

SE(3)-based shape and deformation representations for adaptive control of deformable objects

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1. Introduction

The robotic manipulation of deformable objects (DOs) is a challenging research problem with applications in manufacturing (cables, sheets), robotic surgery (suturing, needle insertion), and household tasks (laundry folding), where slender structures are omnipresent. In this work, we focus on *shape servoing*, the task of controlling an object's deformations to achieve a desired shape, using a closed-loop adaptive control method. Given the infinite configuration space of DOs, a key challenge is designing a feedback signal that accurately captures the shape while remaining simple enough for real-time closed-loop control, making the shape representation a critical choice.

Previous works proposed various methods to approximate DOs' shape, such as object contours or simple geometric features like angles [1]. A standard approach, serving as our comparison baseline, is *position-based servoing* (PBS), which tracks the positions of an ordered set of object points [2]. However, in existing works, PBS representation of deformations is sensitive to rigid-body motions, such that linearized deformation models are not *invariant to the choice of the inertial frame*, thereby increasing their non-linearity.

Our work introduces an *SE(3)*-based servoing (SE3BS) approach which simultaneously combines several key properties that existing works partially fulfill: 1) *completeness* (i.e., being able to capture the object's pose in addition to its intrinsic shape), by representing the configuration through the pose of local frames attached to the DO, 2) *inertial frame invariance* of the deformation model, by expressing deformations as relative transformations between these frames, formulated in the Lie algebra $\mathfrak{se}(3)$, 3) *generality*, as local frames can represent any task and are not restricted to specific objects, and 4) *robustness to features choice*, which is notably improved by the integration of local frame orientations into the feedback loop. The proposed representation is integrated into an adaptive shape controller, evaluated through simulations of both single- and dual-arm manipulation of deformable linear objects (DLOs) and shells (Fig. 1a), and validated on our experimental platform with DLOs (Fig. 1b). A preliminary version of the approach was introduced in [3], demonstrated in limited simulations.

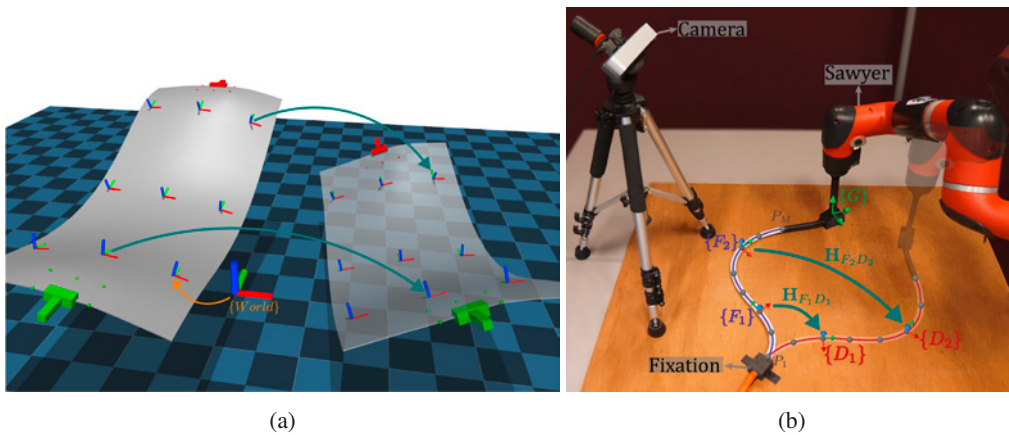


Figure 1: SE(3)-based shape servoing: (a) Simulation with deformable surfaces, (b) Experimental setup with a DLO.

2. Shape servoing using an SE(3)-based representation of shapes and deformations

The current configuration of the DO is represented by the poses $\mathbf{H}_{F_i} \in SE(3)$ of N feature frames $\{F_i\}$ ($i = 1, \dots, N$), and we build the corresponding frames $\{D_i\}$ and their poses $\mathbf{H}_{D_i} \in SE(3)$ using the same points of the DO but on its desired configuration. Frames $\{F_i\}$ and $\{D_i\}$ are respectively indicated in purple and red

in Fig. 1b. We note $\mathbf{H}_F = \text{diag}(\mathbf{H}_{F_1}, \dots, \mathbf{H}_{F_N}) \in SE(3)^N$, $\mathbf{H}_D = \text{diag}(\mathbf{H}_{D_1}, \dots, \mathbf{H}_{D_N}) \in SE(3)^N$ and $\mathbf{H}_{RG} = \text{diag}(\mathbf{H}_{RG_1}, \dots, \mathbf{H}_{RG_M}) \in SE(3)^M$, where $\mathbf{H}_{RG_m}$ denotes the m -th gripper pose with respect to the robot base frame. If the object exhibits elastic deformations only and if quasi-static manipulation is considered, the following linearization holds:

