, which ensures that the solution will be stable, efficient and immune to adversarial behaviors. This disregards the other parts of the fairness definition.

From cooperative game theory, three allocation mechanisms are widely used. In game theory, the core is defined as the set of feasible solutions that provide a stable grand coalition [Gillies, 1953]. By allocating benefits using a solution from the core,

the community ensures that no smaller coalition can provide better incentives for its members. This concept is useful but has limitations: it may not yield a unique solution, and may offer no solution if the core is empty.

- **Marginal utility:** The benefit for each member is proportional to its marginal utility in the EC. To correctly allocate all profits, individual coefficients should be scaled to collectively sum to one.
- **Shapley:** Shapley provides a benefit allocation that works even when the core is empty [Shapley, 1953]. However, this solution is not guaranteed to be in the core, which means it can be unstable. The Shapley value is defined as the player's average marginal contribution to all coalitions containing him. This value is computationally intensive to calculate as it requires finding the optimal solution to all the sub-coalitions, in other words, solving the problem 2^N times, which quickly becomes untractable.
- **Nucleolus:** The nucleolus provides a unique solution located in the core of the game. This solution provides the most stability for the grand coalition by minimizing the incentives of each coalition to deviate. In other words, it maximizes the smallest excess (the difference between pay-offs in the grand coalition and the sub-coalition) in leximin order. This also requires computation of the outcome of the 2^N subcoalitions [Schmeidler, 1969].

7.2.3.1 Fairness for the disadvantaged

Another way to consider fairness is by drifting away from the equality perspective to focus on the equity of the solution. This considers fairness as a way of prioritizing the members who need it the most.

It is based on the Rawlsian principle that inequality is justified only to the extent that it improves the welfare of the worst off [Rawls, 1971]. In Chapter 3, this principle was extended to an utilitarianism point of view which ensures that the worst off is only prioritized if this still achieves a collective optimum.

- **Maximin:** Maximizes the minimum utility. Once maximin is obtained, this does not consider the welfare of other disadvantaged individual.
- **Leximax:** Maximizes the welfare of the worst off, then the second worst off, and so forth. It considers the welfare of all individuals but requires solving a sequence of optimization problems.

7.3 EXAMPLES OF MATHEMATICAL MODELS

An optimal model has to achieve fairness by design. All users should be incentivized to participate in the community [Fornier et al., 2024]. The retribution mech-

anism should also ensure that their individual optimal actions will align to create the global optimal state for the community.

7.3.1 Two-Steps

The predominant way to achieve this in a REC is through keys of repartitions. This heuristic model can have many forms, it simply defines a rule on how to share the energy, and the price is set *ex ante*. Some have been defined in Section 2.1.1, and they have been used throughout this work in Section 5.1.2 and Section 5.2.2.2.

A two-step approach was also used in Section 3.1.2, this time minimizing the inequalities by defining internal prices with predefined quantities. This could be a more versatile strategy as it considers all three of the presented strategies:

- (a) Power profiles are defined *ex ante* by a central optimizer.
- (b) Intra- and extra-community exchanges are defined from the exchanged variables using a KoR.
- (c) Prices are optimized *ex post* to achieve optimal fairness, as chosen by the community and defined by an objective function.

Proposing such a model goes against the idea of *fairness by design*, as members will first be pushed to act optimally and then be remunerated *ex post*. Optimality is not ensured for all members, as the decision variables are defined for the centralized objective of the whole community, then the bill is shared. By having some rule-based decisions (*e.g.*, the direct exchanges), price could become an insufficient lever to allow for all members to make profit.

7.3.2 Bilevel

A promising approach to achieve fairness by design is through the use of bilevel programming. By definition, bilevel is a leader-follower problem, where parties can have diverging objectives. Here, our problem is well-suited, as we have a leader, the CO, whose goal is to achieve a fair allocation of the potential benefits of the community. The members have a simple bill-minimization goal to achieve. Those two objectives are not directly combinable through a weighted sum.

$$\begin{aligned}
 & \max && \text{Fairness} \\
 & \text{subject to} && \text{Price constraints,} \\
 & && \text{Energy exchange constraints,} \\
 & && \min && \text{Members' individual costs} \\
 & && \text{subject to} && \text{Devices constraints,}
 \end{aligned}$$

Exchange constraints,

A problem of this form would represent a single-leader (the CO), multi-followers (the members) Stackelberg game. The upper-level maximizes the fairness, as defined by the objective function. Its constraints are the aggregated costs at community level, price decision variables, and energy exchange allocations. The lower-levels define optimal members responses to the prices, with bounded intra-community exchanges variable that reflect reality of the energy scarcity.

