

Context-dependent manifold learning in dynamical systems: A neuromodulated constrained autoencoder approach

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1 Introduction

Dimensionality reduction (DR) is a key tool for analyzing high-dimensional data from complex dynamical systems. While linear methods such as principal component analysis (PCA) and nonlinear approaches like autoencoders effectively capture structure in fixed settings [1], they typically assume a unique data-generating process. In practice, however, the geometry of the data manifold often depends on external conditions such as physical parameters or environmental context. Simply augmenting the input with contextual variables increases dimensionality and may obscure the underlying structure. We propose a context-dependent DR approach based on the constrained autoencoder (cAE) [2], which enforces smooth manifold structure through geometric constraints. By integrating neuromodulation mechanisms conditioned on a context vector, the model adapts its projection while preserving geometric consistency [3]. We demonstrate the effectiveness of the approach on a 16-degree-of-freedom pendulum system with different context parameters.

2 Neuromodulated Constrained Autoencoder

We propose a context-adaptive constrained autoencoder (cAE) that combines geometric manifold learning with neuromodulation. The cAE framework enforces that the encoder ρ and decoder φ compose into an idempotent projection, *i.e.*, $\rho \circ \varphi = \text{Id}$, so that $P = \varphi \circ \rho$ defines a smooth projection and the learned representation $\hat{M} = \text{Range}(P)$ forms a smooth embedded submanifold of the data space [2]. The encoder and decoder are constructed from paired layers with biorthogonal weights $\vec{\Psi}_l, \vec{\Phi}_l$ and inverse activation functions (σ_-, σ_+) :

$$\begin{aligned}\rho^{(l)}(\vec{x}^{(l)}) &= \sigma_- \left(\vec{\Psi}_l^T (\vec{x}^{(l)} - \vec{b}_l) \right), \\ \varphi^{(l)}(\vec{z}^{(l-1)}) &= \vec{\Phi}_l \sigma_+ \left(\vec{z}^{(l-1)} \right) + \vec{b}_l,\end{aligned}$$

with the constraint $\vec{\Psi}_l^T \vec{\Phi}_l = \vec{I}$. We enforce this constraint via Riemannian optimization on the biorthogonal manifold. To enable adaptation to external conditions, we introduce neuromodulation [3]. Neuromodulation is a fundamental mechanism that uses neuromodulators to tune neuronal properties and circuit function dynamically. It is essential for generating flexible brain states and complex behaviors [4]. Fixed

activation functions $\sigma_{\pm}$ are replaced by parameterized functions $\sigma_{\pm}(\cdot; \vec{\alpha}^{(l)})$, where the parameters depend on a context vector $\vec{c}$. Specifically, the context is mapped to a global modulation signal $\vec{s}$ via a fully connected layer, and each layer parameters are obtained as $\vec{\alpha}^{(l)} = g(\vec{W}_l^T \vec{s})$, where g is a saturating function to keep $\vec{\alpha}^{(l)}$ bounded. This allows the nonlinear transformations to adapt smoothly to the context while preserving the geometric structure of the cAE.

3 Experiments & Results

We evaluate three architectures on two 16-DoF pendulum tasks. The *standard pendulum* randomly samples the lengths of the first four links in $[0.35, 0.65]$ m while keeping the remaining 12 DoF fixed. The *context-dependent pendulum* introduces a coupling between link lengths and joint angles, yielding a context-dependent setting. We compare: (i) a constrained autoencoder (cAE), (ii) a modulated cAE with irrelevant context (IcAE, $\vec{c} = \vec{0}$), and (iii) a modulated cAE with relevant context (McAE). Performance is assessed via RMSE on position and velocity reconstruction on unseen test trajectories. In the standard task, all models perform similarly, as expected for context-independent dynamics. In contrast, in the context-dependent task, McAE outperforms both cAE and IcAE, reducing reconstruction errors by a factor of 2–3. These results demonstrate that neuromodulation effectively exploits relevant contextual information for context-dependent dimensionality reduction.

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References

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