

Abstract

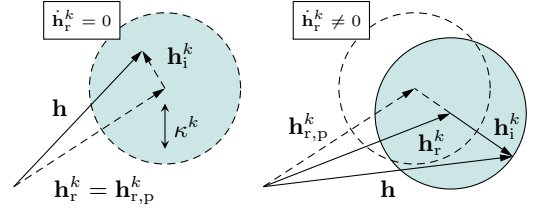
Accounting for hysteresis and eddy currents in finite element simulations of electrical machines is usually computationally expensive. These phenomena have however a significant impact on the machine performance. To account for **iron losses** with a **low computational cost**, we have trained a **Recurrent Neural Network** to serve as material law in **2D finite element simulations**.

Static Mesoscopic Model: EB Hysteresis

N cells subjected to the same input magnetic field $\mathbf{h} = \mathbf{h}^k$, $k = 1, \dots, N$. For each cell, \mathbf{h} is decomposed into a reversible part \mathbf{h}_r^k and an irreversible part \mathbf{h}_i^k . If κ^k denotes the maximum norm of \mathbf{h}_i^k and $\mathbf{h}_{r,p}^k$ the previous \mathbf{h}_r^k , the model update law is:

$$\mathbf{h}_r^k = \begin{cases} \mathbf{h}_{r,p}^k & \text{if } |\mathbf{h} - \mathbf{h}_{r,p}^k| \leq \kappa^k \quad (\dot{\mathbf{h}}_r^k = 0) \\ \mathbf{h} - \kappa^k \frac{\mathbf{h} - \mathbf{h}_{r,p}^k}{|\mathbf{h} - \mathbf{h}_{r,p}^k|} & \text{otherwise} \quad (\dot{\mathbf{h}}_r^k \neq 0) \end{cases}$$

The output magnetic flux density \mathbf{b} is given by $\mu_0 \mathbf{h} + \mathcal{L}(|\mathbf{h}_r|) \frac{\mathbf{h}_r}{|\mathbf{h}_r|}$ with \mathcal{L} the Langevin function and $\mathbf{h}_r = \sum_{k=1}^N w^k \mathbf{h}_r^k$ ($\sum_{k=1}^N w^k = 1$). $\frac{\partial \mathbf{b}}{\partial \mathbf{h}}$ has an analytical expression.



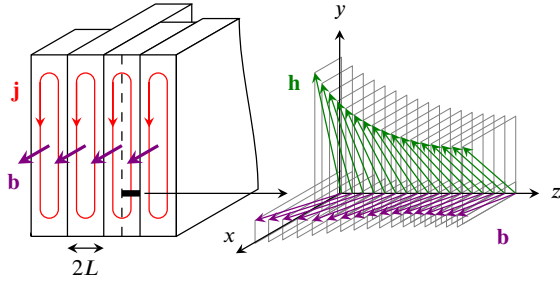
Dynamic Mesoscopic Model: Laminations with Eddy Current and Hysteresis

One-dimensional magneto-quasistatic formulation in terms of the magnetic field: Find $\mathbf{h}(z, t) = (h_x(z, t), h_y(z, t))$ on a half lamination such that, for $z \in [0, L], t > 0$:

$$\begin{cases} \partial_t (b_x(\mathbf{h}), h'_x)_{[0,L]} + (\sigma^{-1} \partial_z h_x, \partial_z h'_x)_{[0,L]} = 0 \\ \partial_t (b_y(\mathbf{h}), h'_y)_{[0,L]} + (\sigma^{-1} \partial_z h_y, \partial_z h'_y)_{[0,L]} = 0 \end{cases}$$

holds for all test functions h'_x, h'_y , with $\partial_z \mathbf{h}(t, 0) = 0$ and $\mathbf{h}(t, L) = \mathbf{H}_{\text{macro}}$. $\mathbf{B} = \mathbf{B}_{\text{macro}}$ is retrieved by averaging \mathbf{b} over the half lamination thickness. $\frac{\partial \mathbf{B}}{\partial \mathbf{H}}$ is obtained by a perturbation method.

The dynamic model accounts for eddy currents, and incorporate the effect of hysteresis if $b_x(\mathbf{h})$ and $b_y(\mathbf{h})$ make use of the static hysteresis model.



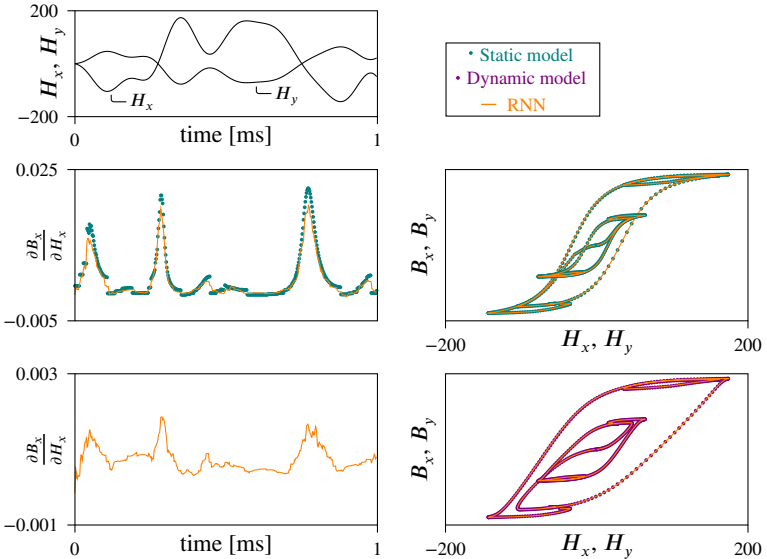
Recurrent Neural Network (RNN)

Architecture: Single-layer gated recurrent unit with a hidden size of 256. Embedding and decoding both use two feed-forward layers with respectively ReLU and Linear activation functions.

Dataset: $5 \cdot 10^5$ \mathbf{H} sequences of 10^3 time steps that mimic fields encountered in electrical machines are generated artificially. The corresponding \mathbf{B} sequences are obtained by solving respectively the static model (50 minutes) and the dynamic model (3 hours) on an AMD EPYC 7763 CPU.

Training: Static and dynamic RNN models are both trained to minimize a mean square error over 10^6 iterations (about 6 hours) on a GPU node (NVIDIA A100 40GB) of the Lucia cluster. Both trainings reach a test error of about $2 \cdot 10^{-6}$.

Inference: The material law $\mathbf{B}(\mathbf{H})$ is evaluated in a forward pass, $\frac{\partial \mathbf{B}}{\partial \mathbf{H}}$ is obtained by **automatic differentiation**.



Macroscopic Finite Element Simulations

Magnetostatic ϕ formulation: $\mathbf{H} = \mathbf{H}_s - \nabla \phi$.

Weak formulation of the magnetic Gauss law $\nabla \cdot \mathbf{B} = 0$ with vanishing Newman boundary condition:

$$\left(\mathbf{B}(\mathbf{H}), -\nabla \phi' \right)_{\Omega} = 0, \quad \forall \phi'.$$

\Rightarrow Linearize the non-linear $\mathbf{B}(\mathbf{H})$ material law, and solve for \mathbf{H}_i with the Newton-Raphson scheme:

$$\left(\mathbf{B}(\mathbf{H}_{i-1}) + \frac{\partial \mathbf{B}}{\partial \mathbf{H}}(\mathbf{H}_{i-1})(\mathbf{H}_i - \mathbf{H}_{i-1}), -\nabla \phi' \right)_{\Omega} = 0, \quad \forall \phi'$$