This can be seen as an action-reaction problem, where the CO decides on the optimal prices for internal exchanges to maximize fairness. Knowing the members' optimal reaction, it also splits their physical power exchanges into intra- and extra-community and prices them accordingly. While it seems to be an adequate solution to a multivariate problem, this problem is non-convex and highly end-user non-friendly.

Another way to model this is by pricing the intra-community exchanges directly at value cost. This is done using the shadow price mechanisms and was introduced by Cornélusse et al. [2019].

$$\begin{aligned}
 & \max \quad \text{Fairness} \\
 & \text{subject to} \quad \text{Benefits sharing,} \\
 & \quad \min \quad \text{Members' individual costs} \\
 & \quad \text{subject to} \quad \text{Devices constraints,} \\
 & \quad \quad \text{Exchange constraints,} \quad \perp \pi^{\text{com}}
 \end{aligned}$$

7.3.3 Iterative

An iterative procedure is well suited for this kind of objective, as it can be seen as a demand-response problem.

A decentralized algorithm with a central coordinator was proposed in Section 5.2.4. The equilibrium point among all participants was found through an iterative strategy similar to bounded bidding. This could be extended to the complete community operation problem, if it is designed to ensure convergence.

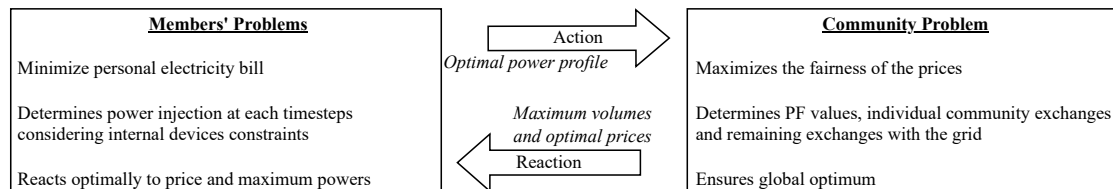


Figure 7.1: Abstract example of an iterative model to ensure fair and efficient operation

Distributed optimization methods like ADMM could ensure individual optimality and privacy while remaining globally optimal. However, they distribute a single objective function that will not correctly grasp the cost-fairness duality of the wanted solution.

Cooperative game theory, such as an iterative Stackelberg game, could also provide adequate results in the form of an equilibrium point between the CO and among members.

In theory, this type of procedure would be transparent, fair, interpretable by members and could achieve optimality. However, manual bidding is not an option in the context of residential EC. Having a daily routine for all members to submit their profiles for the next day is not realistic. This will most definitely need to be fully automatic, which necessitates a complete trust in the system.

7.4 PRACTICALITY OF THE IMPLEMENTED SOLUTION

This chapter did not include any results from simulations to compare the proposed solutions. The goal of this section was to list the different alternatives and to emphasize on the importance of this issue. Fairness is by definition subjective, the results may be optimal regarding the indicator they are optimizing, while being highly suboptimal with respect to another one.

Every community should define its own goal depending on its design, and try to achieve it as well as possible. As already stated, this is a theoretical configuration, optimality is easy to reach using mathematical models. In the real world, humans are involved and do not always act rationally with perfect responses. A more coherent and easy-to-follow set of recommendations and regulations may provide a better result than a confusing optimal-seeking set of rules.

In this idea, several interesting allocation mechanism have also been totally disregarded in Section 7.3 (e.g., Leximax, Shapley, Nucleolus). Indeed, those rely on abstract mathematical concepts, which would have limited social acceptance. They are all combinatorial in nature, meaning that the complete problem needs multiple solving of an equivalent subproblem, which can quickly become computationally intensive.

7.4.1 Limitations

The concept of core was presented in Section 7.2.3. A community could have an empty core, meaning that some members would benefit more from a sub-coalition of the community. In this case, they would be interested in reconstructing a smaller community to maximize their personal interest.

Members with a preponderant importance on the community operation or with adversarial behaviors for self-interest maximization could also ruin the effort for community optimality. The defined set of rules should be immune to individual manipulation of the system.