$$\mathbf{d}_F = \mathbf{J}(\mathbf{H}_{RG}) \mathbf{d}_G \quad (1)$$

where $\mathbf{J}$ is the so-called *shape Jacobian* that relates robots motions to variations of the shape features. Indeed, in the SE3BS approach, we respectively represent *grippers displacements* from a configuration $\mathbf{H}_{RG}^{k_l}$ to a current configuration $\mathbf{H}_{RG}^k$ and *object deformations* from a shape $\mathbf{H}_F^k$ to a current shape $\mathbf{H}_F^k$ as $\tilde{\mathbf{d}}_G^k = \log[(\mathbf{H}_{RG}^{k_l})^{-1} \mathbf{H}_{RG}^k]$ and $\tilde{\mathbf{d}}_F^k = \log[(\mathbf{H}_F^{k_l})^{-1} \mathbf{H}_F^k]$, where the log map, which usually takes an element from $SE(3)$ to $\mathfrak{se}(3)$, is here adapted to $SE(3)^N$ or $SE(3)^M$, k indicates the current iteration and k_l is the last iteration at which the linearization was performed. $\mathbf{J}$ is not constant, and we use sensor feedback (see the camera in Figure 1b) and the Broyden update rule to constantly update its value according to past observations of the deformations. The shape error, computed as $\tilde{\mathbf{e}}_F^k = \log((\mathbf{H}_F^k)^{-1} \mathbf{H}_D)$, is used to build a simple control law: $\mathbf{d}_G^* = \text{sat}(\hat{\mathbf{J}}^\dagger \mathbf{e}_F)$, which can be used to update grippers position. Stability of this controller can be proved using Lyapunov theory. This model-free adaptive control scheme can be applied to different objects and is resilient to external disturbances.

3. Experimental results

Fig. 2 illustrates a 2D manipulation task where a robot controls the position and orientation of a DLO's end. In Fig. 2a, black dots mark PBS feature points, while black arrows in Fig. 2b show SE3BS feature frames. This example illustrates how SE3BS solves the task with significantly fewer iterations (K), and lower total translation (Δx) and rotation ($\Delta \theta$) of the gripper. With PBS, the gripper can move in directions that temporarily decrease the features error by bringing the feature points closer to their target position (see yellow arrows in Fig. 2a and the error plot until iteration 33 in Fig. 2c). However, this comes at the cost of unintended deformations of the rest of the DLO, ultimately leading to a longer trajectory. SE3BS inherently prevents such unnecessary DLO deformations, as they would hinder the reduction of the rotational component of the shape error Φ_{SE3} (or even increase it). Our comprehensive simulation campaign confirms this tendency on a large number of tests in different situations, both in 2D and 3D. Additional benefits of SE3BS, such as reduced non-linearities of $\mathbf{J}$ and better robustness to features selection, are also observed.

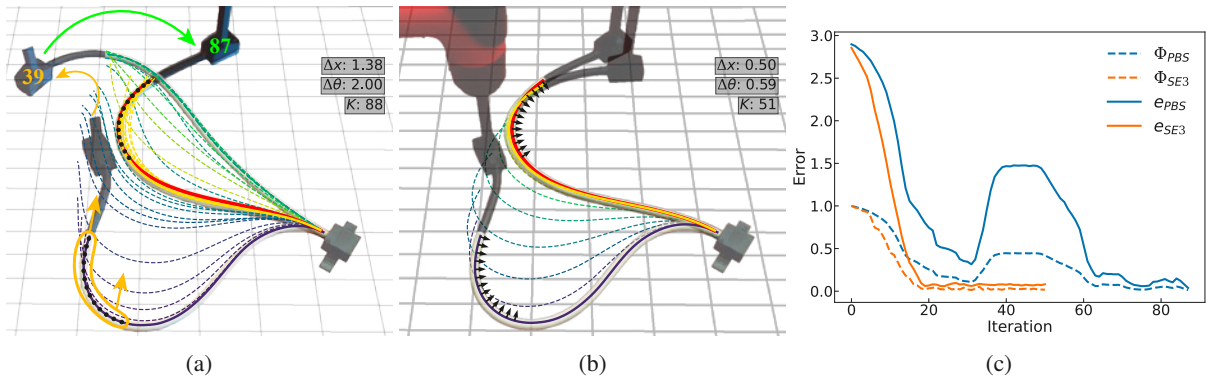


Figure 2: Comparison of trajectories obtained with (a) PBS and (b) SE3BS approaches. (c) Error plot, with $\Phi_{PBS} = \|\mathbf{e}_x\|/\|\mathbf{e}_x^0\|$ and $\Phi_{SE3} = \|\mathbf{e}\|/\|\mathbf{e}^0\|$, where $\mathbf{e}_x^0$ and $\mathbf{e}^0$ are the initial shape errors of the features, and e is a common metric for both method.

References

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