A model-based heuristic for a three-dimensional loading vehicle routing problem with split pickups and time windows

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The global reverse logistics market is expected to reach $954.5 billion by 2029 [6], which highlights the importance of developing efficient logistics operations [3]. In particular, Logistics Service Providers (LSPs) are in charge of the collection and transportation of products, known as the first-mile collection problem, which is a specific part of reverse logistics. The LSP must determine a plan to collect and load boxes of their customers in an urban area and bring them back to a depot, while minimizing their total cost. Complex routing and loading challenges must be considered such as time windows, maximum working duration, box stability and reachability. In this work, we study the three-dimensional loading vehicle routing problem with split pickups and time windows (3L-SPVRP-TW), as presented in [1].

This work proposes a Mixed Integer Linear Programming (MILP) formulation to the 3L-SPVRP-TW, with constant travel durations. This formulation extends those existing in the literature by simultaneously taking into account additional real-life features such as split pickup, maximum working duration, and reachability constraints. Standard exact B&B optimization methods are expected to perform poorly over a restricted computational time.

We decided to adapt the insert-and-fix (I&F) matheuristic introduced by [2]. This matheuristic, able to use the potential offered by the MILP, consists in sequentially solving smaller MILPs. The first MILP is obtained by considering a subset of variables. In the second MILP, some variables of the first MILP are fixed to their values and a subset of variables is added. This process continues until all the variables are considered. I&F has been applied to lot-sizing and vehicle routing problems [2], the three-dimensional bin packing problem solely [4] and to a generalized multi-port container stowage planning problem [5]. To the best of our knowledge, I&F has never been applied to the problem integrating both routing and packing problems. We therefore extend the I&F to the 3L-SPVRP-TW, where the construction of the subproblems relies on the idea of inserting customers, step by step, inside the routes. We carry out some computational experiments to assess the impact of the I&F parameters and to determine the limitations of this method.

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