New members arrival can also be adversarial to the common goal. As shown by [Allard et al. \[2025\]](#) additional members in the community can bring the individual benefits of existing members down. To address this, additional rules could enforce that newer members do not deplete the benefits of previous. However this raises the question of fairness as this solution gives advantage to part of the members.

On the longer term, investments should be considered. If a community member benefited from the community's investment pool for DER at their site, the asset should remain partly community owned. If the community member count decreases, the common investments may also become oversized and too costly to be profitable.

7.4.2 Stepping aside from fairness indicators

A simpler guidance could prove to be more useful and provide overall decent results by encouraging members to consume when the community produces with a simple binary signal. Proposing a constant price for the intra-community energy would remove one variable of decision for the CO and provide more clarity for the members. This could ensure a minimal benefit for collectively self-consumed energy. However, members would also need to know how much energy is available at every time step to adapt their consumption, and this is highly dependant on the overall control of the community.

Another way to simplify this problem would be to pool all the (extra) benefits made by the community and give it to the CO. In this setup, the members would not see any difference in their energy bill, but they would participate in the common project. This would need a trusted and benevolent CO, which could be the cooperative responsible or the community administration, in case of a city-managed EC. The CO would have a specific goal decided by all involved parties, for example:

- Providing free or cheaper energy to selected buildings that are part of the community (social housings, public or group-owned infrastructures,...);
- Reinvesting in a common project that indirectly benefits all members;
- Investing in distributed energy resources to achieve complete energy autarky to then provide energy at cost-price for the members.

However, without direct incentivization through the energy bill, the return on investment should be a big enough motivation for members to act on their energy usage. An EC that groups individuals around a common goal that is more tangible than optimizing their energy would facilitate this kind of approach.

CLOSING REMARKS

This thesis formulated a model and proposed an approach to optimize the operation of a REC with multiple time granularity. The optimization horizon spans from long-term planning to the real-time control of the assets.

The main steps of the approach and their time windows are:

LONG TERM (20 years): Decision about investment in DERs, depending on the REC members' needs, budget and on the grid capacity.

DAY-AHEAD (1 day): Decision about theoretical operation of BSSs and loads, based on forecasts, flexibility and tariffs.

REAL-TIME (1 second): Control of the PV inverters to avoid grid congestion.

POST OPERATION (1 month): Individual billing of the members for fair sharing of the benefits.

8.1 LIMITATIONS OF THE MODELS AND NEXT STEPS

This manuscript tried to propose an overview of the decisional process for REC operation. It used mathematical modeling to prove usefulness of the framework with optimal design. However, the subject is much broader than what was presented. The optimization angle is limiting and many other aspects of energy communities could be studied.

8.1.1 Going further with network planning

This work considered long-term investment planning as a problem centered on community members. A longer-term and broader perspective should also include in the grid. Some of the models presented in this work showcased results proving decrease of the grid impact due to the community.

Section 3.1.1 proposed to ensure that operational costs for loss compensation would be covered by the community operator. This would justify a reduction of the distribution grid fees for local exchanges. A community that owns or rents the grid could also make the community responsible for keeping the grid operation in a safe

point, while ensuring that the participants do not pay for unused external costs. The coordinated control infrastructure of the community could help to achieve grid management.

The peak decrease and reduced losses are short-term effects manifesting reduced grid use. A more complete investment and reinforcement model of the grid would be necessary to assess for complete societal cost reduction.

8.1.2 All models are wrong but some are useful

The mathematical frameworks developed here represent idealized scenarios under specific assumptions: perfect information, rational actors, and negligible forecasting errors. Real energy communities operate in far more uncertain conditions. Human behavior deviates from optimization models, forecast errors accumulate, and regulatory frameworks may not align with the theoretical optimality conditions. Furthermore, the models prioritize technical and economic efficiency without fully addressing social dimensions such as trust, transparency, and perceived fairness. These gaps underscore the importance of empirical validation and iterative refinement of the proposed approaches. Nevertheless, the structured approach offered here provides a roadmap for decision-making, even if actual implementation will require adaptation to local contexts and constraints not captured in these models.

8.1.3 Real world validation

Validating the theoretical results presented in this thesis requires pilot implementation in operating energy communities. Such projects should focus on two critical dimensions:

- operational performance: testing the flexibility models and control algorithms under real forecast errors and actual member behavior;
- social acceptance: measuring how fairly members perceive the benefit distribution and whether they sustain participation over time.

Collaboration with regional regulators in Wallonia and neighboring regions could facilitate controlled experiments that bridge the gap between theory and practice, yielding insights for scaling energy communities effectively.

Additionally, engagement with community members through qualitative methods would provide essential feedback on the fairness and interpretability of the proposed settlement mechanisms, ultimately determining whether the theoretical benefits translate to sustained community participation and acceptance.

8.1.4 Impactful communities

While this thesis demonstrates the technical and economic viability of energy communities, their real-world impact depends fundamentally on behavioral factors currently absent from the models. Energy communities are not merely economic entities but social organizations requiring sustained member engagement, mutual trust, and a shared commitment to collective goals. Current legislative frameworks in Wallonia and across Belgium do little to foster these social dimensions. Moreover, the incentive landscape provides minimal motivation for members to prioritize community benefits over individual savings, and offers no mechanism to reward members for behaviors that benefit the broader energy system, such as flexibility provision or grid support. Without explicit incentive structures, energy communities risk becoming passive vehicles for cost distribution rather than active agents of the energy transition. Developing such structures requires aligning financial incentives with systemic objectives, ensuring transparency in benefit sharing, and building community capacity for governance and decision-making.

8.2 SUMMARY OF THE CONTENTS

This section will give a summary of the content (Part II and Part III) of this manuscript:

- Chapter 3 proposed a way for the community to define adequate sizes and locations of new DER, then made a study on the carbon impact of these investments.
- Chapter 4 described a novel load profile generator tool to be used for flexibilization in residential areas.
- Chapter 5 first estimated the potential for residential flexibility in the community framework. Then it proposed a way to activate it in a decentralized manner.
- Chapter 6 presented a distributed algorithm to ensure safe operation of the grid, which is easily implementable in an EC.

8.2.1 Contributions

This section presents a list of the relevant contributions of the chapters that include peer-reviewed papers.

8.2.1.1 Chapter 3: Sizing

This work proposes a long-term planning to define optimal assets sizes, location, and accounting for local grid management. It compares individual self-investment, community self- and co-investment to isolate the impact of the collective. A benefits allocation scheme that takes into account investment pooling, adequate retribution, and fairness is also proposed.

In the second part, the same problem with CO₂ objectives and budgets is redefined to evaluate the ecological impact of energy communities. Carbon intensity from different countries modelled with their solar intensity is studied to account for locational potential of the community. The CO₂ study differs from most literature by considering investment emissions for renewables instead of operational average in t/kWh.

Additionally, studies on evolving member budgets to define the threshold for over- and under-investment capacities as well as evolving community fees to show the impact of external cost in the intra-community exchanges are conducted.

8.2.1.2 Chapter 4: Profile generator

This work presented a customizable and open source electric load profile generator to model population in residential areas. It outputs a base load profile and flexibility components timeseries. It uses a simple model which can be reused in an optimization framework.

It separates the load and flexibility profiles in five types of assets:

- Baseload: fixed plugged-in appliances;
- Time-shiftable: wet appliances (dishwasher, washing machine);
- Electric vehicle: car type with realistic usage and charging habits;
- Water boiler: a heat storage with envelope losses and usage events;
- Heat pump: thermostatic load with variable power exchanges and dynamic temperature setpoints.

It also allows to make some long-term scenarios with evolving penetration of loads.

8.2.1.3 Chapter 5: Load flexibility

This work proposes a way to quantify the load flexibility for individual and collective self-consumption in residential areas. It compares four frameworks to quantify separately the benefits of a community and the flexibility, before allowing both. It first models the REC as a single entity, and enforces flexibility activation from a

central point of view. Then, *ex post* profit allocation is proposed using optimization or KoR.

In the second part, the emphasis is shifted to improve the realism of the solution, first by using power and flexibility profiles from *ResFlex*, then by redesigning the penalty functions to represent real discomfort felt by the users. A sequential approach to the activation is proposed, where users have priority for flexibility activation to cover their own production, before covering the remaining through the community. It also proposes a decentralized activation and remuneration mechanism, with iterative demand and offers for flexible capacity. This allows for users to respond to a real need in flexibility and to activate it with a transparent rule defined *ex ante*.

8.2.1.4 Chapter 6: Distributed OPF

This work presents a distributed algorithm using ADMM to reach optimal power flow in distribution networks. Unlike many decomposition techniques, it builds upon the graph of the network and the individual objective of each of its nodes. Each subproblem represents one node of the system.

This method uses the iterative nature of ADMM to estimate the square currents, this allows for exact power flow values at convergence while removing non-convexity in the model. The problem remains linear, and the losses from line impedances are not neglected, unlike the linear distflow model (LinDistFlow). It is also superior to the SOCP relaxation of the distflow model for our application. The conic constraint needs to be tight for this relaxation. This is usually achieved through current minimization, which is not the goal here.

It also proposed a feedback method for system state estimation and faster convergence. Voltage levels are measured at each node during the iterative process. The deviation of the local copies from these values is penalized in the objective.

8.2.2 Results

Three of the previous chapters presented numerical results for the proposed models. This section emphasizes the most relevant conclusions of the different peer-studied papers that they are originating from.

8.2.2.1 Chapter 3: Sizing

The EC with tailored DER sizing can be beneficial for the grid and its users. The co-investment scheme can improve further the overall profits. All members can benefit from this with an adequate allocation sharing scheme accounting for all co-investors.

The community can decrease the stress on the grid by localizing the assets at the adequate location. Additional investment budget can be used more efficiently in a community setting. Increased fees for intra-community exchanges (*e.g.*, operator or grid fees, taxes,...) quickly deplete the available benefits, removing any value for energy communities.

Energy costs can provide a good proxy for optimizing the CO₂ impact of the community. As expected, countries with lower irradiance potential benefit less from additional production units while a high carbon-intensity country will allow for a larger reduction.

8.2.2.2 Chapter 5: Load flexibility

The models show that benefits from load flexibilization can be increased by the community framework over individual optimization. The first approach proposes omniscient allocation for flexibility activation, with ensured additional benefits for all members. It demonstrates the high potential for EVs and thermal loads shifting, while mitigating the interest for time shiftable appliances.

The second part of this work improves the inherent modeling of the flexible load using ResFlex and proposes more meaningful discomfort models:

- Delayed energy charged for the EV [€/kWhh];
- Temperature differential duration for thermal appliances [€/°h].

The decentralized algorithm showcases similar results to the optimal central activation, while proposing a more intuitive demand and offer scheme. Rule-based activation is proposed with constant retribution, ensuring partially the fairness of the solution. It also prioritizes individual self-consumption for members' flexible capacity.

8.2.2.3 Chapter 6: Distributed OPF

Compared to the currently implemented decentralized algorithm, this distributed solution limits the amount of voltage violations and allows users to partially degrade their PV production. Additionally, it shares the production curtailment with a utilitarian objective, maximizing the total energy production. This reduces the frequency of automatic disconnections at the end of the feeders, which is commonly observed in high PV-density residential areas.

Whilst it uses reduced communication to avoid single point of failure and enforce data privacy, the proposed algorithm achieved similar efficiency to a central optimization problem.

A REC framework could facilitate this type of algorithm by ensuring community members are cooperative. It could also provide adequate communication systems for coordinated control of specific assets.

8.3 FINAL WORD

This manuscript introduced the concept of EC and proposed mathematical models to optimize the community operation at multiple time-scales. It showed that under the right assumptions (*i.e.*, a favorable regulation framework) and with well-informed, rational and cooperative members, the REC can be beneficial for everyone.

THE MEMBERS: Members can all profit financially from the reduced cost of energy intra-community. They can also profit from the demand aggregation to access more advantageous tariffs for external electricity. When investing in new energy sources, they can also benefit from their numbers to access bigger scale installation at lower capacity price.

THE COMMUNITY: The CO can generate profit through the fee they can impose to intra-community energy exchanges with reduced grid fees. This could cover for losses compensation inside the network with some surplus. A voluntary CO income could be reinvested in the community energy needs or in other community projects.

THE GRID OPERATOR: REC can reduce the demand on the upstream network, the power peaks, energy losses and voltage problems in the network. The DSO could also delegate the management of part of its network to the community which would be able to use more advanced mechanisms to reduce grid stress.

Energy communities could be a tool for achieving technically efficient, economically viable and equitable energy transition. This approach is a bottom-up solution for a global problem. It will not be sufficient to achieve a full transition, but it could help local initiatives achieve their fair share with an impactful multisectorial goal. All stakeholders can benefit from a more cooperative, socially-aware, and optimized framework.

APPENDIX



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